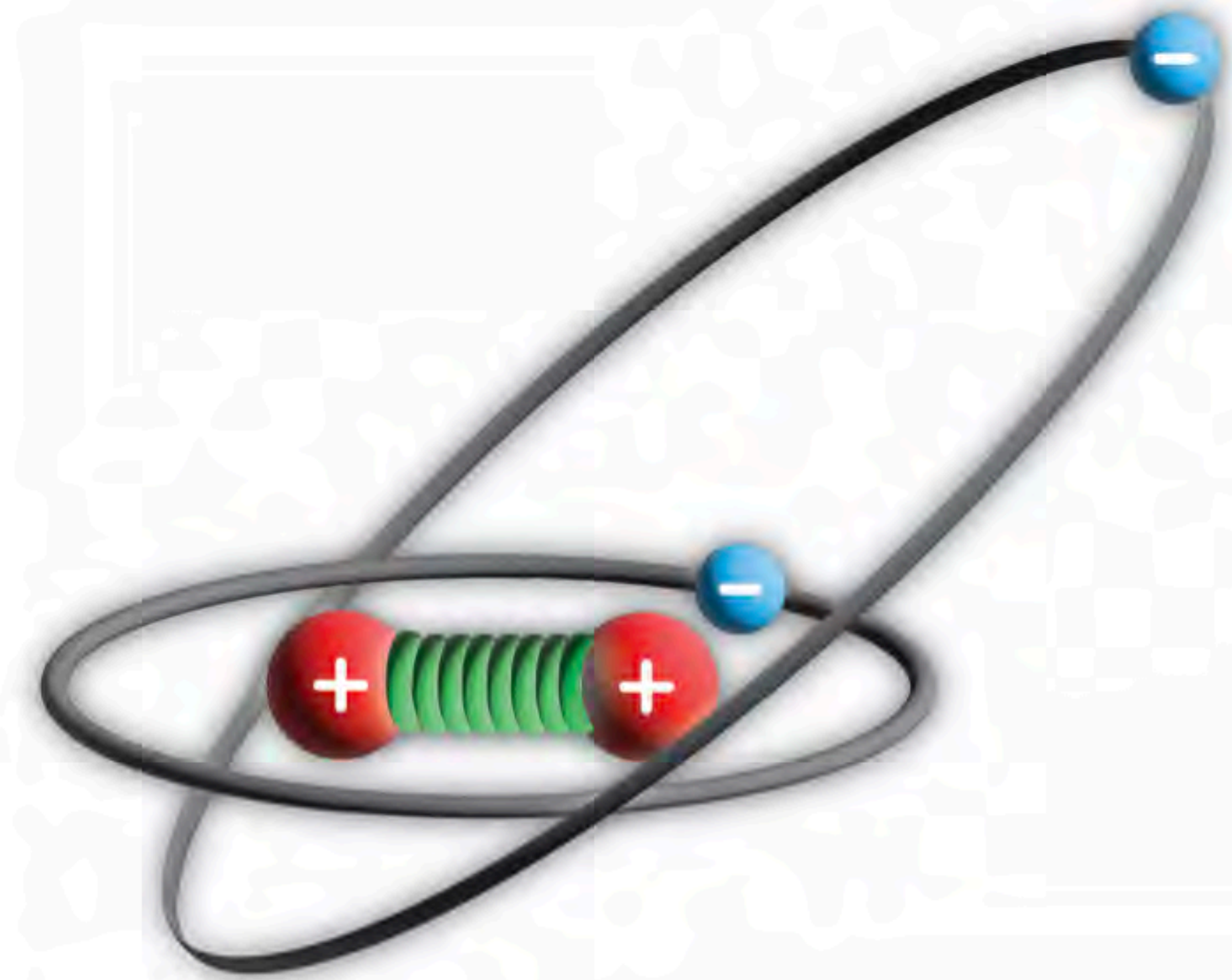


# High-precision spectroscopy of hydrogen molecules

Maximilian Beyer

Quantum Metrology and Laser Applications, VU Amsterdam



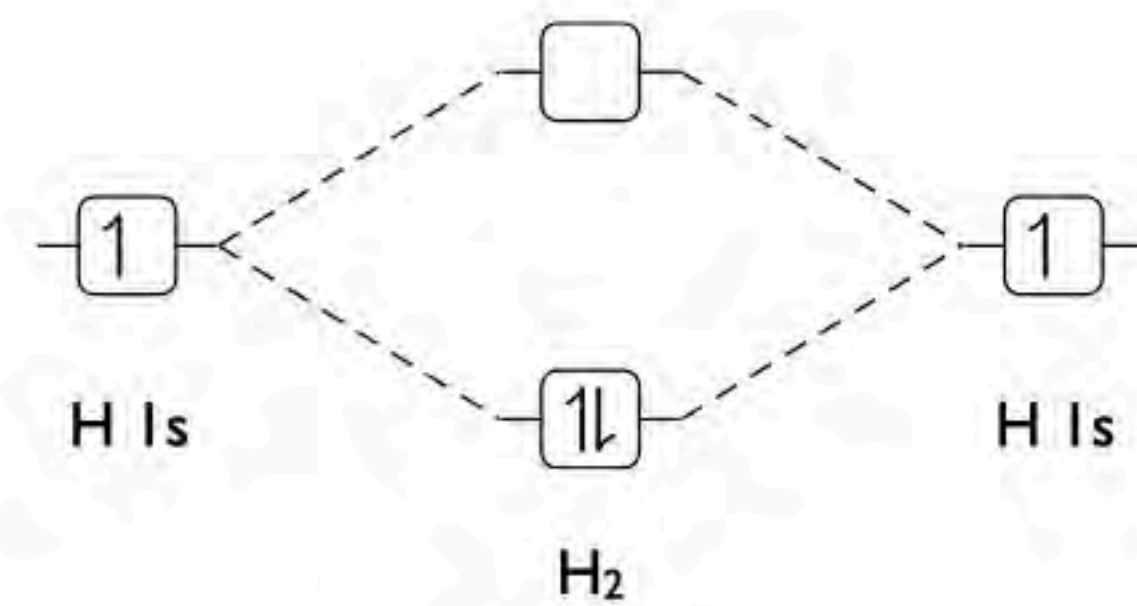
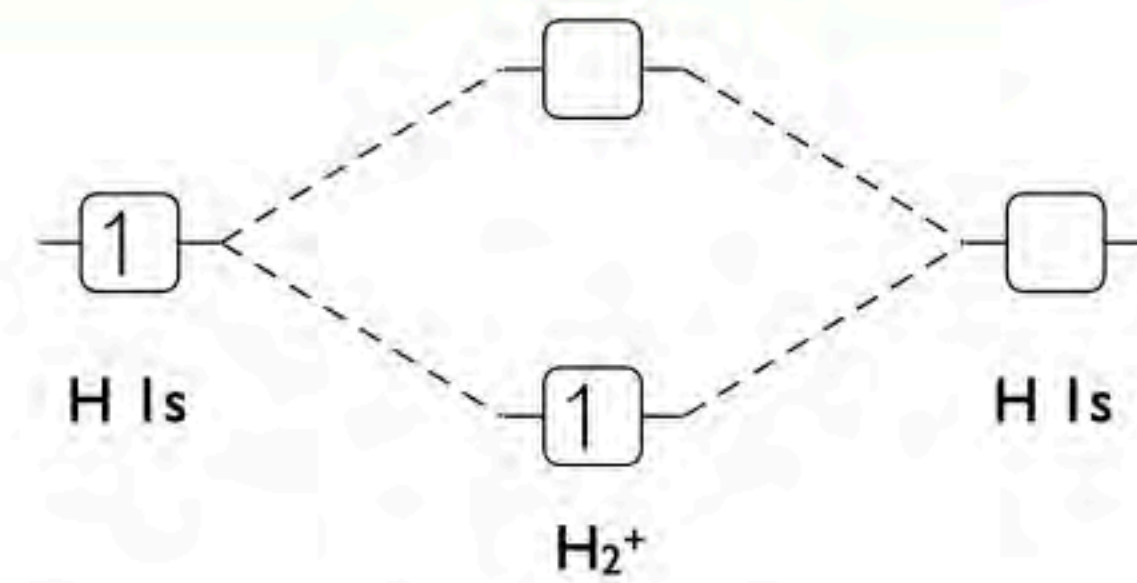
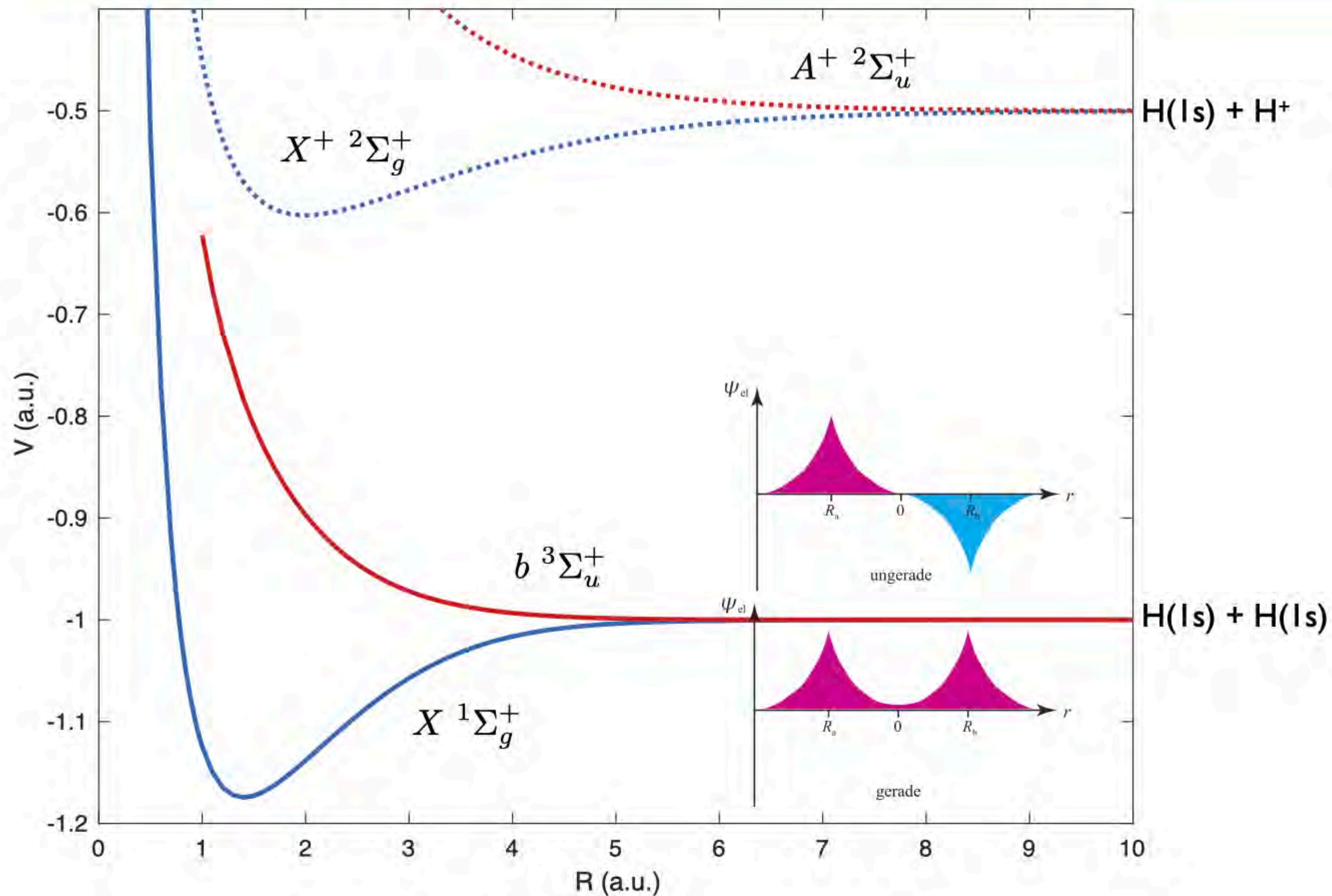
VU



VRIJE  
UNIVERSITEIT  
AMSTERDAM

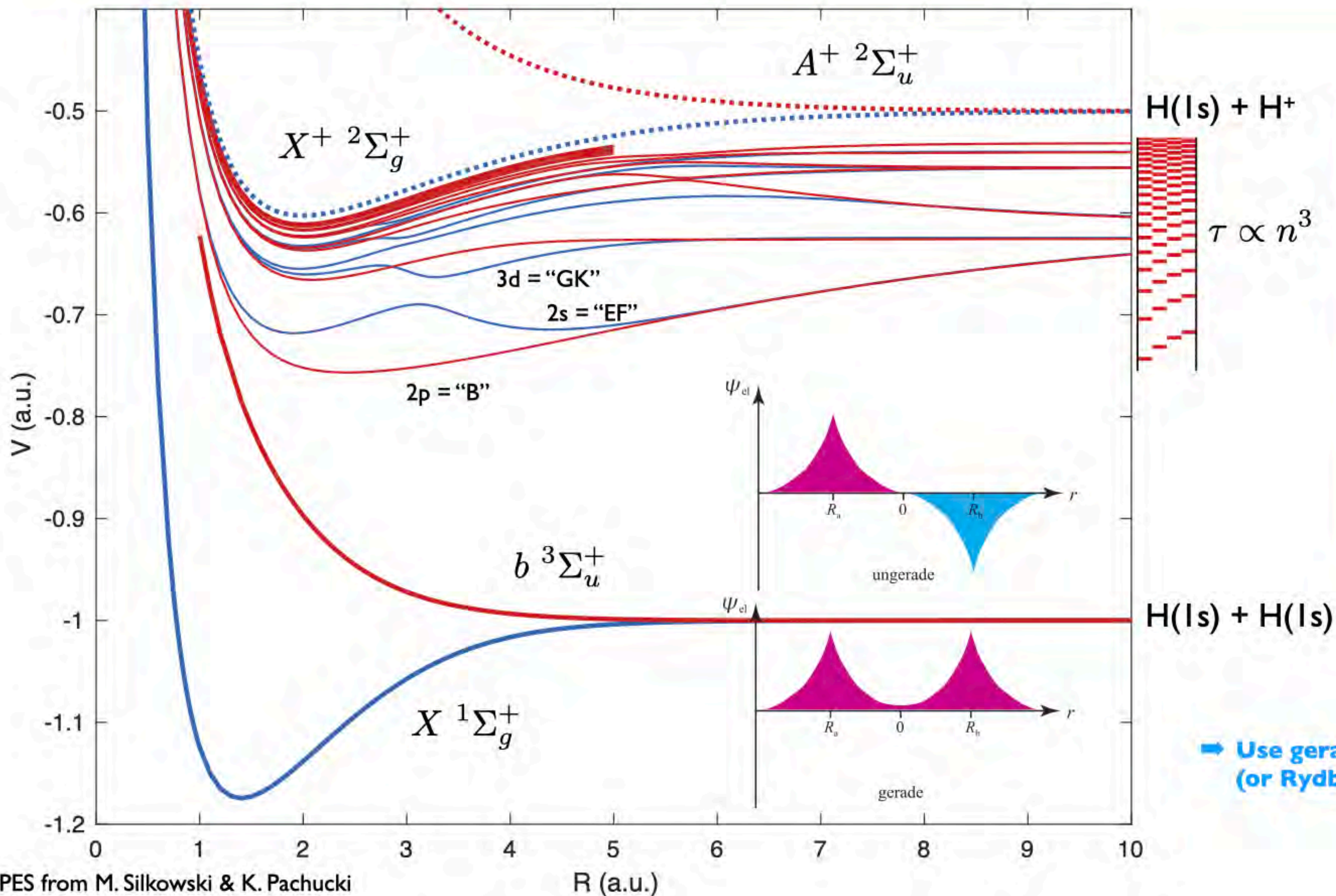


# Molecular Hydrogen





# Molecular Hydrogen



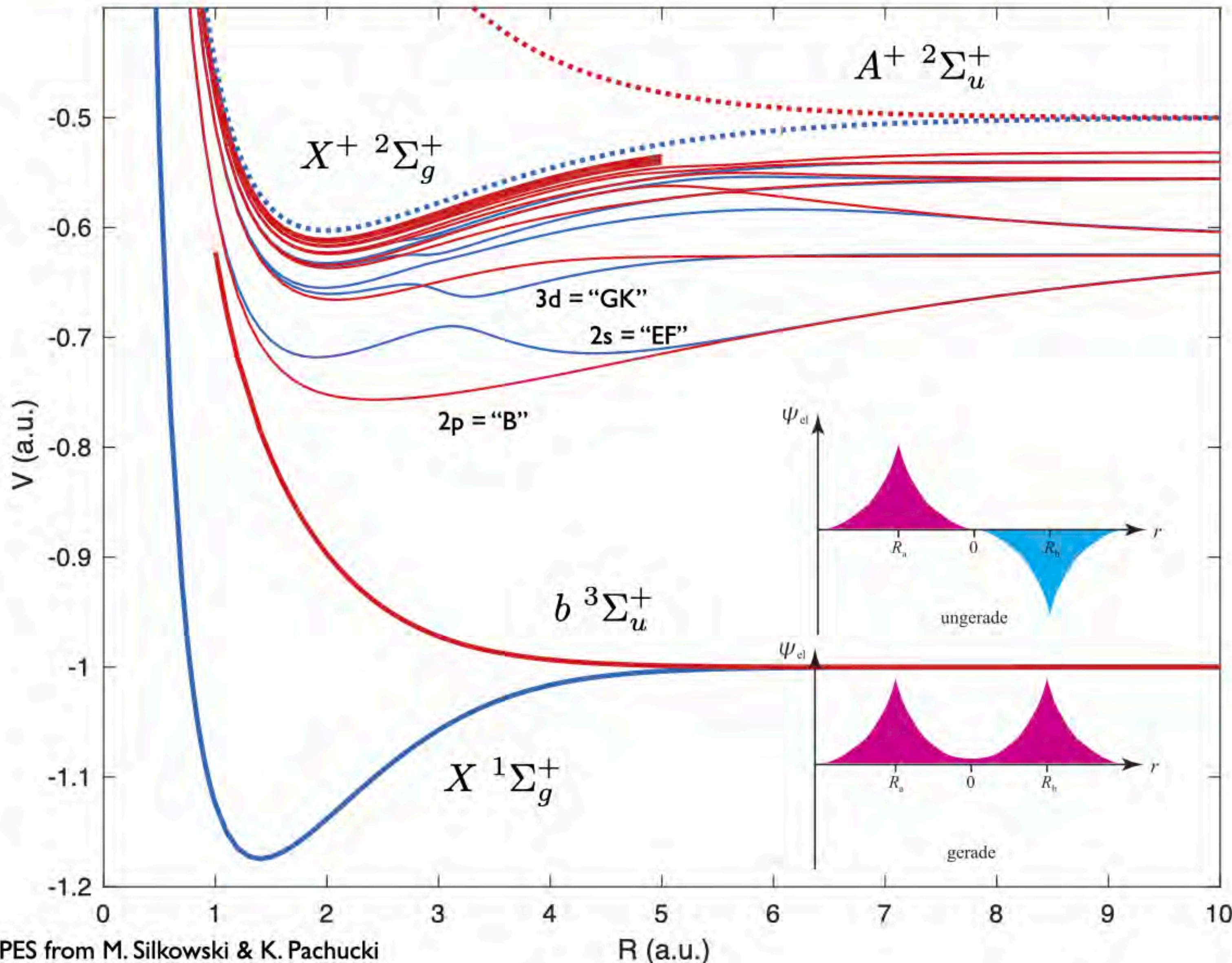
$\tau(B 1\Sigma_u^+) < 1 \text{ ns}$	$2p\sigma$
$\tau(C 1\Pi_u) < 1 \text{ ns}$	$2p\pi$
$\tau(B' 1\Sigma_u^+) \sim 5 \text{ ns}$	$3p\sigma$
$\tau(D 1\Pi_u) \sim 3 \text{ ns}$	$3p\pi$
$\tau \propto n^3$	
$\tau(EF 1\Sigma_g^+) \sim 200 \text{ ns}$	$2s\sigma$
$\tau(GK(v=0) 1\Sigma_g^+) \sim 70 \text{ ns}$	$3d\sigma$
$\tau(GK(v=1) 1\Sigma_g^+) \sim 24 \text{ ns}$	$3d\sigma$
$\tau(H 1\Sigma_g^+) \sim 100 \text{ ns}$	$3s\sigma$
$\tau(I 1\Pi_g^+) \sim 20 \text{ ns}$	$3d\pi$
$\tau(O 1\Sigma_g^+) \sim 170 \text{ ns}$	$4s\sigma$

Astashkevich and Lavrov,  
*J. Phys. Chem. Ref. Data* **44**, 023105-1 (2015)

➔ Use gerade states for precision measurements (or Rydberg states)



# Molecular Hydrogen in this talk



PES from M. Silkowski & K. Pachucki

**ETH Zürich (Merkt)**

- Rydberg states

**VU**

- weakly-bound states

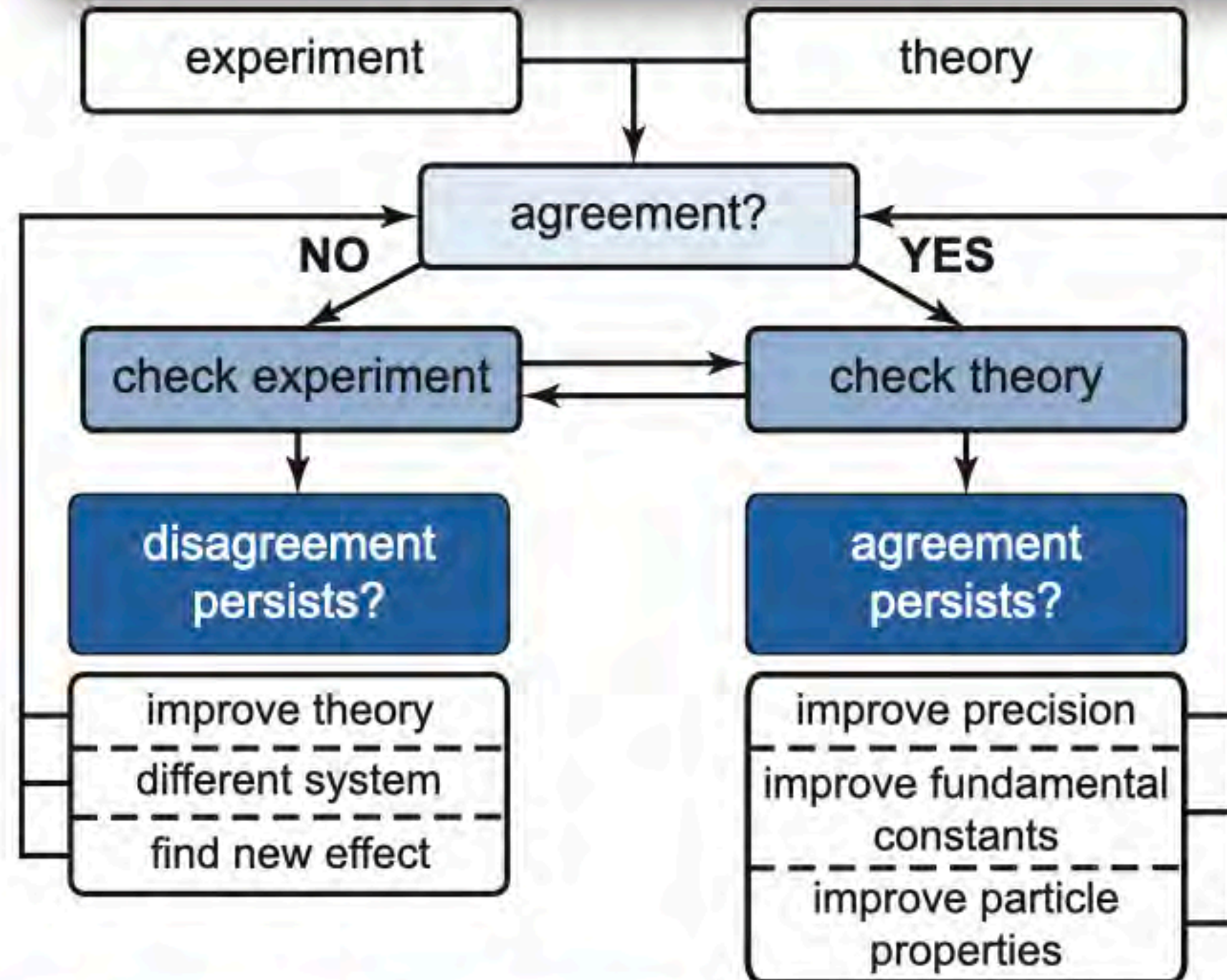
**VU Amsterdam (Ubachs, Eikema)**

- X-GK (= 1s-3d)
- X-EF (= 1s-2s)





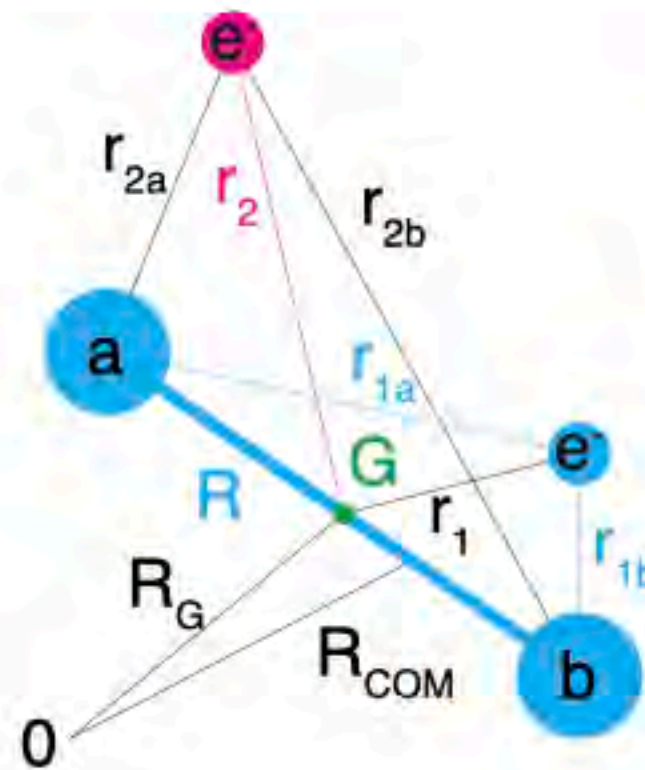
# Theory



**Born-Oppenheimer approximation** (infinite nuclear mass)

- ➔ adiabatic correction (diagonal)
- ➔ nonadiabatic correction (off-diagonal)

} or full NA calculation (pre-BO)



## Goals

- ➔ Provide benchmark to test/improve theory
- ➔ Extract values of fundamental constants ( $R, r_p, m_p/m_e$ )

$$\mathcal{H}_{rve} = \underbrace{-\frac{\nabla_1^2}{2} - \frac{\nabla_2^2}{2} + \frac{1}{R} - \frac{1}{r_{1a}} - \frac{1}{r_{1b}} - \frac{1}{r_{2a}} - \frac{1}{r_{2b}} + \frac{1}{r_{12}}}_{H_{cn}}$$

$$\underbrace{-\frac{\nabla_{R\theta\phi}^2}{2\mu}}_{H'_1} \underbrace{-\frac{\nabla_1^2}{8\mu} - \frac{\nabla_2^2}{8\mu} - \frac{\nabla_1 \cdot \nabla_2}{4\mu}}_{H'_2} \underbrace{-\frac{\nabla_{R\theta\phi} \cdot (\nabla_1 + \nabla_2)}{2\mu\alpha}}_{H'_3}$$

$$E_H^{NA} = \frac{m_p}{m_p + 1} \left(-\frac{1}{2}\right) = \left(1 + \frac{1}{m_p}\right)^{-1} \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2m_p} - \frac{1}{2m_p^2 + 2m_p}$$

59.8 cm<sup>-1</sup>  
1.79 THz

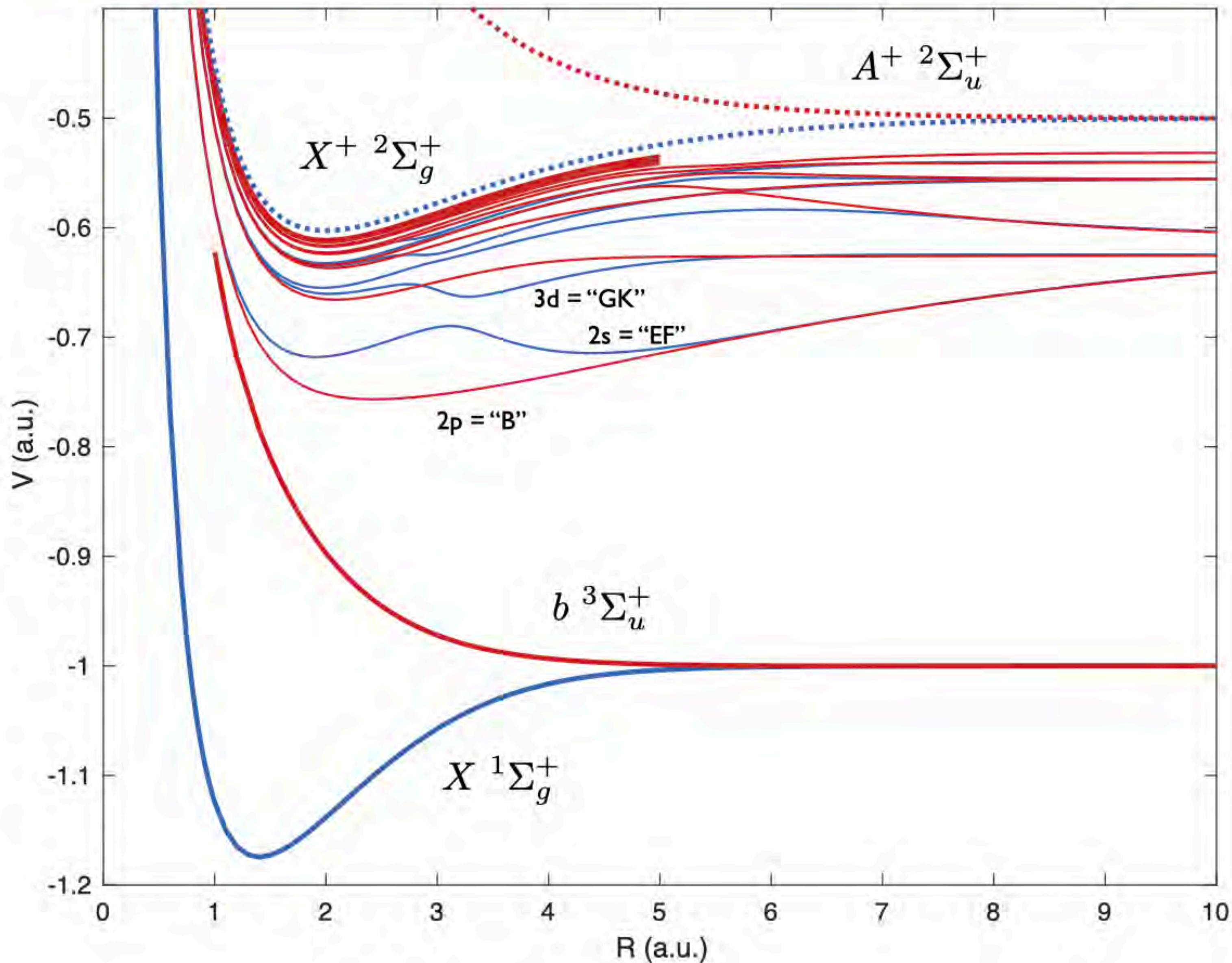
- 0.0325 cm<sup>-1</sup>  
- 975 MHz

$$E(\alpha) = m\alpha^2 E^{(2)} + m\alpha^4 E^{(4)} + m\alpha^5 E^{(5)} + m\alpha^6 E^{(6)} + m\alpha^7 E^{(7)} \dots \quad \text{with}$$

$$E^{(i)} = E^{(i,0)} + \frac{m}{M} E^{(i,1)} + \left(\frac{m}{M}\right)^2 E^{(i,2)} + \dots$$



