Nuclear structure corrections in muonic atoms

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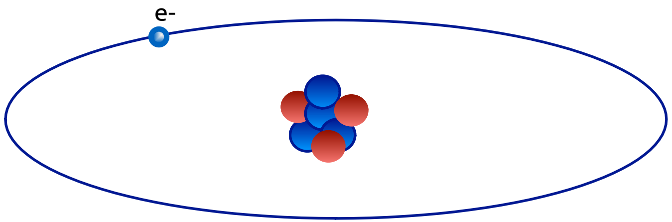
JG|U

Image credit: Oak Ridge National Laboratory, US department of energy;
Conceptual art by LeJean Hardin and Andy Sproles

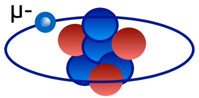
Muonic atoms

Hydrogen-like systems

Ordinary atoms



Muonic atoms



The muon is more sensitive to the nucleus

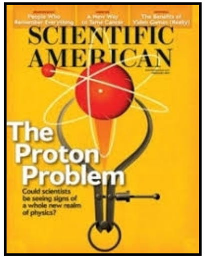
Excellent precision probe for the nucleus

Experimental program at **PSI** of the **CREMA** collaboration

Muonic Hydrogen
- Pohl et al., Nature (2010)
- Antognini et al., Science (2013)

Muonic Deuterium
- Pohl et al., Science (2016)

Muonic Helium-4
- Krauth et al., Nature (2021)



Lamb-shift and charge radius

$$\delta_{\mathrm{LS}} = \delta_{\mathrm{QED+NR}} + \delta_{\mathrm{NS}}^{(4)} \times r_c^2 + \delta_{\mathrm{NS}}^{(5)} + \delta_{\mathrm{NS}}^{(6)} + \ldots$$

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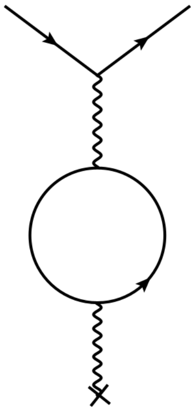
$\delta_{\mathrm{QED}} =$



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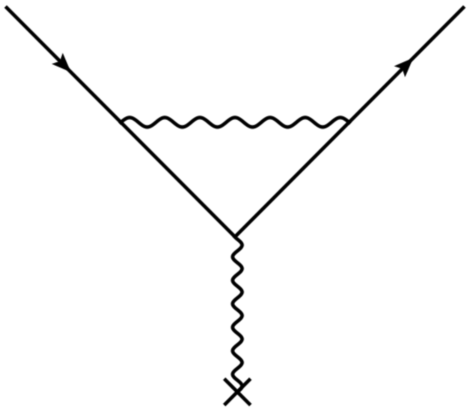

$+ \quad \cdots$

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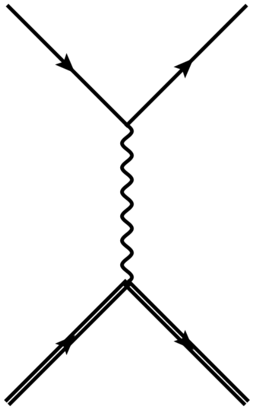
$\delta_{\mathrm{NR}} =$



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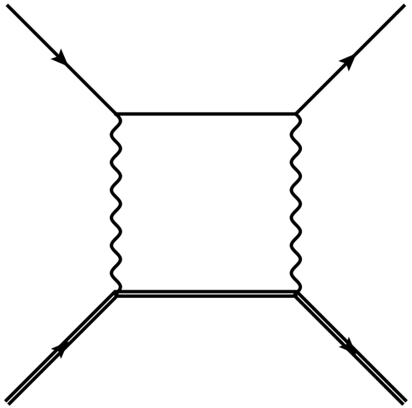
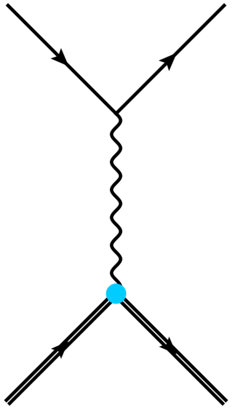
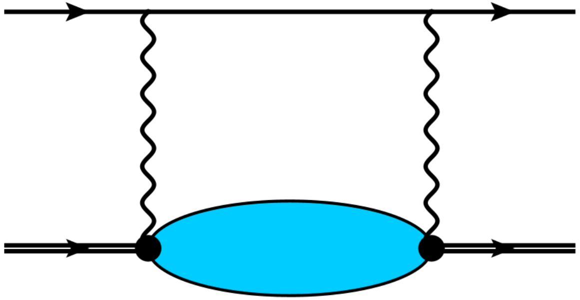


$+ \quad \cdots$

Lamb-shift and charge radius

$$\delta_{\mathrm{LS}} = \delta_{\mathrm{QED+NR}} + \delta^{(4)}_{\mathrm{NS}} \times r_c^2 + \delta^{(5)}_{\mathrm{NS}} + \delta^{(6)}_{\mathrm{NS}} + \ldots$$

