Inclusive $|V_{ub}|$ from SIMBA

Zoltan Ligeti

[On behalf of F. Bernlochner, H. Lacker, ZL, I. Stewart, F. Tackmann, K. Tackmann]

- Model independent shape function treatment
- Fits for $B \to X_s \gamma$
- Fits for $B \to X_u \ell \bar{\nu}$
- Future progress
- Conclusions

V_{ub} — the beginning

CLEO, PRL **64** (1990) 16, Received 8 Nov 1989, $(212+101)/\mathrm{pb}$

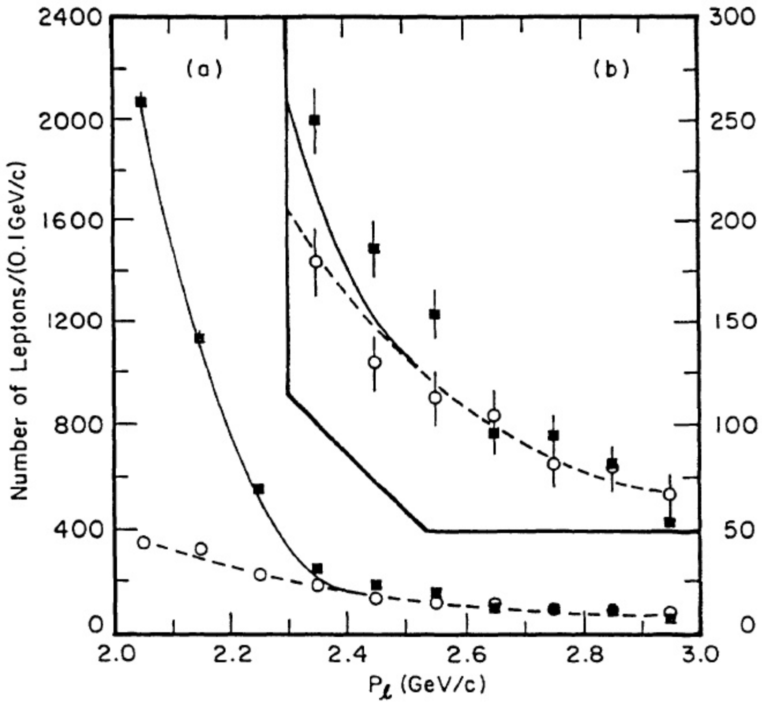


FIG. 1. Sum of the e and μ momentum spectra for ON data (filled squares), scaled OFF data (open circles), the fit to the OFF data (dashed line), and the fit to the OFF data plus the $b \to cl\nu$ yield (solid line). Note the different vertical scales in (a) and (b).

"$|V_{ub}/V_{cb}|$ … is approximately 0.1; it is sensitive to the theoretical model."

ARGUS, PLB **234** (1990) 409, Received 28 Nov 1989, $(201+69)/\mathrm{pb}$

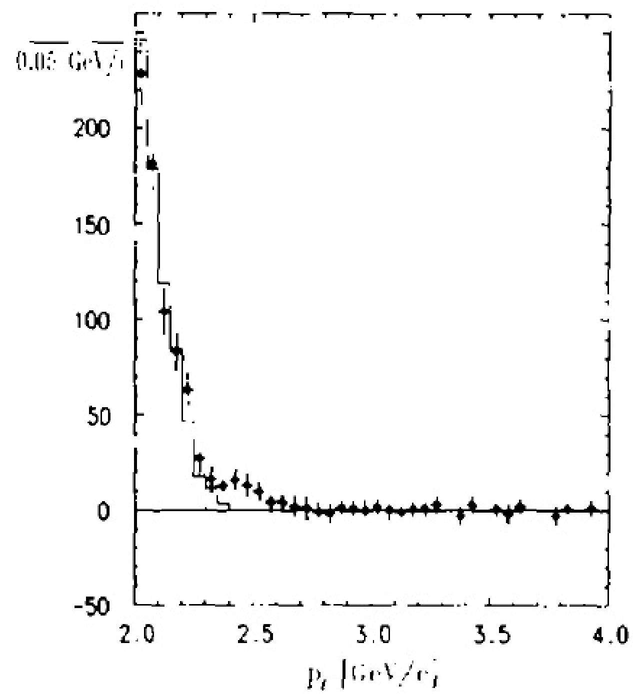


Fig. 5. Combined lepton momentum spectrum for direct $\Upsilon(4S)$ decays: the histogram is a $b \to c$ contribution normalized in the region 2.0–2.3 GeV/c.

"If interpreted as a signal of $b \to u$ coupling … $|V_{ub}/V_{cb}|$ of about 10%."

25 years later — situation still confusing

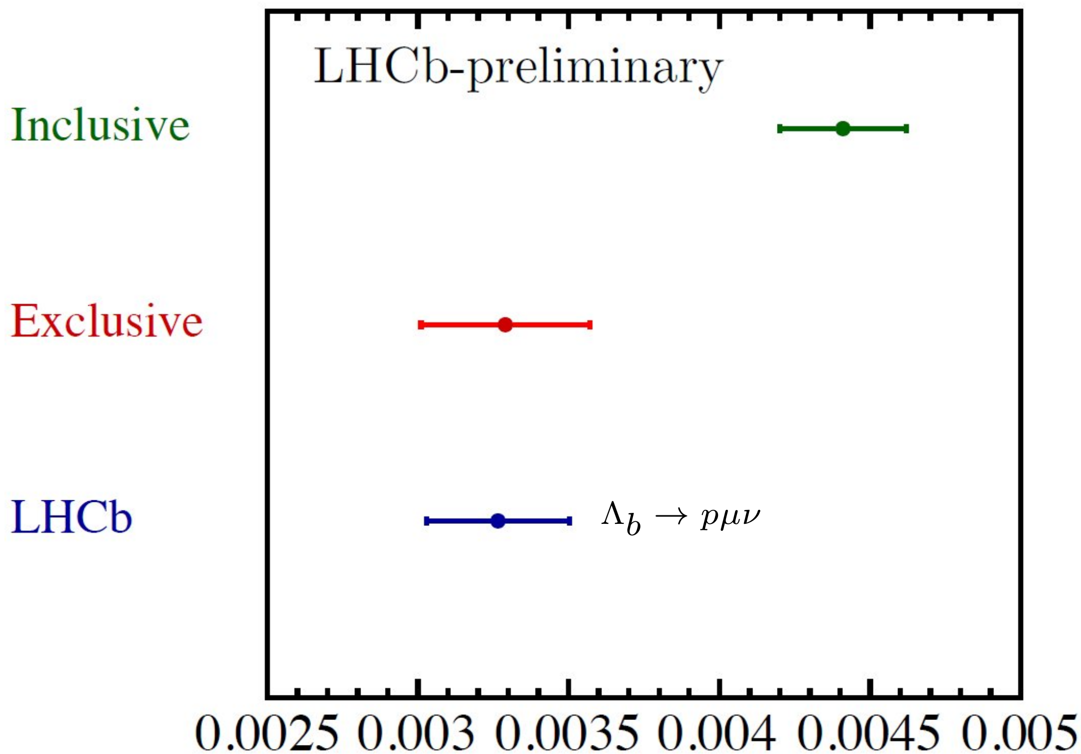
- By now, 5000 times more data, persistent tensions — I think the jury is still out:

HFAG:

$$|V_{ub}|_{\text{incl}-\text{BLL}} = (4.62 \pm 0.35) \times 10^{-3}$$
$$|V_{ub}|_{\text{incl}-\text{BLNP}} = (4.45 \pm 0.27) \times 10^{-3}$$
$$|V_{ub}|_{\text{incl}-\text{GGOU}} = (4.51 \pm 0.22) \times 10^{-3}$$

$$|V_{ub}|_{\pi\ell\bar{\nu}-\text{LQCD}} = (3.4 \pm 0.4) \times 10^{-3}$$
$$|V_{ub}|_{\tau\nu} = ?$$

SM fit: $(3.6 \pm 0.2) \times 10^{-3}$



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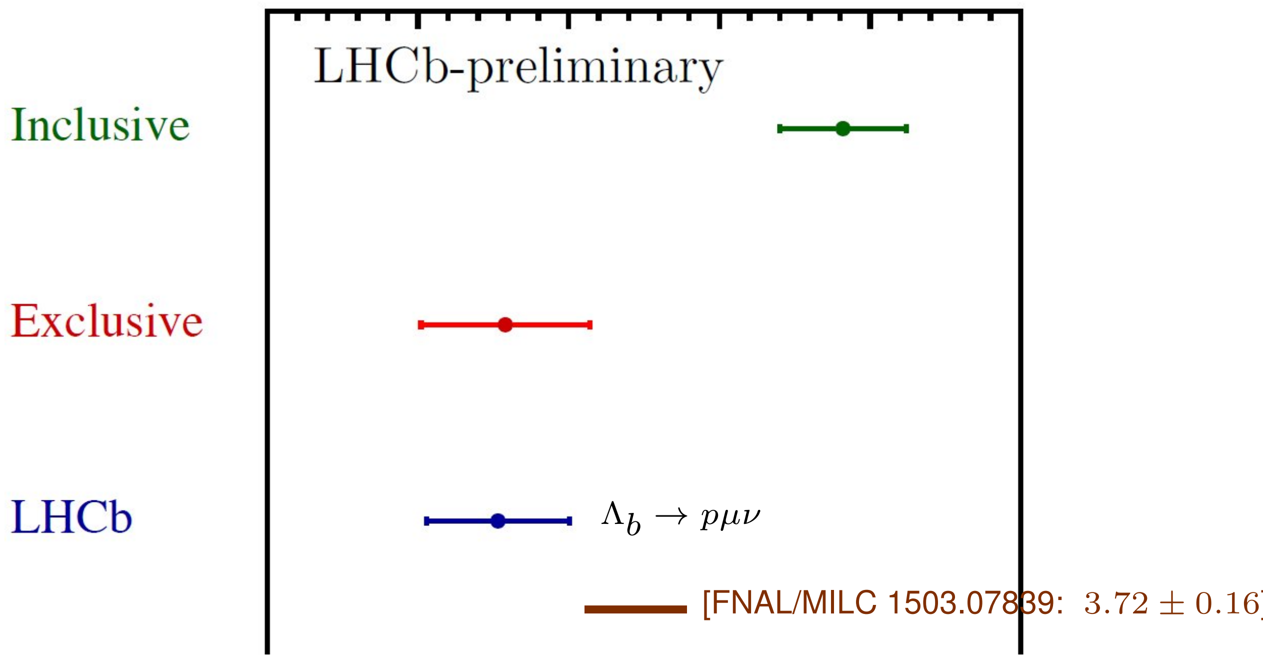


- What would it take to conclude that there is unambiguous evidence for NP?

