status and prospects of lattice QCD input for $|V_{cb}|$

Carlos Pena

Challenges in Semileptonic B decays — MITP, 20-24 April 2015

outline

- relevant channels, form factors

- lattice QCD in the precision era
    - reach of lattice simulations
    - summaries of lattice results: FLAG
    - issues for B-physics

- status
    - pre-2014: form factors at zero recoil
    - recent developments: improved precision, q^2 dependence, baryon channels

- outlook

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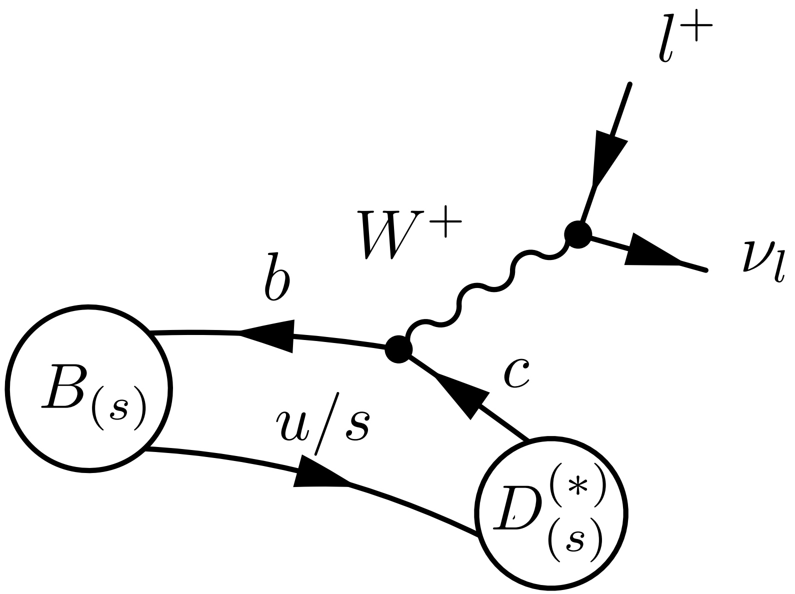
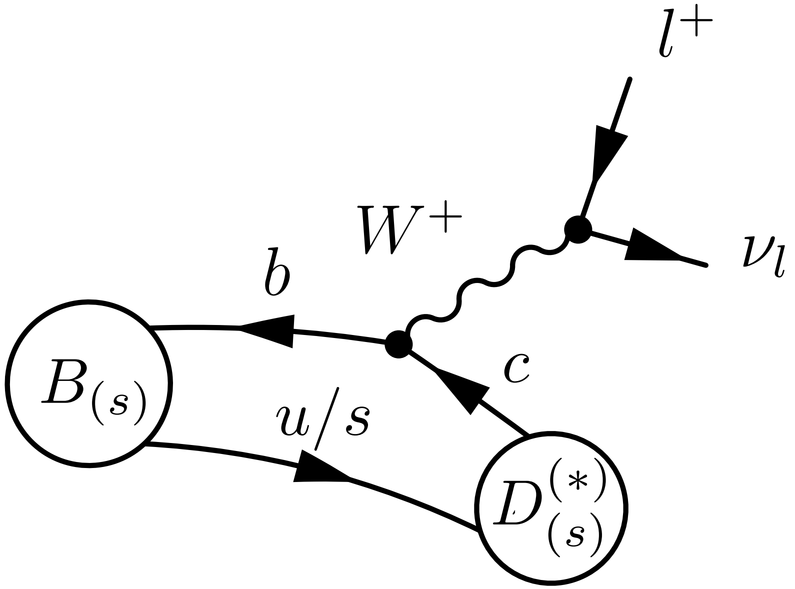
channels, form factors



$$\frac{\mathrm{d}\Gamma(B \to D\ell\nu)}{\mathrm{d}q^2} = \frac{G_{\mathrm{F}}^2|V_{cb}|^2}{24\pi^3}\,\frac{(q^2-m_\ell^2)^2\sqrt{E_D^2-m_D^2}}{q^4 m_B^2} \times \left[\left(1+\frac{m_\ell^2}{2q^2}\right) m_B^2(E_D^2-m_D^2)|f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2}(m_B^2-m_D^2)^2|f_0(q^2)|^2\right]$$

channels, form factors



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$$\frac{\mathrm{d}\Gamma(B\to D\ell\nu)}{\mathrm{d}w} = \frac{G_{\mathrm{F}}^2|V_{cb}|^2}{48\pi^3}(m_B+m_D)^2(w^2-1)^{3/2}|\eta_{\mathrm{EW}}|^2|\mathcal{G}(w)|^2 + \mathcal{O}\left(\frac{m_\ell^2}{q^2}\right)$$

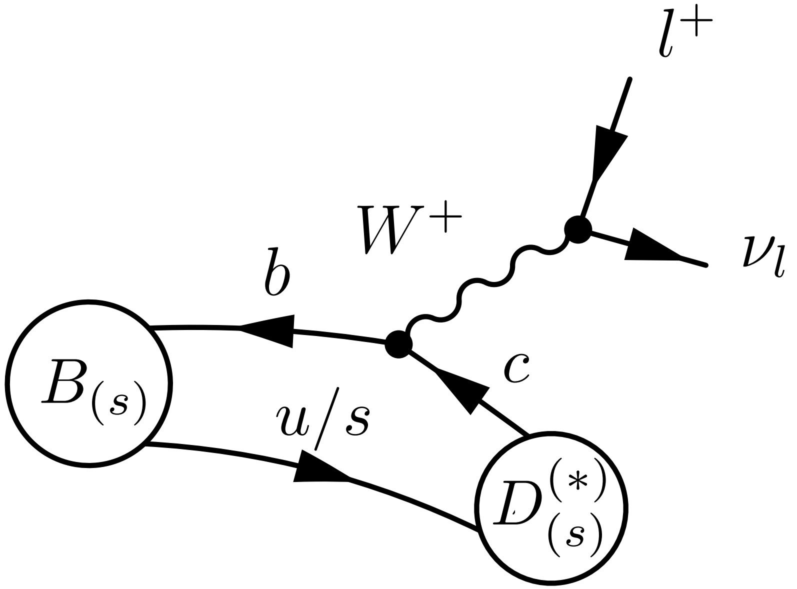
$$\frac{\mathrm{d}\Gamma(B\to D^*\ell\nu)}{\mathrm{d}w} = \frac{G_{\mathrm{F}}^2|V_{cb}|^2}{4\pi^3}(m_B-m_{D^*})^2(w^2-1)^{1/2}|\eta_{\mathrm{EW}}|^2\chi(w)|\mathcal{F}(w)|^2 + \mathcal{O}\left(\frac{m_\ell^2}{q^2}\right)$$

$$w = \frac{p_B\cdot p_{D^{(*)}}}{m_B m_{D^{(*)}}}$$

zero recoil ($w=1$):