$\delta_{\mathrm{NS}} =$  $+$  $+ \quad \cdots$

A matter of precision

$$\delta_{\mathrm{LS}} = \delta_{\mathrm{QED+NR}} + \delta_{\mathrm{NS}}^{(4)} \times r_c^2 + \delta_{\mathrm{NS}}^{(5)} + \delta_{\mathrm{NS}}^{(6)} + \dots$$

For the muonic Helium-4 ion

$$\delta_{\mathrm{QED+NR}} = +1,668.489(14)\ \mathrm{meV}$$
$$\delta_{\mathrm{NS}}^{(4)} = -106.220(8)\ \mathrm{meV\ fm}^{-2}$$
$$\delta_{\mathrm{NS}}^{(5)} = +9.340(250)\ \mathrm{meV}$$
$$\delta_{\mathrm{NS}}^{(6)} = -0.150(150)\ \mathrm{meV}$$

→

$$r_c = 1.67824(13)_{\mathrm{ex}}(82)_{\mathrm{th}}\ \mathrm{fm}$$

J. J. Krauth et. al. Nature 589,527 (2021)

The Uncertainty is largely dominated by the nuclear structure effects.

Outline

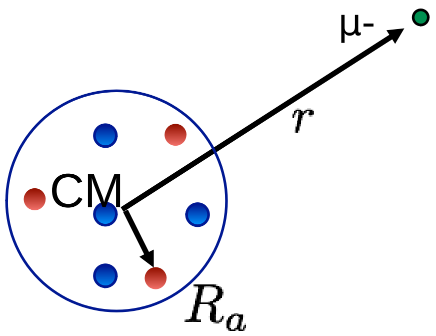
Theory

Evaluation of the NS effects

$$\mathrm{H} = \mathrm{H}_N + H_\mu + \Delta V$$

The Nuclear Hamiltonian is included or not depending on whether we look for nuclear sizes or polarizability corrections

$$H_\mu = \frac{\mathbf{p}^2}{2m_r} - \frac{Z\alpha}{r}$$



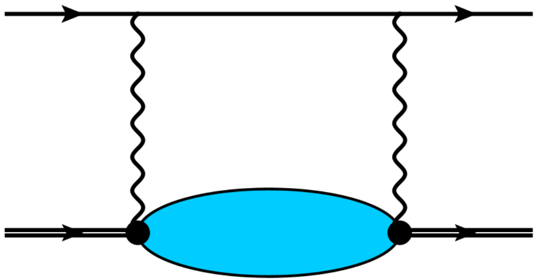
The corrections to the bulk coulomb interactions are included in perturbation theory

$$\Delta V = \sum_a^Z \alpha \left(\frac{1}{r} - \frac{1}{|\mathbf{r} - \mathbf{R}_a|} \right)$$

Evaluation of the NS effects

$$\delta_{\rm NS} = \delta_{\rm FS} + \delta_{\rm pol}$$

$$\delta^{(5)}_{\rm pol} = \langle N_0 \mu | \, \Delta V \; G \; \Delta V \, | N_0 \mu \rangle$$



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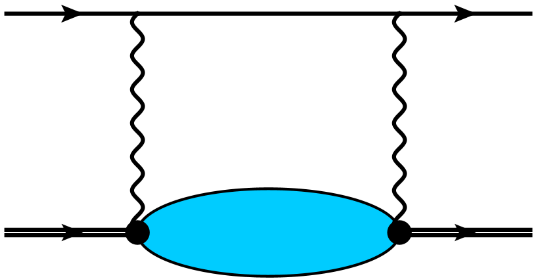
Work in coordinate space

$$= \sum_{N \neq N_0} \int d^3R \, d^3R' \; \rho^p_N(\mathbf{R}) \; W(\mathbf{R}, \mathbf{R}', \omega_N) \; \rho^p_N(\mathbf{R}')$$



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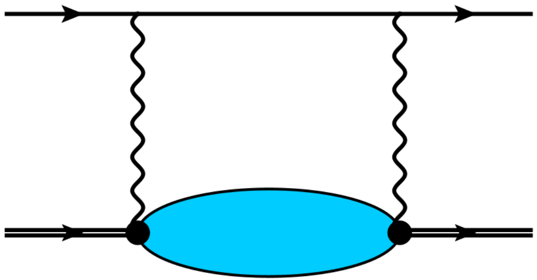

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Taylor expand the lepton matrix element

$$= C \sum_{N \neq N_0} \left(\frac{1}{\omega_N} \right)^{\frac{3}{2}} \int d^3R \; d^3R' \;\; \rho^p_N(\mathbf{R}) \left[\eta^2 - \frac{1}{4}\eta^3 + \frac{1}{20}\eta^4 + \ldots \right] \rho^p_N(\mathbf{R}')$$

Evaluation of the NS effects

$$\delta_{\text{pol}}^{(5)} = C \sum_{N \neq N_0} \left(\frac{1}{\omega_N}\right)^{\frac{3}{2}} \int d^3R \, d^3R' \; \rho_N^p(\mathbf{R}) \left[\eta^2 - \frac{1}{4}\eta^3 + \frac{1}{20}\eta^4 + \ldots\right] \rho_N^p(\mathbf{R}')$$

LO NLO N2LO

$$\eta = \sqrt{2 m_r \omega_N} |\mathbf{R} - \mathbf{R}'| \approx \sqrt{\frac{m_r}{m_N}} \approx 0.33$$