# Status of the theory



$X^+, A^+$	$\sim 3$ MHz	preBO, ma <sup>5</sup>	Moss (1993)
$X^+ (v < 10)$	$\sim 10$ kHz	preBO, ma <sup>8</sup>	Korobov, Karr, Hilico (2017)
Rydberg states ( $n > 40$ )	$\sim 300$ kHz	MQDT	Jungen, Beyer, Merkt
EF, GK, ...	$\sim 1$ MHz	BO	Silkowski, Pachucki
EF	$\sim 150$ MHz	preBO, ma <sup>7</sup>	Matyus (2019)
X	$\sim 0.78$ MHz	preBO, ma <sup>7</sup>	Pachucki + coworkers (2019)

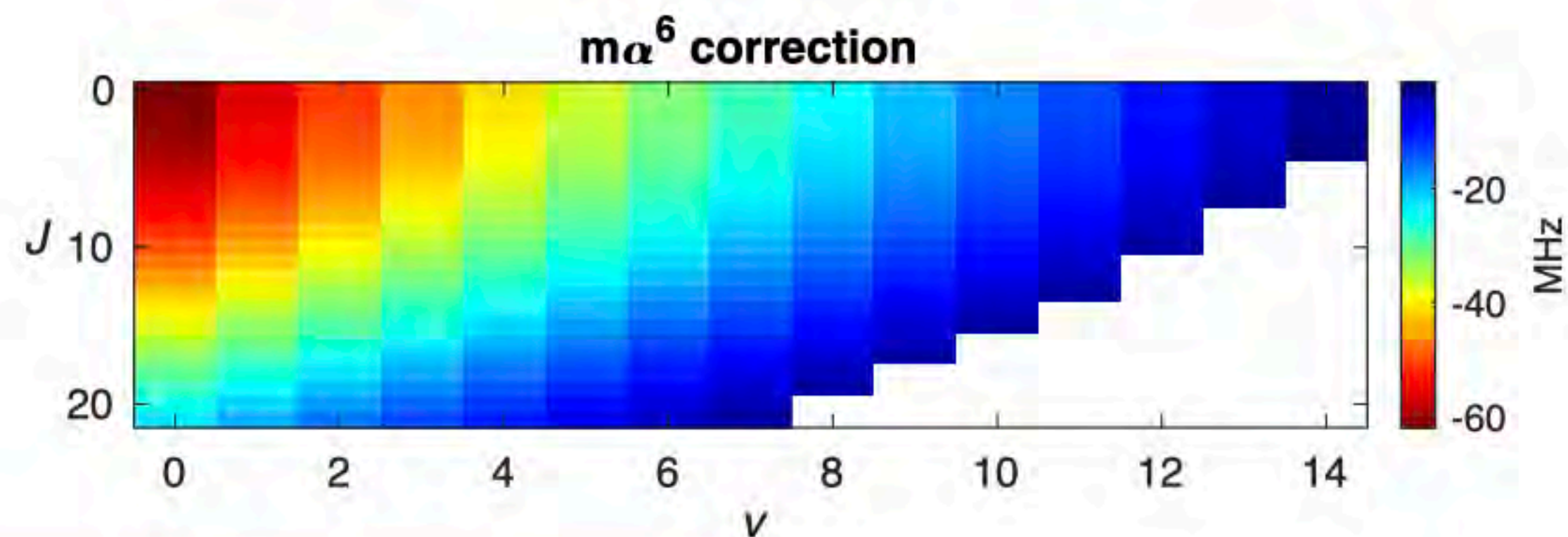
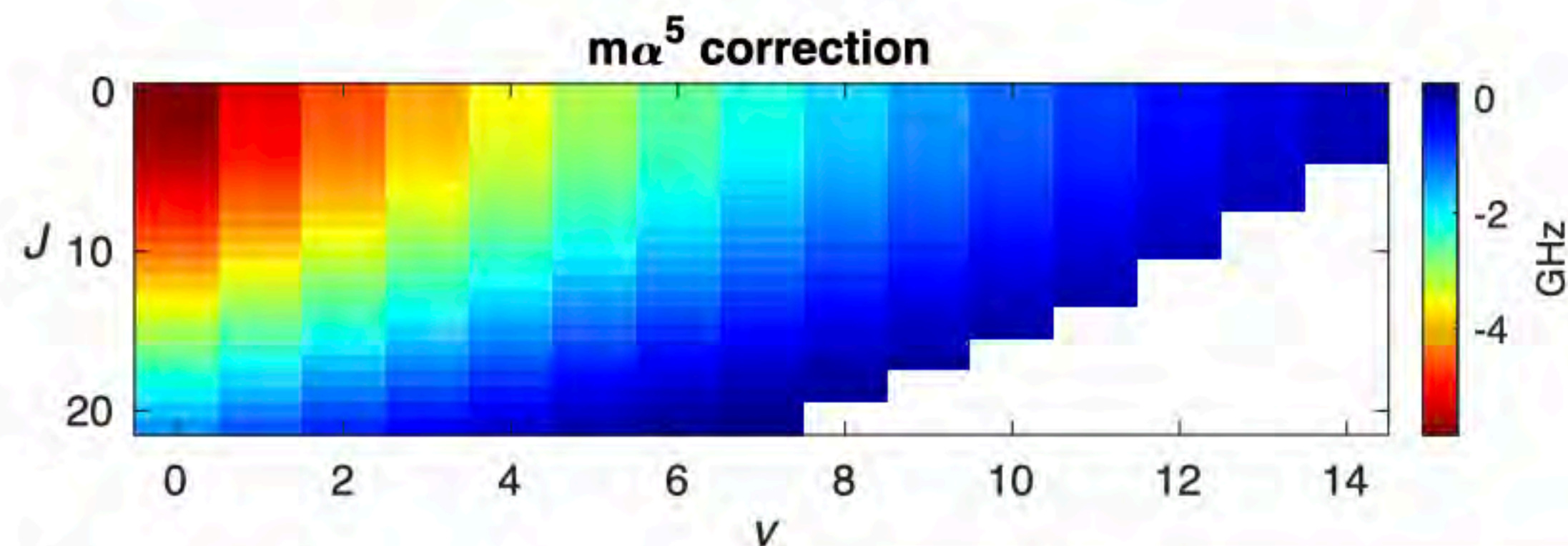
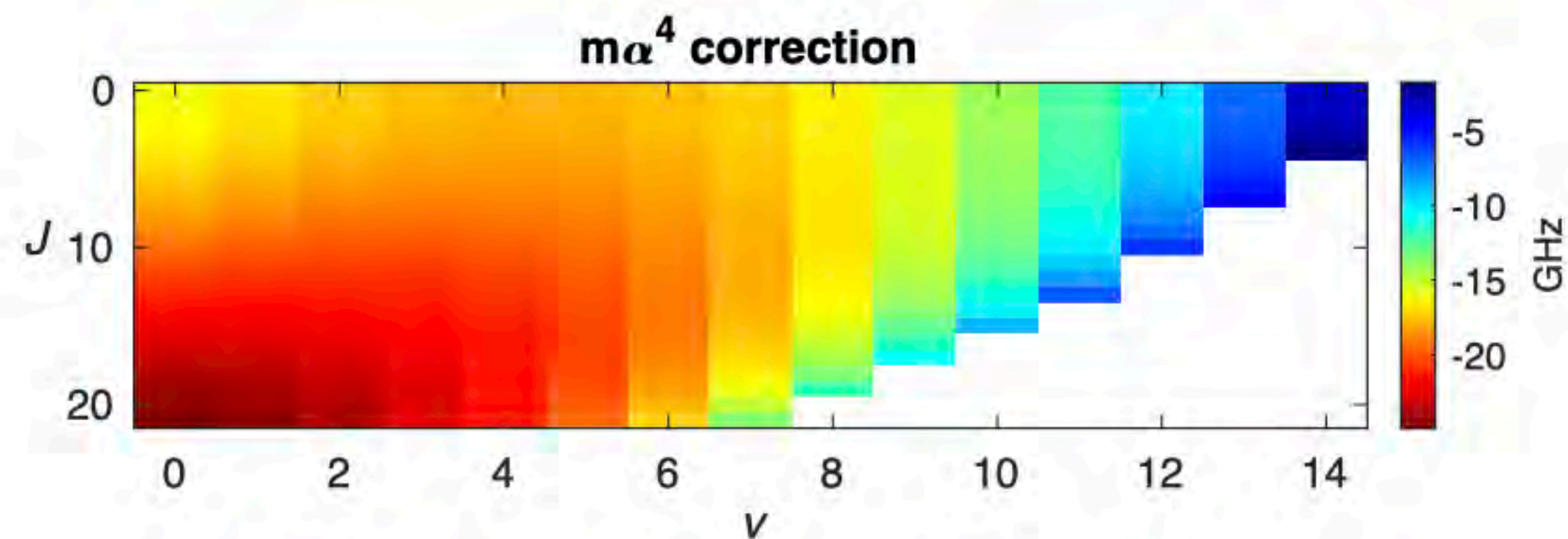
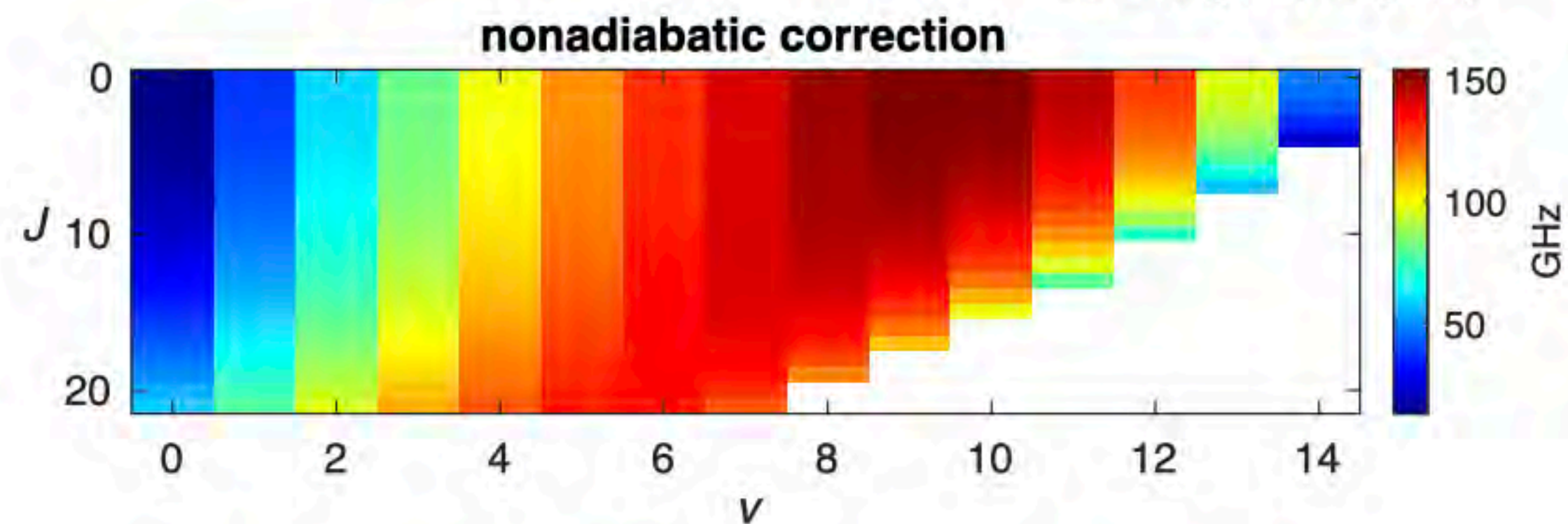


# Corrections for the electronic ground state of H<sub>2</sub>

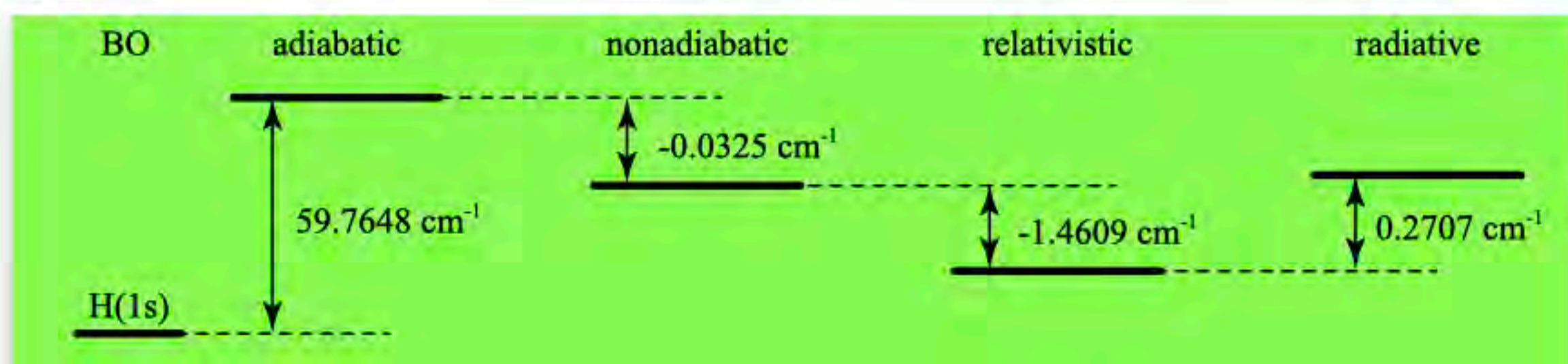
Nonadiabatic correction

$$\Delta E_{\text{na}} = -C \langle T \rangle_v$$

Van Vleck, *JCP* 4, 327 (1936)



Komasa et al., *PRA* **100**, 032519 (2019)  
Lai et al., *PRL* **127**, 183001 (2021)





# Measurement of the dissociation energy

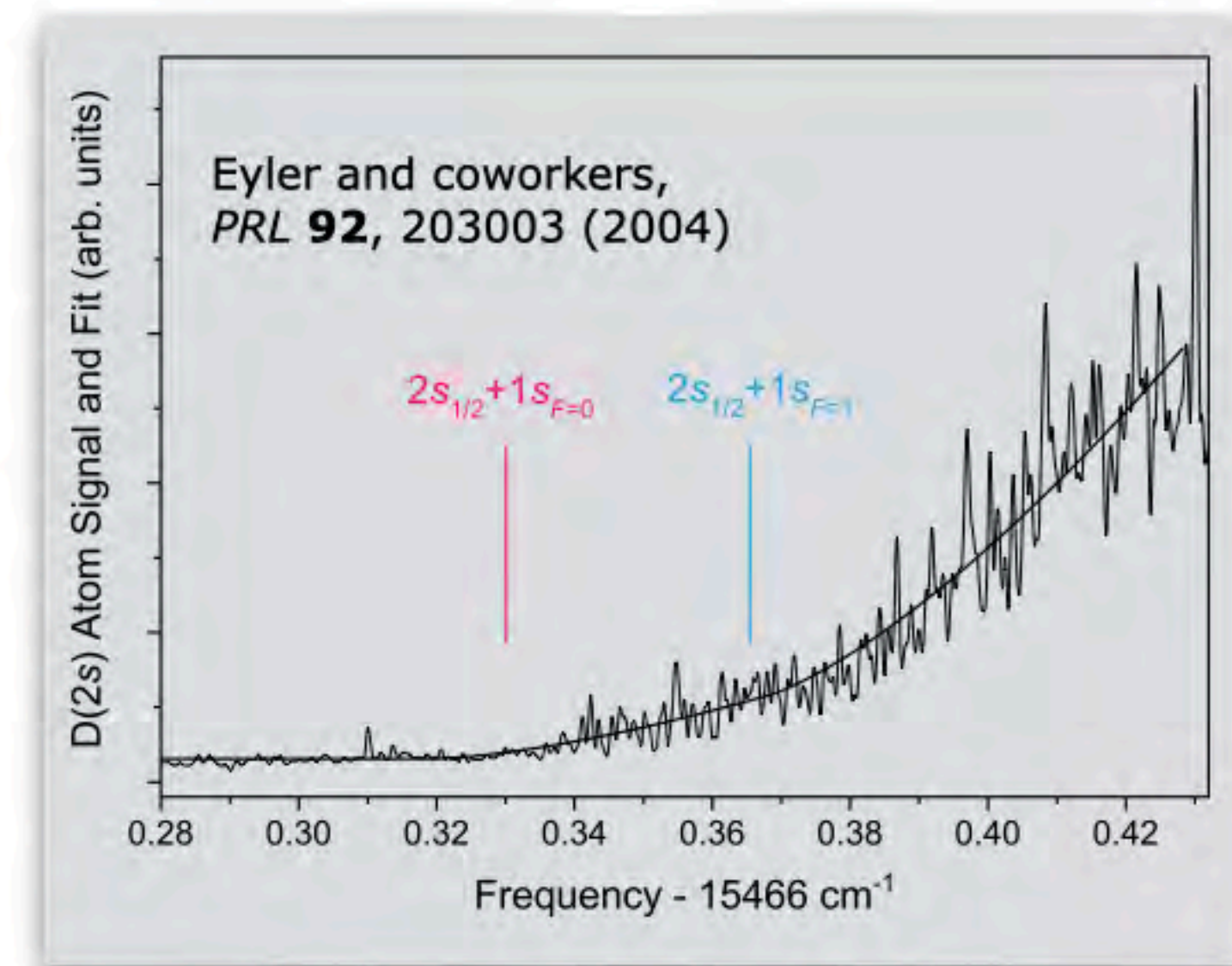
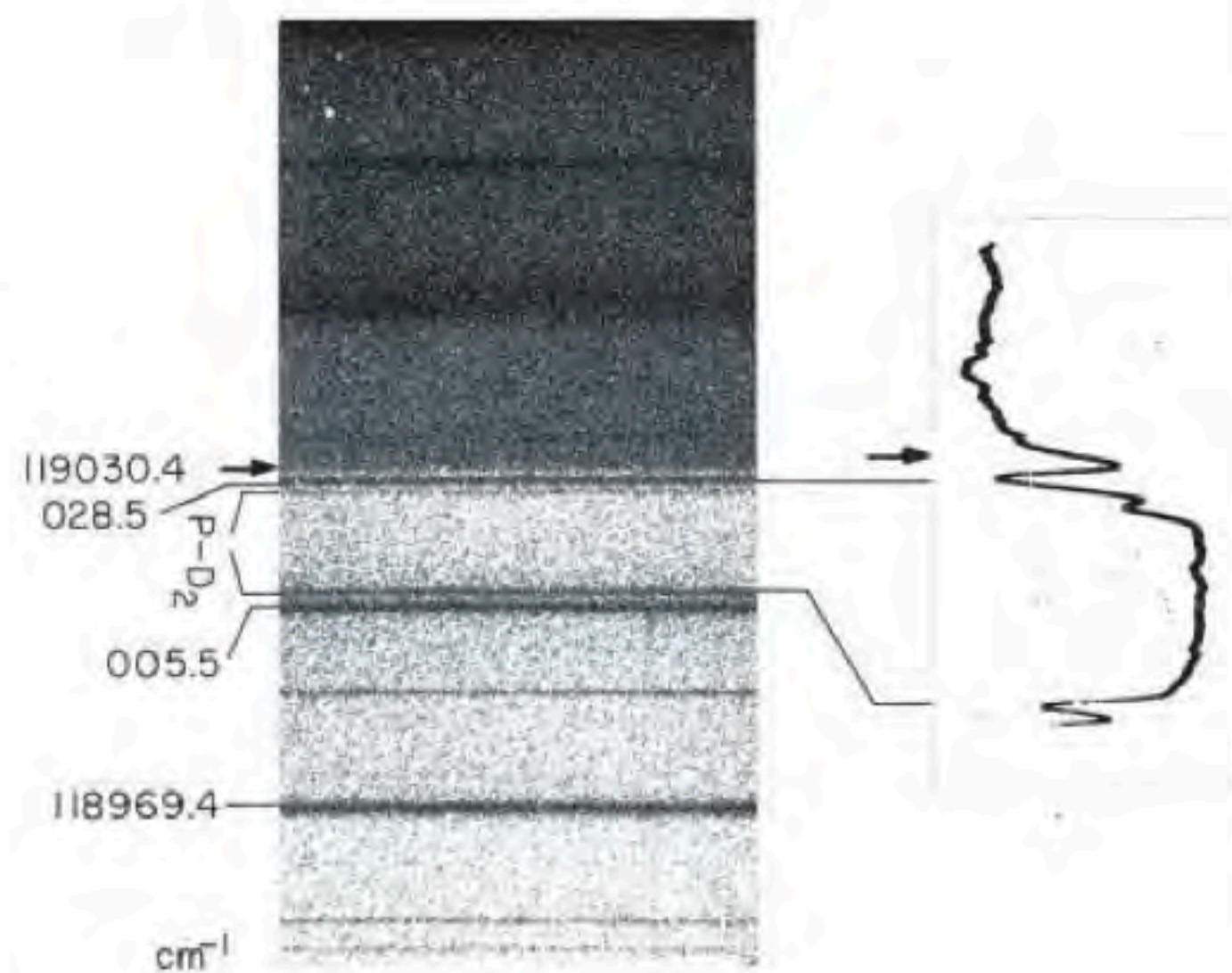
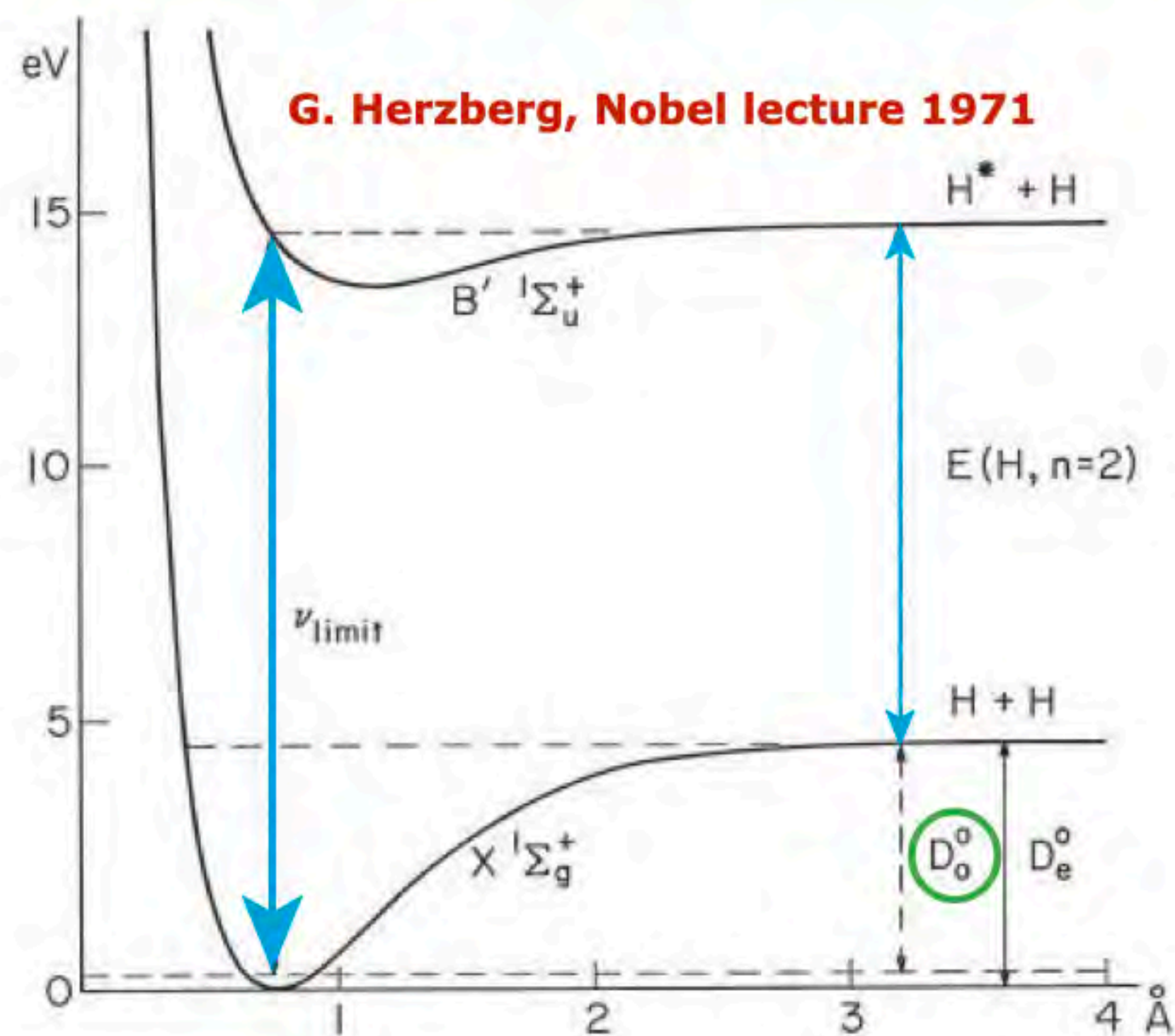
$$\left\langle \Phi'_{\text{elec}} \Phi'_{\text{vib}} \Phi'_{\text{rot}} \left| \sum_{\alpha} \lambda_{\alpha Z} \hat{\mu}_{\alpha} \right| \Phi''_{\text{elec}} \Phi''_{\text{vib}} \Phi''_{\text{rot}} \right\rangle$$

$$= \sum_{\alpha} \langle \Phi'_{\text{rot}} | \lambda_{\alpha Z} | \Phi''_{\text{rot}} \rangle \langle \Phi'_{\text{elec}} \Phi'_{\text{vib}} | \hat{\mu}_{\alpha} | \Phi''_{\text{elec}} \Phi''_{\text{vib}} \rangle$$

$$\langle \Phi'_{\text{vib}} | [\langle \Phi'_{\text{elec}} | \hat{\mu}_{\alpha} | \Phi''_{\text{elec}} \rangle_e] | \Phi''_{\text{vib}} \rangle = \langle \Phi'_{\text{vib}} | \mu_{\alpha}^e(Q) | \Phi''_{\text{vib}} \rangle$$

→ Zero permanent dipole moment for H<sub>2</sub>, D<sub>2</sub>

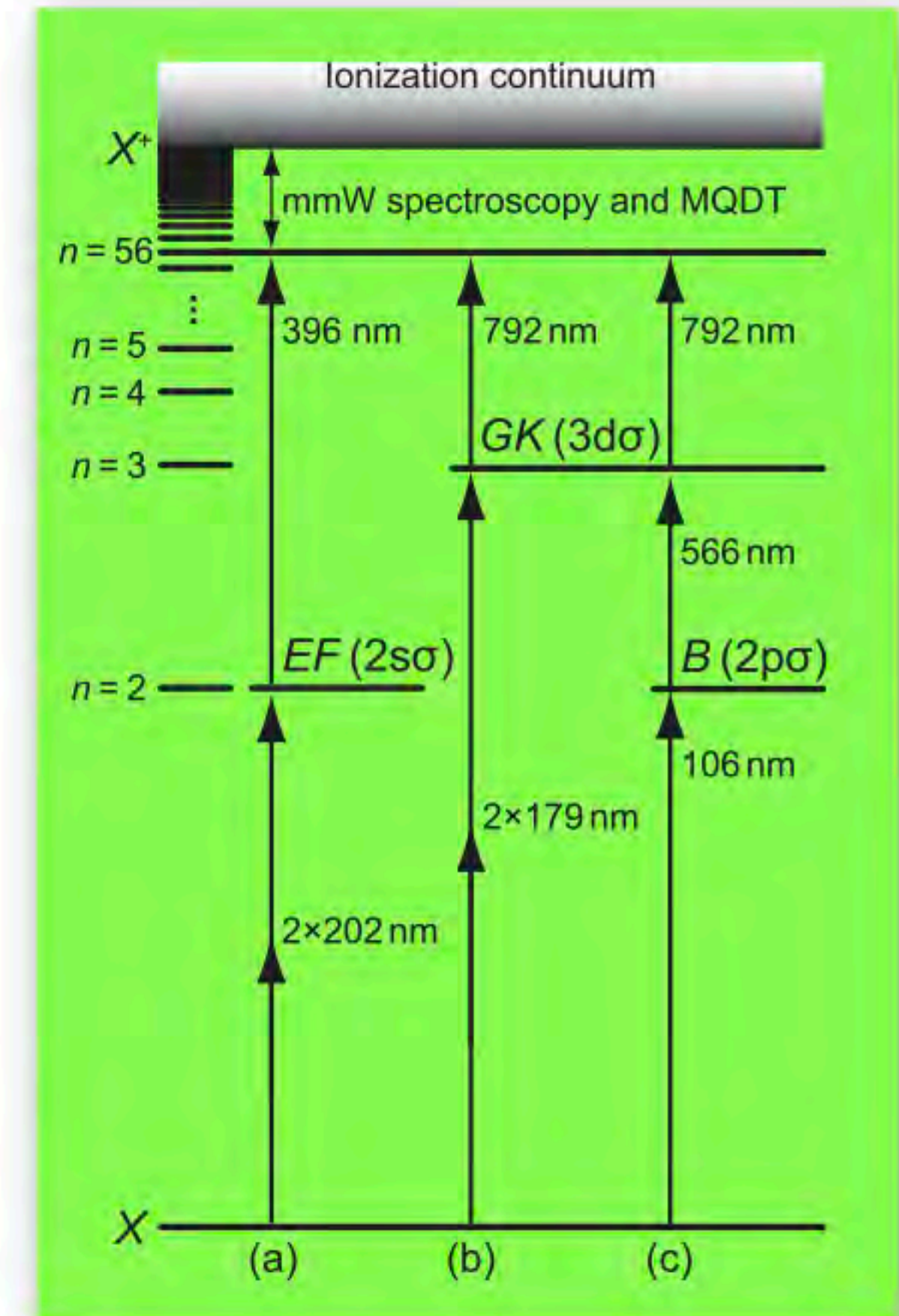
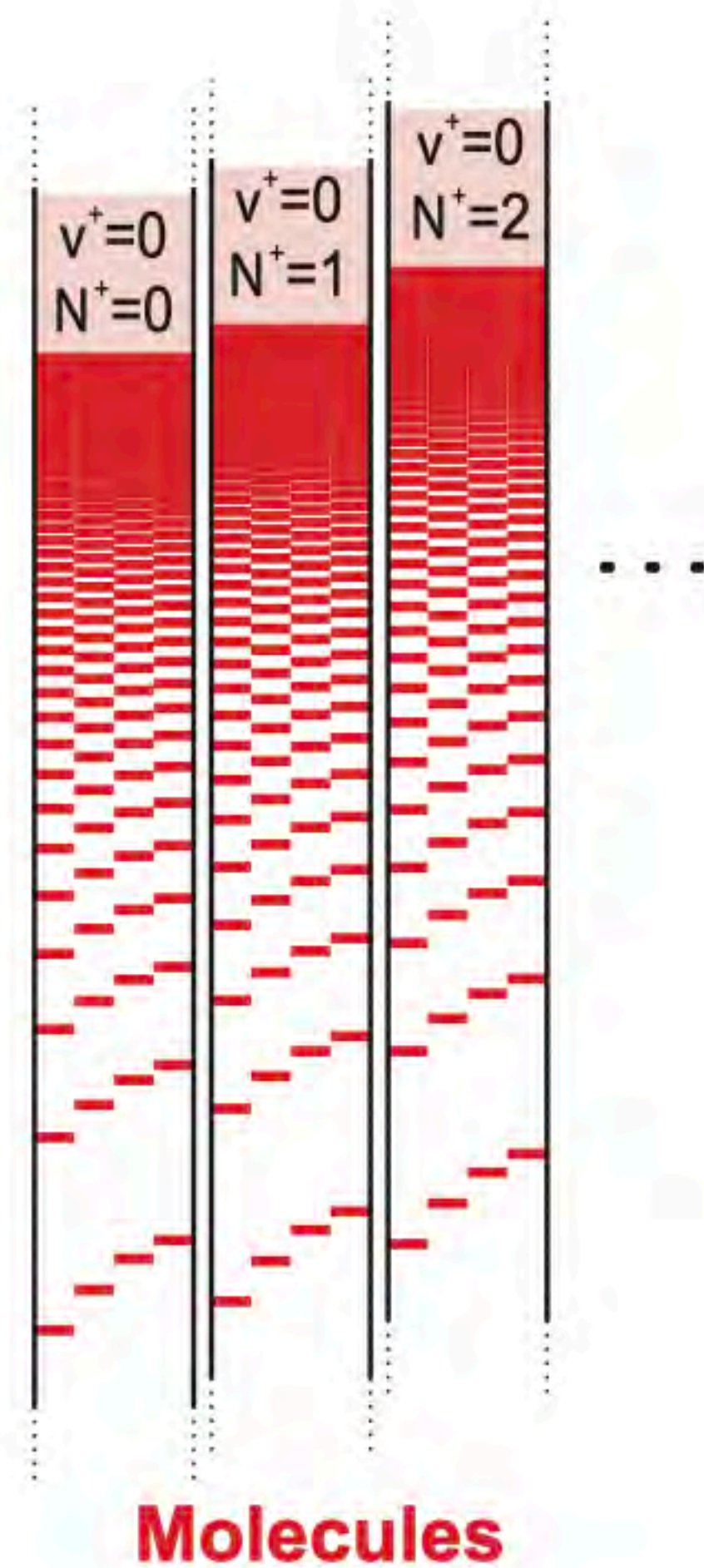
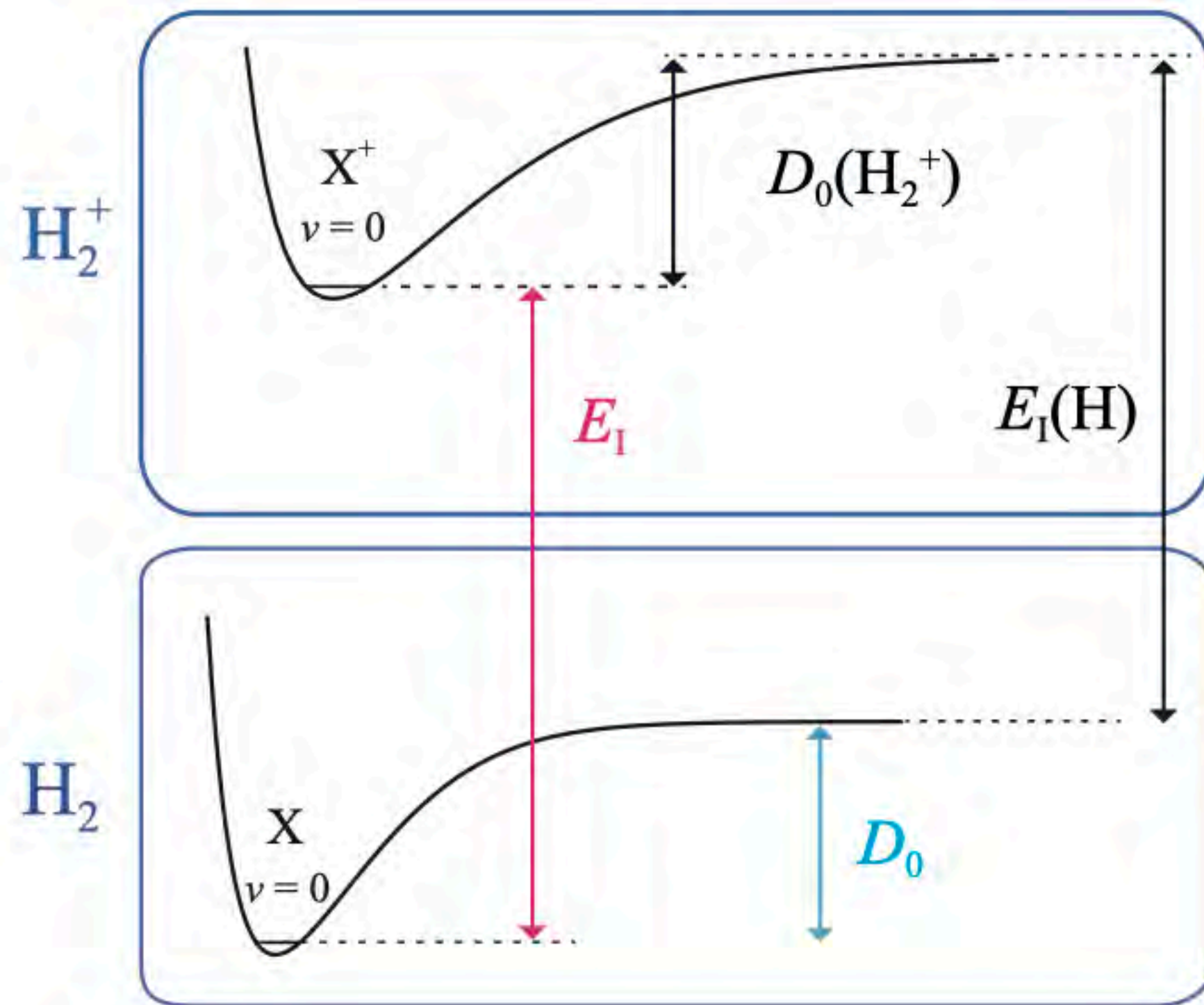
→ No EI rovibrational spectrum



→ Can we measure a sharp feature, instead of an onset of dissociation?



