Theoretical status of the proton radius

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Vaguely based on recent review papers

REVIEWS OF MODERN PHYSICS, VOLUME 94, JANUARY–MARCH 2022

The proton charge radius

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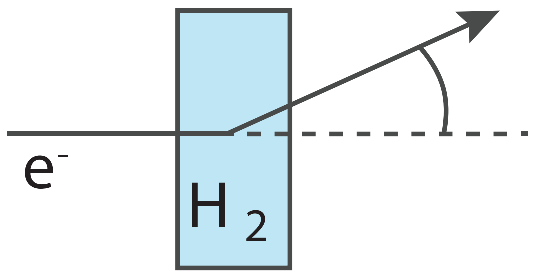
Annu. Rev. Nucl. Part. Sci. 2022. 72:1–31

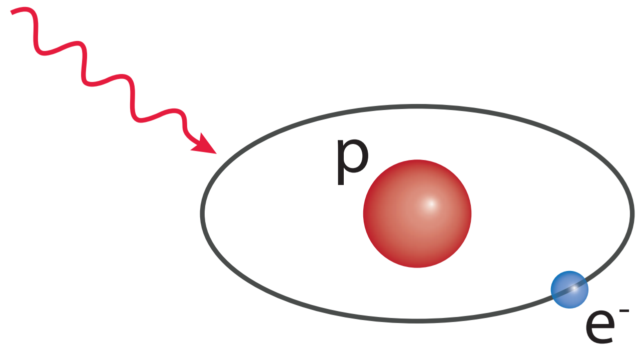
The Proton Structure in and out of Muonic Hydrogen

Aldo Antognini,[1,2] Franziska Hagelstein,[1,3] and Vladimir Pascalutsa[3]

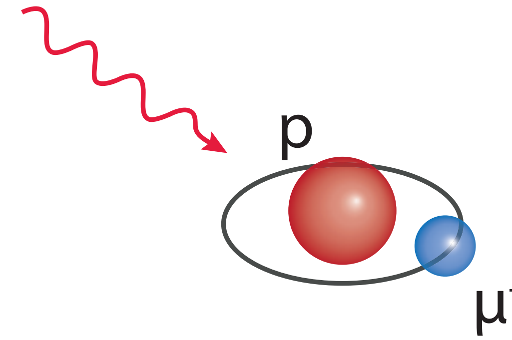
@ PREN2022
Workshop,
Paris
June 20-23, 2022



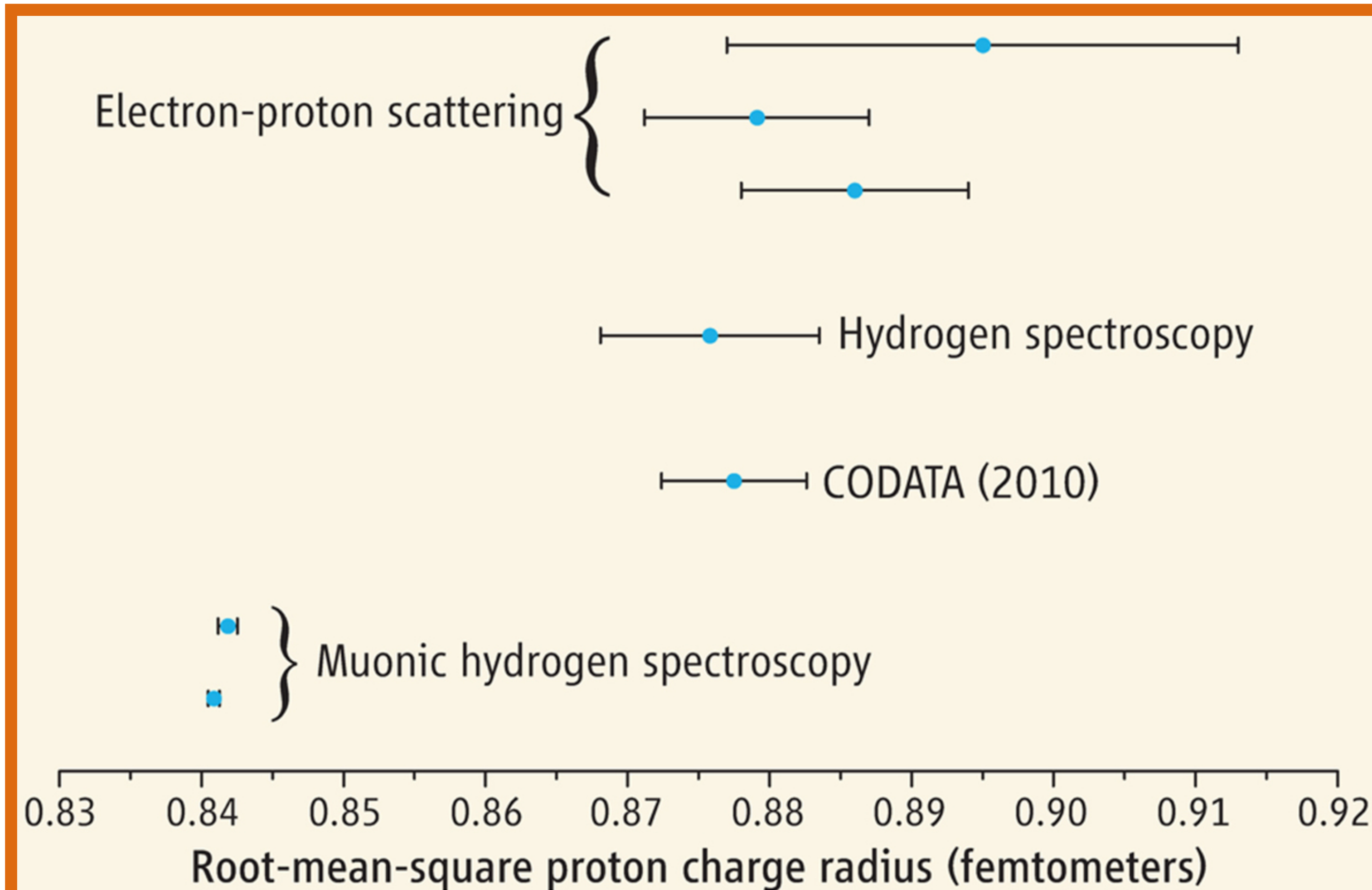
e^--p scattering



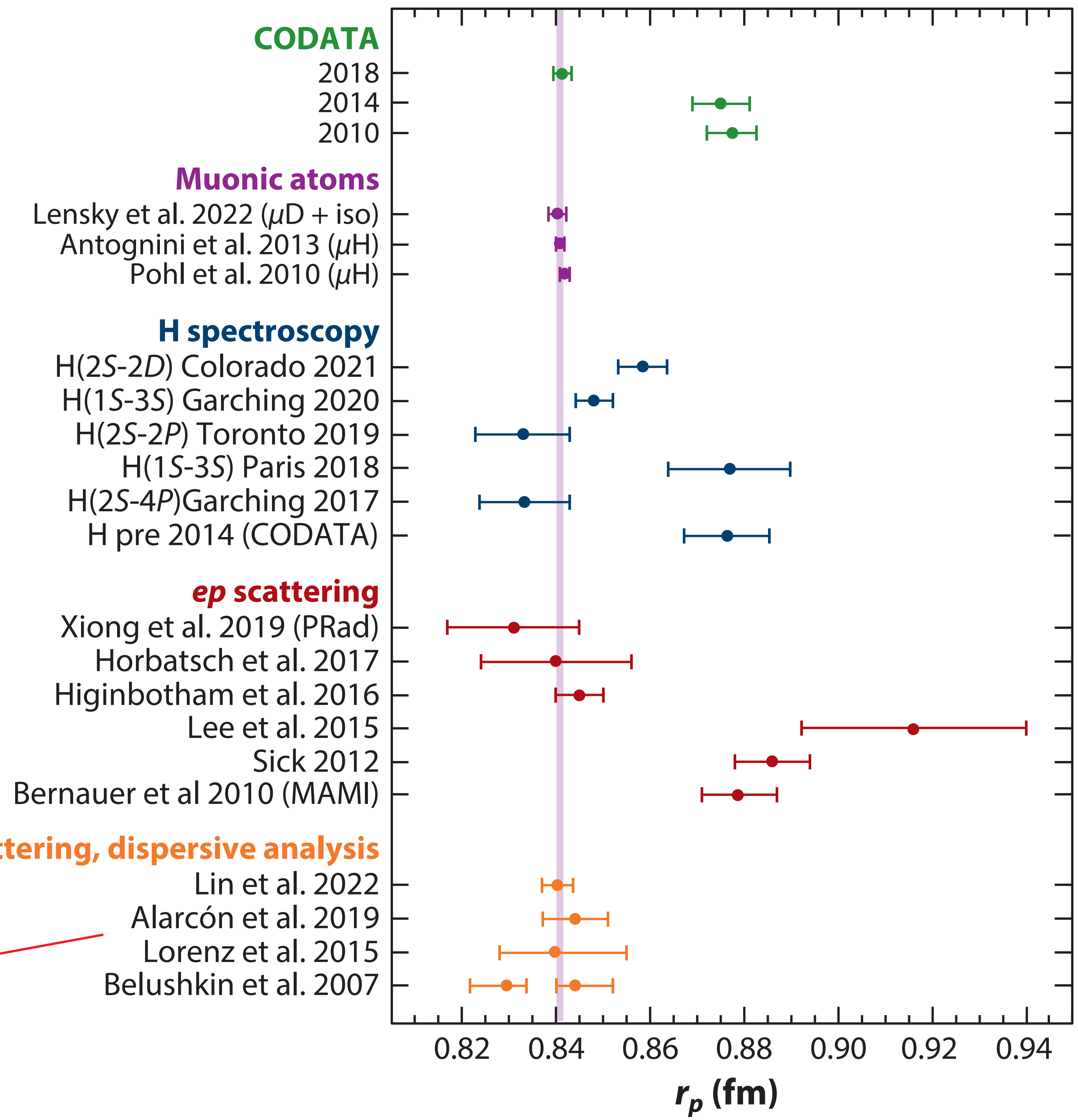
H spectroscopy



μp spectroscopy



Present status of the proton charge radius



Was it ever a puzzle?

U. Meissner

Proton radius puzzle: what could it mean ?

$$\Delta E_{LS} = 206.0336\,(15) \;-\; 5.2275\,(10)\; R_E^2 \;+\; \Delta E_{TPE} \quad \text{meV}$$

different radii

μH expt. wrong ?

eH theory wrong ?

Lamb shift

difference of

≈ 330 μeV

μH theory wrong ?

- QED bound state corrections
- hadronic corrections
- check with different targets

eH expt. wrong ? -> R_∞ wrong

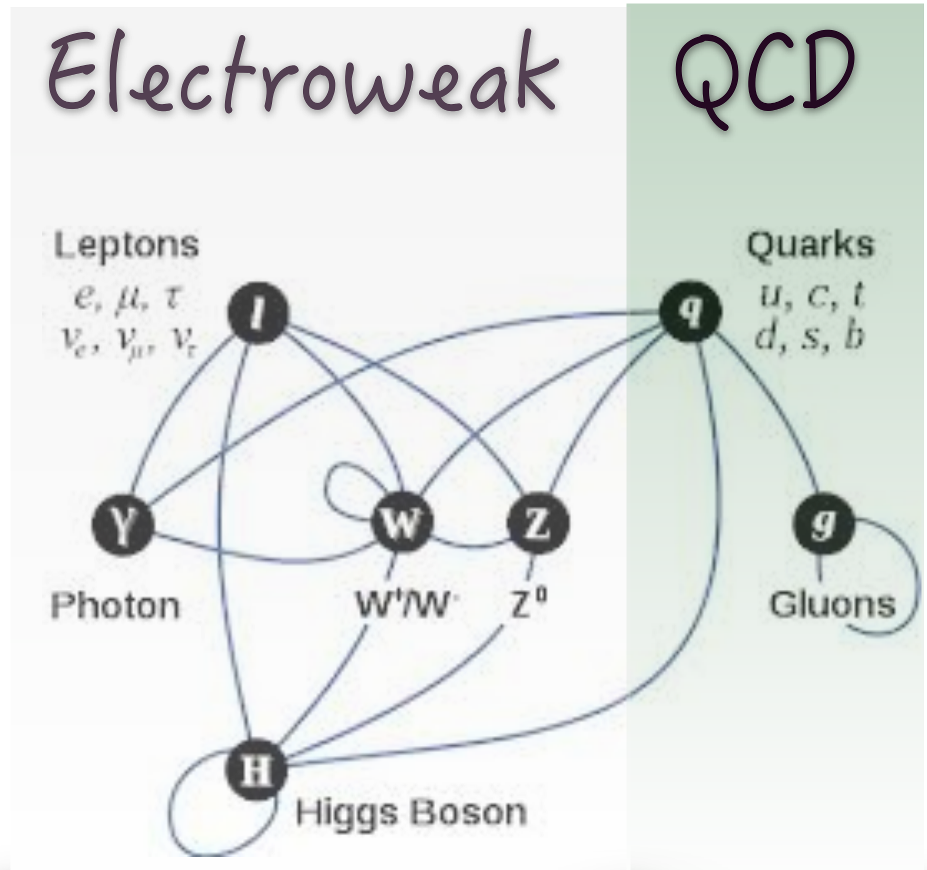
+ ep scattering wrong ?

- radiative corrections
- 2γ corrections
- low Q^2 extrapolation

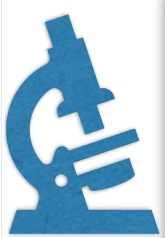
new physics ?

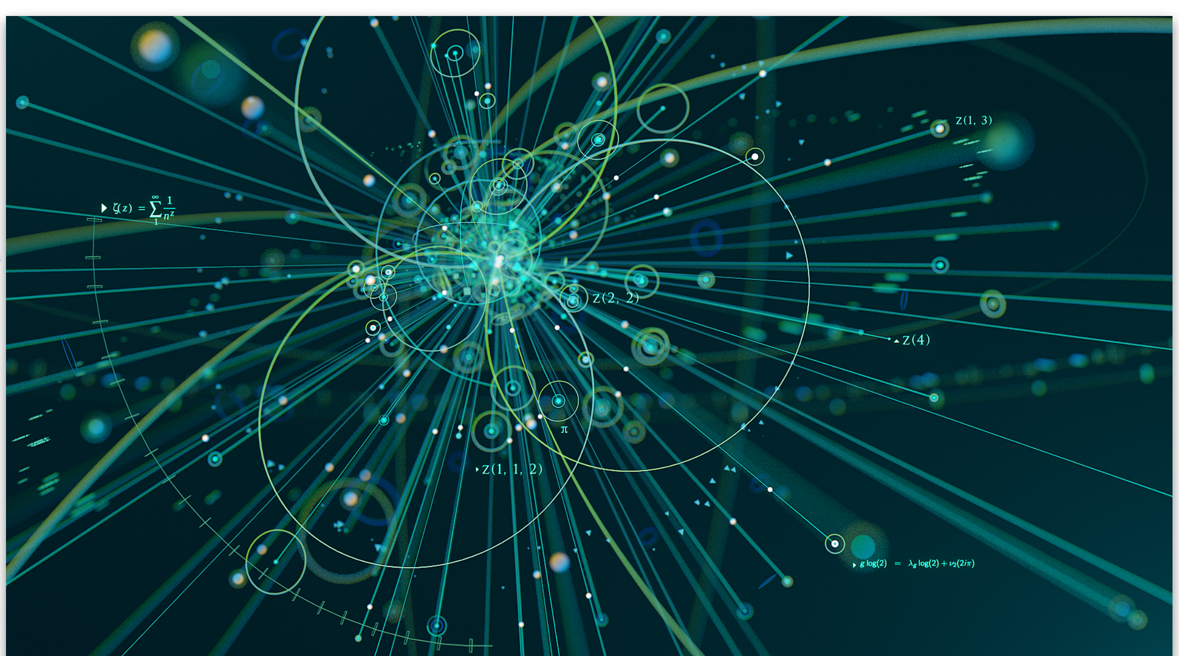
see PPNP review Carlson (2015)

