Status and prospects of lattice form factors and $|V_{ub}|$

Daping Du
Syracuse University

Challenges in Semileptonic B-Decays Workshop
April 20-24, MITP, Mainz, Germany

Outline

- Motivation
- Introduction: semileptonic form factors from lattice
- Recent progress related to $|V_{ub}|$
  - $B \to \pi\ell\nu$
  - $B_s \to K\ell\nu$
  - $\Lambda_b \to p\ell\nu$
- Conclusion and outlook

Why lattice QCD?

- Quantitative understanding of the **non-perturbative** effects is crucial for **precision calculations** of SM and beyond. **Lepage, Mackenzie & Peskin, 1404.0319**
- Lattice QCD is a mature method for the calculations of "**simple quantities**": [with one or zero initial (final) hadron]: masses, decay constants, weak matrix elements… **A. El-Khadra CKM2014**
- Lattice QCD errors are systematically improvable:
  - Emerging simulation techniques combined with hardware improvements (cost/performance drop **100** times in the last decade)
  - Calculations can be **well planned**!

Quantity	CKM element	present expt. error	present lattice error	2009 lattice error	2014 lattice error
f_K/f_π	V_{us}	0.3%	0.9%	0.5 %	0.3%
$f_{K\pi}(0)$	V_{us}	0.4%	0.5%	0.3%	0.2%
$D \to \pi \ell \nu$	V_{cd}	3%	11%	6%	4%
$D \to K \ell \nu$	V_{cs}	1%	11%	5%	2%
$B \to D^* \ell \nu$	V_{cb}	1.8%	2.4%	1.6%	0.8%
$B \to \pi \ell \nu$	V_{ub}	3.2%	14%	10%	4%

R. Sugar: DOE report (2008)

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$D \to \pi\ell\nu$	V_{cd}	3%	11%	6%	4%	**4.3%**
$D \to K\ell\nu$	V_{cs}	1%	11%	5%	2%	**2.5%**
$B \to D^*\ell\nu$	V_{cb}	1.8%	2.4%	1.6%	0.8%	**1.4%**
$B \to \pi\ell\nu$	V_{ub}	3.2%	14%	10%	4%	**3.4%**

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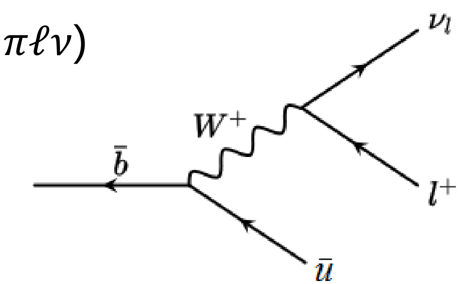
Semileptonic decays and CKM

- Exclusive $B_{(s)} \to M_u \ell \nu$ decays
  - Precise measurements of partial rates are available ($B \to \pi \ell \nu$)
  - Mature theoretical techniques for ME (LQCD or LCSR)



$\pi\ (K)$

$$\frac{(d\Gamma(B_{(s)} \to M_u \ell^- \nu)/dq^2)}{\text{kin. factor}} = |V_{ub}|^2 \, |f_+(q^2)|^2$$

Experiments

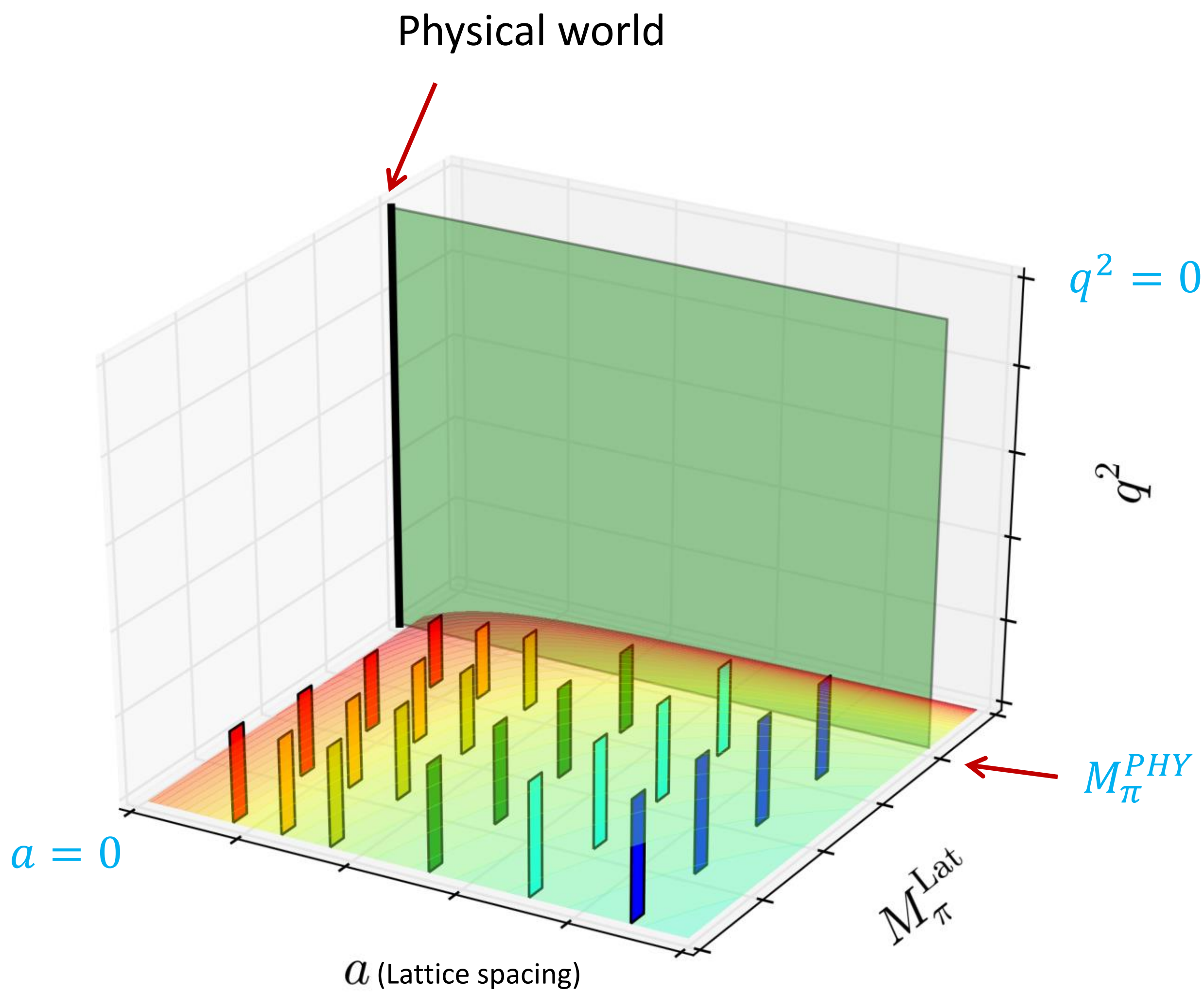
Lattice QCD

- Exclusive $\Lambda_b \to p \ell \nu$ decay
  - Very recent LHCb experiment **LHCb,1504.01568**
  - Possible probe of NP with right-handed currents **Chen et al. 0807.0896**

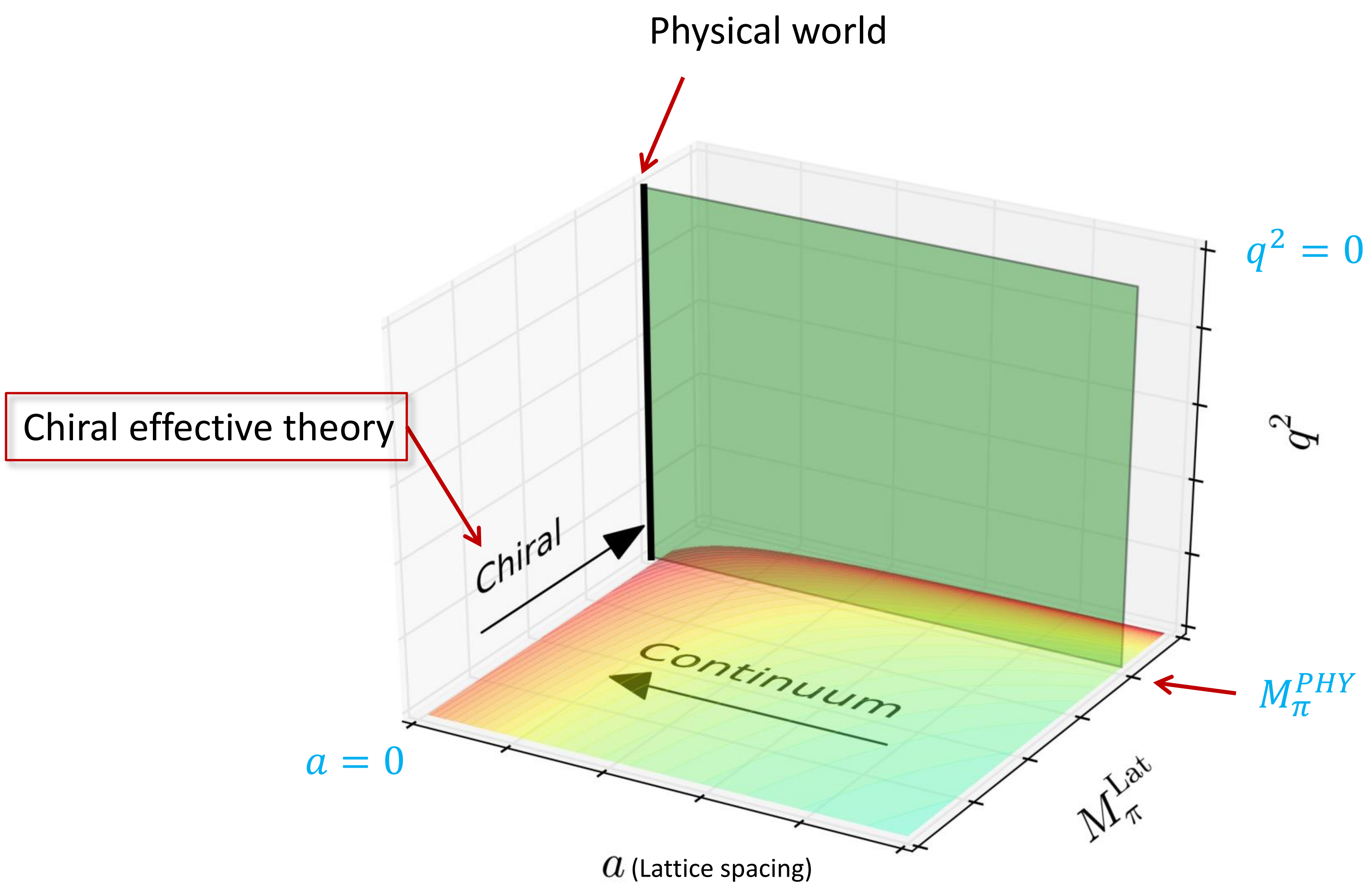
$$\frac{(d\Gamma(\Lambda_b \to p \ell^- \nu)/dq^2)}{\text{kin. factor}} = |V_{ub}|^2 \; F(f_+^2, f_\perp^2, g_+^2, g_\perp^2)$$

Semileptonic decays on the lattice: introduction

The parameter space

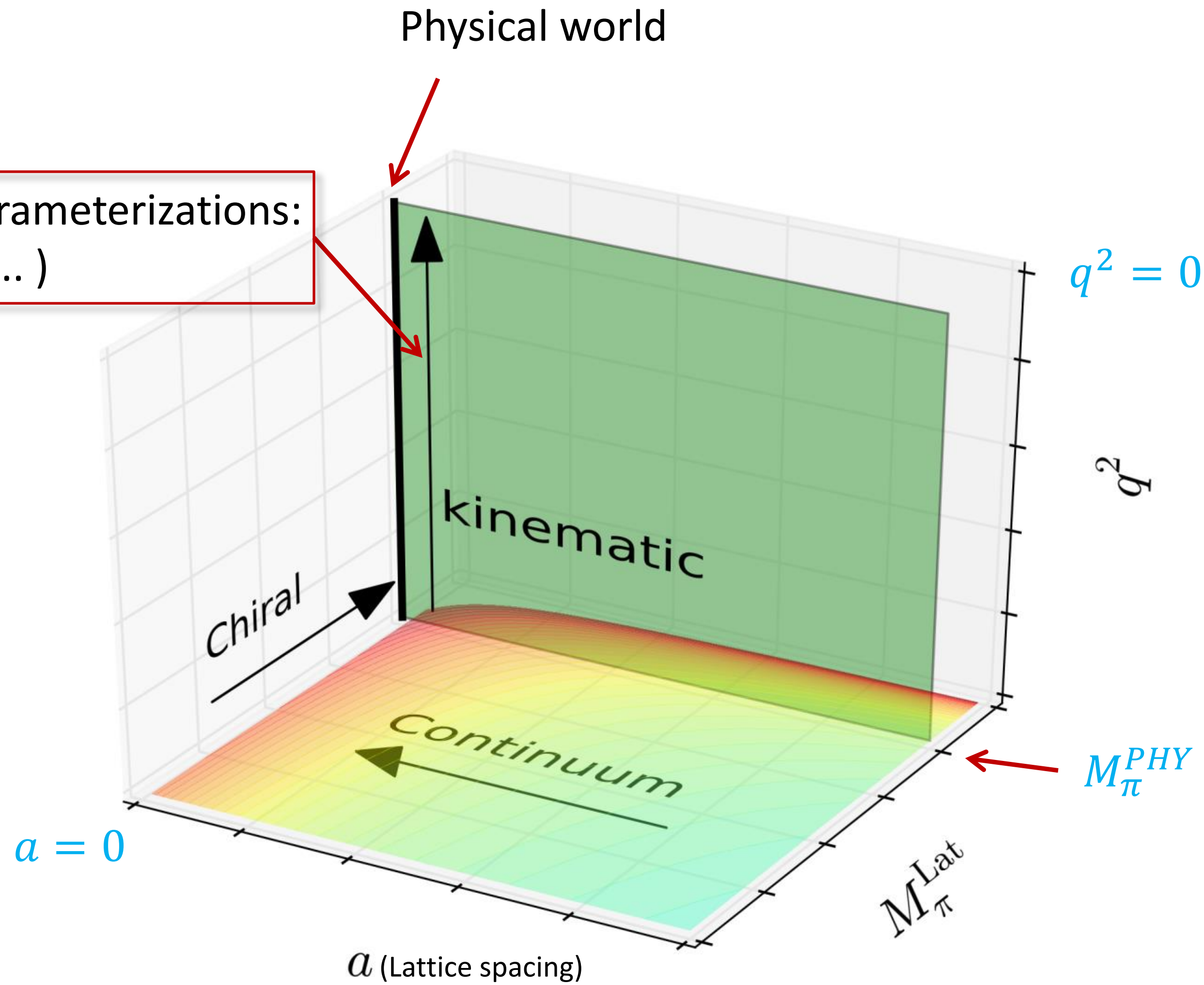


Semileptonic decays on the lattice: extrapolations



Semileptonic decays on the lattice : extrapolations

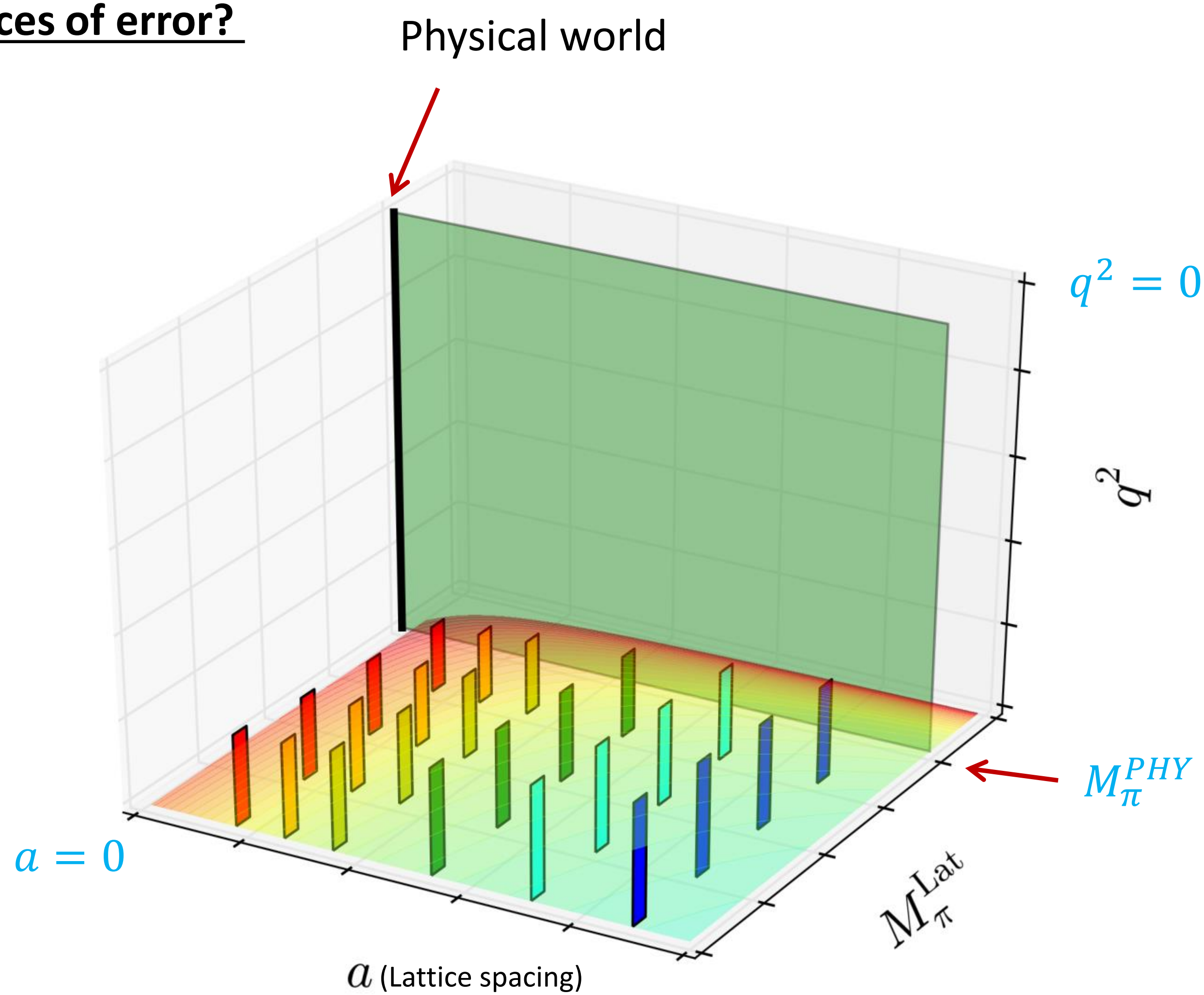
Model-independent parameterizations:
z-expansion (BCL, BGL, ..)



