$b \rightarrow u$ Exclusive Transitions from QCD Light-Cone Sum Rules

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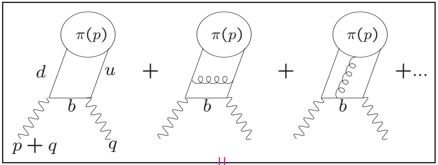
Outline

- QCD light-cone sum rules (LCSR):
  basic elements, sources of the input,
- $B \to \pi$ vector form factor from LCSR:
  first statistical (Bayesian) analysis and $|V_{ub}|$ from $B \to \pi \ell \nu_\ell$
- further improvements in $B \to \pi$ calculation?
- $B \to \pi \tau \nu_\tau$
- $\Lambda_b \to p \ell \nu_\ell$ from LCSR
- LCSR's with B-meson DA's, $B \to \gamma \ell \nu_\ell$
- $B \to \rho$ from LCSR
  disentangling $B \to \rho \ell \nu_\ell$ from $B \to 2\pi \ell \nu_\ell$

□ $B \to \pi$ form factor from LCSR: scheme of derivaiton



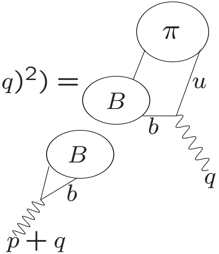
$\Leftarrow$ the correlation function

calculated in terms of
Operator Product Expansion
at $(p+q)^2, q^2 \ll m_b^2$

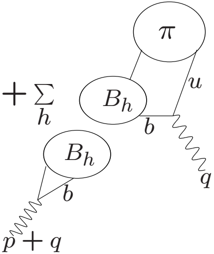
$\Downarrow$

hadronic dispersion relation }

$F(q^2,(p+q)^2) =$



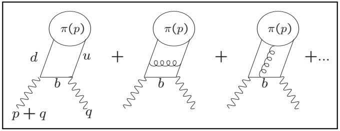
$+\sum_h$



$f_B f^+_{B\pi}(q^2)$

$\sum_{B_h} \to duality\ (s_0^B)$

☐ The correlation function and OPE



$$[F(q^2,(p+q)^2)]_{OPE} =$$

$$= \sum_{t=2,3,4,..} \int\limits_0^1 \mathcal{D}u \, T^{(t)}(\alpha_s, m_b; q^2, (p+q)^2, u, \mu) \, \varphi_\pi^{(t)}(u,\mu)$$

$$\Uparrow \qquad\qquad\qquad \Uparrow$$

{ perturb. diagrams with b-propagator} $\otimes$ {pion Distribution Amplitudes}

- pion DA's: twist $\otimes$ multiplicity$\otimes$ polynomial (conformal partial wave) expansion:

$$\varphi_\pi^{(t)}(u,\mu) = f_\pi^{(t)}(\mu)\left\{ C_0(u) + \sum_{n=2,4,..} a_n^{\pi(t)}(\mu) C_n(u) \right\}$$

- OPE and input
  - expansion level: $O(\alpha_s)$, $t \leq 4$, $\{\bar{q}q, \bar{q}qG\}$, $n \leq 4$
  - $\mathcal{L}_{QCD}$ parameters: $\alpha_s, \bar{m}_b$
  - pion DA parameters: $f_\pi^{(t)}(\mu_0)$, $a_n^{\pi(t)}(\mu_0)$
  - variable scales: μ, $(p+q)^2 \to M^2 \sim m_b\chi$, $m_b \gg \chi \gg \Lambda_{QCD}$

Distribution amplitudes (DA's) of the pion

- standard definition: expansion near light-cone, $\langle x^2\rangle \sim 1/(m_b\chi)$

$$\langle 0|\bar{u}(x)[x,0]\gamma_\mu\gamma_5 d(0)|\pi^-(p)\rangle_{x^2\to 0} = ip_\mu f_\pi \int\limits_0^1 du e^{-iupx}\varphi_\pi(u,\mu) + twist\ 4 + ...$$

- twist 2 DA: $\varphi_\pi^{(2)}(u,\mu) \equiv f_\pi\varphi_\pi(u,\mu)$, expansion in Gegenbauer polynomials:

$$\varphi_\pi(u,\mu) = 6u(1-u)\left[1+\sum_{n=2,4,..} \underbrace{a_n^\pi(\mu)}_{\Downarrow} C_n^{3/2}(2u-1)\right],$$

$\sim [Log(\mu/\Lambda_{QCD})]^{-\gamma n} \to 0$ at $\mu\to\infty$

[Efremov-Radyushkin-Brodsky-Lepage evolution]

- input: the pattern $a^\pi_{2,4,..}(\mu_0)$

  obtained fitting LCSR's for pion e.m. form factors to experiment,

□ LCSRs for pion e.m. form factors $\to$ pion DAs

$\gamma\gamma^*(Q^2) \to \pi^0$ form factor

[A. K. (1999)], ...

