The proton radius from electron scattering measurements
(and other thoughts about form factors)

Jan C. Bernauer

PREN 2022, PARIS

Center for Frontiers in Nuclear Science

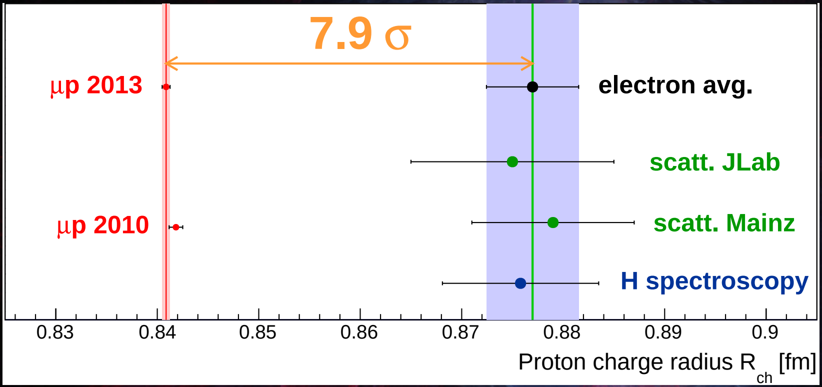
RBRC
RIKEN BNL Research Center

Stony Brook University

Dr. Bernauer is supported by NSF grant PHY 2012114

Reminder: The Proton Radius puzzle

Reminder: The Proton Radius puzzle



Elastic lepton-proton scattering

Method of choice: Lepton-proton scattering

- Point-like probe
- No strong force
- Lepton interaction "straight-forward"

Measure cross sections and reconstruct form factors.

Cross section for elastic scattering

$$\frac{\left(\frac{d\sigma}{d\Omega}\right)}{\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}} = \frac{1}{\varepsilon(1+\tau)}\left[\varepsilon G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)\right]$$

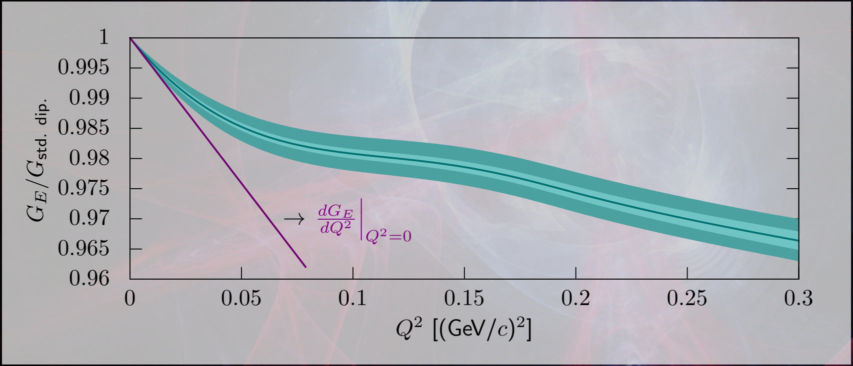
with:

$$\tau = \frac{Q^2}{4m_p^2}, \quad \varepsilon = \left(1 + 2(1+\tau)\tan^2\frac{\theta_e}{2}\right)^{-1}$$

- Rosenbluth formula
- Electric and magnetic form factor encode the shape of the proton
- Fourier transform (almost) gives the spatial distribution, in the Breit frame