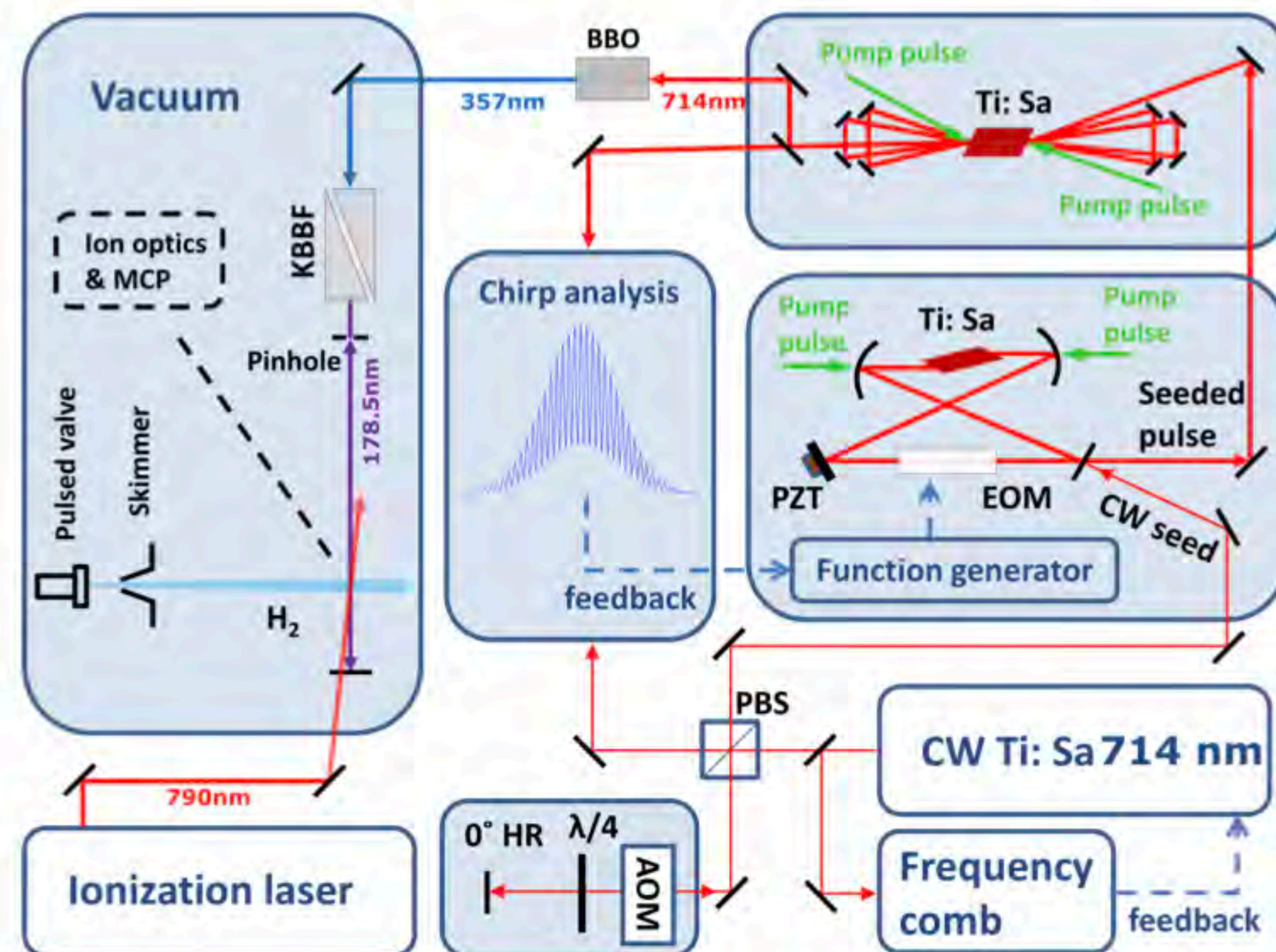
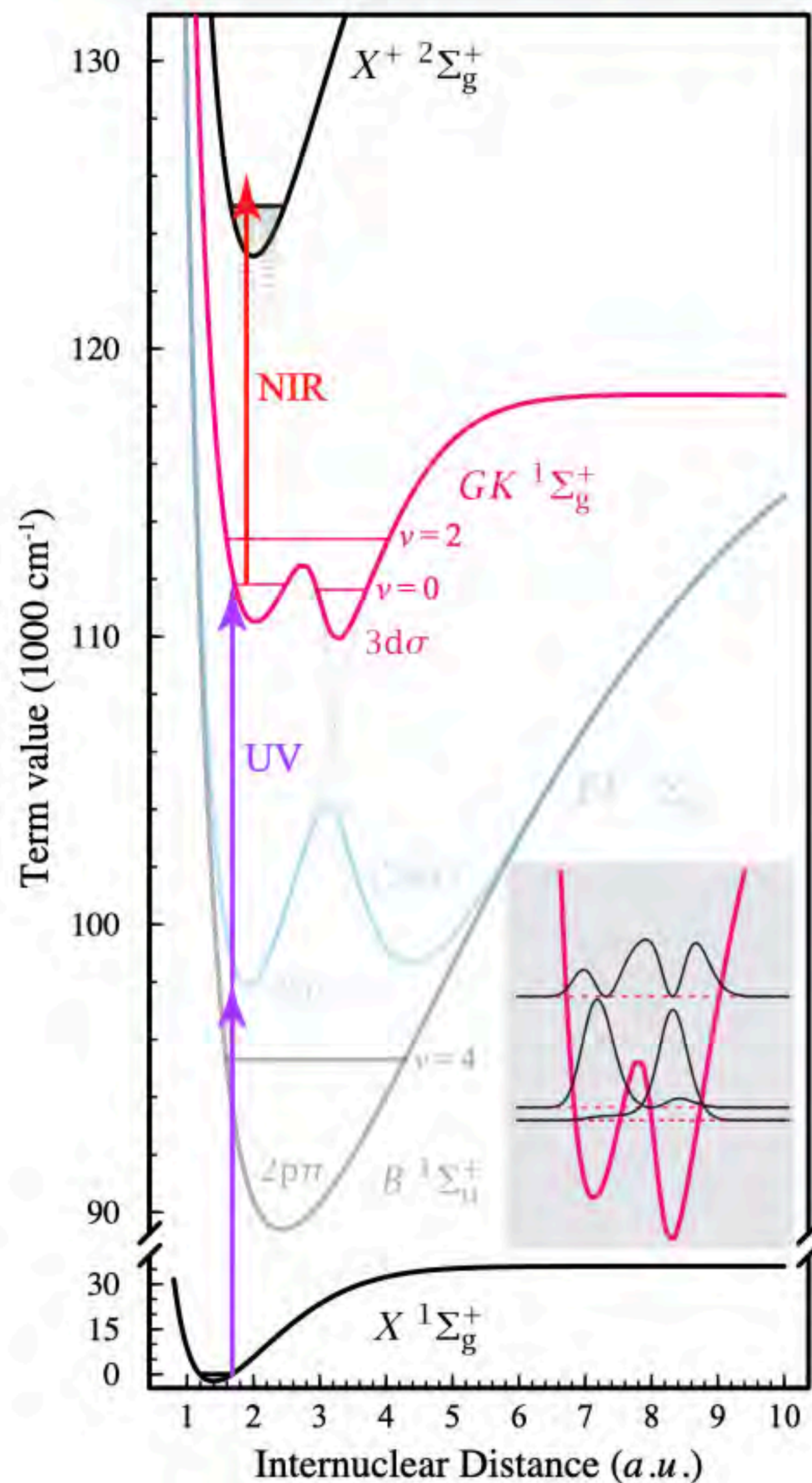
# Measurement of the ionization energy



- ➔ Measure transitions from gerade intermediate states to long-living Rydberg states
- ➔ Extrapolate Rydberg series to the ionization limit using multichannel quantum-defect theory (MQDT)

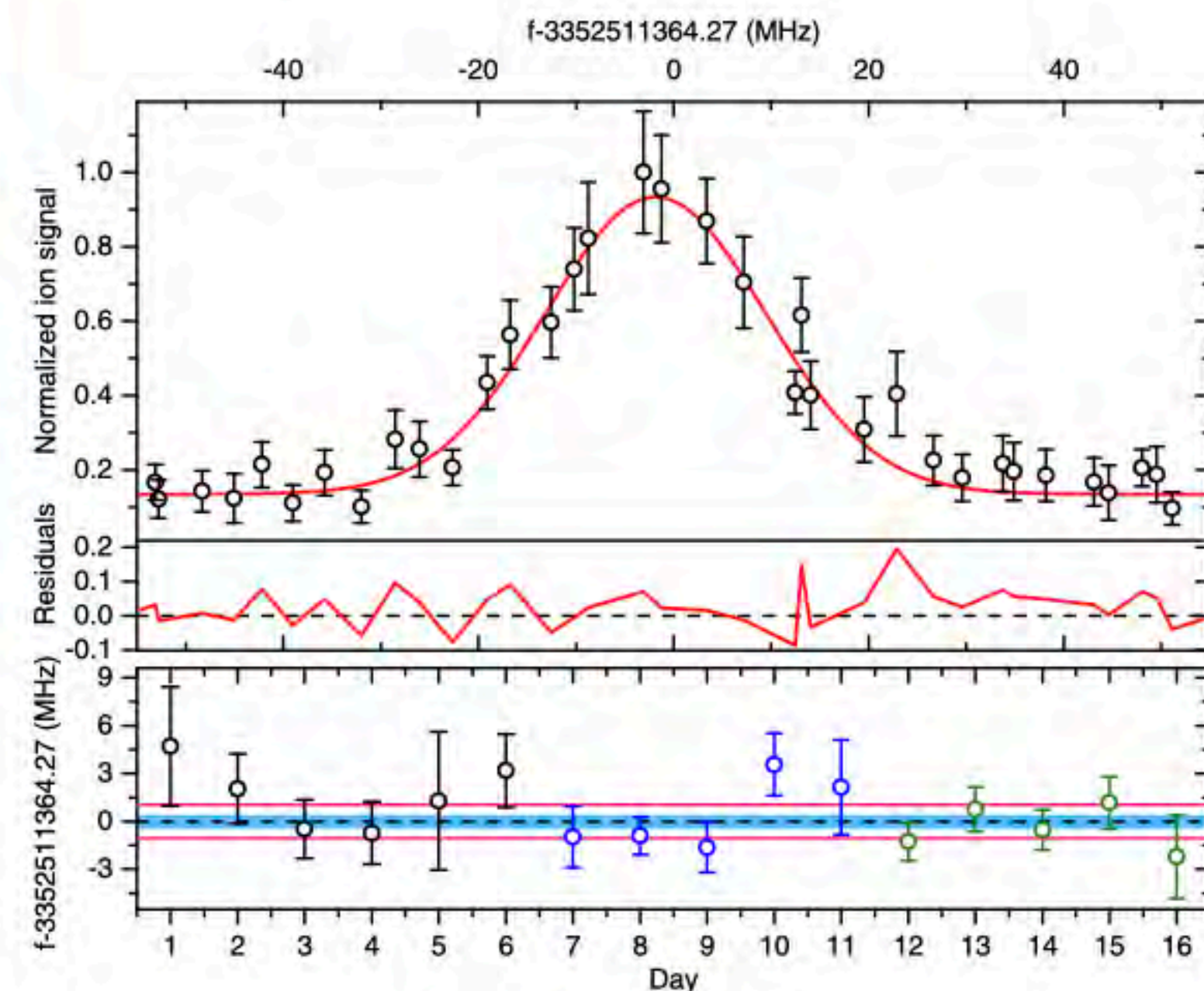


# VU: two-photon deep-UV spectroscopy for X-GK interval



J. Hussels, C. Cheng, E. J. Salumbides, and W. Ubachs, *Opt. Lett.* 45, 5909-5912 (2020)

- Dominating systematic uncertainties:
- Residual first-order Doppler shift
  - ac-Stark effect from the two lasers

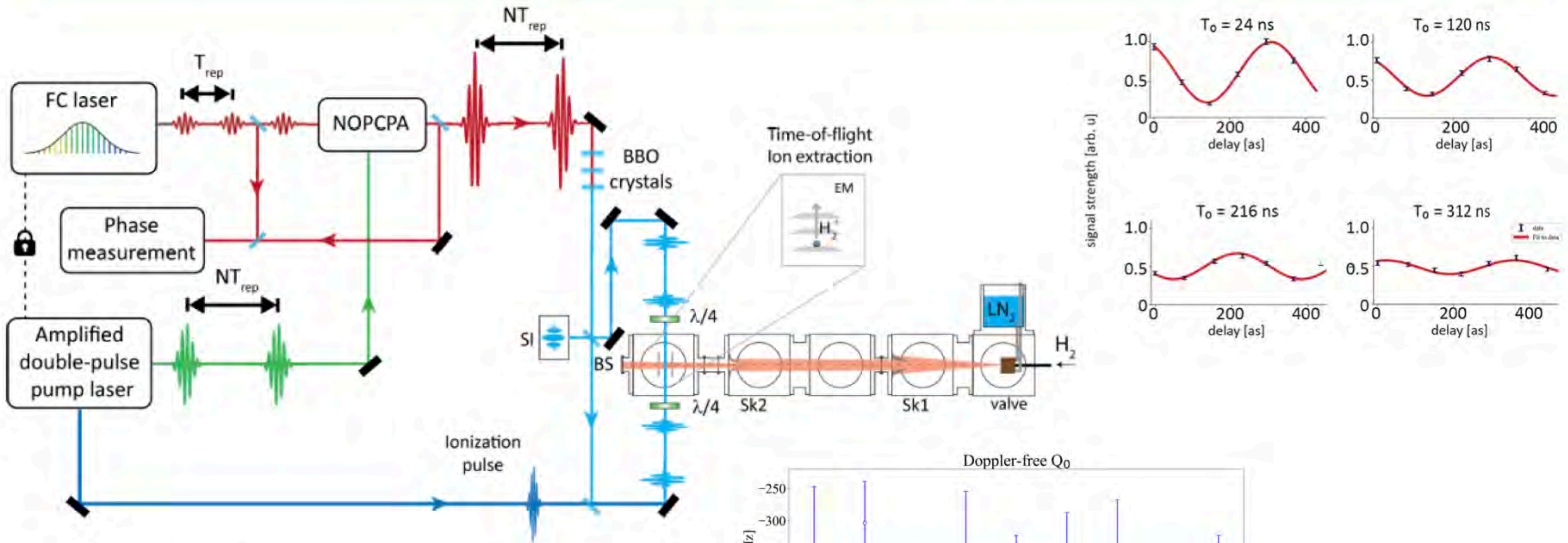


Work by:

J. Hussels, C. Cheng, E. Salumbides, W. Ubachs



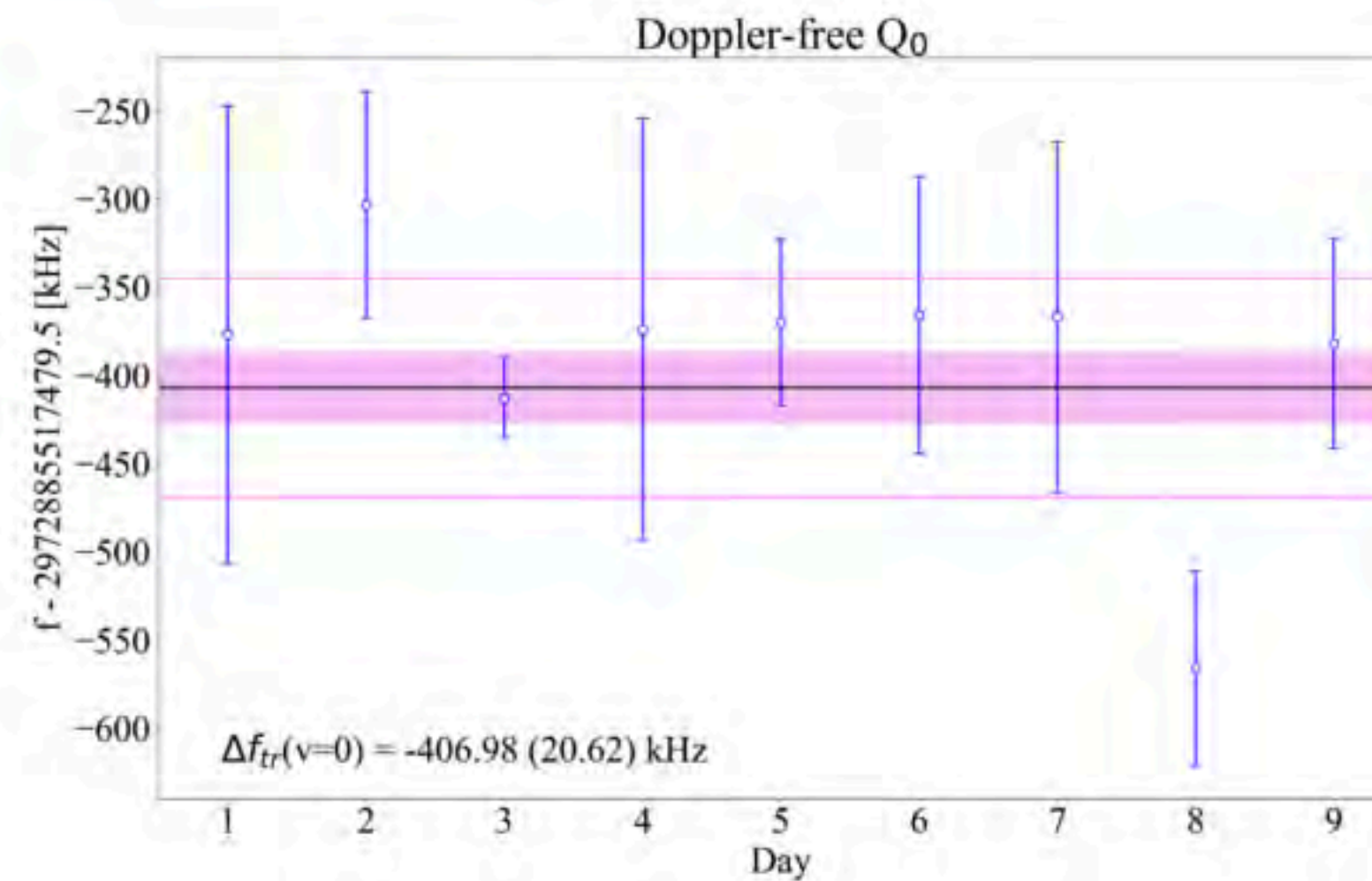
# VU: Ramsey-comb spectroscopy for X-EF interval



Dominating systematic uncertainties:  
i) ac-Stark effect

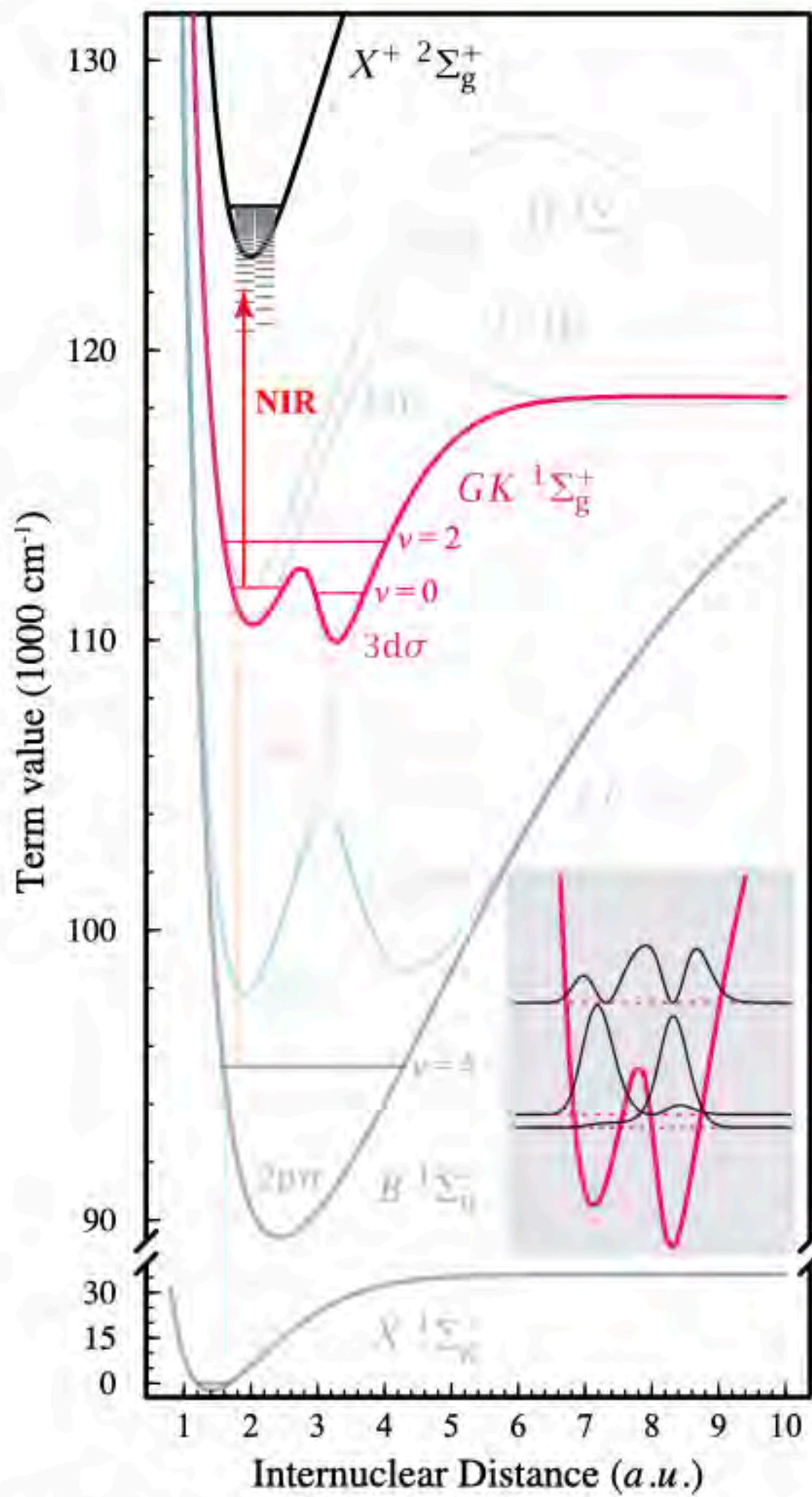
Work by:

Ch. Roth, A. Martínez de Velasco, K. Eikema



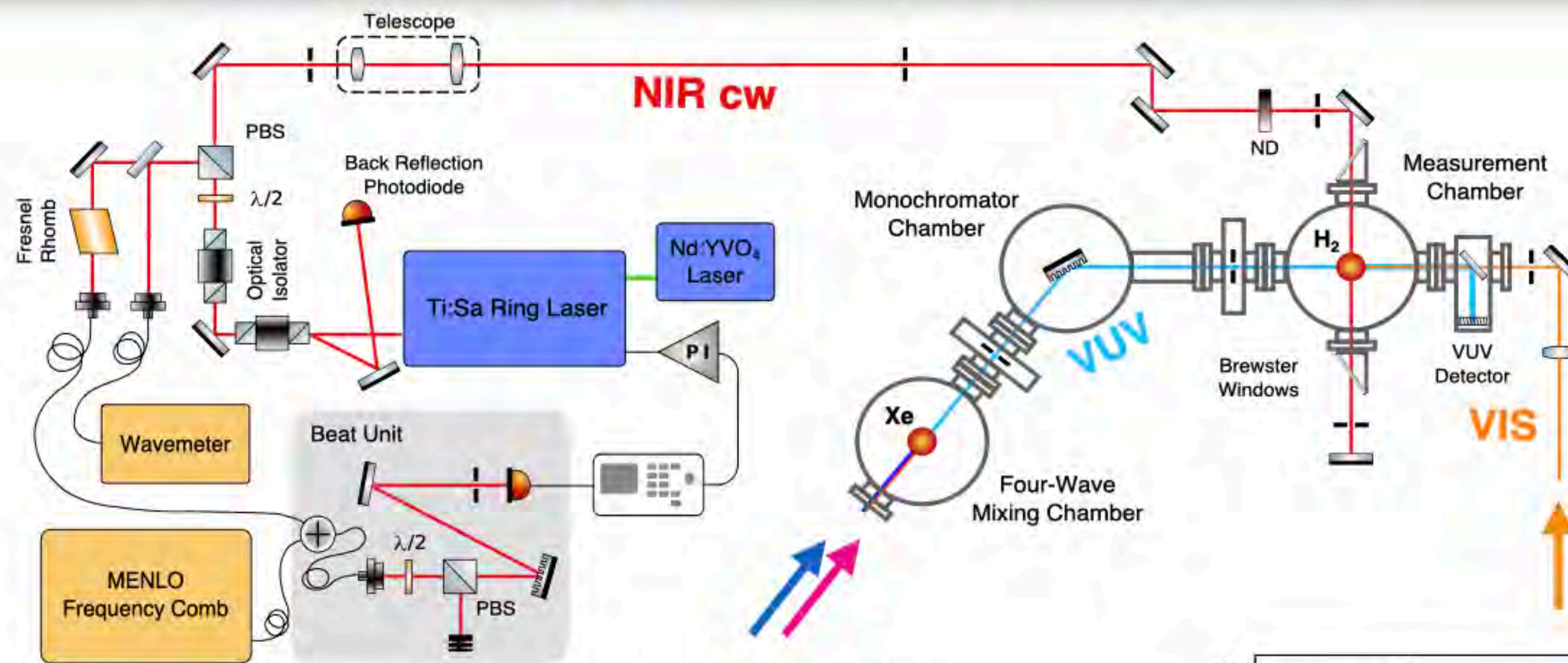


# ETH: Rydberg spectroscopy



Work by:

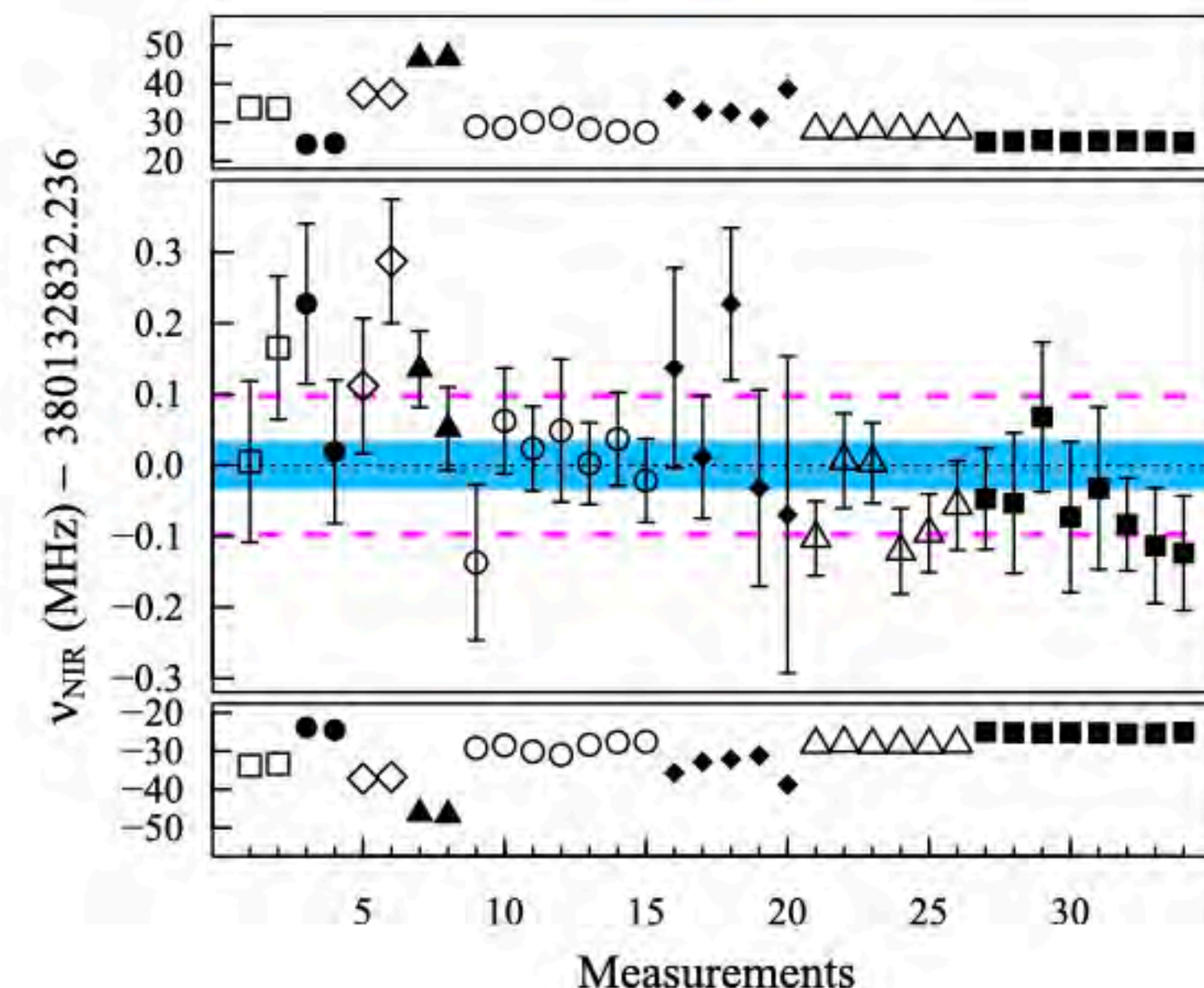
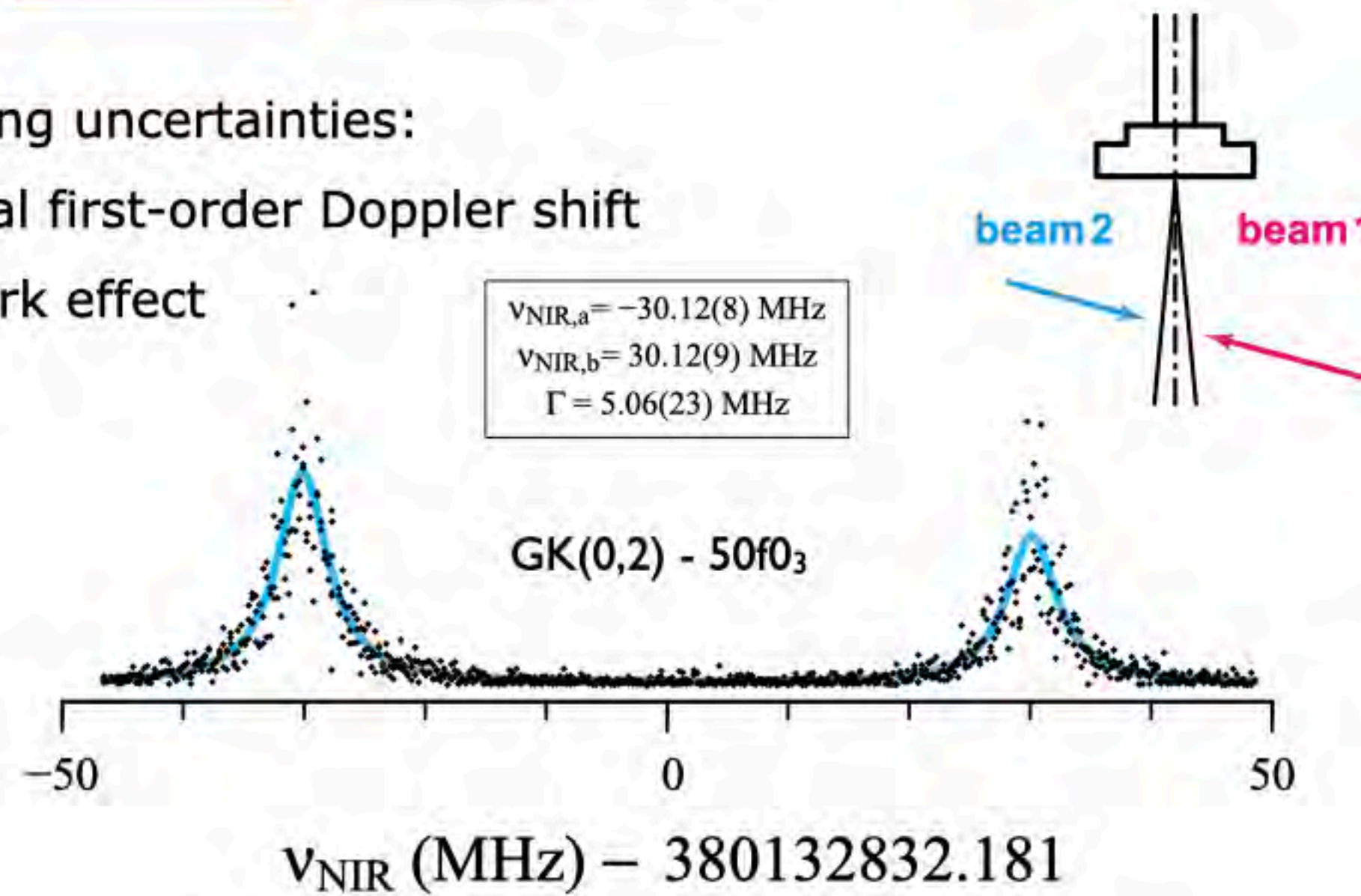
MB, N. Hölsch, F. Merkt



Dominating uncertainties:

- i) Residual first-order Doppler shift
- ii) dc-Stark effect

$$\begin{aligned} \nu_{\text{NIR},a} &= -30.12(8) \text{ MHz} \\ \nu_{\text{NIR},b} &= 30.12(9) \text{ MHz} \\ \Gamma &= 5.06(23) \text{ MHz} \end{aligned}$$



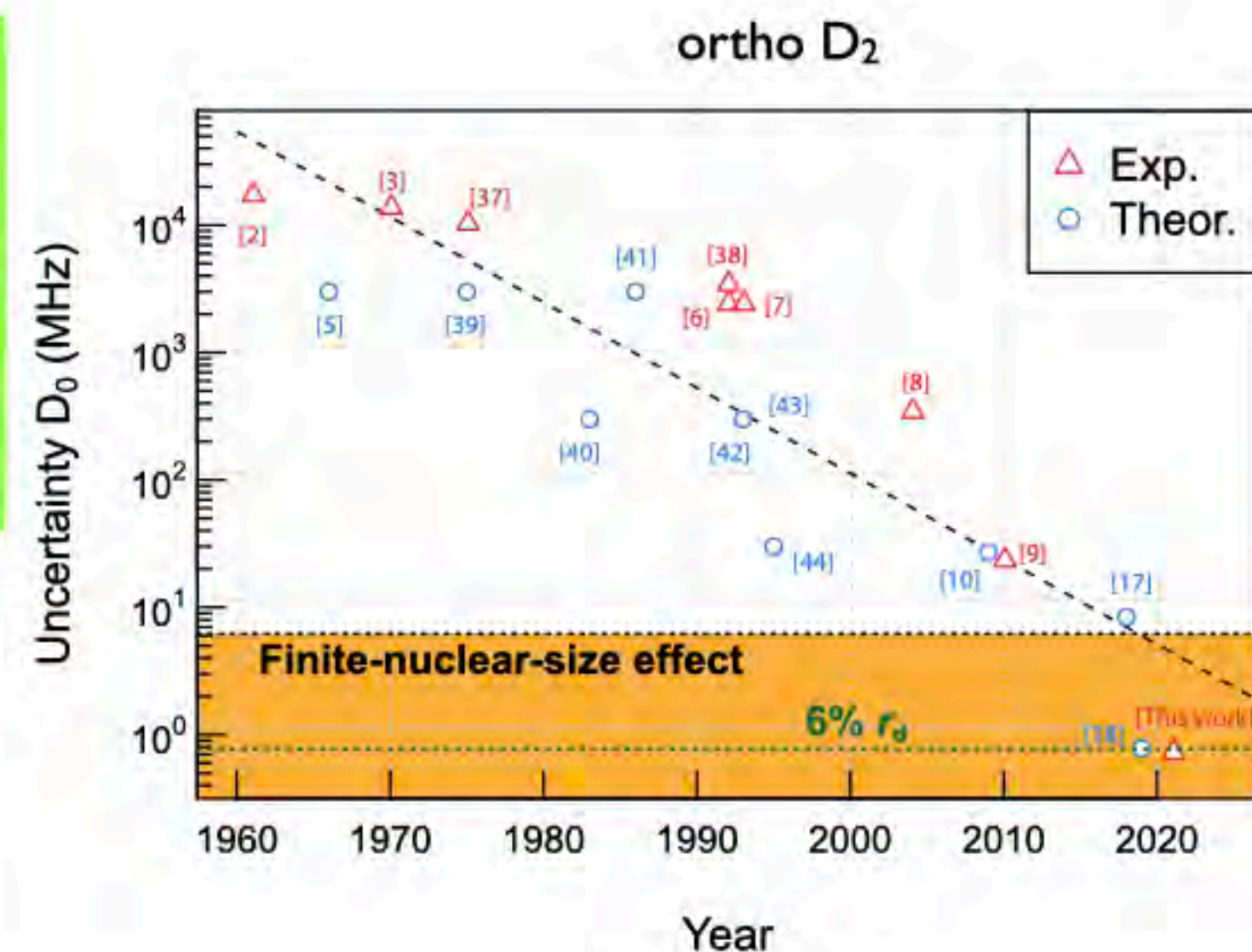
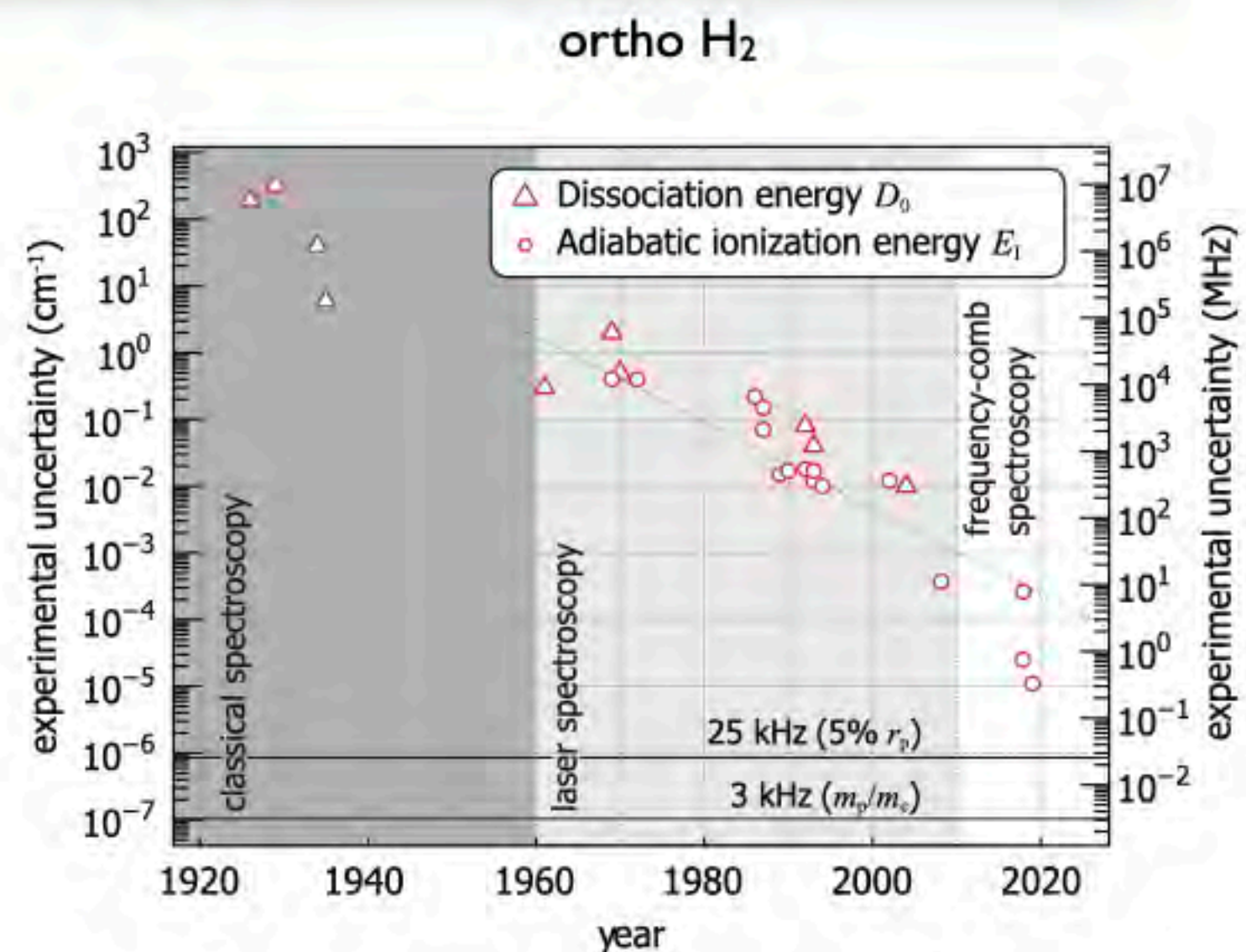


# Measured ionization and dissociation energies

Values in  $\text{cm}^{-1}$

	<b>H<sub>2</sub></b>		<b>D<sub>2</sub></b>	<b>HD</b>	
	<b>para (N = 0)</b>	<b>ortho (N = 1)</b>	<b>ortho (N = 0)</b>	<b>N = 0</b>	
X-GK/EF	111 827.741 986(21) 630 kHz	111 686.632 836(22) 660 kHz	99 109.731 204 9(24) 73 kHz	112 099.087 712(15) 450 kHz	400 kHz
GK/EF-Rydberg	12 723.757 461(23) 700 kHz	12 635.724 114(12) 360 kHz	25 209.997 785(10) 300 kHz	12 979.344 802(20) 600 kHz	...
Rydberg extrapolation		150 kHz	150 kHz		...
IP	124 417.491 098(31) 940 kHz	124 357.238 062(25) 750 kHz (2E-10)	124 357.238 003(11) 340 kHz (9E-11)	124 745.393 739(26) 780 kHz	...
D <sub>0</sub>	36 118.069 605(31) 940 kHz	35 999.582 894(25) 750 kHz (7E-10)	35 999.582 834(11) 340 kHz (3E-10)	36 748.362 282(26) 780 kHz	[36 405.783 66(36)]
D <sub>0</sub> (theory)	36 118.069 632(26) 780 kHz		35 999.582 820(26) 780 kHz	36 748.362 342(26) 780 kHz	36 405.782 477(26) 780 kHz

H<sub>2</sub> ortho: PRL **121**, 013001 (2018), PRL **122**, 103002 (2019),  
H<sub>2</sub> para: PRL **123**, 163002 (2019),  
D<sub>2</sub> ortho: PRA **105**, 022820 (2022)  
Theory: PRL **122**, 103003 (2019), PRA **100**, 020503(R) (2019),





## New results for X-EF in H<sub>2</sub> (para) and D<sub>2</sub> (ortho)

Contribution	Value [kHz]	Uncertainty [kHz]
Transition frequency, 1 <sup>st</sup> and 2 <sup>nd</sup> order Doppler-	2 972 885 517 058	21
AC-Stark shift correction	28	23
Phase measurement setup correction	-8	1
Zeeman shift correction	0	1
DC-Stark shift correction	1	1
Clock	0	1
Total	2 972 885 517 079	31

H<sub>2</sub>

Contribution	Value [kHz]	Uncertainty [kHz]
Transition frequency, 1 <sup>st</sup> and 2 <sup>nd</sup> order Doppler-	2 981 779 227 576	13
AC-Stark shift correction	9	14
Phase measurement setup correction	-6	2
Zeeman shift correction	0	1
DC-Stark shift correction	0	1
Clock	0	1
Total	2 981 779 227 579	19

D<sub>2</sub>

**Work by:**

**Ch. Roth, A. Martínez de Velasco, K. Eikema**



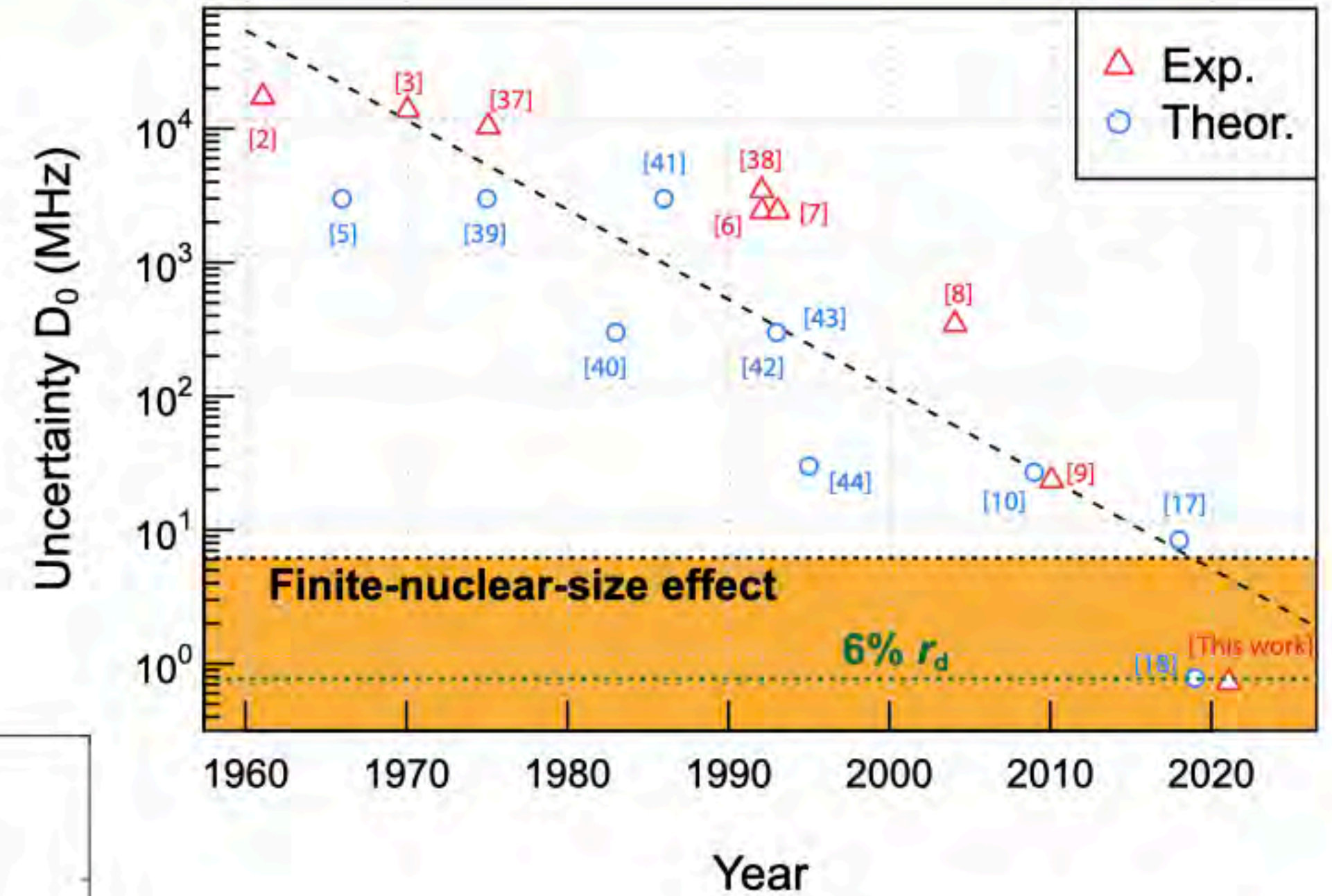
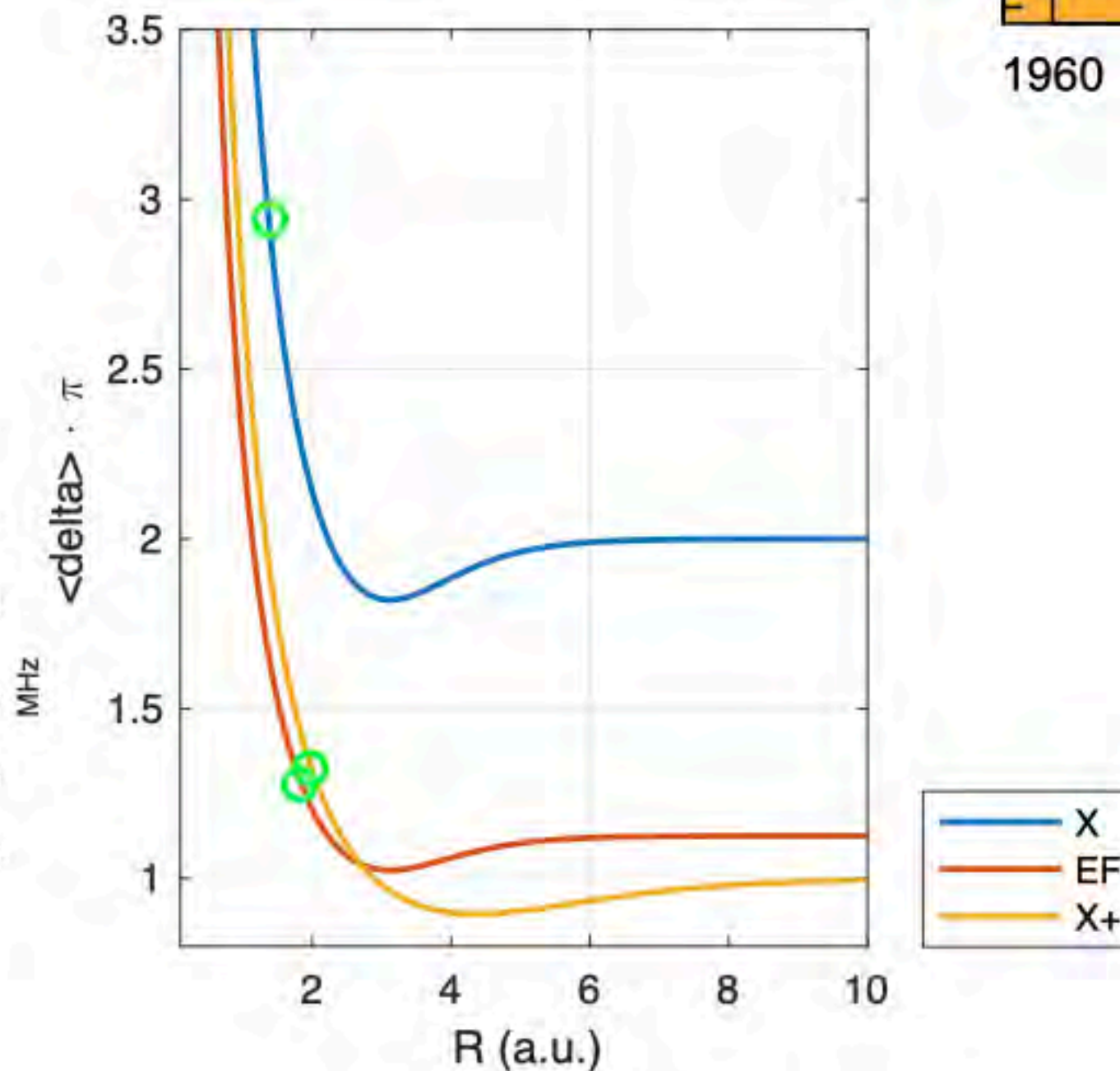
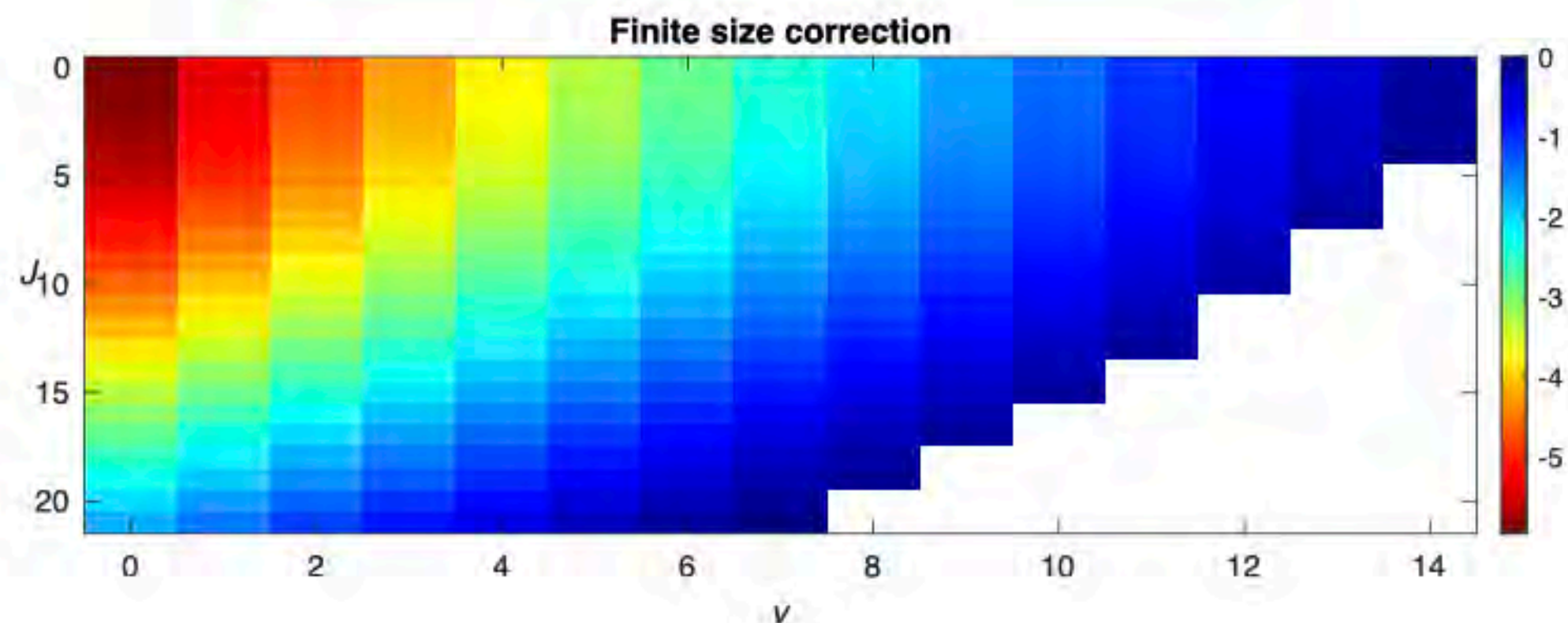
# Effect of charge radius: D<sub>0</sub> vs X-EF vs IP

Contribution	H <sub>2</sub>	D <sub>2</sub>	HD
$E^{(2)}$	36 118.797 746 10(3)	36 749.090 990 00(1)	36 406.510 890 07(1)
$E^{(4)}$	-0.531 215 6(5)	-0.528 206 05(9)	-0.529 887 5(2)
$E^{(5)}$	-0.194 910 43(15)	-0.198 256(3)	-0.196 441(4)
$E^{(6)}$	-0.002 067(6)	-0.002 096(6)	-0.002 080(6)
$E_{\text{sec}}^{(6)}$	0.000 009 2	0.000 009 4	0.000 009 3
$E^{(7)}$	0.000 101(25)	0.000 103(25)	0.000 102(25)
$E_{\text{FS}}^{(4)}$	-0.000 031	-0.000 202	-0.000 116
Total	36 118.069 632(26)	36 748.362 342(26)	36 405.782 477(26)

PRL **122**, 103003 (2019), PRA **100**, 020503(R) (2019)

$$\mathcal{E}_{\text{FS}}^{(4)}(R) = \frac{2\pi}{3} \langle \phi_{\text{el}} | \sum_{a,X} \delta^3(r_{aX}) | \phi_{\text{el}} \rangle_{\text{el}} \frac{(r_{C,A}^2 + r_{C,B}^2)}{2\lambda^2}$$

$$r_d = 2.12799(74) \text{ fm}$$



- ➔ Effect of FS in D<sub>2</sub> D<sub>0</sub>: 6.056 MHz
- ➔ FS uncertainty on D<sub>2</sub> D<sub>0</sub>: 4 kHz
- ➔ FS uncertainty on D<sub>2</sub> IP: 11 kHz
- ➔ FS uncertainty on D<sub>2</sub> X-EF: 12 kHz



# Collisions of (ground state) Hydrogen atoms

- Frequency shift (H maser, 1s-2s spectroscopy, ...)