[S. Agaev, V. Braun, N. Offen and F. Porkert, (2011)]

$e^+e^- \to e^+e^- + \pi^0$

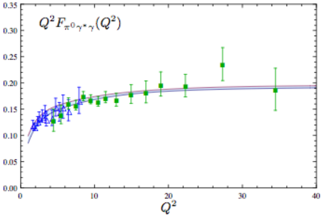


FIG. 2: The pion transition form factor for the model of the pion DA described in the text. The estimated theoretical uncertainty is shown by the shaded area. The experimental data are from [2] (squares) and [4] (open triangles).

- Belle
- CLEO

$\gamma^*(Q^2)\pi^\pm \to \pi^\pm$ form factor

[V.Braun, A.K., M.Maul (2000)]

[AK, T.Mannel, N.Offen, Y-M.Wang (2011)]

$eN \to e\pi N$

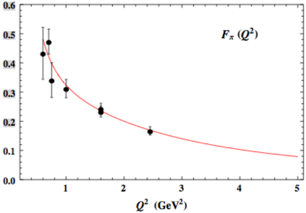


FIG. 1 (color online). The pion e.m. form factor calculated from LCSR [17,18] as a function of Gegenbauer moments $a_2^\pi(1\text{ GeV})$ and $a_4^\pi(1\text{ GeV})$ and fitted (solid line) to the experimental data points taken from [19].

- JLab

both sum rules sensitive to the twist-2 pion DA

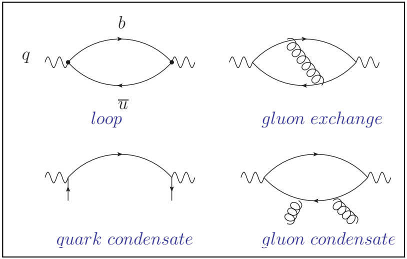
□ Use of 2-point QCD sum rules

- correlation function $\langle 0|T\{\bar{b}\gamma_\mu b(x)\,,\bar{b}\gamma_\mu b(0)\}|0\rangle$ saturated by Υ states

  $\Rightarrow$ non-lattice determination of m_b (in $\overline{MS}$ scheme)

  [K. Chetyrkin et al (2011); A. Hoang et al (2012)]

- correlation function $\langle 0|T\{\bar{b}\gamma_5 u(x), \bar{u}\gamma_5 b(0)\}|0\rangle$:

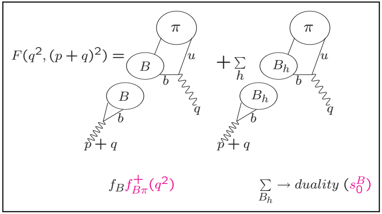


$$= \frac{\overbrace{\langle 0|\bar{b}\gamma_5 u|B\rangle\langle B|\bar{u}\gamma_5 b|0\rangle}^{f_B^2}}{m_B^2 - q^2} + \underbrace{\sum_{B_h} \frac{\langle 0|\bar{b}\gamma_5 u|B_h\rangle\langle B_h|\bar{u}\gamma_5 b|0\rangle}{m_{B_h}^2 - q^2}}_{\text{quark-hadron duality}}$$

- input: $m_b, \alpha_s, \langle\bar{q}q\rangle$, ...(condensate densities)
- various 2pt sum rules with pion currents: $\Rightarrow f_\pi^{(t)}, a_n^{\pi(t)}$, $t = 2, 3, 4$

  [P. Ball, V.Braun, A.Lenz (2006)]

□ Hadronic dispersion relation



- hadronic dispersion relation (analyticity $\oplus$ unitarity in QFT)

$$[F(q^2,(p+q)^2)]_{OPE} = \frac{m_B^2 f_B f^+_{B\pi}(q^2)}{m_B^2-(p+q)^2} + \int\limits_{(m_{B^*}+m_\pi)^2}^{\infty} ds\, \frac{\rho_h(s)}{s-(p+q)^2}$$

$$\Uparrow$$

- quark-hadron "semilocal" duality

$$\boxed{\int\limits_{(m_{B^*}+m_\pi)^2}^{\infty} ds\, \frac{\rho_h(s)}{s-(p+q)^2} = \int\limits_{s_0^B}^{\infty} ds\, \frac{[\mathrm{Im} F(q^2,s)]_{OPE}]}{s-(p+q)^2}}$$

- input:
  - f_B from 2pt SR, in NLO
  - variable scale: $(p+q)^2 \to M^2 \sim m_b\chi \to$ optimal interval of M^2
- s_0^b determined by calculating m_B^2 from LCSR

