AXION COUPLINGS IN GRAND UNIFIED THEORIES*

*: together with P. Agrawal, M. Nee

based on: 2206.07053

MARIO REIG

mario.reiglopez@physics.ox.ac.uk

WHAT DO WE LEARN?

i) Quantization of axion-photon coupling allows knowledge about UV: charge quantization, GUTs, ...

ii) In (all??) unified theories the only axion* coupled to photons is the QCD axion.

$m \ll \frac{\Lambda_{QCD}^2}{g_a}$ ← LIGHT ~ SIZEABLE → $g_{a\gamma} \sim \frac{\alpha_{em}}{g_a}$

*: the only LIGHT axion with SIZEABLE coupling to $F\tilde{F}$

(SOME) STANDARD MODEL PUZZLES

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

* 3 independent (?) interactions @ low E

  Quarks, leptons, LH, RH fields, 3 families .... !?

* Plethora of independent charges & quantum numbers

but still:

CHIRALITY + ANOMALY FREEDOM

g_{strong}, g_{weak}, g_Y $\sim \mathcal{O}(1)$ @ TeV

very different from Yukawa couplings

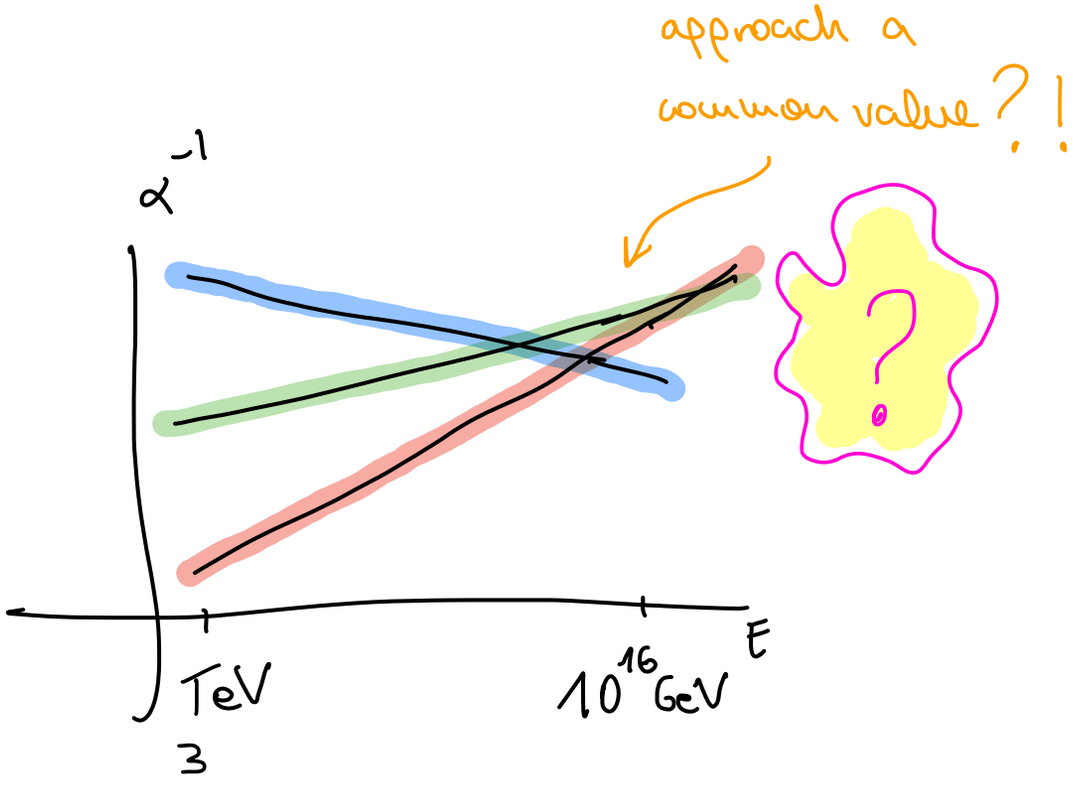
$Y_{1st} \ll Y_{2nd} \ll Y_{3rd}$

THE STANDARD MODEL

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

$g_s, g_{weak}, g_y \sim O(1)$ at low energies →

↳ looks even better at high E!