Tensions in $|V_{ub}|$ determinations

- $\sim 3\,\sigma$ tension among $|V_{ub}|$ measurements

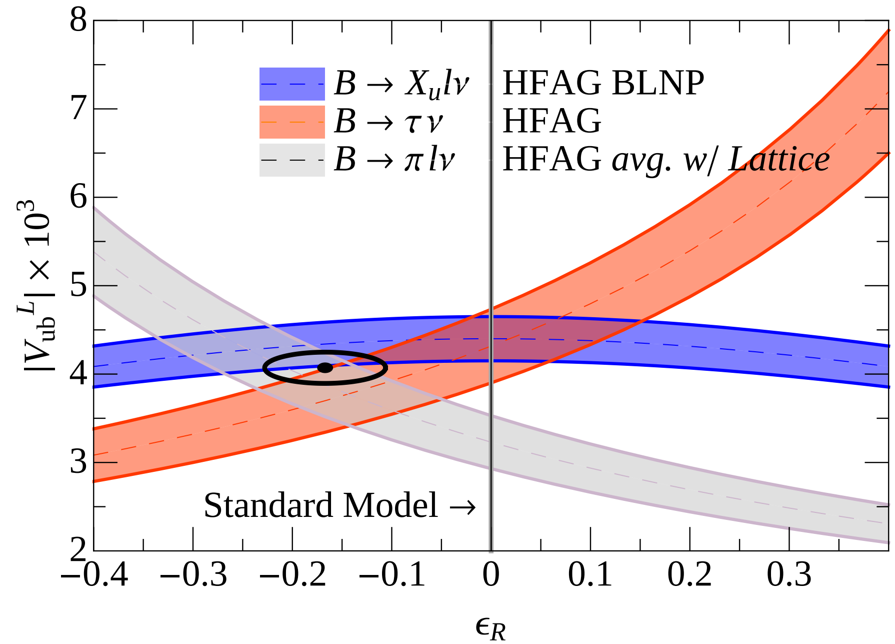
  Tim Gershon @ FPCP 2014: "Understanding this will involve a great deal of effort, but is essential for continued progress in the field"

- Too early to conclude:
  - Inclusive determination can improve
  - Exclusive measured better with full reco
  - Lattice QCD results will improve

- A BSM possibility:

$$\mathcal{L} = -\frac{4G_F}{\sqrt{2}} V_{ub}^L \left(\bar{u}\gamma_\mu P_L b + \epsilon_R\, \bar{u}\gamma_\mu P_R b\right)(\bar{\nu}_\ell \gamma^\mu P_L \ell)$$

Can we construct observables which give "more vertical" constraints?



[Bernlochner, ZL, Turczyk, 1408.2516]

Decay	$\lvert V_{ub}\rvert \times 10^4$	adm.
$B \to \pi\,\ell\,\bar{\nu}_\ell$	3.23 ± 0.30	$(1+\epsilon_R)$
$B \to X_u\,\ell\,\bar{\nu}_\ell$	4.39 ± 0.21	$\sqrt{1+\epsilon_R^2}$
$B \to \tau\,\bar{\nu}_\tau$	4.32 ± 0.42	$(1-\epsilon_R)$

- NB: Cleanest $|V_{ub}|$ I know, only isospin, $\mathcal{B}(B_u \to \ell\bar{\nu})/\mathcal{B}(B_d \to \mu^+\mu^-)$ [Grinstein, CKM'06 (?)]

Features of SIMBA

- Optimally combine all information on $B \to X_u \ell \bar{\nu}$ & $B \to X_s \gamma$
  Consistently treat uncertainties and their correlations (exp, theo, parameters)

- Simultaneously determine:
  - Overall normalization: $\mathcal{B}(B \to X_s \gamma)$, $|V_{ub}|$
  - Parameters: m_b, shape function(s)

- Utilize all measurements:
  - Different $B \to X_s \gamma$ spectra, or partial rates
  - Different $B \to X_u \ell \bar{\nu}$ spectra, or partial rates
  - Include other constraints on m_b, λ_1, etc.
  - Eventually use or predict $B \to X_s \ell^+ \ell^-$

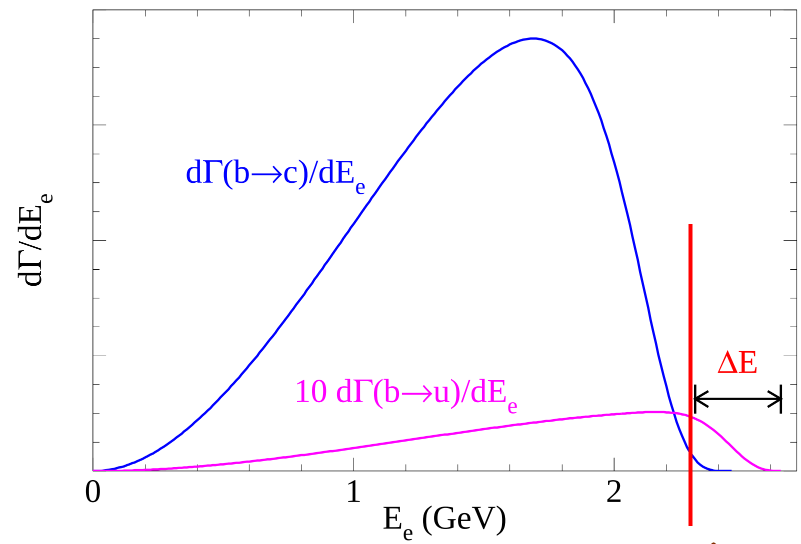
- Same strategy as for inclusive $|V_{cb}|$, just a lot more complicated...

Shape function

The challenge of inclusive $|V_{ub}|$ measurements

- Total rate calculable with $\sim 4\%$ uncertainty, similar to $\mathcal{B}(B \to X_c \ell \bar\nu)$
- To remove the huge charm background $(|V_{cb}/V_{ub}|^2 \sim 100)$, need phase space cuts

  Phase space cuts can enhance perturbative and nonperturbative corrections drastically



- Hadronic parameters are functions (like PDFs)

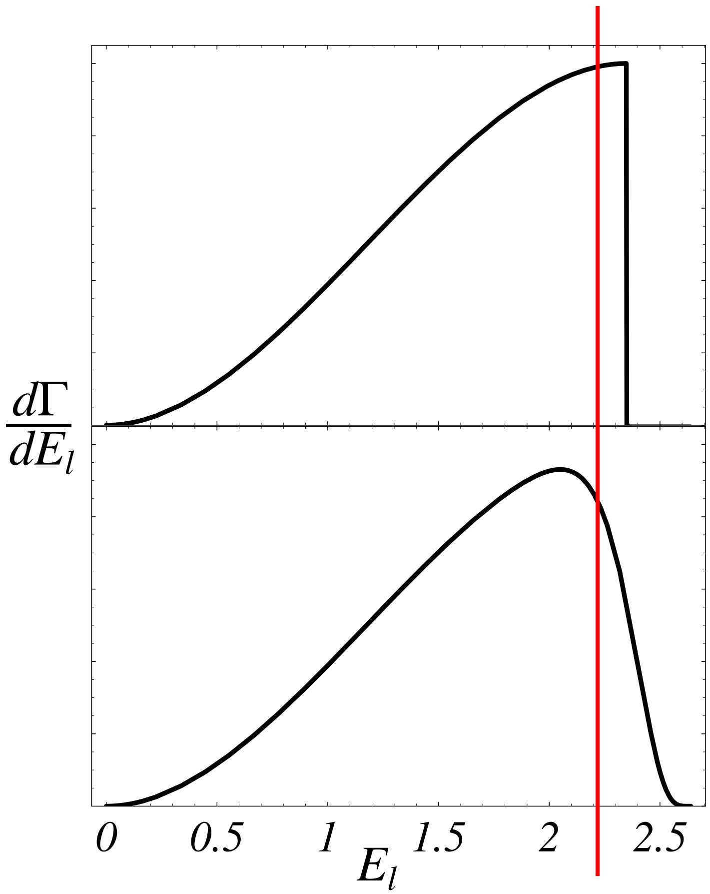
  Leading order: universal & related to $B \to X_s \gamma$;
  $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$: several new unknown functions

- Nonperturbative effects shift endpoint $\frac{1}{2} m_b \to \frac{1}{2} m_B$ & determine its shape
- Shape in the endpoint region is determined by b quark PDF in B ["shape function"]
  Related to $B \to X_s \gamma$ photon spectrum at lowest order [Bigi, Shifman, Uraltsev, Vainshtein; Neubert]

Shape function: lepton endpoint vs. $B \to X_s\gamma$

b quark decay spectrum

with a model for b quark PDF



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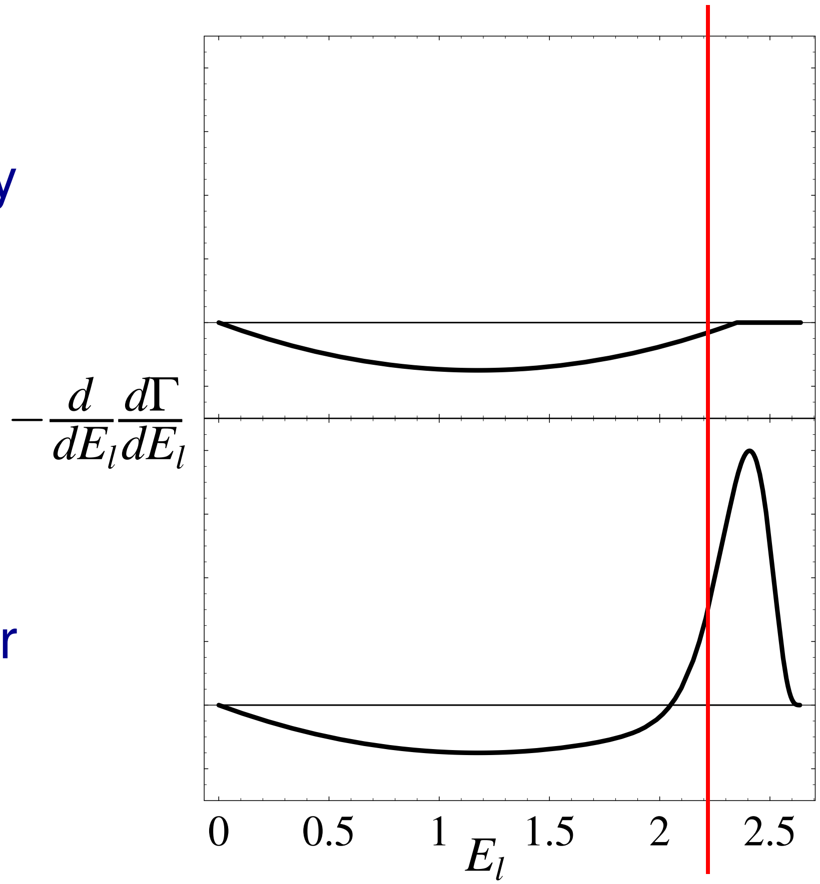