- single form factor
- in D* channel: $\chi(1)=1$, no $\mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)$ corrections

channels, form factors



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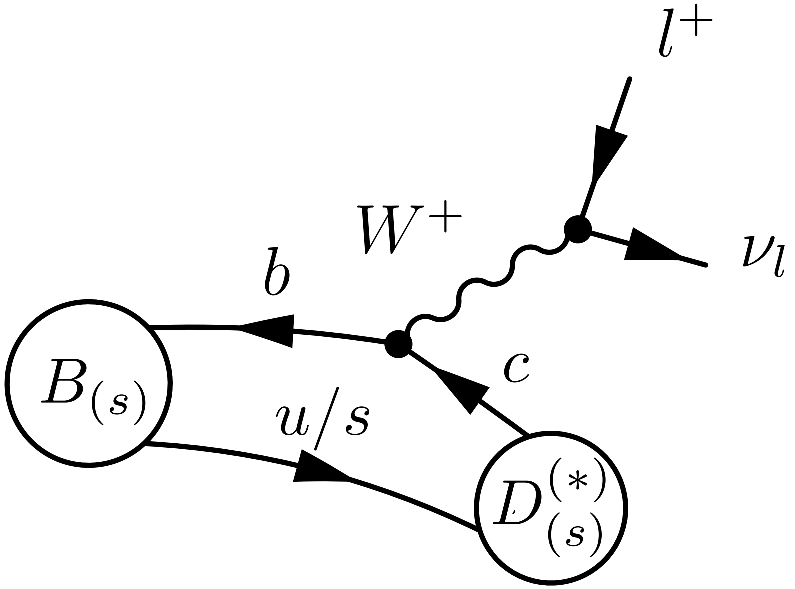
$$w = \frac{p_B\cdot p_{D^{(*)}}}{m_Bm_{D^{(*)}}}$$

zero recoil ($w=1$):

- ○ not good enough for τ final state
- ○ improved precision requires shape knowledge

channels, form factors



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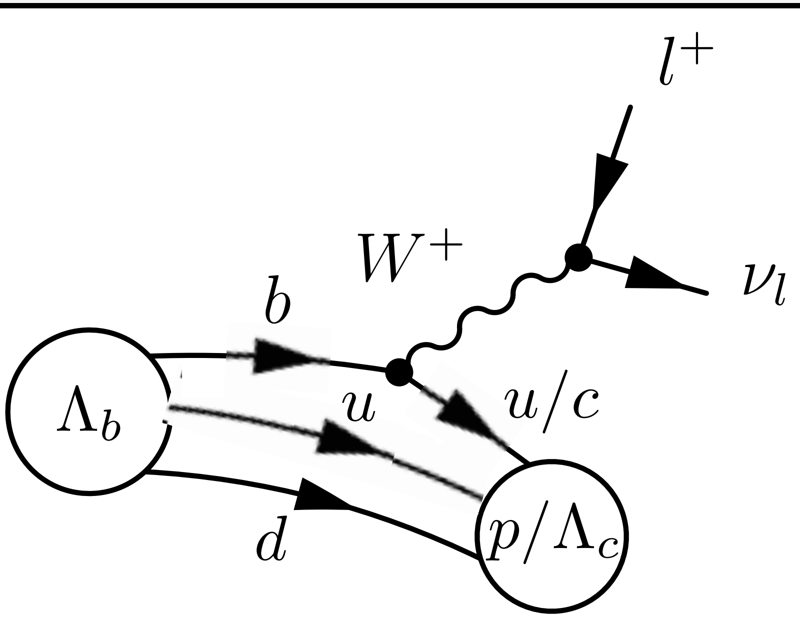
standard observable to study relative τ channel effect:

$$R(D^{(*)}) = \frac{\mathcal{B}(B\to D^{(*)}\tau\nu)}{\mathcal{B}(B\to D^{(*)}\ell\nu)}\,, \qquad \ell = e,\mu$$

Λ_b decays



"helicity-based" FF parametrisation

[Feldmann, Yip, PRD 85 (2012) 014035]

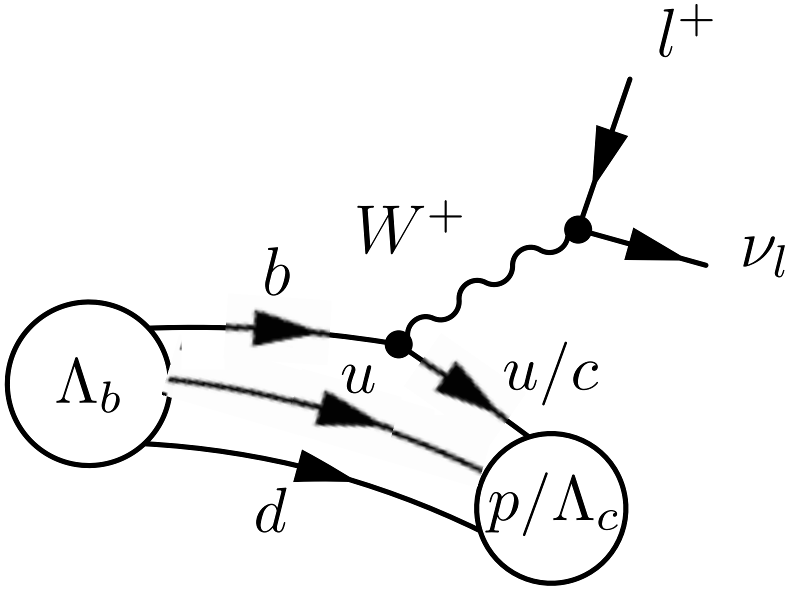
$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} = \frac{G_F^2 |V_{qb}^L|^2 \sqrt{s_+ s_-}}{768\pi^3 m_{\Lambda_b}^3} \left(1 - \frac{m_\ell^2}{q^2}\right)^2$$