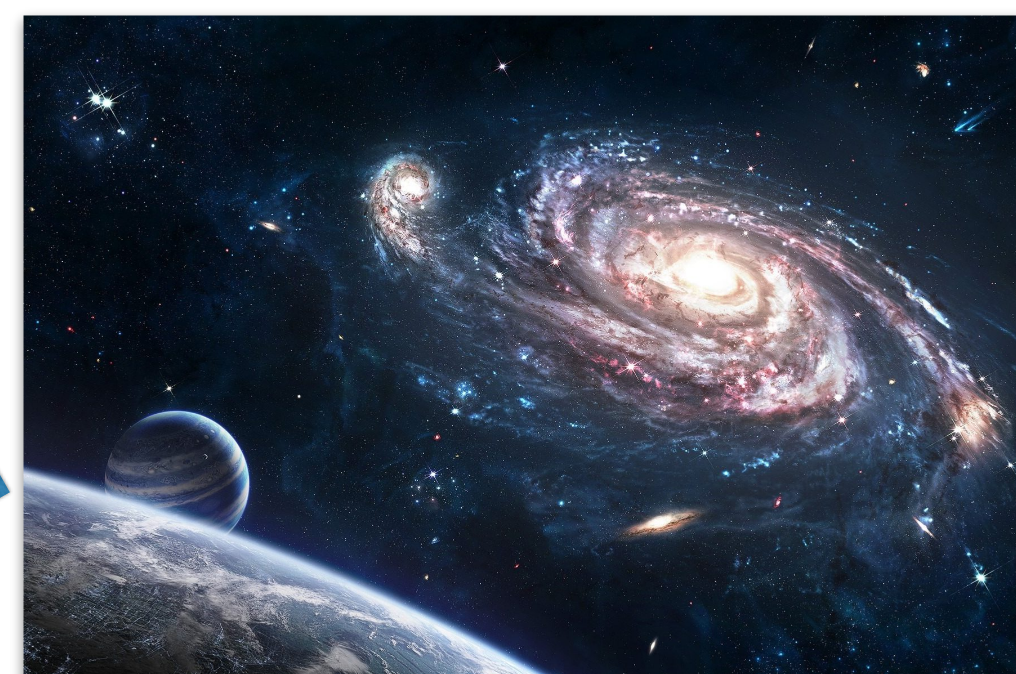
Standard Model



is presently the best theory of (nearly) everything

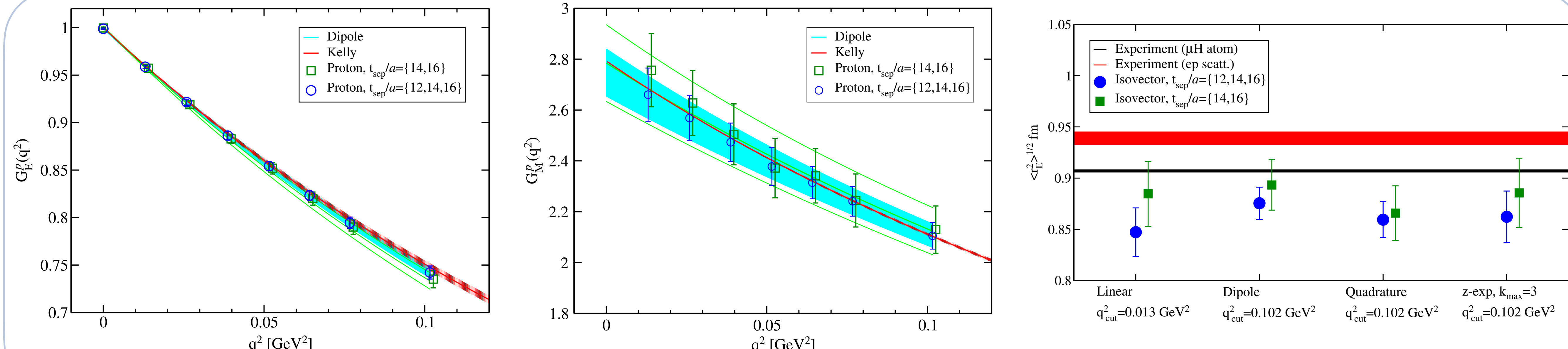




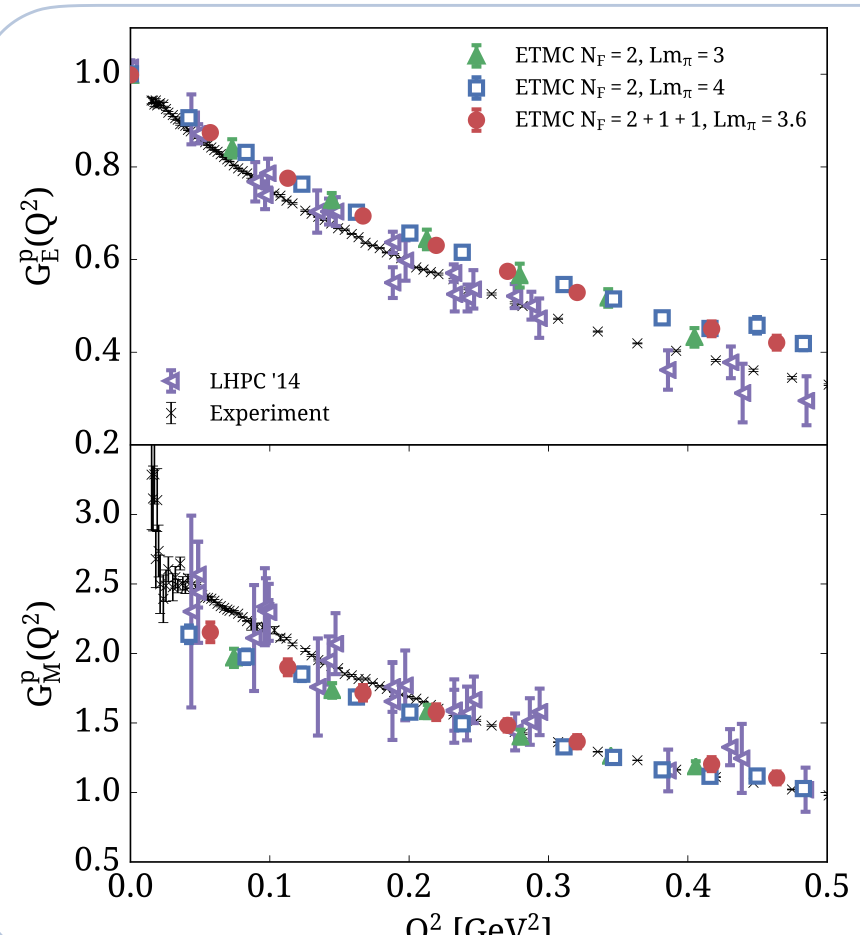


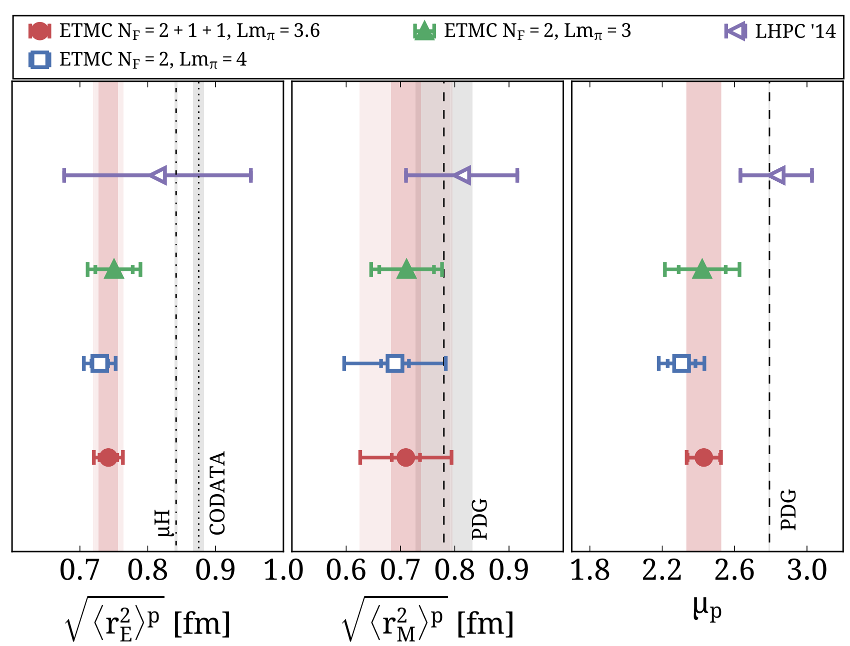
Approaches to low-energy QCD

LATTICE QCD



PACS Coll., Phys. Rev. D **99**, 014510 (2019)



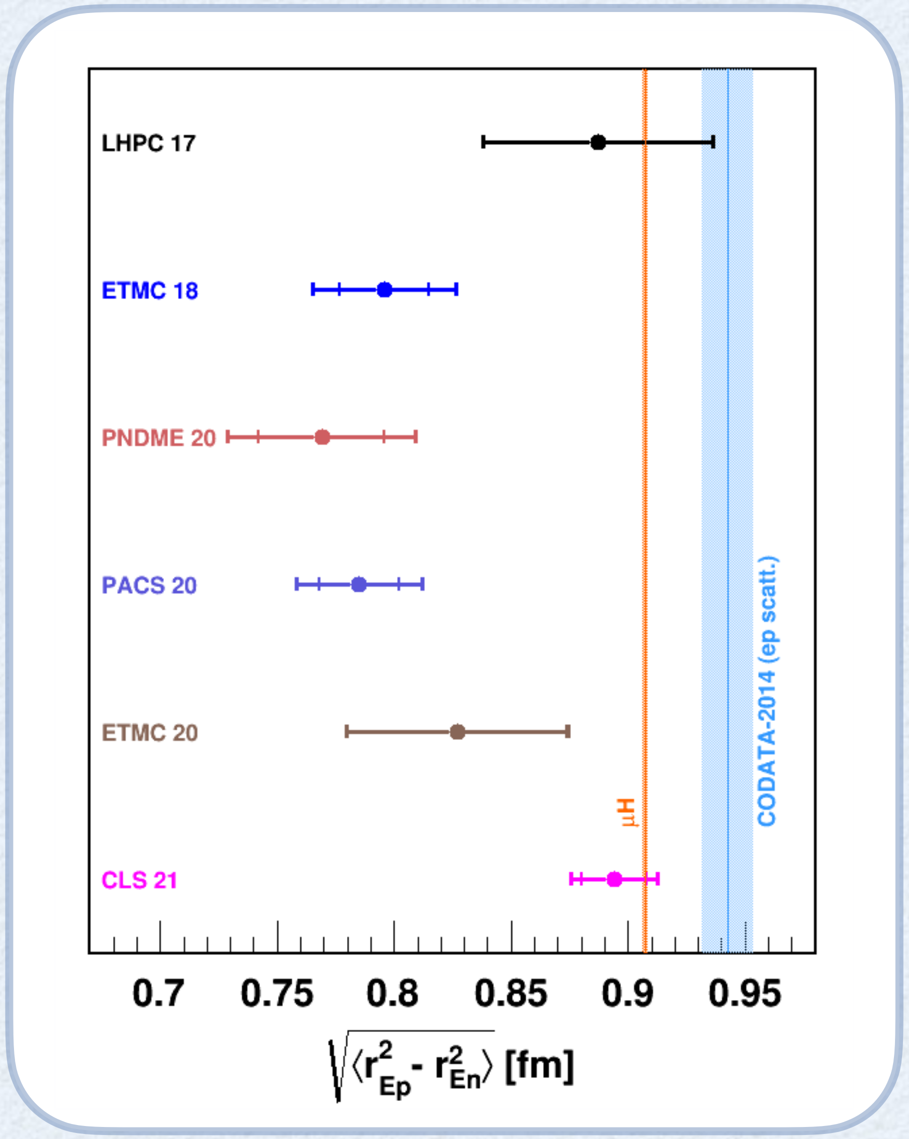


C. Alexandrou *et al.*, Phys. Rev. D **100**, 014509 (2019)

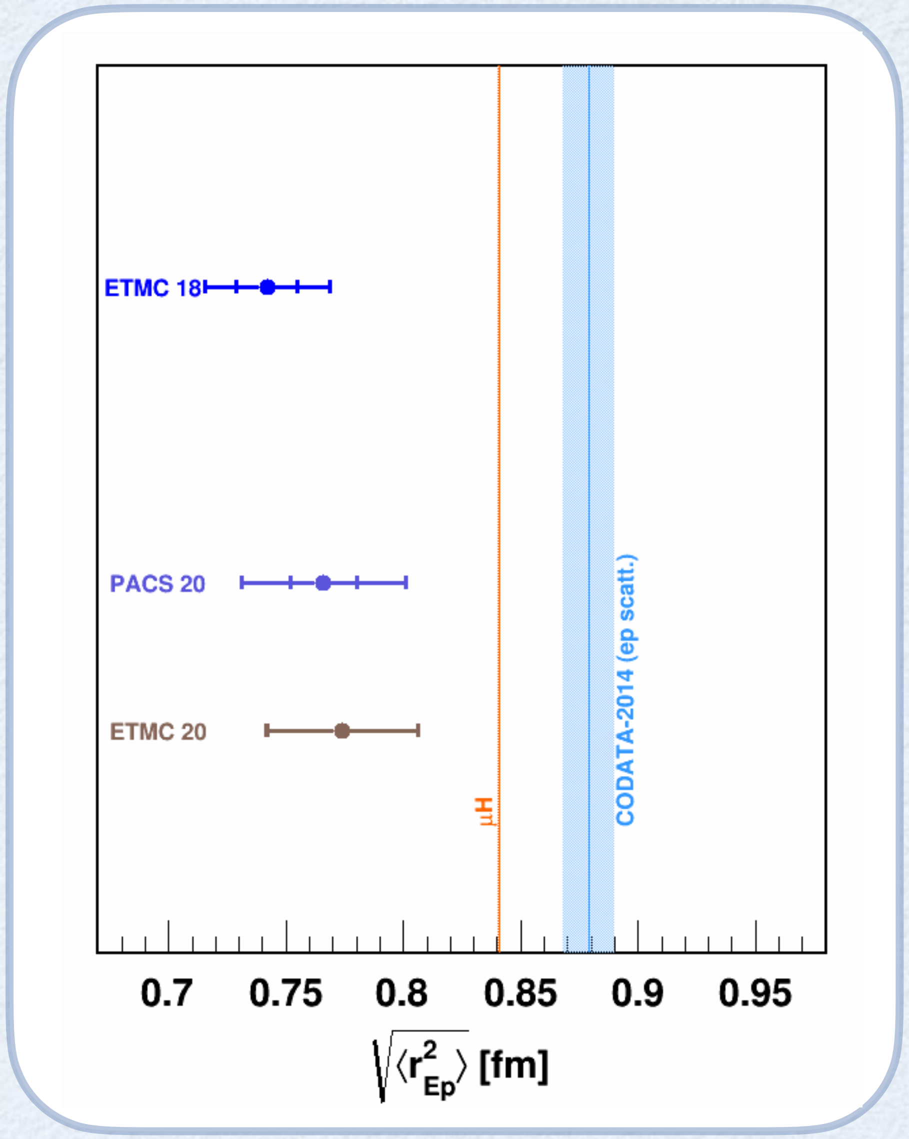
Proton charge radius from lattice QCD

- ➢ Lattice QCD results at (or near) physical pion mass
- ➢ Control of excited state contaminations
- ➢ Proton electromagnetic FFs require disconnected contributions (found to be ~1%)
- ➢ Low Q^2 -> very large lattice volumes, radius extraction requires extrapolation using FF fit