VOLUME 81, NUMBER 18

PHYSICAL REVIEW LETTERS

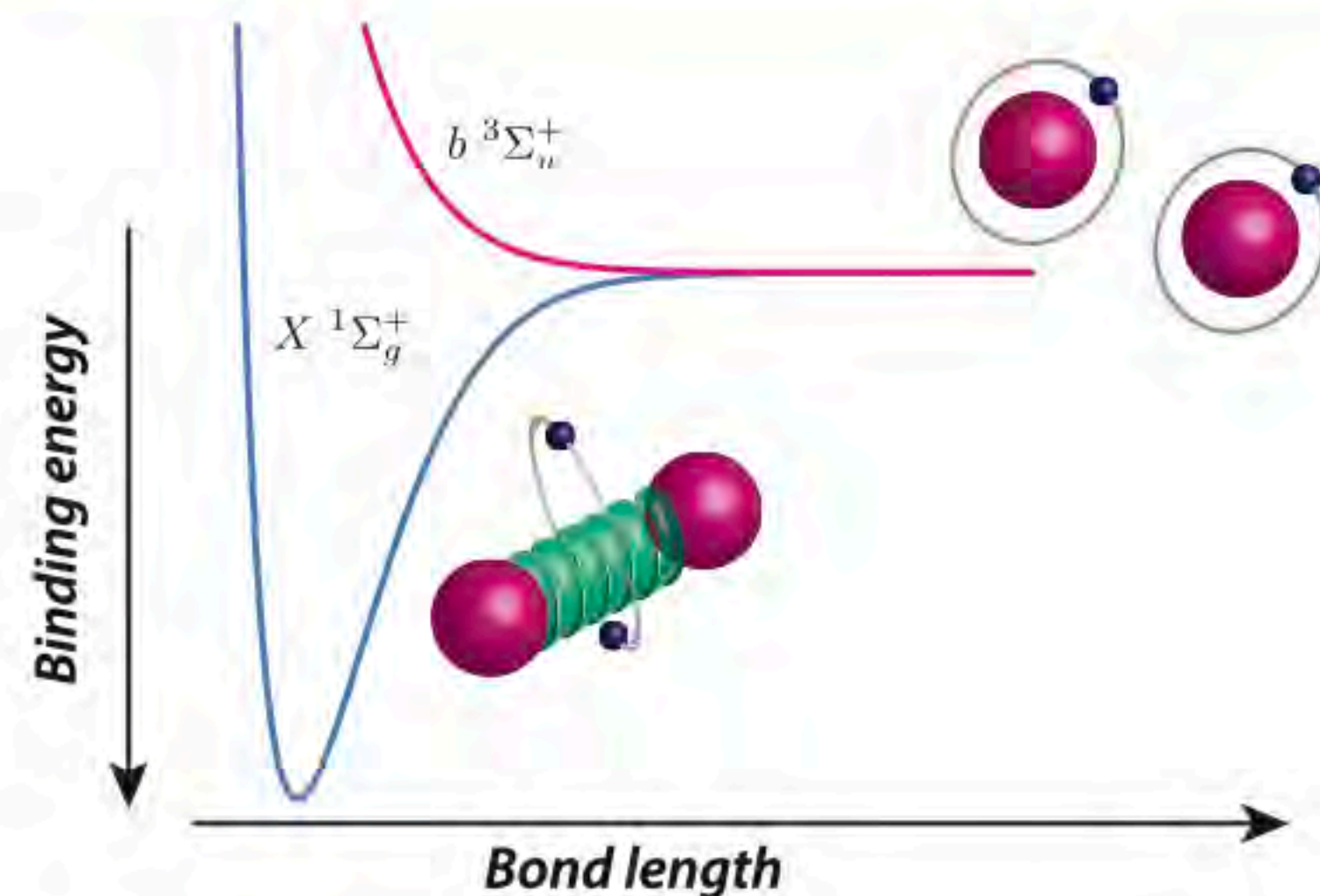
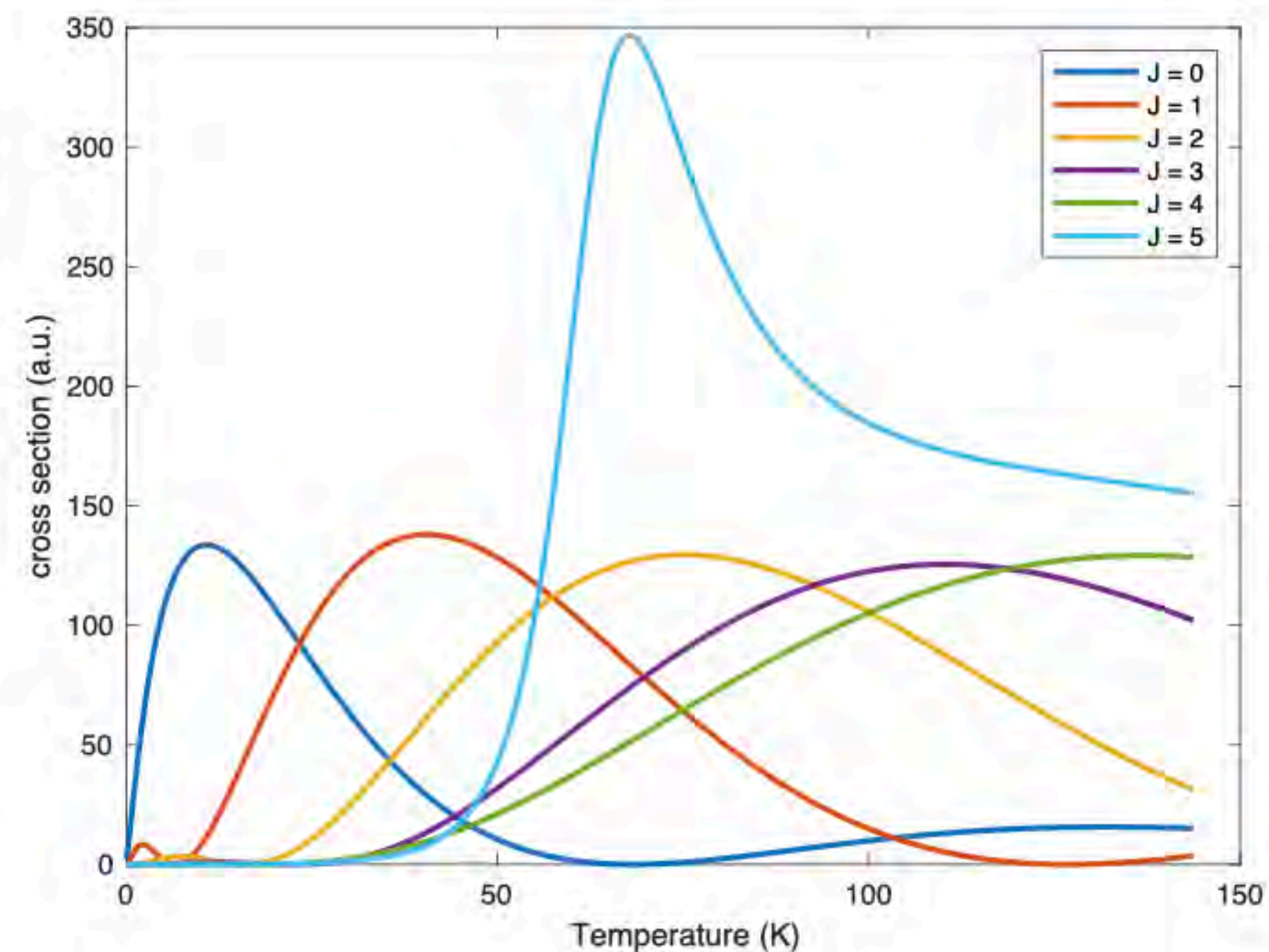
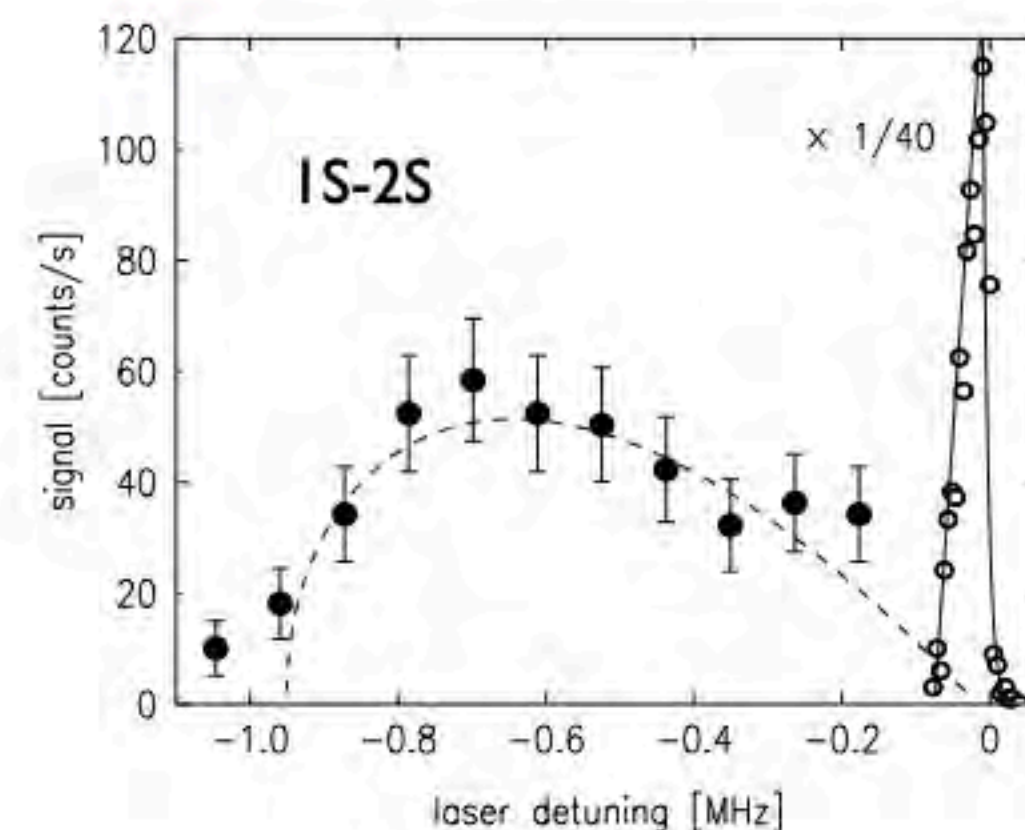
2 NOVEMBER 1998

- BEC

## Bose-Einstein Condensation of Atomic Hydrogen

Dale G. Fried, Thomas C. Killian, Lorenz Willmann, David Landhuis, Stephen C. Moss, Daniel Kleppner, and Thomas J. Greytak

Department of Physics and Center for Materials Science and Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139  
(Received 11 September 1998)



- Three-body recombination

THE JOURNAL OF CHEMICAL PHYSICS

VOLUME 50, NUMBER 12

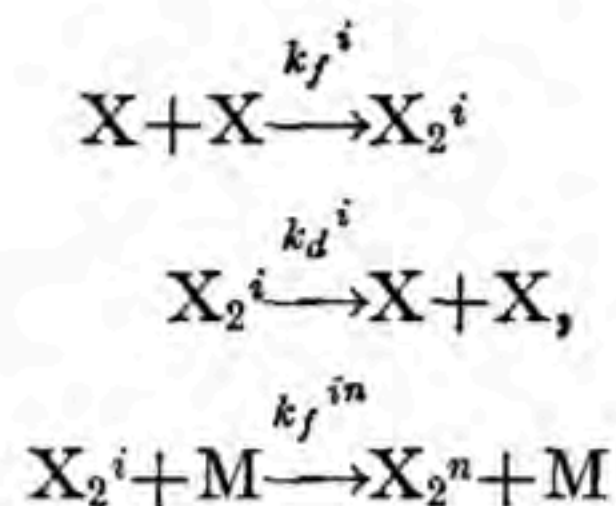
15 JUNE 1969

## Resonance Theory of Termolecular Recombination Kinetics\*: $H + H + M \rightarrow H_2 + M$

ROBERT E. ROBERTS,† R. B. BERNSTEIN, AND C. F. CURTISS

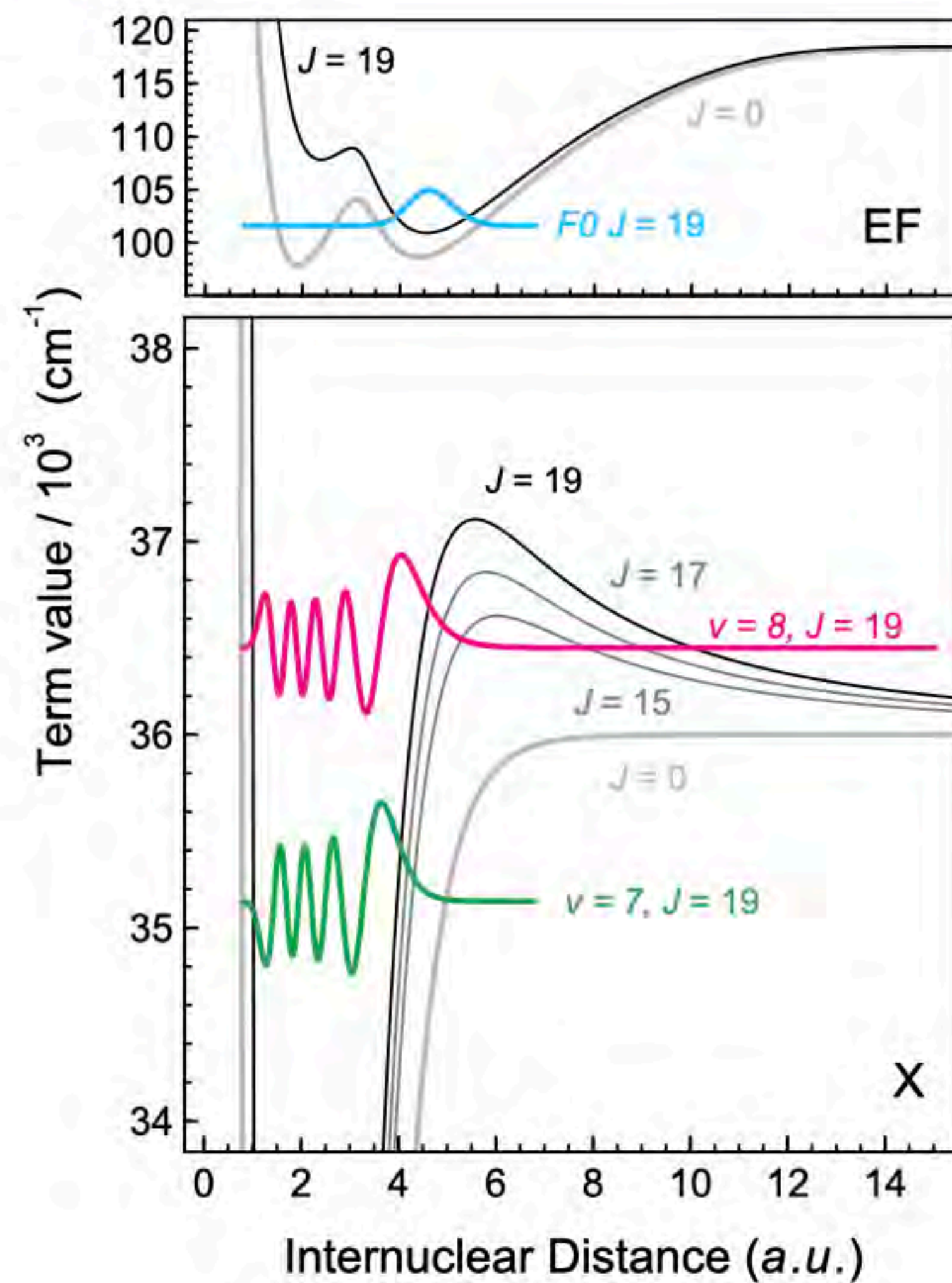
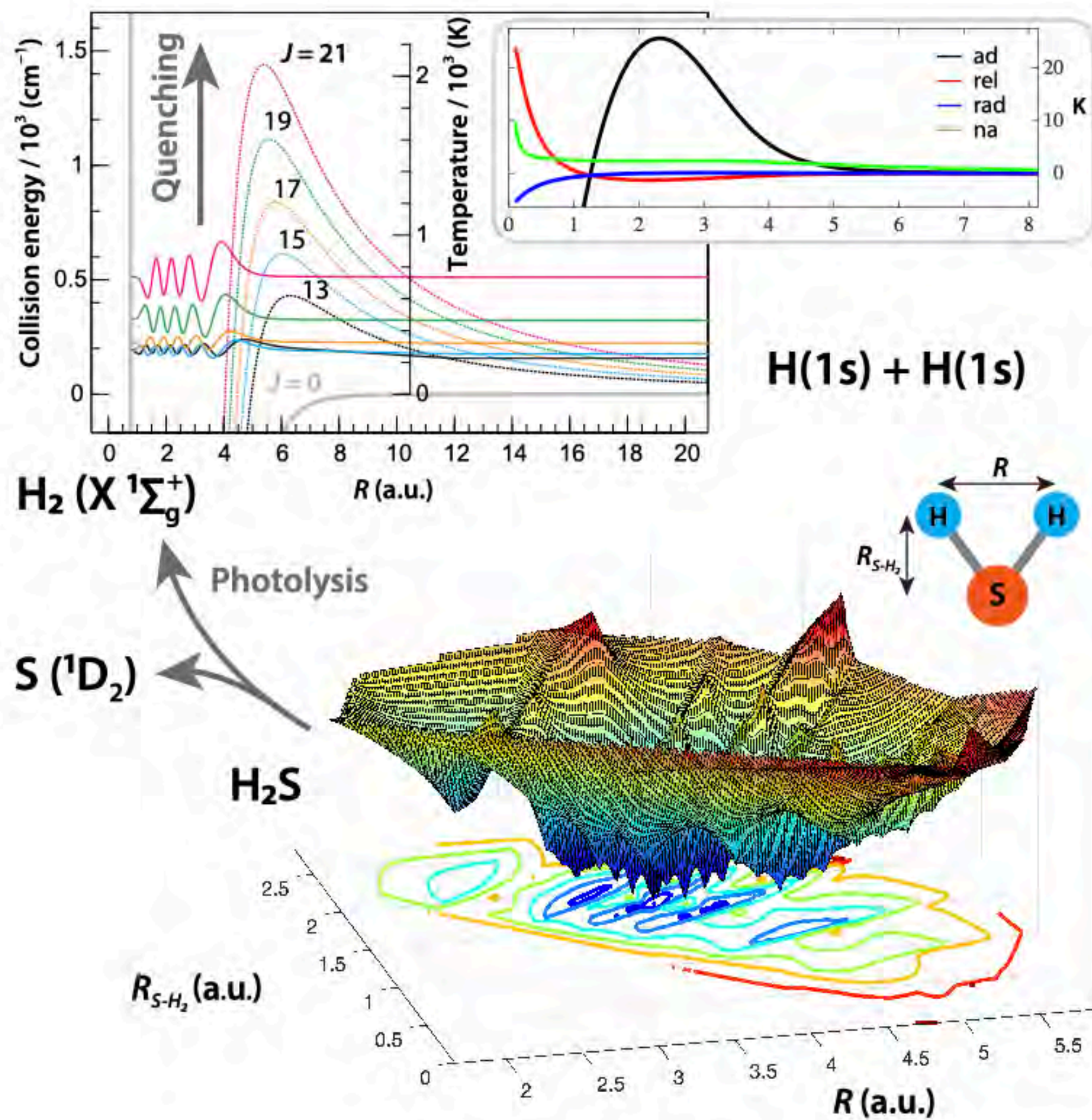
Theoretical Chemistry Institute and Chemistry Department, University of Wisconsin, Madison, Wisconsin 53706

(Received 14 February 1969)



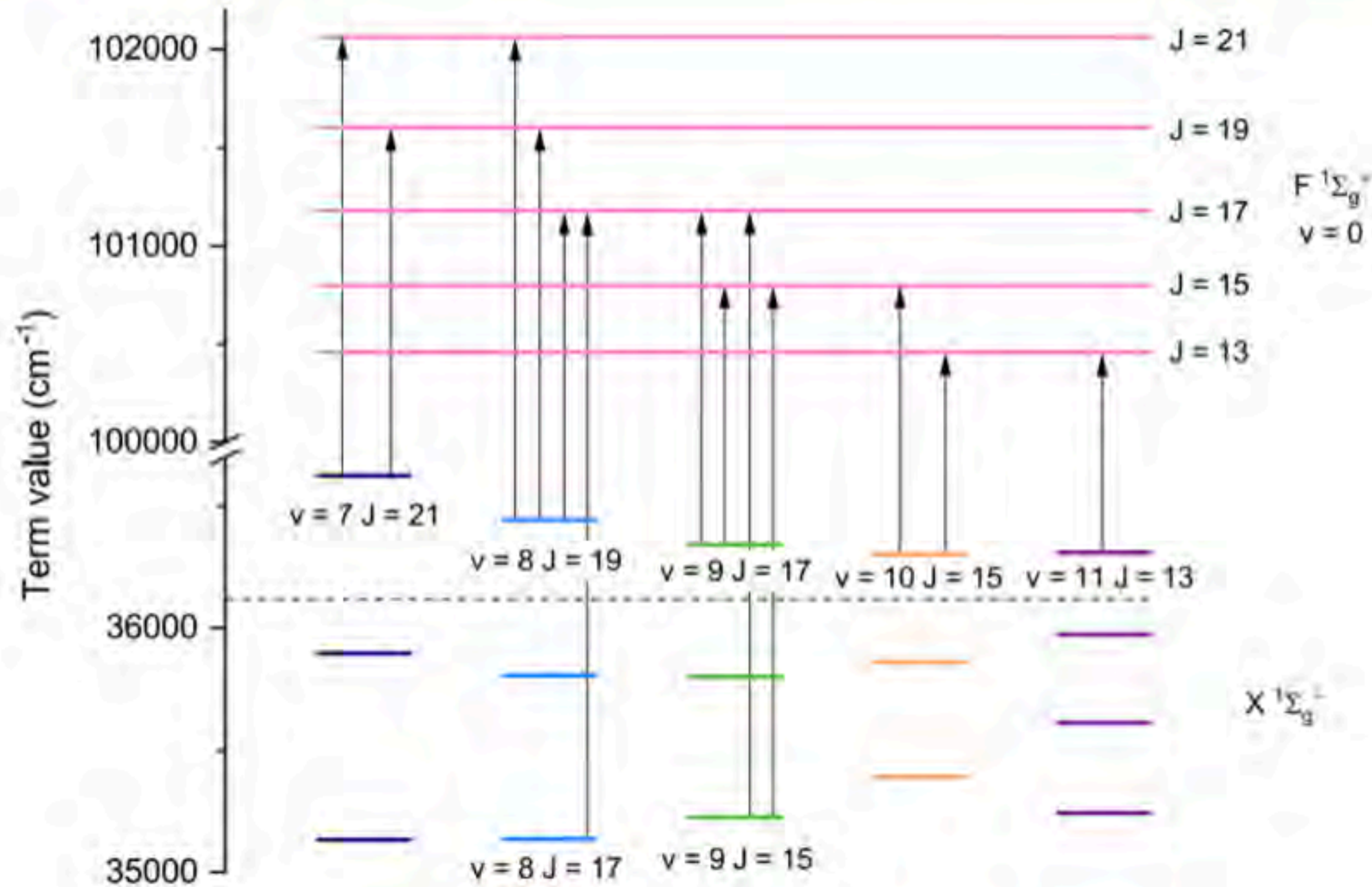


# Production of highly rovibrationally excited states





# Dissociation energies of shape resonances



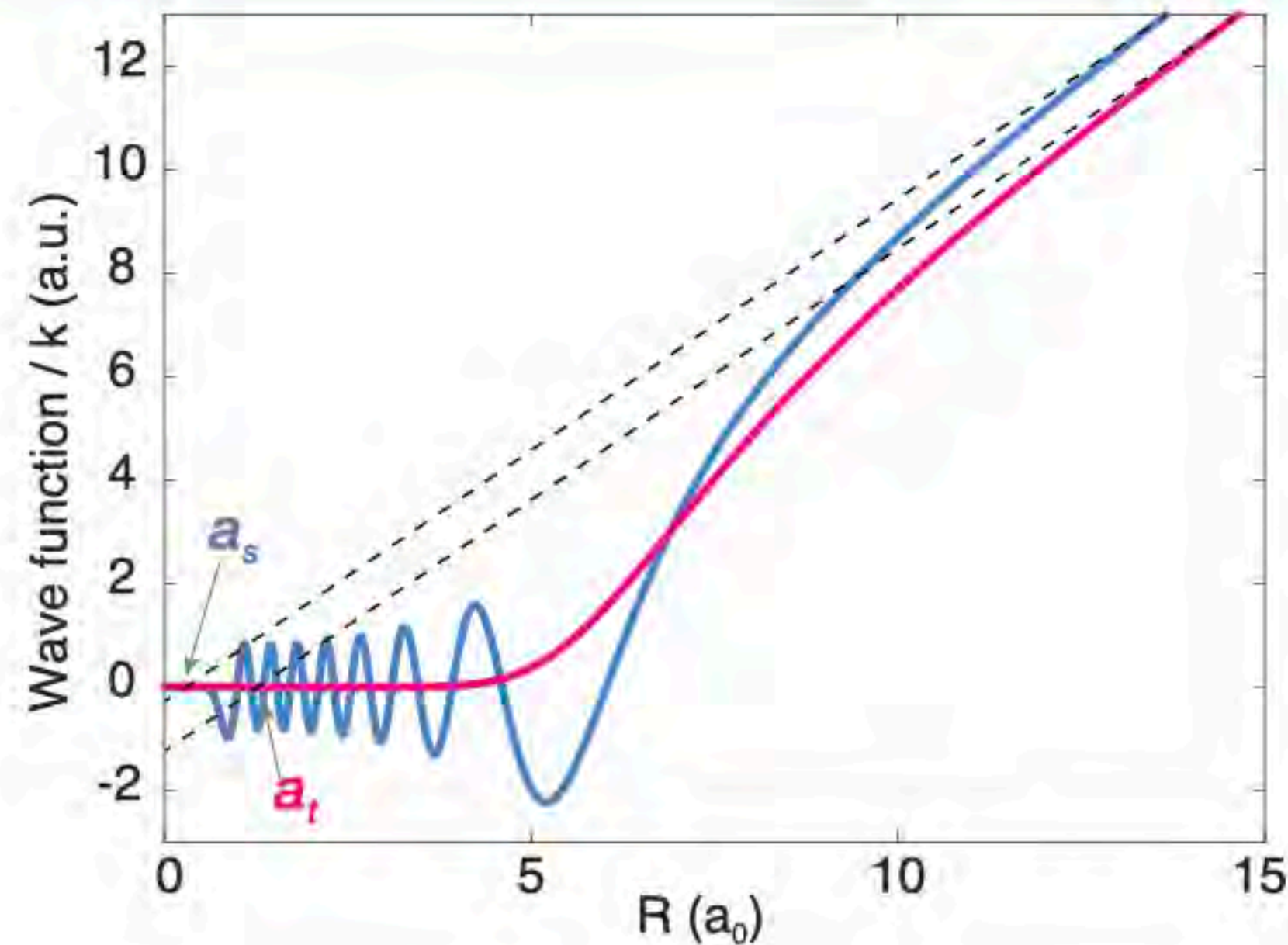
Contribution / $(v, J)$	(11,13)	(10,15)
$m\alpha^2$	-192.2683(29)	-186.1598(36)
adiabatic	3.1895	4.2857
nonadiabatic	2.6847	3.2666
$m\alpha^4$	-0.2254	-0.2945
BO	-0.2261	-0.2952
nonadiabatic	0.0007	0.0007
$m\alpha^5$	-0.0008	0.0002
$m\alpha^6$	-0.0001	-0.0001
Total	-192.4961(29)	-186.4555(36)
$\Gamma_{\text{FWHM}}$ (kHz)	$1 \times 10^5$	73.6
Lifetime (s)	$1.59 \times 10^{-9}$	$2.16 \times 10^{-6}$

➔ Determination of ro-vibrational intervals

$(v, J)$	$T_{\text{rel}}^{\text{exp}}$	$T_{\text{rel}}^{\text{calc}}$	$\Delta T$
(7,21)*	319.4772 (28)	319.4768 (9)	0.0004 (29)
(8,17)	-1168.9200 (25)	-1168.9200 (12)	0.0000 (28)
(8,19)*	140.9748 (25)	140.9750 (7)	-0.0002 (26)
(9,15)	-1078.6659 (21)	-1078.6671 (11)	0.0012 (24)
(9,17)*	38.4872 (30)	38.4869 (4)	0.0003 (30)
(10,15)*	0	0	0
(11,13)*	6.041 (28)	6.0404 (6)	0.001 (28)



# Determination of the scattering length



$$a = \begin{cases} 0.5699_{697}^{700} a_0 & \text{BO} \\ 0.4160_{58}^{61} a_0 & \text{AD} \\ 0.2572_{68}^{76} a_0 & \text{NA} \\ 0.2735_{31}^{39} a_0 & \text{NA, } m\alpha^4, m\alpha^5, m\alpha^6. \end{cases}$$

Using the atomic mass:  $a_{\text{NA}} = 0.2651 a_0$  3% difference

A calculation for the triplet PES gives:  $a_t = 1.331 a_0$   
using Wolniewicz, PRA 61, 042705, 2000.

Calculation	Mass ( $m_e$ )	$a$
atomic mass [5]	1837.1527	0.3006
nuclear mass [5]	1836.1527	0.4503
Bunker et al. [7]	1837.0777	0.3121
new mass	1837.0318	0.3191
Williams & Julienne [4]		0.3159
variable mass [8]		0.4042
close coupling [2]		0.564

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PRL 98, 043004 (2007)

PHYSICAL REVIEW LETTERS

week ending  
26 JANUARY 2007

## Cold Collision Frequency Shift in Two-Dimensional Atomic Hydrogen

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(Received 2 October 2006; published 24 January 2007)

We report a measurement of the cold collision frequency shift in atomic hydrogen gas adsorbed on the surface of superfluid  $^4\text{He}$  at  $T \lesssim 90$  mK. Using two-photon electron and nuclear magnetic resonance in 4.6 T field we separate the resonance line shifts due to the dipolar and exchange interactions, both proportional to surface density  $\sigma$ . We find the clock shift  $\Delta\nu_c = -1.0(1) \times 10^{-7} \text{ Hz cm}^{-2} \times \sigma$ , which is about 100 times smaller than the value predicted by the mean field theory and known scattering lengths in the three-dimensional case.



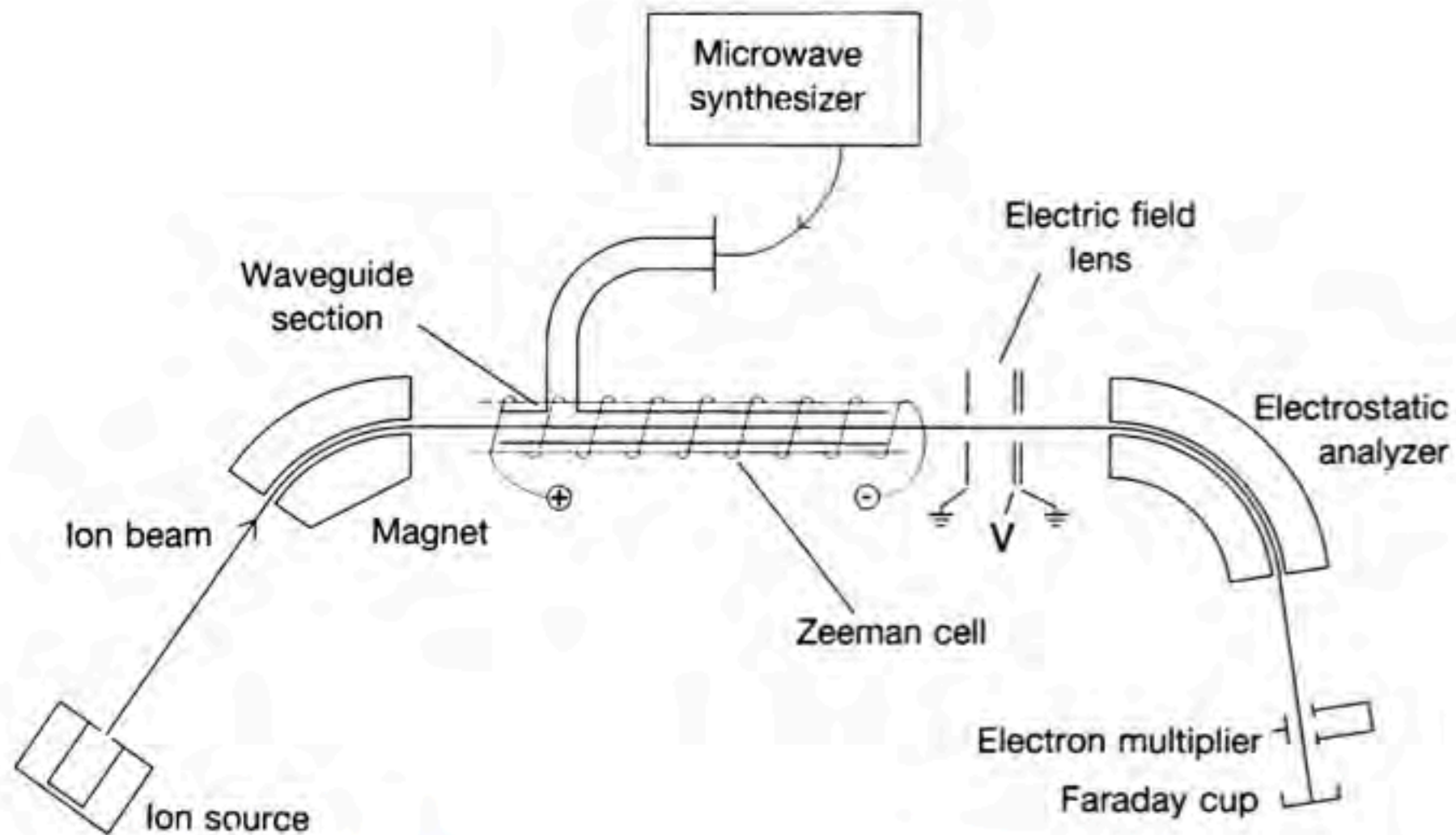
# Weakly-bound states of hydrogen molecular ions