$$\times \Bigg\{ 4\left(m_\ell^2 + 2q^2\right) \left(s_+ \left[(1-\epsilon_q^R) g_\perp\right]^2 + s_- \left[(1+\epsilon_q^R) f_\perp\right]^2\right)$$

$$+2\frac{m_\ell^2 + 2q^2}{q^2} \left(s_+ \left[(m_{\Lambda_b} - m_X)(1-\epsilon_q^R) g_+\right]^2 + s_- \left[(m_{\Lambda_b} + m_X)(1+\epsilon_q^R) f_+\right]^2\right)$$

$$+\frac{6m_\ell^2}{q^2} \left(s_+ \left[(m_{\Lambda_b} - m_X)(1+\epsilon_q^R) f_0\right]^2 + s_- \left[(m_{\Lambda_b} + m_X)(1-\epsilon_q^R) g_0\right]^2\right) \Bigg\},$$

Λ_b decays



"helicity-based" FF parametrisation

[Feldmann, Yip, PRD 85 (2012) 014035]

$$\langle X(p',s')|\bar{q}\,\gamma^\mu\, b|\Lambda_b(p,s)\rangle = \bar{u}_X(p',s')\Bigg[f_0(q^2)\,(m_{\Lambda_b}-m_X)\frac{q^\mu}{q^2} + f_+(q^2)\frac{m_{\Lambda_b}+m_X}{s_+}\left(p^\mu+p'^\mu-(m_{\Lambda_b}^2-m_X^2)\frac{q^\mu}{q^2}\right) + f_\perp(q^2)\left(\gamma^\mu-\frac{2m_X}{s_+}p^\mu-\frac{2m_{\Lambda_b}}{s_+}p'^\mu\right)\Bigg]u_{\Lambda_b}(p,s),$$

$$\langle X(p',s')|\bar{q}\,\gamma^\mu\gamma_5\, b|\Lambda_b(p,s)\rangle = -\bar{u}_X(p',s')\,\gamma_5\Bigg[g_0(q^2)\,(m_{\Lambda_b}+m_X)\frac{q^\mu}{q^2} + g_+(q^2)\frac{m_{\Lambda_b}-m_X}{s_-}\left(p^\mu+p'^\mu-(m_{\Lambda_b}^2-m_X^2)\frac{q^\mu}{q^2}\right) + g_\perp(q^2)\left(\gamma^\mu+\frac{2m_X}{s_-}p^\mu-\frac{2m_{\Lambda_b}}{s_-}p'^\mu\right)\Bigg]u_{\Lambda_b}(p,s).$$

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hadronic effects in flavour physics

we are in an era of precision flavour physics, where hadronic effects can be ...

- mostly irrelevant: $\mu \to e\gamma,\ \mathbf{d}_\mathrm{n}$
- under good theoretical control: $K \to \pi\nu\bar{\nu}$
- relevant, difficult, but measured indirectly: $(g-2)_\mu$
- relevant and difficult to compute: $V_{xy},\ K \to \pi\pi,\ \Delta m_{d,s},\ \ldots$

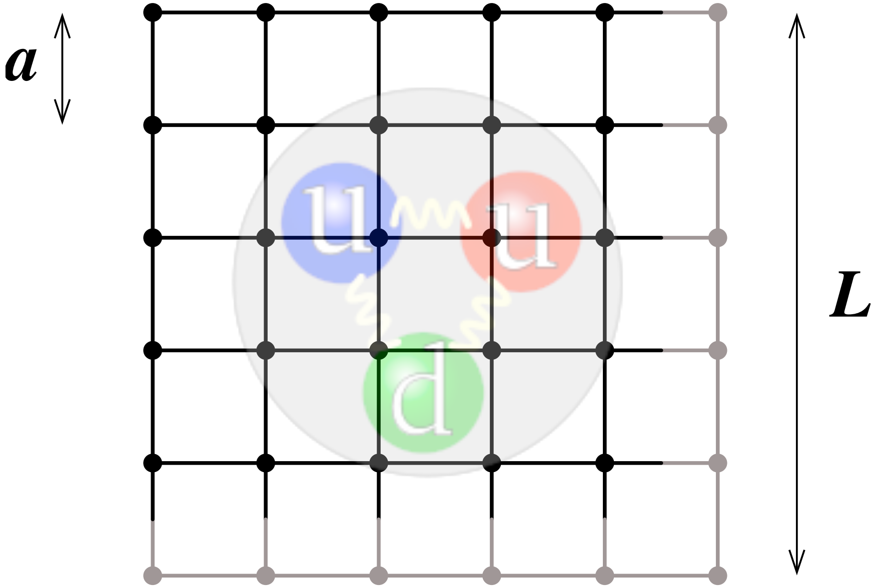
use first-principles technique to deal with low-energy hadronic physics: lattice QCD

(complement with other first-principles/systematic approaches: effective theories, dispersion relations, ...)

lattice QCD

first-principles, systematically improvable approach to strongly coupled quantum field theories