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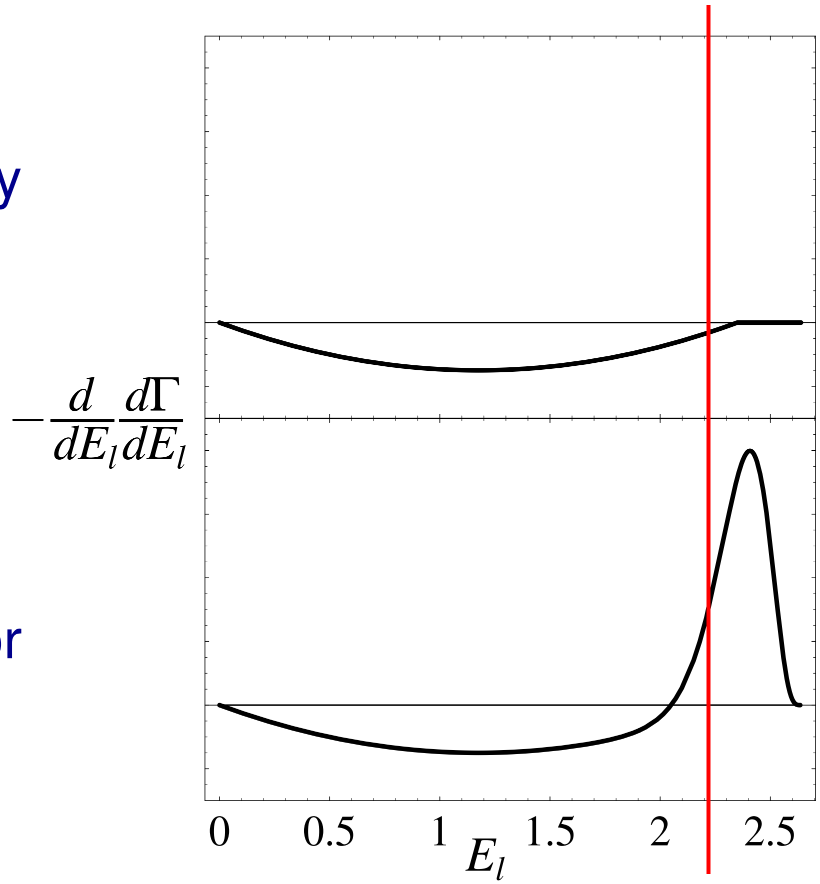
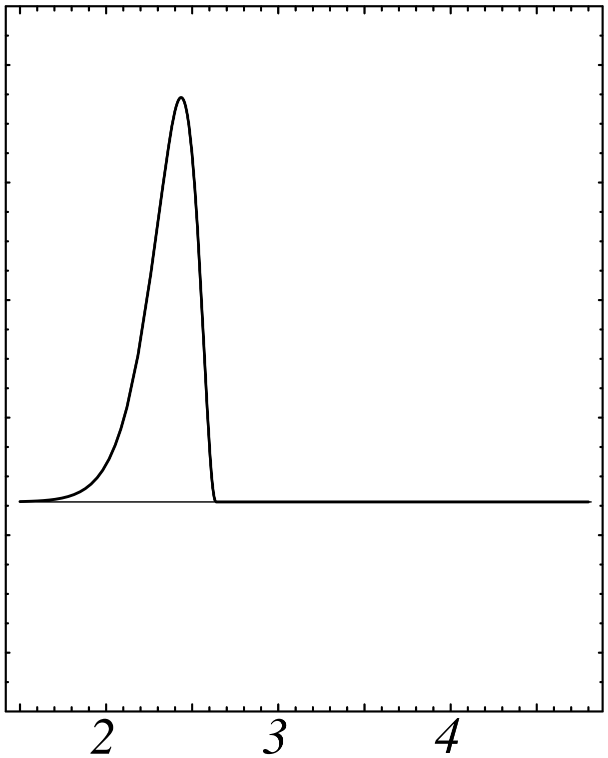


difference:



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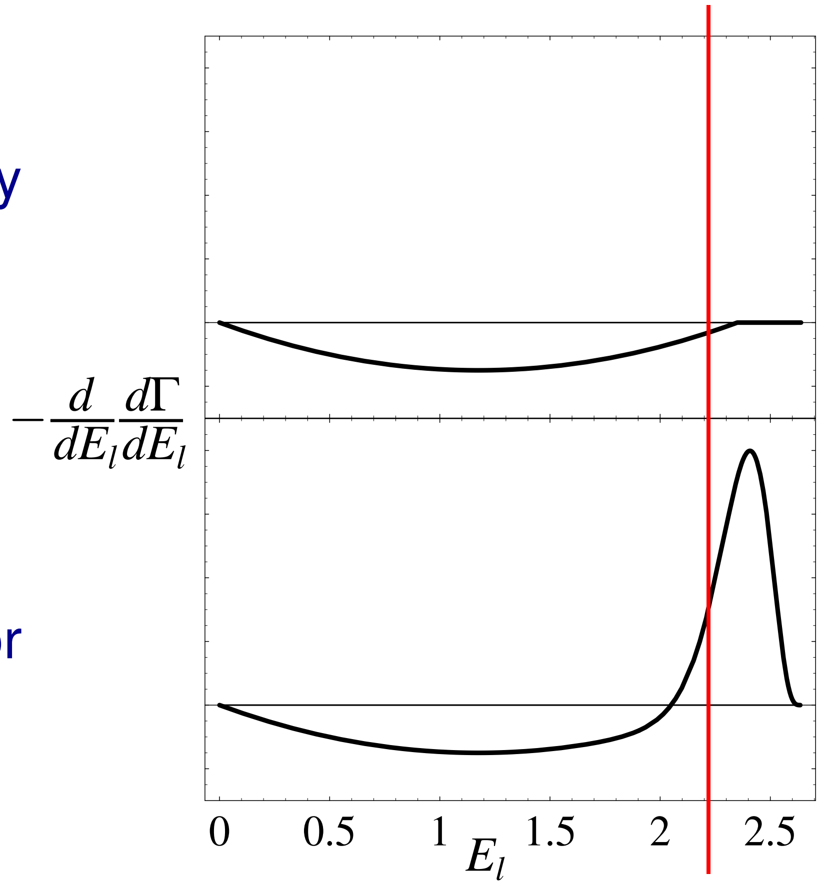

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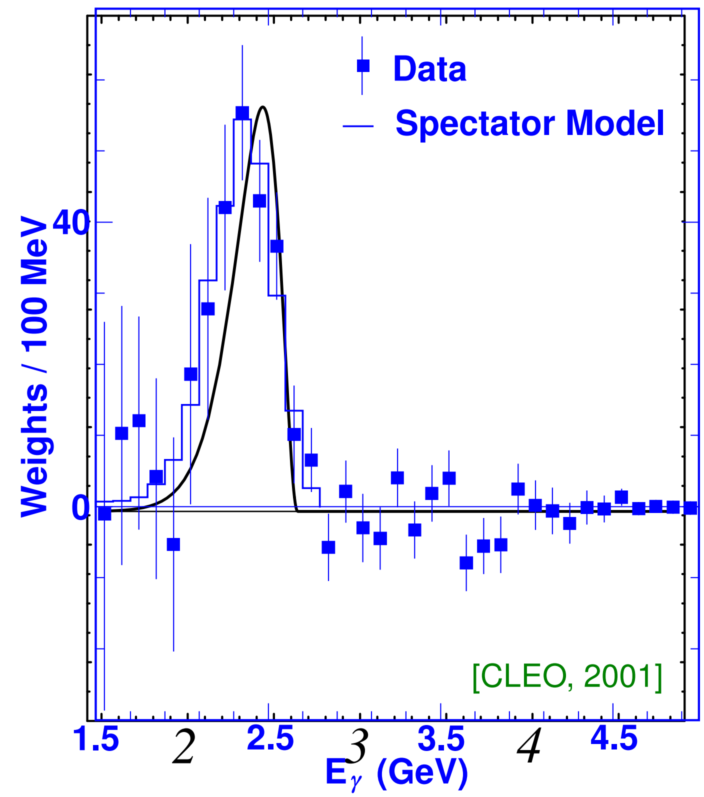

- Both spectra determined at lowest order by the b quark PDF in B meson

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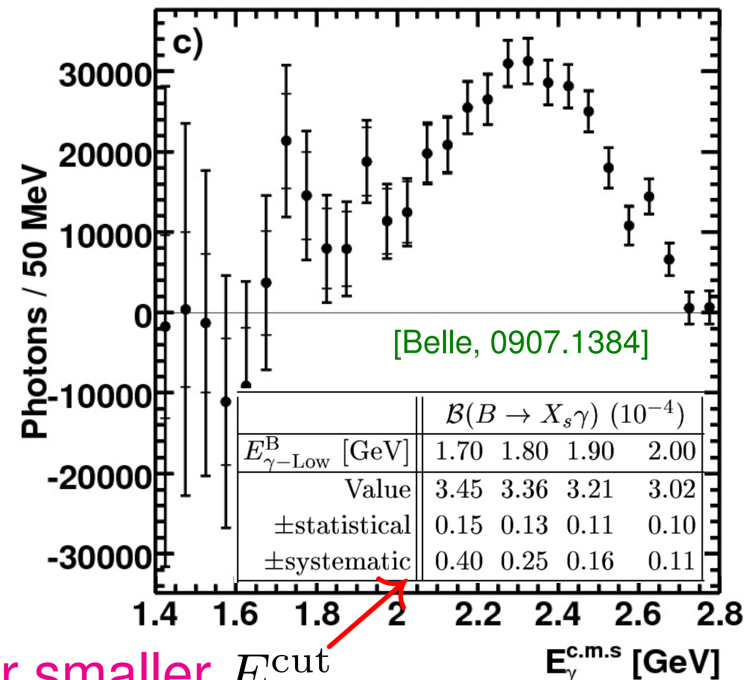
Start with $B \to X_s\gamma$

Regions of $B \to X_s\gamma$ photon spectrum

- Important both for $|V_{ub}|$ and constraining NP



- Peak around $E_\gamma \sim 2.3\,\text{GeV}$ $\quad (m_B - 2E_\gamma \sim 0.8\,\text{GeV})$

  Three cases: 1) $\Lambda_{\rm QCD} \sim m_B - 2E_\gamma \ll m_B$ ["SCET"]

  2) $\Lambda_{\rm QCD} \ll m_B - 2E_\gamma \ll m_B$ ["MSOPE"]

  3) $\Lambda_{\rm QCD} \ll m_B - 2E_\gamma \sim m_B$

  Expansions and theory uncertainties differ in the 3 regions

  Neither 1) nor 2) is fully appropriate

- Experimental systematic error rapidly increases for smaller $E_\gamma^{\rm cut}$

- Current practice: Compare rate extrapolated to $1.6\,\text{GeV}$ with theoretical prediction

  Con: (i) extrapolation uses theory, so comparison of theory and data is effectively done at the measured values; (ii) best use of the most precise measurements?

