Semileptonic *B*-meson width: NLO corrections to chromomagnetic operator coefficient

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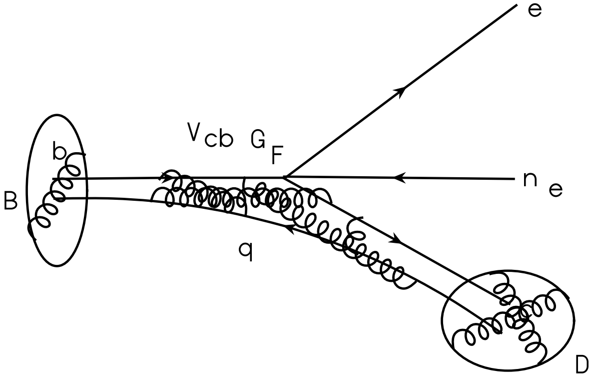
Theory precision in SM: hadrons

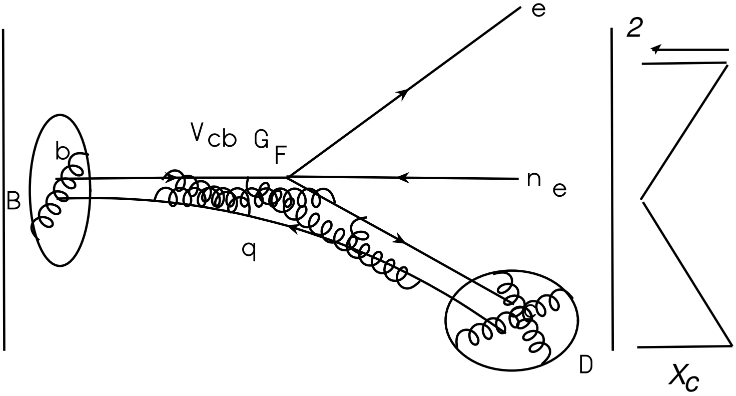
In experiment - *B*-meson decay

“Easy” to measure - difficult (impossible?) to compute



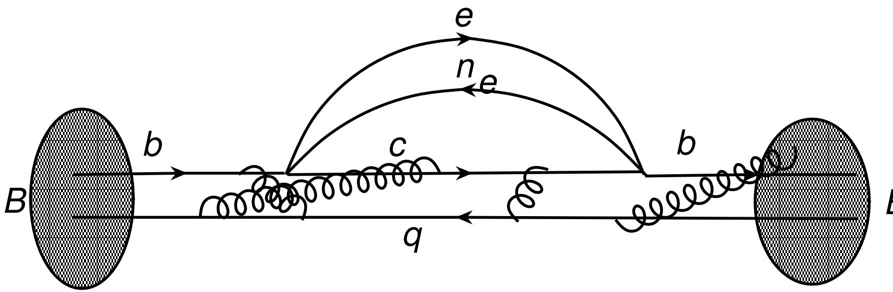
Theory precision in SM: hadrons

Simplification for theory - inclusive setup



Hadron weak decay story

The problem of dealing with final hadronic states is solved (by-passed). Initial state: forward matrix element



Theory tool - HQE

Further simplification for large m_b:
$m_b \gg \Lambda$ one can use HQE to make the dependence on m_b explicit: separation of scales m_b and Λ
Indeed, one needs

$$\mathrm{Im} \int dx \langle B(p)| iT \bar{b}(x) ... \{\text{light stuff}\} ... b(0) | B(p) \rangle$$

using $b(x) \to e^{imvx} h_v(x)$ ($\langle 0|b(x)|b(\vec{p})\rangle = e^{-ipx} u(p)$)
one gets large phase as in usual OPE

$$\mathrm{Im} \int dx e^{-imvx} iT\{\bar{h}_v(x) ... h_v(0)\}$$

Theory tool - HQE

Theory expression is then

$$\Gamma(B \to X_c \ell \bar{\nu})/\Gamma_b^0 = |V_{cb}|^2 \left[a_0 (1 + \frac{\mu_\pi^2}{2m_b^2}) + a_2 \frac{\mu_G^2}{m_b^2} + a_3 \frac{\bar{\rho}^3}{m_b^3} + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^4}{m_b^4} \right) \right]$$

μ_π - kinetic, μ_G - chromo-magnetic, $\bar{\rho}^3$ - Darwin term
For $\Lambda_{\mathrm{QCD}} \sim 1\ \mathrm{GeV}$, $m_b \sim 5\ \mathrm{GeV}$ and $\alpha_s/\pi \sim 0.1$

$$\frac{\alpha_s}{\pi} \sim \frac{\Lambda_{\mathrm{QCD}}}{m_b}$$

Theory tool - HQE - matching to HQET

$$T = i \int dx T \left[H_{\rm eff}(x) H_{\rm eff}(0) \right]$$

where $H_{\rm eff} = 2\sqrt{2} G_F V_{cb} (\bar{b}_L \gamma_\mu c_L)(\bar{\nu}_L \gamma^\mu \ell_L)$

$$\mathrm{Im}\, T / T_0 = C_0 \mathcal{O}_0 + C_v \frac{\mathcal{O}_v}{m_b} + C_\pi \frac{\mathcal{O}_\pi}{2m_b^2} + C_G \frac{\mathcal{O}_G}{2m_b^2}$$

where $T_0 = \pi \Gamma_b |V_{cb}|^2$. The local operators in the expansion are ordered by their dimensionality

$$dim\, 3: \; \mathcal{O}_0 = \bar{h}_v h_v, \quad dim\, 4: \; \mathcal{O}_v = \bar{h}_v (ivD) h_v,$$

$$dim\, 5: \mathcal{O}_\pi = \bar{h}_v (iD_\perp)^2 h_v, \quad \mathcal{O}_G = \bar{h}_v \frac{1}{2} [i\not{D}_\perp, i\not{D}_\perp] h_v$$