[Wilson 1974]



lattice QCD

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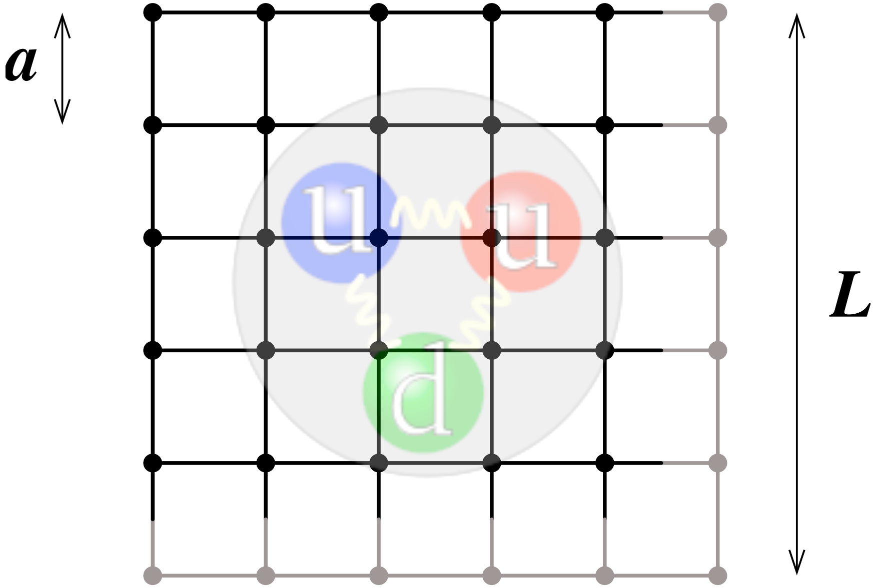
[Wilson 1974]



- take continuum, infinite volume limits
- tune irrelevant couplings to preserve symmetries, improve scaling to CL ...

$$S_{\text{lat}} = S_0 + aS_1 + a^2 S_2 + \dots$$
$$\mathcal{O}_{\text{lat}} = \mathcal{O}_0 + a\mathcal{O}_1 + a^2 \mathcal{O}_2 + \dots$$

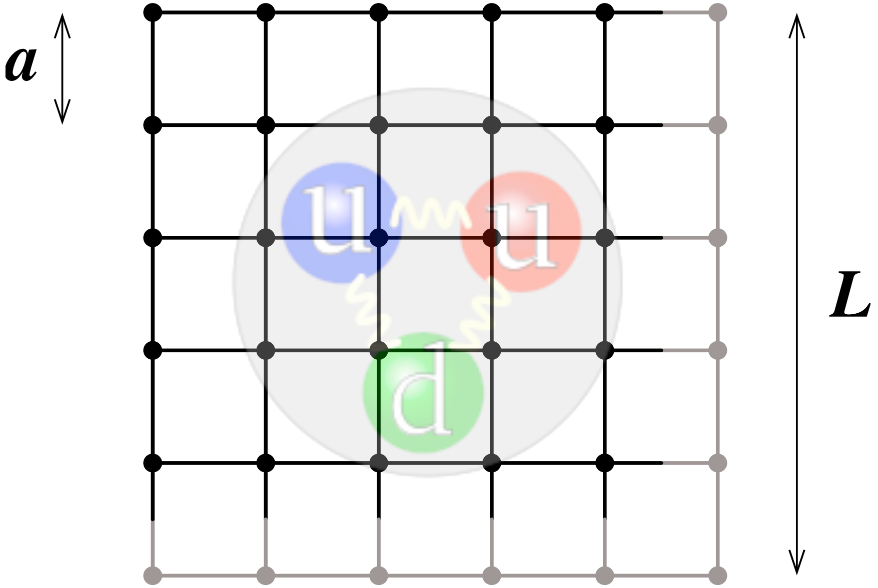
several different lattice actions: universality

fermion actions: (improved) Wilson, (improved) staggered, domain-wall, perfect actions, Neuberger fermions, twisted-mass QCD, ...

lattice QCD

first-principles, systematically improvable approach to strongly coupled quantum field theories

[Wilson 1974]



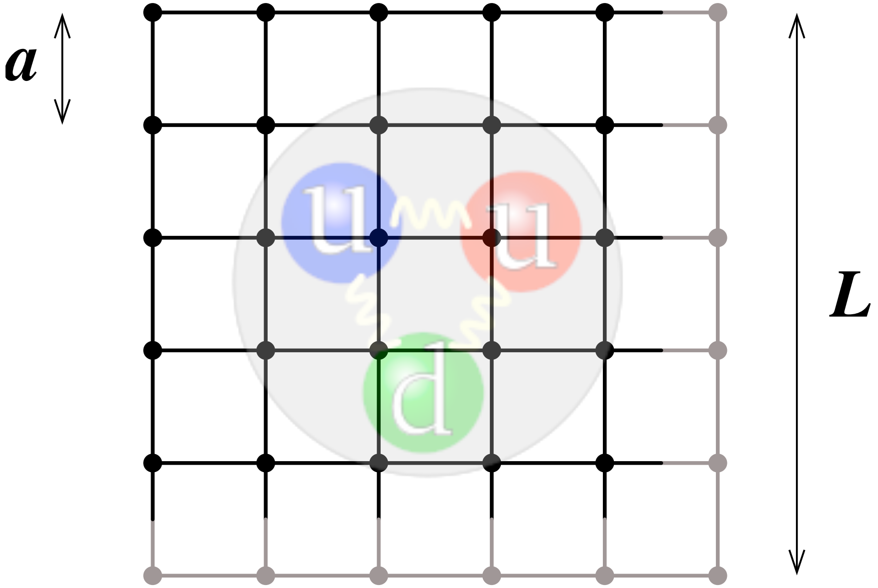
many tools developed along the last 20+ years:

- control scaling (Symanzik improvement)
- non-perturbative renormalisation and matching (e.g. to effective theories)
- lattice regularisations with exact chiral symmetry
- ...

lattice QCD

first-principles, systematically improvable approach to strongly coupled quantum field theories

[Wilson 1974]

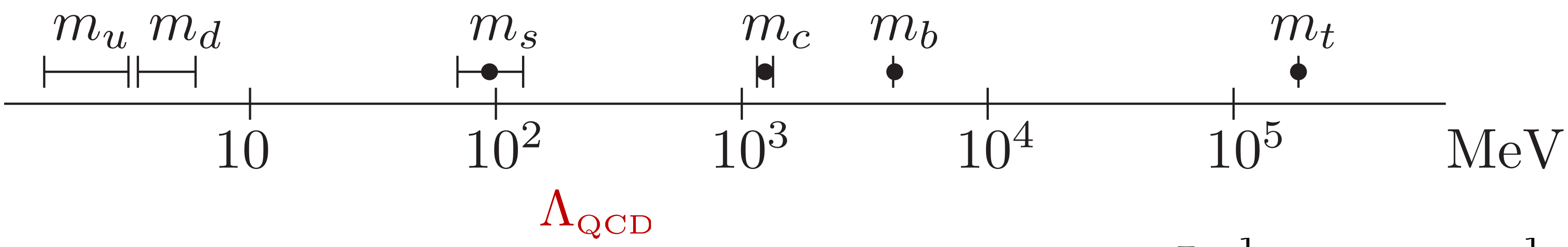


crucial: control systematic uncertainties

- get rid of cutoffs ($a \to 0,\ L \to \infty$)
- compute in / extrapolate to physical SSB regime (light quarks, isospin breaking)
- keep all relevant scales far from cutoffs