isovector

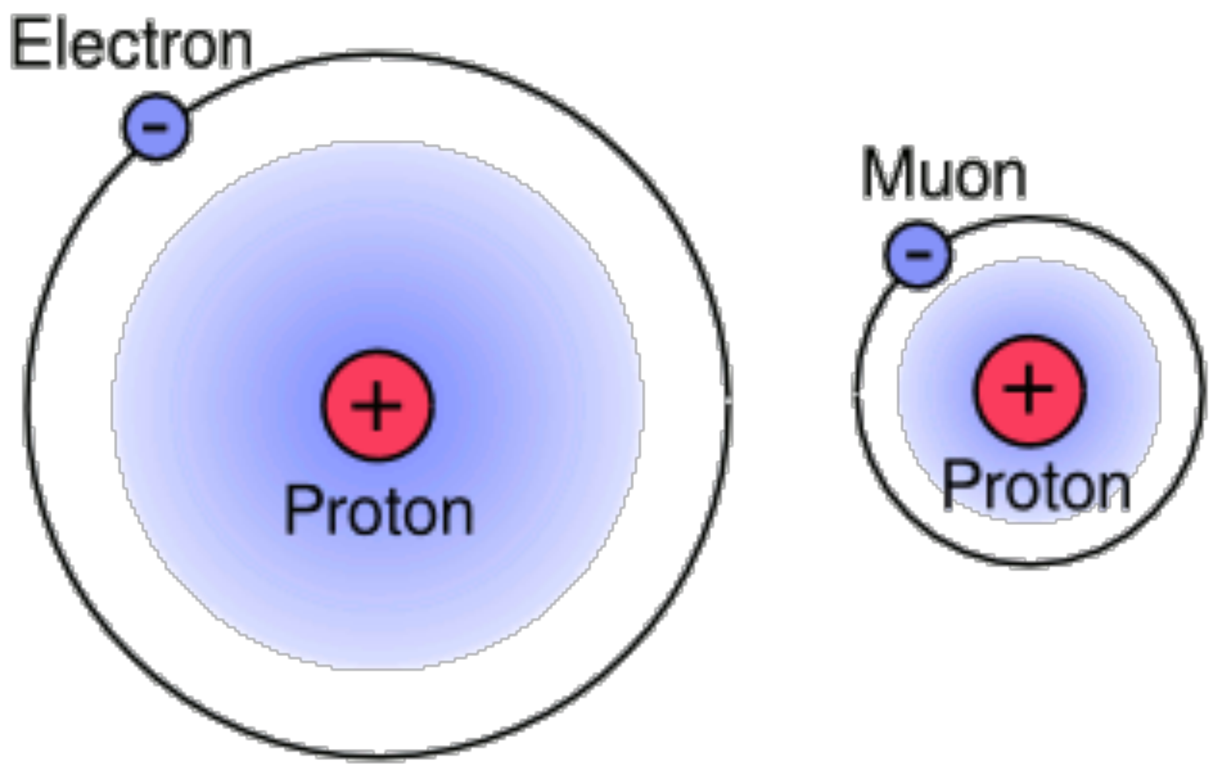


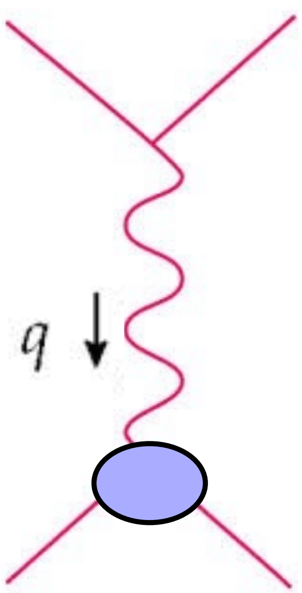
proton



Proton charge radius in hydrogens

Hydrogens sensitive to proton structure





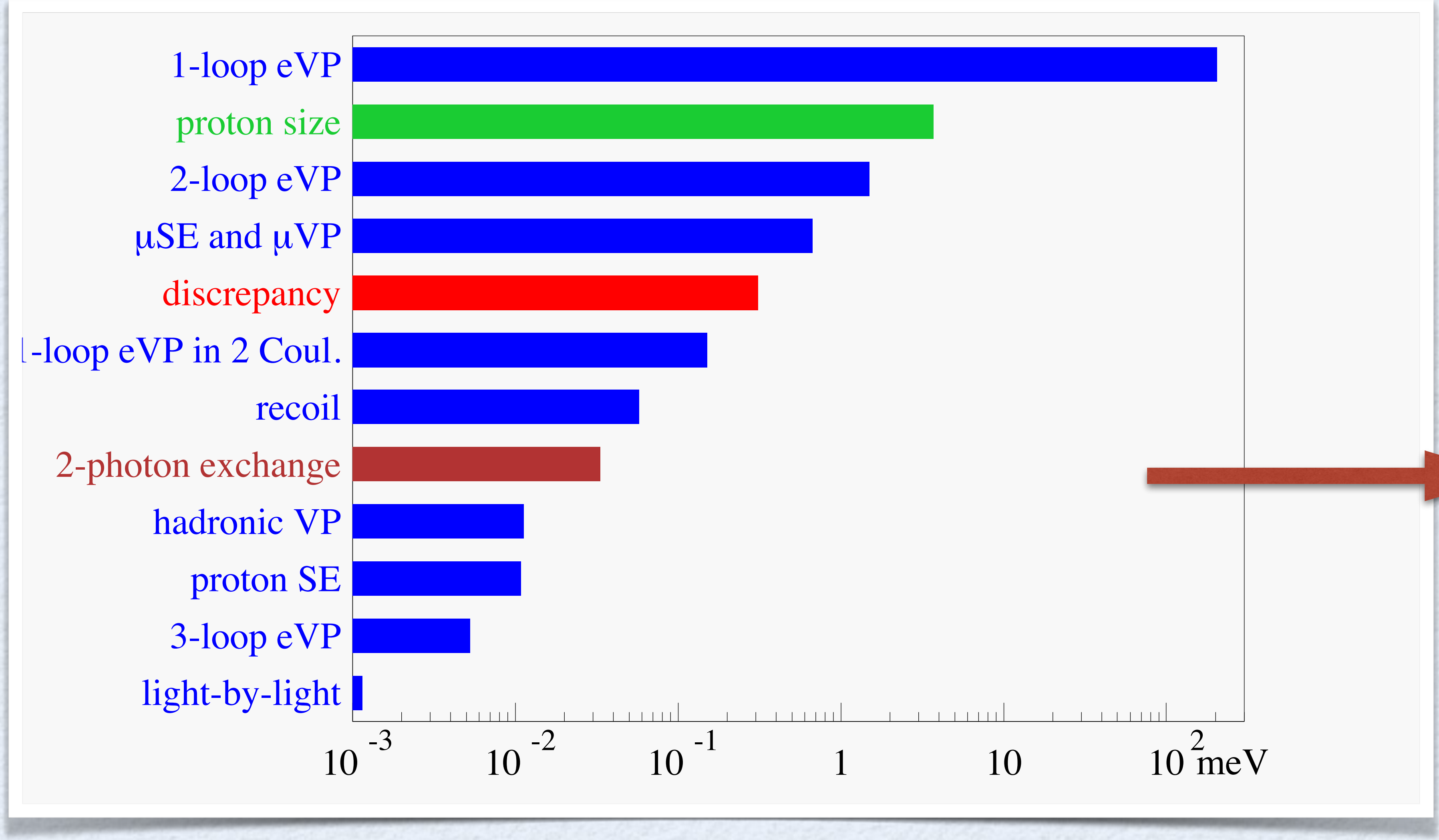
$$\delta V^{(1\gamma)} = -\frac{4\pi\alpha}{\vec{q}^{\,2}}[G_E(-\vec{q}^{\,2}) - 1] = \frac{2}{3}\pi\alpha\, r_E^2 + O(\vec{q}^{\,2})$$

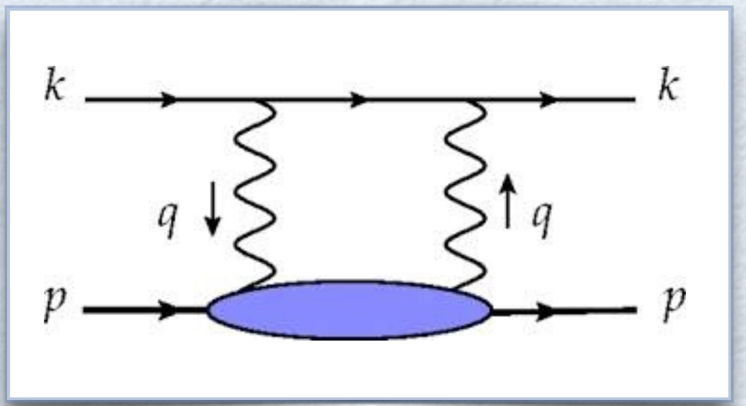
$$\Delta E_{nl}^{(\mathrm{FS})} = \left\langle nlm \right| \delta V^{(1\gamma)} \left| nlm \right\rangle = \delta_{l0}\frac{2}{3}\pi\alpha\, r_E^2\, \frac{\alpha^3 m_r^3}{\pi n^3} + O(\alpha^5)$$

wave function
at origin

Subleading proton effects in the Lamb shift

μH Lamb shift: summary of corrections



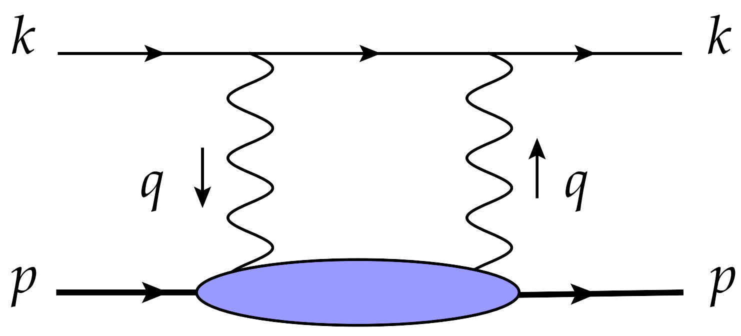


largest theoretical uncertainty

ΔE_{TPE} (2P-2S) = (33 ± 2) μeV

- elastic contribution on 2S level: ΔE_{2S} = -23 μeV
- inelastic contribution: Carlson & Vanderhaeghen (2011) Birse & McGovern (2012)

Two-photon exchange: hadronic corrections



$$T^{\mu\nu}(p,q) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1(\nu, Q^2) + \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2} q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2} q^\nu\right) T_2(\nu, Q^2)$$

- Two-photon exchange (TPE): lower blob contains both elastic (nucleon) and inelastic states
- Lamb shift: described by unpolarized amplitudes T_1, T_2 : functions of energy ν and Q^2
- Hyperfine splitting: described by polarized amplitudes S_1, S_2
- Imaginary parts: directly proportional to nucleon structure functions F_1, F_2 resp. g_1, g_2
- Real parts: obtained as dispersion integral over the imaginary parts modulo a subtraction function in case of T_1

$$\Delta E = \Delta E^{el} + \Delta E^{subtr} + \Delta E^{inel}$$

Elastic: involves **nucleon form factors**

Subtraction: involves **nucleon polarizabilities**

Inelastic state: involves **nucleon structure functions**

Data-driven evaluation possible, except for the subtraction function (in the Lamb shift)

Two-Photon Exchange (TPE) in Lamb shift

wave function at the origin

$$\Delta E(nS) = 8\pi\alpha m\,\phi_n^2\,\frac{1}{i}\int_{-\infty}^{\infty}\frac{\mathrm{d}\nu}{2\pi}\int\frac{\mathrm{d}\mathbf{q}}{(2\pi)^3}\,\frac{\left(Q^2-2\nu^2\right)T_1(\nu,Q^2)-(Q^2+\nu^2)\,T_2(\nu,Q^2)}{Q^4(Q^4-4m^2\nu^2)}$$

dispersion relation
& optical theorem

$$T_1(\nu,Q^2) = T_1(0,Q^2) + \frac{32\pi Z^2\alpha M\nu^2}{Q^4}\int_0^1 \mathrm{d}x\,\frac{xF_1(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2-i0^+}$$

$$T_2(\nu,Q^2) = \frac{16\pi Z^2\alpha M}{Q^2}\int_0^1 \mathrm{d}x\,\frac{F_2(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2-i0^+}$$

data-driven dispersive calculations:

low-energy expansion:

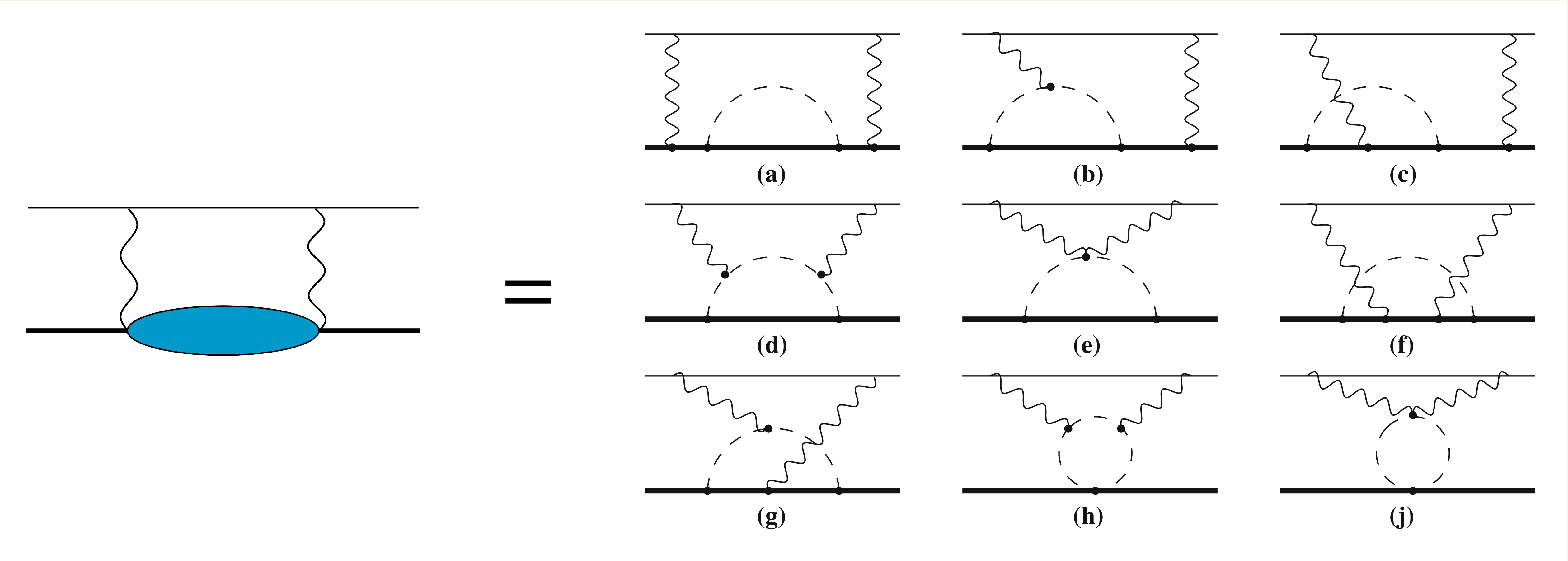
$$\lim_{Q^2\to 0}\overline{T}_1(0,Q^2)/Q^2 = 4\pi\beta_{M1}$$

the subtraction function
can be calculated in ChPT

e.g., Pachucki modeled Q^2 behavior as:

$$\overline{T}_1(0,Q^2) = 4\pi\beta_{M1}\,Q^2/\left(1+Q^2/\Lambda^2\right)^4$$

forward TPE in muonic hydrogen



Lamb shift

LO: J. M. Alarcon, V. Lensky & V.P., Eur. Phys. J. C **74** (2014) 2852

NLO: F. Hagelstein, V. Lensky & V.P., in prep.

HFS

LO: F. Hagelstein & V.P., PoS (2015)

NLO: F. Hagelstein, V. Lensky & V.P., in prep.