SCIENCE • VOL. 274 • 22 NOVEMBER 1996

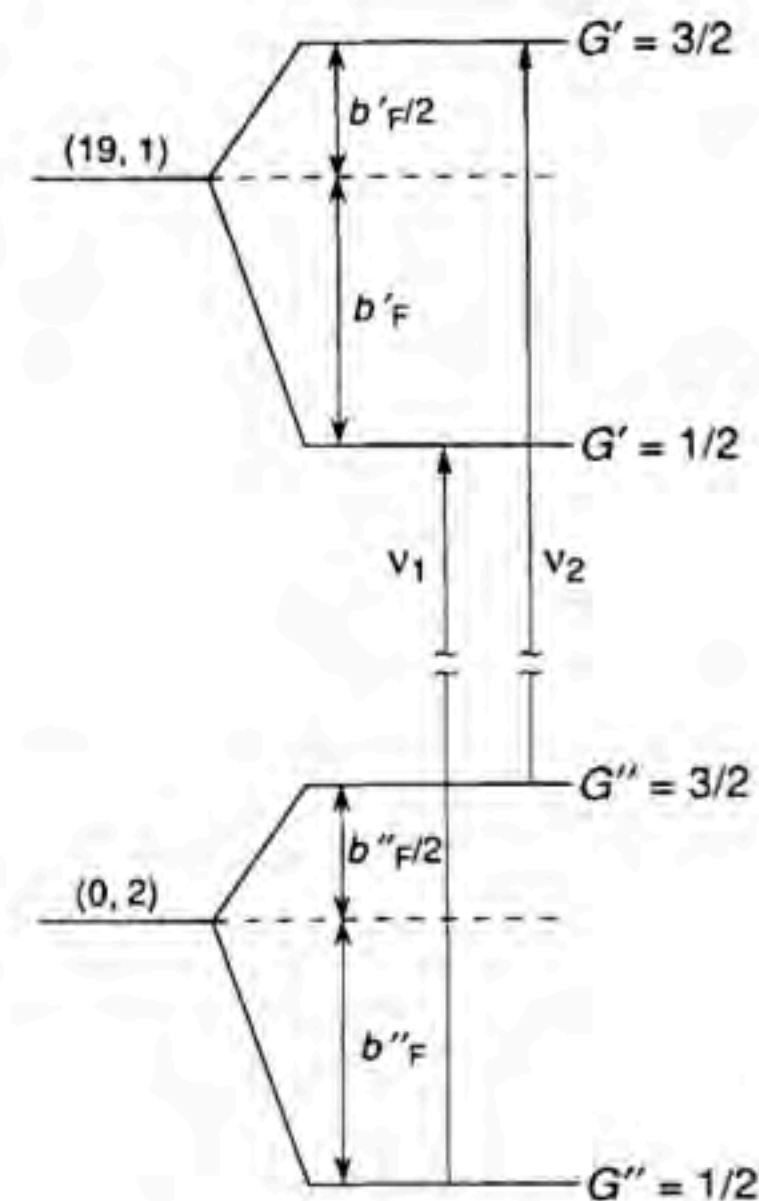
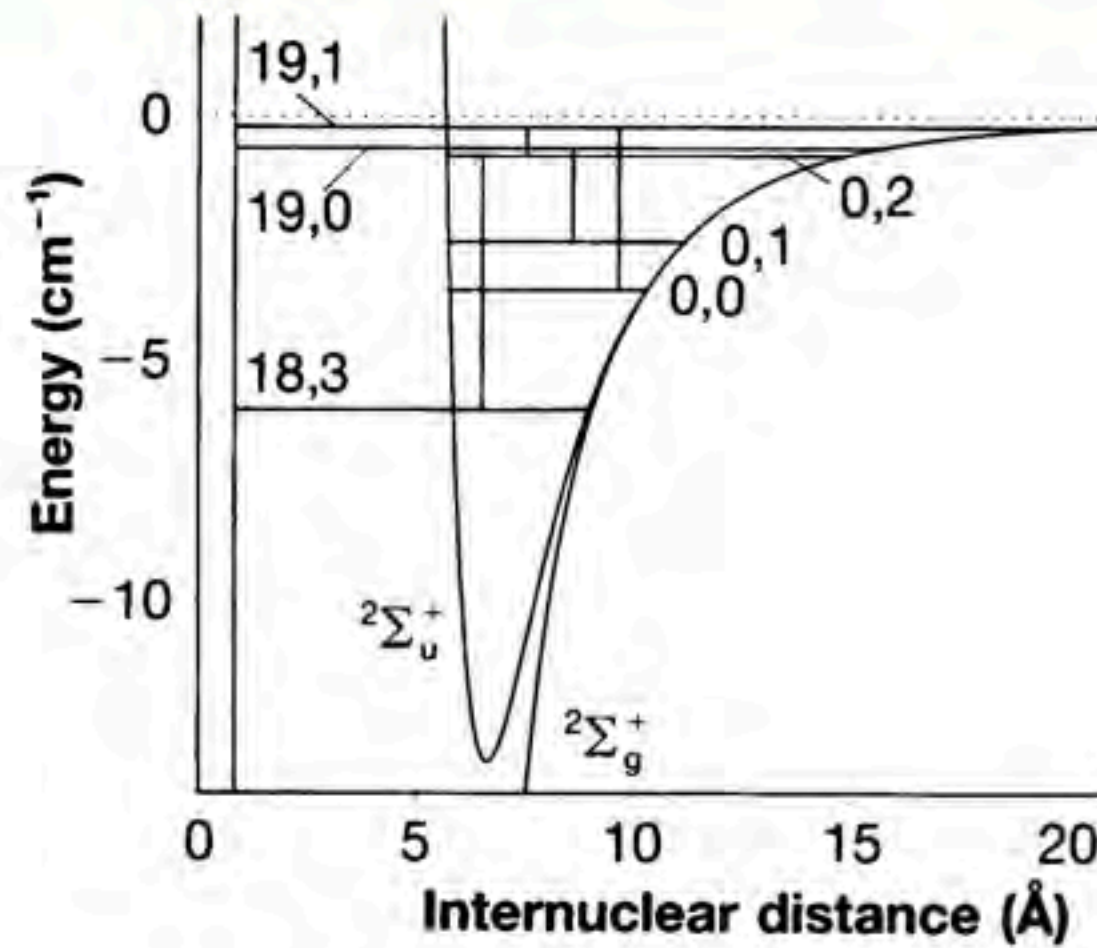
ARTICLE

## Microwave Spectroscopy at the Dissociation Limit

Alan Carrington

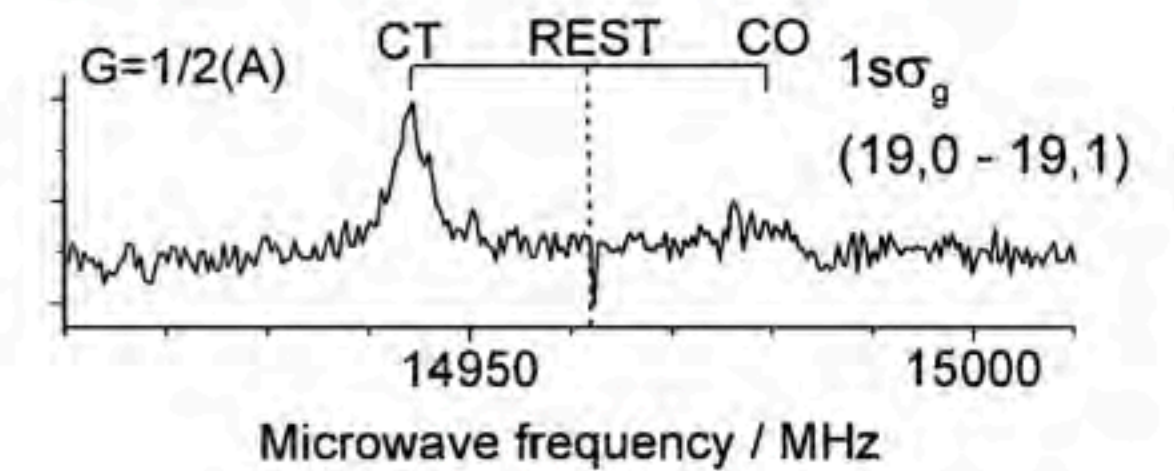


600 kHz linewidth due to transit time broadening

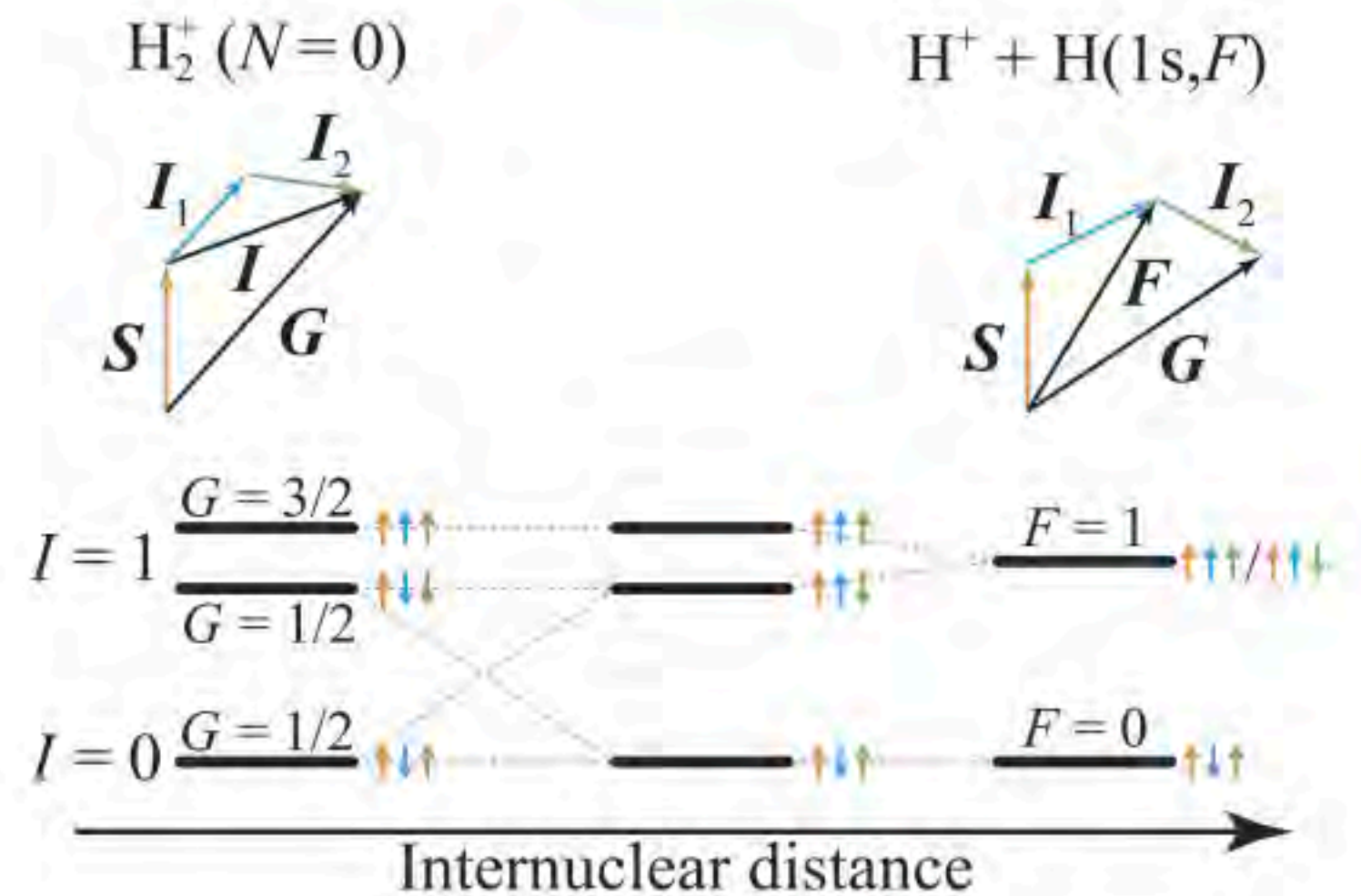


Mixing of ortho and para (= gerade and ungerade)!

Critchley *et al.*, PRL **86**, 1725 (2001)

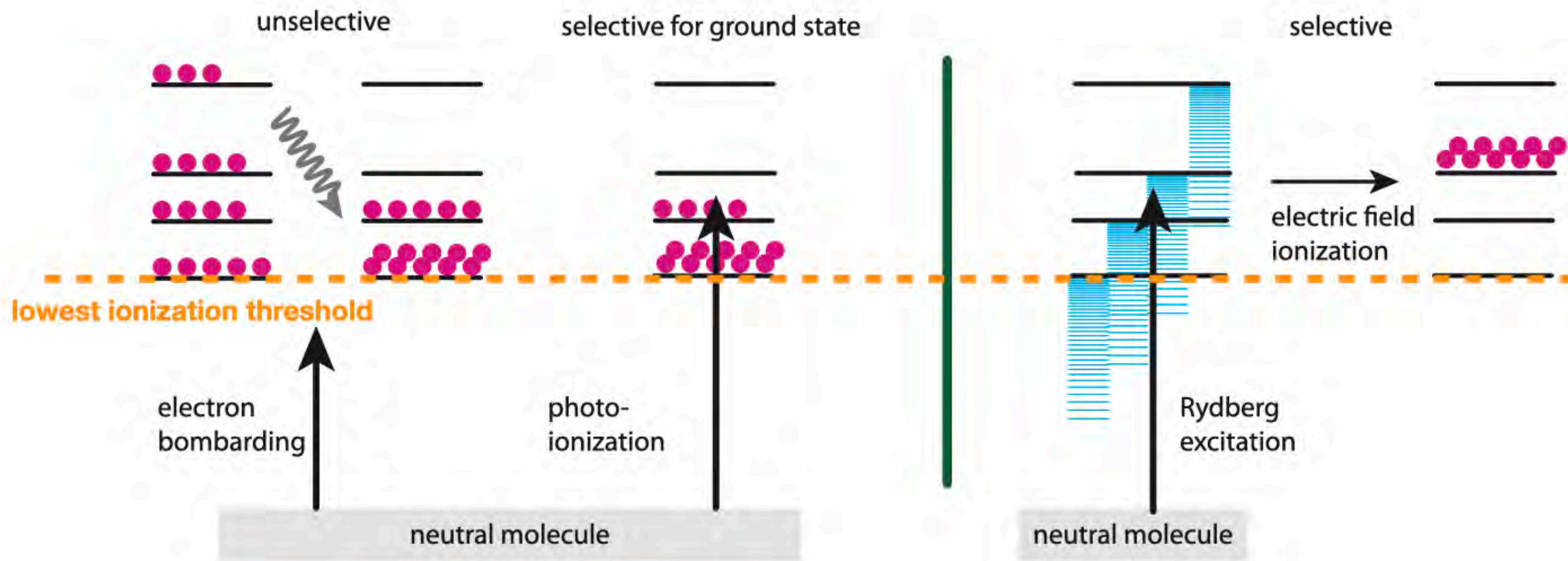


550 kHz

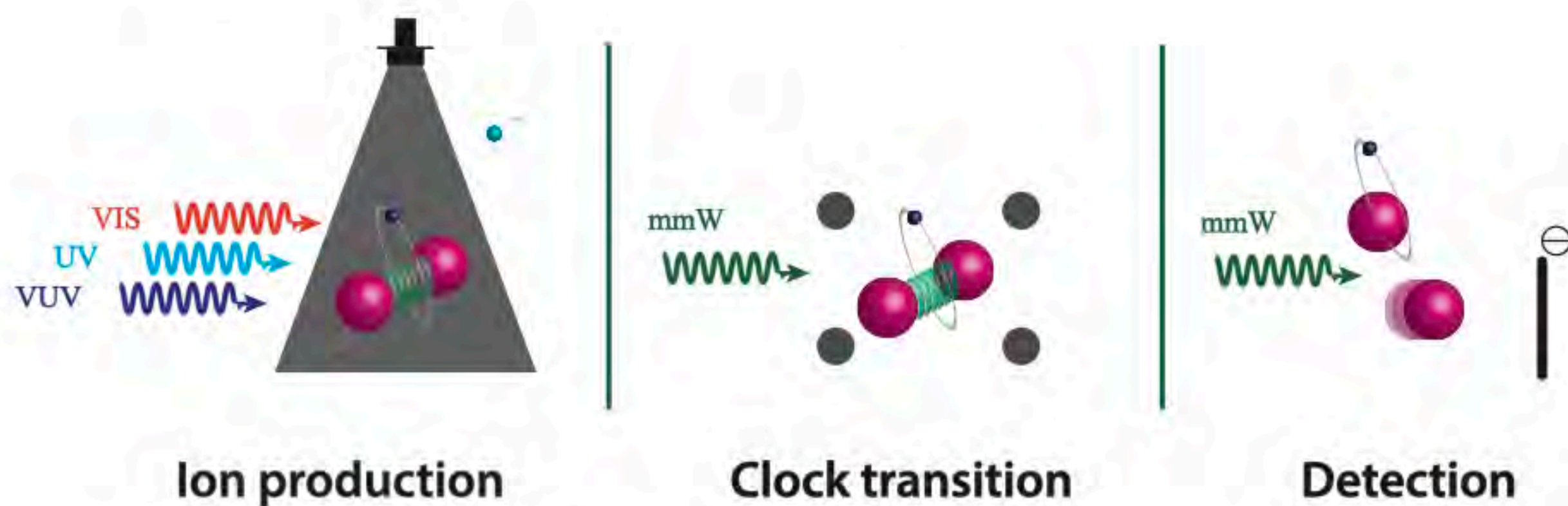




# Weakly-bound states of hydrogen molecular ions



*Works only for polar molecules!*

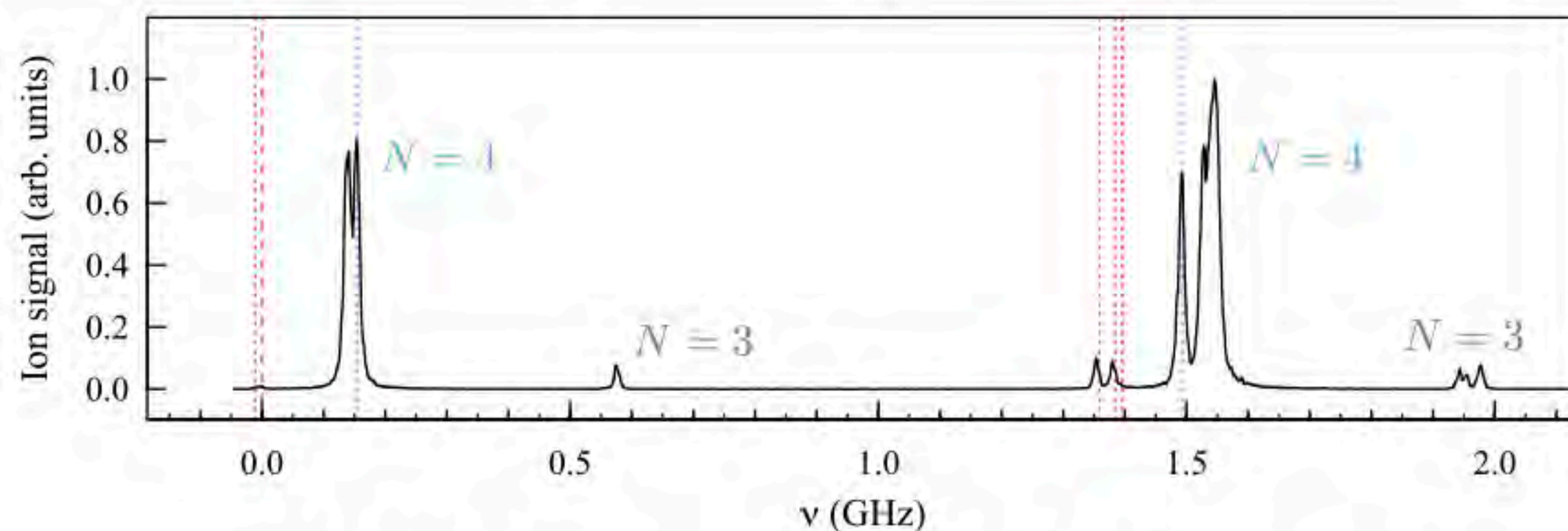
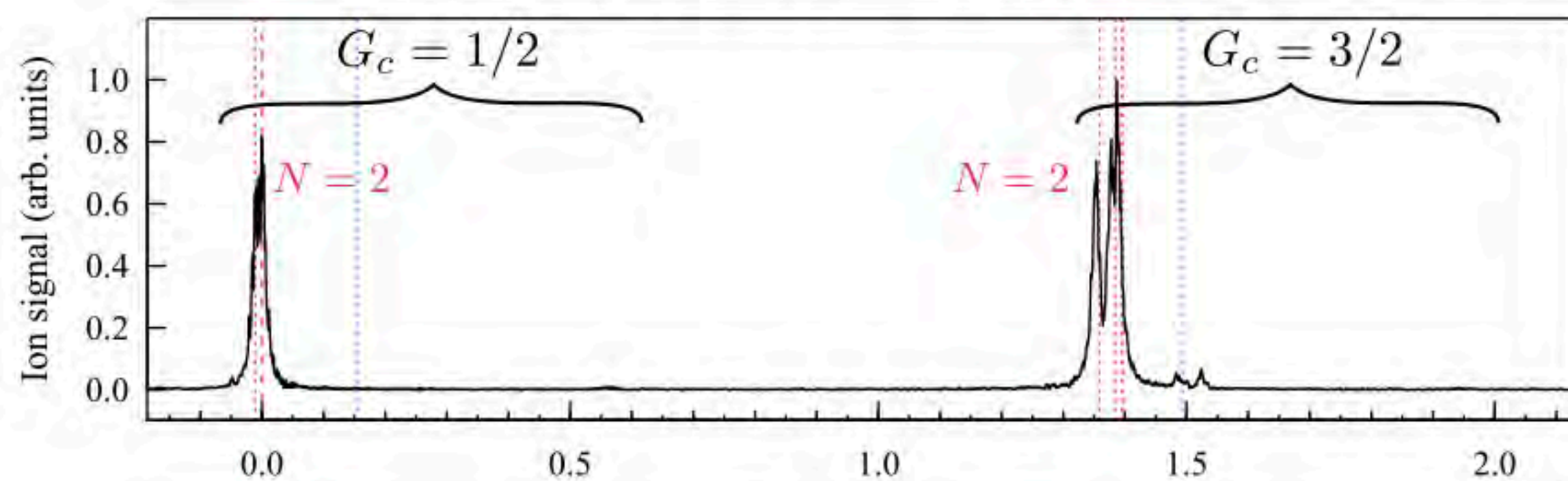




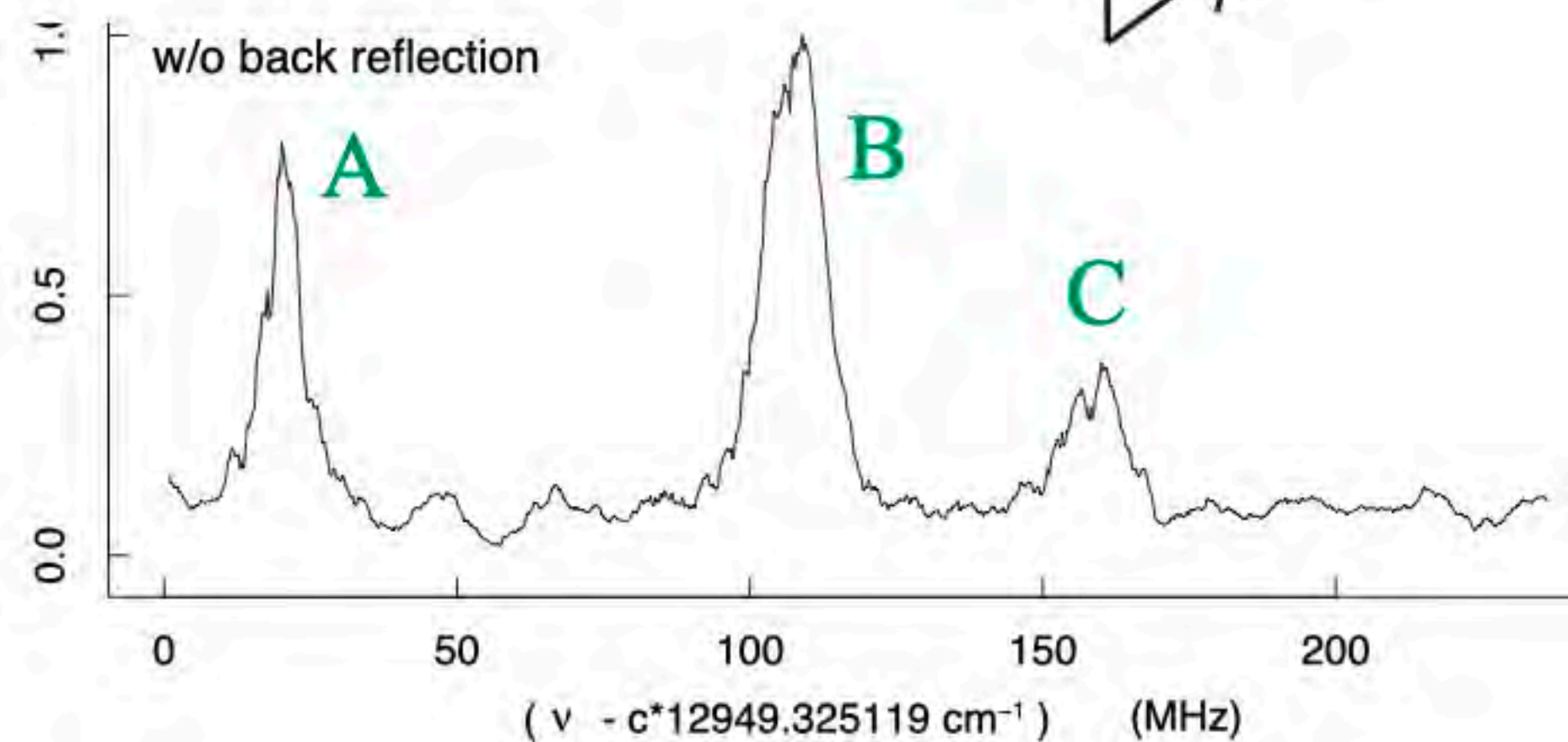
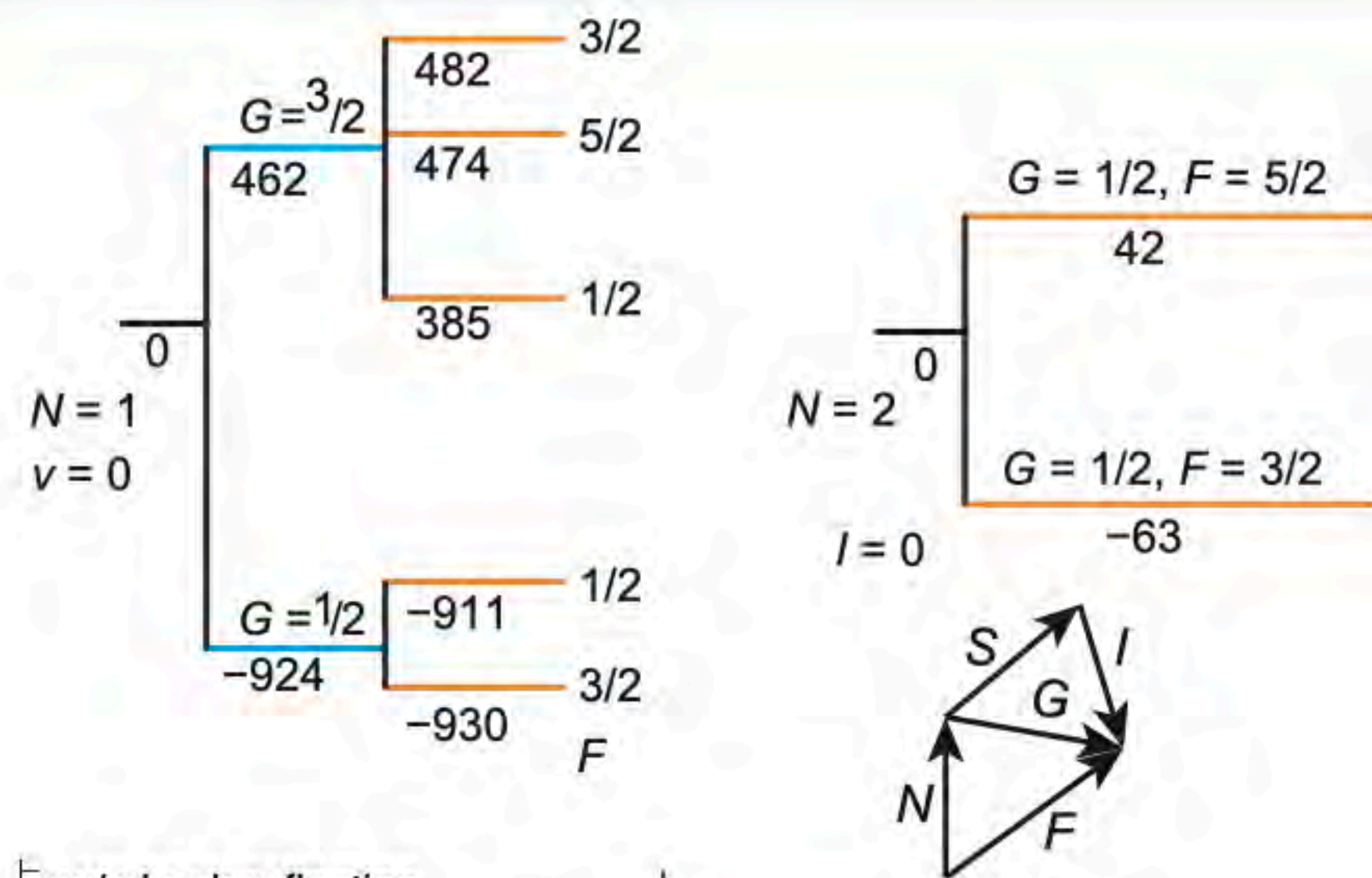
# Molecular Rydberg states with resolved hfs

➔ Excitation of f Rydberg states (small exchange interaction)