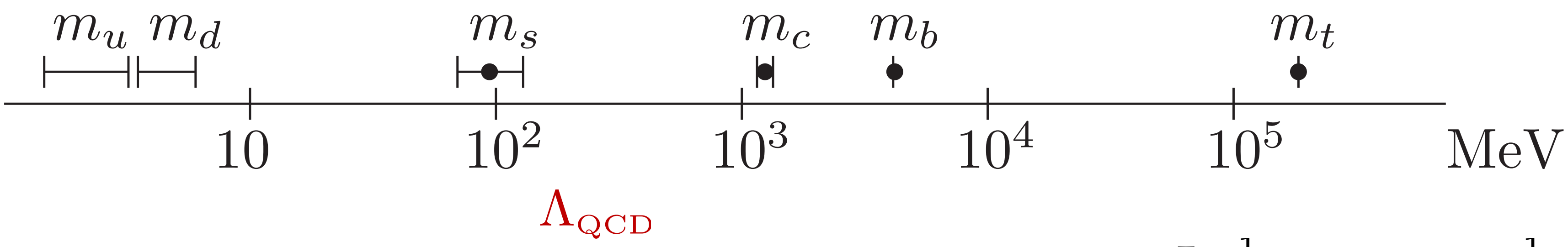
what is the current physics reach of LQCD?

lattice QCD reach: scales and cost



$$L^{-1} \ll \mu \ll a^{-1}$$

lattice QCD reach: scales and cost



$$L^{-1} \ll \mu \ll a^{-1}$$

main cost factor: reiterated inversion of lattice Dirac operator on fixed gauge field

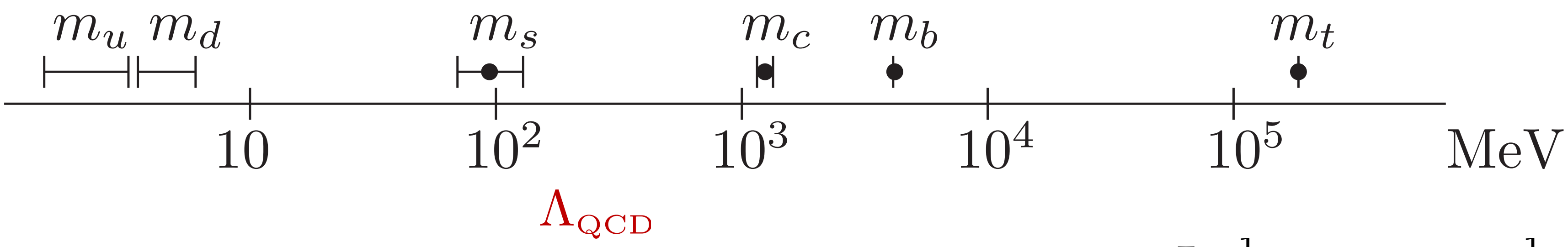
$$\mathrm{cost} \; = \; N \left(\frac{20\ \mathrm{MeV}}{m}\right)^{\alpha} \left(\frac{L}{3\ \mathrm{fm}}\right)^{\beta} \left(\frac{0.1\ \mathrm{fm}}{a}\right)^{\gamma}$$

overall cost (⇒ cpu power)

physics reach

for a long time: serious difficulties in reaching light dynamical quark masses

lattice QCD reach: scales and cost



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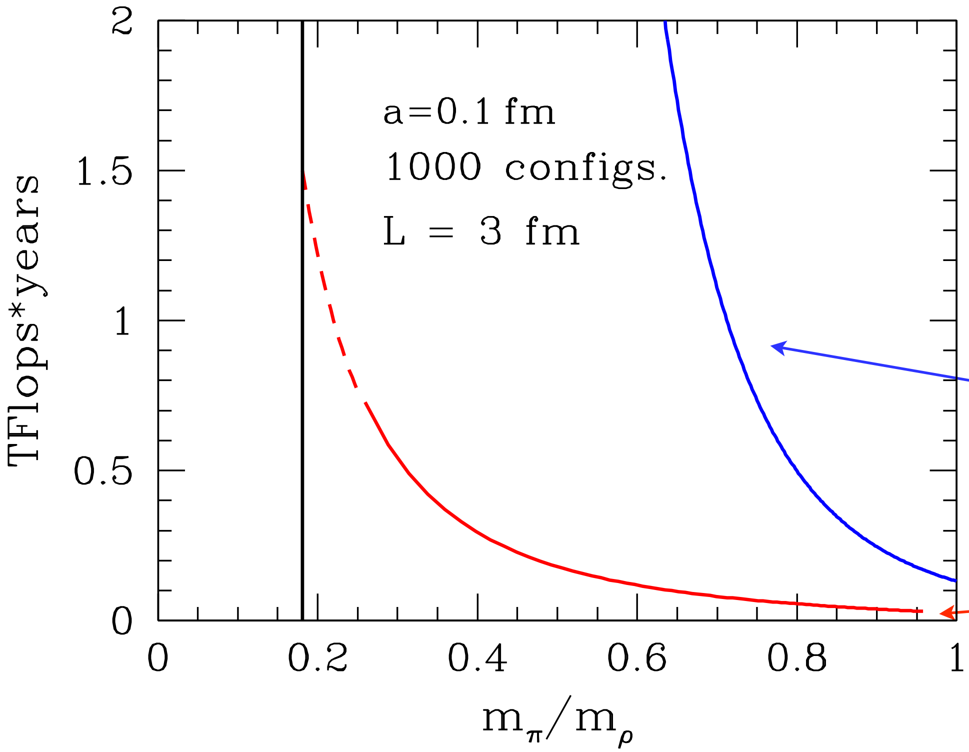