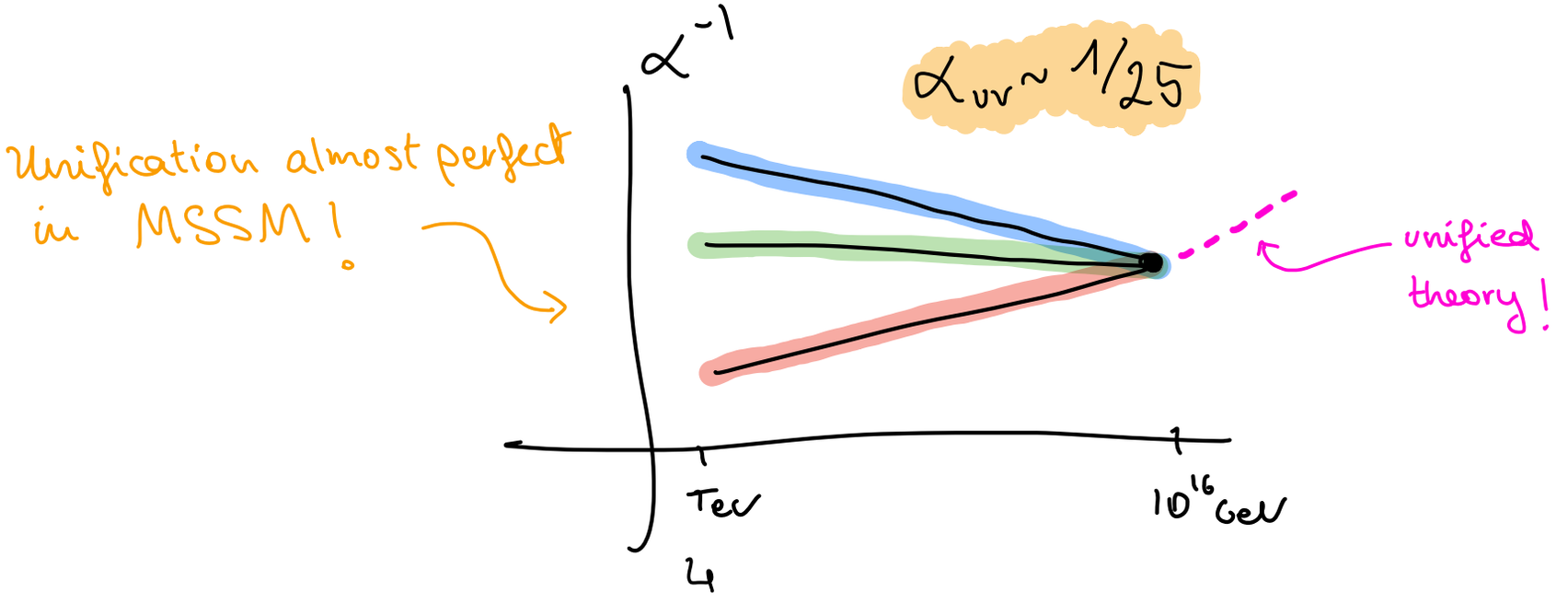


THE STANDARD MODEL

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

* To keep couplings together we need a SIMPLE GROUP!

G_{GUT}



GEORGI-GLASHOW SU(5)

* **minimal GUT**: $SU(5) \longrightarrow SU(3) \times SU(2) \times U(1)$

$$\underbrace{5 + \overline{10}}_{\text{anomaly free}} \longrightarrow \underbrace{q, \ell, u^c, d^c, e^c}_{\text{SM family}}$$

* $\sin^2\theta_W = \dfrac{g'^2}{g^2 + g'^2} = \dfrac{3}{8}$ @ M_{GUT} $\longrightarrow$ @ low E $\sim$ $\sin^2\theta_W \simeq 0.23$

$g' = \sqrt{\frac{3}{5}}\, g_1$ ⟶

* Other GUTs (contain SU(5) as subgroup): $SO(10)$, $SO(18)$, E_6, E_8 ...

spinor unification ↑⟶

particularly appealing: CHIRAL + ANOMALY FREE

SO(10) spinor: $16 \rightarrow q + u^c + d^c + \ell + e^c + \nu^c$

DISCLAIMER

I will be talking about general features of GUTs but will be using all the time SU(5) as particular example.