The shape function (b quark PDF in B)

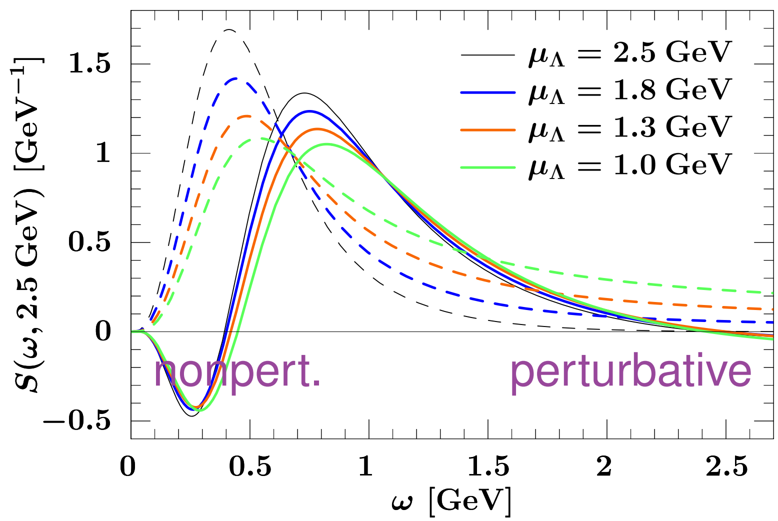
- The shape function $S(\omega, \mu)$ contains nonperturbative physics and obeys a RGE

  Even if $S(\omega, \mu_\Lambda)$ has exponentially small tail, RGE running gives long tail and divergent moments

  $$S(\omega, \mu_i) = \int d\omega' \, U_S(\omega - \omega', \mu_i, \mu_\Lambda) \, S(\omega', \mu_\Lambda)$$

  [Balzereit, Mannel, Kilian]

  Constraint: moments (OPE) + $B \to X_s \gamma$ shape



Model $\begin{cases} S & \text{(dash)} \\ F & \text{(solid)} \end{cases}$ run to 2.5GeV

- Derive: $S(\omega, \mu_\Lambda) = \int dk \, C_0(\omega - k, \mu_\Lambda) F(k)$

  [ZL, Stewart, Tackmann, 0807.1926]

  – Can use any (mass) scheme, work to any order

  – Stable results for varying μ_Λ (SF modeling scale, part of uncertainty, often ignored)

  – Similar to how all matrix elements are defined [e.g., $B_K(\mu) = \widehat{B}_K \times [\alpha_s(\mu)]^{2/9}(1 + \ldots)$]

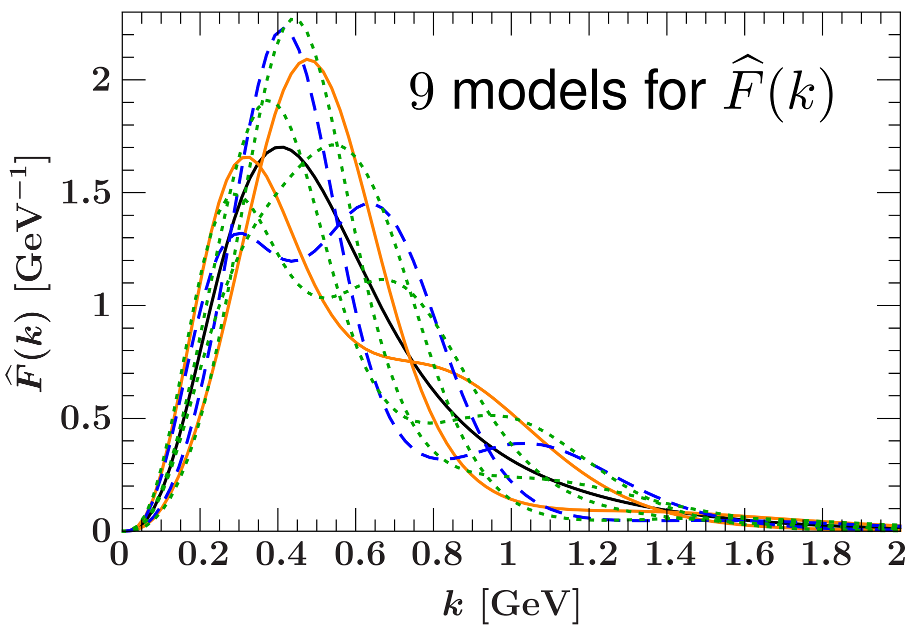
- Consistent to impose moment constraints on $F(k)$, but not on $S(\omega, \mu_\Lambda)$ w/o cutoff

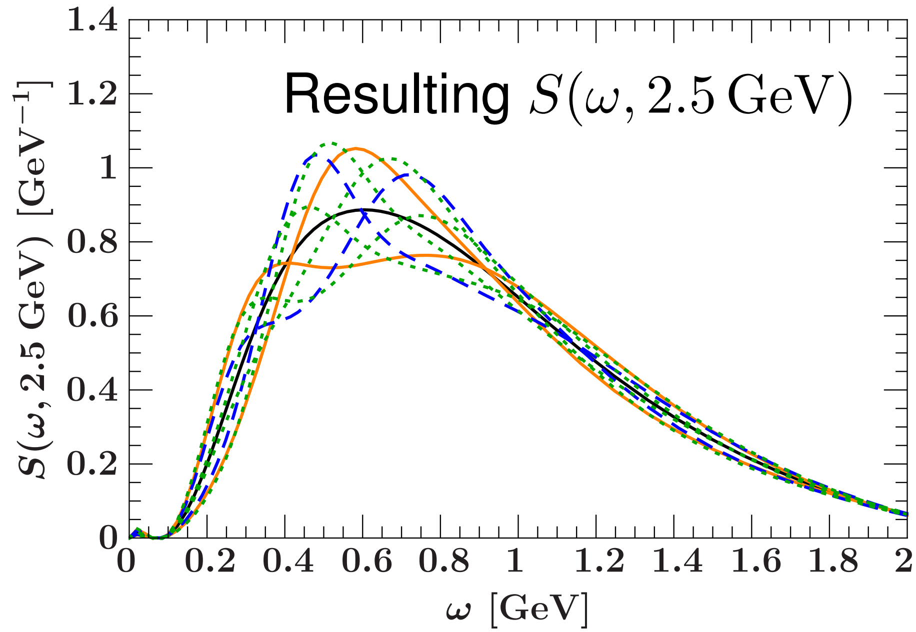
Shape function: the bottom line

$$S(\omega, \mu_\Lambda) = \int \mathrm{d}k\, \widehat{F}(k)\, \widehat{C}_0(\omega - k, \mu_\Lambda)$$

$\widehat{F}$: nonperturbative
- determines peak region
- well-defined moments
- fit from data

$\widehat{C}_0$: perturbative
- generates tail consistent with RGE
- divergent moments
- calculable



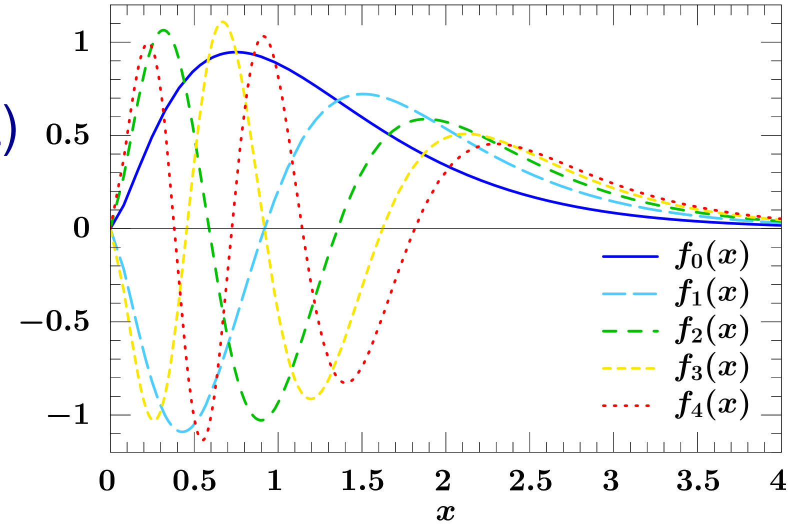


Designer orthonormal functions

- Devise suitable orthonormal basis functions
  (avoid: fit parameters of model functions to data)

$$\widehat{F}(\lambda x) = \frac{1}{\lambda}\Big[\sum c_n f_n(x)\Big]^2,\ n\text{ th moment} \sim \Lambda_{\rm QCD}^n$$

$$f_n(x) \sim P_n[y(x)] \leftarrow \text{Legendre polynomials}$$



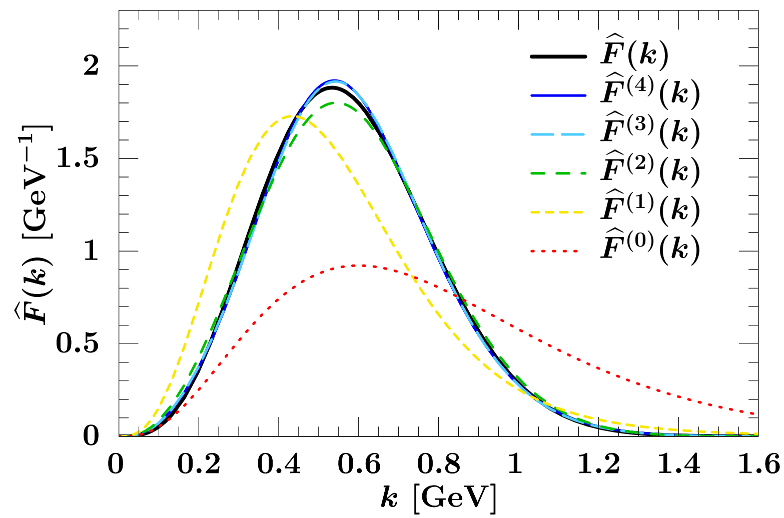
[ZL, Stewart, Tackmann, 0807.1926]