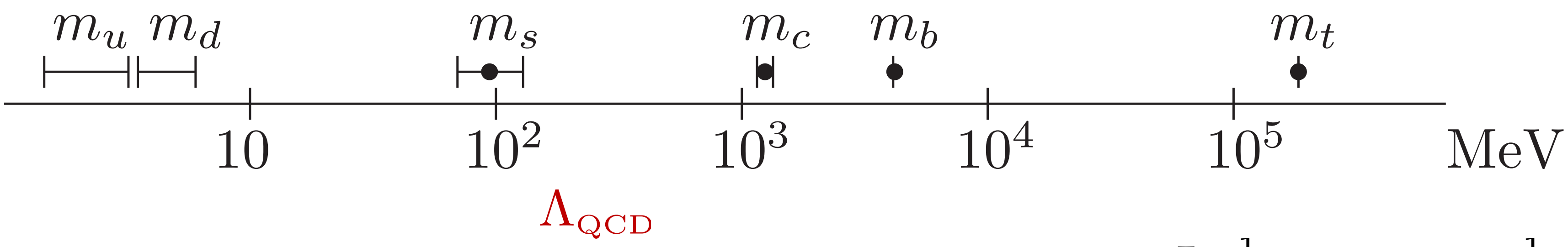
physics "taught" to algorithms

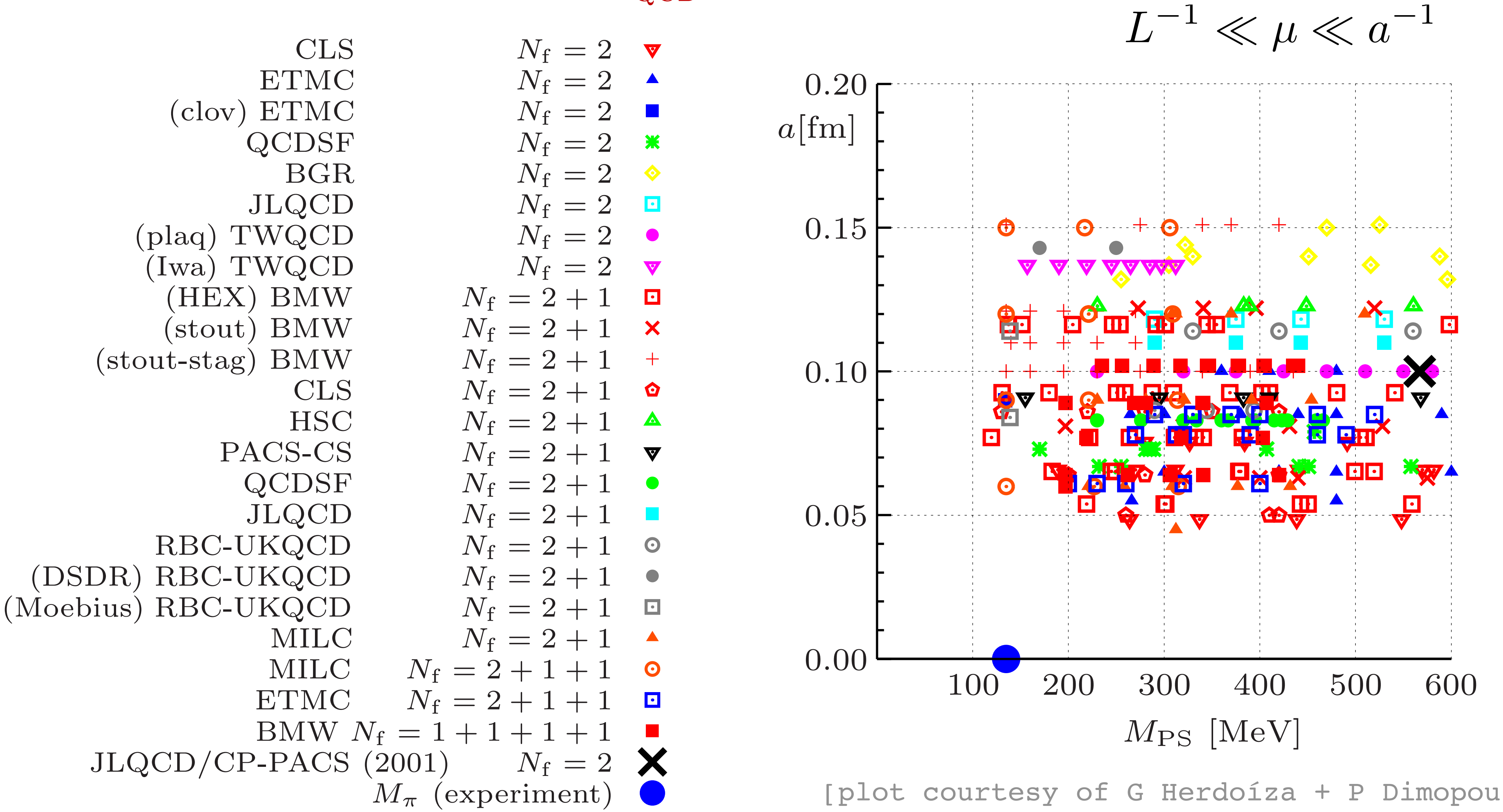
[Sexton-Weingarten 1990s]
[Hasenbusch, Lüscher 2000s]

$$5 \left[\frac{20\ \text{MeV}}{m}\right]^3 \left[\frac{L}{3\ \text{fm}}\right]^5 \left[\frac{0.1\ \text{fm}}{a}\right]^7$$ [Ukawa 2001]

$$0.05 \left[\frac{20\ \text{MeV}}{m}\right]^1 \left[\frac{L}{3\ \text{fm}}\right]^5 \left[\frac{0.1\ \text{fm}}{a}\right]^6$$ [Giusti 2006]