□ resulting LCSR for $B \to \pi$ form factor:

quark-hadron duality

$\downarrow$

Borel transf. $(p+q)^2 \to M^2 \sim m_b \chi$

$$f^+_{B\pi}(q^2;\vec{\theta}) = \frac{e^{m_B^2/M^2}}{2m_B^2 [f_B]_{\rm 2ptSR}} \int\limits_{m_b^2}^{s_0^B} ds \, \frac{1}{\pi} \mathrm{Im} F(s, q^2, \alpha_s, \mu, m_b, \vec{\theta}^{(2,3,4)}_{\rm DA}) \, e^{-s/M^2}$$

$\Uparrow$ $\Uparrow$

QCD SR for f_B — calculated from light-cone OPE

$$\left[f_B^2\right]_{\rm 2ptSR} = \left(\frac{e^{m_B^2/\overline{M}^2}}{m_B^4}\right) \overline{\mathcal{F}}(\overline{M}^2, \bar{s}_0^B, \alpha_s, \mu, m_b, \vec{\theta}_{\rm cond})$$

- set of inputs

$$\vec{\theta} \equiv \left(\alpha_s(M_Z), \overline{m}_b(m_b), \vec{\theta}^{(2,3,4)}_{DA}, \vec{\theta}_{\rm cond}, M^2, s_0^B, \overline{M}^2, \bar{s}_0^B\right).$$

- possibility to calculate derivatives of the form factor
- m_B calculation (differentiating the sum rules) constrains s_0^B, $\bar{s}_0^B$

Recent analysis of LCSR for $f^+_{B\pi}(q^2)$

I. S. Imsong, A. Khodjamirian, T. Mannel and D. van Dyk, arXiv:1409.7816 [hep-ph]

- calculate the form factor $f^+_{B\pi}(q^2, \vec{\theta})$ from LCSR;
  use 2pt SR for f_B

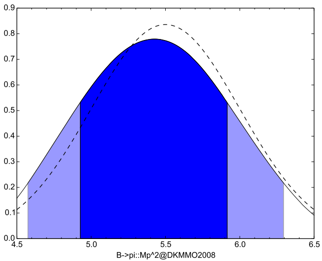
- input parameters $\vec{\theta}$ include:
  - α_s, b-quark mass
  - vacuum condensate densities
  - coefficients of pion DA's
  - Borel parameters
  - effective thresholds

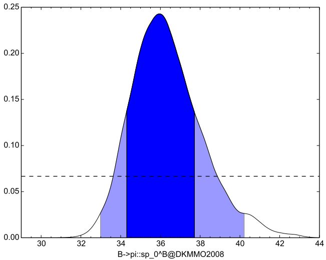
- statistical (Bayesian) analysis:
  inputs (assumed uncorrelated) taken as priors,
  constructing theoretical likelihood by imposing $[m_B]_{SR}$ within 1% of m_B

Some results

- Posterior of parameter space: one-dimension marginal PDF's

M^2 (GeV^2) s_0^B (GeV^2)





(prior: dashed lines, blue: 68%, light-blue: 95%)

- 6 quantities obtained from LCSR:
  $f^+_{B\pi}(q^2)$ + first + second derivative (value, slope, curvature) at $q^2 = 0, 10$ GeV^2 ,
  output approximately gaussian with large correlations

□ LCSR results fitted to BCL parameterization

- z-series parameterization, including $f^+_{B\pi}(0)$ $\;q^2 \to z(q^2, t_0)$, mapping SL region to small z, *the BCL-version [Bourrely, Caprini, Lellouch, (2008)]*

$$f^+_{B\pi}(q^2) = \frac{f^+_{B\pi}(0)}{1 - q^2/m^2_{B*}} \Big\{ 1 + b_1^+ \big[z(q^2, t_0) - z(0, t_0) - \frac{1}{3}\big(z(q^2, t_0)^3 - z(0, t_0)^3\big)\big]$$

$$+ b_2^+ \big[z(q^2, t_0)^2 - z(0, t_0)^2 + \frac{2}{3}\big(z(q^2, t_0)^3 - z(0, t_0)^3\big)\big] \Big\},$$

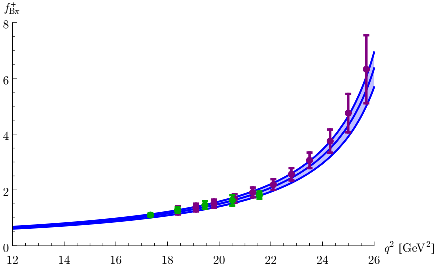
$f_{B\pi}(0) = 0.307 \pm 0.02$

$b_1^+ = -1.31 \pm 0.42$

$b_2^+ = -0.904 \pm 0.444$

$$\rho^{BCL} = \begin{pmatrix} 1.000 & 0.503 & -0.391 \\ 0.503 & 1.000 & -0.824 \\ -0.391 & -0.824 & 1.000 \end{pmatrix}$$

- extrapolation beyond the LCSR region
  lattice results (< 2015):
  • -HPQCD , • -Fermilab-MILC