Semileptonic decays on the lattice: errors

What are the major sources of error?

Statistical error



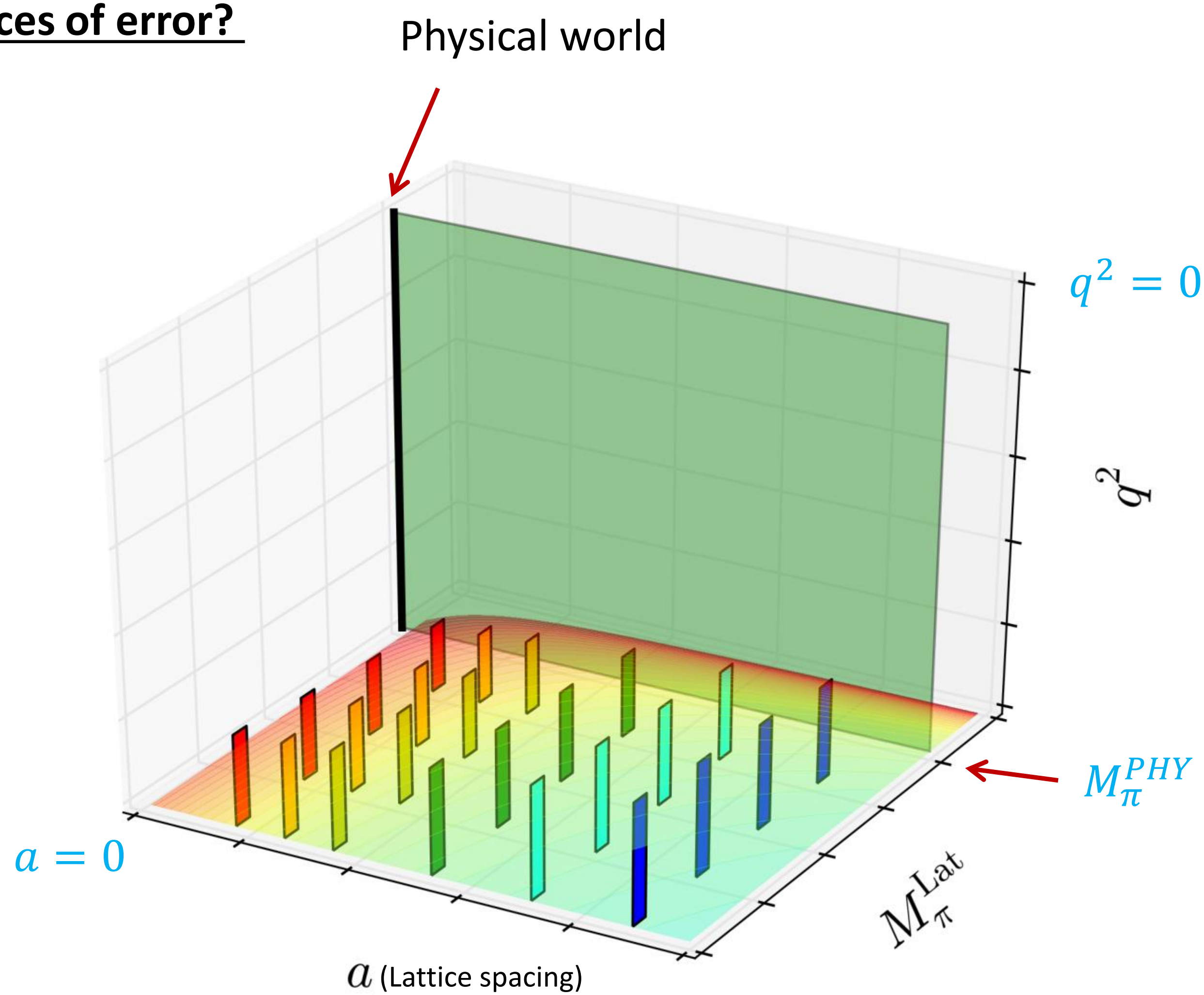
Semileptonic decays on the lattice: errors

What are the major sources of error?

Statistical error

Discretization effects

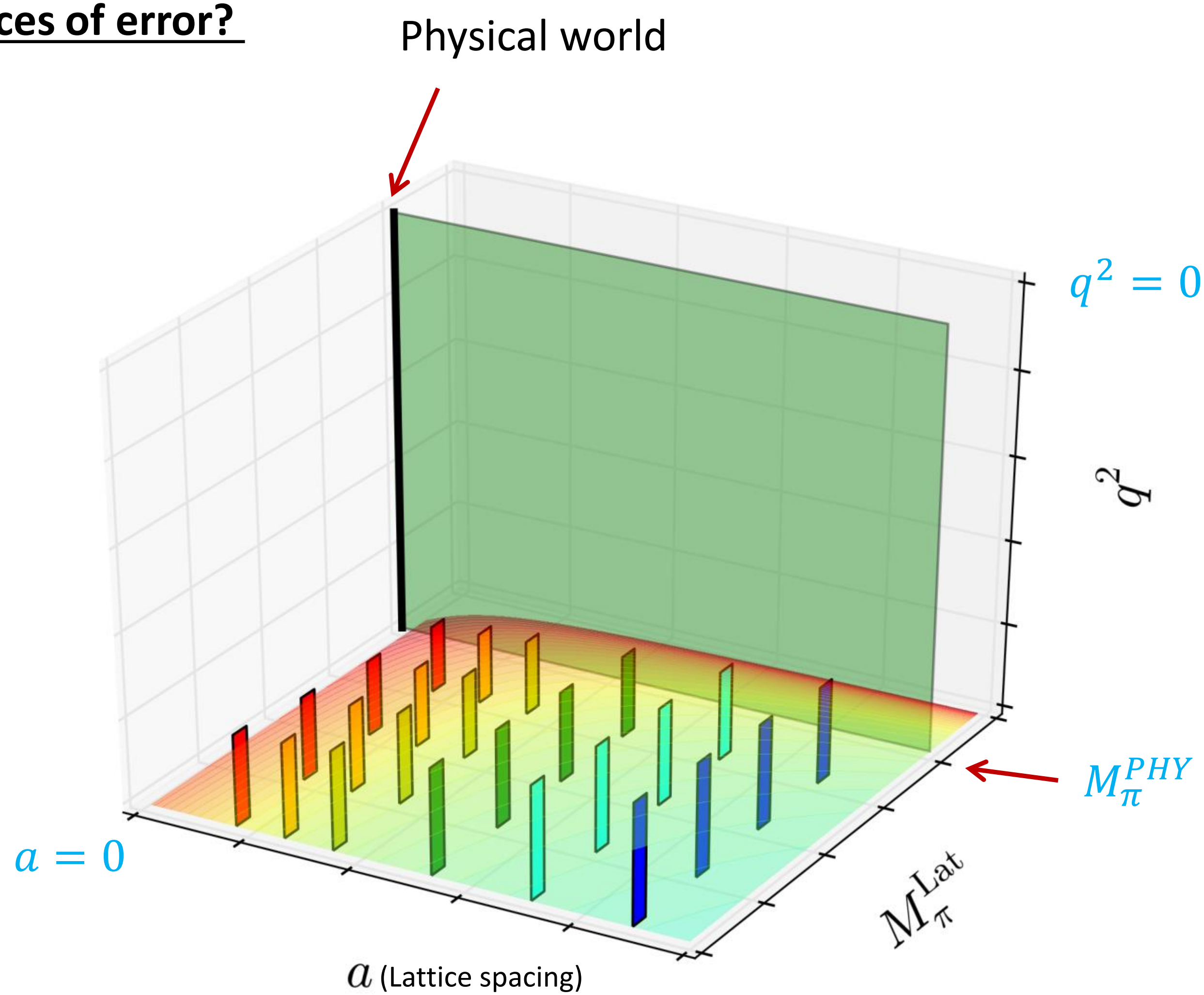
Chiral extrapolation



Semileptonic decays on the lattice: errors

What are the major sources of error?

- **Statistical error**
- **Discretization effects**
- **Chiral extrapolation**
- **Finite volume effects**
- **Tuning: m_q, a ...**



Semileptonic decays on the lattice: errors

What are the major sources of error?

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Finite volume effects

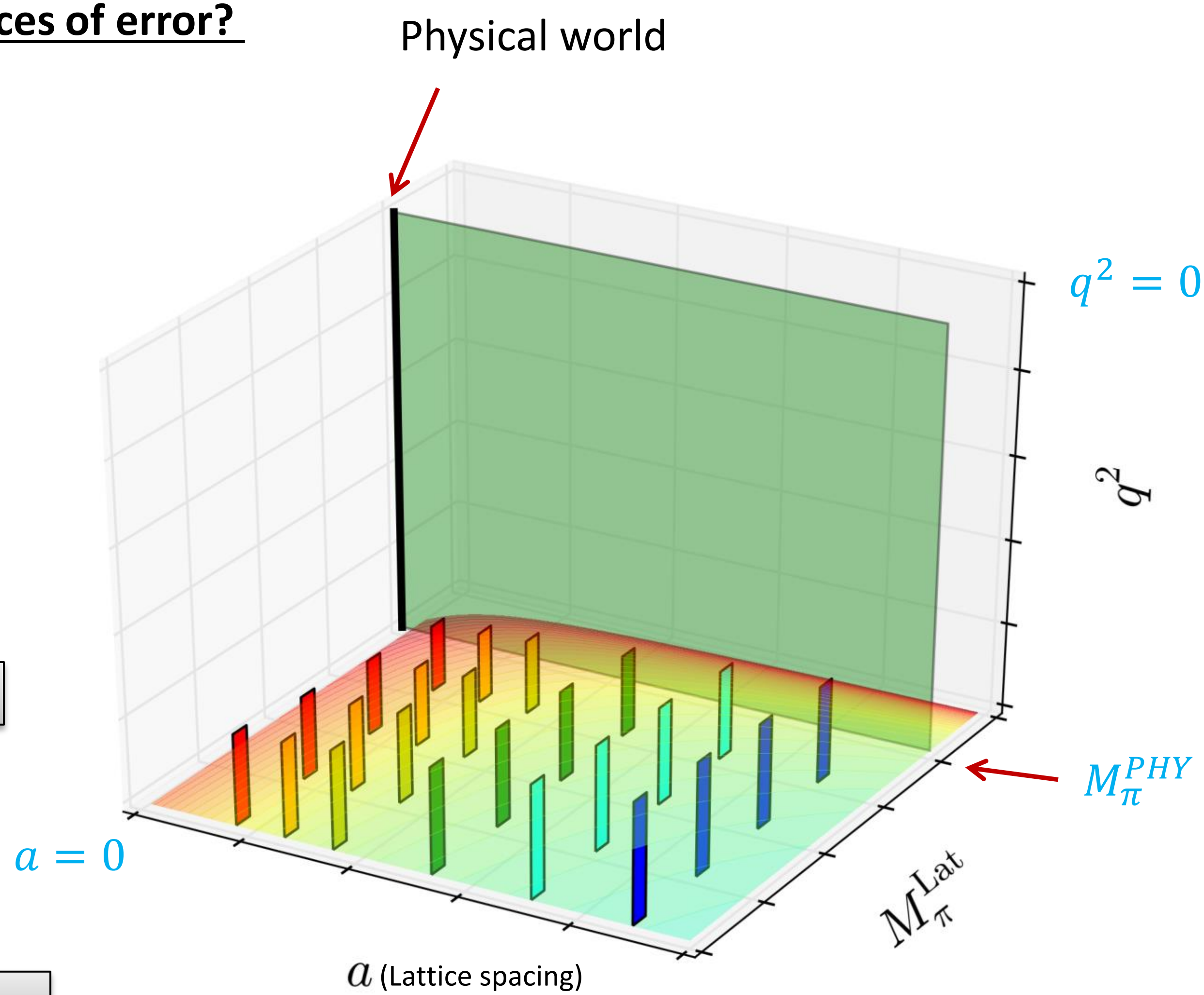
Tuning: m_q, a ...

Current renormalization

(When a heavy quark is used)

Heavy quark mass

Heavy quark discretization



LQCD calculations for $|V_{ub}|$: recent progress

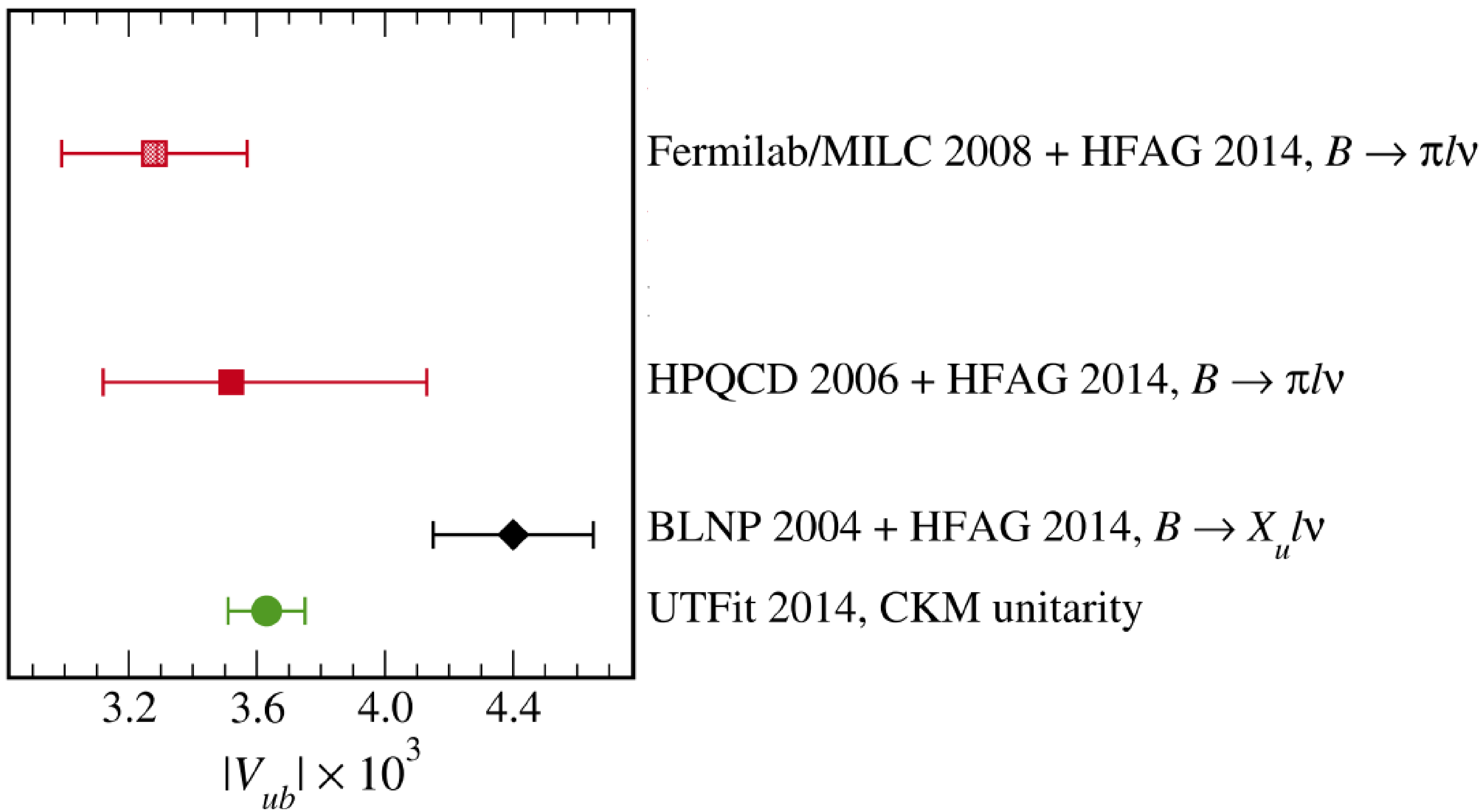
- Disclaimer: the list is not meant to be inclusive. I am focusing on the publicized results.