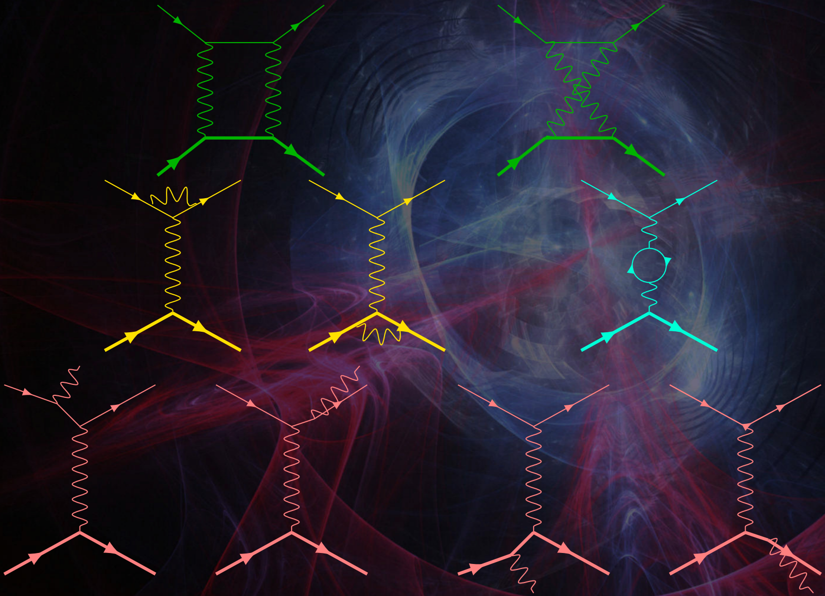
How to measure the proton radius

$$\left\langle r_E^2 \right\rangle = -6\hbar^2 \left.\frac{dG_E}{dQ^2}\right|_{Q^2=0} \qquad \left\langle r_M^2 \right\rangle = -6\hbar^2 \left.\frac{d\left(G_M/\mu_p\right)}{dQ^2}\right|_{Q^2=0}$$

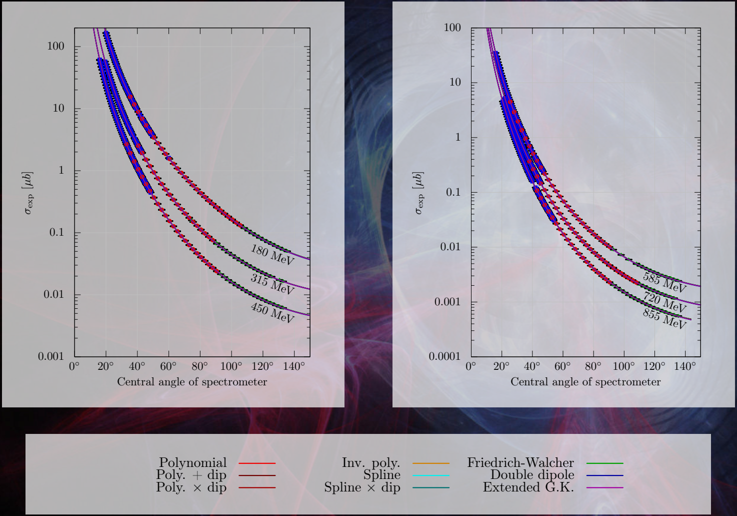


Complications

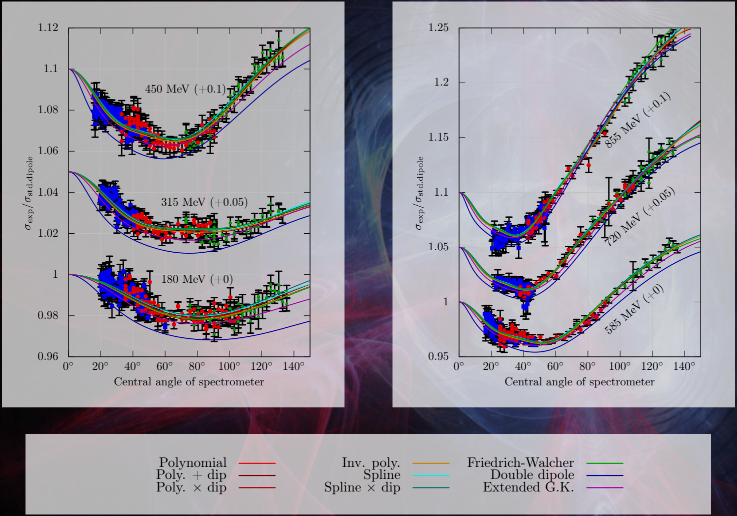
We are actually measuring $ep \to ep\gamma^N$



Cross sections



Cross sections over standard dipole



Why is it hard to extract the radius

- Need to extrapolate slope to $Q^2 = 0$ (This is actually harder than extrapolating G_E)
- Shape not a priori known. Model dependence.
- N.B:
  - All fits are model dependent (they have to, as the number of parameters must be finite)
  - A polynomial fit has nothing to do with a Taylor expansion (except that it's also a polynomial)

Does low Q^2 help?