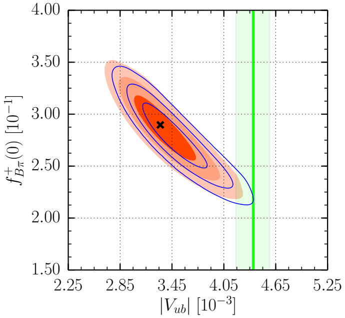


□ Determination of $|V_{ub}|$ from $B \to \pi \ell \nu_\ell$ data

fit of LCSR with the combined BaBar/Belle data at $0 < q^2 < 12~\text{GeV}^2$

(2010): $|V_{ub}| = \left(3.43^{+0.27}_{-0.23}\right) \cdot 10^{-3}$

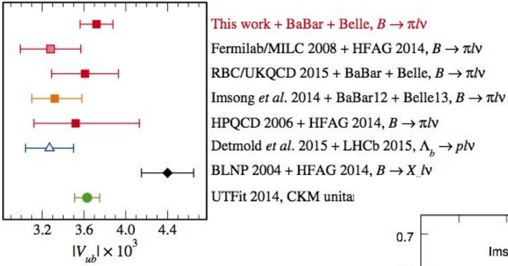
(2013): $|V_{ub}| = \left(3.32^{+0.26}_{-0.22}\right) \cdot 10^{-3}$

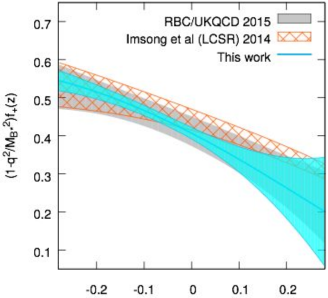


blue lines: 68%, 95%, 99% prob. contours for 2010 data
red area: 68%, 95%, 99% prob. contours for 2013 data

green line/area - inclusive determination:
central value / 68% CL interval for GGOU/HFAG

- Comparison with 2015 lattice results

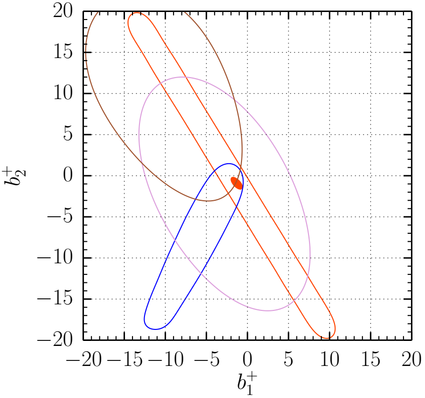




figs. from [J. A. Bailey *et al.* [Fermilab Lattice and MILC Collaborations], arXiv:1503.07839 [hep-lat].]

□ Comparison of shape parameters (preliminary !)

- z-parameterizations of experiment (lattice) fitted (transformed) to the $N = 3$ BCL (modified) form to compare with our results



orange: LCSR
(full with derivatives; lines without)
blue : Lattice with 3 data points
violet: 2010 BaBar+Belle
brown: 2013 BaBar+Belle

- only parametrical uncertainties of LCSR result are taken into account
- future Belle-2 data on the shape will be crucial

□ Future improvements in LCSR for $B \to \pi$

- statistical Bayesian analysis of LCSR for $f^+_{B\pi}(q^2)$ improves error estimate (within adopted theoretical approximation !)

  - analogous treatment of other LCSR's planned for

$$f^0_{B\to\pi},\ f^T_{B\to\pi},\quad B \to K\ ,\ B_s \to K \text{ FFs}$$

global Bayesian analysis including LCSRs for pion FFs and 2pt SR's

- further improvement of OPE and input?
  - $O(\alpha_s)$ to the nonasymptotic twist-3 part , may further soften μ-dependence
  - complete NNLO correction to twist-2, difficult, not a priority task

    very small $O(\alpha_s^2\beta_0)$ [A. Bharucha (2012)]

  - tw 5,6 in factorizable approximation
  - pion DAs: more accurate pion e.m. form factors

  data from BESS and Belle-2 on $\gamma^*\gamma \to \pi^0$

  and from JLab on the pion e.m. form factor

- further improvement of hadronic dispersion relation ?:
  - ansatz of semi-local quark-hadron duality:

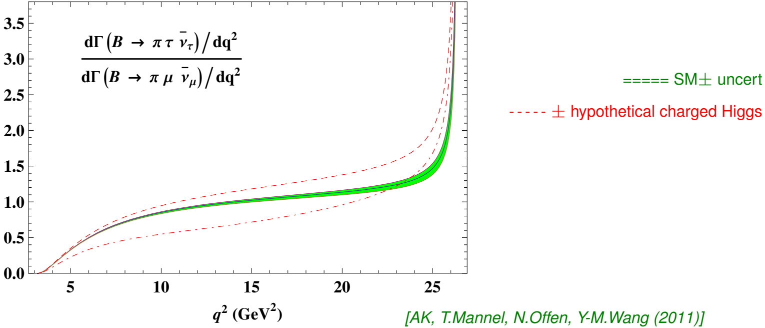
  works well in 2pt SR for f_B, LCSR for $D \to \pi$; improving OPE $\to$ larger M^2