. . . Some more algebra . . .

$$\delta_{\text{pol}}^{(5)} = \sum_i \left[C_i(Z\alpha, m_r) \int_0^\infty \mathcal{F}_i(\omega/m_r) \, S_{O_i}(\omega) \, d\omega \right]$$

Nuclear response function

Ab-initio Nuclear Theory

Ab-initio methods: Solutions of the time-independent Schrödinger equation for the nuclear states

$$\widehat{H}_{\mathrm{N}}\Psi_{\mathrm{N}} = E_{\mathrm{N}}\Psi_{\mathrm{N}}$$

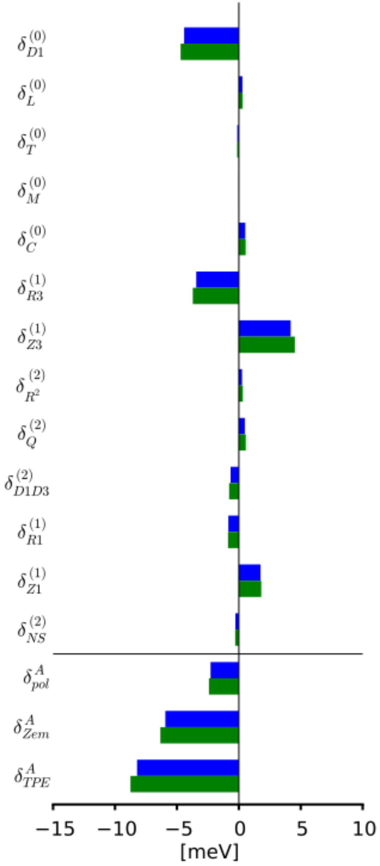
With **controlled approximations.**

Solutions of Schrödinger Eq. $\longrightarrow$ $S_{O_i}(\omega)$ $\longrightarrow$ $\delta^{(5)}_{\mathrm{pol}}$

The nuclear Hamiltonian is an input and a big source of uncertainty.