THE GOOD: HINTS FOR UNIFICATION

* **Charge quantisation:** ALL isolated states have integer electric charge.

* **Anomaly freedom:** SM quantum numbers "conspire" to cancel gauge anomalies.

* **Unification of couplings; $\sin^2\theta_W$ & $\frac{m_b}{m_\tau}$**

$$\sin^2\theta_W = \frac{g'^2}{g^2 + g'^2} = \frac{3}{8} \quad , \quad \frac{m_b}{m_\tau} \simeq 3 \text{ at low E}$$

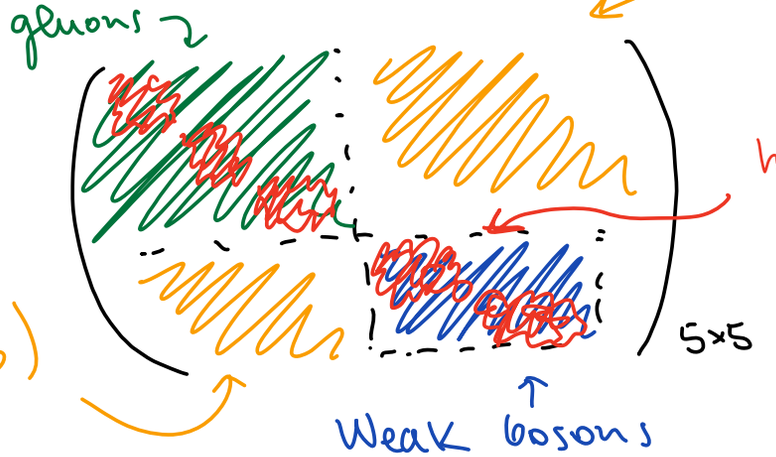
$g' = \sqrt{\frac{3}{5}}\, g_1$ ⟶

THE BAD: PROTON DECAY

* Generic GUT prediction

SU(5) ADJOINT MATRIX: 24

* GUT bosons:



mediate proton decay

hypercharge boson
$\propto \mathrm{diag}(-\frac{1}{3},-\frac{1}{3},-\frac{1}{3},\frac{1}{2},\frac{1}{2})$

$X \sim (3,2,-5/6)$

$$\tau_{p\to\pi e^+} \sim \frac{m_p^5}{M_{GUT}^4}$$

$M_{GUT} \geq 10^{16}$ GeV

$\tau_{proton} > 1.6\times10^{34}$ yr.

Hyper-K expected to improve by factor of 10!

(data-taking ~ 2027)

Current limits from Super-K

THE UGLY: GUT DRAWBACKS

* Doublet-triplet splitting:

$$\overline{5} \sim \begin{pmatrix} H_3 \\ H_{SM} \end{pmatrix}$$

→ lead to proton decay unless $m_{H_3} \sim M_{GUT}$

* Mass ratios: $\frac{m_b}{m_\tau}$ is OK... but $\frac{m_s}{m_\mu}$ or $\frac{m_d}{m_e}$?!

NOT EVERYTHING IS LOST...

↳ requires model building effort: orbifold GUTs, flavor sym...

! Our results will be independent of GUT model building details...