$$\frac{d\sigma}{d\Omega} \propto 1 - \underbrace{A}_{\mathcal{O}(6)} \cdot Q^2 + \underbrace{B}_{\mathcal{O}(30)} \cdot Q^4 + \ldots$$

(Q in units of GeV/c)

We want to measure the radius ($\propto \sqrt{A/2}$) to within 0.5%, without knowing B. So:

$$B/A \cdot Q^2 \ll 0.02 \longrightarrow Q^2 \ll 0.004\,(\mathrm{GeV}/c)^2$$

Does low Q^2 help?

$$\frac{d\sigma}{d\Omega} \propto 1 - \underbrace{A}_{\mathcal{O}(6)} \cdot Q^2 + \underbrace{B}_{\mathcal{O}(30)} \cdot Q^4 + \ldots$$

(Q in units of GeV/c)

We want to measure the radius ($\propto \sqrt{A/2}$) to within 0.5%, without knowing B. So:

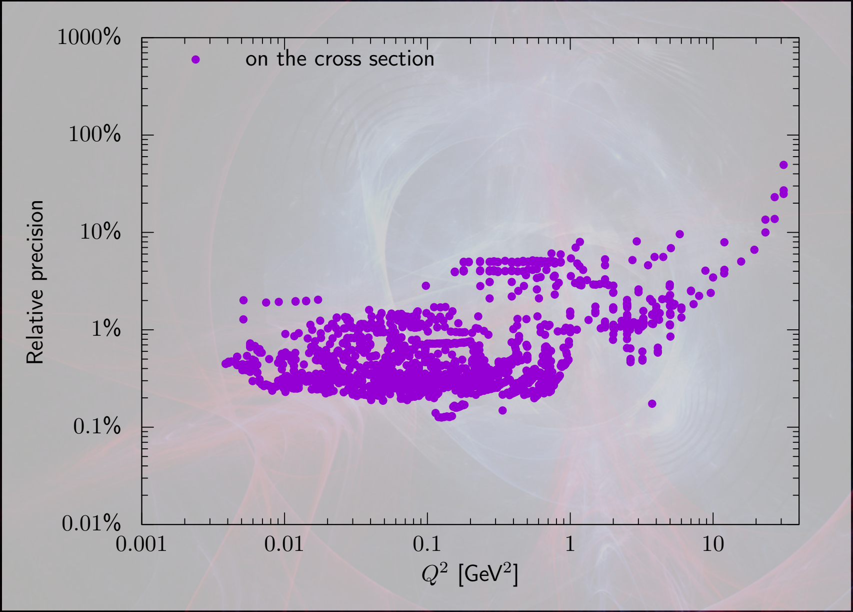
$$B/A \cdot Q^2 \ll 0.02 \longrightarrow Q^2 \ll 0.004\,(\mathrm{GeV}/c)^2$$

But: Need to measure A to 2%, so measure $\frac{d\sigma}{d\Omega}$ to $6 \cdot 0.004 \cdot 0.02 = 0.048\%$.

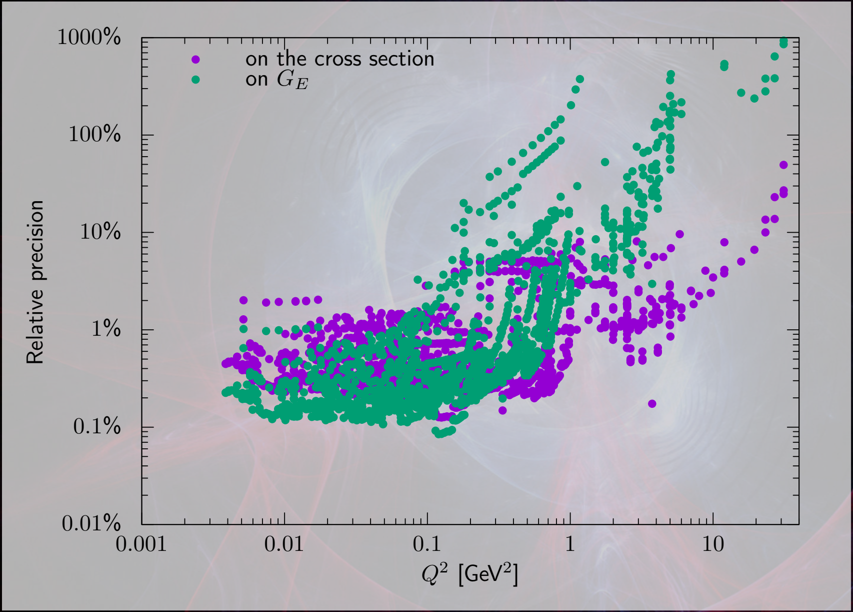
In principle also true for spectroscopy

- De Rujula, Phys.Lett.B693:555-558,2010
  - Can "fix" muon result by assuming a different third Zemach moment $< r_p^3 >_2$.
- Ruled out by scattering measurements. Full form factors, so we get "all" moments.