Lattice Group	Fermilab/MILC	HPQCD	RBC/UKQCD	Alpha	Detmold et al.
Process	$B \to \pi \ell \nu$	$B_s \to K \ell \nu$	$B \to \pi \ell \nu$	$(B_s \to K \ell \nu)$	$\Lambda_b \to p \ell \nu$
	$(B_s \to K \ell \nu)$	$(B \to \pi \ell \nu)$	$B_s \to K \ell \nu$		
Gauge ensembles	MILC asqtad	MILC asqtad	Domain-Wall	CLS	Domain-Wall
Sea flavors	2+1	2+1	2+1	2	2+1
a (fm)	0.045–0.12	0.09–12	0.086–0.11	0.049–0.076	0.086–0.11
M_π	≥ 177 MeV	≥ 354 MeV	≥ 289 MeV	≥ 310 MeV	≥ 295 MeV
l-quark action	asqtad	HISQ	Domain-Wall	Imprv. Wilson	Domain-Wall
b-quark action	Fermilab Clover	NRQCD	RHQ	Lat. HQET	RHQ
χPT	NNLO,SU(2), hard-π	HPχPT+	NLO,SU(2), hard-π		
q^2-extrapolation	functional BCL	modified z	synthetic BCL		modified-z
Ref.	arXiv:1503.07839	arXiv:1406.2279	arXiv:1501.05373v2	arXiv:1411.3916	arXiv:1306.0446
	arXiv:1312.3197				arXiv:1503.01421v2
					arXiv:1504.01568

- (): work in progress

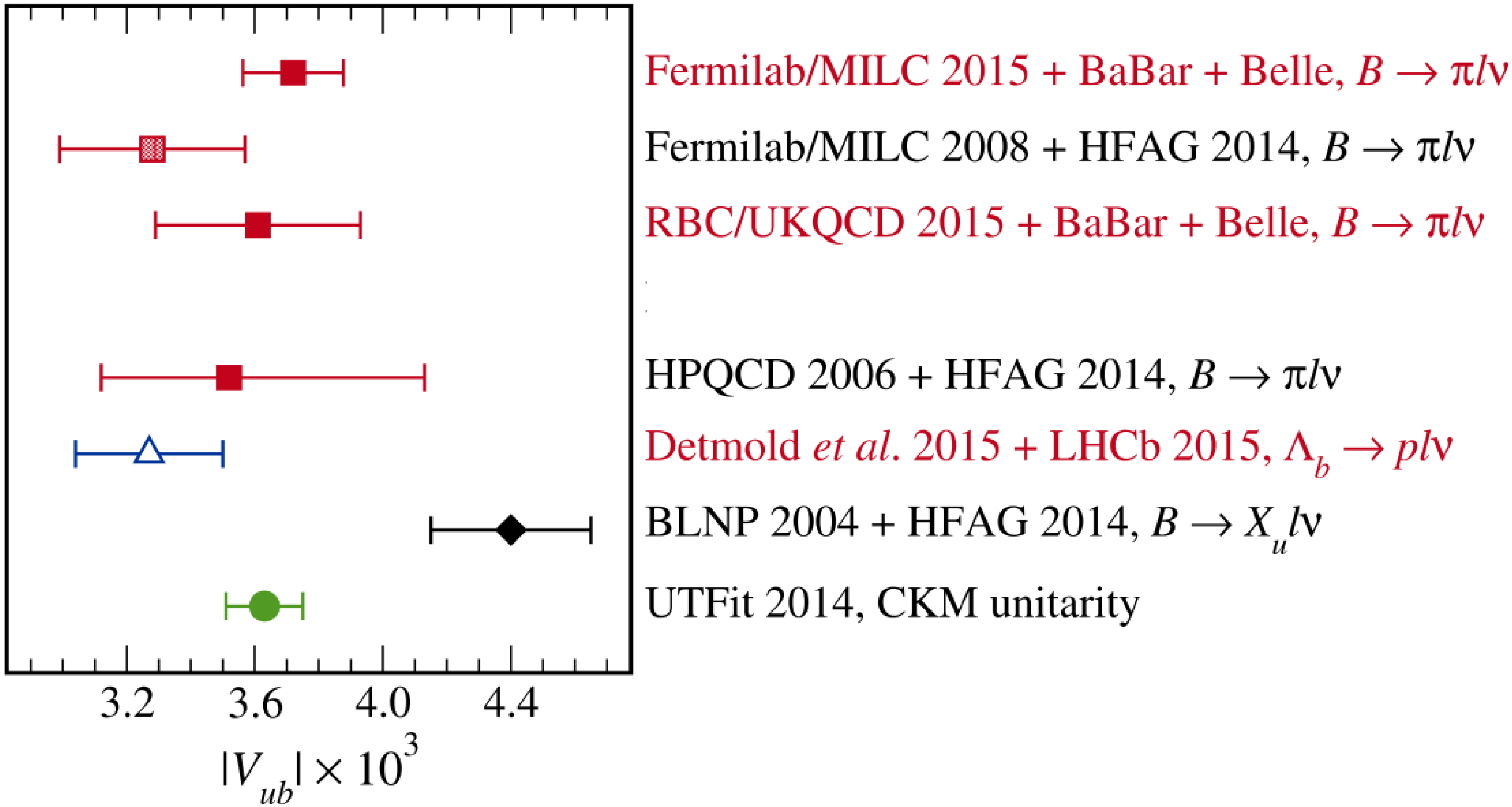
Results for $|V_{ub}|$

➢ From lattice **semileptonic form factors** (2008 and earlier)



Results for $|V_{ub}|$ (most recent development)

- From lattice **semileptonic form factors** (2015)



$$B \to \pi \ell \nu$$

$B \to \pi \ell \nu$ form factors

- Vector current matrix element

$$\langle \pi | \mathcal{V}^\mu | B \rangle = f_+(q^2) \left(p_B^\mu + p_\pi^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu \right) + f_0(q^2) \frac{M_B^2 - M_\pi^2}{q^2} q^\mu$$

$$= \sqrt{2M_B} [v^\mu f_{||}(E_\pi) + p_\perp^\mu f_\perp(E_\pi)]$$

$$v^\mu = p_B^\mu / M_B$$

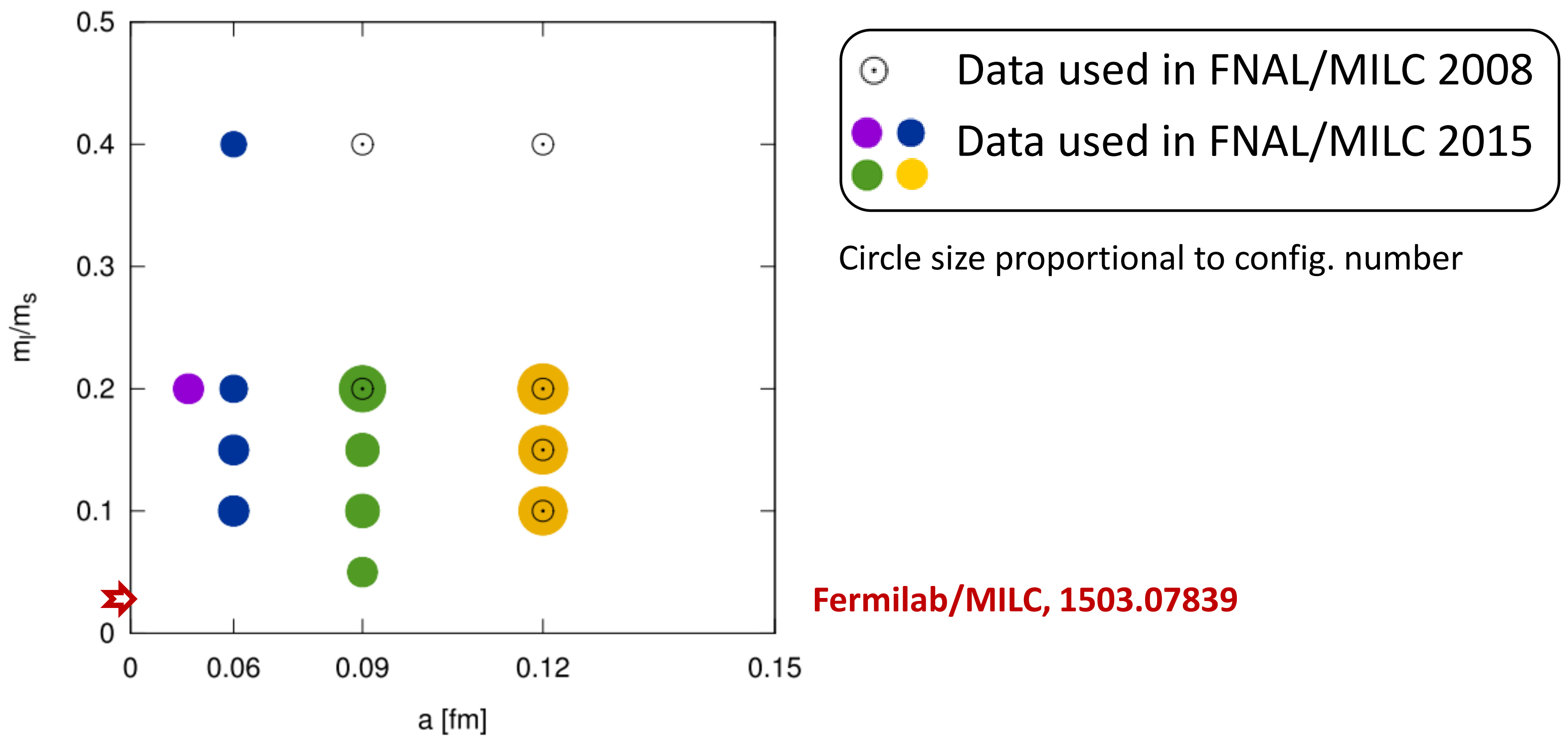
$$p_\perp^\mu = p_\pi^\mu - (p_\pi \cdot v) v^\mu$$

Easier to extract on the lattice

- Last published result is dated back to Fermilab/MILC 2008
- Major source of error in $|V_{ub}|$: ~8% compared to ~3% from experiment!

$B \to \pi \ell \nu$ (Fermilab/MILC 2015)

- Improvements with respect to FNAL/MILC 2008 **Fermilab/MILC, 0811.3640**
  - **Increased statistics**: > 3X number of configurations
  - **Finer lattice spacing**: $a_{min} = 0.09$ fm $\to 0.045$ fm
  - **Smaller light quark masses**: (M_π=177~450 MeV)
  - Improved (non-perturb. and perturb.) renormalization factors
  - b-quark mass mistuning correction
  - Functional z-expansion for q^2 extrapolation

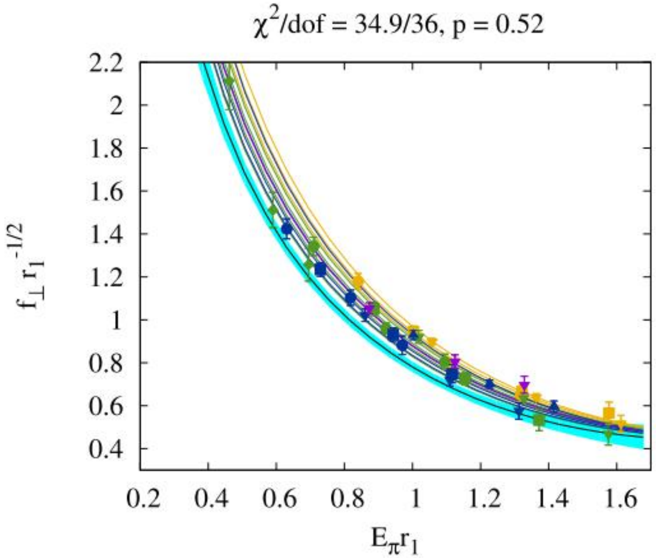


Circle size proportional to config. number

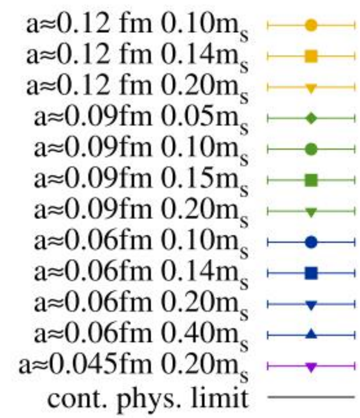
Fermilab/MILC, 1503.07839

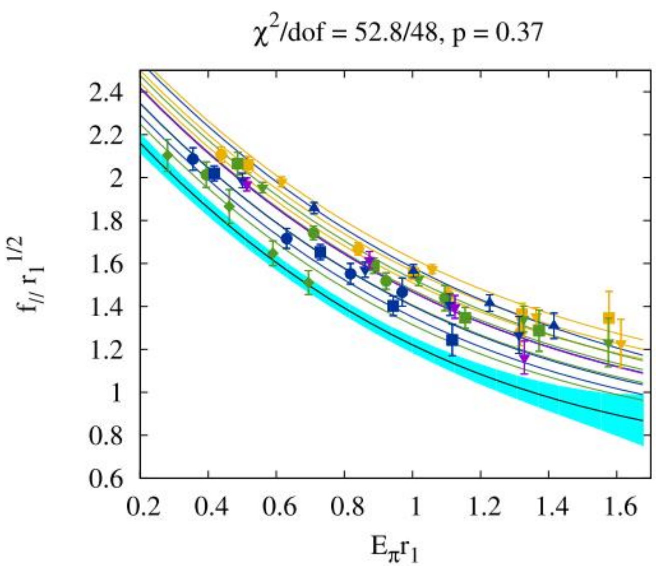
$B \to \pi \ell \nu$ (Fermilab/MILC 2015)

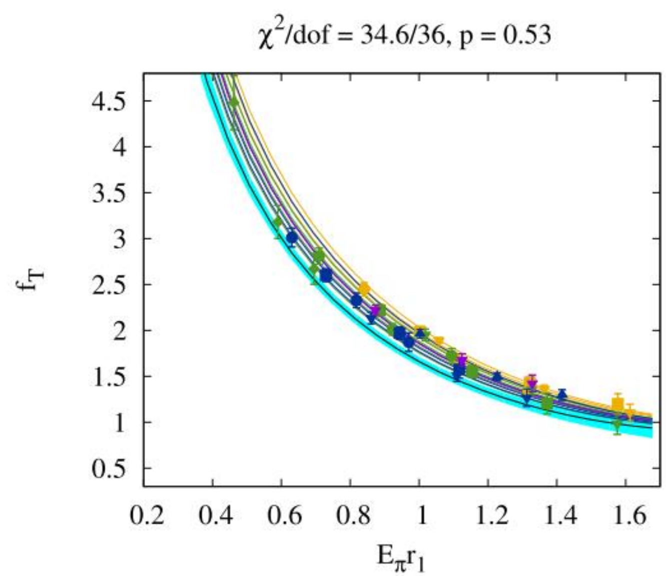
- SU(2) Hard-pion HMsχPT is used for the **χPT/continuum extrapolation**



Fermilab/MILC, 1503.07839



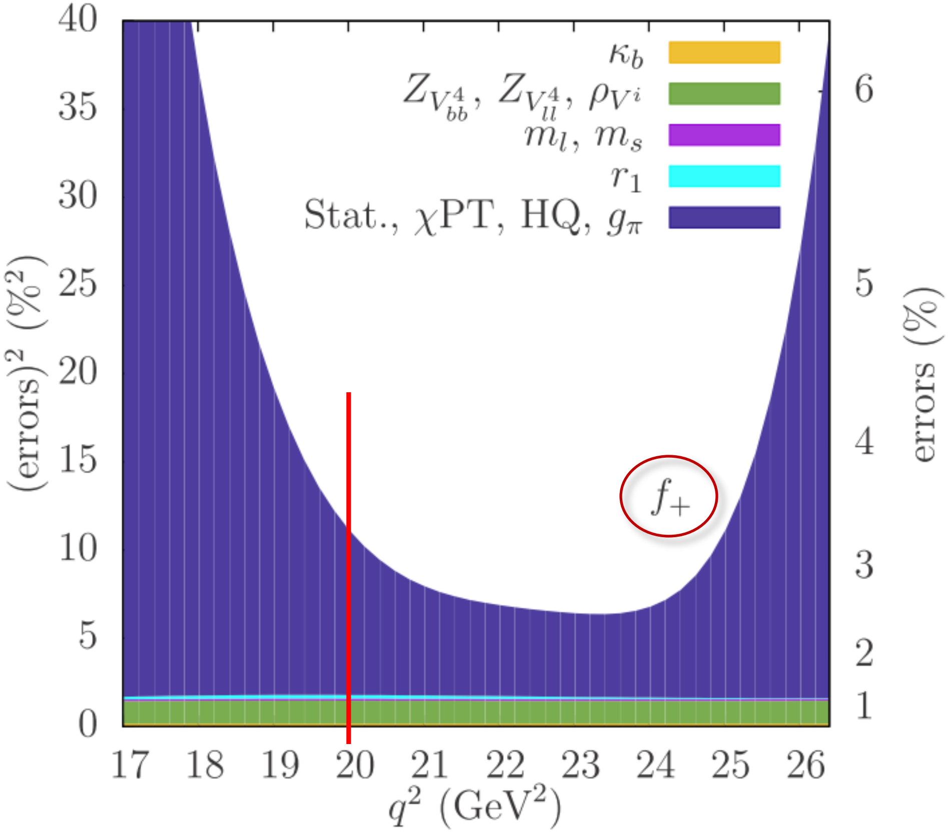




$B \to \pi \ell \nu$ (Fermilab/MILC 2015)

- Full error budget

Fermilab/MILC, 1503.07839



Error budgets of form factors f_+ at $q^2 = 20\text{GeV}^2$.

Uncertainty	δf_+
Statistical+χPT+HQ+$g_{B^*B\pi}$	3.1
Scale r_1	0.4
Non-perturbative $Z_{V^4_{bb}}$	0.4
Non-perturbative $Z_{V^4_{ll}}$	0.3
Perturbative ρ	1.0
Heavy-quark mass mistuning	0.4
Light-quark mass tuning	0.3
Total	3.4

Improvement over Fermilab/MILC 2008 is about factor of 3

$B \to \pi \ell \nu$ (Fermilab/MILC 2015)

- Functional BCL z-expansion is used for the kinematic extrapolation

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$t_+ = (M_B + M_\pi)^2$

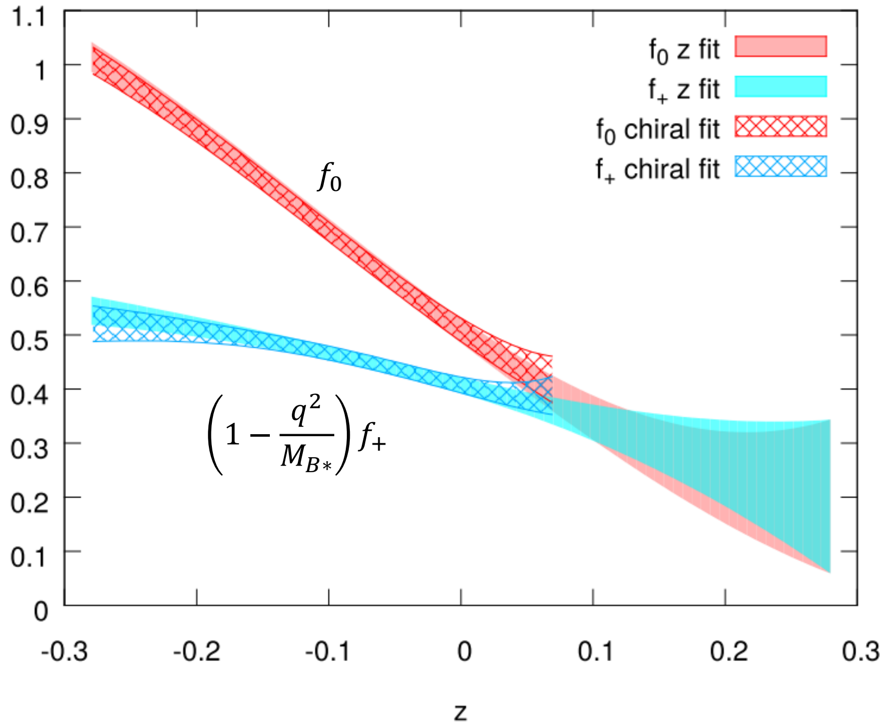
(t_0 is chosen to symmetrize the whole range)

$$f_i(z) = \frac{1}{P_i(z)} \sum_{n=0}^{N_z - 1} b_n \left[z^n - (-1)^{n - N_z} \frac{n}{N_z} z^{N_z} \right],$$