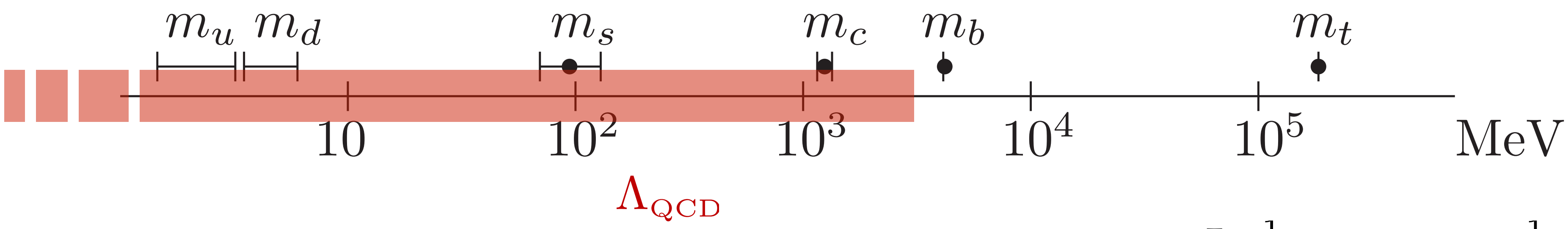
lattice QCD reach: simulation landscape



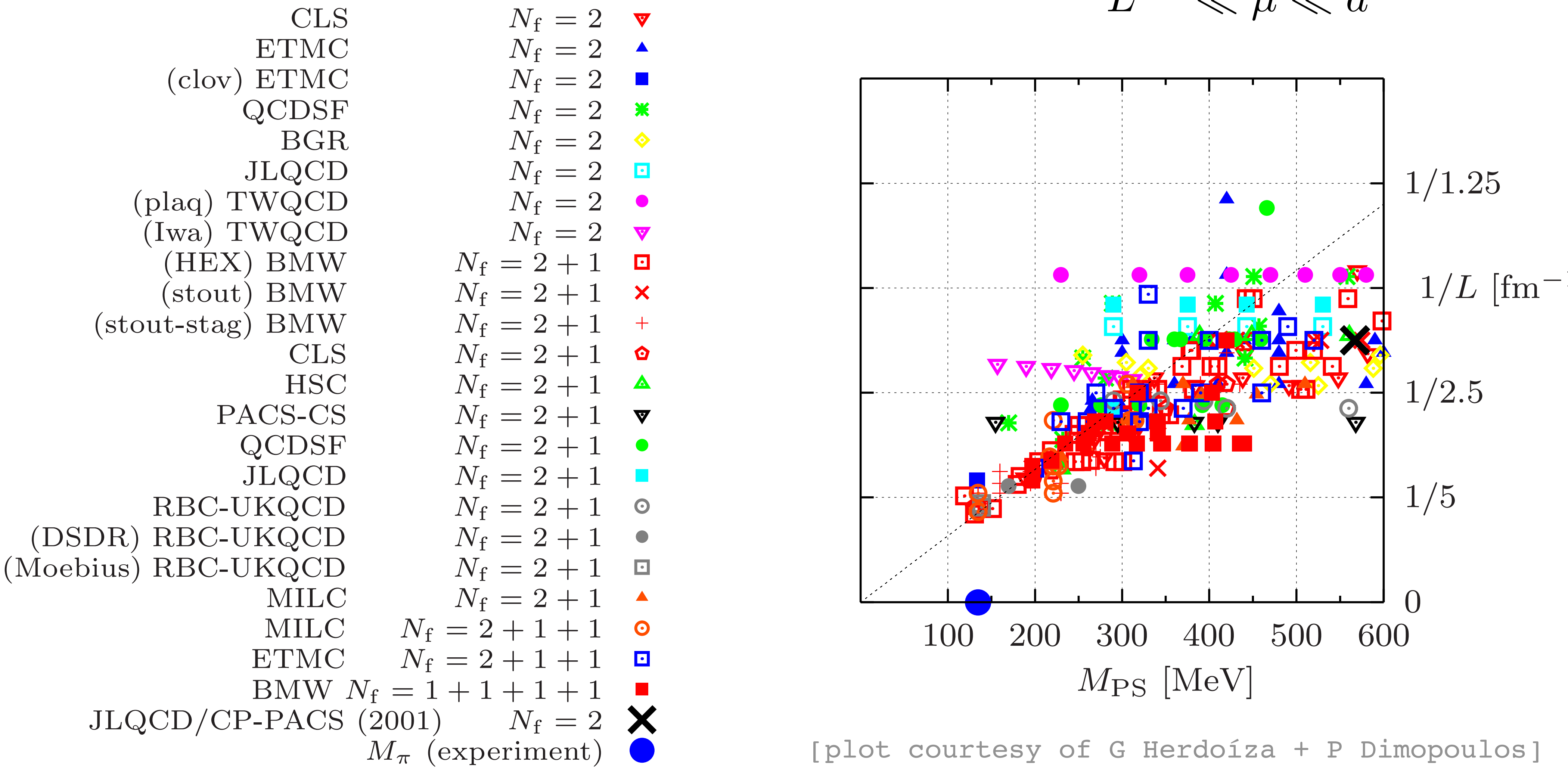


[plot courtesy of G Herdoíza + P Dimopoulos]

lattice QCD reach: simulation landscape



$$L^{-1} \ll \mu \ll a^{-1}$$



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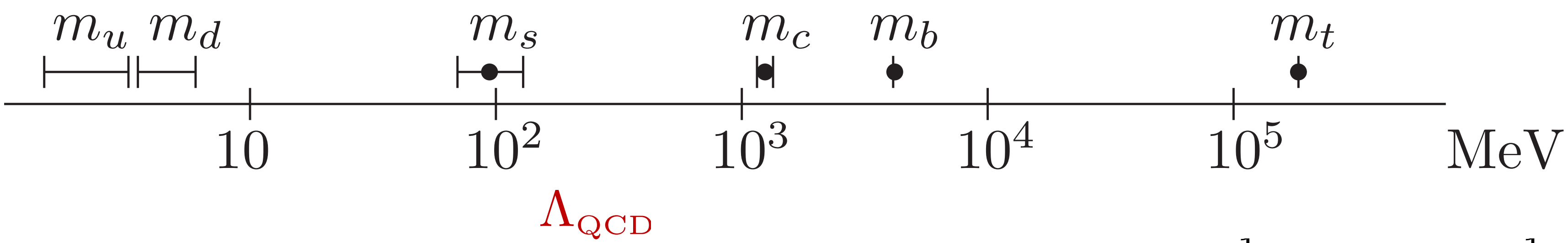