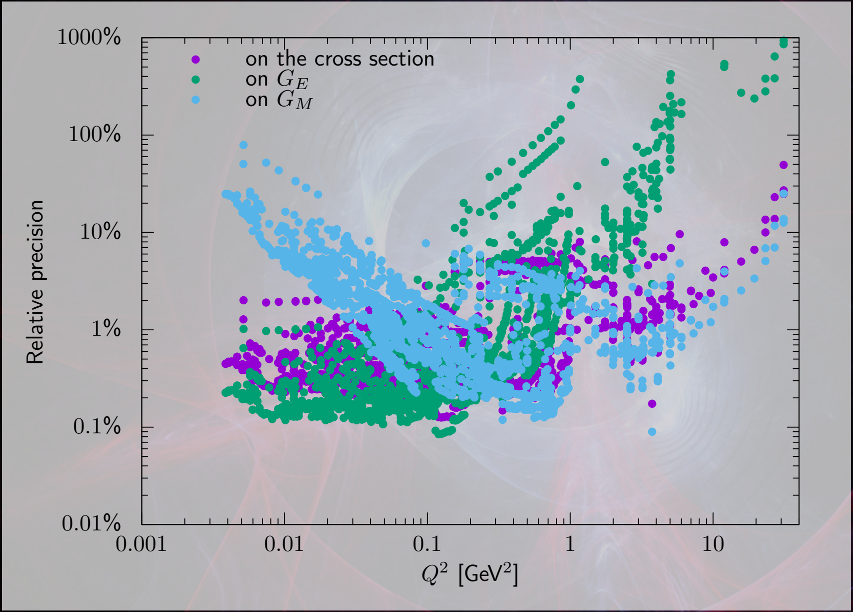
World data set 2010



World data set 2010



World data set 2010



Magnetic radius

I do not believe we have a reliable magnetic radius with currently available data without model assumption.

Fits (as reported in papers)

Fitting a selected, small Q^2 range: often small radius.
Fitting a large Q^2 radius

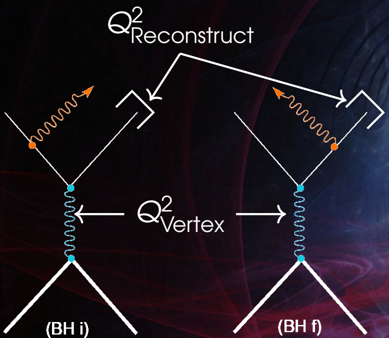
- Flexible fits: large radius (including those that use physics constraints)
- (Strongly) Physics motivated fits:
  - Dispersion relation (see talk by Ulf-G. Meissner tomorrow)
  - Dispersively improved χpt

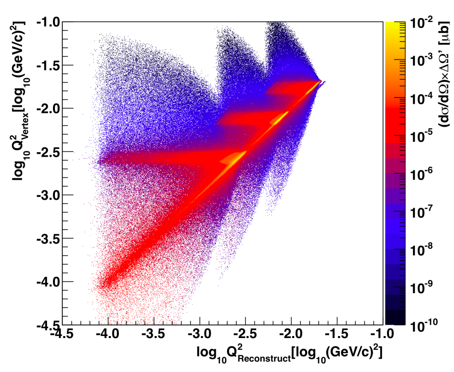
I stopped looking at papers which don't properly discuss χ^2. Low bar.