- Approximating a model shape function

Better to add a new term in an orthonormal basis than a new parameter to a model:

– less parameter correlations

– uncertainties easier to quantify



"With four parameters I can fit an elephant, and with five I can make him wiggle his trunk." (John von Neumann)

Details of fitting the data

- $\widehat{F}(k)$ enters the spectra linearly
  $\Rightarrow$ can calculate independently the contribution of $f_m\, f_n$ in the expansion of $\widehat{F}(k)$:

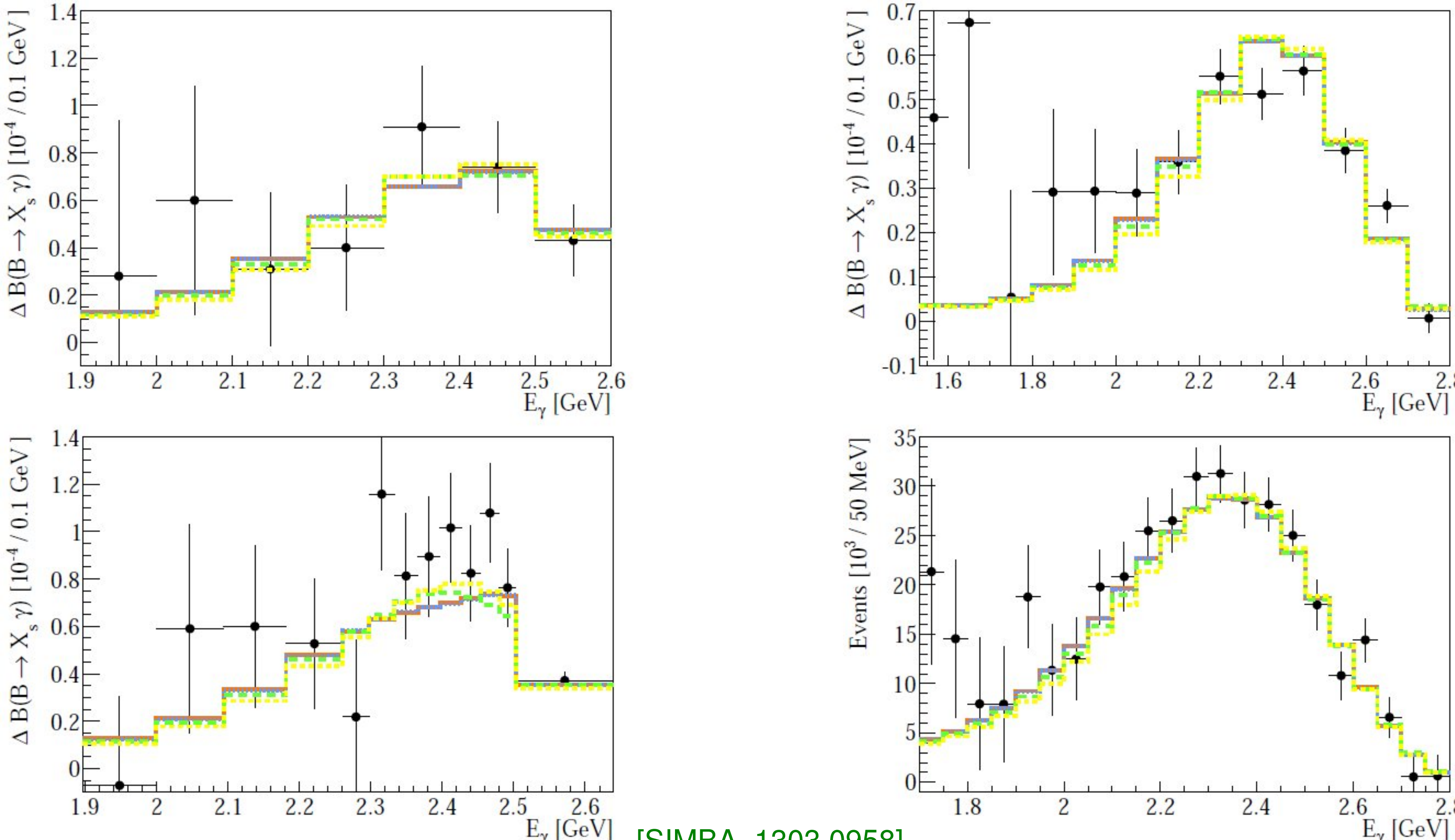
$$\mathrm{d}\Gamma = \sum \underbrace{c_m\, c_n}_{\text{fit}}\; \underbrace{\mathrm{d}\Gamma_{mn}}_{\text{compute}}$$

$$\mathrm{d}\Gamma_{mn} = \Gamma_0\, H(p_X^\pm) \int_0^{p_X^+} \mathrm{d}k\, \frac{\widehat{P}(p^-, k)}{\lambda}\, \underbrace{f_m\!\left(\frac{p_X^+ - k}{\lambda}\right) f_n\!\left(\frac{p_X^+ - k}{\lambda}\right)}_{\text{basis functions}}$$

  Fit the c_i coefficients from all measured (binned) spectra (similar to $|V_{cb}|$ fit)

- SIMBA includes:
  - Simultaneous fit using all available information
  - Correlations in data, propagation of SF uncertainties
  - Validate the fits with pseudo-experiments
  - Check model independence by varying number of basis functions in fit (up to 5)

Fit results for $B \to X_s\gamma$



[SIMBA, 1303.0958]

- Fit with $\lambda = 0.5\,\mathrm{GeV}$ with 2 (yellow), 3 (green), 4 (blue), and 5 (orange) coefficients

  Can change the basis by varying λ — find consistent results

Comments on uncertainties

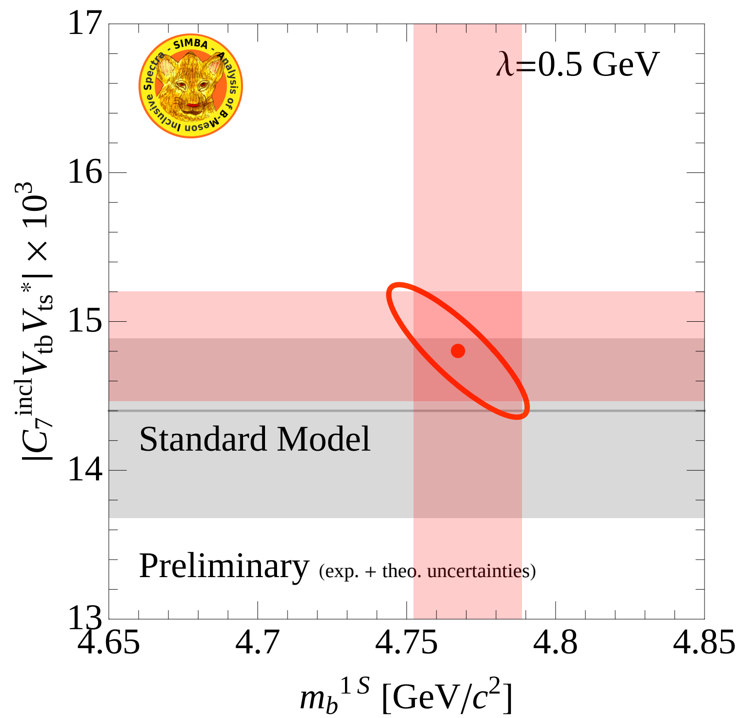
- Theoretical inputs:
  - Scale variations: μ_i and profiles
  - Subleading SF: tree level C_7^2 terms absorbed in $C_7^{\rm incl}$
    estimate uncertainty due to $\mathcal{O}(\alpha_s\Lambda_{\rm QCD}/m_b)$ shape functions
  - Non-C_7^2 subleading SF (4-quark) less important than sometimes claimed, since $\mathcal{O}(\alpha_s\Lambda_{\rm QCD}/m_b)$ in peak region, which dominates the fit
  - λ_2, ρ_2, m_c — mild sensitivity
- Fit procedure and validation:
  - Measurements with all available correlations
  - Shape function basis, number of terms in fit, test with toys

Fit results for $B \to X_s\gamma$ (2)

- Have complete NNLO + NNLL' (2-loop matching & running, 3-loop cusp) [1303.0958]
- $\chi^2/\mathrm{ndf} = 41.7/48$
- SM prediction: $\left|C_7^{\mathrm{incl}}\right|^{\mathrm{SM}} = 0.354^{+0.011}_{-0.012}$

$$|V_{tb}V_{ts}| = (40.4 \pm 0.1) \times 10^{-3}$$

- Fit: $\left|C_7^{\mathrm{incl}}\, V_{tb}V_{ts}\right| = (14.83{\pm}0.53_{[\mathrm{exp}]}{\pm}0.37_{[\mathrm{th}]}){\times}10^{-3}$
- Data slightly above SM prediction, as in HFAG combination vs. Misiak *et al.*



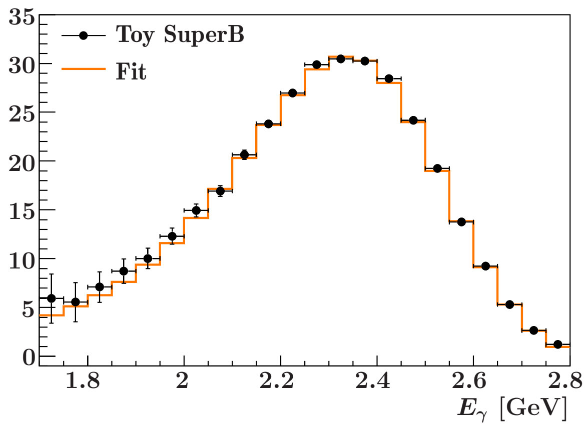
Future of $B \to X_s\gamma$

- Toy fits few years ago for 75/ab:

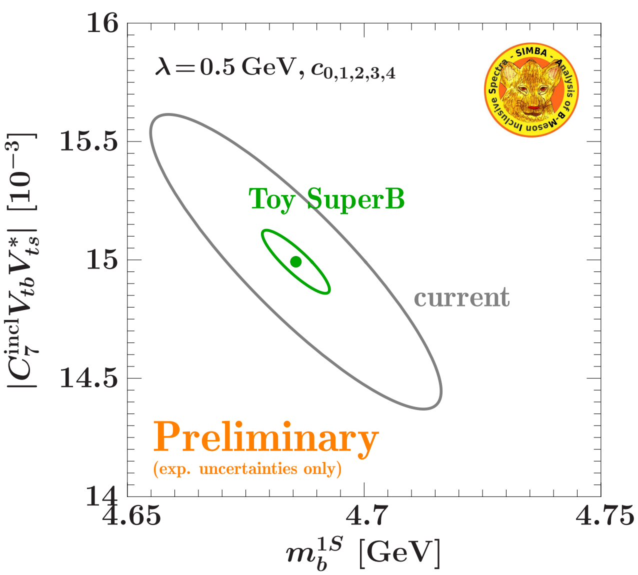
5 coefficients

$\lambda = 0.5\,\text{GeV}$

Theory uncert. will dominate



[BELLE2-NOTE-0021]	Statistical	Systematic (reducible, irreducible)	Total Exp
$\mathcal{B}(B \to X_s\gamma)$ inclusive (untagged)			
605 fb^{-1}	4.2	(10.3, 5.3)	12.3
5 ab^{-1}	1.5	(3.6, 5.3)	6.6
50 ab^{-1}	0.5	(1.1, 5.3)	5.4
$\mathcal{B}(B \to X_s\gamma)$ inclusive (hadron tagged)			
210 $\text{fb}^{-1\dagger}$	23.2	(15.7, 4.8)	28.4
5 ab^{-1}	4.8	(3.2, 4.8)	7.5
50 ab^{-1}	1.5	(1.0, 4.8)	5.1



We assumed factor of 3 reduction in systematic uncertainty, slightly (but not wastly) optimistic

High precision data can be used to fit with more coefficients and constrain subleading effects

$$B \to X_u \ell \bar{\nu}$$

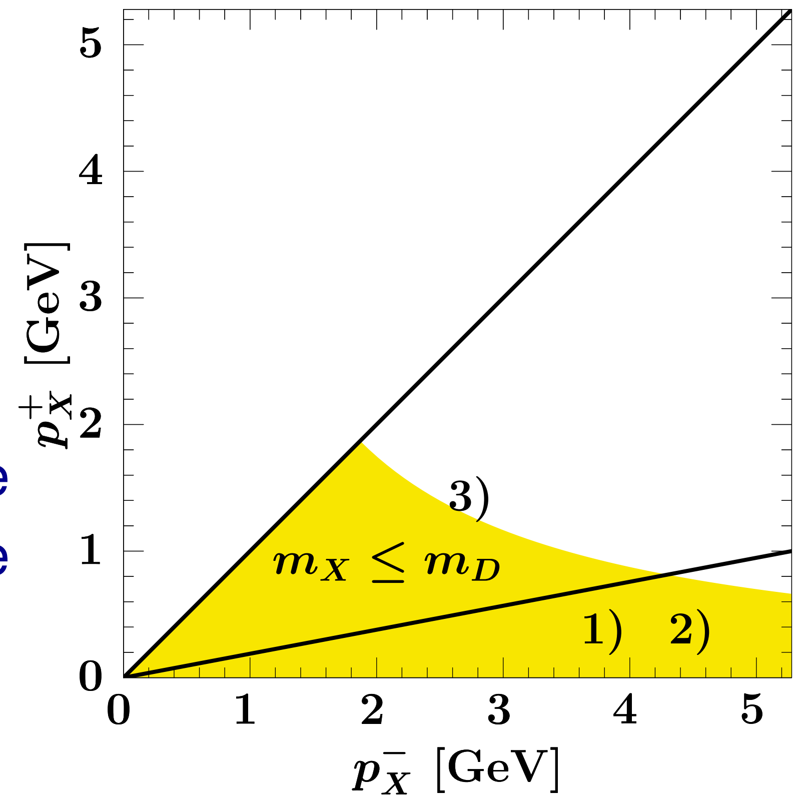
$B \to X_u \ell \bar\nu$ is more complicated

- "Natural" kinematic variables: $p_X^\pm = E_X \mp |\vec p_X|$ (ratio is "jettiness" of hadrons)

  $B \to X_s\gamma$: $p_X^+ = m_B - 2E_\gamma$ & $p_X^- \equiv m_B$ — independent variables in $B \to X_u \ell \bar\nu$

- Three cases: 1) $\Lambda \sim p_X^+ \ll p_X^-$
  2) $\Lambda \ll p_X^+ \ll p_X^-$ } Shape Fn region
  3) $\Lambda \ll p_X^+ \sim p_X^-$ local OPE region

  Want to make no assumptions how p_X^- compares to m_B



- $B \to X_u \ell \bar\nu$: 3-body final state, appreciable rate in region 3), where hadronic final state not jet-like

  E.g., $m_X^2 < m_D^2$ does not imply $p_X^+ \ll p_X^-$

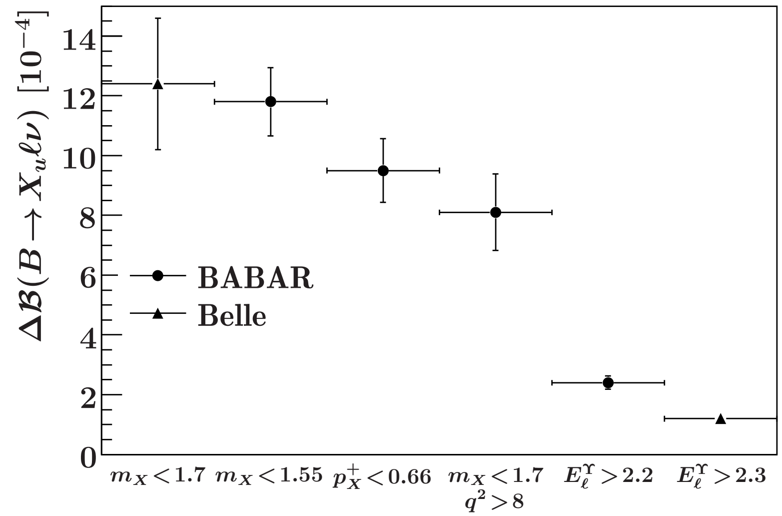
- Existing results based on theory in one region, extrapolated / modeled to rest

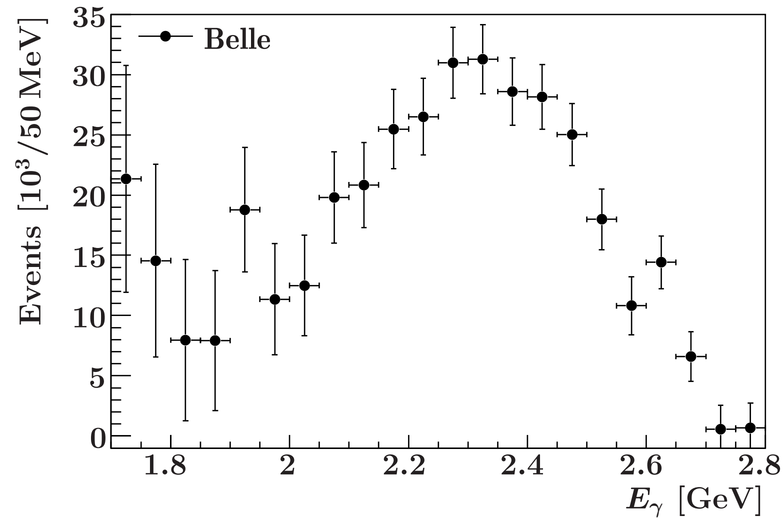
Other approaches

- BLNP [Bosch *et al.*] — based on SCET region, tied to “shape function scheme”
- DGE [Andersen & Gardi] — based on SCET region + perturbative model for the SF
- GGOU [Gambino *et al.*] — based on local OPE region + SF smearing
- BLL [Bauer, ZL, Luke] — based on local OPE at large q^2 (but expansion scale is smaller)
  - combine q^2 and m_X cuts, such that SF effect is kept small
- Shape function independent relations [Leibovich, Low, Rothstein; Hoang, ZL, Luke; Lange, Neubert, Paz; Lange]
  - beautiful at leading order, less so when $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$ included

Exploratory: $|V_{ub}|$ w/ NLO + NLL' only

- $B \to X_u \ell \bar{\nu}$ hadronic tag
  - BaBar m_X, $m_X - q^2$, p_X^+
  - Belle m_X
- $B \to X_u \ell \bar{\nu}$ lepton endpoint
  - BaBar $E_\ell^\Upsilon > 2.2\,\mathrm{GeV}$
  - Belle $E_\ell^\Upsilon > 2.3\,\mathrm{GeV}$
- $B \to X_s \gamma$ spectra
  - Belle latest result (shown)
  - BaBar sum over exclusive + hadronic tag
- m_b^{1S}, λ_1 from $B \to X_c \ell \bar{\nu}$ fit
  - $m_b^{1S} = (4.66 \pm 0.05)\,\mathrm{GeV}$
  - $\lambda_1 = (-0.34 \pm 0.05)\,\mathrm{GeV}^2$