$P_+ = 1 - q^2/M_{B^*}^2,$

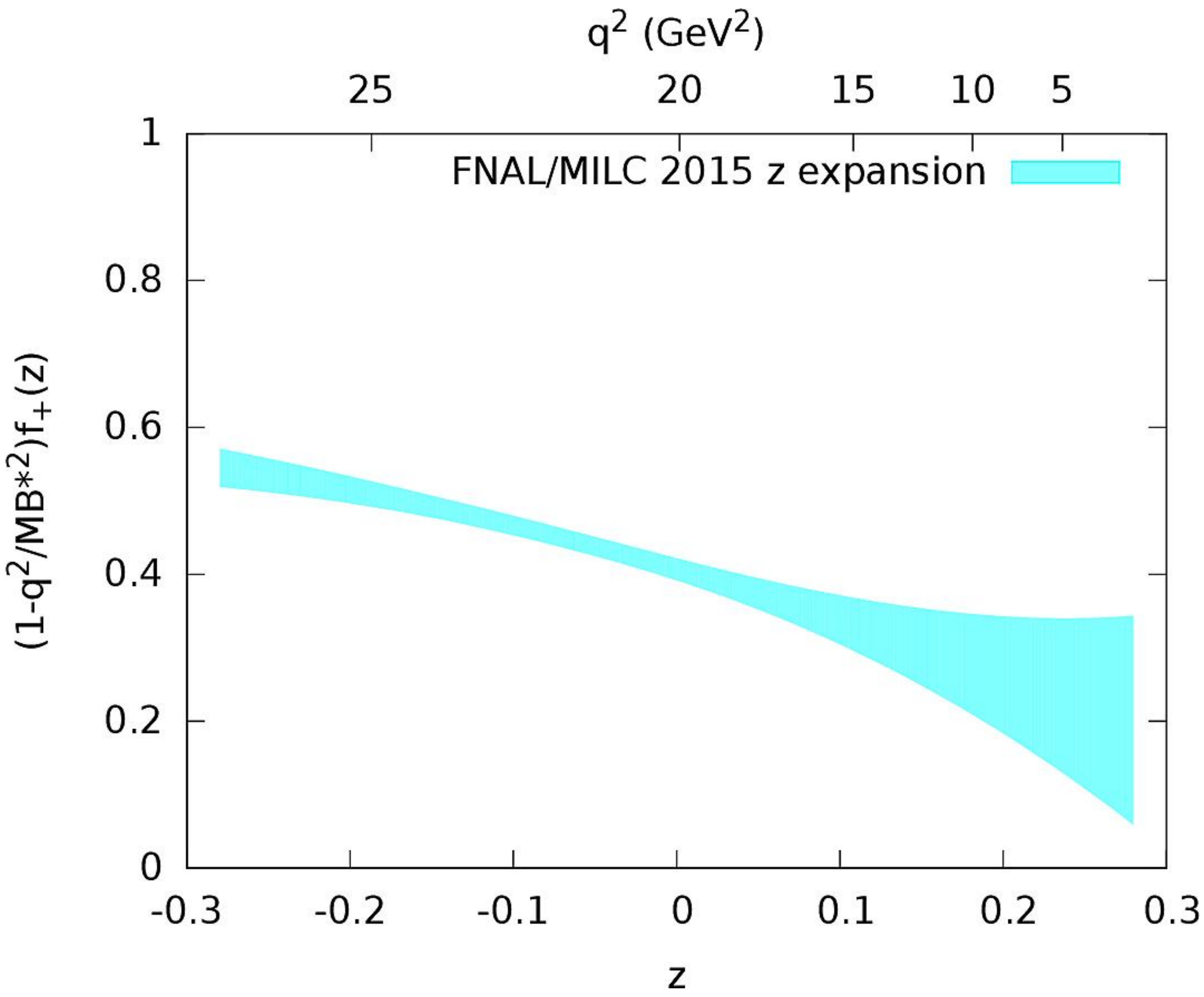
$P_0 = 1$

- Kinematic constraint $f_+(0) = f_0(0)$, up to z^3 (actually z^4, but it is not free)



$B \to \pi \ell \nu$ (Fermilab/MILC 2015)

- Combined fit using lattice + experiments for $|V_{ub}|$
  - Experimental data are converted using combined-fit $|V_{ub}|$



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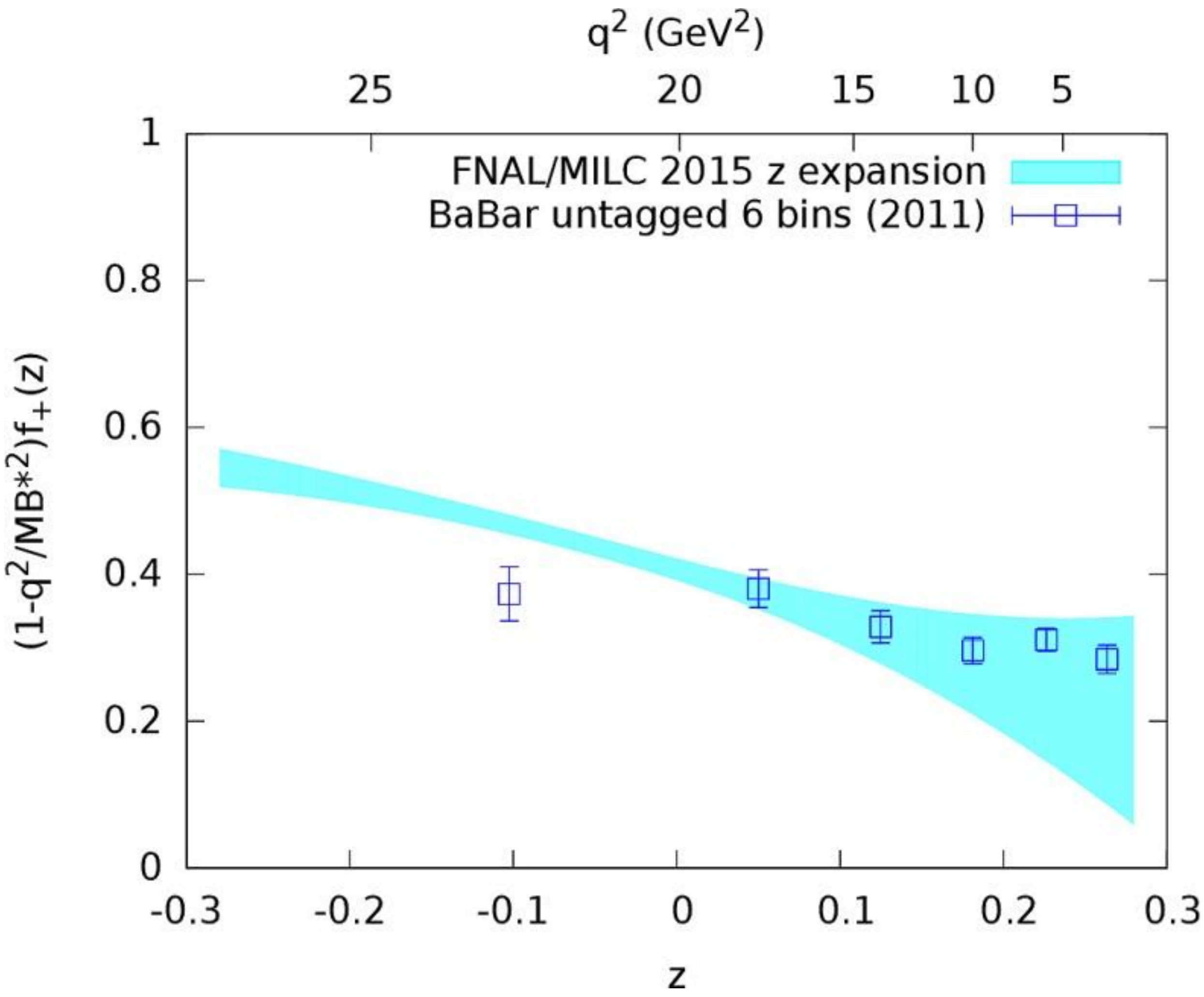

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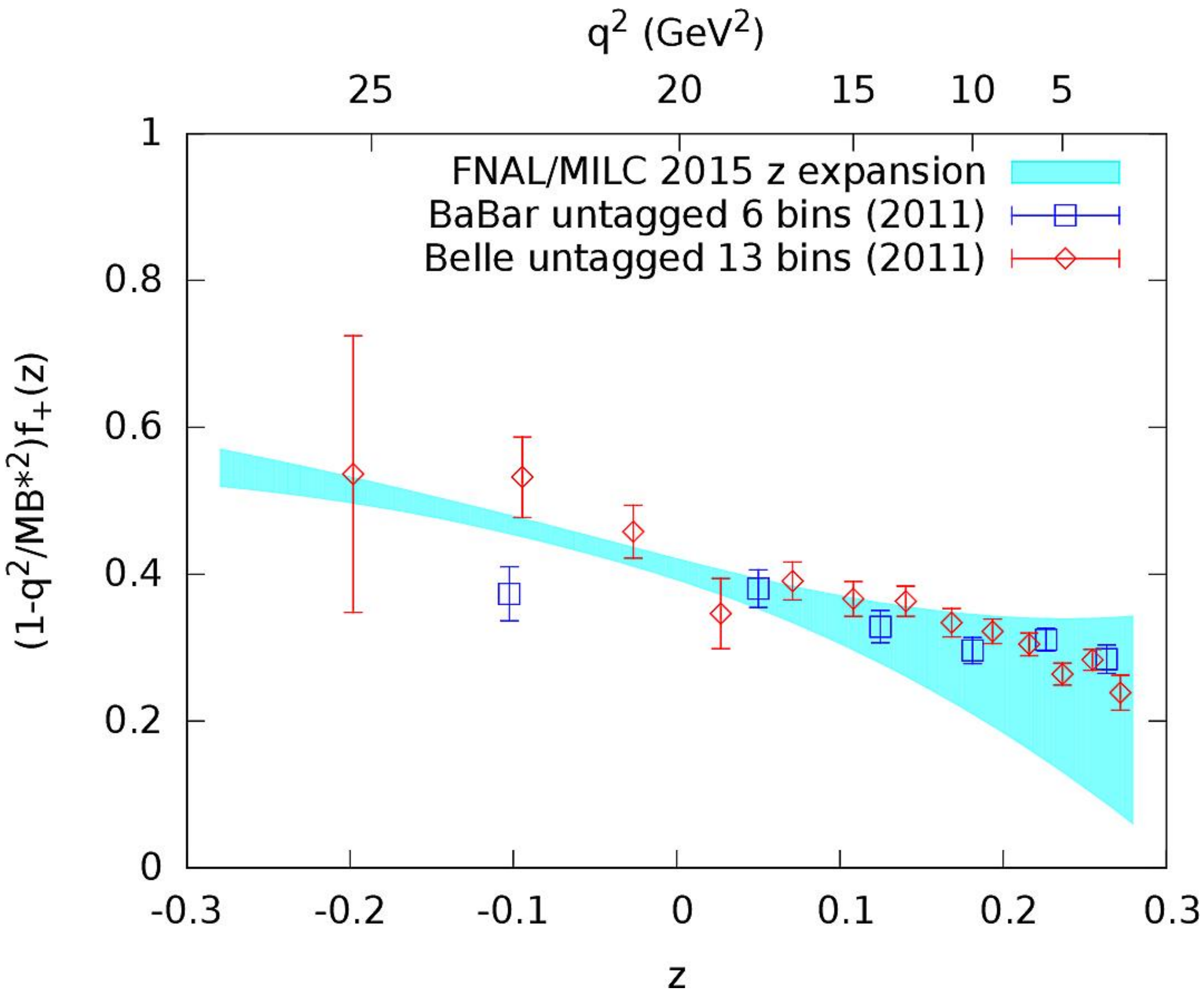

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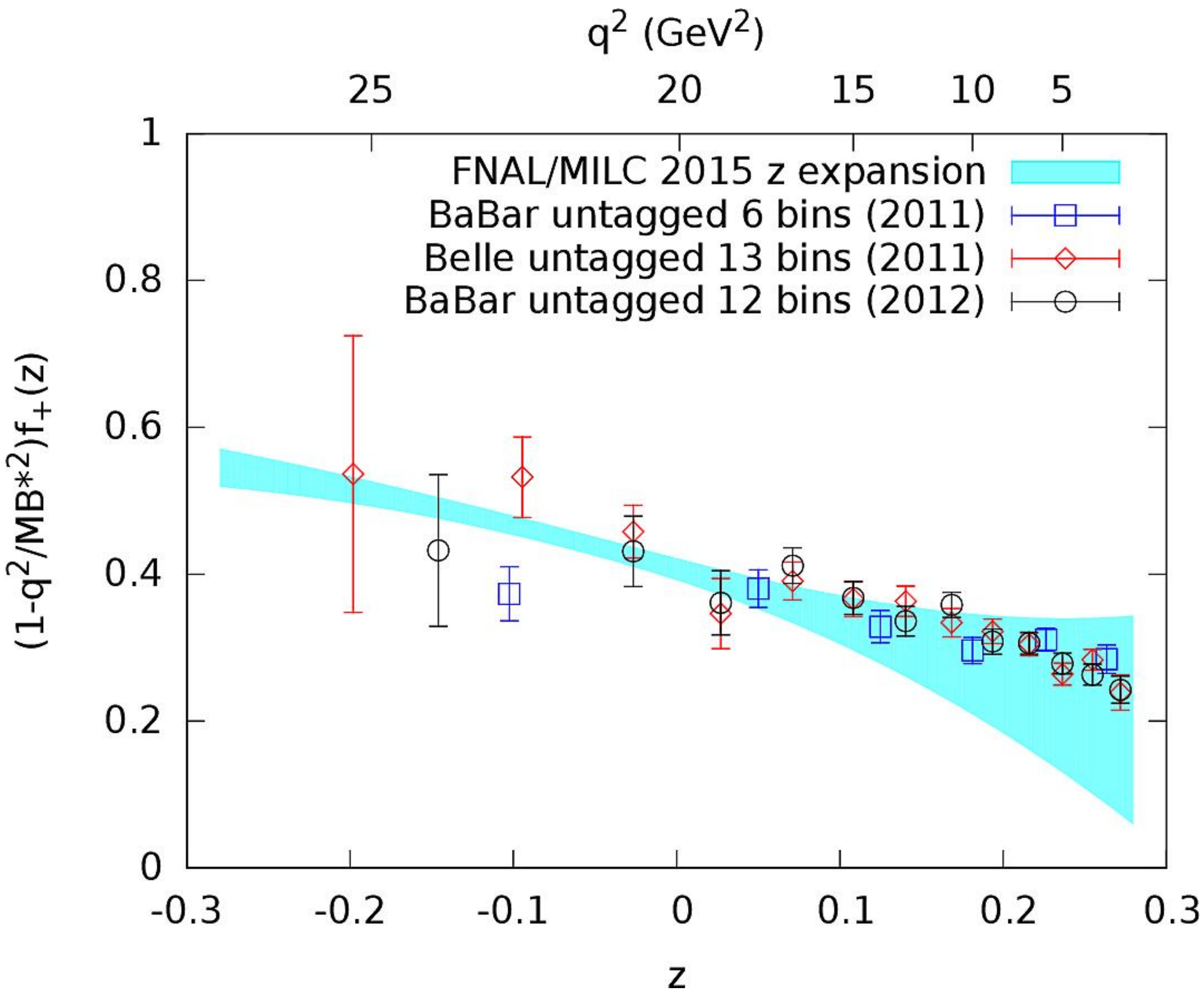

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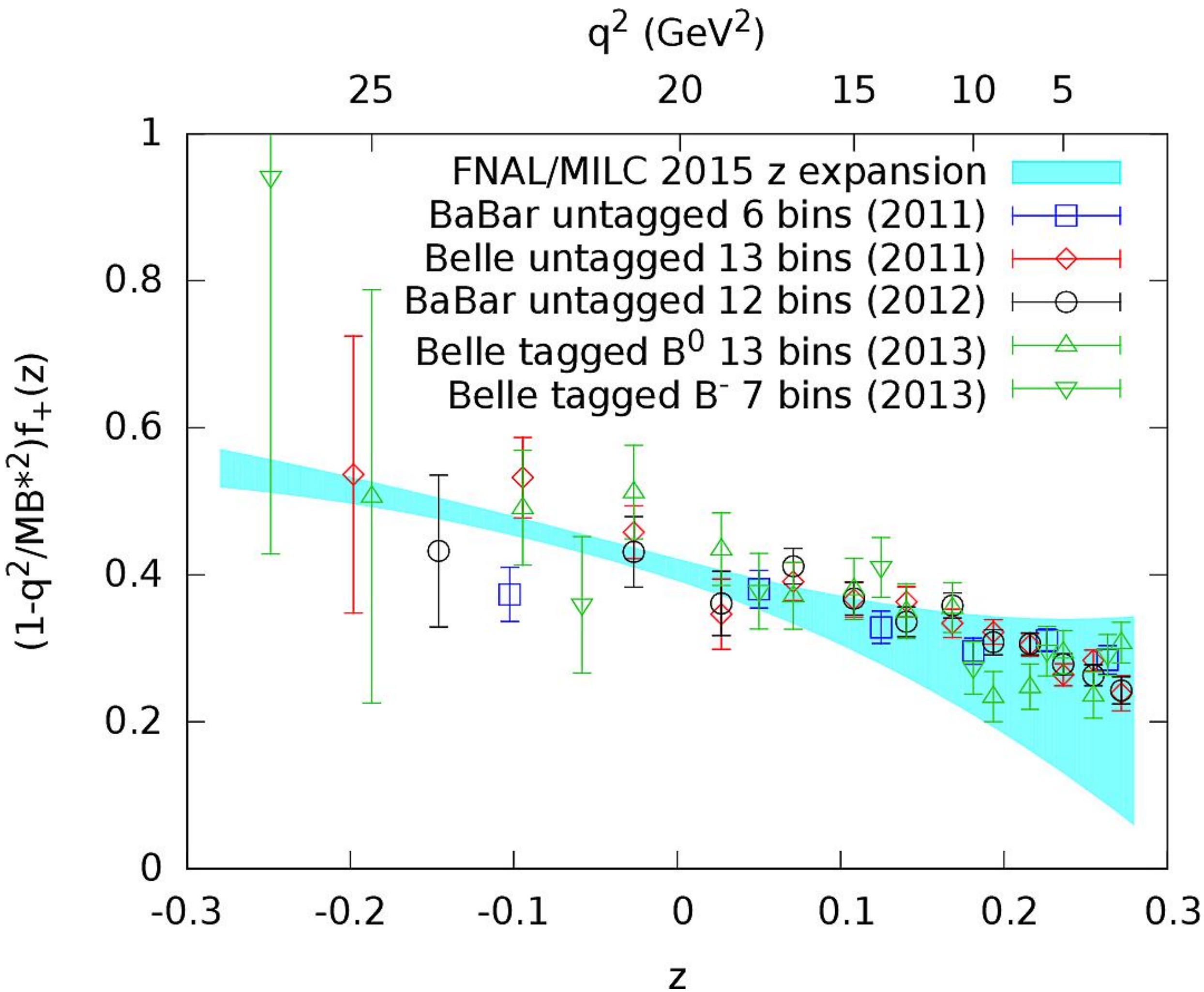