New data

New data

ISR method

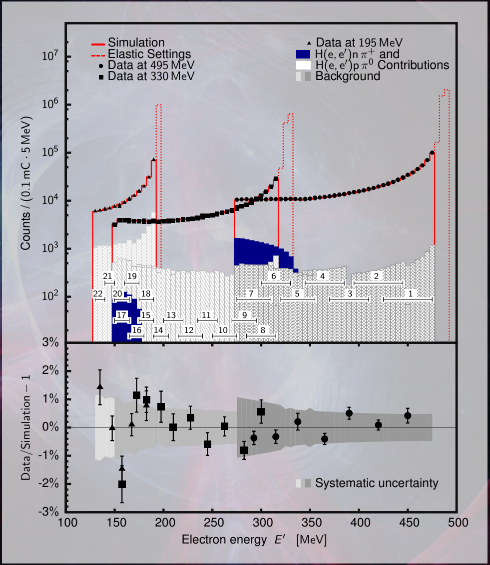




- Use initial state radiation to reduce effective beam energy
- Have to subtract FSR

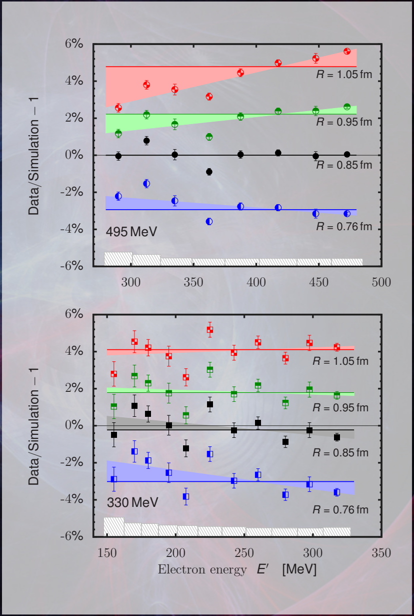
ISR at MAMI

- Published: Miha Mihovilovič et al. PLB 771:194-198
- Radiative correction correct on the 1% level deep in the tail!
- Radius extraction not competitive in precision
- In principle: Larger scattering angle for G_M



Updated analysis of ISR

- Miha Mihovilovič et al. arXiv: 1905.11182
- Focuses on cs instead of FF
- $r_p = 0.870 \pm 0.014_{stat} \pm 0.024_{sys} \pm 0.003_{mod}$ fm
- Slightly prefers large radius



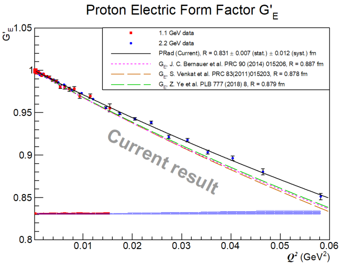
PRad

- @JLAB,
- 1 and 2 GeV beam, very forward angles
- "open" cell, so less background
- Calorimeter
  - worse energy resolution
  - but only 1 setting per energy
  - calibration with Møller scattering
- Fit using function determined before data was available!
- See more in Ashot Gasparian's talk later today

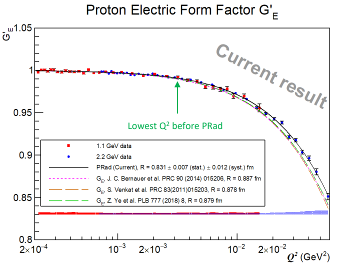
Proton electric form factor G'_E

PRoton Radius

Fit G_E to $\begin{cases} n_1\, f(Q^2) & \text{for 1.1 GeV} \\ n_2\, f(Q^2) & \text{for 2.2 GeV} \end{cases}$ $\quad f(Q^2) = \dfrac{1 + p_1 Q^2}{1 + p_2 Q^2}$ $\quad G'_E = \begin{cases} G_E/n_1 \\ G_E/n_2 \end{cases}$