Lamb shift: subtraction function

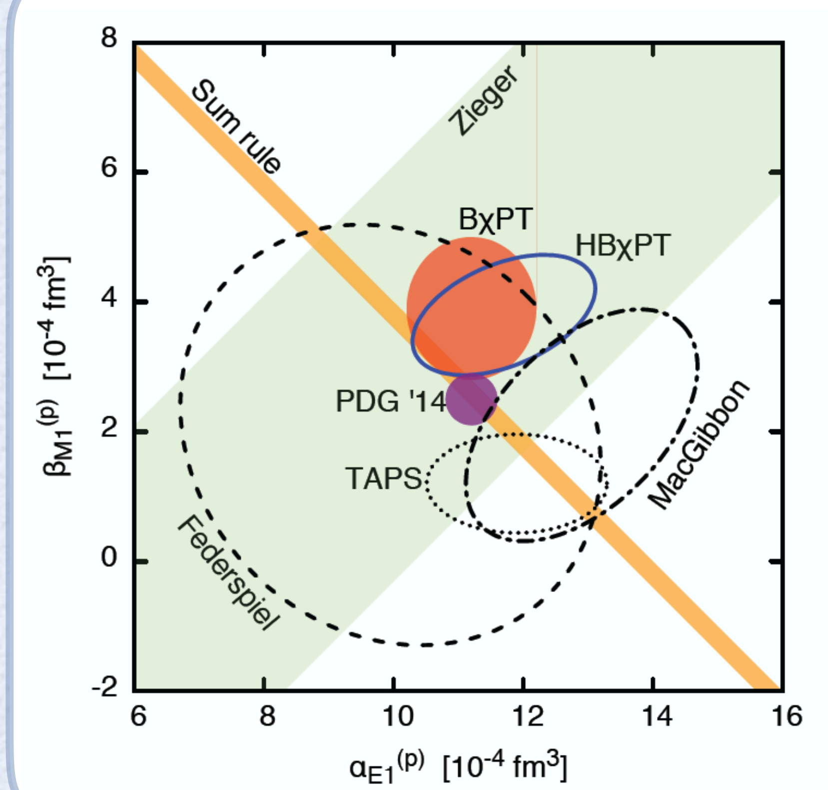
- low-energy expansion of forward, doubly virtual Compton scattering contains a subtraction term $T_1(0,Q^2)$

effective Hamiltonian:

$$\mathcal{H} = -\frac{1}{2} 4\pi\alpha_E \vec{E}^{\,2} - \frac{1}{2} 4\pi\beta_M \vec{B}^{\,2}$$

electric — magnetic

polarizabilities



Theory analyses:

BChPT

Lensky, Pascalutsa (2010)

HBChPT

Griesshammer, McGovern, Phillips (2013)

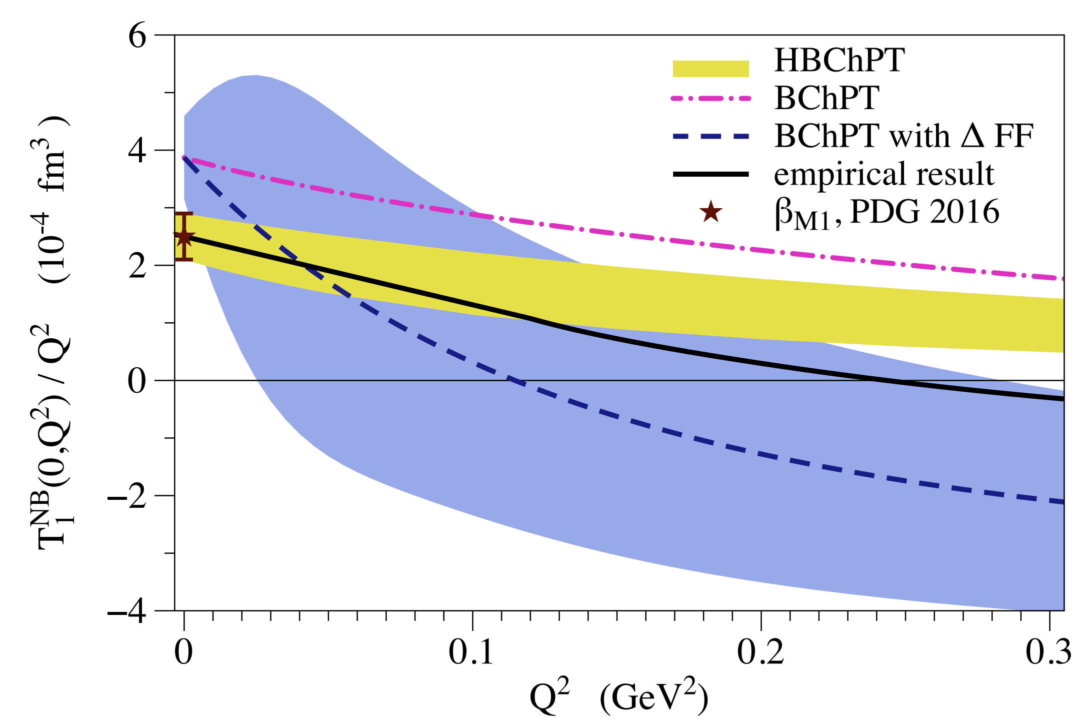
PDG '14 values:

α_E = (11.2 ± 0.2) × 10^{-4} fm^3

β_M = (2.5 ± 0.4) × 10^{-4} fm^3

- subtraction term

$$T_1^{\mathrm{non-Born}}(0,Q^2) = Q^2 \beta_M + \mathcal{O}(Q^4)$$



HBChPT

Birse, McGovern (2012)

BChPT

Lensky, Hagelstein, Pascalutsa, Vdh (2018)

Empirical result (based on HERA data)

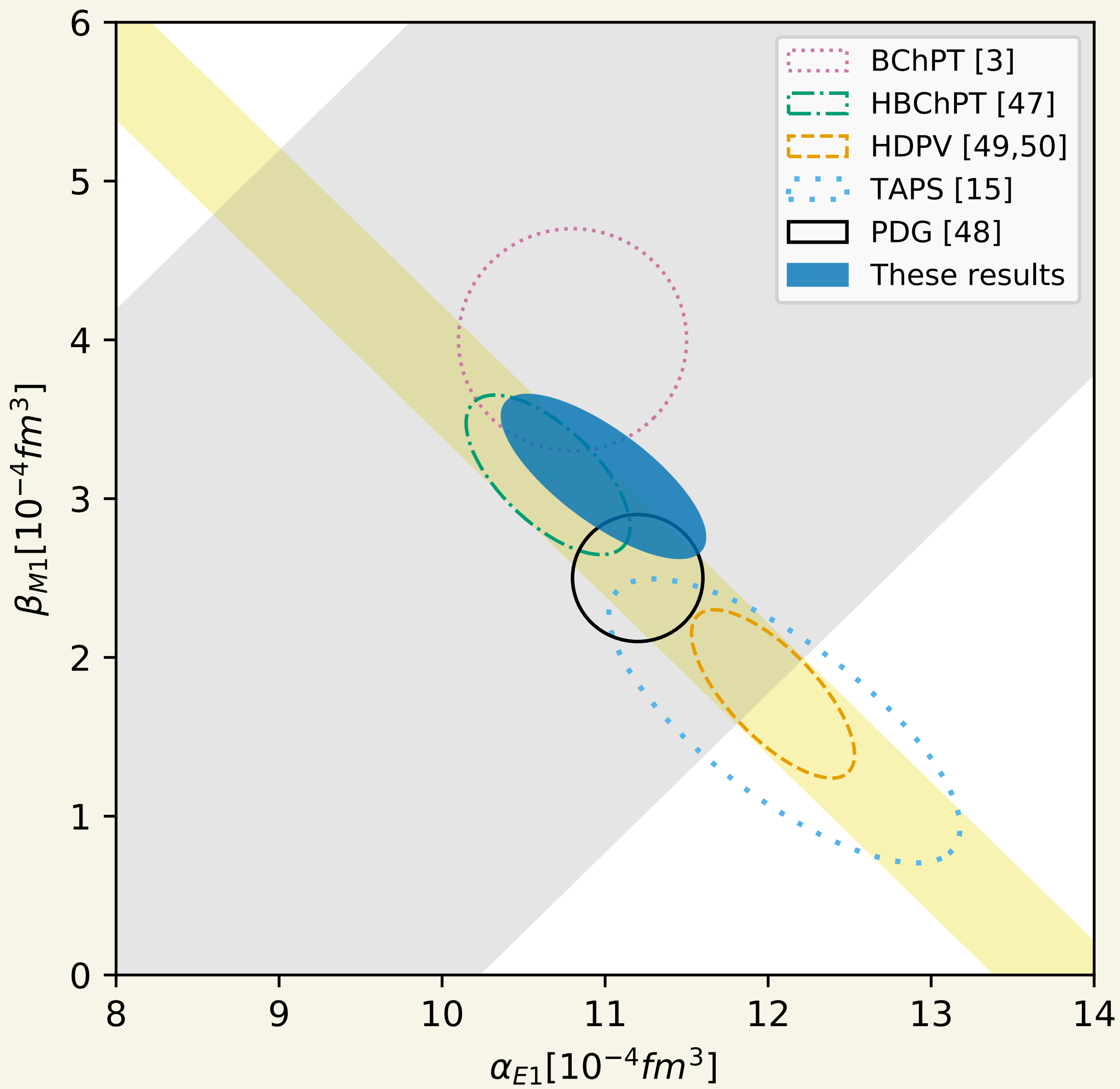
Tomalak, Vdh (2016)

New measurement of proton polarizabilities

Measurement of Compton scattering at MAMI for the extraction of the electric and magnetic polarizabilities of the proton

E. Mornacchi,[1] P.P. Martel,[1,2] S. Abt,[3] P. Achenbach,[1] P. Adlarson,[1] F. Afzal,[4] Z. Ahmed,[5] J.R.M. Annand,[6] H.J. Arends,[1] M. Bashkanov,[7] R. Beck,[4] M. Biroth,[1] N. Borisov,[8] A. Braghieri,[9] W.J. Briscoe,[10] F. Cividini,[1] C. Collicott,[1] S. Costanza,[9] A. Denig,[1] A.S. Dolzhikov,[8] E.J. Downie,[10] P. Drexler,[1] S. Fegan,[7] S. Gardner,[6] D. Ghosal,[3] D.I. Glazier,[6] I. Gorodnov,[8] W. Gradl,[1] M. Günther,[3] D. Gurevich,[11] L. Heijkenskjöld,[1] D. Hornidge,[2] G.M. Huber,[5] A. Käser,[3] V.L. Kashevarov,[1,8] S.J.D. Kay,[5] M. Korolija,[12] B. Krusche,[3] A. Lazarev,[8] K. Livingston,[6] S. Lutterer,[3] I.J.D. MacGregor,[6] D.M. Manley,[13] R. Miskimen,[14] M. Mocanu,[7] C. Mullen,[6] A. Neganov,[8] A. Neiser,[1] M. Ostrick,[1] D. Paudyal,[5] P. Pedroni,[9] A. Powell,[6] T. Rostomyan,[3] V. Sokhoyan,[1] K. Spieker,[4] O. Steffen,[1] I. Strakovsky,[10] T. Strub,[3] M. Thiel,[1] A. Thomas,[1] Yu.A. Usov,[8] S. Wagner,[1] D.P. Watts,[7] D. Werthmüller,[7,15] J. Wettig,[1] M. Wolfes,[1] and N. Zachariou[7]
(A2 Collaboration at MAMI)

Phys.Rev.Lett. 128 (2022) 13, 132503
arXiv: 2110.15691 [nucl-ex]

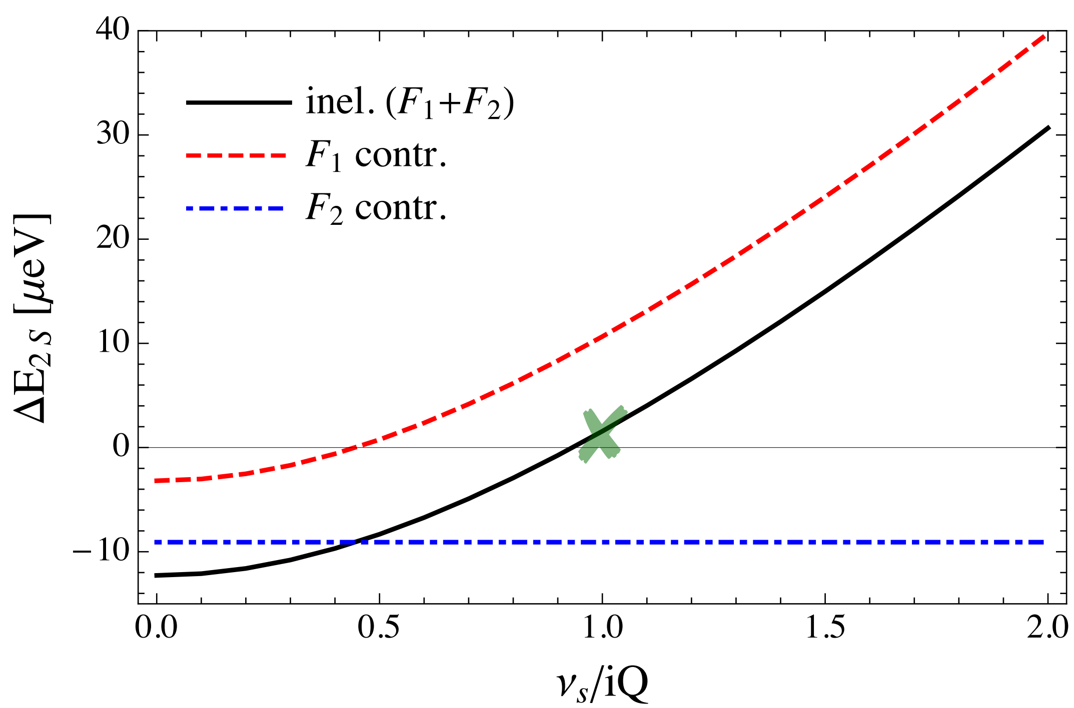


EUCLIDEAN SUBTRACTION FUNCTION

- Once-subtracted dispersion relation for $\overline{T}_1(\nu, Q^2)$ with subtraction at $\nu_s = iQ$
- Dominant part of polarizability contribution:

$$\Delta E_{nS}^{\prime\,(\mathrm{subt})} = \frac{2\alpha m}{\pi}\phi_n^2 \int_0^\infty \frac{\mathrm{d}Q}{Q^3}\,\frac{2+v_l}{(1+v_l)^2}\,\overline{T}_1(iQ, Q^2) \text{ with } v_l = \sqrt{1+4m^2/Q^2}$$

- Inelastic contribution for $\nu_s = iQ$ is order of magnitude smaller than for $\nu_s = 0$
- Prospects for future lattice QCD and EFT calculations



Hagelstein & VP, Nucl. Phys. A **1016** (2021) 122323

based on Bosted-Christy parametrization:

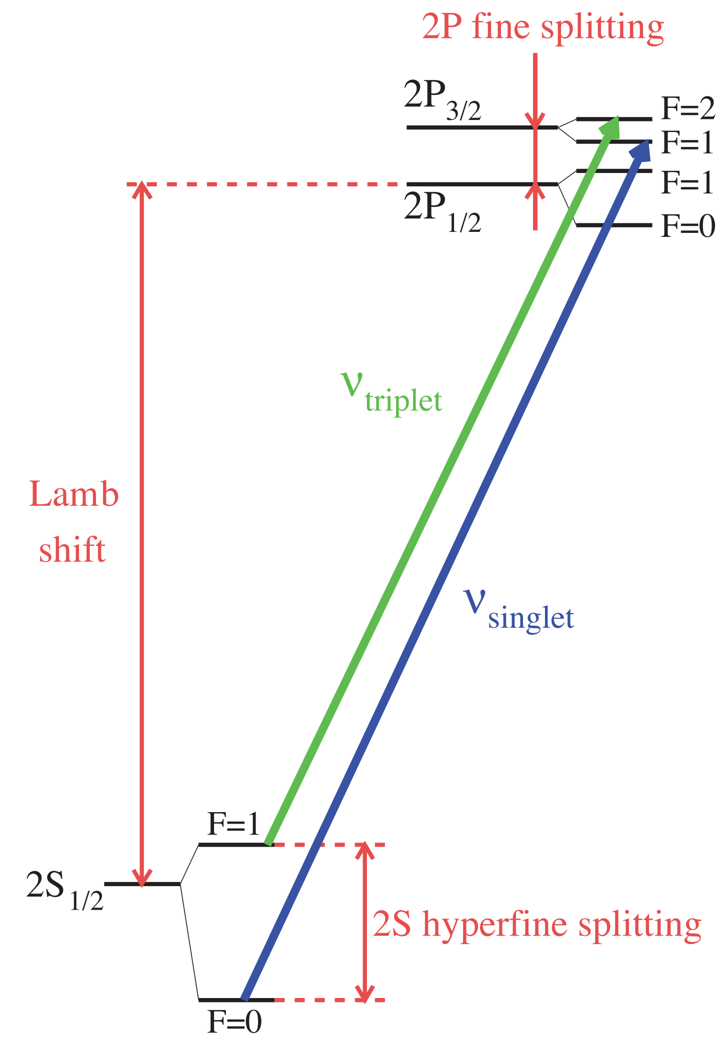
$$\Delta E_{2S}^{(\mathrm{inel})}(\nu_s = 0) \simeq -12.3\,\mu\mathrm{eV}$$

$$\Delta E_{2S}^{\prime\,(\mathrm{inel})}(\nu_s = iQ) \simeq 1.6\,\mu\mathrm{eV}$$

HYPERFINE SPLITTING IN μH

$$\Delta E_{\mathrm{HFS}}(nS) = [1 + \Delta_{\mathrm{QED}} + \Delta_{\mathrm{weak}} + \Delta_{\mathrm{structure}}]\, E_F(nS)$$

with $\Delta_{\mathrm{structure}} = \Delta_Z + \Delta_{\mathrm{recoil}} + \Delta_{\mathrm{pol}}$



Zemach radius:

$$\Delta_Z = \frac{8Z\alpha m_r}{\pi} \int_0^\infty \frac{\mathrm{d}Q}{Q^2} \left[\frac{G_E(Q^2) G_M(Q^2)}{1+\kappa} - 1 \right] \equiv -2Z\alpha m_r R_Z$$

experimental value: $R_Z = 1.082(37)\ \mathrm{fm}$

A. Antognini, et al., Science **339** (2013) 417–420

⏰ Measurements of the μH ground-state HFS planned by the CREMA, FAMU and J-PARC / Riken-RAL collaborations

- Very precise input for the 2γ polarizability effect needed to find the μH ground-state HFS transition in experiment
- Zemach radius involves magnetic properties of the proton

Proton Zemach radius from hyperfine splittings

THEORY OF HYPERFINE SPLITTING

Antognini, Hagelstein & VP, Ann. Rev. Nucl. Part. **72** (2022)

The hyperfine splitting of μH (theory update):

$$E_{1S\text{-hfs}} = \Big[\underbrace{182.443}_{E_F}\ \underbrace{+1.350(7)}_{\text{QED+weak}}\ \underbrace{+0.004}_{\text{hVP}}\ \underbrace{-1.30653(17)\left(\frac{r_{Zp}}{\text{fm}}\right) + E_F\left(1.01656(4)\,\Delta_{\text{recoil}} + 1.00402\,\Delta_{\text{pol}}\right)}_{2\gamma \text{ incl. radiative corr.}}\Big]\,\text{meV}$$

The hyperfine splitting of H (theory update):

$$E_{1S\text{-hfs}}(\text{H}) = \Big[\underbrace{1\,418\,840.082(9)}_{E_F}\ \underbrace{+1\,612.673(3)}_{\text{QED+weak}}\ \underbrace{+0.274}_{\mu\text{VP}}\ \underbrace{+0.077}_{\text{hVP}}$$
$$\underbrace{-54.430(7)\left(\frac{r_{Zp}}{\text{fm}}\right) + E_F\left(0.99807(13)\,\Delta_{\text{recoil}} + 1.00002\,\Delta_{\text{pol}}\right)}_{2\gamma \text{ incl. radiative corr.}}\Big]\,\text{kHz}$$

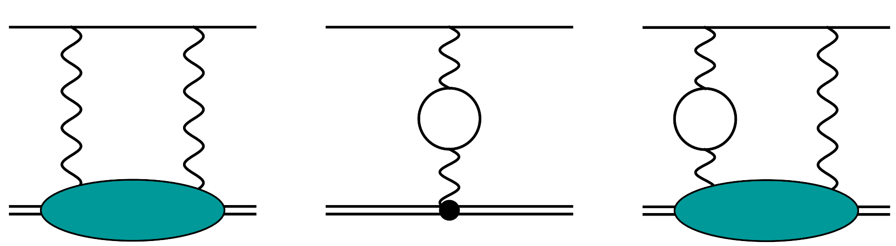
High-precision measurement of the "21cm line" in H:

$$\delta\left(E^{\text{exp.}}_{1S-\text{hfs}}(\text{H})\right) = 10 \times 10^{-13}$$

Hellwig et al., 1970

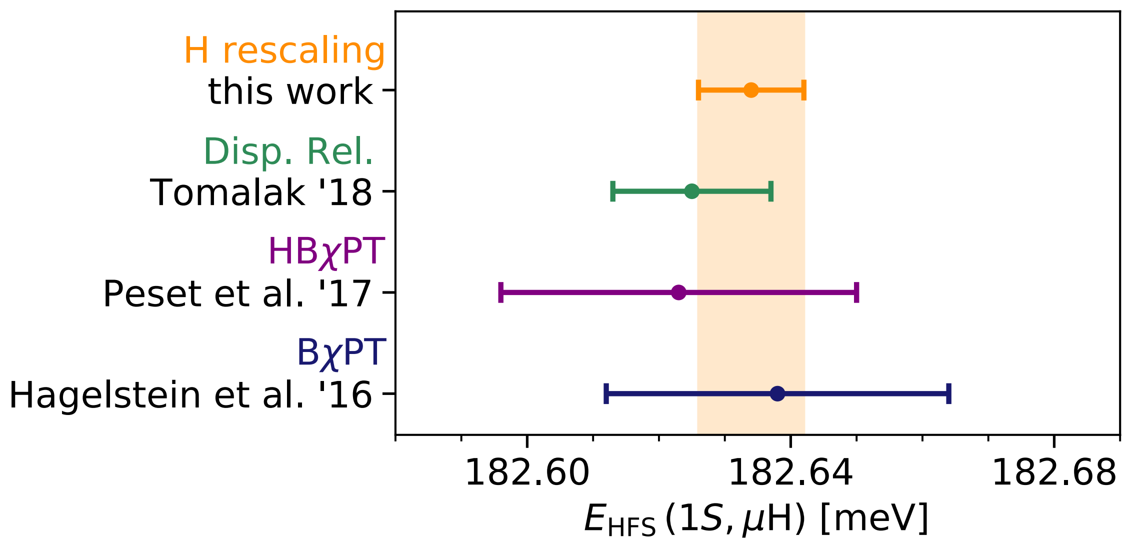
$$E^{\text{hadr}}_{1S-\text{hfs}}(\text{H}) = E_\text{F}(\text{H})\Big[b_{1S}(\text{H})\,\Delta_\text{Z}(\text{H}) + c_{1S}(\text{H})\,\Delta_{\text{pol}}(\text{H}) + \Delta_{\text{hVP}}(\text{H})\Big] = -\,54.823(71)\,\text{kHz}$$

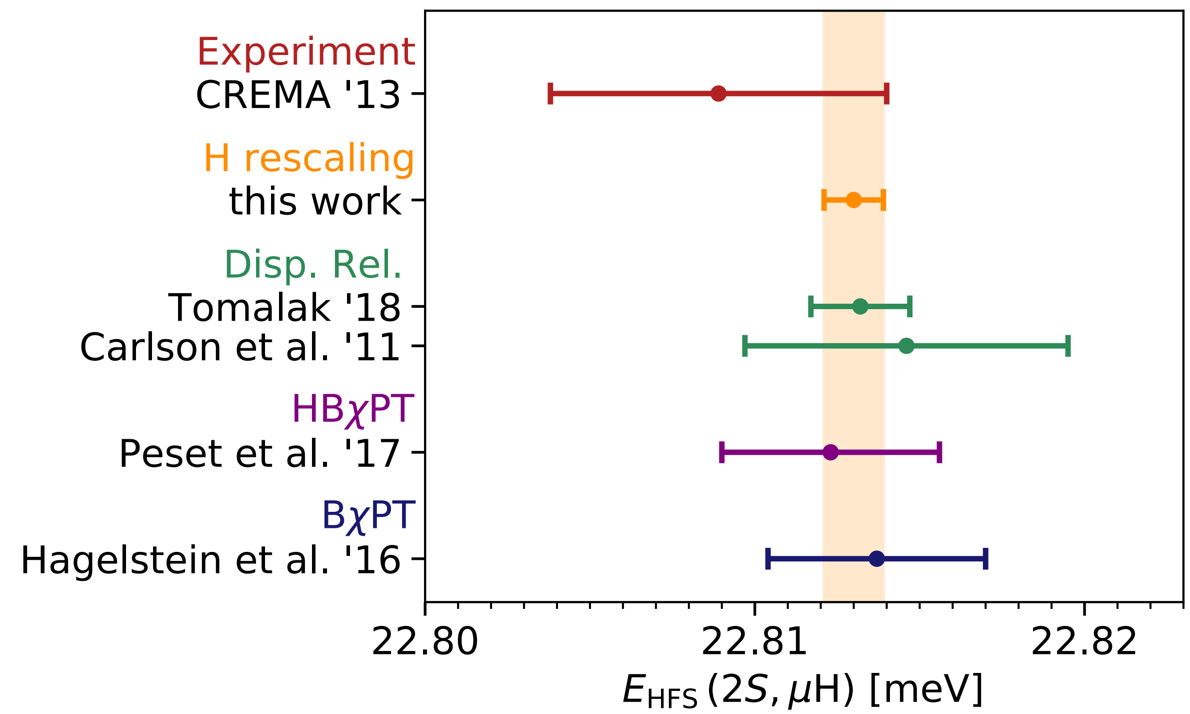
- 2γ + radiative corrections $\Longrightarrow$ differ for H vs. μH and 1S vs. 2S



+ …

2γ EFFECT IN THE HFS





- Leverage radiative corrections:
  1. Prediction for μH HFS from empirical 1S HFS in H

$$E^{\mathrm{hadr}}_{nS-\mathrm{hfs}}(\mu\mathrm{H}) = \frac{E_\mathrm{F}(\mu\mathrm{H})\, m_r(\mu\mathrm{H})\, b_{nS}(\mu\mathrm{H})}{n^3 E_\mathrm{F}(\mathrm{H})\, m_r(\mathrm{H})\, b_{1S}(\mathrm{H})} E^{\mathrm{hadr}}_{1S-\mathrm{hfs}}(\mathrm{H}) - \frac{E_\mathrm{F}(\mu\mathrm{H})}{n^3}\Delta_{\mathrm{pol}}(\mu\mathrm{H}) \underbrace{\left[c_{1S}(\mathrm{H})\frac{b_{nS}(\mu\mathrm{H})}{b_{1S}(\mathrm{H})} - c_{nS}(\mu\mathrm{H})\right]}_{\substack{= -6\times10^{-5} \quad \mathrm{n}=1 \\ = -5\times10^{-5} \quad \mathrm{n}=2}}$$

  2. Disentangle Zemach radius and polarizability contribution

POLARIZABILITY EFFECT IN THE HFS

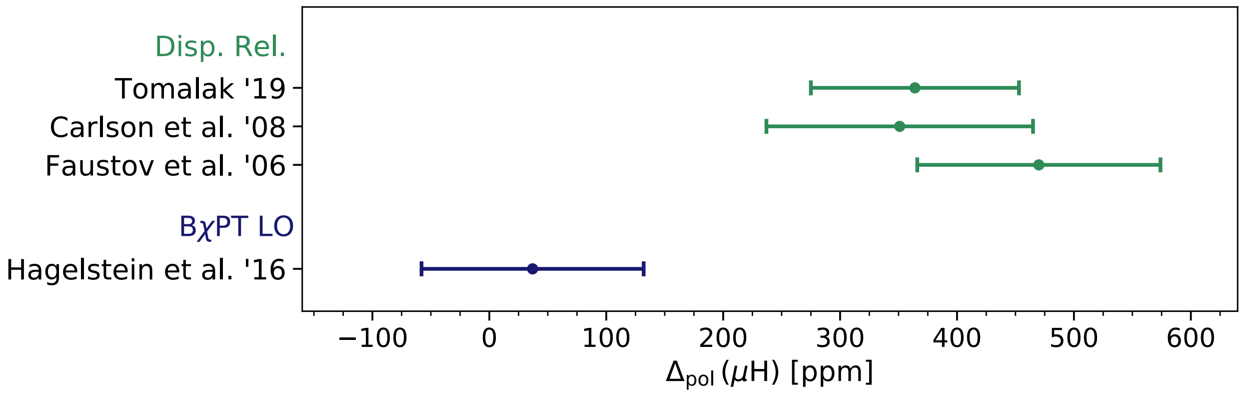
$$\Delta_{\mathrm{pol}} = \frac{\alpha m}{2\pi(1+\kappa)M}\left[\Delta_1 + \Delta_2\right]$$

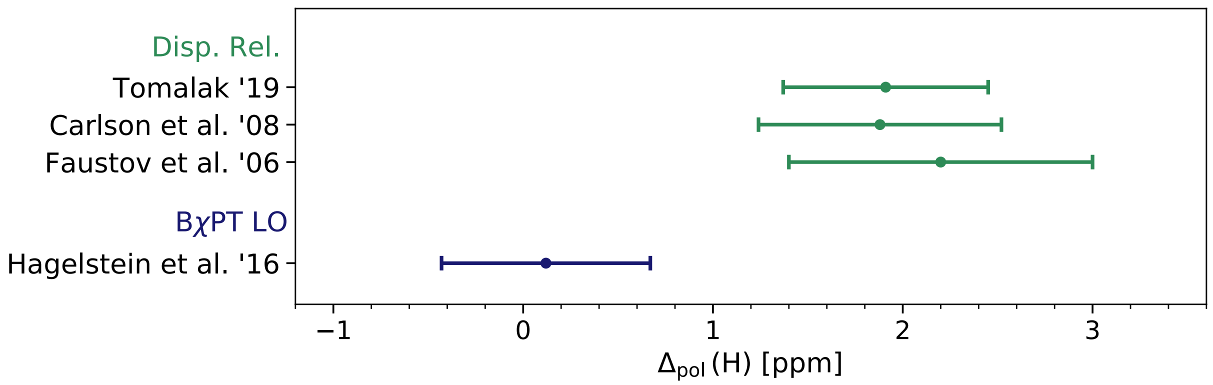
$$\Delta_1 = 2\int_0^\infty \frac{\mathrm{d}Q}{Q}\left(\frac{5+4v_l}{(v_l+1)^2}\left[4I_1(Q^2)+F_2^2(Q^2)\right] - \frac{32M^4}{Q^4}\int_0^{x_0}\mathrm{d}x\, x^2 g_1(x,Q^2)\right.$$
$$\left.\times\left\{\frac{1}{(v_l+\sqrt{1+x^2\tau^{-1}})(1+\sqrt{1+x^2\tau^{-1}})(1+v_l)}\left[4+\frac{1}{1+\sqrt{1+x^2\tau^{-1}}}+\frac{1}{v_l+1}\right]\right\}\right)$$

$$\Delta_2 = 96M^2\int_0^\infty \frac{\mathrm{d}Q}{Q^3}\int_0^{x_0}\mathrm{d}x\, g_2(x,Q^2)\left\{\frac{1}{v_l+\sqrt{1+x^2\tau^{-1}}} - \frac{1}{v_l+1}\right\}$$

- Polarizability effect on the HFS is completely constrained by empirical information
- ChPT calculation puts the reliability of dispersive calculations (and ChPT) to the test ?!