$\square$ $B \to \pi\tau\nu_\tau$

accessible at Belle-2

- observable independent of V_{ub}: ($\ell = e$ or μ, $m_\ell = 0$)

$$\frac{d\Gamma(B\to\pi\tau\nu_\tau)/dq^2}{d\Gamma(B\to\pi\ell\nu_\ell)/dq^2} = \frac{(q^2-m_\tau^2)^2}{(q^2)^2}\left(1+\frac{m_\tau^2}{2q^2}\right)\left\{1+\frac{3m_\tau^2(m_B^2-m_\pi^2)^2}{4(m_\tau^2+2q^2)m_B^2p_\pi^2}\frac{|f^0_{B\pi}(q^2)|^2}{|f^+_{B\pi}(q^2)|^2}\right\}$$



[AK, T.Mannel, N.Offen, Y-M.Wang (2011)]

- scalar form factor $f^0_{B\pi}(q^2)$ from LCSR, with the same accuracy

to be updated with statistical analysis

$\Lambda_b \to p\ell\nu_\ell$

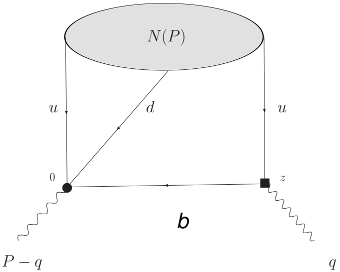
[AK, Ch.Klein, Th. Mannel and Y.M.Wang (2011)]

- vacuum-to-nucleon correlation function:
  LCSRs with nucleon DAs provided by [V.Braun et al]

- Λ_b interpolating 3-quark currents:

  $$\eta_{\Lambda_b}^{(\mathcal{P})} = (u\, C\, \gamma_5\, d)\, b$$
  $$\eta_{\Lambda_b}^{(\mathcal{A})} = (u\, C\, \gamma_5 \gamma_\lambda\, d)\, \gamma^\lambda\, b$$

- calculating $\Lambda_b \to p$ form factors,
  predicting the $\Lambda_b \to p\ell\nu_\ell$ width

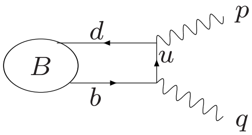


$$\Delta\zeta(0, q^2_{max}) = \frac{1}{|V_{ub}|^2} \int_0^{q^2_{max}} dq^2 \, \frac{d\Gamma}{dq^2}(\Lambda_b \to p l \nu_l) = 5.5^{+2.5}_{-2.0}\ \mathrm{ps}^{-1} \left(= 5.6^{+3.2}_{-2.9}\ \mathrm{ps}^{-1} \right)$$

uncertainties still large

□ Modifying the method: LCSR's with B-meson DA's

- a different correlation function:
  B -meson on-shell,
  pion interpolated with a current
  [A.K., T. Mannel, N.Offen,(2005) ,
  F.De Fazio, Th. Feldmann and T. Hurth, (2005)]



- B-meson distribution amplitude (DA), defined in HQET:

$$\langle 0|\bar{q}(x)[x,0]h_v(0)|\bar{B}_v\rangle = f_B m_B \int_0^\infty d\omega e^{-i\omega v\cdot x}\left\{\phi_+^B(\omega) + ...\right\}$$

- limited accuracy:
  - $O(\alpha_s)$ corrections not yet calculated,
  - the inverse moment of B-meson DA only approximately known

- $B \to \gamma\ell\nu_\ell$, a key process to determine the inverse moment
  also involves $b \to u$ transition !
  recent update in SCET [M.Beneke, J.Rohrwild (2011)],
  soft form factor estimated from LCSR [V.Braun, AK(2013)]
  Belle failed to detect this process (talk at Moriond-15), hopefully Belle-2 will do

$B \to \rho \ell \nu_\ell$ from LCSR

- LCSR's for $B \to V$ form factors, $V = \rho, K^*, \omega, \phi$

  [P. Ball, V.Braun (1998), P.Ball, R. Zwicky (2004,...)]

- using OPE for correlation functions with DA's of vector mesons

  special treatment of m_V-corrections

  DA's of vector mesons, worked out up to twist 4

  [P. Ball, V. M. Braun, Y. Koike and K. Tanaka, (1998),...]

- $\Gamma_V = 0$ approximation ("quenched")
- can the $B \to \rho \ell \nu_\ell$ determination of $|V_{ub}|$ be made as accurate as from $B \to \pi \ell \nu_\ell$?
- most recent analysis of $B \to \rho, \omega, K^*$ form factors in

  [A. Bharucha, D. M. Straub and R. Zwicky, arXiv:1503.05534 [hep-ph]]

  - the same OPE as in Ball-Zwicky '04 paper
  - statistical analysis, correlations, errors comparable to $B \to \pi$ LCSRs
  - the claim: "In summary, in practice the $\rho(\to \pi\pi)$ -meson state includes the non-resonant background in the experimental as well as the LCSR prediction".