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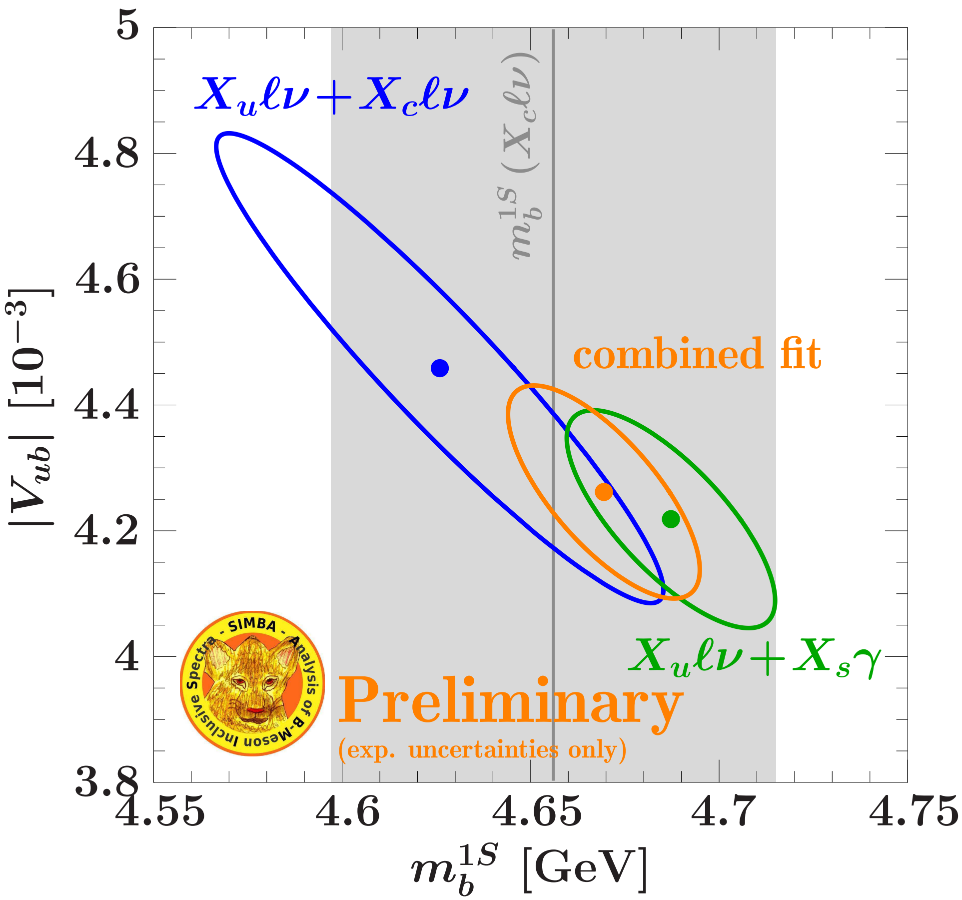

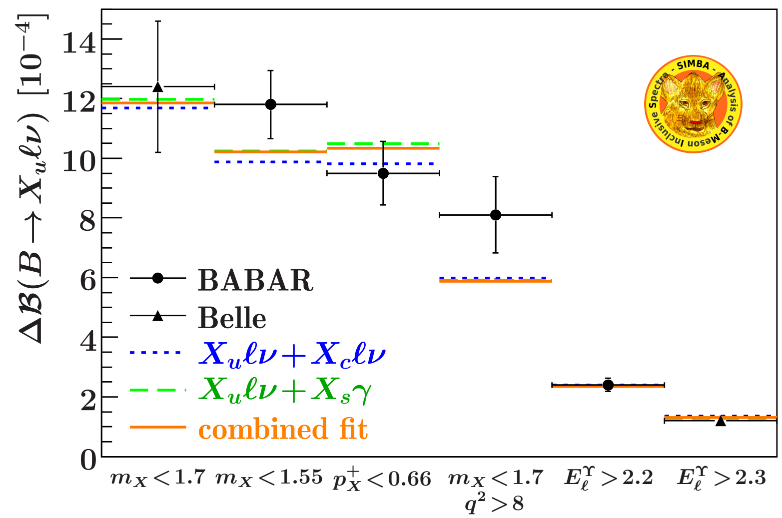

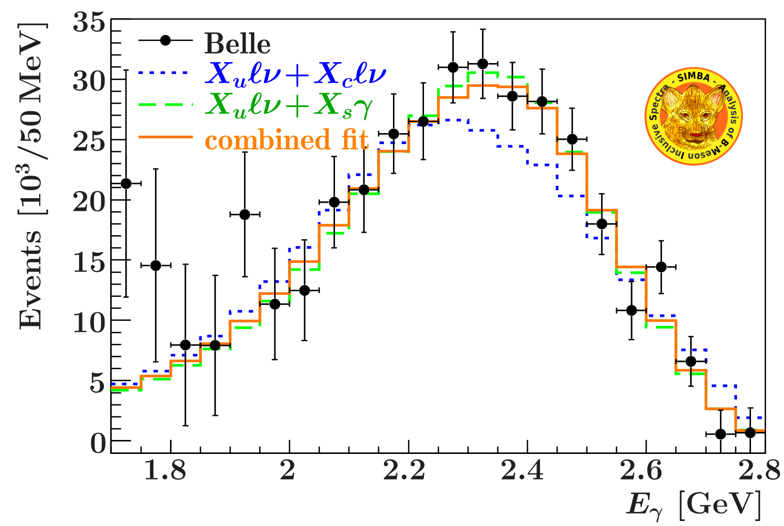

- E_γ spectrum is off without $B \to X_s\gamma$ in the fit
- Including it, favors lower values of $|V_{ub}|$

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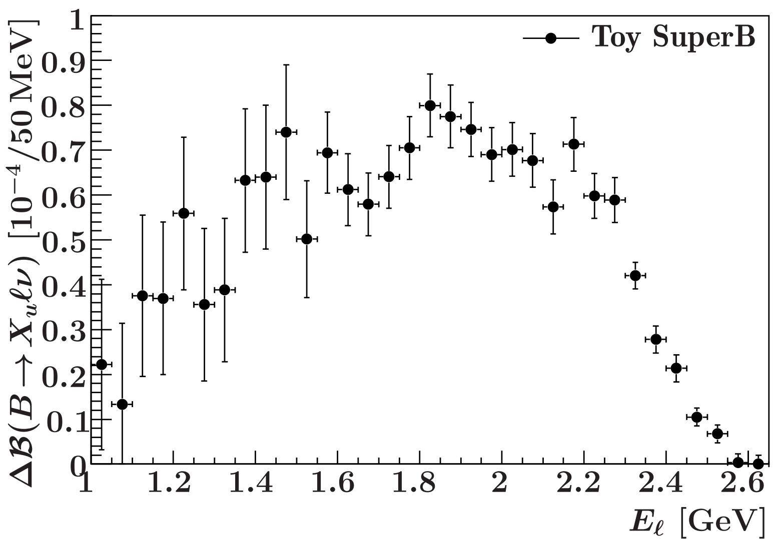
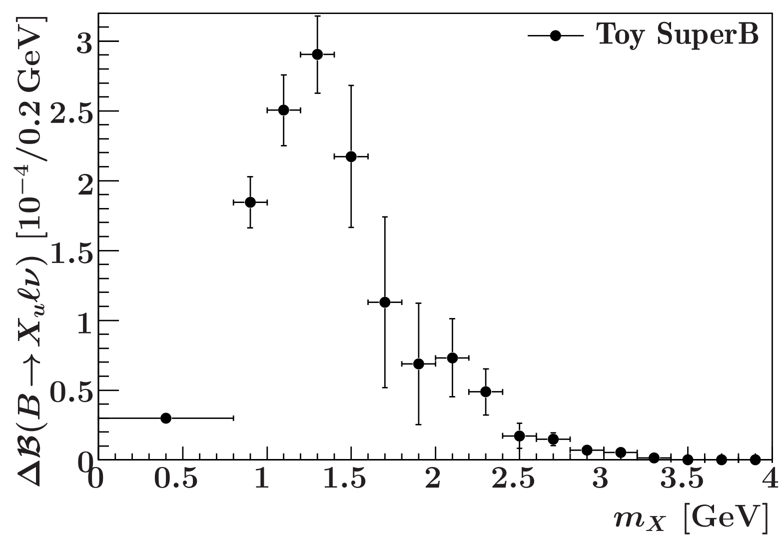
Future of $B \to X_u \ell\bar{\nu}$

- Spectra generated with $\lambda = 0.6\,\mathrm{GeV}$ and $c_0 = 1$ (Assumed uncertainties & correlations similar to BaBar full reco analysis, 1112.0702 — by now Belle hadronic tagging efficiency is better)





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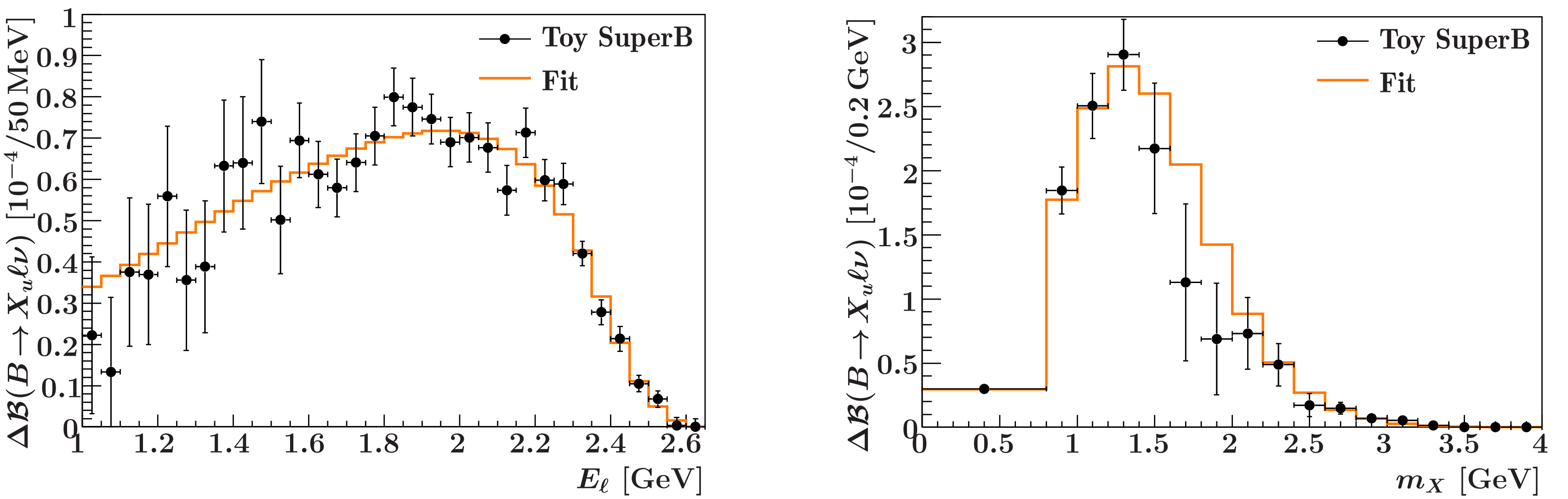
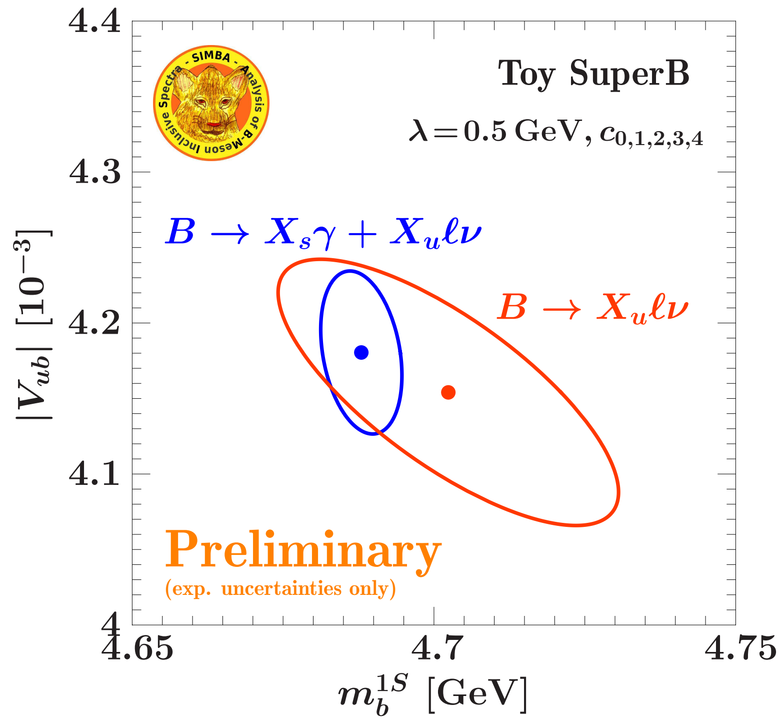