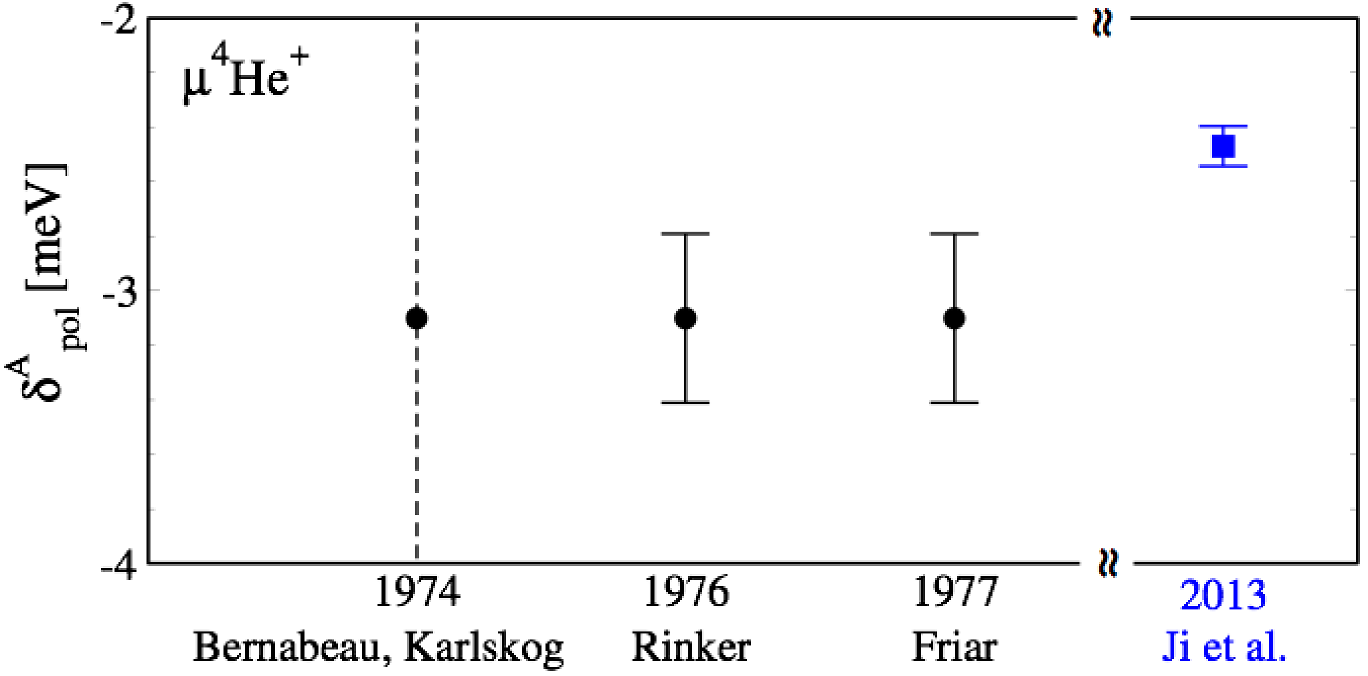
Previous work on Helium-4

C. Ji, et al. PRL 143402-1 (2013)
C. Ji, et al. J. Phys. G: Nucl. Part. Phys. 45 (2018)

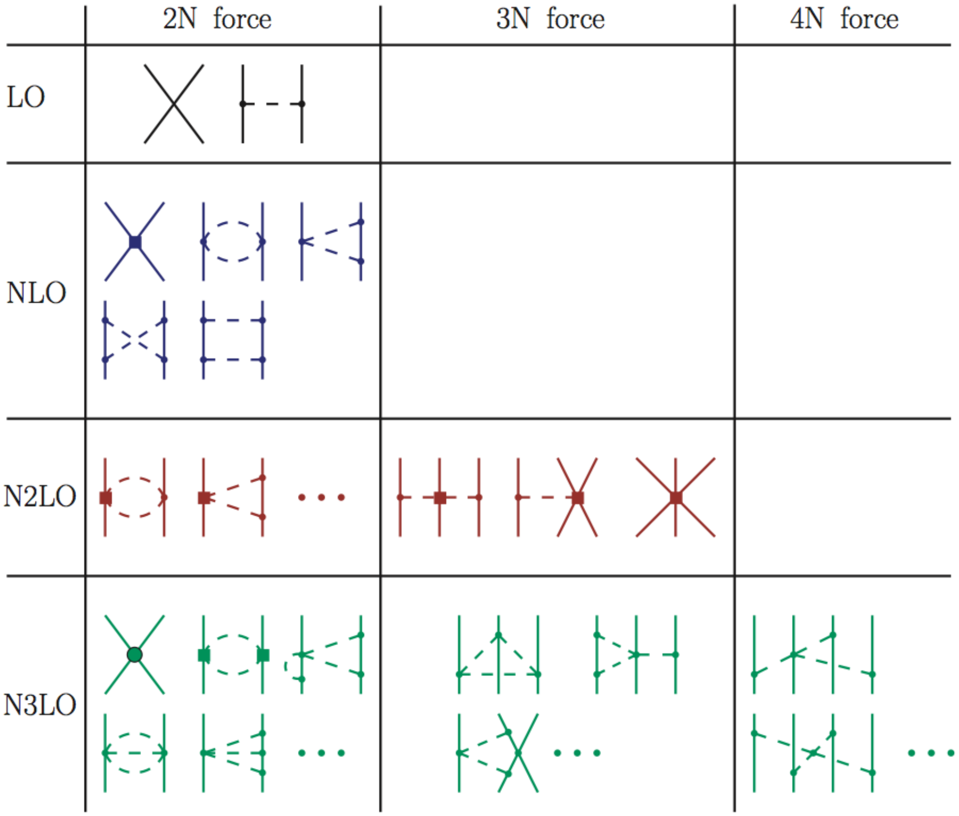


	AV18/UIX	χEFT
$\delta_{D1}^{(0)}$	-4.418	-4.701
$\delta_{L}^{(0)}$	0.289	0.308
$\delta_{T}^{(0)}$	-0.126	-0.134
$\delta_{M}^{(0)}$	0.008	0.006
$\delta_{C}^{(0)}$	0.519	0.553
$\delta_{R3}^{(1)}$	-3.442	-3.717
$\delta_{Z3}^{(1)}$	4.183	4.526
$\delta_{R^2}^{(2)}$	0.259	0.324
$\delta_{Q}^{(2)}$	0.484	0.561
$\delta_{D1D3}^{(2)}$	-0.666	-0.784
$\delta_{R1}^{(1)}$	-0.848	-0.874
$\delta_{Z1}^{(1)}$	1.753	1.811
$\delta_{NS}^{(2)}$	-0.279	-0.293
δ_{pol}^{A}	-2.284	-2.414
δ_{Zem}^{A}	-5.936	-6.337
δ_{TPE}^{A}	-8.220	-8.751

Impact of ab-initio theory



The Hamiltonians



Interactions derived from the Chiral effective field theory and written in coordinate space.

Hierarchy among different operators decided after specifying a power counting scheme.