AXION REVIEW

* **Axion**: periodic (compact) scalar with discrete shift-symmetry. $a \to a + 2\pi f_a$

  ↳ AKA axion-like particle (ALP)

* (periodic) Interactions shaped by shift-symmetry

$$\frac{\partial_\mu a}{f_a} \bar{f} \gamma^\mu \gamma^5 f \; ; \; \frac{a}{f_a} F\tilde{F} \; ; \; V(a) = \Lambda^4 \cos(a/f_a)$$

* Field theory language: pNGB of (anomalous) symmetries

  ↳ $U(1)_{PQ}$ for QCD axion

$$[SU(3)_c]^2 \times U(1)_{PQ} = \mathcal{A}_{QCD}$$

↖ anomaly coefficient

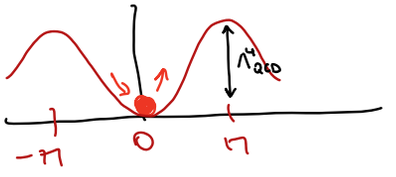
WHY AXIONS?

* Appear BSM models & String Theory (i.e. AXIVERSE)
* solve strong CP problem: QCD axion
* Dark matter candidates
* Dark energy, or even inflation (?)

Ex: QCD AXION

$\theta_{QCD}\, G\tilde{G} \rightarrow \frac{a}{F_a} G\tilde{G}$

* solves strong CP: $\frac{\langle a \rangle}{F_a} = 0$
* $V(a) = \Lambda_{QCD}^4 \left(1 - \cos\left(\frac{a}{F_a}\right)\right) \Rightarrow m_a \sim \frac{\Lambda_{QCD}^2}{F_a}$



WHY AXIONS ?

- Appear in many BSM constructions
- solve strong CP problem: QCD axion
- Dark matter candidates
- Dark energy, or even inflation (?)
- Topological, quantized couplings to gauge bosons

$$\mathcal{L}_a = \frac{(\partial_\mu a)^2}{2} + \mathcal{A}\,\frac{a}{F_a}\,\frac{\alpha_{GUT}}{8\pi}\,G_{GUT}\,\tilde{G}_{GUT}$$

Chern-Simons-like coupling

↳ QUANTISATION:

Anomaly coefficient $\mathcal{A} \in \mathbb{Z}$, an integer!

TOPOLOGICAL COUPLINGS TO GAUGE BOSONS

* Anomaly coeff. **unaffected by renormalization**

$$\mathcal{A}_{UV} = \mathcal{A}_{IR}$$

→ directly probing the far UV!

↳ Together with **gauge invariance** offers info about gauge group.

WAIT!! WHAT ABOUT MIXING?

* AXION-PHOTON COUPLING (QCD axion case)

topological nature ↓ , axion-pion mixing ↓

$$\frac{\alpha_{em}}{F_a}\left(E/N - 1.92\right) a F\tilde{F}$$

↳ we obtain solid information in the massless axion limit:

MIXING EFFECTS VANISH IN THIS LIMIT → $m^2 \ll m^2_{QCD} \sim \Lambda_{QCD}^4 / f_a^2$

WILL BE SHOWN LATER...

MINIMAL 4dim GUT

* Starting point: $G_{GUT} \times \prod_i U(1)_{PQ_i}$

simple gauge group

e.g. SU(5)

Set of commuting, global unbroken symmetries

↳ Analogy with SM: $\underbrace{U(1)_B \text{ and } U(1)_L}_{\text{weak interaction } SU(2)}$ → $\begin{cases} U(1)_{B-L} \text{ anomaly-free} \\ U(1)_{B+L} \text{ ANOMALOUS!} \end{cases}$

applications for baryogenesis etc.

* After symmetry redefinition:

Important!!

$[G_{GUT}]^2 \times U(1)_{PQ} = \mathcal{A}$

$[G_{GUT}]^2 \times U(1)_i = 0$

$$G_{GUT} \times U(1)_{PQ} \times \prod_i U(1)_i$$

axion

non anom.

exact or decoupled Goldstone bosons

MINIMAL 4dim GUT

$$\prod_i U(1)_{PQ\,i} \longrightarrow U(1)_{PQ} \times \prod_i^{\text{non-anom}} \tilde{U}(1)_i$$

field redef.

↳ only this linear combination gives an axion coupled to gauge bosons.