n_1 = 1.0002 +/- 0.0002(stat.) +/- 0.0020 (syst.),

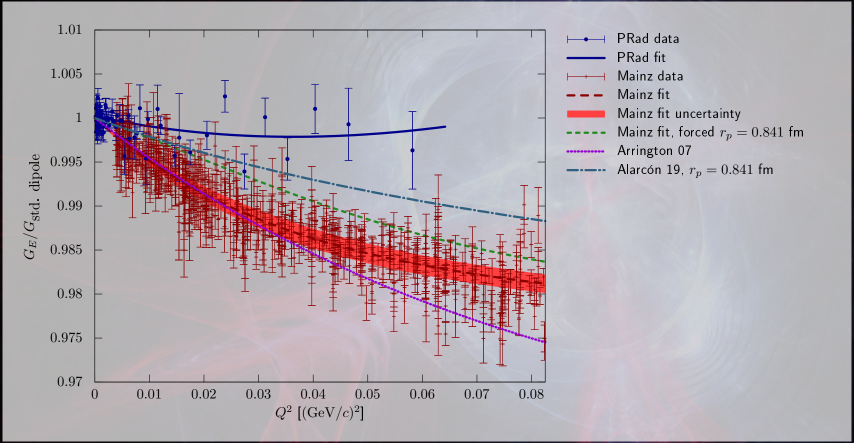


n_2 = 0.9983 +/- 0.0002(stat.) +/- 0.0013 (syst.)

Current result

r_p = 0.831 +/- 0.007 (stat.) +/- 0.012 (syst.) fm

No agreement on form factor level



Take aways

- Getting the same radius in fits to Mainz and PRad does not mean the data is in agreement.
- Hard to see how both results can be right
  - At least one of the experiments wrong.
  - But is it a problem in the experimental part or in theory?
- If PRad is fully right, what do we know about FFs after all?

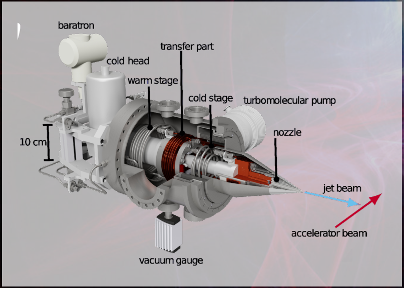
What could have gone wrong

- Will not speculate on the experimental part
- What is different?
  - Momentum resolution (tail shape!)
  - Kinematics: PRad is very forward, all other are not.
    $\Rightarrow$ Radiative corrections?

RadCorr workshop

https://indico.ectstar.eu/event/146/

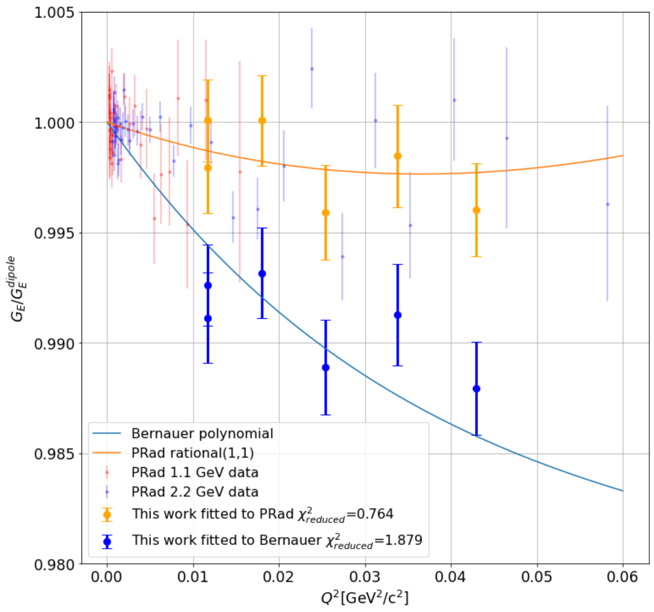
Mainz new results (PhD. thesis Yimin Wang)



Collaboration with Muenster group

Results of pilot experiment

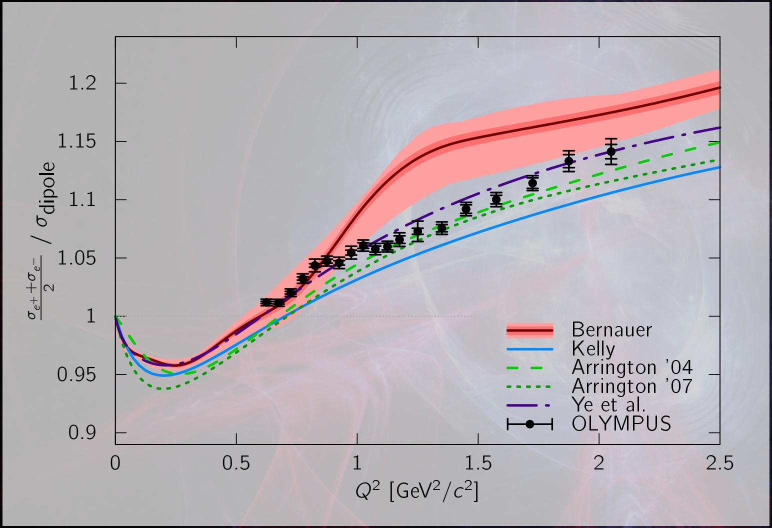
Two data groups. Fit two norms to PRad and Mainz fits