$GK(\nu = 1, N = 1) \rightarrow 56f1(\nu^+ = 0)$



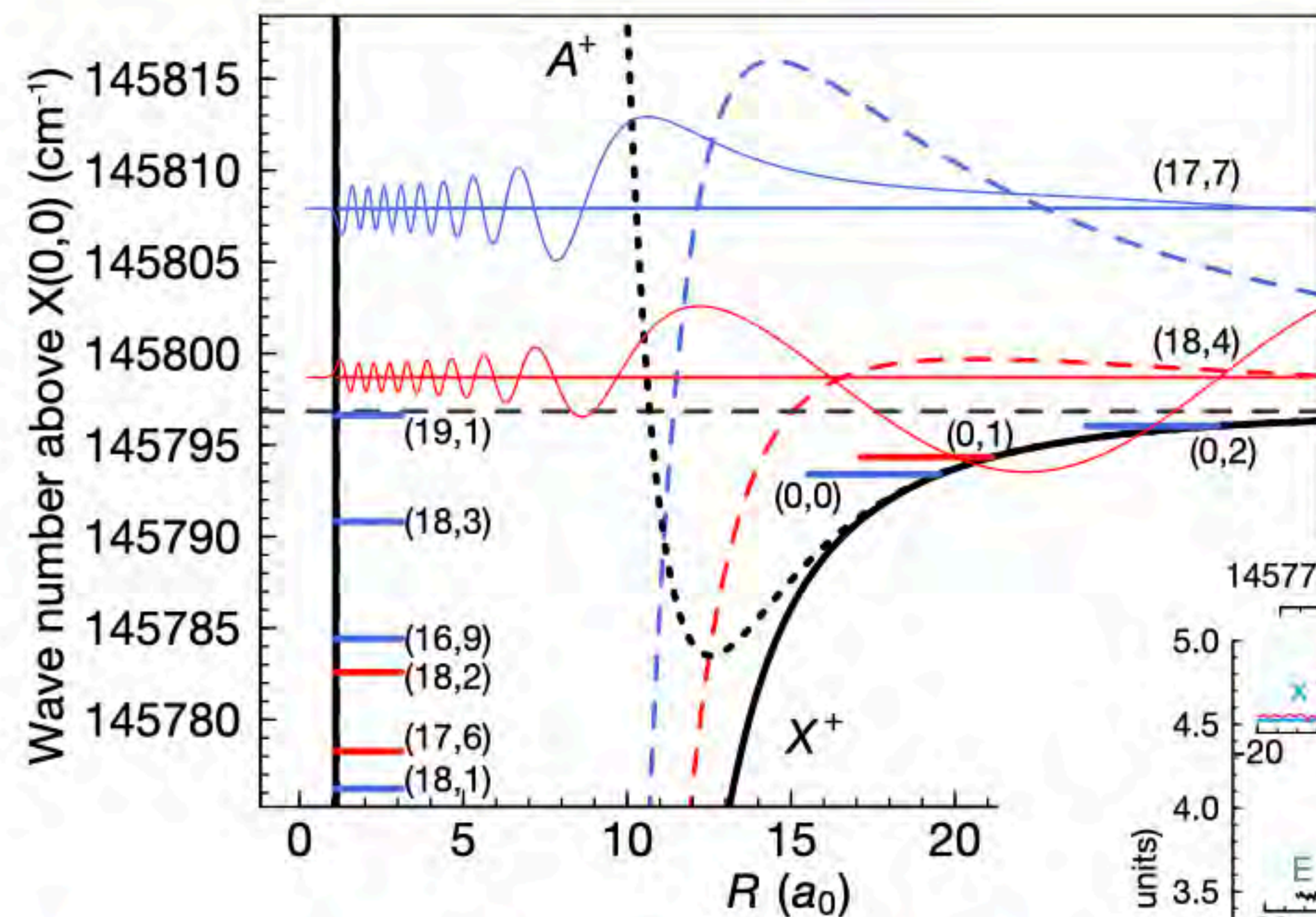
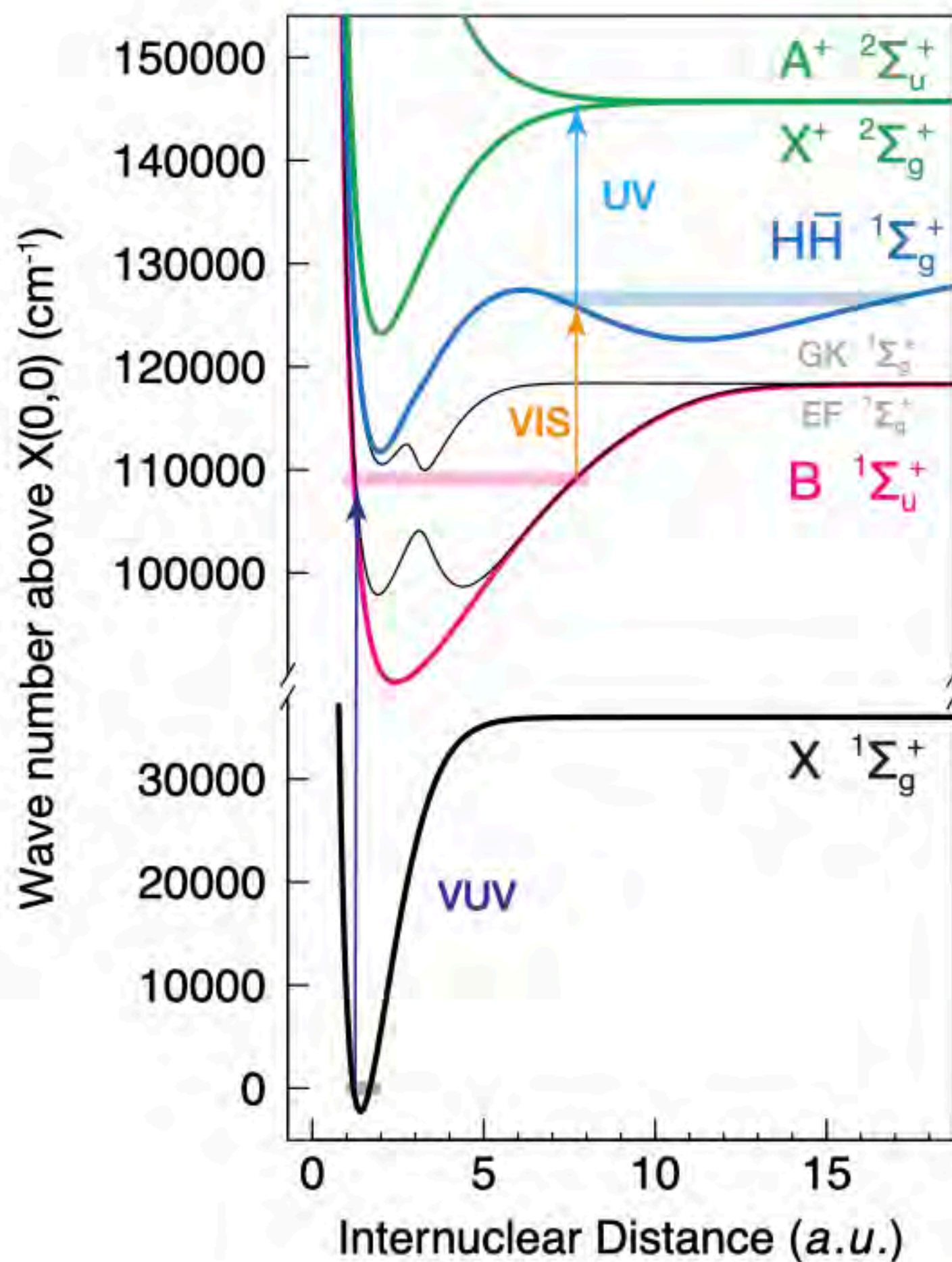
$GK(\nu = 0, N = 3) \rightarrow 56f1(\nu^+ = 0)$



$GK(0,0) \rightarrow 90f2 (J=1)$

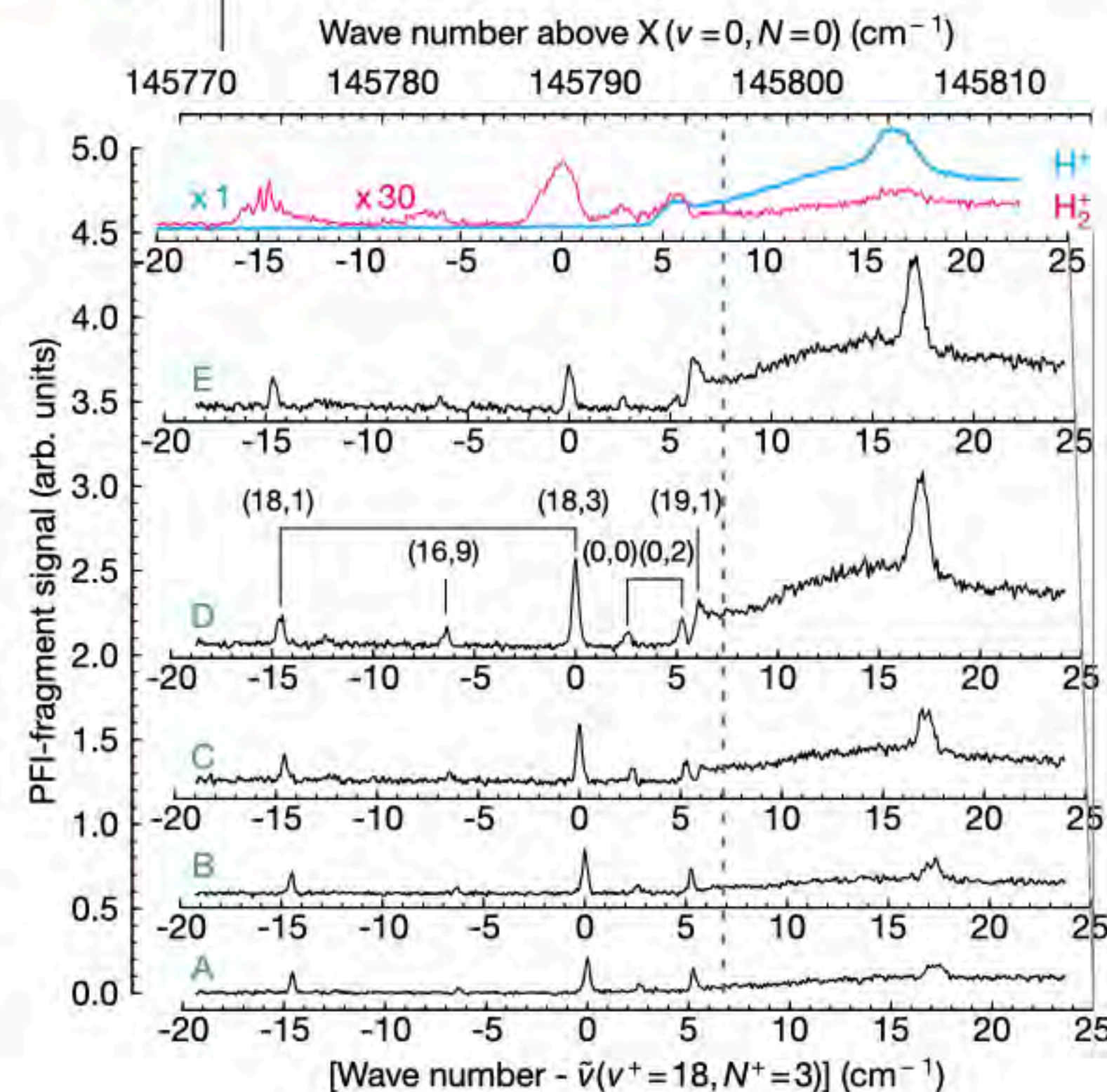


# Weakly-bound states of hydrogen molecular ions



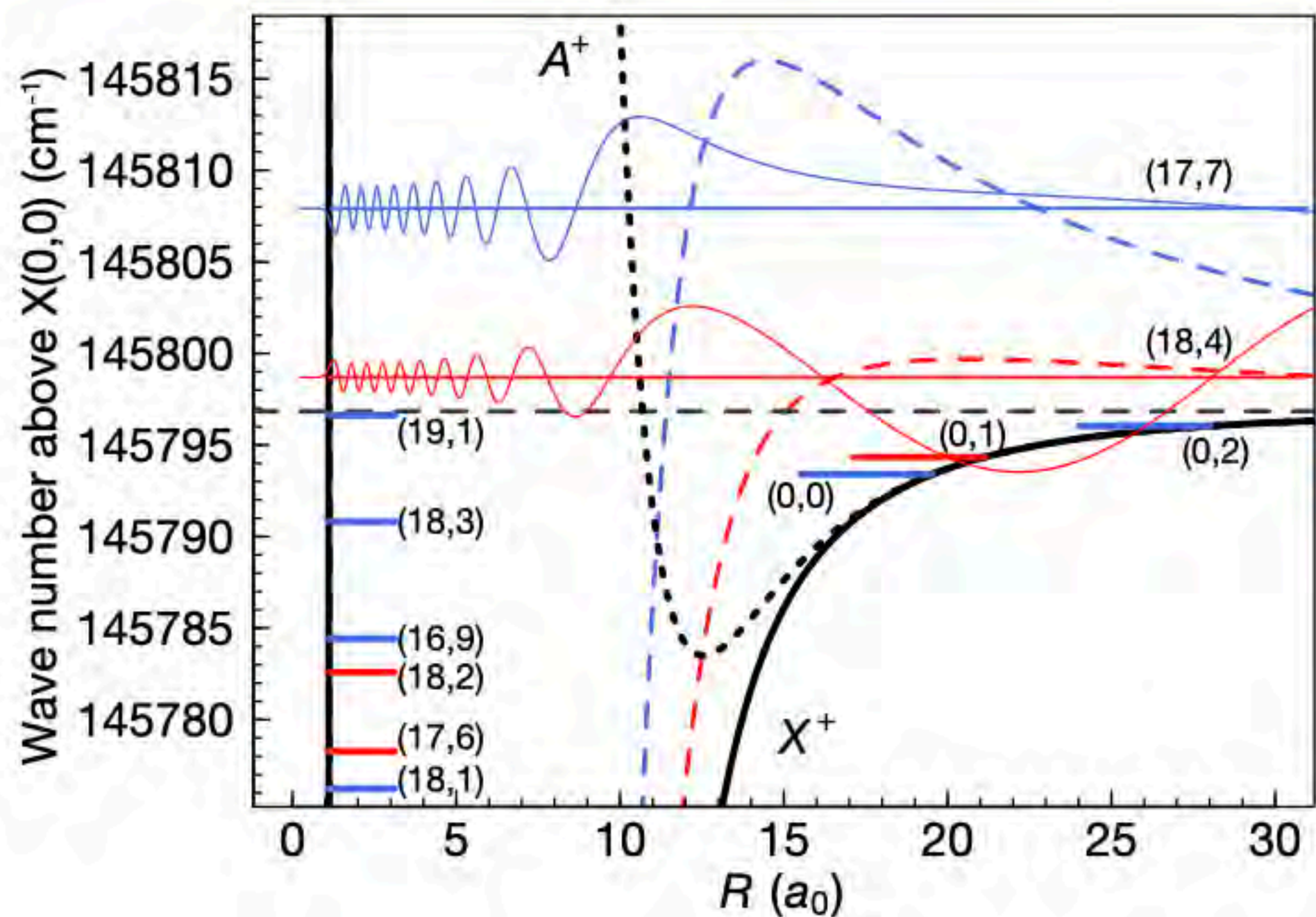
Beyer and Merkt, *Phys. Rev. Lett.* **116**, 093001 (2016)  
 Beyer and Merkt, *J. Mol. Spectrosc.* **330**, 147 (2016)

- ➔ Enhanced sensitivity for  $m_p/m_e$ :  $K \sim 65$
- ➔ Enhanced sensitivity for g/u-mixing (= ortho/para mixing)
- ➔ Measurement of pure rotational transitions

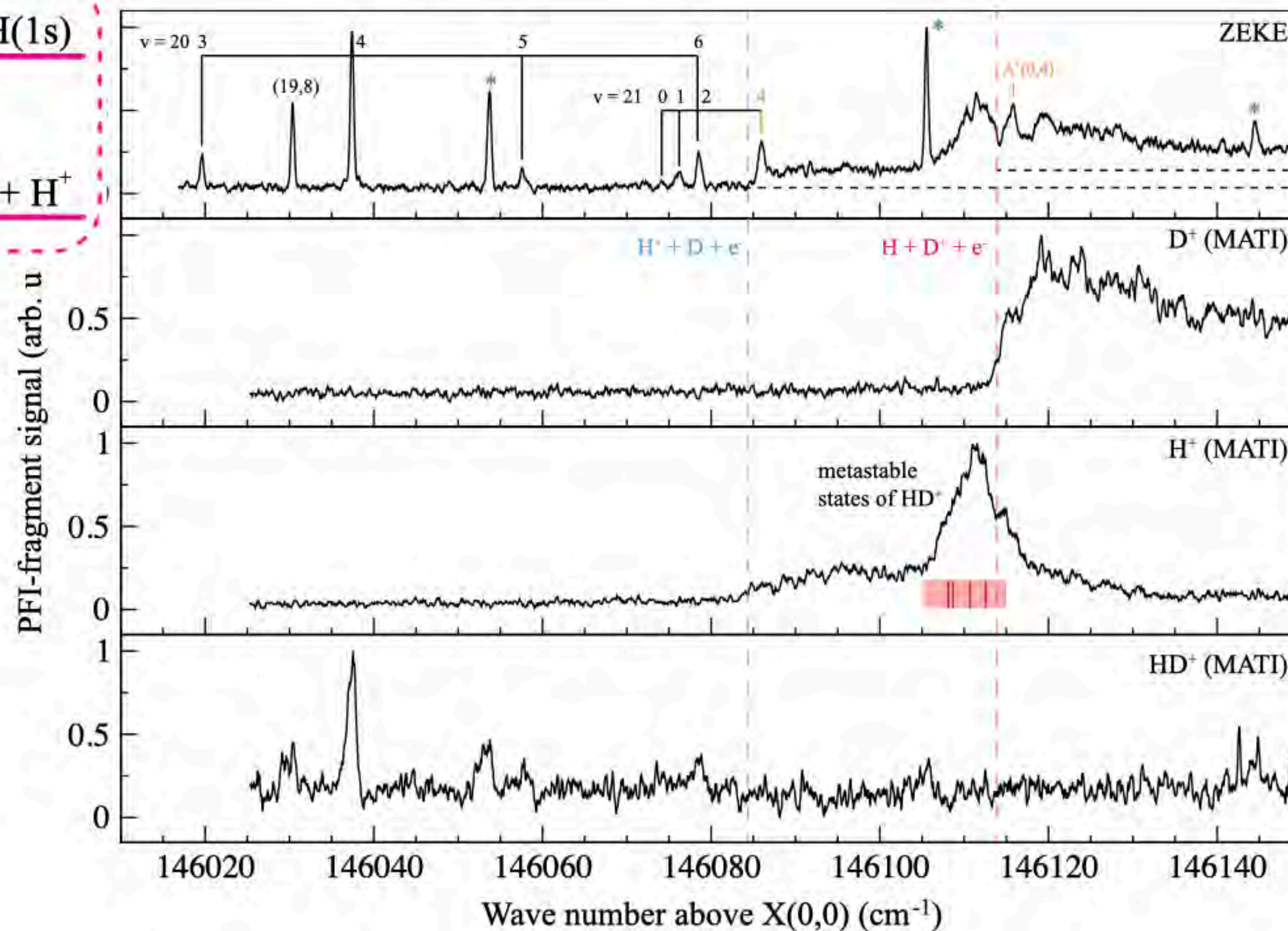




# Weakly-bound states and Feshbach resonances in HD<sup>+</sup>

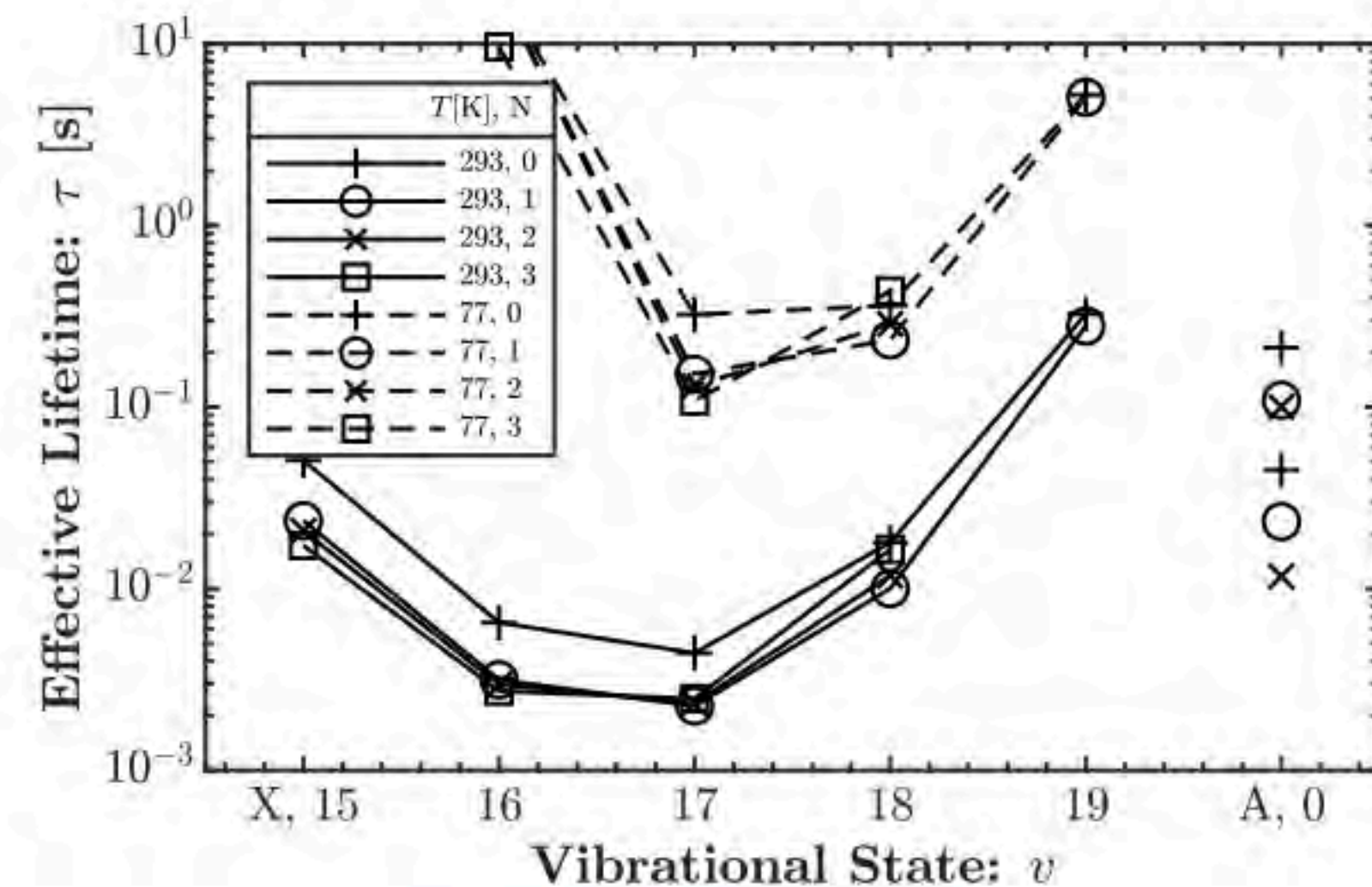
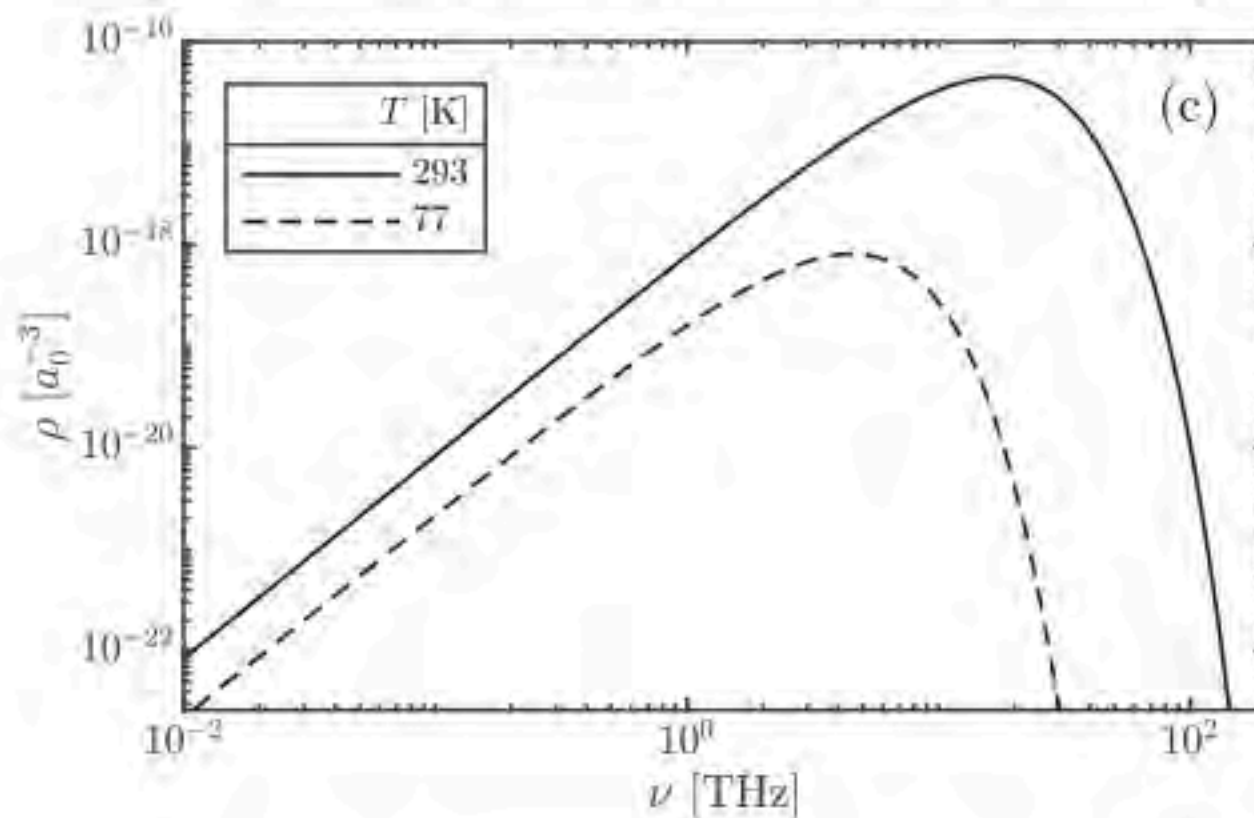
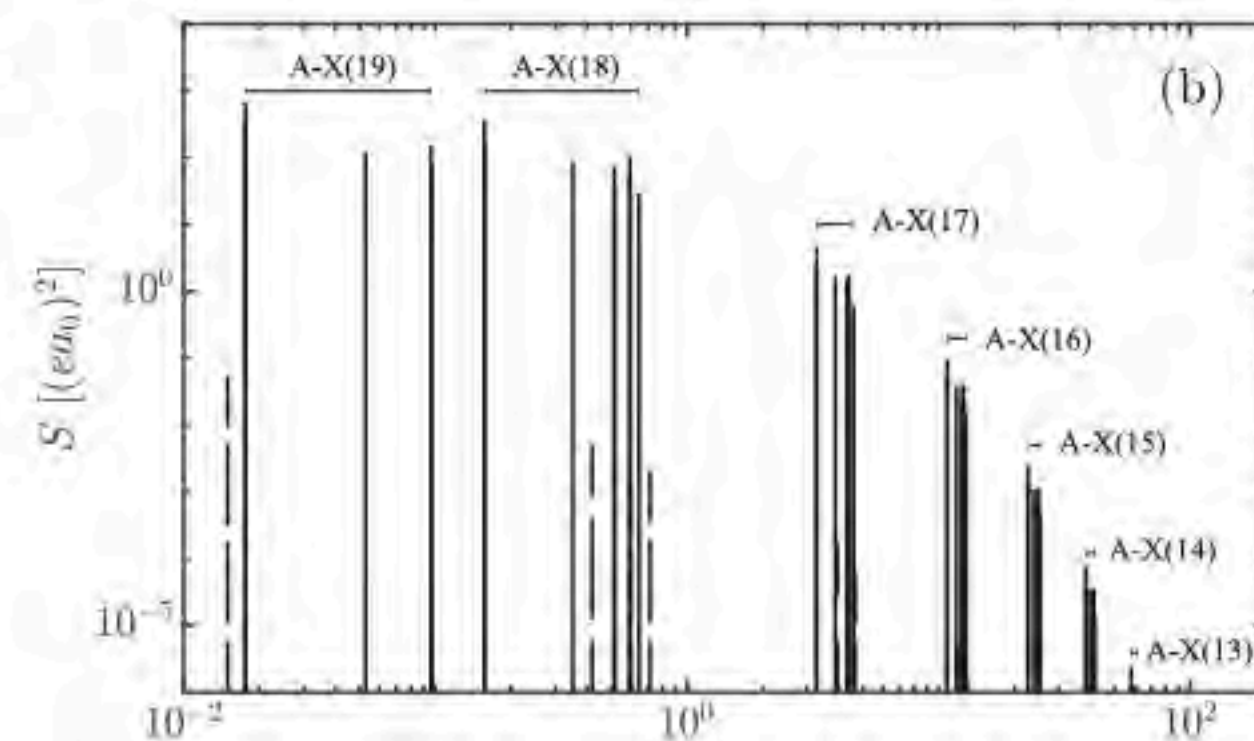
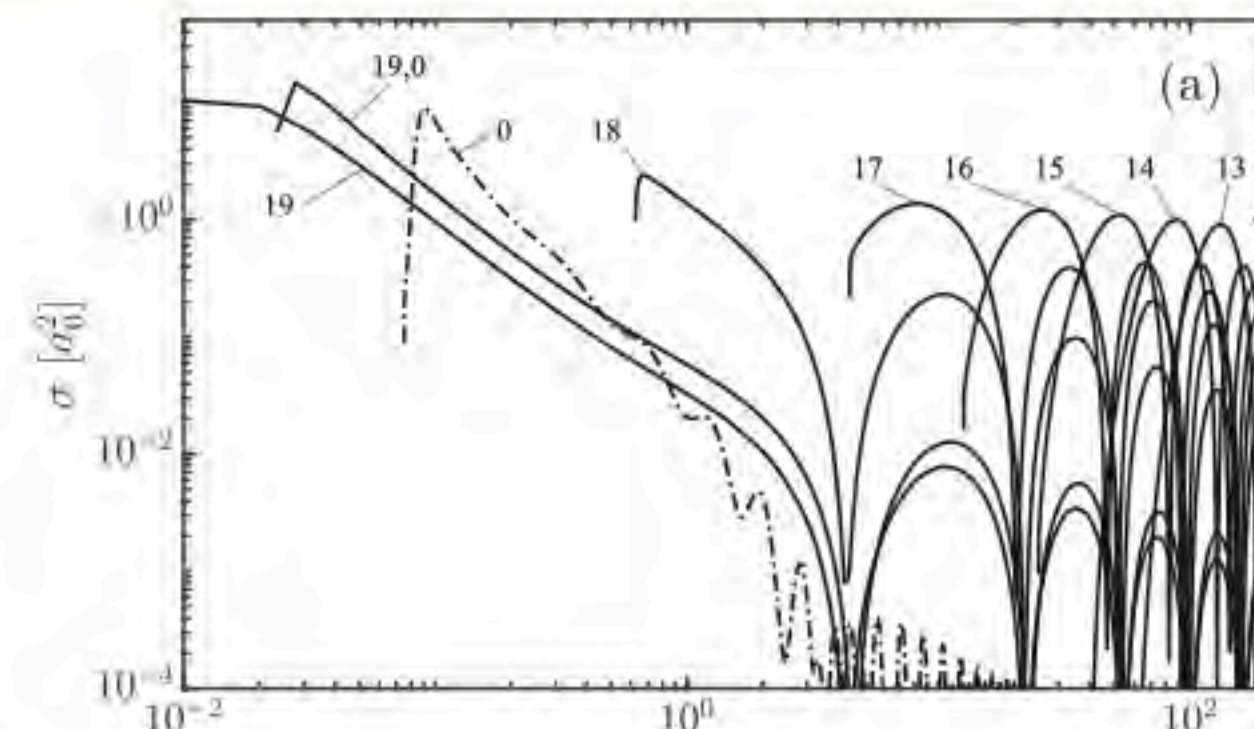
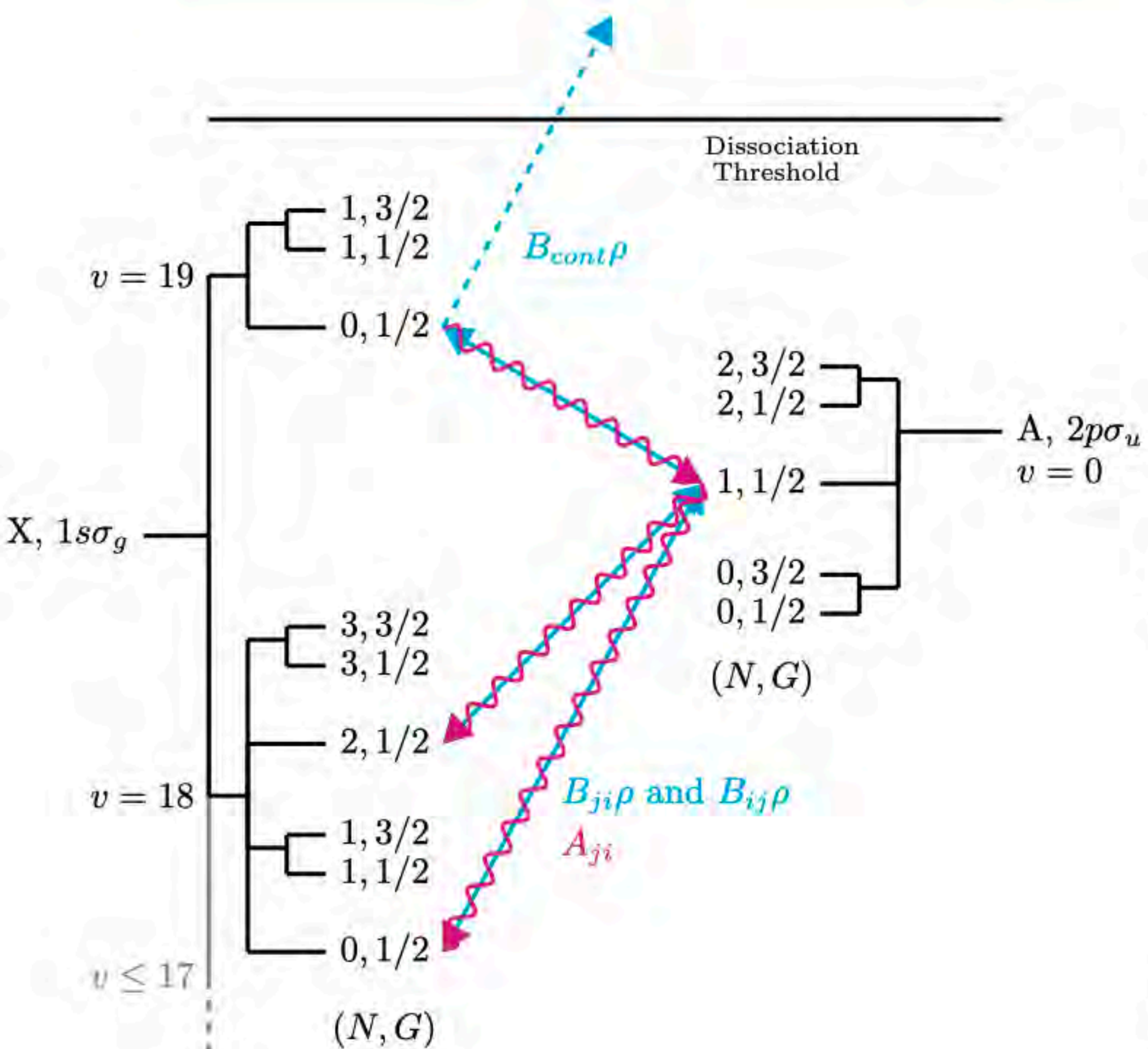