* $\left.\begin{array}{l} \mathcal{A}_{PQ} \neq 0 \\ \mathcal{A}_i = 0 \end{array}\right\}$ and due to quantisation $\mathcal{A}^{UV} = \mathcal{A}^{IR}$

↳ CURRENTS:

Above PQ & GUT SSB scales

$$\begin{cases} U(1)_{PQ}: & \partial^\mu J^{PQ}_\mu = \mathcal{A}_{PQ} \dfrac{\alpha_{GUT}}{8\pi} G\tilde{G}_{GUT} \\ \tilde{U}(1)_i: & \partial^\mu J^{\tilde{U}(1)_i}_\mu = 0 \end{cases}$$

↳ This axion couples to both, photons and gluons!!

decoupled Goldstones! (from gauge bosons)

DEPENDENCE ON PQ SCALE ?

$$\partial^\mu J^{PQ}_\mu = \mathcal{A}_{PQ}\,\frac{\alpha_{GUT}}{8\pi}\,G\tilde{G}_{GUT} \rightarrow \text{What if } F_a < M_{GUT}\text{?}$$

A) $F_a > M_{GUT}$: effects of anomaly captured by dim-5 op.

$$\mathcal{A}_{PQ}\,\frac{a}{F_a}\,\frac{\alpha_{GUT}}{8\pi}\,G\tilde{G}_{GUT}$$

contains both photons and gluons!

K_3, K_2, K_1 levels of embedding of $SU(3)$, $SU(2)$, $U(1)$

B) $F_a < M_{GUT}$:

$$\partial^\mu J^{PQ}_\mu = \mathcal{A}_{PQ}\left\{K_3\,\frac{\alpha_3}{8\pi}\,G\tilde{G}_{QCD} + K_2\,\frac{\alpha_2}{8\pi}\,W\tilde{W} + K_1\,\frac{\alpha_1}{8\pi}\,B\tilde{B}\right\}$$

↓ After PQ breaking...

$$\mathcal{A}_{PQ}\,\frac{a}{F_a}\left\{K_3\,\frac{\alpha_3}{8\pi}\,G\tilde{G}_{QCD} + K_2\,\frac{\alpha_2}{8\pi}\,W\tilde{W} + K_1\,\frac{\alpha_1}{8\pi}\,B\tilde{B}\right\}$$

↳ Again, axion couples to both photons & gluons!

MINIMAL 4dim GUT

TOPOLOGY + GAUGE INVARIANCE $\rightarrow$ $\frac{a}{f_a}\left[\alpha_{em} E F_{em}\tilde{F}_{em} + \alpha_s N G\tilde{G}_{QCD}\right]$

↳ generates QCD potential!

$V(a) \simeq \Lambda_{QCD}^4 (1 - \cos(a/f_a))$

* Generic GUT prediction:

↳ $\frac{E}{N} = \frac{k_1 + k_2}{k_3}$ Standard GUT embedding $\begin{cases} k_3 = k_2 = 1 \\ k_1 = 5/3 \end{cases}$

↳ Only one axion coupled to photons: the QCD axion!