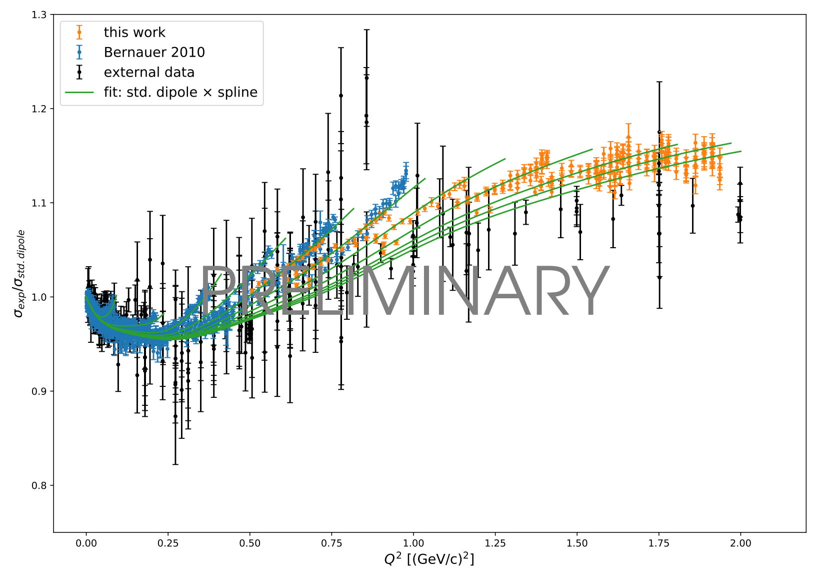
Why I believe Mainz high Q^2 is right

- OLYMPUS yields
  - TPE measurement via ratio of e^+p to e^-p
  - But can use charge average to cancel TPE.
- New Mainz high energy proton ff. measurement
  - Same machine but partially double coincidence
  - Not analyzed by me

OLYMPUS yields



Mainz high Q^2 ff (PhD. thesis Julian Mueller)



Future experiments: PRad II

- Different energies
- Better outer calorimeter
- Please don't concentrate on lowest Q^2 only
- See talk by Ashot Gasparian later today

Future experiments: ULQ2

- Aims for absolute cs on per-mille level!
- 60 MeV beam at Tohoku
- Please also think about G_M!
- See talk by Yuki Honda on Wednesday

Future experiments: AMBER

- Using muons at CERN
- Both charges
- Ultra-high energy, very small scattering angle.
- Measuring proton recoil $\rightarrow$ different systematics, rad corr.
- See talk by Stephan Paul on Wednesday

Future experiments: MUSE

- Electrons, positrons, muons, pions at PSI
- Separated by ToF
- Direct test of lepton universality, rad. cor., TPE
- See talk by Tigran Rostomyan on Wednesday

Future experiments: Mainz

- Hopefully have chance to redo ISR and jet target with A1
- Jet target will be the work horse for MAGIX@MESA
- Data on G_M relevant for the radius!
- See talk by Soeren Schlimme on Wednesday

Proton CS/FF database

- World fits have to normalize data to same level of radiative corrections
- Needs meta data beyond published CS, FF etc.
- Better fit CS then FF (correlations!)
- Ethan Cline, Axel Schmidt, Craig McRae and I are working on open database with this meta information.
  - Few clicks to download selected datasets
  - Check for independence of selected sets
  - Auto-normalized to selected radiative corrections
  - Auto-fill of kinematic variables
- Who wants to help?

Conclusion

- The PRad$\leftrightarrow$Mainz discrepancy has me worried
  - If you discard PRad high Q^2, why believe the low Q^2
  - If you discard Mainz low Q^2, why believe the high Q^2
- Future experiments will illuminate puzzle from many directions.
- Magnetic radius is hard
- Look at all data before you claim victory/agreement.
- There is a world beyond the proton radius:
  - See talks bei Michael Paolone, Toshimi Suda and Tyler Kutz, on Thursday