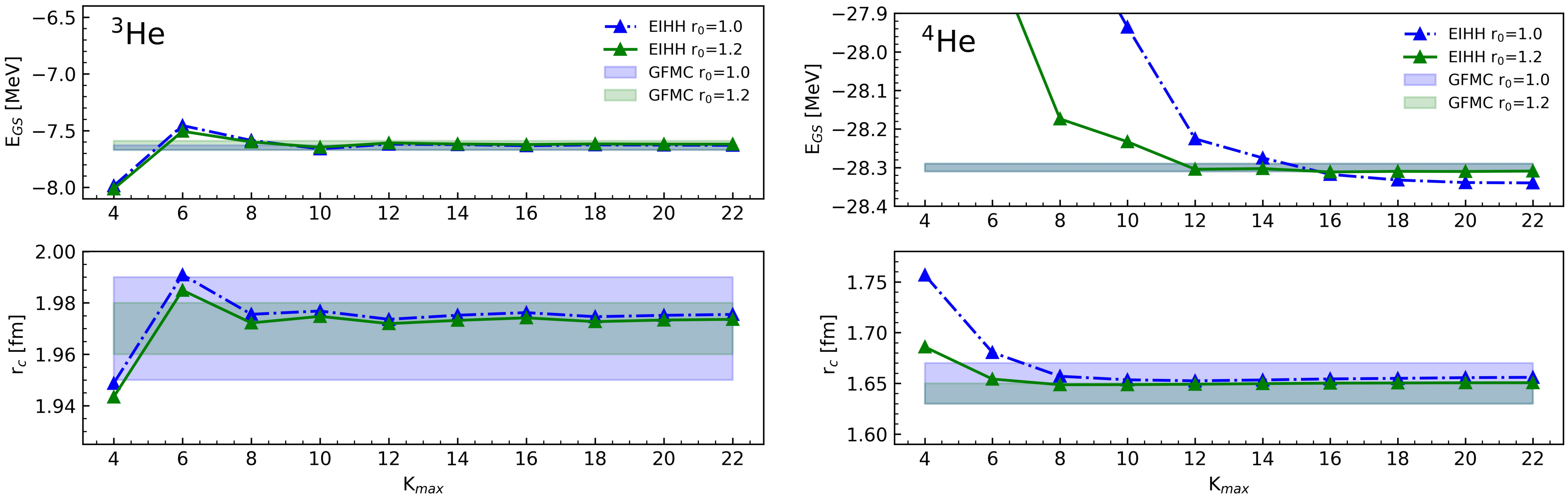
The separation of scales of the operators makes it possible to reliably quantify the uncertainties due to the truncation of the expansion (Ex. With Bayesian Statistics).

Outline

Results

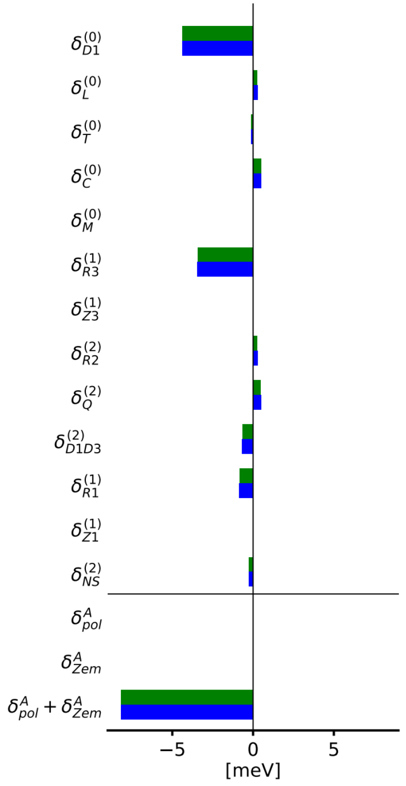
Benchmark tests

S.S. LM, S. Bacca , N. Barnea, Front. Phys. 9, 671869 (2021)

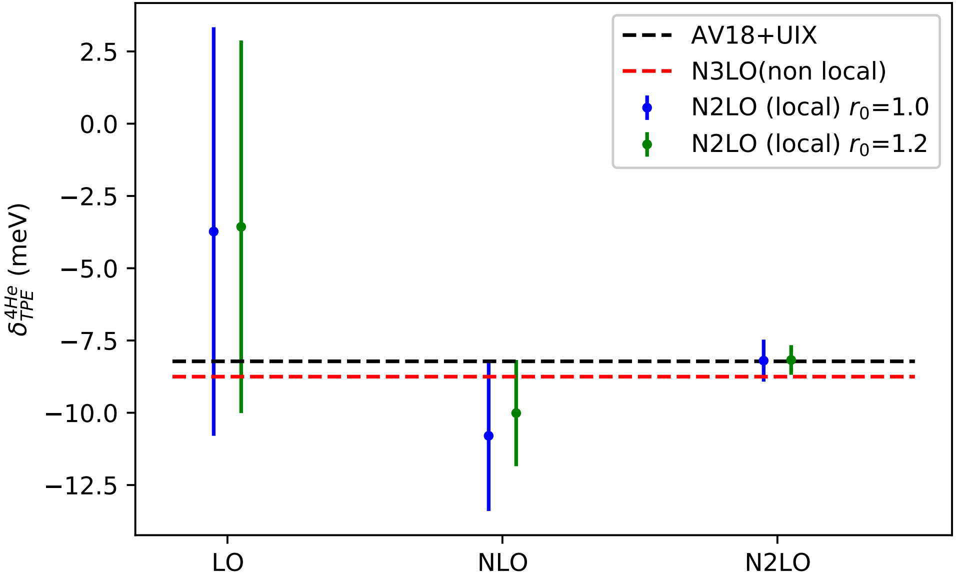


NS corrections in μ4He+ (EKM)