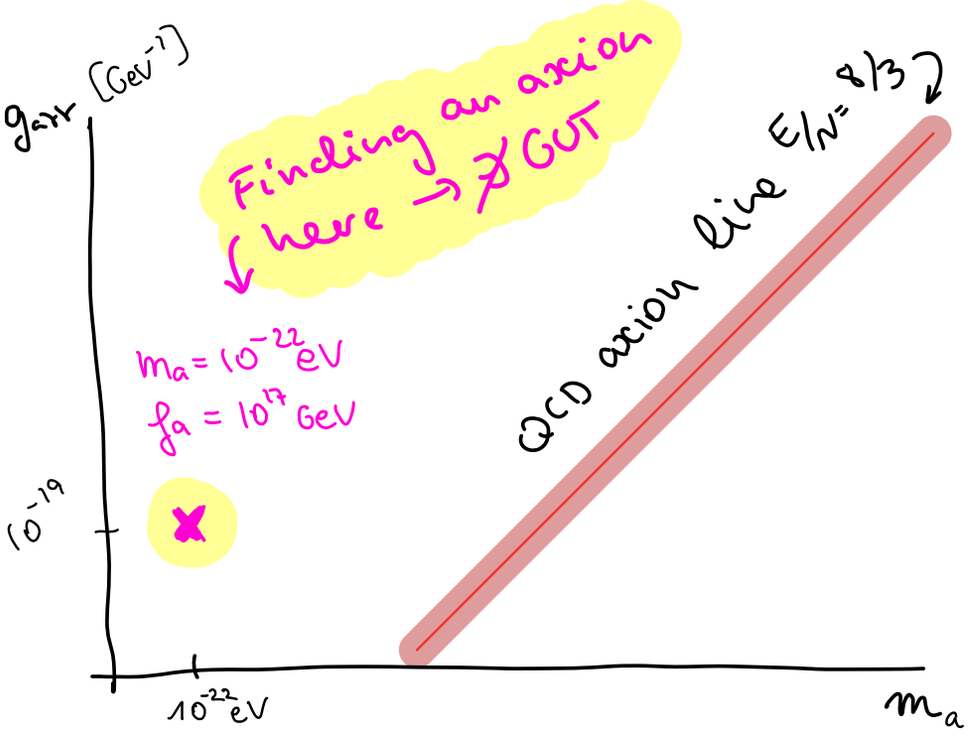
↳ CAN WE TEST GUTs WITH AXIONS?

↳ How do we look for them?

TESTING GUTs WITH AXIONS?

ZERO ORDER: CLAIM

In the presence of a unified gauge group, any axion coupled to photons must lie on the QCD line.