$B \to \pi\pi\ell\nu_\ell$

- general $B \to \pi\pi$ form factors :
  - partial wave expansion & resonances: ρ (P-wave) , f_0 (S-wave)
  - regions of Dalitz plot with specific QCD (effective theory) dynamics

    [S. Faller, T. Feldmann, A. Khodjamirian, T. Mannel and D. van Dyk, (2013)]

- suppose we separate 2π system in P-wave from S-wave

  fitting angular distribution with improved statistics at Belle-2

- non-$\rho(770)$ contributions will contribute

  due to hadronic dispersion relation in the two-pion channel

- a sum over ρ resonances in Breit-Wigner /Gounaris-Sakurai form is a standard solution for timelike pion form factor,

  e.g. in $\tau \to \pi\pi\nu_\tau$

- no model-independent definition of $\rho(770)$, $\rho'(1450)$

  [C. Bruch, A. K. and J. H. Kuhn, [hep-ph/0409080]]

$\square$ $B \to \pi\pi\ell\nu_\ell$

- approach without explicit ρ modelling

  available only at small 2π mass and at large q^2 with the help of HQ χ PT

  [X. W. Kang, B. Kubis, C. Hanhart and U. G. Meissner (2013)

- $B \to \pi\pi$ form factors at $m_{2\pi} \lesssim 1$ GeV and small q^2 from LCSR with two-pion DA's
  - new nonperturbative input: timelike pion form factors (including resonances)

  [D.Mueller et al,(1994); M.Diehl et al. (1998); M.Polyakov (1999), M.Polyakov, Ch.Weiss (1999)]
  - first attempts to apply LCSRs to $B \to 2\pi\ell\nu_\ell$, [M.Maul (2001)]

  $B \to (\pi\pi)_0, (K\pi)_0$ form factors, using a model for the scalar timelike form factors

  [U.-G.Meißner, W.Wang, (2013)]

Concluding:

- $B \to \pi \ell \nu_\ell$ remains the best mode to extract $|V_{ub}|$
- $B \to \pi \tau \nu_\tau$ is a high priority task (due to the situation in $B \to D\tau\nu$)
- LCSR and lattice QCD are two independent methods to extract $|V_{ub}|$ and the results seem to agree.
- expecting from experiment:
  - accurate measurement of the q^2-shape in $B \to \pi \ell \nu_\ell$
  - improved measurements of pion e.m. form factors
- $B \to \pi\pi \ell \nu_\ell$ remains a useful tool to analyse angular observables,
- LCSR calculation of $B \to \rho$ form factors with $\Gamma_\rho = 0$ is insufficient, should be complemented by the calculation of $B \to (\pi\pi)_P \ell \nu_\ell$ and $B \to (\pi\pi)_S \ell \nu_\ell$ to assess the non-resonant background , modelling is unavoidable

BACKUP SLIDES

The OPE result

- currently achieved accuracy

$$F(q^2,(p+q)^2) = \left(T_0^{(2)} + (\alpha_s/\pi)T_1^{(2)} + (\alpha_s/\pi)^2 T_2^{(2)}\right) \otimes \varphi_\pi^{(2)}$$
$$+\frac{\mu_\pi}{m_b}\left(T_0^{(3)} + (\alpha_s/\pi)T_1^{(3)}\right) \otimes \varphi_\pi^{(3)}$$
$$+\frac{\delta_\pi^2}{m_b\chi} T^{(4)} \otimes \varphi_\pi^{(4)} + \ldots$$

- LO twist 2,3,4 $q\bar{q}$ and $\bar{q}qG$ terms:

[V.Belyaev, A.K., R.Rückl (1993); V.Braun, V.Belyaev, A.K., R.Rückl (1996)]

-NLO $O(\alpha_s)$ twist 2, (collinear factorization)

[A.K., R.Rückl, S.Weinzierl, O. Yakovlev (1997); E.Bagan, P.Ball, V.Braun (1997);]

-NLO $O(\alpha_s)$ twist 3 (coll.factorization for asympt. DA)

[P. Ball, R. Zwicky (2001); G.Duplancic,A.K.,B.Melic, Th.Mannel,N.Offen (2007)]

-part of NNLO $O(\alpha_s^2\beta_0)$ twist 2

[A. Bharucha (2012)]

□ LCSR for $D \to \pi, K$ form factors

[AK, Ch. Klein, Th. Mannel, N.Offen (2009)]

- important cross-check of the LCSR method
- $b \to c$ in the correlation function (finite quark masses !)