S.S.LM, et al. In preparation for 2022

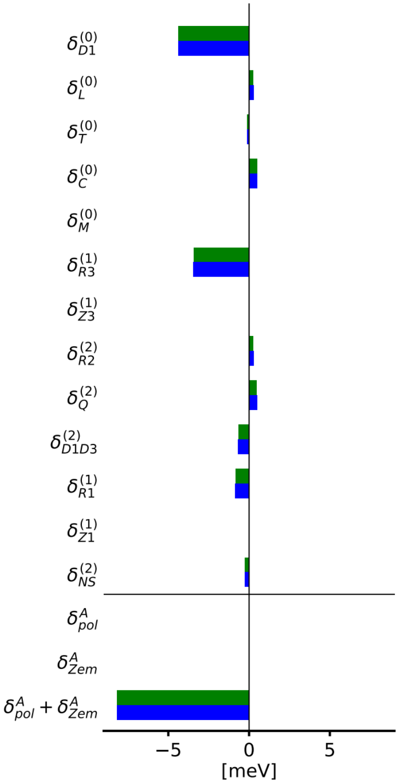


	$r_0 = 1.2$	$r_0 = 1.0$
$\delta_{D1}^{(0)}$	-4.386	-4.373
$\delta_{L}^{(0)}$	0.272	0.287
$\delta_{T}^{(0)}$	-0.124	-0.125
$\delta_{C}^{(0)}$	0.517	0.514
$\delta_{M}^{(0)}$	0.011	0.011
$\delta_{R3}^{(1)}$	-3.422	-3.477
$\delta_{Z3}^{(1)}$	-	-
$\delta_{R2}^{(2)}$	0.267	0.285
$\delta_{Q}^{(2)}$	0.484	0.505
$\delta_{D1D3}^{(2)}$	-0.668	-0.69
$\delta_{R1}^{(1)}$	-0.846	-0.856
$\delta_{Z1}^{(1)}$	-	-
$\delta_{NS}^{(2)}$	-0.272	-0.277
δ_{pol}^{A}	-	-
δ_{Zem}^{A}	-	-
$\delta_{pol}^{A} + \delta_{Zem}^{A}$	-8.174	-8.196



NS corrections in μ4He+ (Bayes)

S.S.LM, et al. In preparation for 2022



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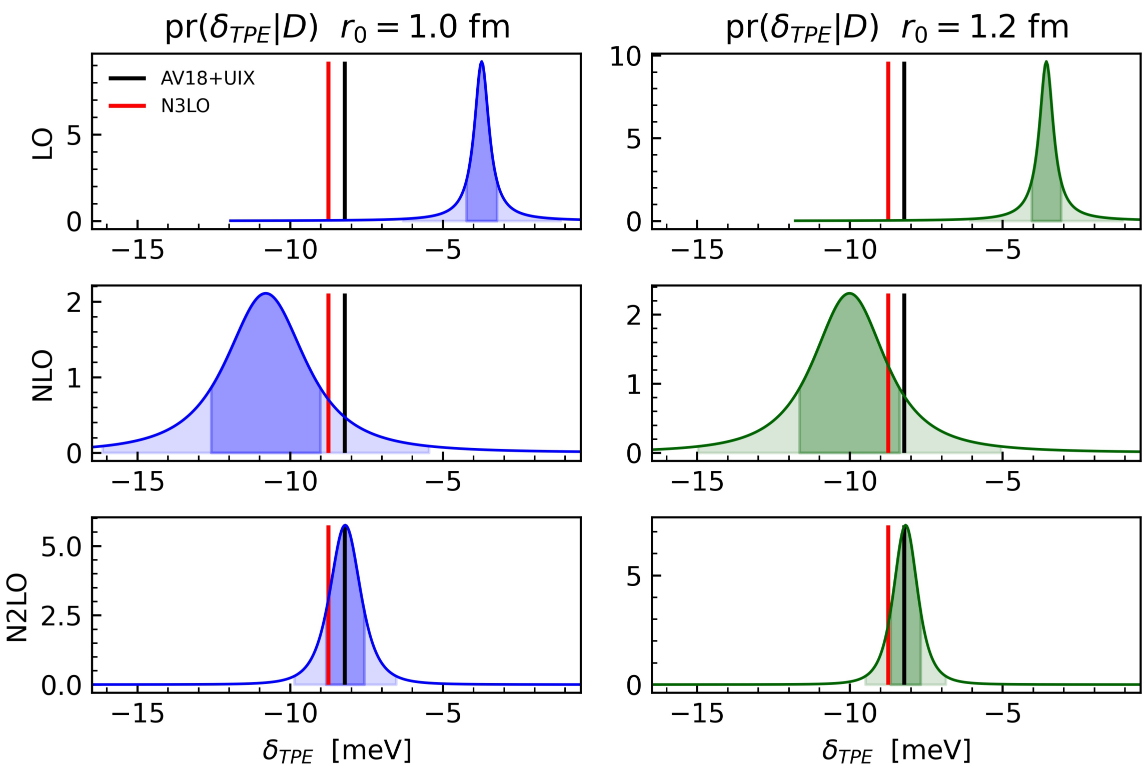
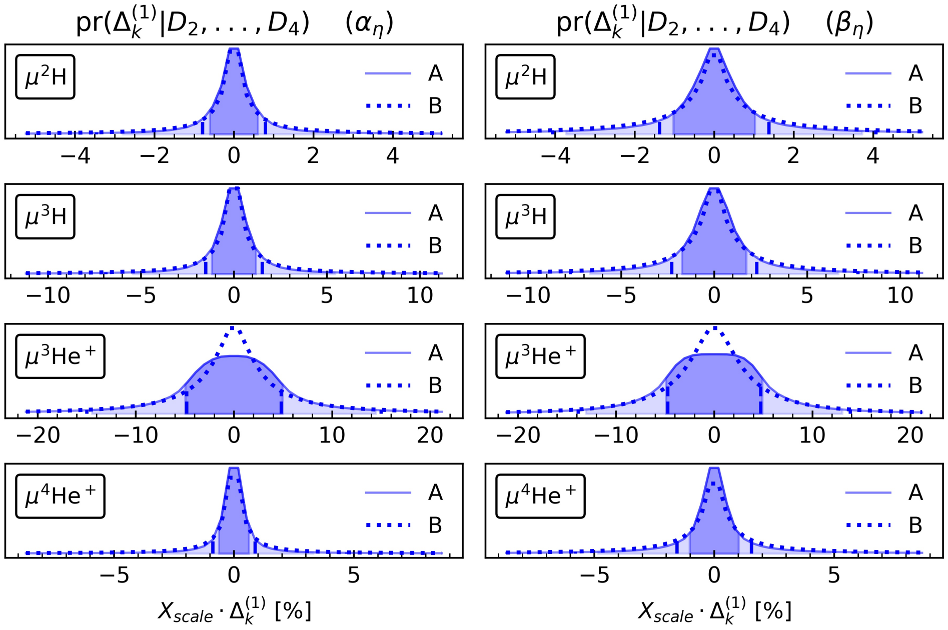