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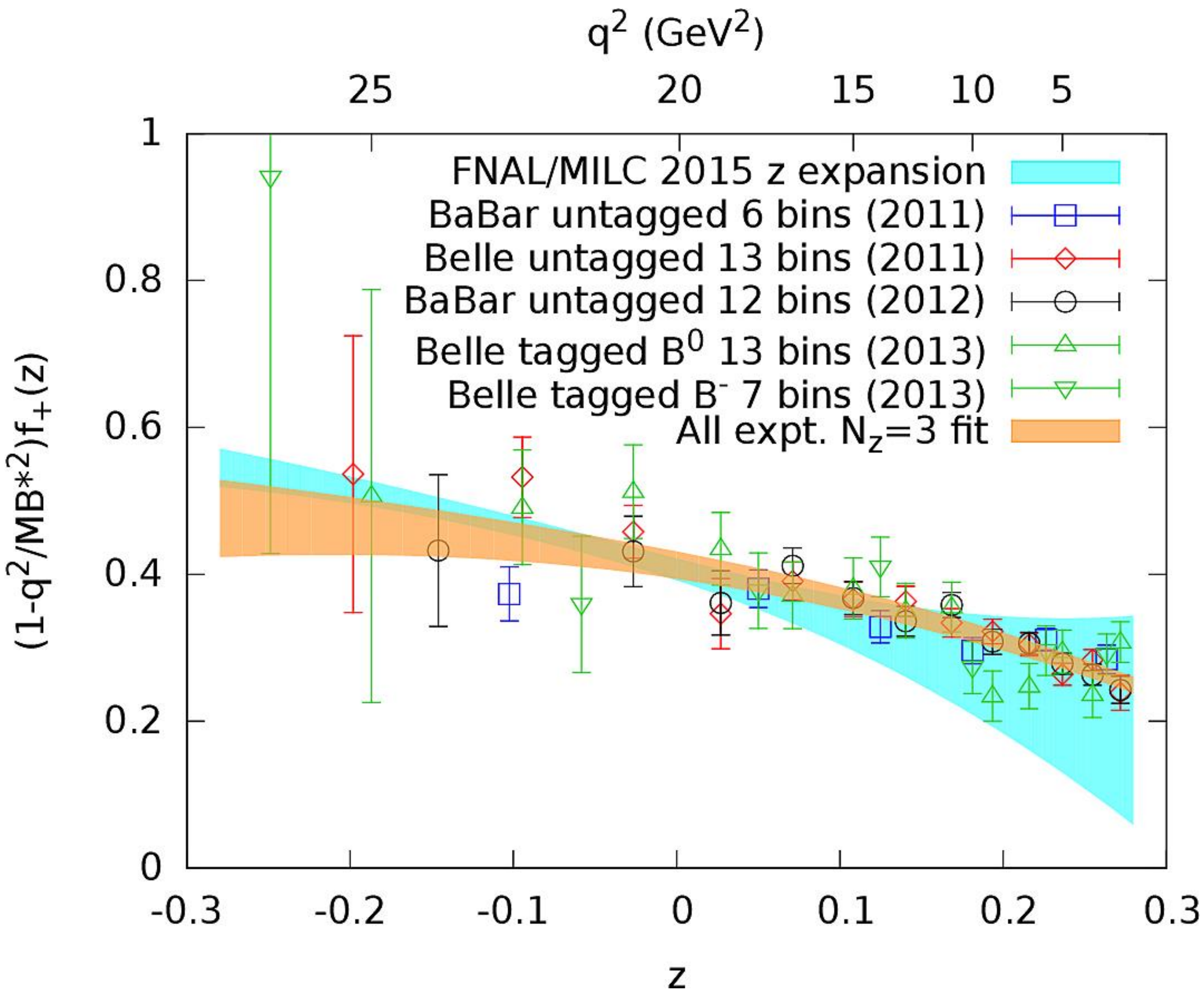

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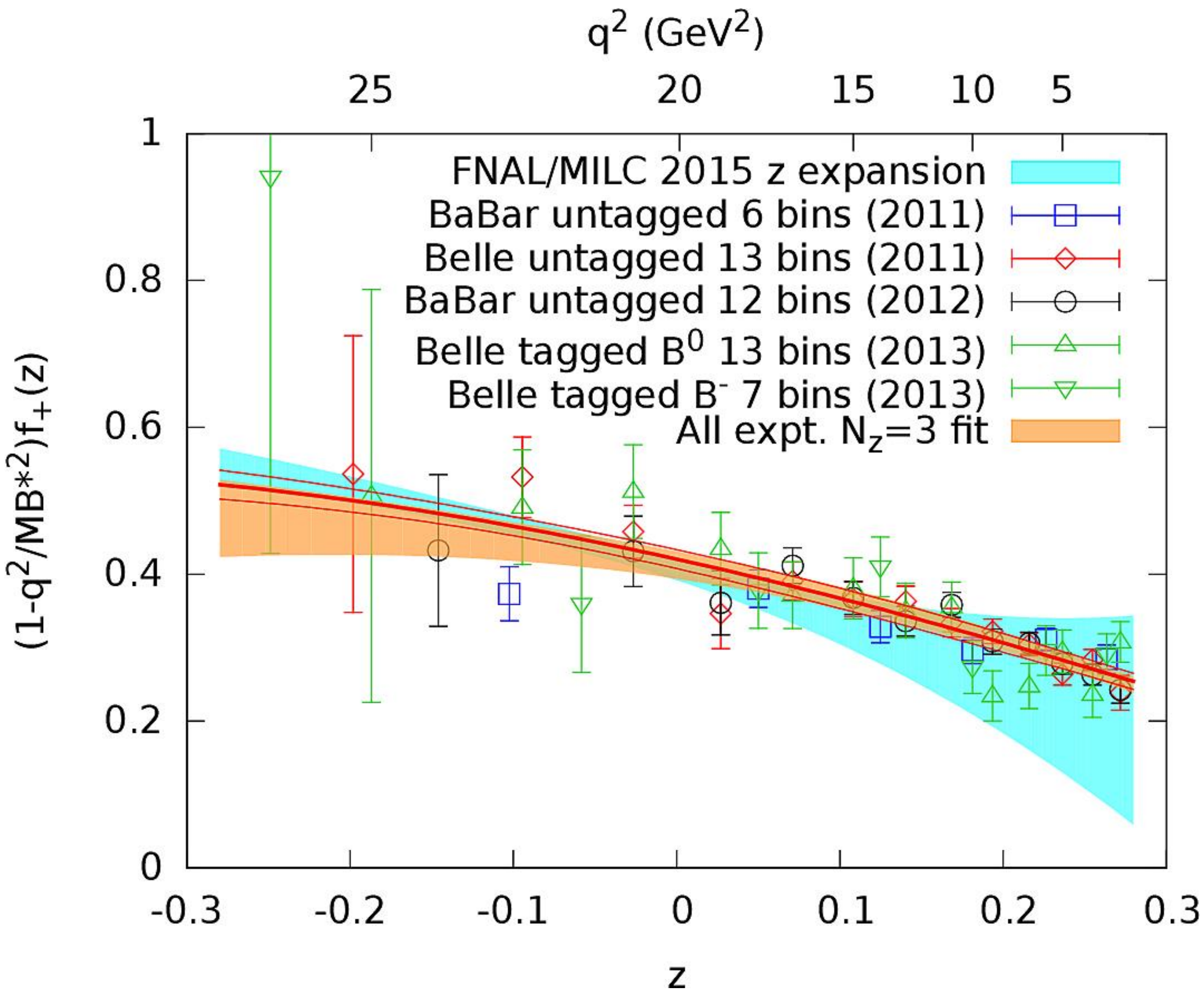


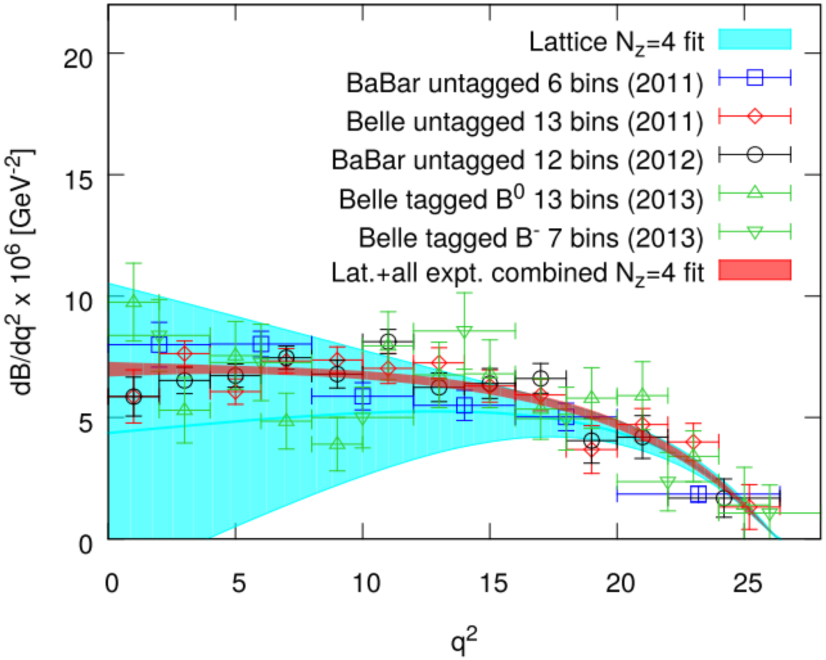
$B \to \pi \ell \nu$ (Fermilab/MILC 2015)

- Combined fit using lattice + experiments for $|V_{ub}|$

Table XVIII. Combined lattice+experiments z fits with $N_z = 3, 4$ and 5.

N_z	χ^2/dof	dof	p value	b_0^+	b_1^+	b_2^+	b_3^+	b_4^+	$\vert V_{ub}\vert$
3	2.5	56	0.0	0.425(12)	−0.424(31)	−0.59(9)			3.63(11)
4	1.4	54	0.02	0.419(13)	−0.495(55)	−0.43(14)	0.22(31)		3.72(16)
5	1.5	52	0.01	0.418(13)	−0.491(56)	−0.31(30)	0.01(55)	−0.6(1.9)	3.72(16)

Fermilab/MILC, 1503.07839



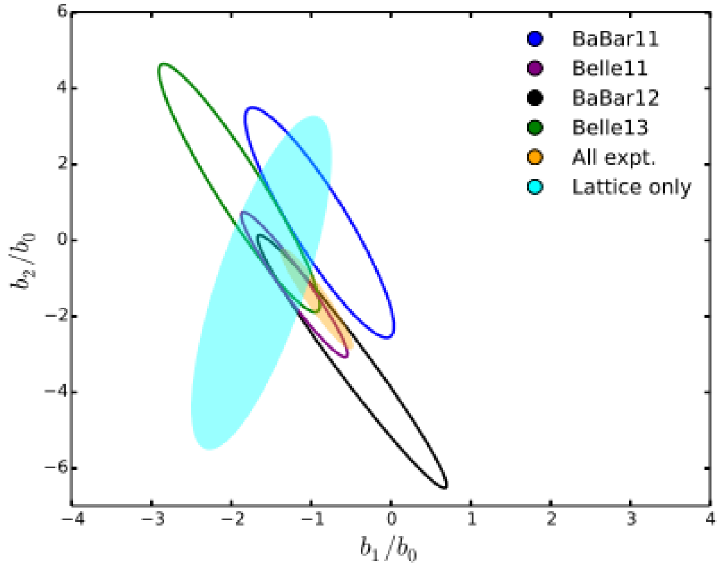
$B\to\pi\ell\nu$ (Fermilab/MILC 2015)

➢ Result

$$|V_{ub}| = (3.72 \pm 0.16) \times 10^{-3}$$

- The error includes full uncertainties from both experiment and lattice
- Lattice error is now **comparable** to the experimental error: looking at $q^2 = 20\ GeV^2$
  Experiment combined fit: 2.8%
  Lattice: 3.4%
  Total: 4.4% (same as the result from full-q^2 fit, 4.3%)
- The fit has a small p-value, $p = 0.02$, which is due to a tension among the experimental datasets.

data set	# data	χ^2	$\chi^2/\#$ data
Lattice	11	4.8	0.44
BaBar11 [7]	6	20.9	3.5
BaBar12 [8]	12	15.1	1.3
Belle11 [9]	13	13.8	1.1
Belle13 [10]	20	23.5	1.2
Total	62	78.2	1.26

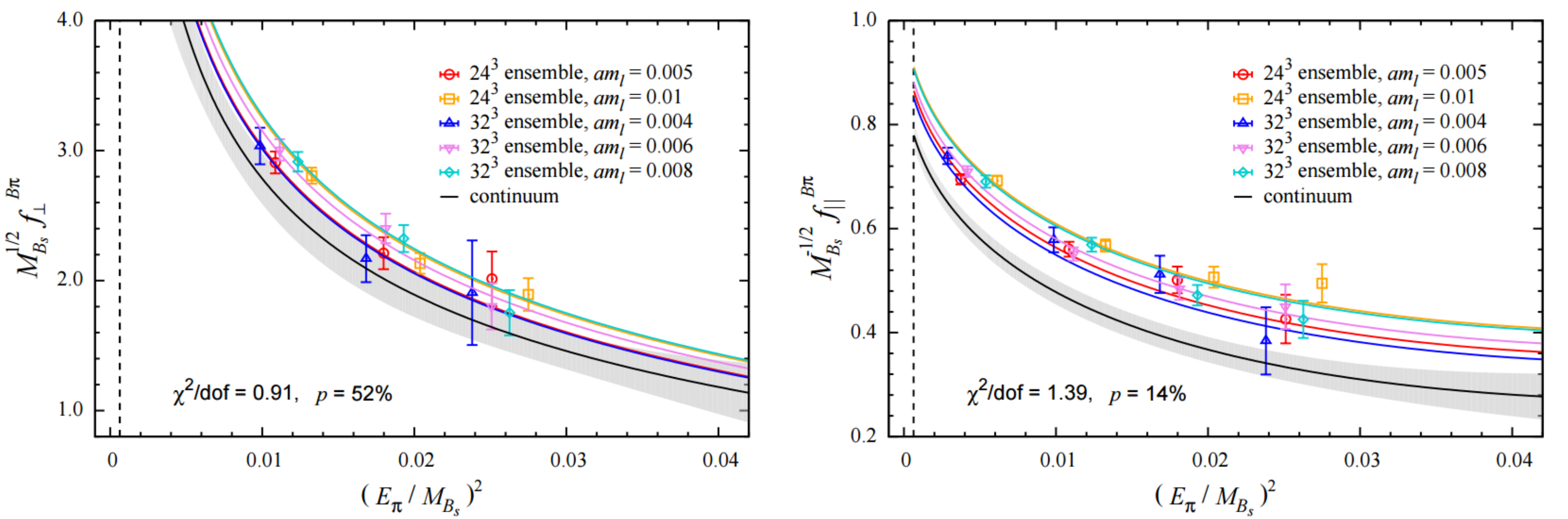


$B \to \pi \ell \nu$ (RBC/UKQCD 2015)

$\left(\frac{L}{a}\right)^3 \times \left(\frac{T}{a}\right)$	$\approx a$(fm)	am_l	am_h	M_π[MeV]	# configs.
$24^3 \times 64$	0.11	0.005	0.040	329	1636
$24^3 \times 64$	0.11	0.010	0.040	422	1419
$32^3 \times 64$	0.086	0.004	0.030	289	628
$32^3 \times 64$	0.086	0.006	0.030	345	889
$32^3 \times 64$	0.086	0.008	0.030	394	544

RBC/UKQCD, 1501.05373

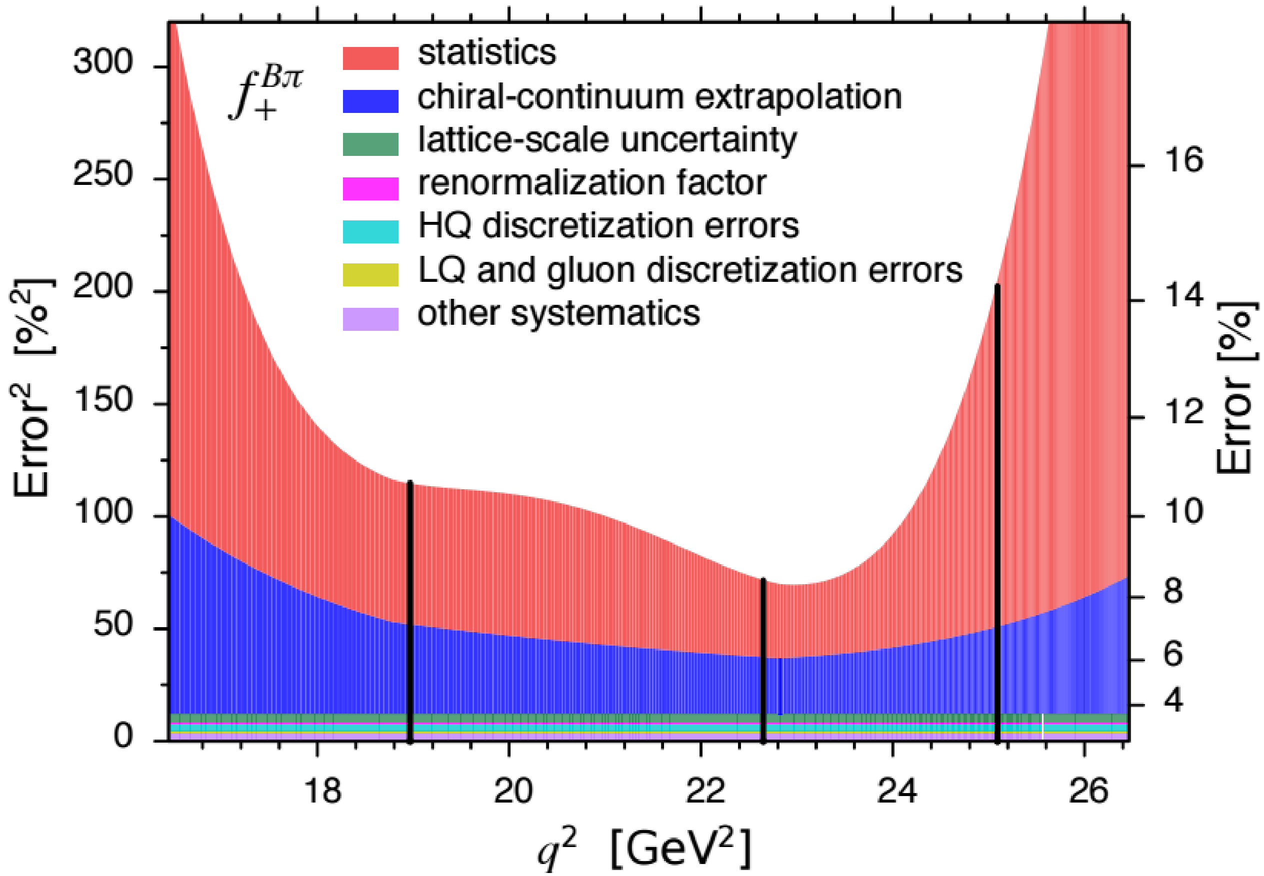
- Simpler χPT formula with Domain-Wall fermions:
  continuum-like NLO SU(2) χPT in hard-pion limit