- Measure spectra — the rate with low E_ℓ or high m_X cut cannot give optimal $|V_{ub}|$
  - Uncertainties grow, as for $\mathrm{d}\Gamma(B \to X_s \gamma)/\mathrm{d}E_\gamma$
  - Experimental analysis needs input on shape in any case
- Large data sets will push analysis to the limits, constain subleading SF effects

Future of $B \to X_u \ell \bar{\nu}$ (2)

- Toy fit with 5 coefficients for $75/\mathrm{ab}$:



- With Belle II data sets:
  - Combination with $B \to X_s\gamma$ will be essential for ultimate sensitivity
  - Combination with $B \to X_c\ell\bar{\nu}$ (moments, shapes?) also possible

Final comments

Some comments on $|V_{ub}|$ inclusive

- Is $\{$in/ex$\}$clusive tension a nuisance or tip of an iceberg? (right-handed currents?)

- Recently $\Gamma(D_s \to X\ell\bar{\nu})$ gave some indication of what the resolution is not

- Qualitatively better analyses are possible than those implemented so far
  - Fitting $F(k)$ instead of modeling $S(\omega, \mu)$
  - Designer orthonormal functions — reduce role of shape function modeling
  - Fully consistent combination of all phase space regions
  - Decouple SF shape variation from m_b variation

- Inclusive $|V_{cb}|$ uses a combined fit; clearly the right method for $|V_{ub}|$ as well
  Combine all $B \to X_s\gamma$, $X_u\ell\bar{\nu}$, $X_c\ell\bar{\nu}$ data to constrain short distance physics & SFs
  Need all available spectra and correlations

- $|V_{ub}|$ is tricky: to draw conclusions about new physics, we'll want ≥ 2 extractions with different uncertainties to agree well (inclusive, exclusive, leptonic)

Conclusions

- Current status of $|V_{ub}|$ unsettled — improvement crucial to better constrain NP
  Hope to see measurements w/ different uncertainties agree (incl., excl., leptonic)

- Qualitatively better inclusive $|V_{ub}|$ analysis possible than those implemented so far
  Full hadronic reconstruction B sample at Belle II is crucial for this

- Measure all possible **spectra** with best possible precision

- SIMBA allows: eliminate shape function modeling
  constrain subleading shape functions
  consistently combine all $B \to X_u \ell \bar{\nu}$ measurements
  consistently include $B \to X_s \gamma$ measurements
  consistently include $B \to X_c \ell \bar{\nu}$ constraints
  eventually include/predict $B \to X_s \ell^+ \ell^-$

Backup slides

Derivation of the magic formula (1)

- The shape function is the matrix element of a nonlocal operator:

$$S(\omega,\mu) = \langle B|\, \underbrace{\bar b_v\, \delta(iD_+ - \delta + \omega)\, b_v}_{O_0(\omega,\mu)} \,|B\rangle, \qquad \delta = m_B - m_b$$

Integrated over a large enough region, $0 \le \omega \le \Lambda$, one can expand O_0 as

$$O_0(\omega,\mu) = \sum C_n(\omega,\mu)\, \underbrace{\bar b_v\,(iD_+ - \delta)^n\, b_v}_{Q_n} + \ldots = \sum C_n(\omega-\delta,\mu)\, \underbrace{\bar b_v\,(iD_+)^n\, b_v}_{\widetilde Q_n} + \ldots$$

The C_n are the same for Q_n and $\widetilde Q_n$ (since O_0 only depends on $\omega - \delta$)

- Matching: $\langle b_v|O_0(\omega{+}\delta,\mu)|b_v\rangle = \sum C_n(\omega,\mu)\,\langle b_v|\widetilde Q_n|b_v\rangle = C_0(\omega,\mu), \quad \langle b_v|\widetilde Q_n|b_v\rangle = \delta_{0n}$

$$\langle b_v(k_+)|O_0(\omega+\delta,\mu)|b_v(k_+)\rangle = C_0(\omega+k_+,\mu) = \sum \frac{k_+^n}{n!}\,\frac{\mathrm{d}^n C_0(\omega,\mu)}{\mathrm{d}\omega^n}$$

$$\langle b_v(k_+)|O_0(\omega+\delta,\mu)|b_v(k_+)\rangle = \sum C_n(\omega,\mu)\langle b_v|\widetilde Q_n|b_v\rangle = \sum C_n(\omega,\mu)\, k_+^n$$

- Comparing last two lines: $C_n(\omega,\mu) = \dfrac{1}{n!}\,\dfrac{\mathrm{d}^n C_0(\omega,\mu)}{\mathrm{d}\omega^n}$ [Bauer & Manohar]

Derivation of the magic formula (2)

- Define the nonperturbative function $F(k)$ by: [ZL, Stewart, Tackmann; Lee, ZL, Stewart, Tackmann]

$$S(\omega, \mu_\Lambda) = \int \mathrm{d}k\, C_0(\omega - k, \mu_\Lambda)\, F(k), \qquad C_0(\omega, \mu) = \langle b_v | O_0(\omega + \delta, \mu) | b_v \rangle$$

uniquely defines $F(k)$: $\widetilde{F}(y) = \widetilde{S}(y, \mu) / \widetilde{C}_0(y, \mu)$

- Expand in k: $S(\omega, \mu) = \sum_n \frac{1}{n!} \frac{\mathrm{d}^n C_0(\omega, \mu)}{\mathrm{d}\omega^n} \int \mathrm{d}k\, (-k)^n F(k)$

Compare with previous page $\Rightarrow \int \mathrm{d}k\, k^n F(k) = (-1)^n \langle B | Q_n | B \rangle$

$$\langle B | Q_0 | B \rangle = 1\,, \quad \langle B | Q_1 | B \rangle = -\delta\,, \quad \langle B | Q_2 | B \rangle = -\frac{\lambda_1}{3} + \delta^2$$

More complicated situation for higher moments, so stop here

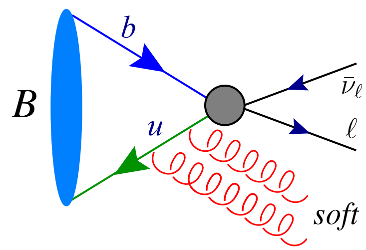
- This treatment is fully consistent with the OPE

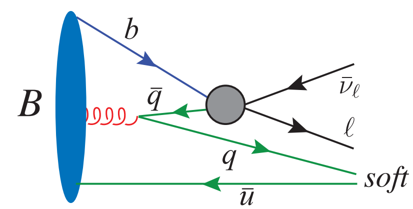
Weak annihilation

- Hard to estimate: $(16\pi^2)\,(\Lambda_{\rm QCD}^3/m_b^3)\,\varepsilon$, centered near $q^2 = m_B^2$ and $E_\ell = m_B/2$

$$\langle B|(\bar b\gamma^\mu P_L u)\,(\bar u\gamma_\mu P_L b)|B\rangle = \frac{f_B^2\, m_B}{8}\, B_1$$

$$\langle B|(\bar b P_L u)\,(\bar u P_L b)|B\rangle = \frac{f_B^2\, m_B}{8}\, B_2$$





Overall shift vs. splitting between $B^\pm$ and B^0

Factorization + vacuum saturation: $B_{1,2} = \begin{cases} 1\,, & B^\pm \\ 0\,, & B^0 \end{cases}$ assume $\varepsilon \equiv B_1 - B_2 \sim 0.1$

Rate: $\Gamma_{\rm WA} = \dfrac{G_F^2 m_b^2 |V_{ub}|^2}{12\pi}\, f_B^2\, m_B (B_2 - B_1) \sim 3\%$ of $\Gamma(B \to X_u \ell\bar\nu)$ [Voloshin, hep-ph/0106040]

- Enters all $|V_{ub}|$ measurements, enhanced by $(m_b/m_c)^3 \sim 30$ in $D_{u,d,s}$ decays

$\Gamma(D^0 \to X\ell\bar\nu) \approx \Gamma(D^\pm \to X\ell\bar\nu)$ to $\lesssim 3\%$, recently $\Gamma(D_s \to X\ell\bar\nu)$ [CLEO-c, arXiv:0912.4232]

No evidence that WA is bigger when light quark in operator = spectator flavor

- Probably a smaller effect in the determination of $|V_{ub}|$ than typically assumed

[ZL, Luke, Manohar, arXiv:1003.1351; Gambino & Kamenik, arXiv:1004.0114]