C. Ji, et al. J. Phys. G: Nucl. Part. Phys. 45 (2018)

Eta-expansion uncertainty

$$\delta^{(5)}_{\text{pol}} = C \sum_{N \neq N_0} \left(\frac{1}{\omega_N}\right)^{\frac{3}{2}} \int d^3R \, d^3R' \, \rho^p_N(\mathbf{R}) \left[\eta^2 - \frac{1}{4}\eta^3 + \frac{1}{20}\eta^4 + \ldots\right] \rho^p_N(\mathbf{R}')$$



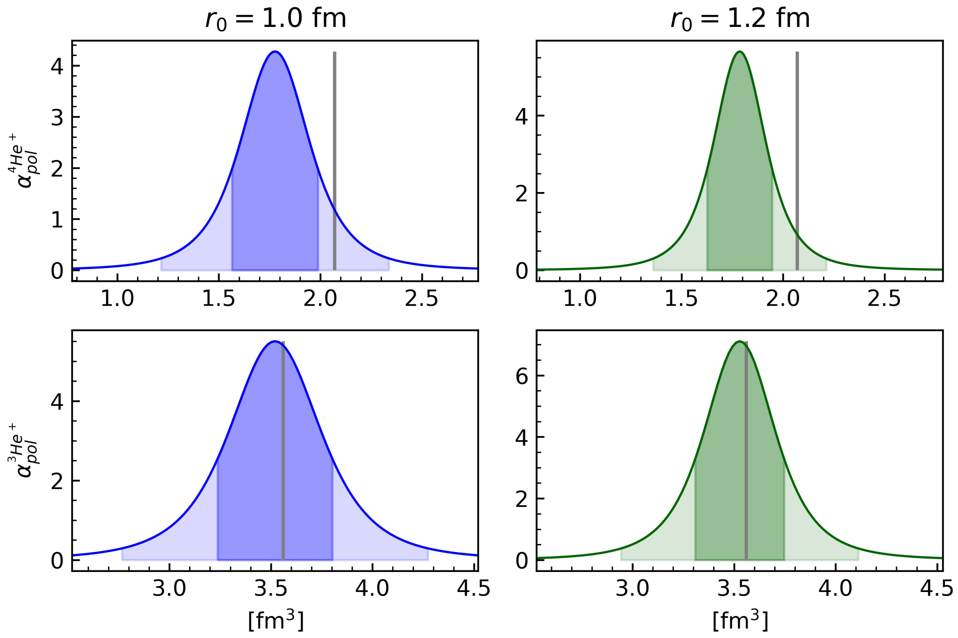
S.S.LM, et al. ArXiv:2203.10792

	μ^2H	μ^3H	μ^3He$^+$	μ^4He$^+$
[1]	0.4%	1.3%	1.1%	0.8%
This work	0.8%	1.5%	4.8%	0.9%

[1] C. Ji, et al. J. Phys. G: Nucl. Part. Phys. 45 (2018)

NS effects in 3He+ and 4He+

$$\delta^{(5)}_{\rm pol} = -m\alpha^4 \left\langle \sum_a \delta^3(r_a) \right\rangle (m^3 \alpha_{\rm pol})$$



S.S.LM, et al. In preparation for 2022

1S-2S	$^3He^+$	$^4He^+$
[1]	48(5)kHz	28(3)kHz
This work	48(6)kHz	24(4)kHz

— [1] K. Pachucki, A.M. Moro Phys.Rev.A 75,032521(2007)

Conclusions

- Ab-initio theories represent an excellent framework to calculate nuclear structure effects in hydrogen-like atoms.

- We re-evaluated the uncertainties coming from the nuclear model and from the η -expansion using chiral perturbation theory and Bayesian techniques.

- The local chiral interaction at N2LO used here is not able to reduce the uncertainty of $\delta_{\mathrm{NS}}^{(5)}$ we need interactions at higher order.