Tension between the BChPT prediction and data-driven dispersive results:



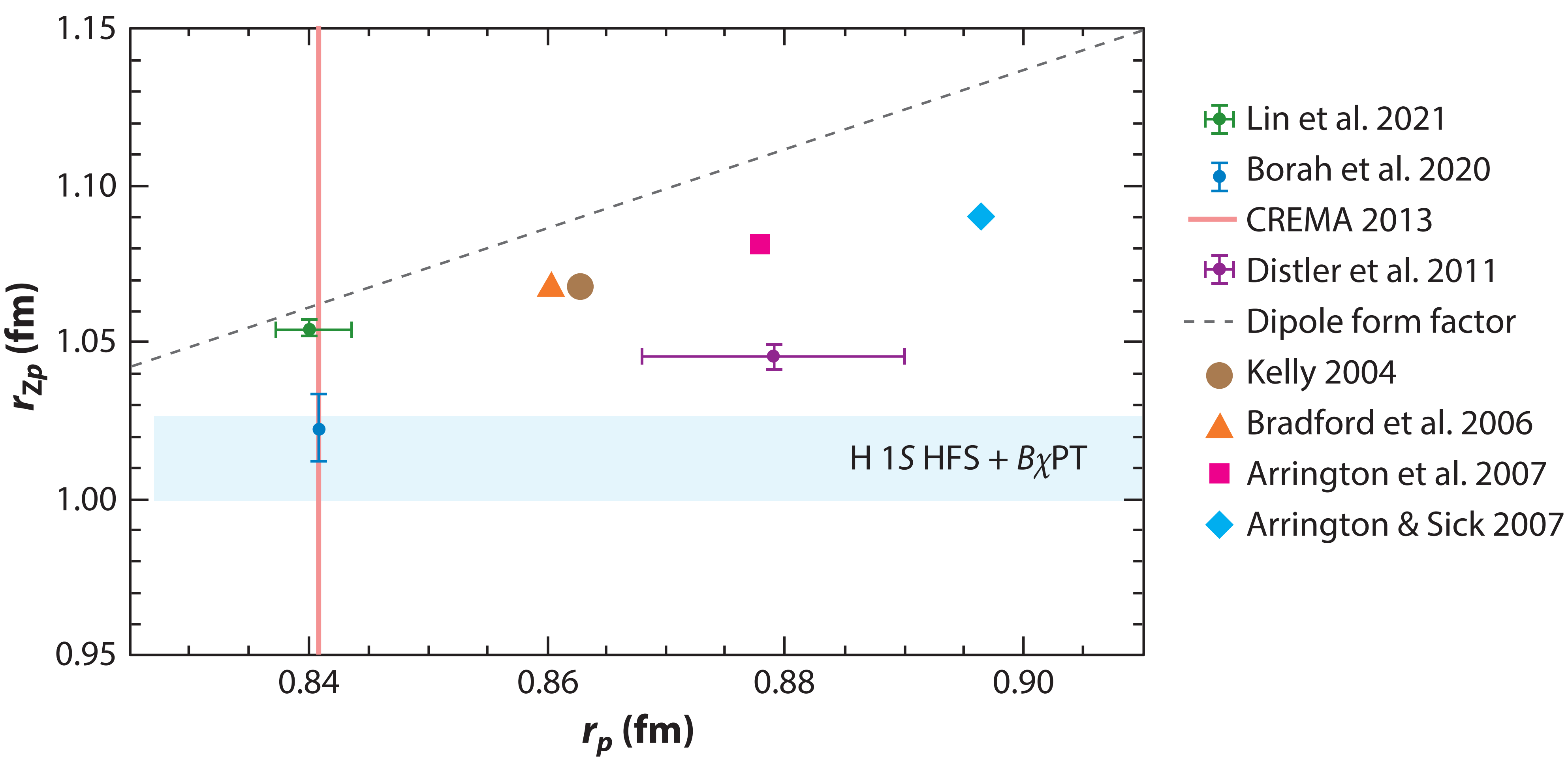


PROTON ZEMACH RADIUS

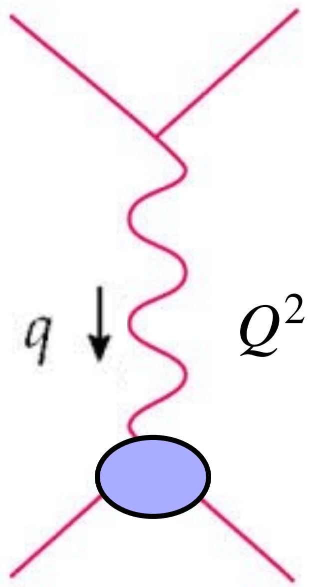
- Changes Zemach radius (smaller, just like r_p)

Table 2 Determinations of the proton Zemach radius r_{Zp}, in units of fm.

ep scattering		μH 2*S* hfs		H 1*S* hfs	
Lin *et al.* (26)	Borah *et al.* (91)	Antognini *et al.* (2)	BχPT (62)	Volotka *et al.* (92)	BχPT (62)
$1.054^{+0.003}_{-0.002}$	1.0227(107)	1.082(37)	1.041(31)	1.045(16)	1.012(14)



Radius from elastic e-p scattering

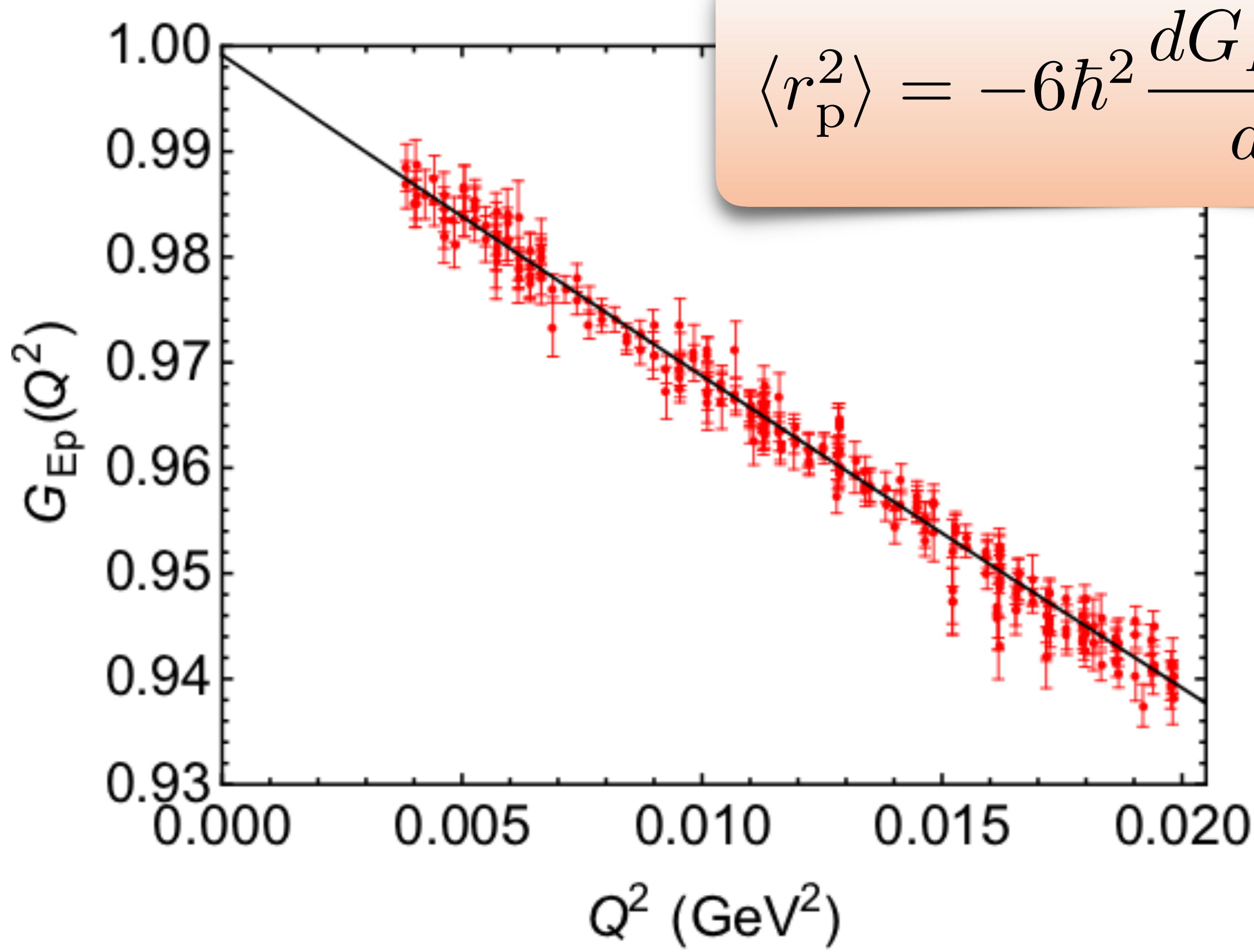


$G_E(Q^2), G_M(Q^2)$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Ros.}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{1}{(1+\tau)}\left(\varepsilon G_E^2(Q^2) + \tau G_M^2(Q^2)\right)$$

with $\tau = Q^2/4M_p^2, \quad \varepsilon \lesssim 1$

$$\langle r_{\rm p}^2 \rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2}\bigg|_{Q^2=0}$$



Caveat:
Radius extraction involves extrapolation to 0

data points: J. C. Bernauer *et al.*, Phys. Rev. C**90**,015206 (2014).

Ongoing and planned scattering experiments

Experiment	Beam	Laboratory	Q^2 $[(\mathrm{GeV}/c)^2]$	δr_p (fm)	Status
MUSE	$e^{\pm}$, $\mu^{\pm}$	PSI	0.0015–0.08	0.01	Ongoing
AMBER	$\mu^{\pm}$	CERN	0.001–0.04	0.01	Future
PRad-II	e^{-}	Jefferson Lab	4×10^{-5}–6×10^{-2}	0.0036	Future
PRES	e^{-}	Mainz	0.001–0.04	0.6% (relative)	Future
A1@MAMI (jet target)	e^{-}	Mainz	0.004–0.085		Ongoing
MAGIX@MESA	e^{-}	Mainz	$\geq 10^{-4} - 0.085$		Future
ULQ^2	e^{-}	Tohoku University	3×10^{-4}–8×10^{-3}	$\sim$1% (relative)	Future

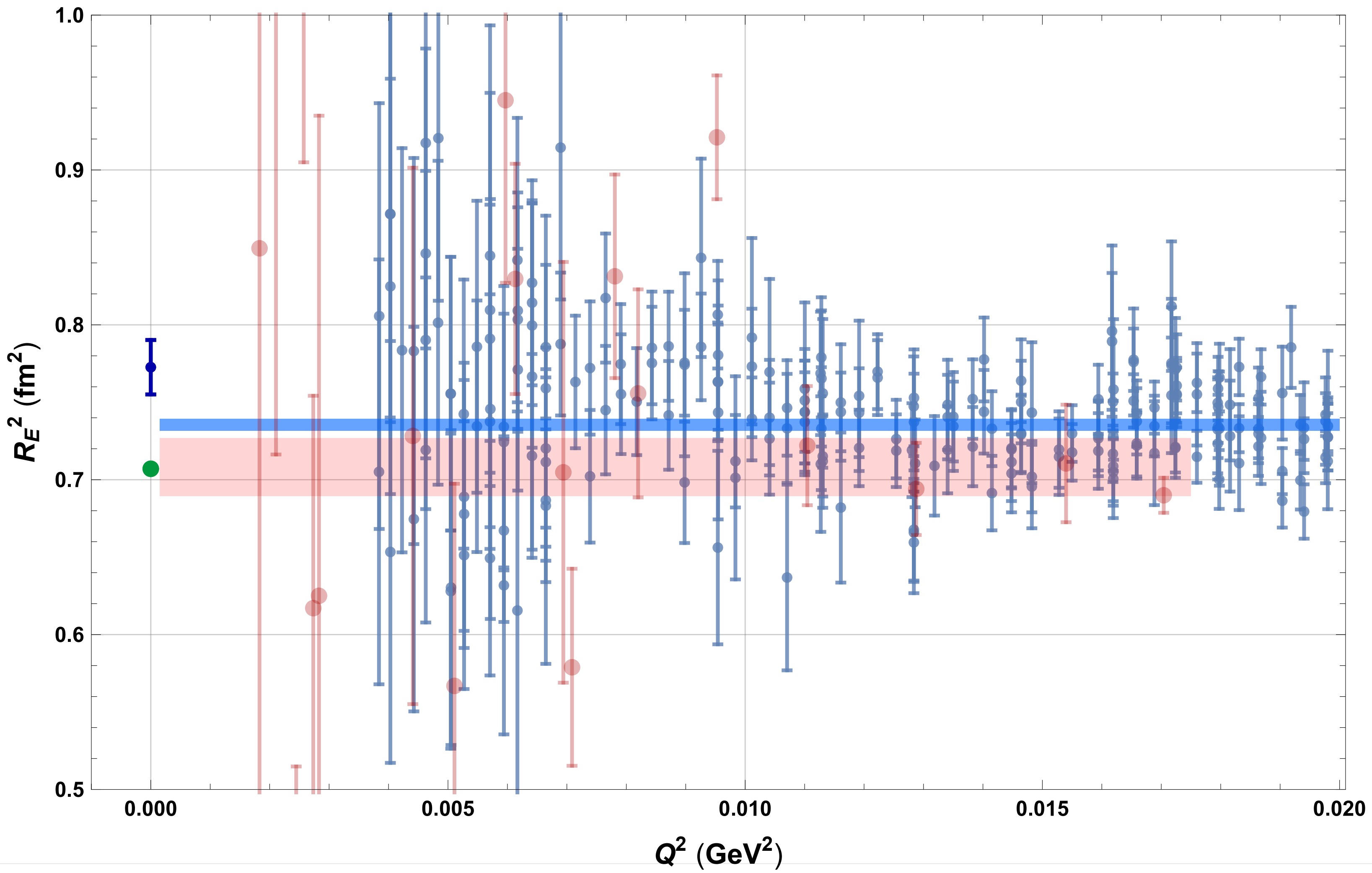
Lower bound directly from e-p data

$$R_E^2(Q^2) = -\frac{6}{Q^2}\log G_E(Q^2) \quad \xrightarrow[Q^2=0]{} R_E^2$$

This function sets a lower bound:

$$R_E^2(Q^2) \leq R_E^2\,, \quad \text{for } Q^2 \geq 0$$

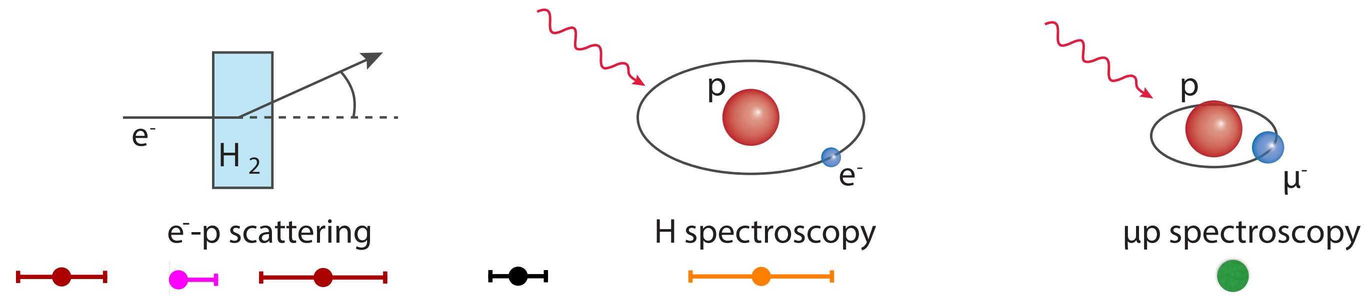
Hagelstein & VP, Phys. Lett. B (2019).