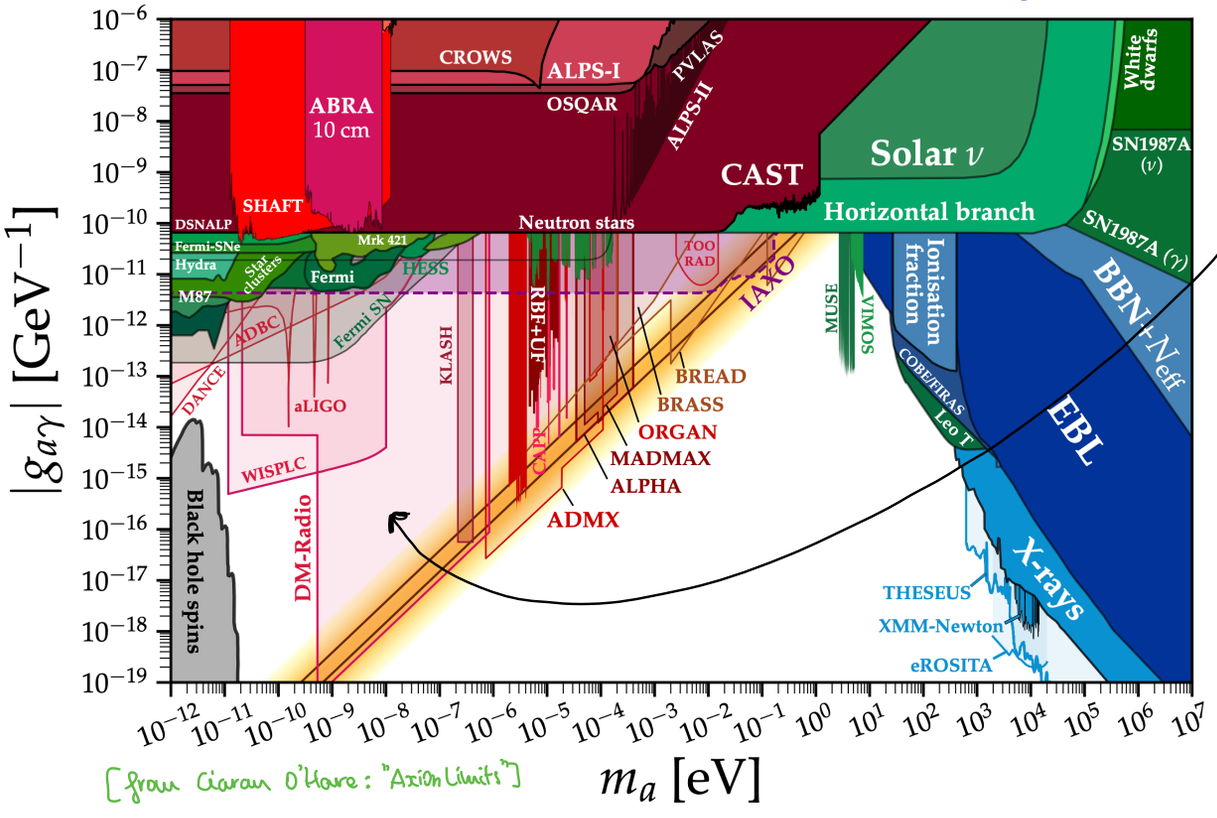
GUT PREDICTION

* $m_{aQCD} = \frac{\Lambda_{QCD}^2}{f_a}$
* $g_{a\gamma\gamma} = \frac{\alpha_{em}}{f_a}\left(\frac{E}{N} - 1.92\right)$
* $\frac{E}{N} = \frac{k_1 + k_2}{k_3}$

Axion-Photon Coupling: a bright future...

... or why I think all this is exciting!

* lab & haloscopes * astrophysics * cosmology



[from Ciaran O'Hare: "AxionLimits"]

* Many ongoing and planned experiments looking for axions coupled to photons!

specially for: $m^2 \ll m_{QCD}^2$

mixing effects vanish in this limit!!

Go to 2206.07053 for details!

COUNTEREXAMPLES?

$$\partial^\mu J^{PQ}_\mu = \mathcal{A}_{PQ}\,\frac{\alpha_{GUT}}{8\pi}\,G\tilde{G}_{GUT} + (\,?\,)$$

$$\partial^\mu J^{\tilde{U}(1)_\mu}_\mu = 0 + (\,?\,)$$

A) GUT-CHARGED AXIONS

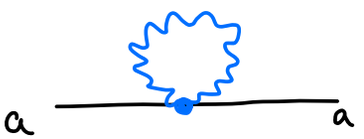
↳ PQ symmetry does NOT commute with G_{GUT}

↳ E.g. in SU(5)

$$\frac{a^Y}{F_a} \propto \mathrm{diag}\left(-\frac{1}{3},-\frac{1}{3},-\frac{1}{3},\frac{1}{2},\frac{1}{2}\right)$$

$\frac{a^Y}{F_a} F_{U(1)_Y}\tilde{F}_{U(1)}$

$\bar{X} \sim (\bar{3},2,5/6)$



UV divergent!

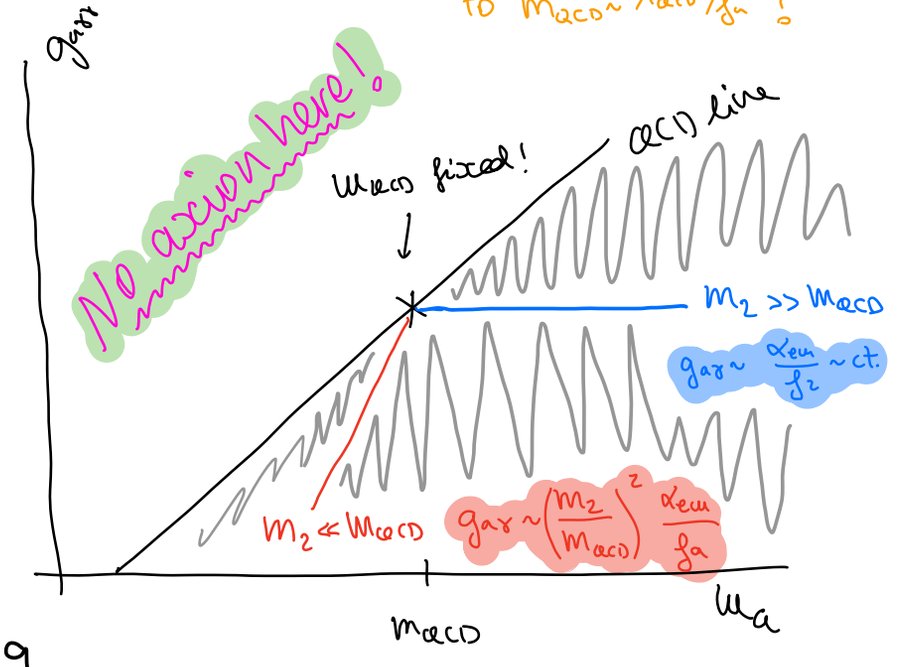
HEAVY perturbative mass from GUT gauge bosons