- It is crucial to compute the nuclear structure effects at order $\delta_{\mathrm{NS}}^{(6)}$

Backup

Uncertainty

Uncertainty sources

- **Numerical**
- **Nuclear model**
- **Nucleon model**
- **Truncation of EM multipoles**
- **η-expansion**
- **Expansion in $(Z\alpha)$**

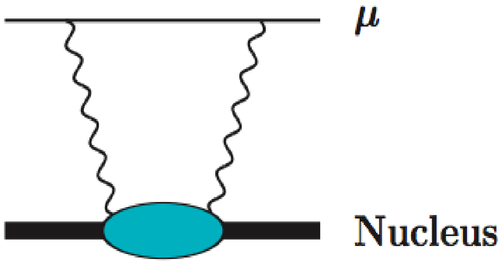
Evaluation of the TPE amplitude

- **Coulomb Distortion**

Contribution coming from intermediate interactions between lepton and nucleus during the TPE process.

We include only the correction of order $(Z\alpha)^6 log(Z\alpha)$

Since we work in the leading dipole approximation, the correction is related to the electric dipole response



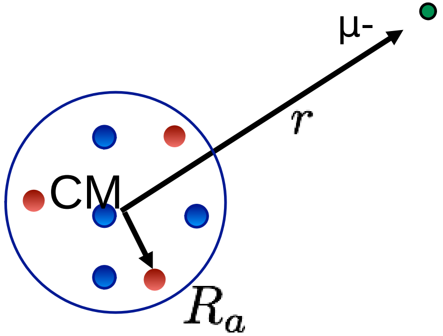
- **Relativistic effects**

They are smaller by factors $\frac{\omega_{th}}{m_r}$

Includes also first effects from electromagnetic currents. Are evaluated in the leading dipole approximation

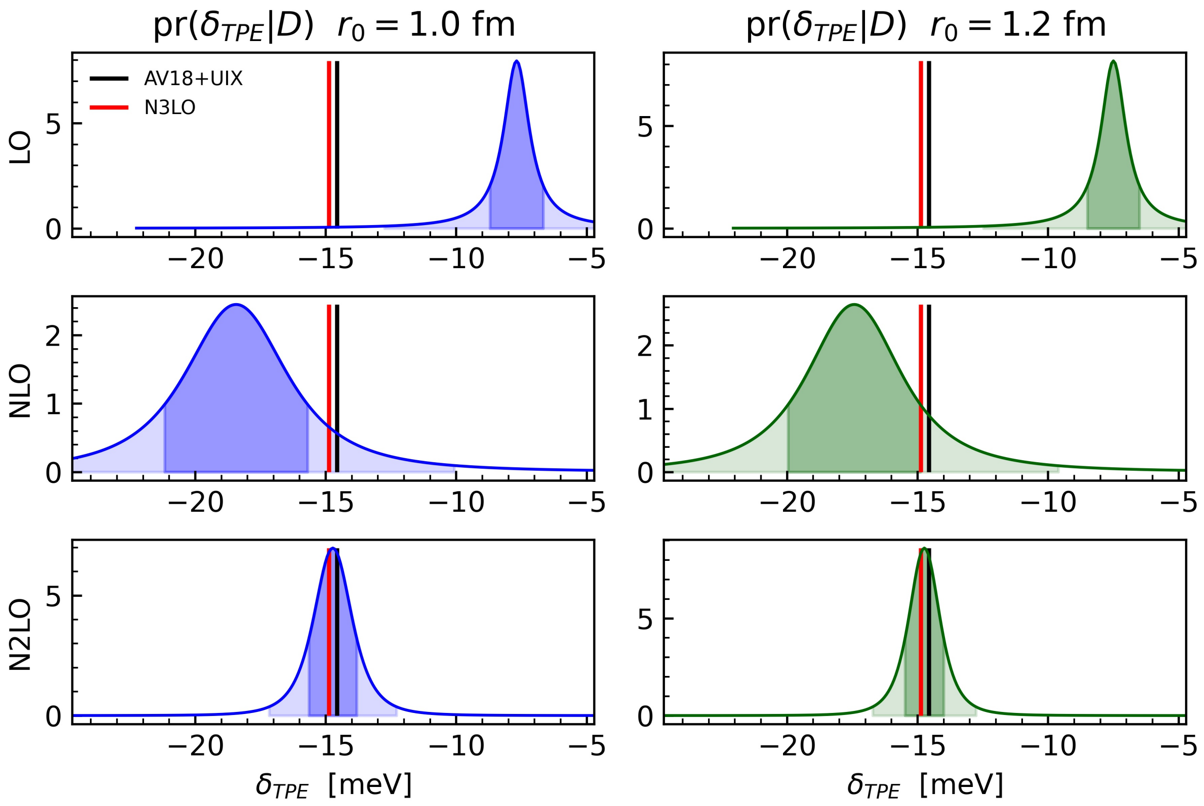
$$\delta^{(0)}_{L(T)} = \frac{2m_r^3}{9}(Z\alpha)^5 \int_{\omega_{th}}^{\infty} K_{L(T)}\left(\frac{\omega}{m_r}\right) S_{D1}(\omega)$$

- **Nucleon size effects**



TPE in He-3

S.S. LM, et al. In preparation



C. Ji, et al. J. Phys. G: Nucl. Part. Phys. 45 (2018)