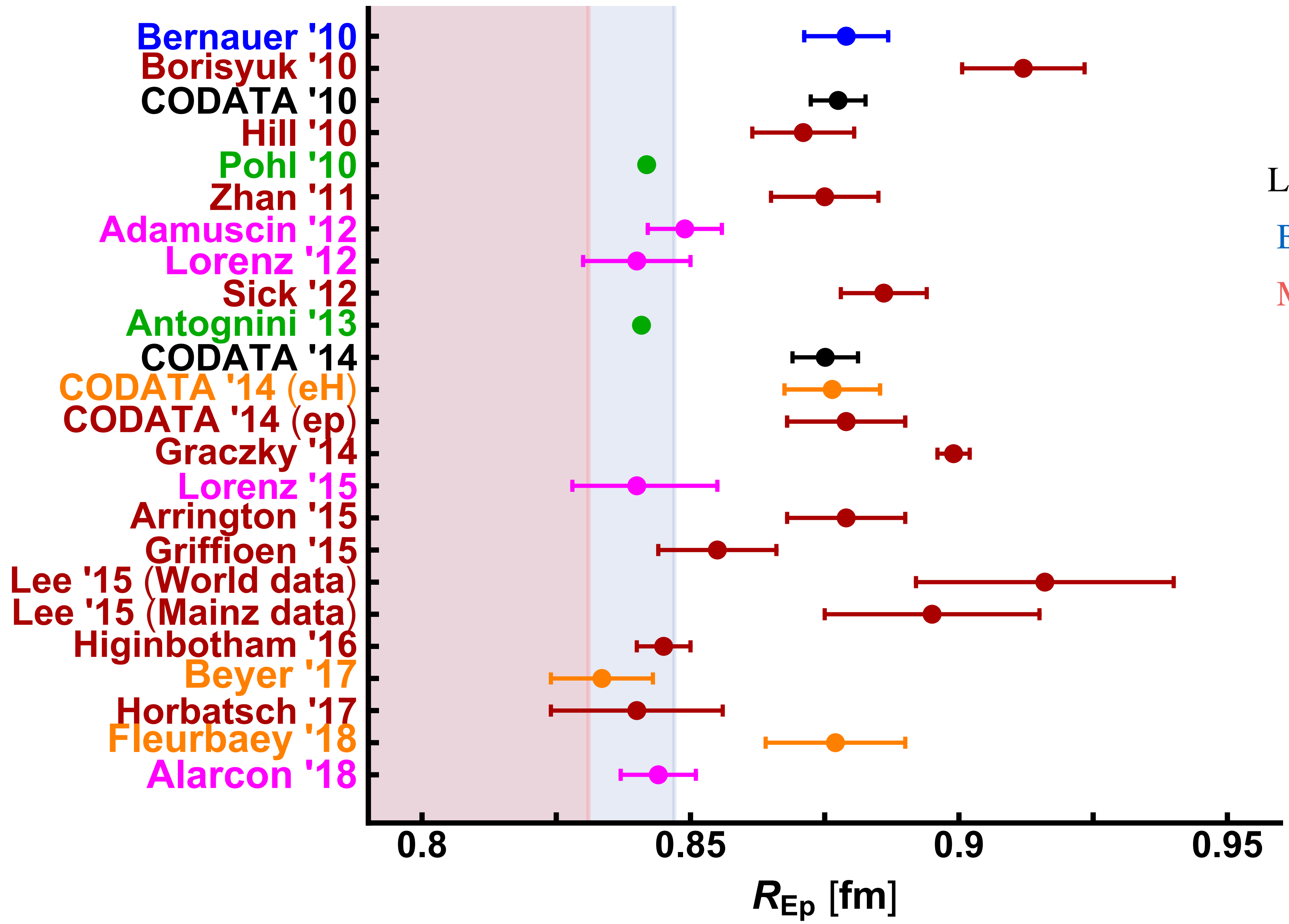


Data points from A1 Coll.:
Bernauer et al (2010)
Mihovilovic et al (2017)

No extrapolation required

Various extractions





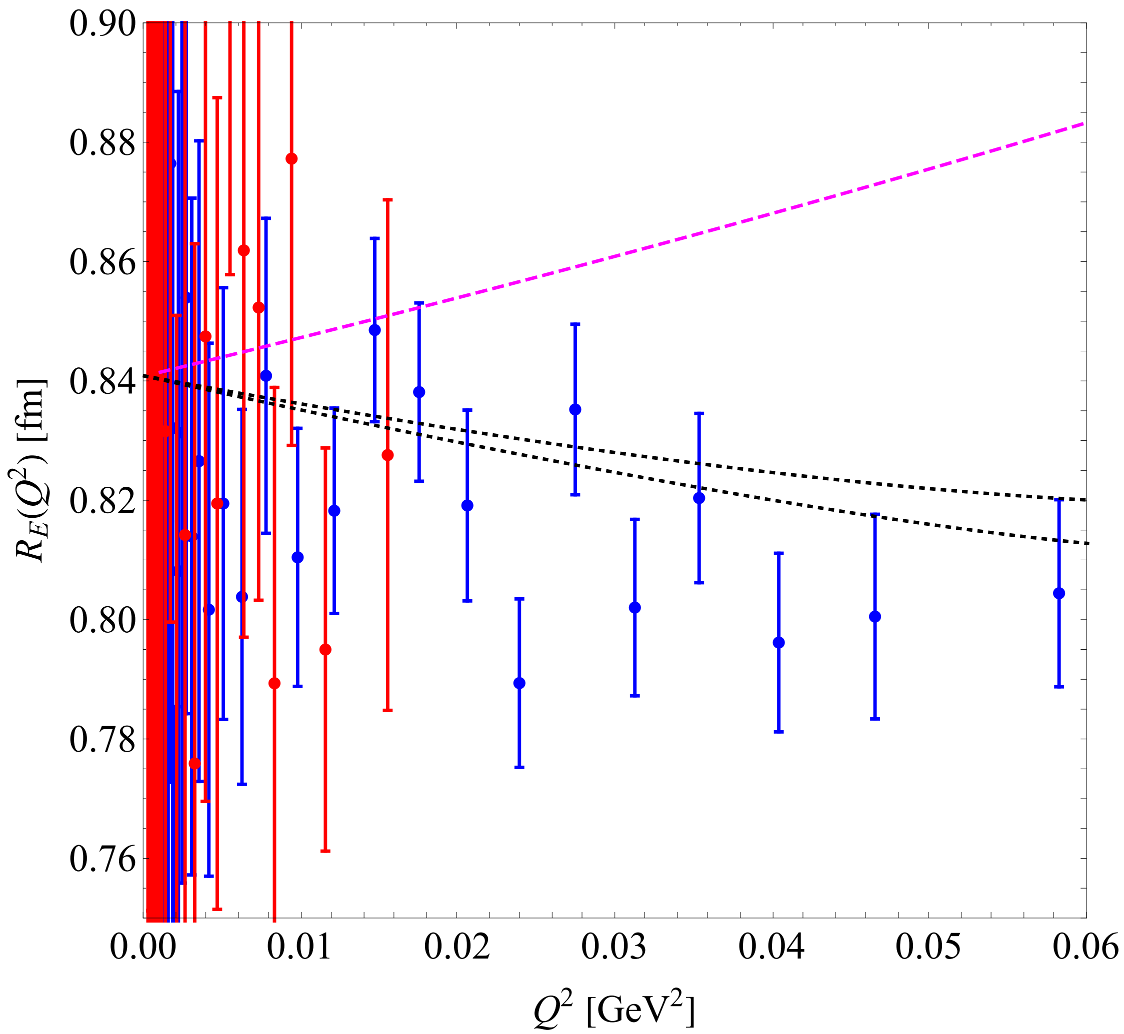
Lower bounds based on:

Bernauer et al (2010)

Mihovilovic et al (2017)

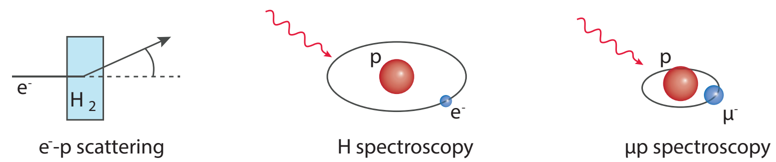
Lower bound from PRad data — uncertain?