Beyer and Merkt, *Mol. Phys.* e2048108 (2022)





# Weakly-bound states: blackbody redistribution



➔ > 1 ms lifetime of

Work by:

D. Ochoa, MB



# Summary



- *Most accurate determination of the dissociation energy of a molecule.*
- *Experimental and theoretical accuracy sufficient to see the effect of the charge radius.*
- *Experimental uncertainty for  $D_0$  on the way towards  $\sim 100$  kHz.*
- *Rydberg states can be used to create molecular ions selectively.*
- *Measurements of weakly-bound molecular ions as  $m_p/m_e$  and ortho/para mixing probe*



**Christian Jungen  
(CNRS, Orsay)**



**Nicolas Hölsch**



**Frédéric Merkt**



@ **ETH**

**Ioanna Doran**

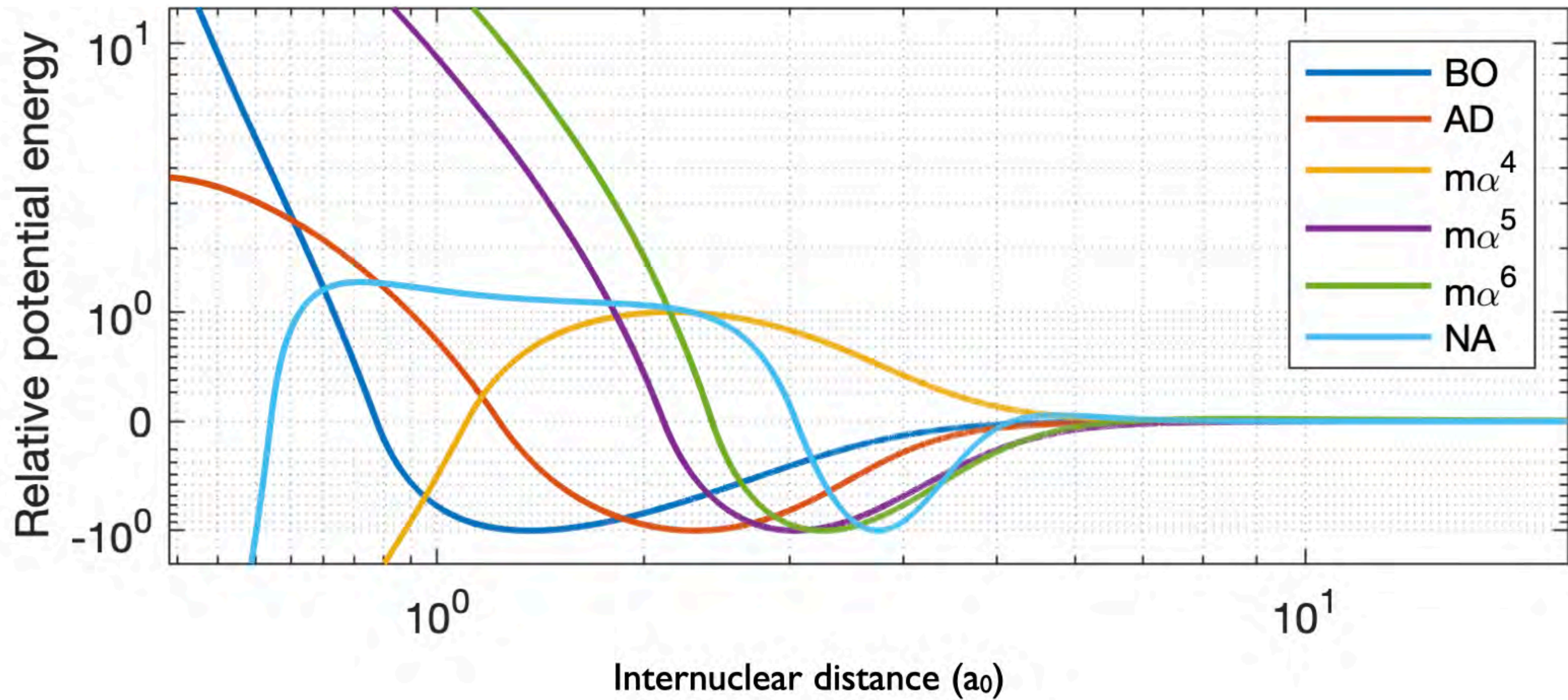
**Josef Agner**

**Hansjürg Schmutz**



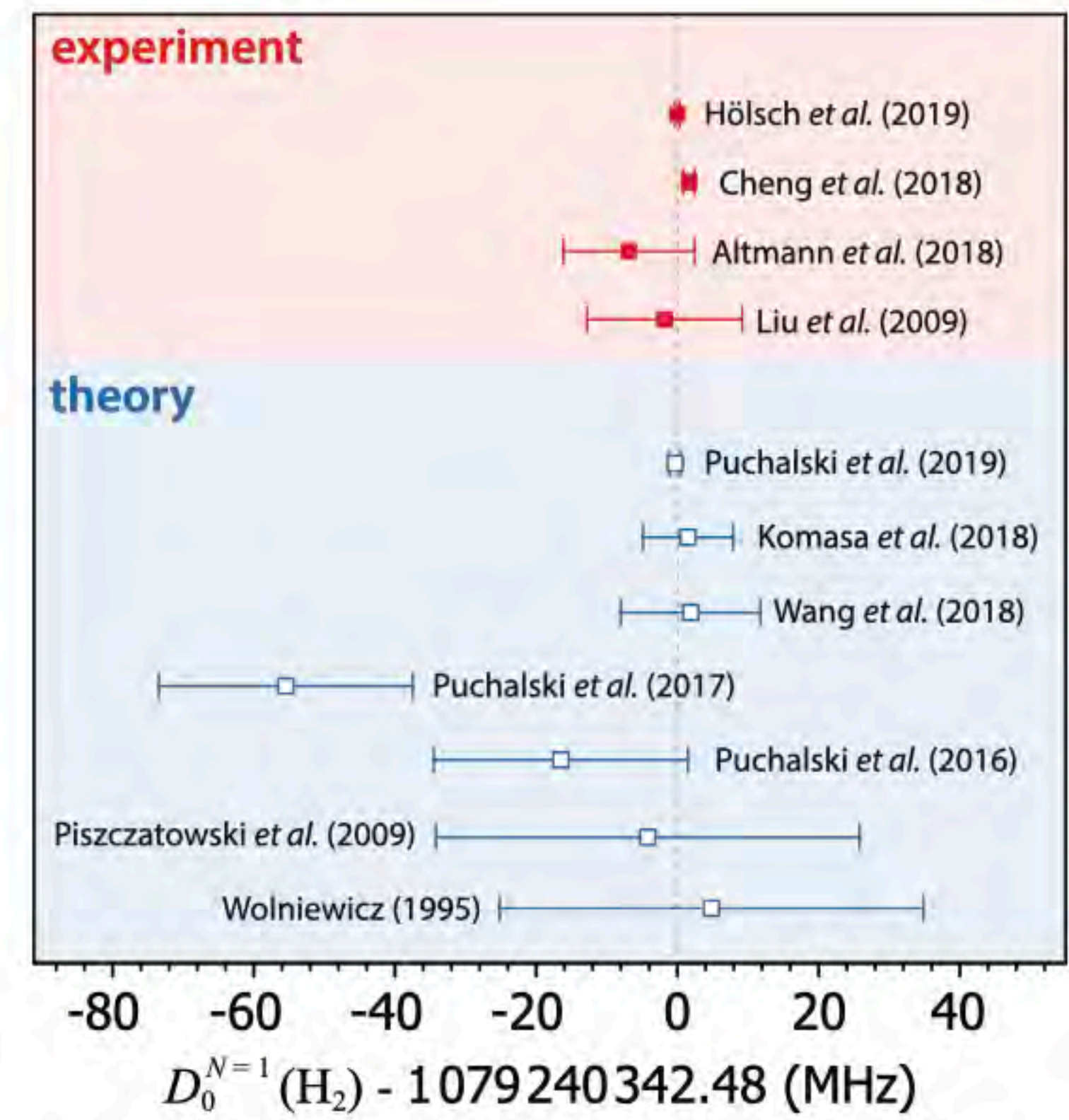
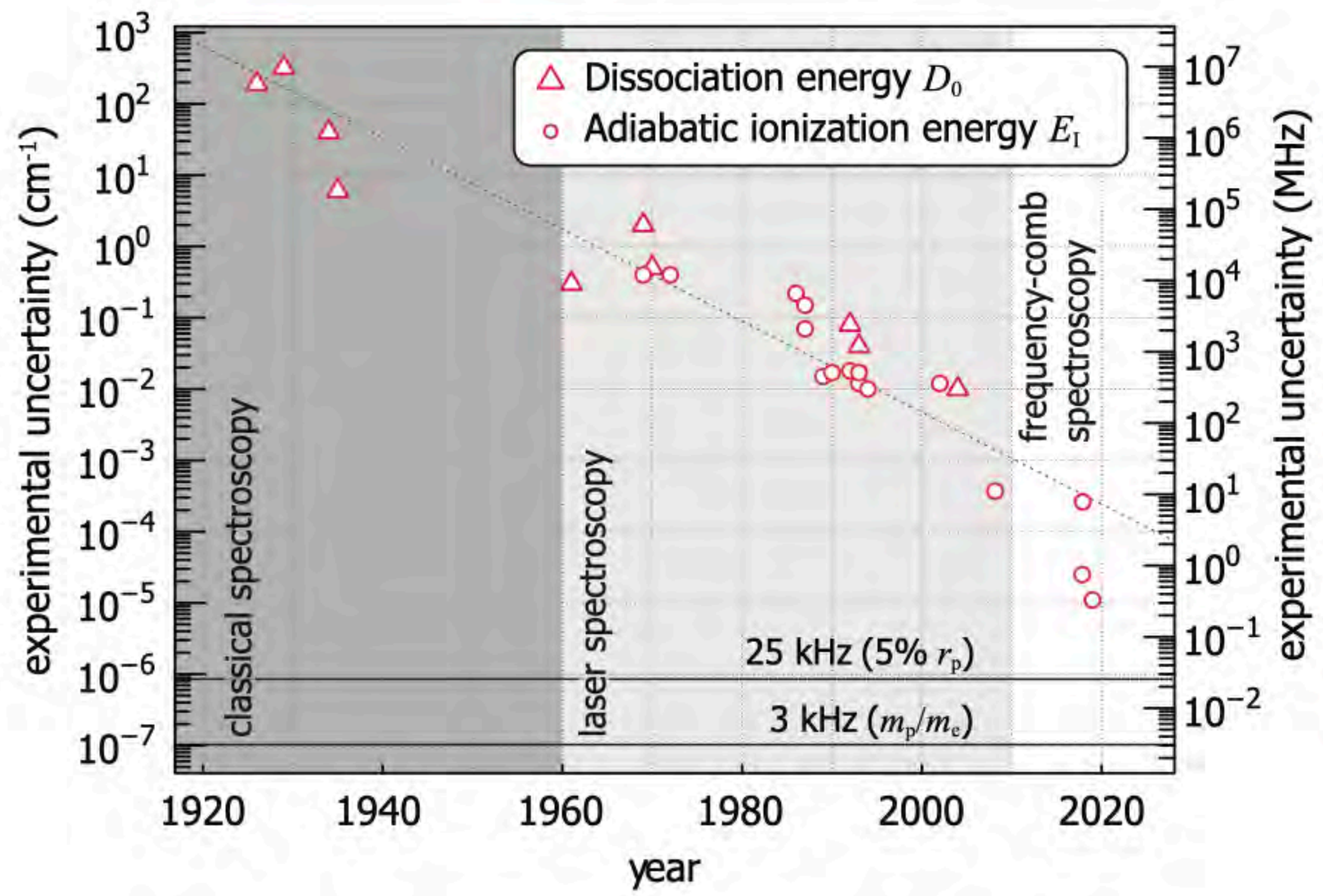


# Electronic ground state of H<sub>2</sub> - corrections





# D<sub>0</sub> in H<sub>2</sub> - theory vs. experiment





# Ionization energy in D<sub>2</sub>

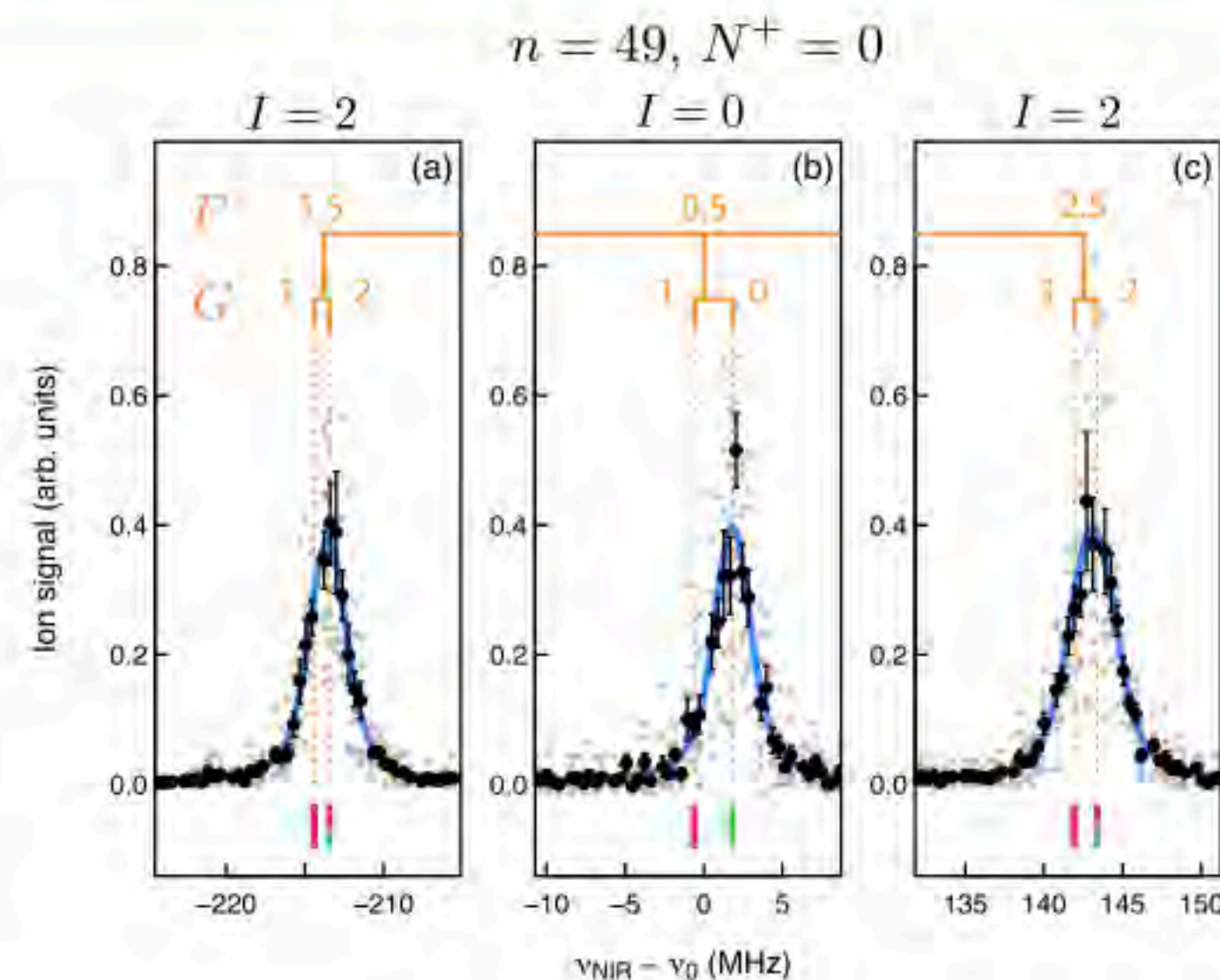
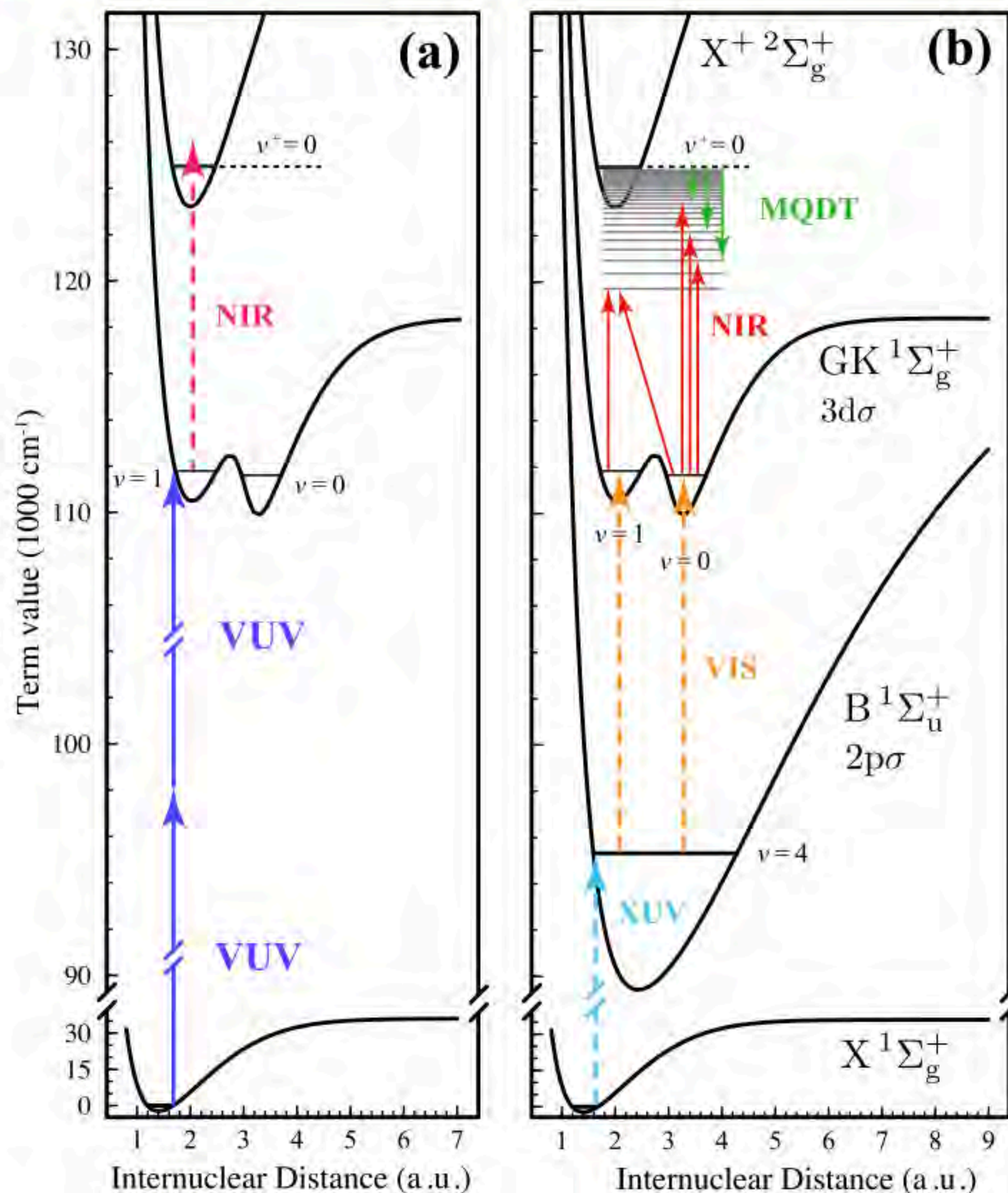


TABLE I. Error budget of the measurement of the  $GK^1\Sigma_g^+(1, 2) \leftarrow X^1\Sigma_g^+(0,0)$  transition in D<sub>2</sub>.

Measured frequency	3 360 646 104.46 MHz
Effect	Uncertainty
Residual first-order Doppler	(350 kHz) <sub>stat</sub>
Second-order Doppler	30 kHz
ac-Stark ionization laser	100 kHz
ac-Stark VUV laser	240 kHz
Hyperfine structure	100 kHz
Final frequency	3 360 646 104.46(45) MHz

TABLE II. Error budget for the transition between the GK(0,2) state of D<sub>2</sub> and the  $62f0_3(I = 0, S = 0)$  Rydberg state, resulting from a series of independent measurements. All systematic uncertainties are the same for the other measured transitions except the uncertainty in the dc Stark shift which is  $n$ -dependent (see text).

Measured frequency	388 255 115.129(16) MHz	
	Correction	Uncertainty
dc Stark shift		60 kHz
ac Stark shift		~5 kHz
Zeeman shift		~10 kHz
Pressure shift		~1 kHz
Residual first-order Doppler shift		(<200 kHz) <sub>stat</sub>
Second-order Doppler shift	+2 kHz	0.5 kHz
Line-shape model		50 kHz
Photon-recoil shift	-83 kHz	
Systematic uncertainty		80 kHz
Final frequency	388 255 115.048(82) MHz	



# Rydberg spectroscopy - characteristics

H <sub>2</sub>			
Initial state	$GK \ ^1\Sigma_g^+ (3s/d)$	Electric field (mV/cm)	<0.8
Final state	$50 \ f0_3(S = 0, v^+ = 0)^2 \Sigma_g^+$	Magnetic field (mG)	$\ll 40$
$\tilde{\nu}$ (cm <sup>-1</sup> )	12 700	Line-shape model (kHz)	<50
$\alpha_{nl}$ [MHz/(V/cm) <sup>2</sup> ]	$5.47(11) \times 10^4$	dc Stark shift (kHz)	18
Lifetime initial state (ns)	69(6)	Zeeman shift/broadening (kHz)	<10
Lifetime final state ( $\mu$ s)	>5	ac Stark shift (kHz)	<4
$\Gamma_{\text{natural}}$ (MHz)	2.31(20)	Rydberg-Rydberg interaction (kHz)	$\ll 1$
Density (particles/cm <sup>3</sup> )	$10^{11}$	Pressure shift (kHz)	<1
Sample volume (mm <sup>3</sup> )	1	First-order Doppler shift (kHz)	<110 <sup>e</sup>
Mass (u)	2.0	Second-order Doppler shift (kHz)	4.1(5)
Temperature ( $\mu$ K)	100 <sup>b</sup>	$\sigma_{\text{stat}}$ (kHz)	91
Velocity (m/s)	1390 <sup>c</sup>	$\sigma_{\text{sys}}$ (kHz)	54
$\Gamma_{\text{laser}}$ (MHz)	1.2		
$\Gamma_{\text{Doppler}}$ (MHz)	<2		
$\Gamma_{\text{FWHM}}$ (MHz)	5		

<sup>a</sup>Calculated using the approach described in Ref. [39].

<sup>b</sup>Corresponds to the width of the velocity distribution in the direction of the NIR laser beam.

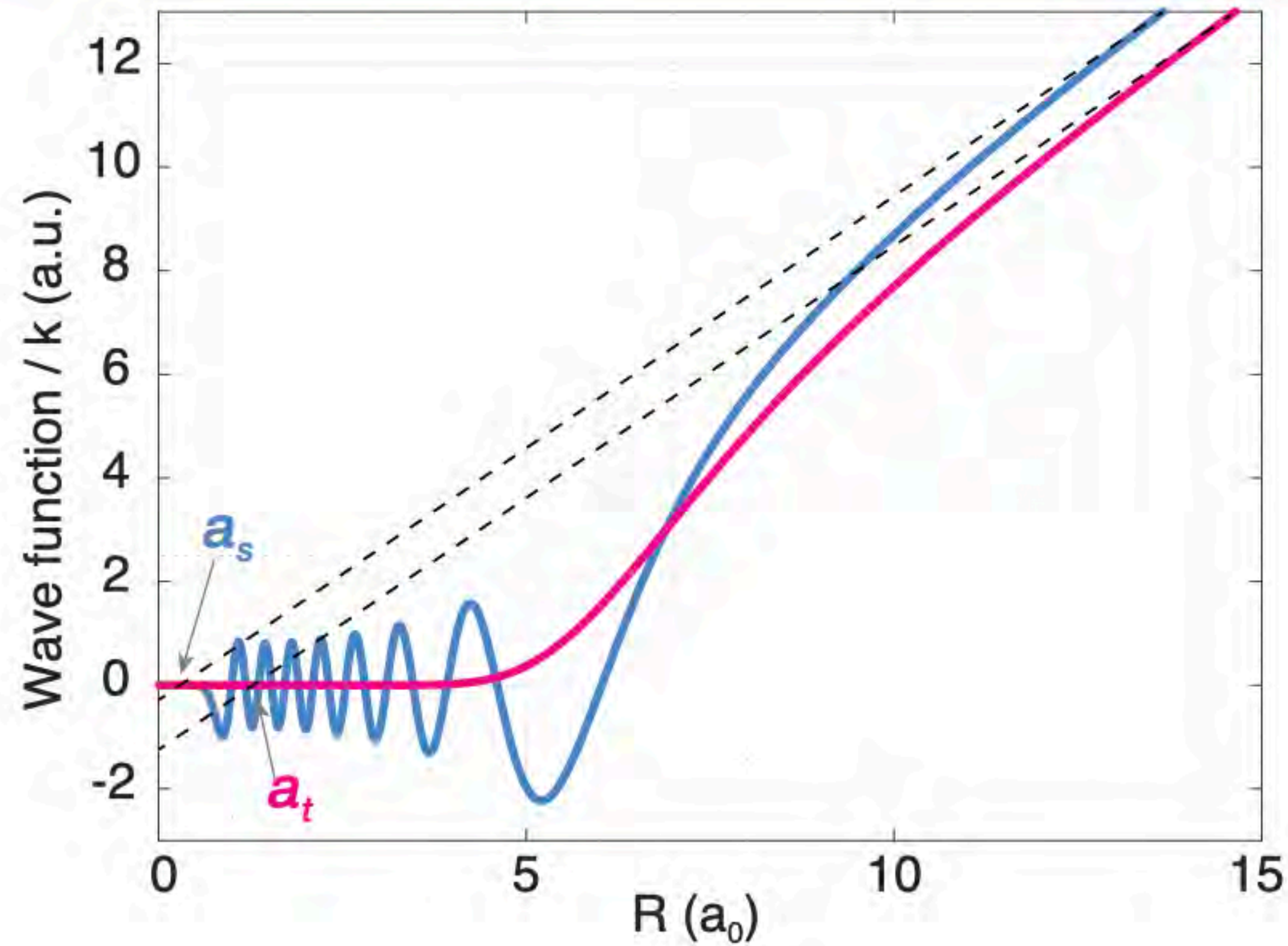
<sup>c</sup>For a valve temperature of 88 K.

<sup>d</sup>ac Stark shift caused by residual light from the optical-dipole-trapping laser.

<sup>e</sup>The uncertainty averages out over several measurements and contributes to  $\sigma_{\text{stat}}$  (see text).



# Determination of the scattering length

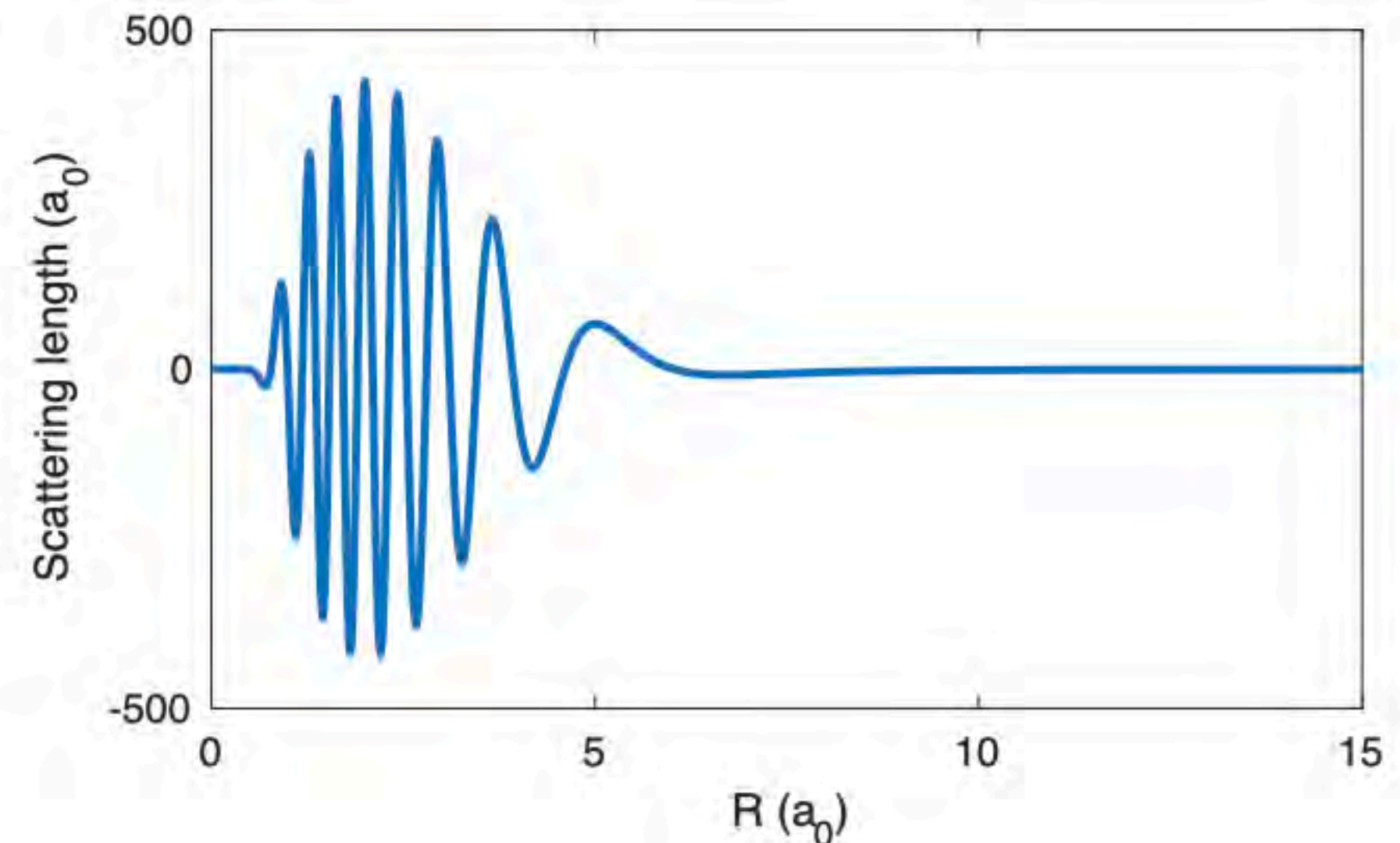


$$a = - \lim_{k \rightarrow 0} \frac{\tan \eta_{J=0}}{k}$$

$$k = \sqrt{2\mu(E - \mathcal{V}(\infty))}$$

$$\lim_{k \rightarrow 0} \chi(R; k) \propto \sin(kR) + \tan \eta_{J=0} \cos(kR) \\ \propto k(R - a)$$

$$a = \frac{2\mu}{k^2} \int_0^\infty \sin(kR) \mathcal{V}(R) \chi(R; k = k_0) dR$$



Universal model: scattering length +  $C_6$   $\longleftrightarrow$  resonance position + width

PHYSICAL REVIEW A **82**, 012510 (2010)

Shape resonances in ground-state diatomic molecules: General trends and the example of RbCs

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