see $\pi^\pm$ mass from U(1)$_{em}$ effects

B) AXION MIXING

$$V(a_1,a_2) = \left(\frac{a_1}{f_1}+\frac{a_2}{f_2}\right)G\tilde{G} + \frac{1}{2}m_2^2 a_2^2$$

How big is m_2 compared to $m_{aCD} \sim \Lambda_{aCD}^2/f_a$?



Go to 2206.07053 for details!

COUNTEREXAMPLES?

* Give up unified gauge group assumption!

C) "FLIPPED" GUTs

* Theories based on $SU(5) \times U(1)_X$

↳ hypercharge: $Y = T_{24} + \beta X$

* SU(5) and $U(1)_X$ are not necessarily unified.

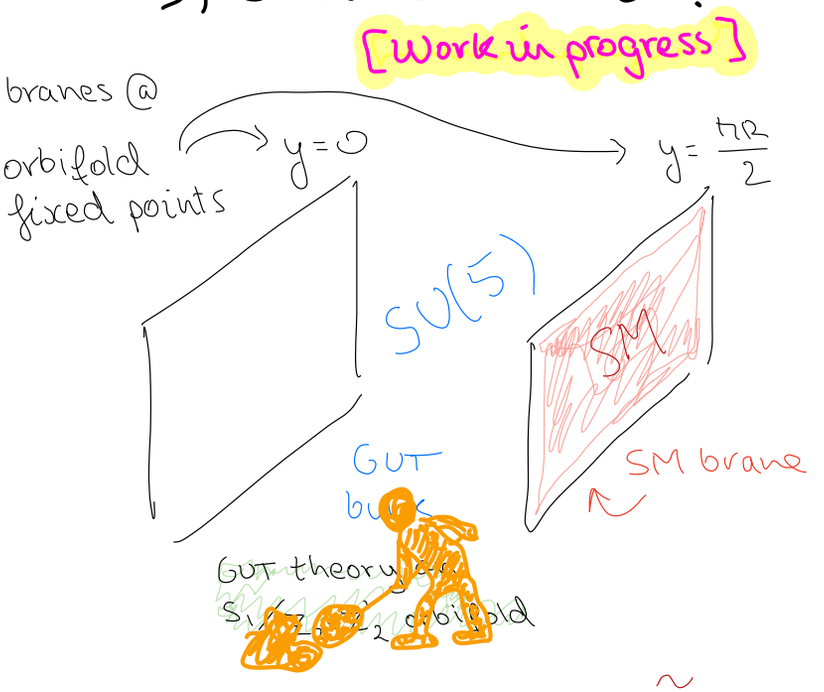
↳ Axion coupled to $U(1)_X$ without SU(5)

Light ALP $\frac{a^X}{F_a} F \tilde{F}$

BUT
* no charge quantisation
* no prediction for $\sin^2\theta_W$

D) STRING AXIONS ?

[Work in progress]



↳ Brane localized D-instanton appears inducing large axion mass!

WHAT DO WE LEARN?

based on: 2206.07053 ← GO HERE FOR MORE DETAILS!

i) Quantization of axion-photon coupling allows knowledge about UV: charge quantization, GUTs, ...

ii) In (all??) unified theories the only axion* coupled to photons is the QCD axion.

$m \ll \frac{\Lambda_{QCD}^2}{g_a}$ ← LIGHT ~ SIZEABLE → $g_{a\gamma} \sim \frac{\alpha_{em}}{g_a}$

*: the only LIGHT axion with SIZEABLE coupling to $F\tilde{F}$