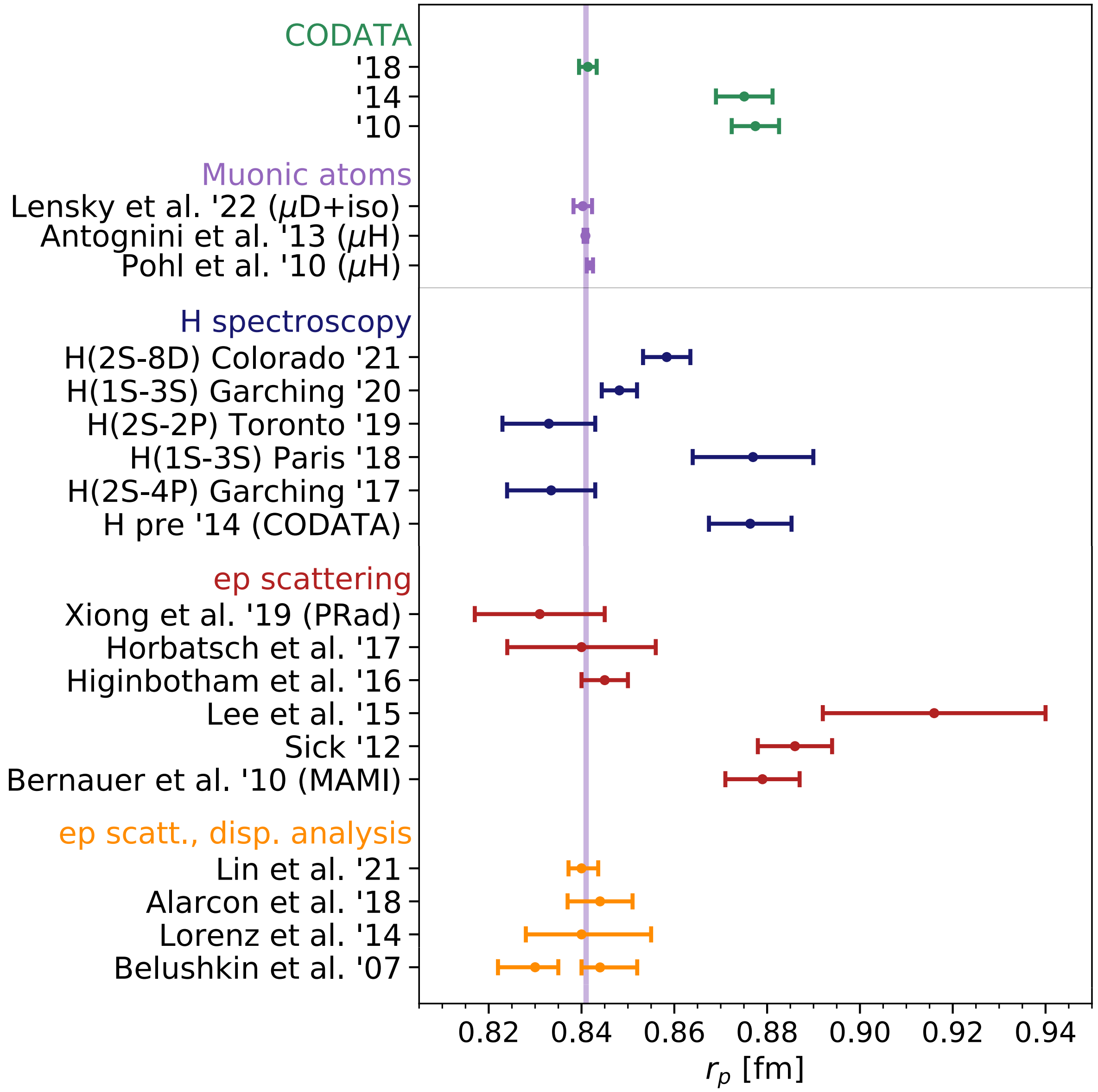
PRad data: 1.1 GeV and 2.2 GeV

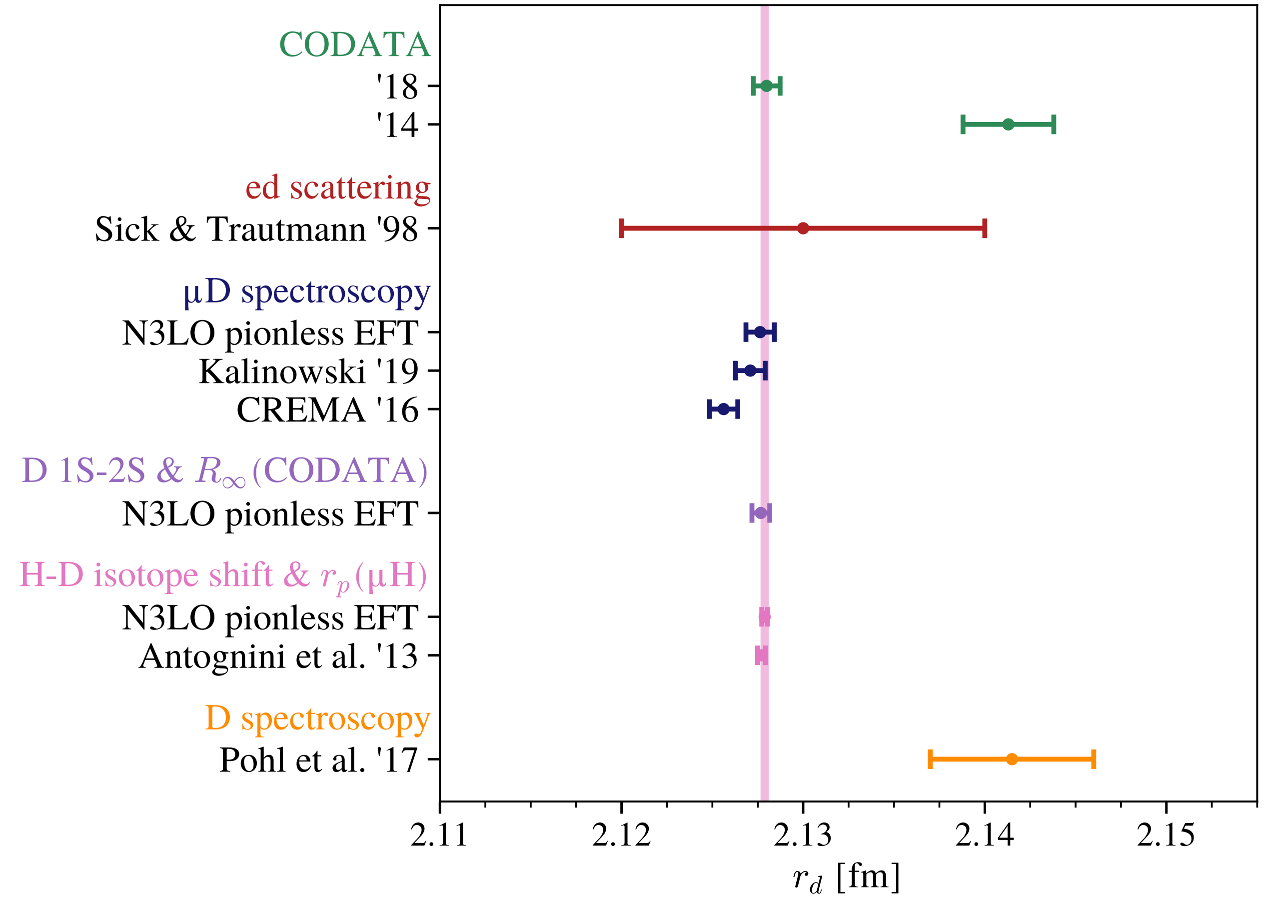


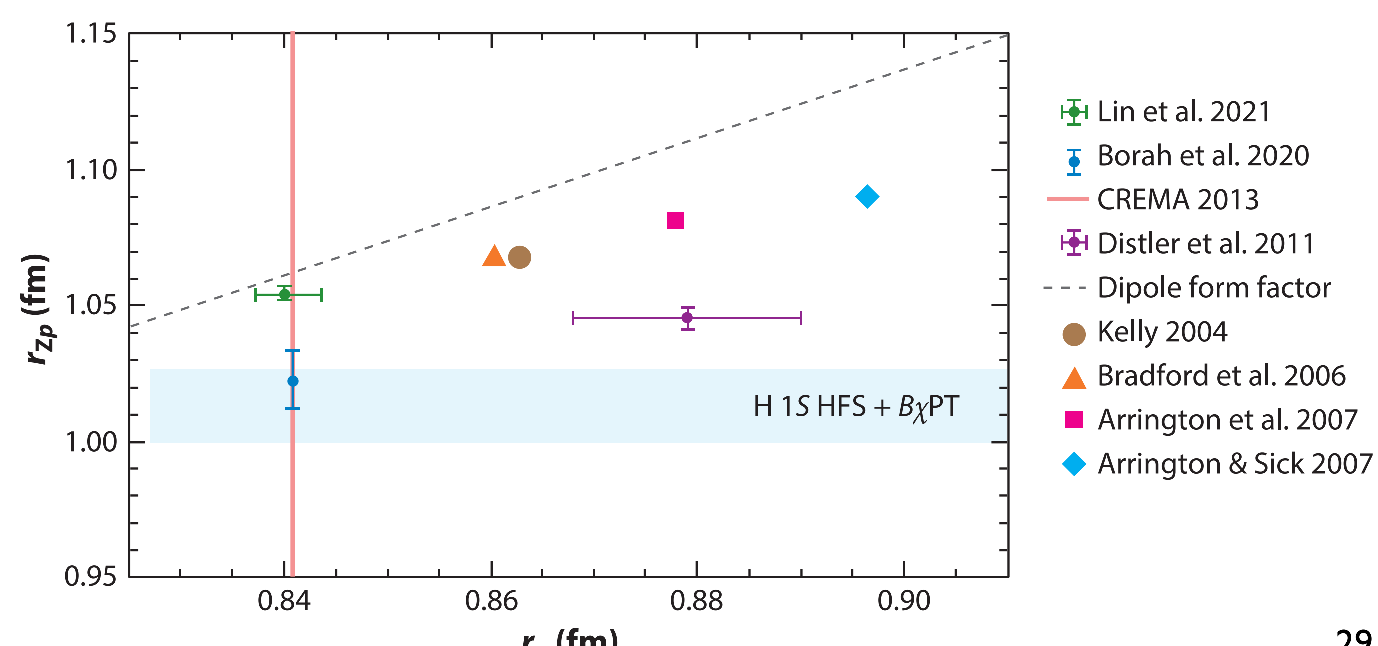
M. Horbatsch, Phys. Lett. B **804** (2020) 135373

Summary and conclusion



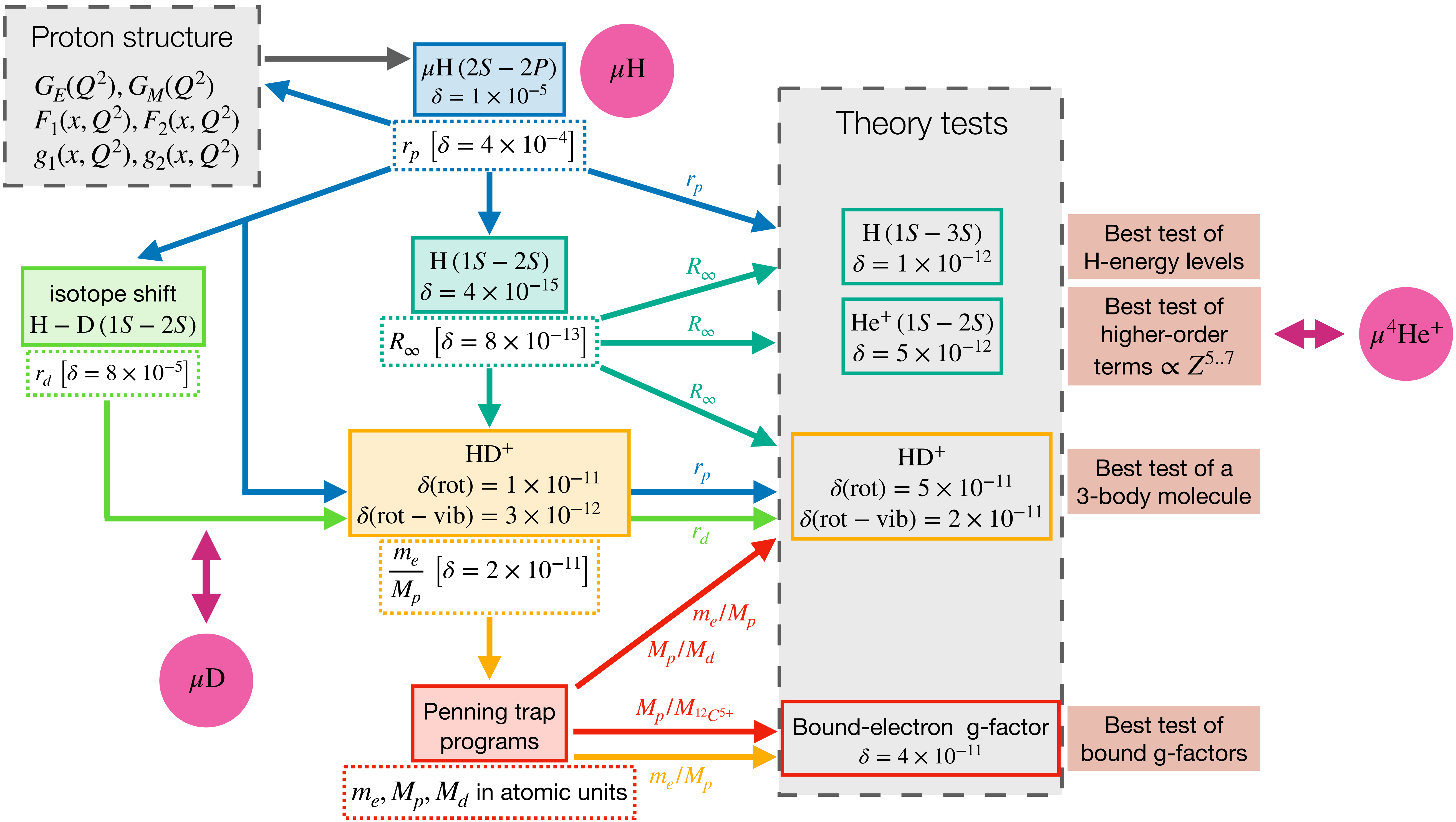






IMPACT MUONIC ATOMS

Antognini, Hagelstein & VP, Ann. Rev. Nucl. Part. **72** (2022) [arXiv:2205.10076]



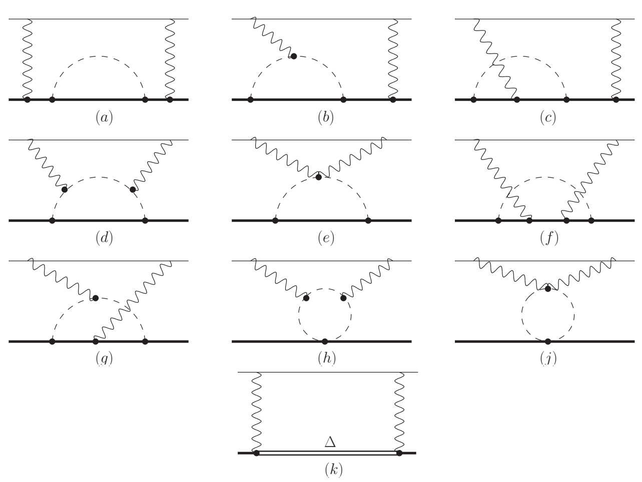
Backup

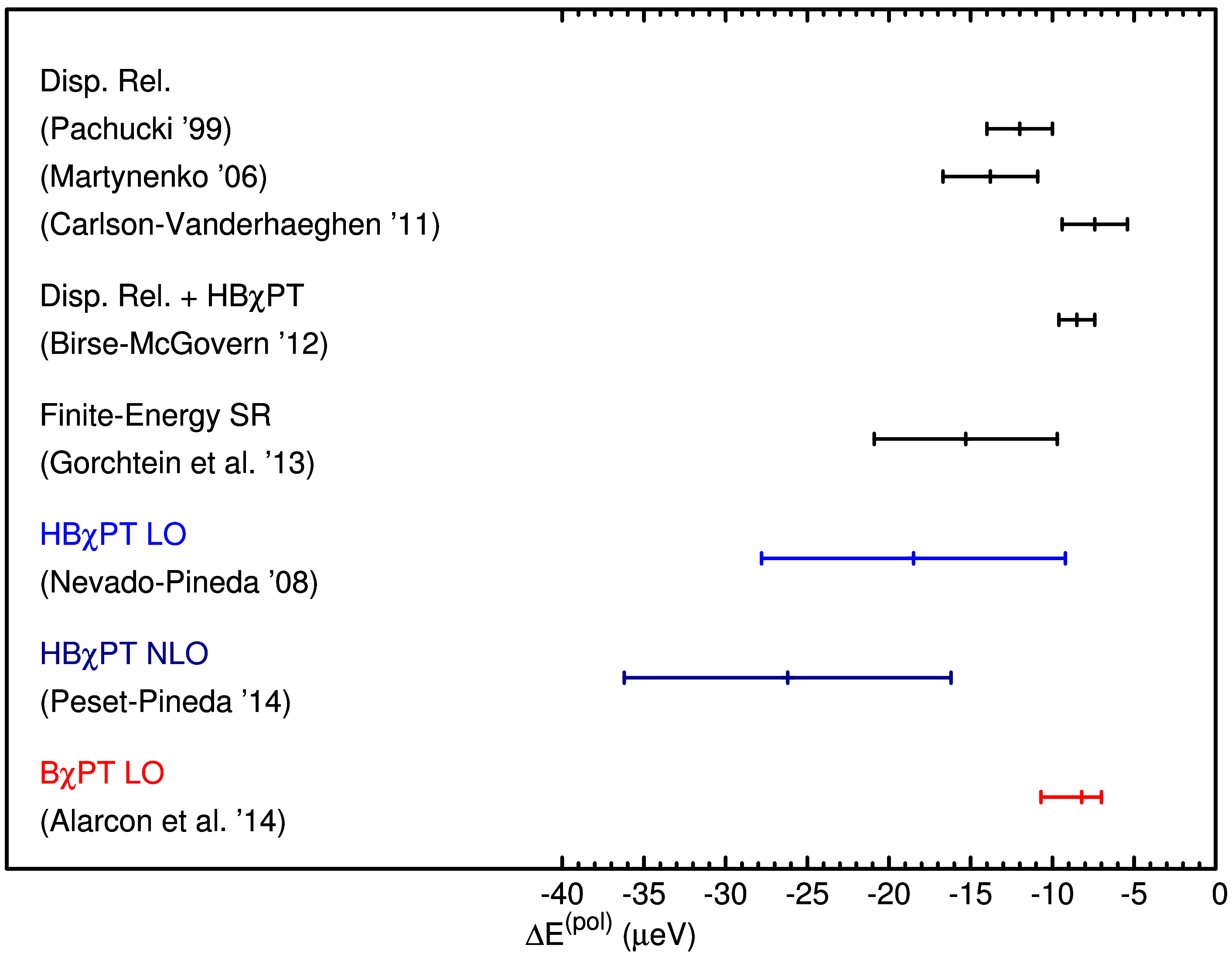
Proton polarizability in muonic-H Lamb shift

Can be computed with dispersion th. + data

But subtraction term is needed — model dependent

vs.
Chiral perturbation theory predictive at LO

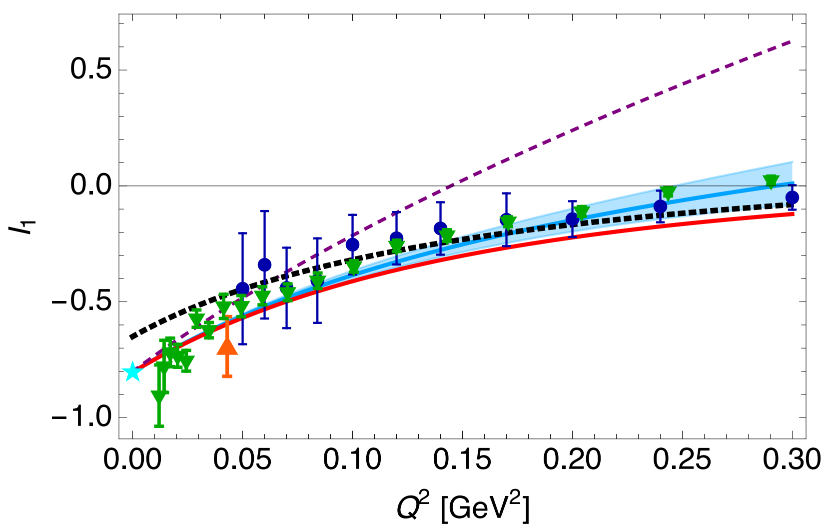


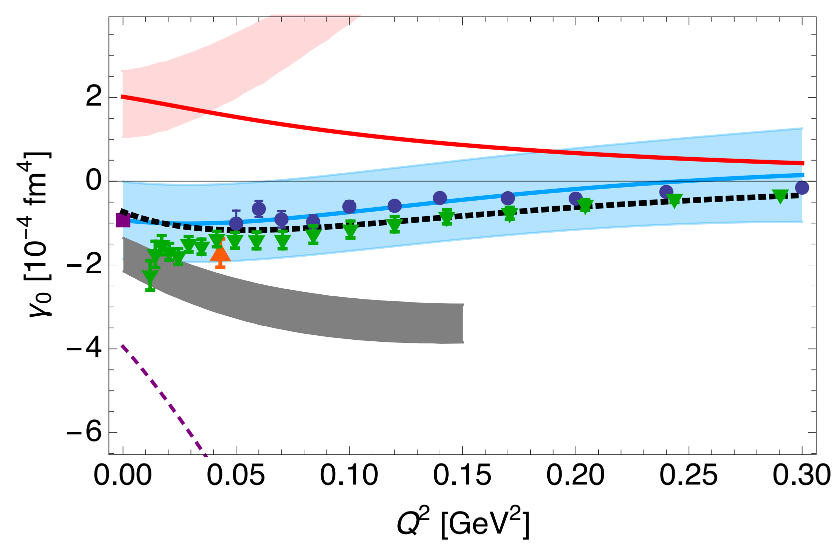


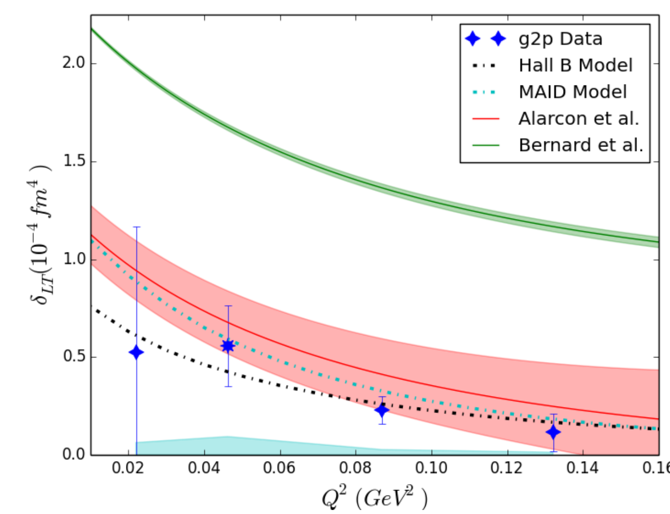
Compiled by: Hagelstein, Miskimen & VP, *Prog. Part. Nucl. Phys. (2016)*

DISCREPANCY IN THE HFS

- Empirical information on spin structure functions is limited
- Low-Q region is very important (cancelation between $I_1(Q^2)$ and $F_2(Q^2)$)







New data JLab Spin Physics Programme, e.g., g2p 2204.10224.