HQE

Thus,

$$\begin{aligned} T/T_0 &= C_0 \left\{ \bar{b} \not{v} b - \frac{\mathcal{O}_\pi}{2m_b^2} \right\} \\ &+ \left\{ -C_v C_m + C_G - \tilde{C}_G C_0 \right\} \frac{\mathcal{O}_G}{2m_b^2} \end{aligned}$$

$\tilde{C}_G$ comes from the conversion

$$\bar{b} \not{v} b = \bar{h}_v h_v + \ldots + \tilde{C}_G C \frac{\mathcal{O}_G}{2m_b^2}$$

$C_m(\mu)\mathcal{O}_G(\mu)$ is RG invariant (μ independent)
Mass splitting between the B and B^*

$$m_{B^*}^2 - m_B^2 \sim \langle B(p_B) | C_m(\mu) \mathcal{O}_G(\mu) | B(p_B) \rangle$$

Width representation

The final representation

$$\Gamma(B \to X_c \nu \ell) = \Gamma_b |V_{cb}|^2 \left\{ C_0 \left(1 + \frac{\mu_\pi^2}{2m_b^2} \right) + \left(-C_v + \frac{C_G - \tilde{C}_G C_0}{C_m} \right) \frac{3\Delta m_B^2}{8m_b^2} \right\}$$

Here $\tilde{C}_G$ comes from the conversion

$$C_G^{fin} = \left(-C_v + \frac{C_G - \tilde{C}_G}{C_m} \right)$$

Results

We reproduced the known result

$$C_0 = 1 + \Delta_0^{(0)}(\rho) + C_F \frac{\alpha_s}{\pi}\{\left(\frac{25}{8} - \frac{\pi^2}{2}\right) + \Delta_0^{(1)}(\rho)\}$$

with full dependence of $\Delta_0^{(0)}(\rho)$, $\Delta_0^{(1)}(\rho)$ on c-quark mass analytically, $\rho = m_c^2/m_b^2$, $\Delta_0^{(0)}(0) = \Delta_0^{(1)}(0) = 0$.

New: C_G^{fin} with full mass dependence ($\Delta_G^{(1)}(m_c)$)

$$C_G^{fin} = -3 + \Delta_G^{(0)}(m_c) + \frac{\alpha_s}{\pi}\Delta_G^{(1)}(m_c)$$
$$+\frac{\alpha_s}{\pi}\left\{C_A\left(\frac{31}{18} - \frac{\pi^2}{9}\right) + C_F\left(\frac{-91}{72} - \frac{5\pi^2}{18}\right)\right\}$$

(massless limit - ThM, AAP, DR, arXiv:1405.5072, Phys.Lett.(2015))

Final number

Final result with mass correction ($r = m_c^2/m_b^2$)
(preprint SI-HEP-2015-15)

$$
\begin{aligned}
C_G^{fin} &= -3 + 8r + \tfrac{\alpha_s}{\pi}\Big\{ C_A\Big(\tfrac{31}{18} - \tfrac{\pi^2}{9}\Big) + C_F\Big(- \tfrac{91}{72} - \tfrac{5\pi^2}{18}\Big) \\
&\quad + \sqrt{r}\Big(C_A\Big(- \tfrac{8\pi^2}{3}\Big) + C_F \tfrac{32\pi^2}{3}\Big)\Big\} + \mathcal{O}(r) \\
&= -3 + 8r + \tfrac{\alpha_s}{\pi}(-3.46 + 61.4109\sqrt{r}) \\
&= -3\left(1 - 2.7r + \tfrac{\alpha_s}{\pi}(1.15 - 20.5\sqrt{r})\right)
\end{aligned}
$$

Huge coeff in front of $\sqrt{r}$ —- $(56\pi^2/9)$

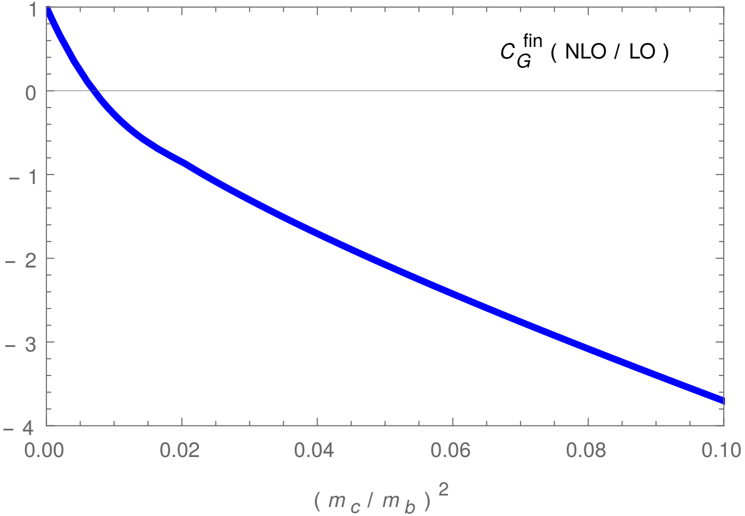
Final number

For $m_c = 0$ we have

$$C_G^{fin} \sim 1 + \frac{\alpha_s}{\pi}(1.15)$$

while for the physical value $r = 0.07$ it is

$$C_G^{fin} \sim 1 + \frac{\alpha_s}{\pi}(-0.44)$$



Dependence on m_c: the ratio of NLO to LO coefficient C_G (norm to 1 at $mC = 0$) as a function of m_c