$B\to\pi\ell\nu$ (RBC/UKQCD 2015)

- Error budget

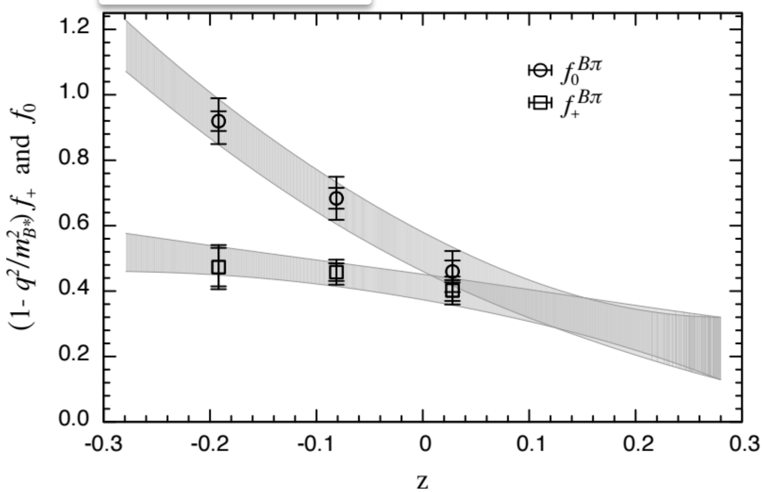
RBC/UKQCD, 1501.05373



$B \to \pi \ell \nu$ (RBC/UKQCD 2015)

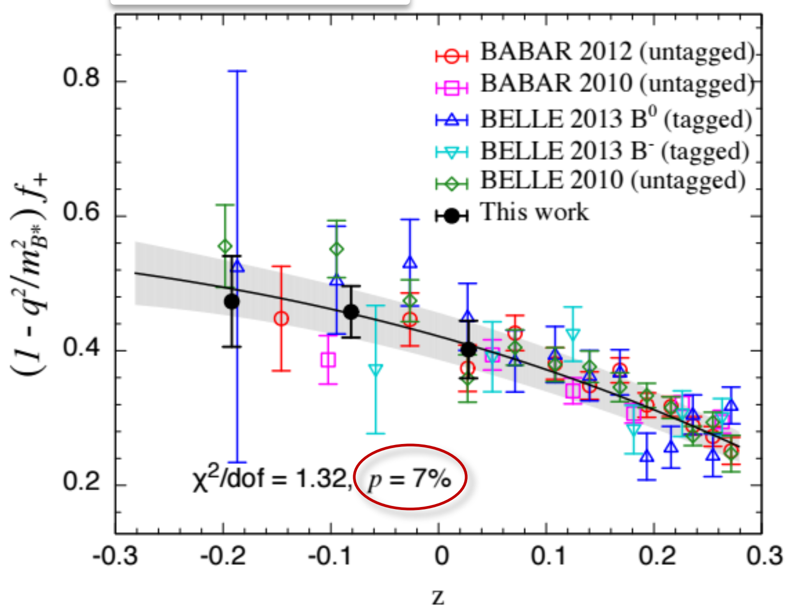
- BCL z-expansion (up to z^2) using 3+3 synthetic data points from χPT
- Kinematic constraint

Lattice only



RBC/UKQCD, 1501.05373

Lattice+expt.



- Result

$$|V_{ub}| = (3.61 \pm 0.31) \times 10^{-3}$$

estimated error breakdown: lattice 8.3%, experiment 2.8%

$B\to\pi\ell\nu$ (HPQCD recent results)

ens	$L^3\times N_t$	$\approx a$ [fm]	m_l^{sea}/m_s^{sea}	N_{conf}	N_{tsrc}	am_l^{val}	am_s^{val}	T
C1	$24^3\times64$	0.12	0.005/0.050	1200	2	0.007	0.0489	13, 14, 15
C2	$20^3\times64$	0.12	0.010/0.050	1200	2	0.0123	0.0492	13, 14, 15
C3	$20^3\times64$	0.12	0.020/0.050	600	2	0.0246	0.0491	13, 14, 15
F1	$28^3\times96$	0.09	0.0062/0.031	1200	4	0.00674	0.0337	23, 24
F2	$28^3\times96$	0.09	0.0124/0.031	600	4	0.0135	0.0336	21, 22, 24

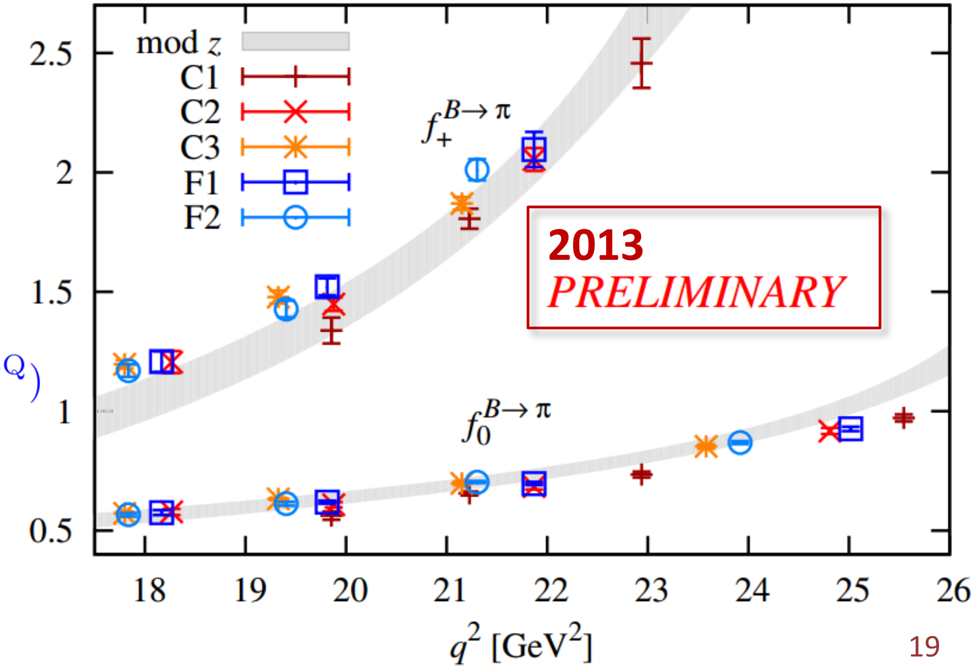
HPQCD, 1310.3207

- Improved valence action (HISQ)
- BCL Modified z-expansion:
  a and m_q dependence in z
- Dominant error: current matching

  Future improvement:

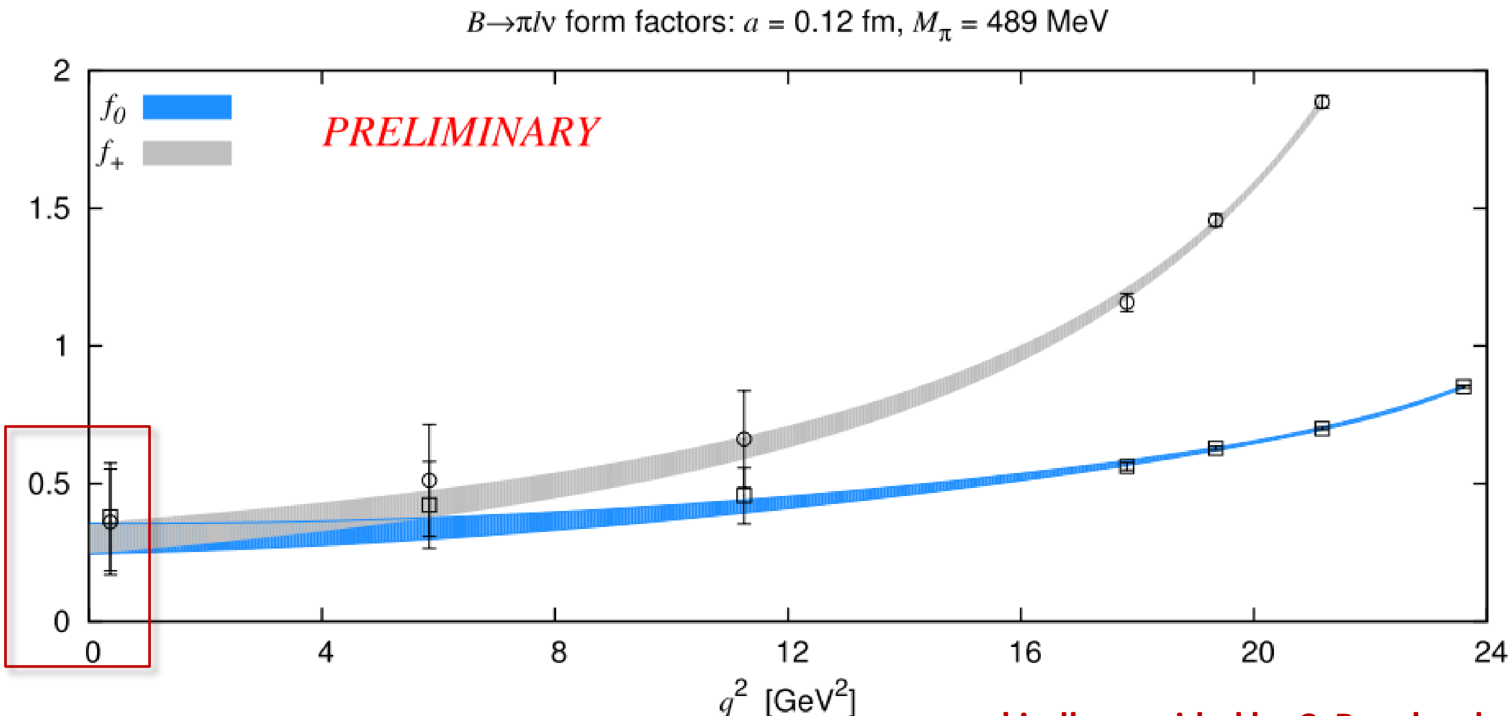
$$\frac{f^{B\to\pi}(m_b^{\rm NRQCD}, m_\ell^{\rm HISQ})}{f^{B_s\to\eta_s}(m_b^{\rm NRQCD}, m_\ell^{\rm HISQ})} \times f^{B_s\to\eta_s}(\mathrm{m_b^{HISQ}}, \mathrm{m}_\ell^{\rm HISQ})$$

Matching done nonperturbatively



$B \to \pi \ell \nu$ (HPQCD recent progress)

- Simulation at very high recoil using HISQ
- Could reduce the error due to q^2 extrapolation



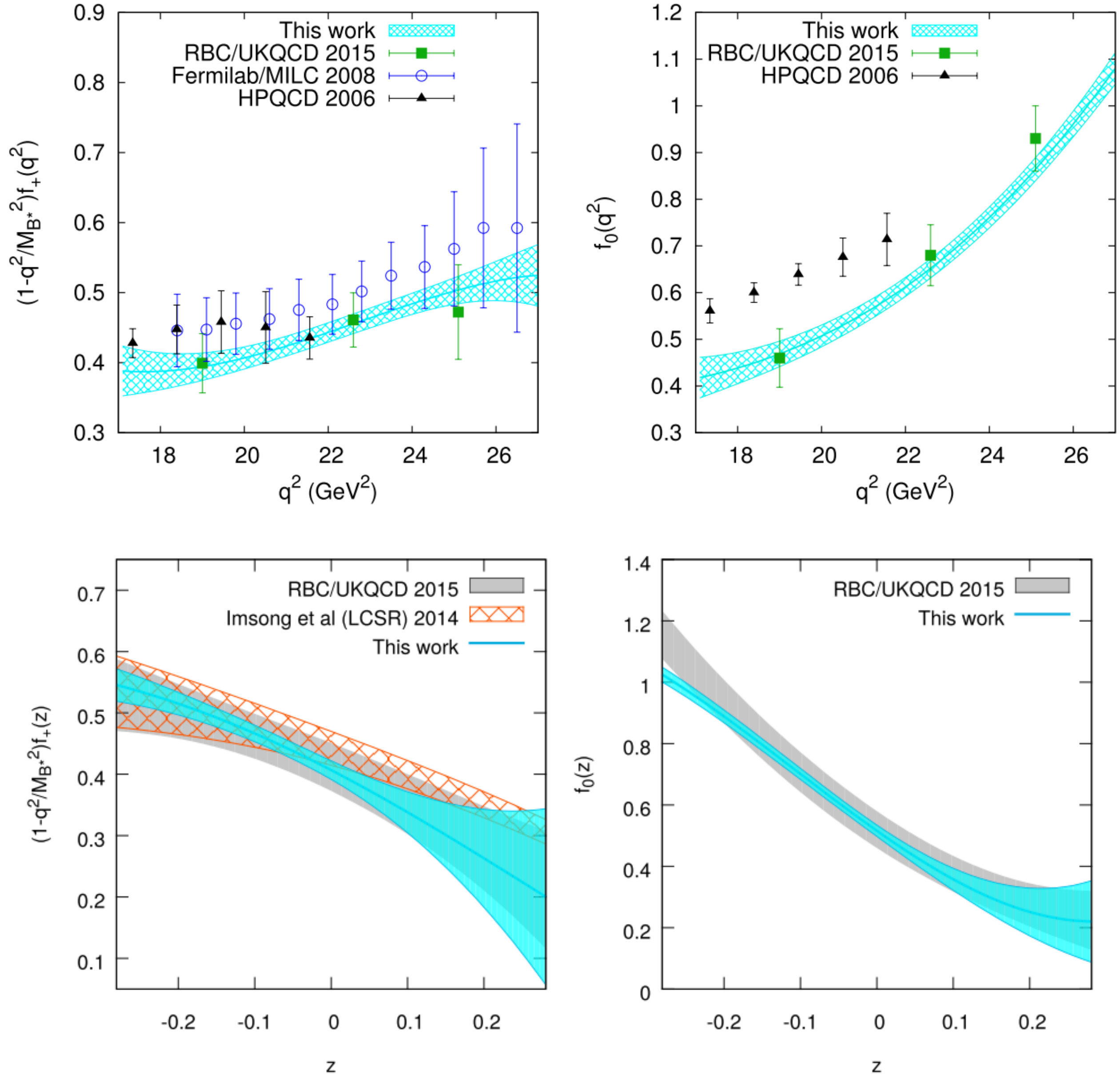
kindly provided by C. Bouchard

Comparison of lattice results (with full errors)

Fermilab/MILC, 1503.07839 ("this work")

χPT fit results

z fit result



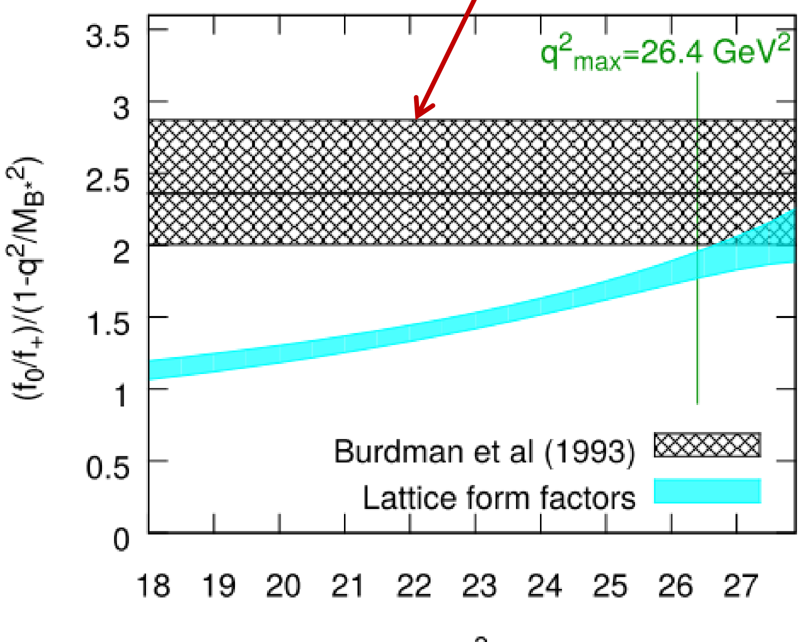
Ratio of $f_0^{B\to\pi}/f_+^{B\to\pi}$

- In the soft-pion limit, HQ symmetry gives **Burdman et al., hep-ph/9309212**

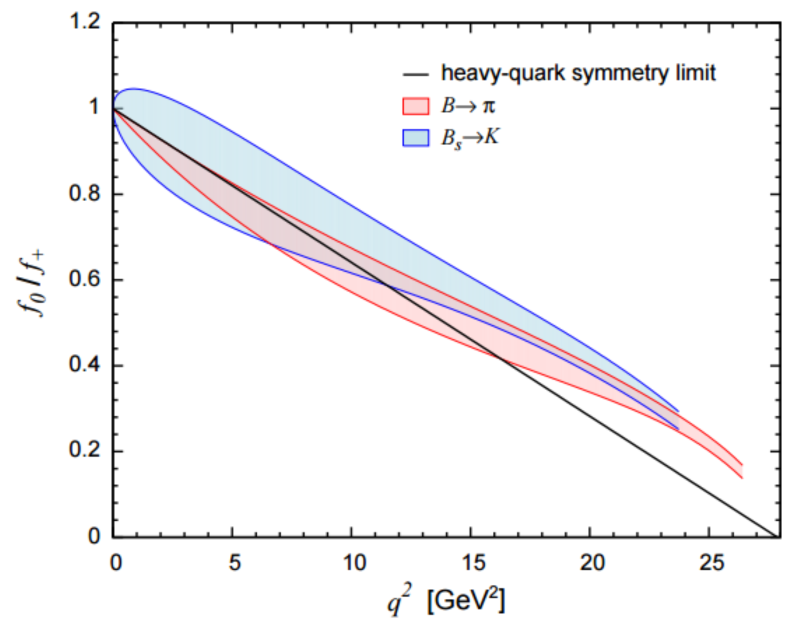
$$\lim_{q^2\to M_B^2} \frac{f_0(q^2)}{f_+(q^2)} = \left(\frac{f_B}{f_{B^*}}\right) \frac{1-q^2/M_{B^*}^2}{g_{BB^*\pi}} \qquad O(1/m_b^2)$$