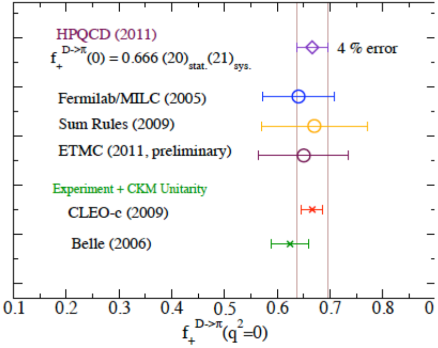


FIG. 6: The $D \to \pi$ form factor $f_+^{D\to\pi}(0)$ from this work an comparisons with other determinations [12, 13, 23–25].

taken from [HPQCD Collab. hep-lat 109.1501(2011)]

Second Gegenbauer moment of the pion DA

Method	$\mu = 1$ GeV	$\mu = 2$ GeV	Reference
LO QCDSR, CZ model	0.56	0.39	[35, 36]
QCDSR	$0.26^{+0.21}_{-0.09}$	$0.18^{+0.15}_{-0.06}$	[37]
QCDSR	0.28 ± 0.08	0.19 ± 0.06	[38]
QCDSR, NLC	0.19 ± 0.06	0.13 ± 0.04	[19, 39, 40]
$F_{\pi\gamma\gamma^*}$, LCSR	0.19 ± 0.05	0.12 ± 0.03 ($\mu = 2.4$)	[18]
$F_{\pi\gamma\gamma^*}$, LCSR	0.32	0.21 ($\mu = 2.4$)	[20]
$F_{\pi\gamma\gamma^*}$, LCSR, R	0.44	0.31	[41]
$F_{\pi\gamma\gamma^*}$, LCSR, R	0.27	0.19	[22]
F_π^{em},LCSR	$0.24 \pm 0.14 \pm 0.08$	$0.17 \pm 0.10 \pm 0.05$	[42, 43]
F_π^{em},LCSR, R	0.20 ± 0.03	0.14 ± 0.02	[44]
$F_{B\to\pi\ell\nu}$, LCSR	0.19 ± 0.19	0.13 ± 0.13	[45]
$F_{B\to\pi\ell\nu}$, LCSR	0.16	0.11	[46]
LQCD, $N_f = 2$, CW	0.289 ± 0.166	0.201 ± 0.114	QCDSF/UKQCD [47]
LQCD, $N_f = 2{+}1$, DWF	0.334 ± 0.129	0.233 ± 0.088	RBS/UKQCD [48]

TABLE I: The Gegenbauer moment $a_2(\mu^2)$. The CZ model involves $a_2 = 2/3$ at the low scale $\mu = 500$ MeV; for the discussion of the extrapolation to higher scales, see Ref. [20]. The abbreviations stand for: QCDSR: QCD sum rules; NLC: non-local condensates; LCSR: light-cone sum rules; R: renormalon model for twist-4 corrections; LQCD: lattice calculation; $N_f = 2(+1)$: calculation using $N_f = 2(+1)$ dynamical quarks; CW: non-perturbatively $\mathcal{O}(a)$ improved Clover–Wilson fermion action; DWF: domain wall fermions. For convenience we present the results for two scales, $\mu = 1$ GeV and $\mu = 2$ GeV, the relation is calculated in NLO.

$$a_2^\pi(\mu = 2GeV) = 0.1364 \pm 0.0154 \pm 0.0145$$ V.Braun et al, 15.03.03656 [hep-lat]

Table of inputs in LCSR for $B \to \pi$ form factor

Parameter	value/interval	unit	prior	source/comments
quark-gluon coupling and quark masses				
$\alpha_s(m_Z)$	0.1184 ± 0.0007	—	gaussian @ 68%	[6]
$\overline{m}_b(\overline{m}_b)$	4.18 ± 0.03	GeV	gaussian @ 68%	[6]
m_s	95 ± 10	MeV	—	[6] (error doubled)
$R \equiv 2m_s/(m_u + m_d)$	24.4 ± 1.5	—	—	ChPT, [25]
$m_u + m_d$	7.8 ± 0.9	MeV	gaussian @ 68%	$2m_s/R$
hadron masses				
m_B	5279.58	MeV	—	[6]
m_π	139.57	MeV	—	[6]
vacuum condensate densities				
$\langle \bar{q}q(2\mathrm{GeV})\rangle$	$-(277^{+12}_{-10})^3$	MeV^3	—	$m_\pi^2 f_\pi^2/2(m_u + m_d)$
$\langle \frac{\alpha_s}{\pi} G^2\rangle$	$[0.000, 0.018]$	GeV^4	uniform @ 100%	[26]
m_0^2	$[0.6, 1.0]$	GeV^2	uniform @ 100%	[26]
r_{vac}	$[0.1, 1.0]$	—	uniform @ 100%	[26]
parameters of the pion DAs				
f_π	130.4	MeV	—	[6]
$a_{2\pi}(1\mathrm{GeV})$	$[0.09, 0.25]$	—	uniform @ 100%	[10]
$a_{4\pi}(1\mathrm{GeV})$	$[-0.04, 0.16]$	—	uniform @ 100%	[10]
$\mu_\pi(2\mathrm{GeV})$	2.5 ± 0.3	GeV	—	$m_\pi^2/(m_u + m_d)$
$f_{3\pi}(1\mathrm{GeV})$	$[0.003, 0.006]$	GeV^2	uniform @ 100%	[22]
$\omega_{3\pi}(1\mathrm{GeV})$	$[-2.2, -0.8]$	—	uniform @ 100%	[22]
$\delta_\pi^2(1\mathrm{GeV})$	$[0.12, 0.24]$	GeV^2	uniform @ 100%	[22]
$\omega_{4\pi}(1\mathrm{GeV})$	$[0.1, 0.3]$	—	uniform @ 100%	[22]
sum rule parameters and scales				
μ	3.0	GeV	—	[10, 24]
M^2	16.0 ± 4.0	GeV^2	gaussian @ 68%	[10]
s_0^B	$[30.0, 45.0]$	GeV^2	uniform @ 100%	
$\overline{M}^2$	5.5 ± 1.0	GeV^2	gaussian @ 68%	[24]
$\overline{s}_0^B$	$[29.0, 44.0]$	GeV^2	uniform @ 100%	

Table 1. Input parameters used in the numerical analysis. The prior distribution $P_0(\vec{\theta})$ is a product

$B_{(s)}$ and $D_{(s)}$ decay constants from 2-point SR

[P.Gelhausen, AK, A.A.Pivovarov, D.Rosenthal, (2013)]

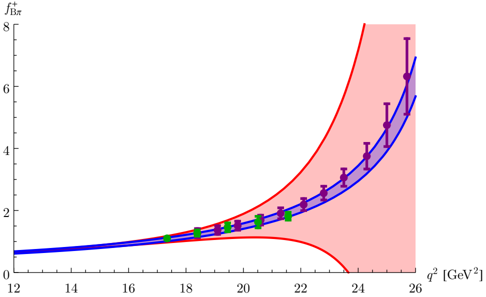
Decay constant	Lattice QCD [ref.]	this work
f_B[MeV]	196.9 ± 9.1 [1] 186 ± 4 [2]	207^{+17}_{-9}
f_{B_s}[MeV]	242.0 ± 10.0 [1] 224 ± 5 [2]	242^{+17}_{-12}
f_D[MeV]	218.9 ± 11.3 [1] 213 ± 4 [2]	201^{+12}_{-13}
f_{D_s}[MeV]	260.1 ± 10.8 [1] 248.0 ± 2.5 [2]	238^{+13}_{-23}

[1]-Fermilab/MILC, [2]-HPQCD

- calculated in NNLO, used for LCSR in NLO

Bounds for $B \to \pi$ form factor

- Parameterization-independent bounds following from the analytical properties of the form factor and from the unitarity of two-point correlation function [...., L.Lellouch (1996),...]
- form factor value, slope and curvature at one point yield the best constraints: [Th. Mannel, B.Postler (1998)]
- we use our results of statistical analysis at $q^2 = 10\ \mathrm{GeV}^2$



- bounds critically constraining lattice results up to $q^2 = 20\ \mathrm{GeV}^2$.

☐ $B \to \pi \ell \nu_\ell$ vs $B \to \tau \nu_\tau$

- update of the integrated LCSR form factor squared
  used previously for $|V_{ub}|$ determination

$$\Delta\zeta\,(0, q^2_{max}) \equiv \frac{G_F^2}{24\pi^3}\int\limits_0^{q^2_{max}} dq^2 p_\pi^3 |f^+_{B\pi}(q^2)|^2 = \frac{1}{|V_{ub}|^2\tau_{B^0}}\int\limits_0^{q^2_{max}} dq^2 \frac{d\mathcal{B}(B\to\pi\ell\nu_\ell)}{dq^2}\,,$$

$$\Delta\zeta(0, 12GeV^2) = 5.25^{+0.68}_{-0.54}\ \mathrm{ps}^{-1}\,,$$

- $|V_{ub}|$-independent ratio -test of QCD/SM

[AK, T.Mannel, N.Offen, Y-M.Wang (2011)]

$$R_{s/l}(q_1^2, q_2^2) \equiv \frac{\Delta\mathcal{B}_{B\to\pi\ell\nu_\ell}(q_1^2, q_2^2)}{\mathcal{B}(B\to\tau\nu_\tau)}\left(\frac{\tau_{B^-}}{\tau_{B^0}}\right) = \frac{\Delta\zeta(q_1^2, q_2^2)}{(G_F^2/8\pi)m_\tau^2 m_B(1 - m_\tau^2/m_B^2)^2 f_B^2}\,,$$