HPQCD, 1503.05764

$g_{BB^*\pi} = 0.45 \pm 0.08$



Fermilab/MILC, 1503.07839



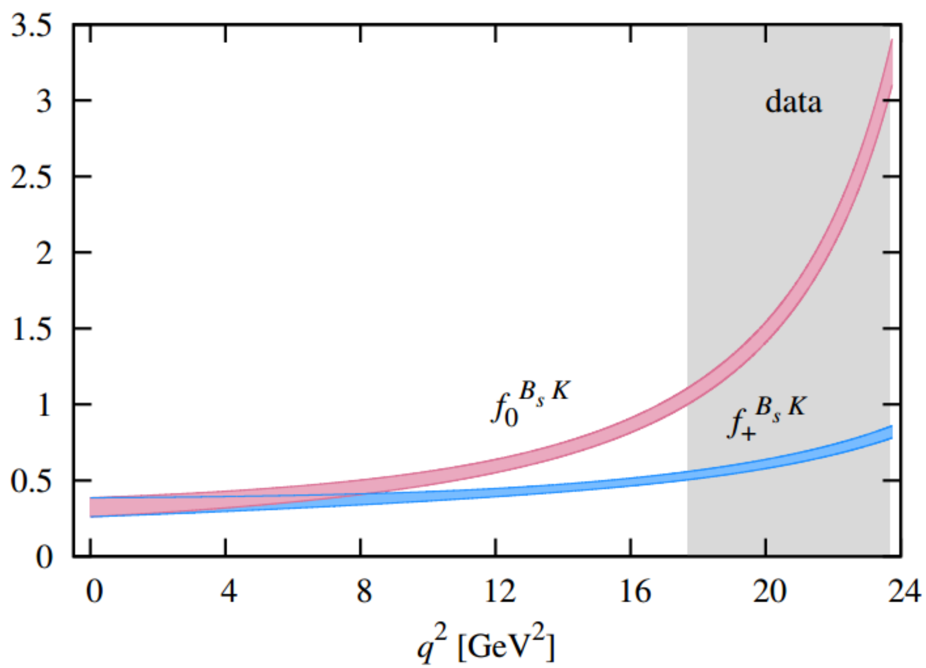
RBC/UKQCD, 1501.05373

$$B_s \to K\ell\nu$$

$B_s \to K\ell\nu$ (HPQCD 2014)

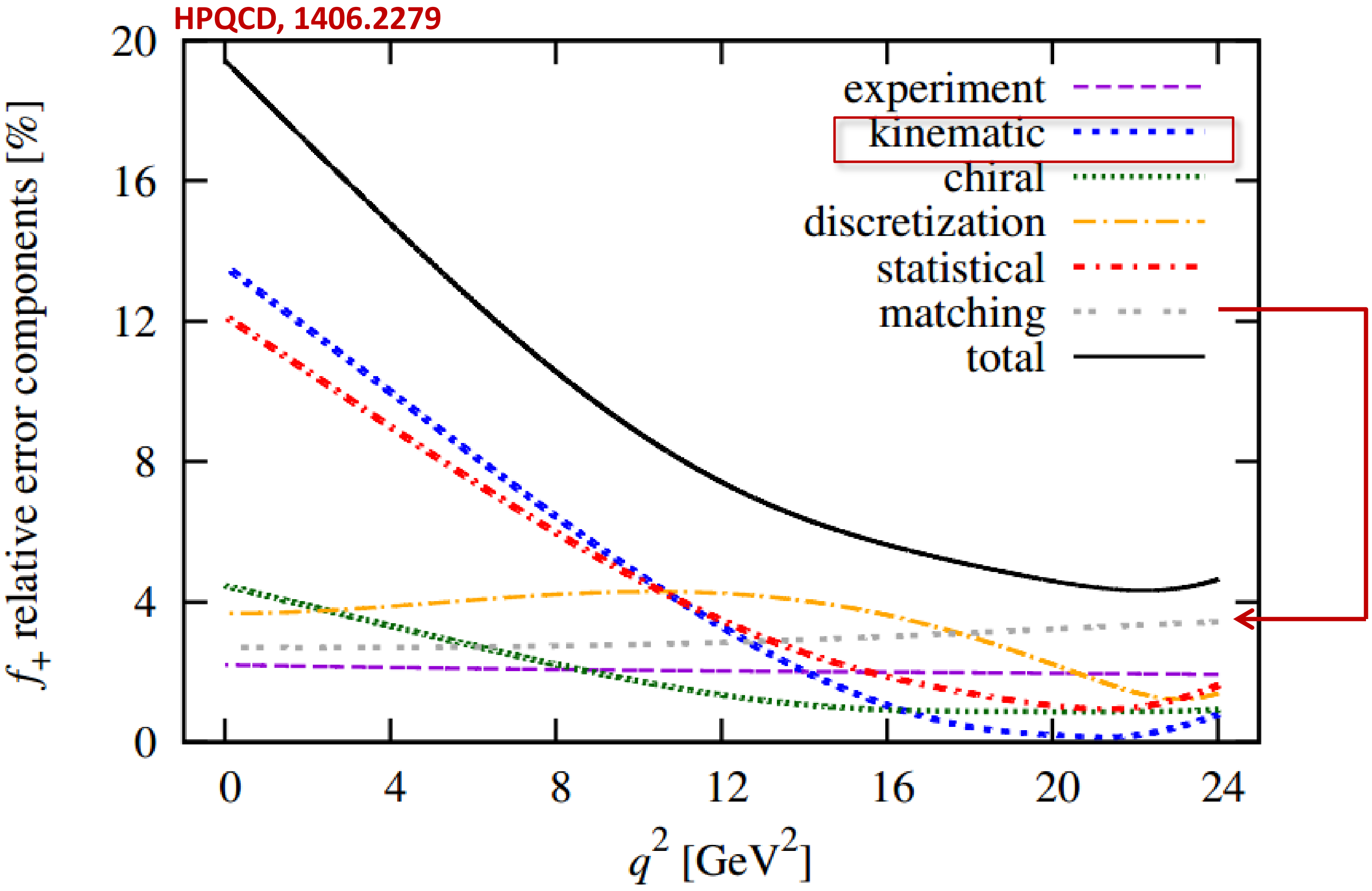
Ensemble	$L^3 \times N_t$	r_1/a	$au_0 m_{\rm sea}$	u_0	$N_{\rm conf}$
C1	$24^3 \times 64$	2.647(3)	0.005/0.05	0.8678	1200
C2	$20^3 \times 64$	2.618(3)	0.01/0.05	0.8677	1200
C3	$20^3 \times 64$	2.644(3)	0.02/0.05	0.8688	600
F1	$28^3 \times 96$	3.699(3)	0.0062/0.031	0.8782	1200
F2	$28^3 \times 96$	3.712(4)	0.0124/0.031	0.8788	600

HPQCD, 1406.2279



$B_s \to K\ell\nu$ (HPQCD 2014)

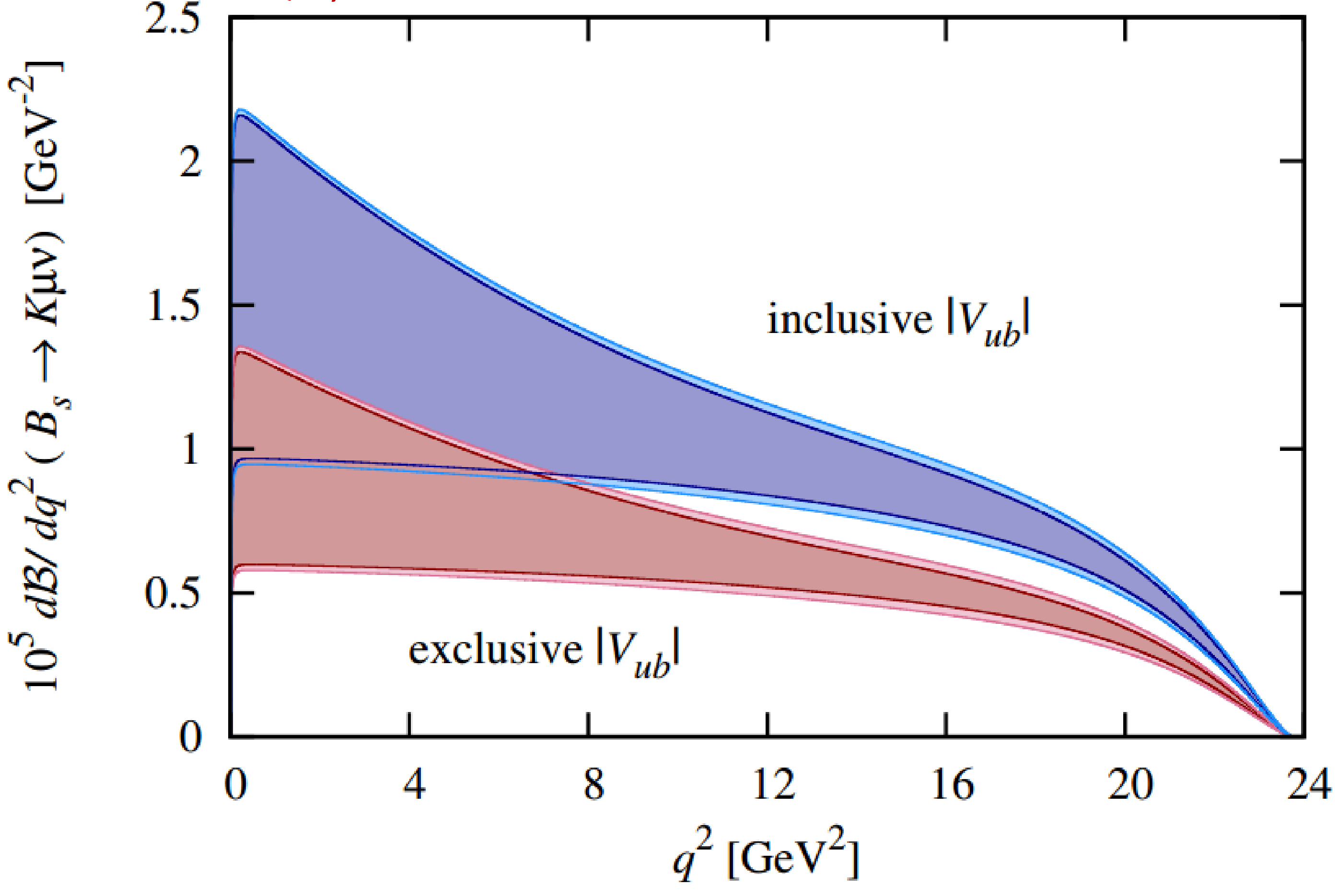
- Error budget



$B_s \to K\ell\nu$ (HPQCD 2014)

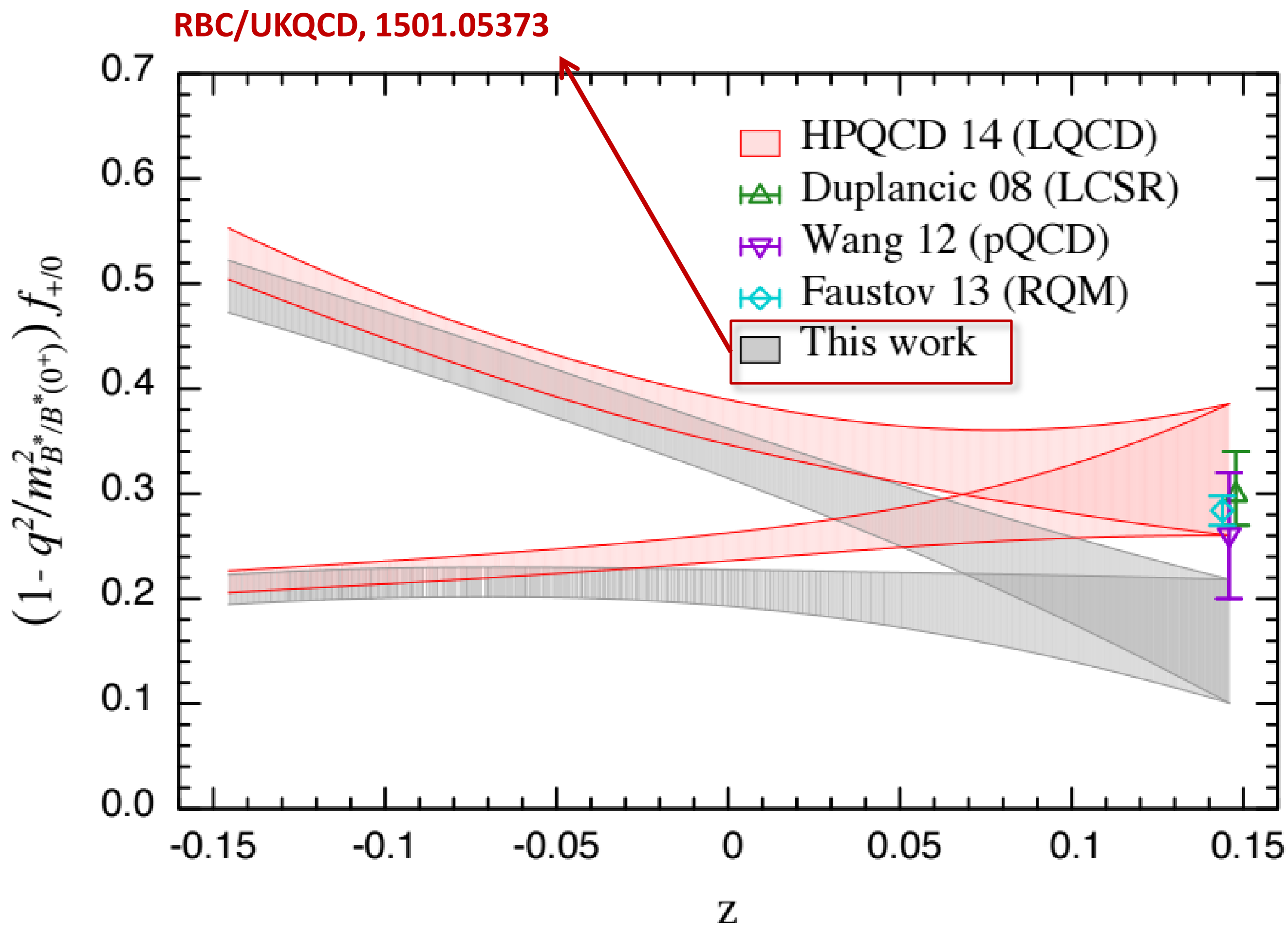
- Prediction of BR($B_s \to K\mu\nu$)

HPQCD, 1406.2279



$B_s \to K\ell\nu$ (RBC/UKQCD 2015)

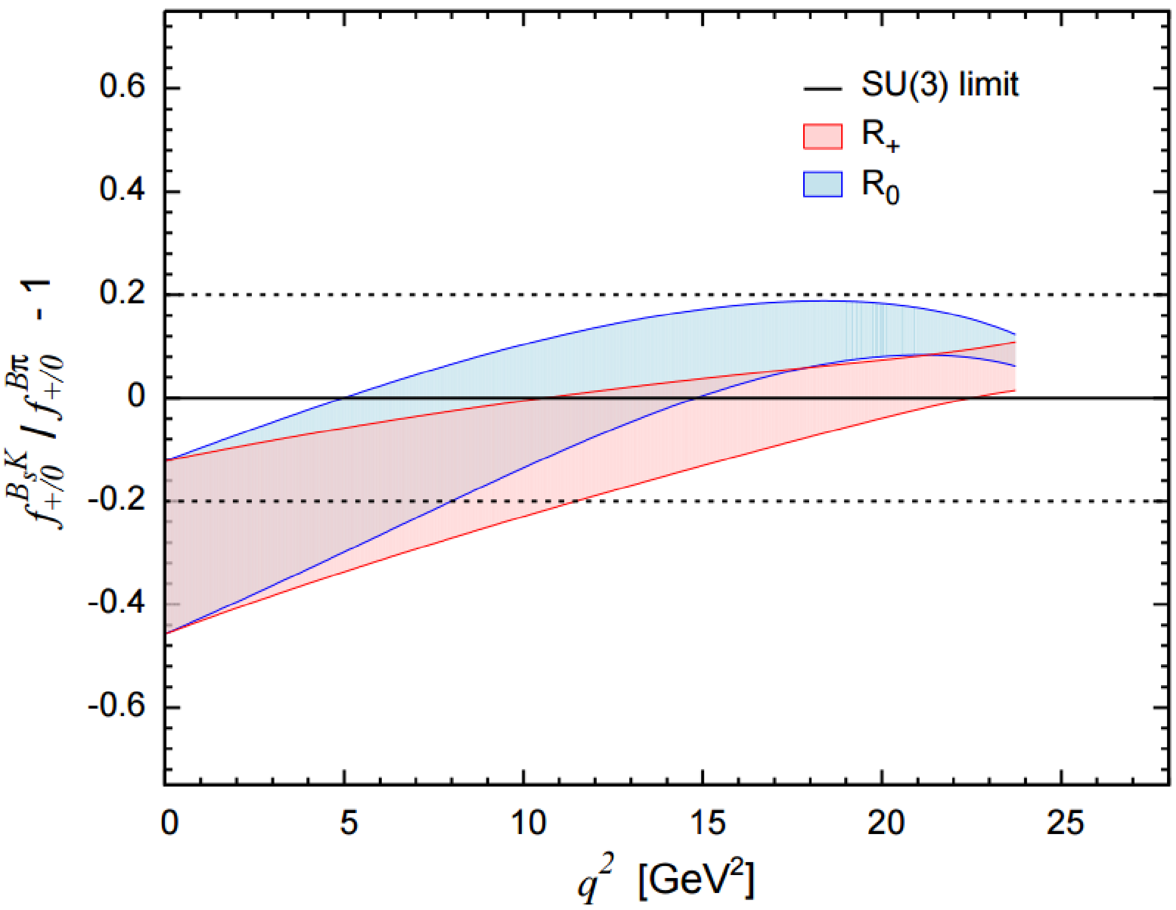
- Comparison of $f_{+,0}^{B_s\to K}$



$B_s \to K\ell\nu$ (RBC/UKQCD 2015)

- SU(3) breaking effects

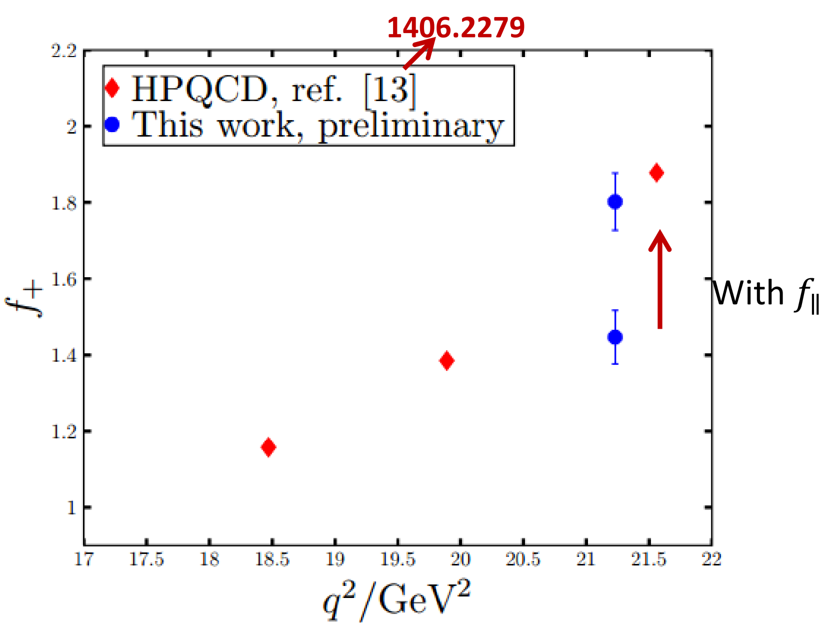
RBC/UKQCD, 1501.05373



$B_s \to K\ell\nu$ (other ongoing efforts)

- Fermilab/MILC: (**Y. Liu et al., 1312.3194**)
  - Use a subset of the MILC asqtad ensembles

- Alpha: (**Bahr et al., 1411.3916**)



$$\Lambda_b \to p \ell \nu$$

$\Lambda_b \to p\ell\nu$

- A new alternative method to determine $|V_{ub}|$
- LHCb has just reported their results **1504.01568**
- Axial vector current form factors: probe to right-handed currents

Feldmann&Yip, 1111.1844

$X = p$

$$\langle X(p',s')|\bar{q}\,\gamma^\mu\, b|\Lambda_b(p,s)\rangle = \bar{u}_X(p',s')\Big[f_0(q^2)\,(m_{\Lambda_b}-m_X)\frac{q^\mu}{q^2} + f_+(q^2)\frac{m_{\Lambda_b}+m_X}{s_+}\Big(p^\mu + p'^\mu - (m^2_{\Lambda_b}-m^2_X)\frac{q^\mu}{q^2}\Big) + f_\perp(q^2)\Big(\gamma^\mu - \frac{2m_X}{s_+}p^\mu - \frac{2m_{\Lambda_b}}{s_+}p'^\mu\Big)\Big]u_{\Lambda_b}(p,s),$$

$$\langle X(p',s')|\bar{q}\,\gamma^\mu\gamma_5\, b|\Lambda_b(p,s)\rangle = -\bar{u}_X(p',s')\,\gamma_5\Big[g_0(q^2)\,(m_{\Lambda_b}+m_X)\frac{q^\mu}{q^2} + g_+(q^2)\frac{m_{\Lambda_b}-m_X}{s_-}\Big(p^\mu + p'^\mu - (m^2_{\Lambda_b}-m^2_X)\frac{q^\mu}{q^2}\Big) + g_\perp(q^2)\Big(\gamma^\mu + \frac{2m_X}{s_-}p^\mu - \frac{2m_{\Lambda_b}}{s_-}p'^\mu\Big)\Big]u_{\Lambda_b}(p,s).$$

$\Lambda_b \to p \ell \nu$ (Detmold et al. 2015)

- Improves upon their earlier calculation **Detmold, Lin, Meinel & Wingate, 1306.0446**
  b-quark action: **static limit** → **RHQ** (large error reduction)

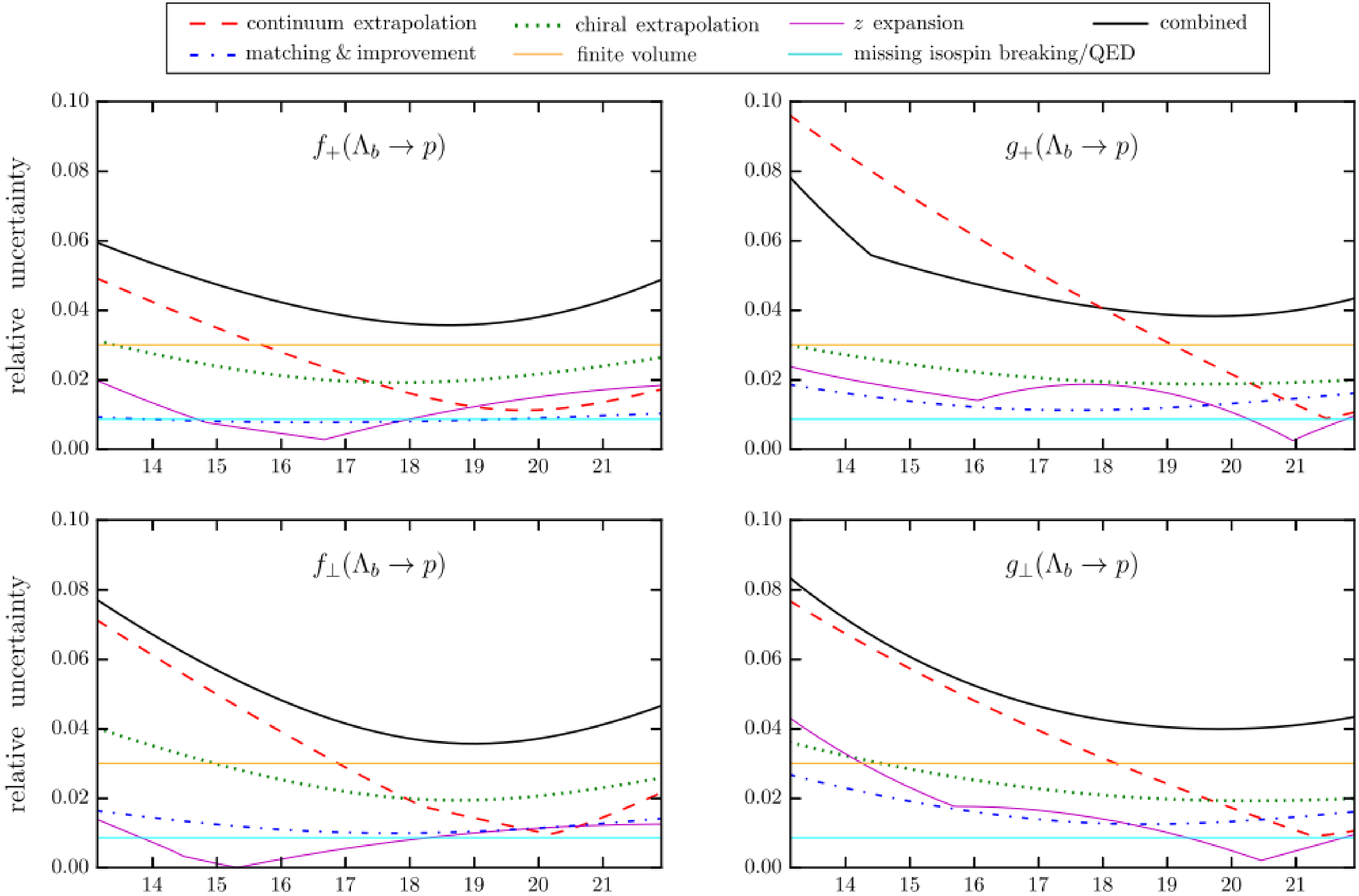
Set	β	$N_s^3 \times N_t \times N_5$	am_5	$am_s^{(\mathrm{sea})}$	$am_{u,d}^{(\mathrm{sea})}$	a (fm)	$am_{u,d}^{(\mathrm{val})}$	$m_\pi^{(\mathrm{val})}$ (MeV)	N_{meas}
C14	2.13	$24^3 \times 64 \times 16$	1.8	0.04	0.005	0.1119(17)	0.001	245(4)	2672
C24	2.13	$24^3 \times 64 \times 16$	1.8	0.04	0.005	0.1119(17)	0.002	270(4)	2676
C54	2.13	$24^3 \times 64 \times 16$	1.8	0.04	0.005	0.1119(17)	0.005	336(5)	2782
F23	2.25	$32^3 \times 64 \times 16$	1.8	0.03	0.004	0.0849(12)	0.002	227(3)	1907
F43	2.25	$32^3 \times 64 \times 16$	1.8	0.03	0.004	0.0849(12)	0.004	295(4)	1917
F63	2.25	$32^3 \times 64 \times 16$	1.8	0.03	0.006	0.0848(17)	0.006	352(7)	2782

Detmold et al., 1503.01421v2

- The modification of b-quark action enables z expansion:
  - Incorporates the a and m_q dependence in the fit (similar to HPQCD)
  - z and poles are evaluated at the experimental values

$\Lambda_b \to p\ell\nu$ (Detmold et al. 2015)

- Error budget **1503.01421v2**

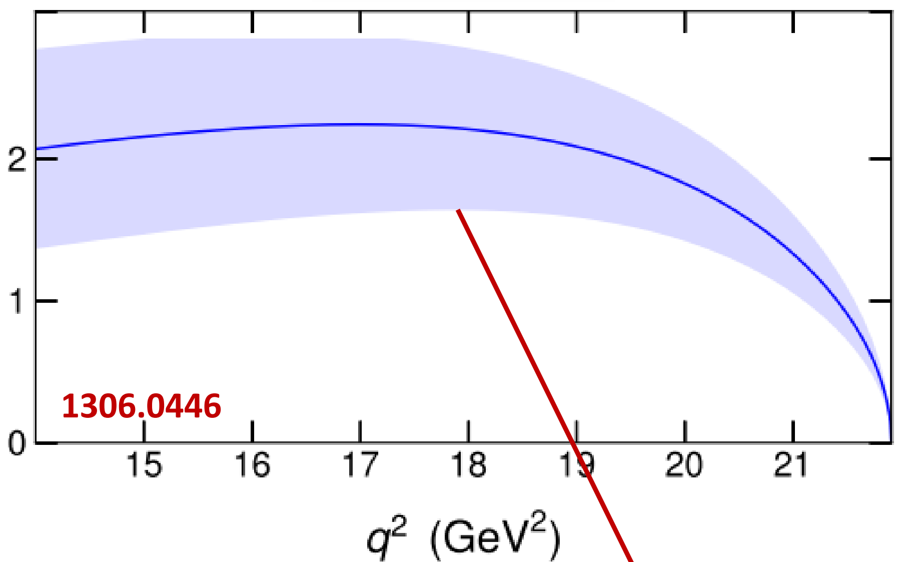


$\Lambda_b \to p\ell\nu$ (Detmold et al. 2015)

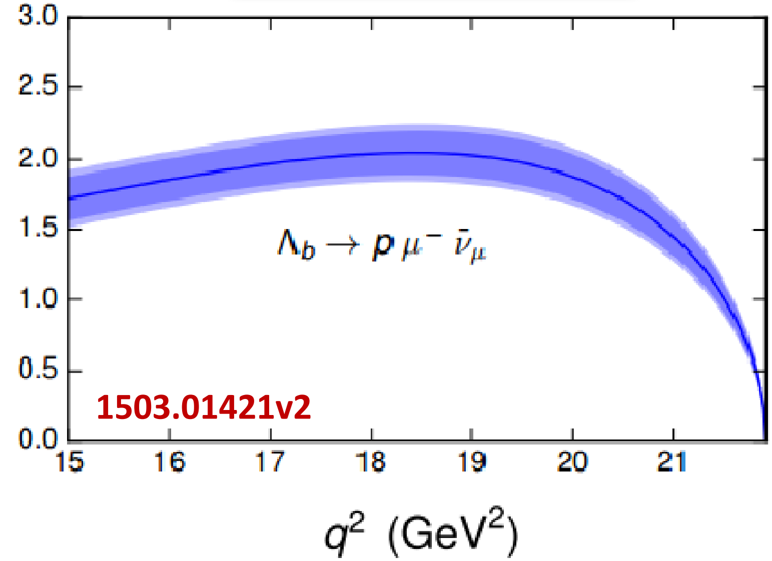
- The partial rate

$$\frac{d\Gamma}{dq^2}(\mathrm{ps}^{-1}\mathrm{GeV}^{-2}|V_{ub}|^2)$$

static b quark



relativistic b quark



Includes systematic due to $\sqrt{|p'|^2 + \Lambda_{\rm QCD}}/m_b$

$\Lambda_b \to p\ell\nu$ (LHCb 2015 + Detmold et al. 2015)

- Determination of $|V_{ub}|$

$$\frac{|V_{ub}|}{|V_{cb}|} = \frac{\mathrm{BR}(\Lambda_b^0 \to p\mu^-\bar{\nu}_\mu)_{\geq 15\mathrm{GeV}^2}}{\mathrm{BR}(\Lambda_b^0 \to \Lambda_c\mu^-\bar{\nu}_\mu)_{\geq 7\mathrm{GeV}^2}} \times R_{\mathrm{FF}}$$

Use excl. avg: $(39.5 \pm 0.8) \times 10^{-3}$

LHCb 2015

Form factors ratio (Lattice)

- Result:

$$|V_{ub}| = (3.27 \pm 0.15_{\mathrm{expt}} \pm 0.17_{\mathrm{lattice}} \pm 0.06_{|V_{cb}|}) \times 10^{-3}$$

  - Comparable uncertainties from experiment and lattice
  - Relies on the value of exclusive $|V_{cb}|$

Summary and outlook

- Major updates from lattice-QCD on semileptonic form factors for $|V_{ub}|$
  - Progress in $B \to \pi\ell\nu$, $B_s \to K\ell\nu$ and $\Lambda_b \to p\ell\nu$ decays
  - For $B \to \pi\ell\nu$, the lattice error shrinks to 3.4%; now comparable to experiment
  - The exclusive $|V_{ub}|$ (from Fermilab/MILC 2015) is updated $(3.72 \pm 0.16) \times 10^{-3}$
  - $B_s \to K\ell\nu$ is awaiting experimental measurements
  - More lattice QCD calculations are coming!

Lattice Group	lat%	Curr. expt%	Curr. $\|V_{ub}\|$%
Fermilab/MILC($B \to \pi\ell\nu$)	9%→ 3.4%	2.8%	4.3%
RBC/UKQCD($B \to \pi\ell\nu$)	8.4%	2.8%	8.9%
HPQCD($B_s \to K\ell\nu$)	~4.5%	-	-
RBC/UKQCD($B_s \to K\ell\nu$)	~5%	-	-
Detmold et al. ($\Lambda_b \to p\ell\nu$)	5.2%	4.6%	7.2%

Summary and outlook

- Multiple lattice groups on the same quantities (averaging: LLV, FLAG)
- Beyond simple quantities (vector meson decay channels)?

	Current	Future	
	2015	2018	2020+
Lattice	3.4%	2% *	
Experiment	3∼4%	3∼4% (Belle II 5 ab^{-1})	$<$ 2%(Belle II 50 ab^{-1}) **
	4.3%	3∼4%	2%

* Projection 2018: **Snowmass 2013 Quars Flavor Working Group, 1311.1076**

** Belle II: **Phillip Urquijo, CKM 2014**

Thank You

Backup slides:

- Form factor shape from experiment: fit details

Table XV. The results of fits to experimental data only.

Fit	χ^2/dof	dof	p	b_1/b_0	b_2/b_0	$b_0\|V_{ub}\| \times 10^{-3}$
All exp.	1.5	48	0.02	−0.93(22)	−1.54(65)	1.53(4)
BaBar11 [7]	2	3	0.12	−0.89(47)	0.5(1.5)	1.36(7)
BaBar12 [8]	1.2	9	0.31	−0.48(59)	−3.2(1.7)	1.54(9)
Belle11 [9]	1.1	10	0.36	−1.21(33)	−1.18(95)	1.63(7)
Belle13 [10]	1.2	17	0.23	−1.89(50)	1.4(1.6)	1.56(8)

Backup slides:

- Fermilab/MILC 2015 + experiments: fit details

Lattice+	χ^2/dof	dof	p value	b_0^+	b_1^+	b_2^+	b_3^+	$\lvert V_{ub}\rvert(\times 10^3)$
All exp.	1.4	54	0.02	0.419(13)	−0.495(55)	−0.43(14)	0.22(31)	3.72(15)
BaBar11	1.1	9	0.38	0.414(14)	−0.490(74)	−0.250(22)	1.35(45)	3.37(21)
BaBar12	1.1	15	0.34	0.415(14)	−0.551(72)	−0.45(18)	0.27(41)	3.97(22)
Belle11	0.9	16	0.55	0.412(13)	−0.574(65)	−0.40(16)	0.39(36)	4.03(21)
Belle13	1.0	23	0.42	0.405(14)	−0.628(74)	−0.12(22)	0.95(45)	3.82(25)
All-BaBar11	1.1	48	0.29	0.415(13)	-0.548(58)	-0.42(14)	0.30(32)	3.91(17)
All-BaBar12	1.5	42	0.016	0.412(14)	-0.596(53)	-0.320(14)	0.44(30)	3.72(17)
All-Belle11	1.6	41	0.01	0.417(14)	-0.468(53)	-0.49(15)	0.0.10(29)	3.75(17)
All-Belle13	1.6	34	0.01	0.414(14)	-0.489(56)	-0.33(15)	0.40(30)	3.69(17)

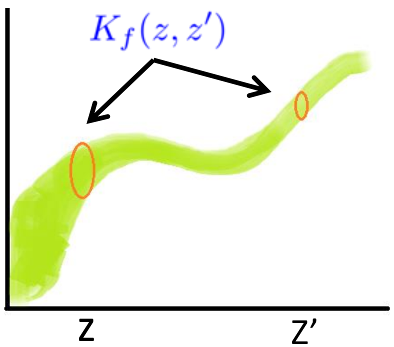
Extrapolation in q^2: functional z-expansion

- The q^2 extrapolation is another extrapolation in addition to the chiral/continuum extrapolation. Match a new function form (better) to the old function form (insufficient).
- The number of independent functions in the χPT extrapolated results determines the degrees of freedom of the fit (singular modes in the FNAL/MILC 2008)
- Functional z-expansion:

Covariance function $K_f(z, z') = \langle\, \delta f^{\chi PT}(z)\, \delta f^{\chi PT}(z')\, \rangle$

Mercer's Theorem:

$$K_f(z, z') = \sum_i \lambda_i \psi_i(z) \psi_i(z')$$



Minimizing "χ^2" to find expansion coefficients b_n.

$$\begin{aligned} \chi^2 &= \int_{z_1}^{z_2} dz \int_{z_1}^{z_2} dz' \, [f^{\chi PT}(z) - f^{BCL}(z)] K_f^{-1}(z, z') [f^{\chi PT}(z') - f^{BCL}(z')], \\ &= \sum_i (1/\lambda_i) \left[\int_{z_1}^{z_2} [f^{\chi PT}(z) - f^{BCL}(z)] \psi_i(z) \right]^2 \end{aligned}$$