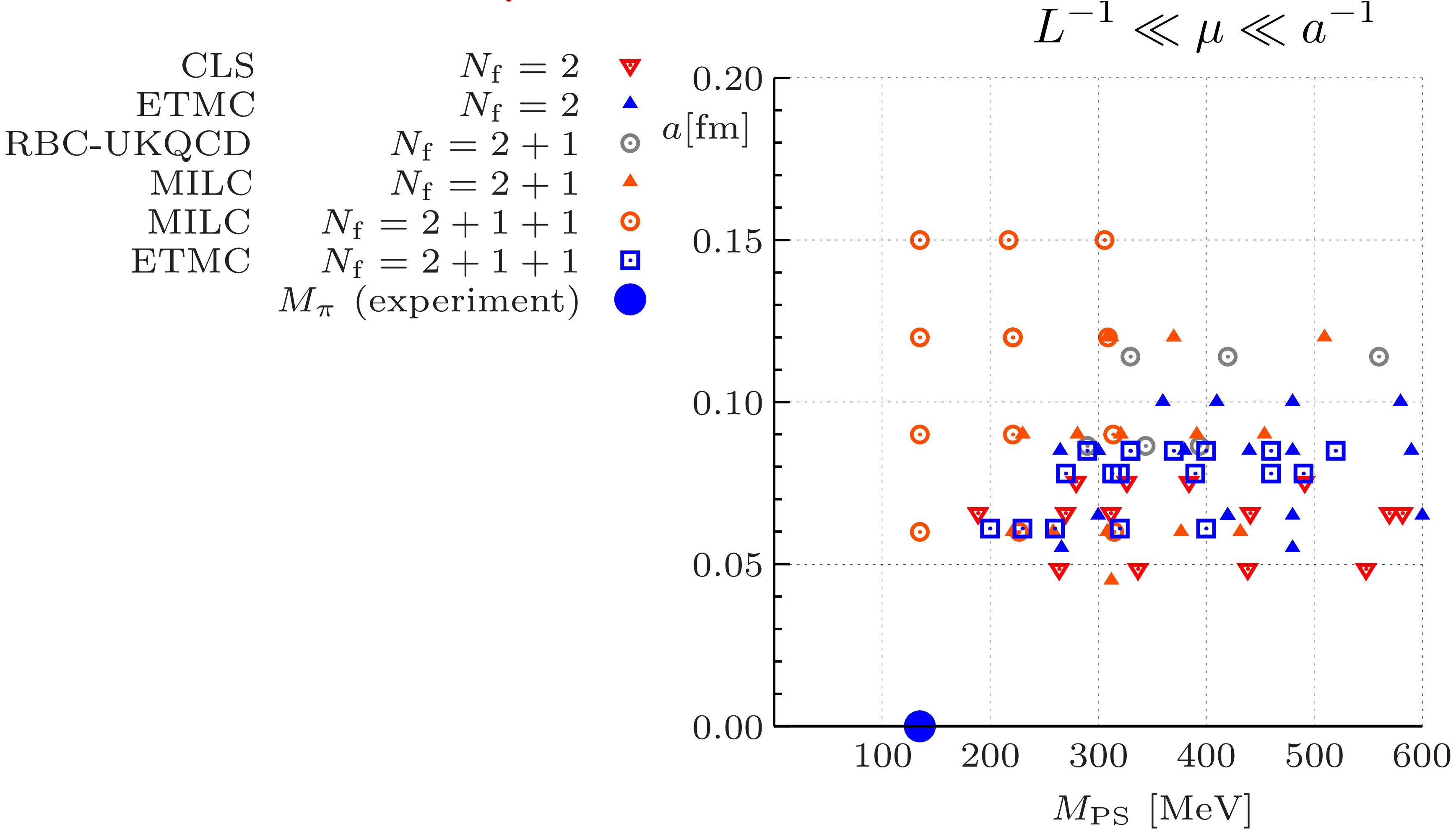

[plot courtesy of G Herdoíza + P Dimopoulos]

Flavour Lattice Averaging Group

FLAG: your one-stop repository of lattice results, world averages / estimates

covers several phenomenologically relevant quantities, big effort to maximise representativity across lattice collaborations / geographical regions

advisory board: S. Aoki, C. Bernard, C. Sachrajda

editorial board: G. Colangelo, H. Leutwyler, A. Vladikas, U. Wenger

working groups:

quark masses	T. Blum, L. Lellouch, V. Lubicz
V_{ud}, V_{us}	A. Jüttner, T. Kaneko, S. Simula
LECs	S. Dürr, H. Fukaya, S. Necco
B_K	J. Laiho, S. Sharpe, H. Wittig
α_s	R. Horsley, T. Onogi, R. Sommer
$f_D, \ f_B, \ B_B$	Y. Aoki, M. Della Morte, A. El Khadra
$D, B \to P\ell\nu \ + \ \text{other}$	E. Lunghi, CP, R. Van de Water

FLAG-2 review published in 2014, includes results up to Nov 2013

[S Aoki et al, Eur Phys J C (2014) 74]

FLAG-3

what FLAG provides (for each quantity):

- complete list of references
- summary of relevant formulae and notation
- summary of essential aspects of each computation, in easily readable colour-coded tables
- averages / estimates (if sensible)
- a “lattice dictionary” for non-experts
- thorough appendix tables with details of all computations

what FLAG begs readers for:

- always quote original references too

FLAG-3

update scheduled for end-2015, extended to include heavy quark masses and BSM matrix elements for ϵ_K

advisory board: S. Aoki, C. Bernard, H. Leutwyler, C. Sachrajda

editorial board: G. Colangelo, S. Hashimoto, A. Jüttner, S. Sharpe, A. Vladikas, U. Wenger

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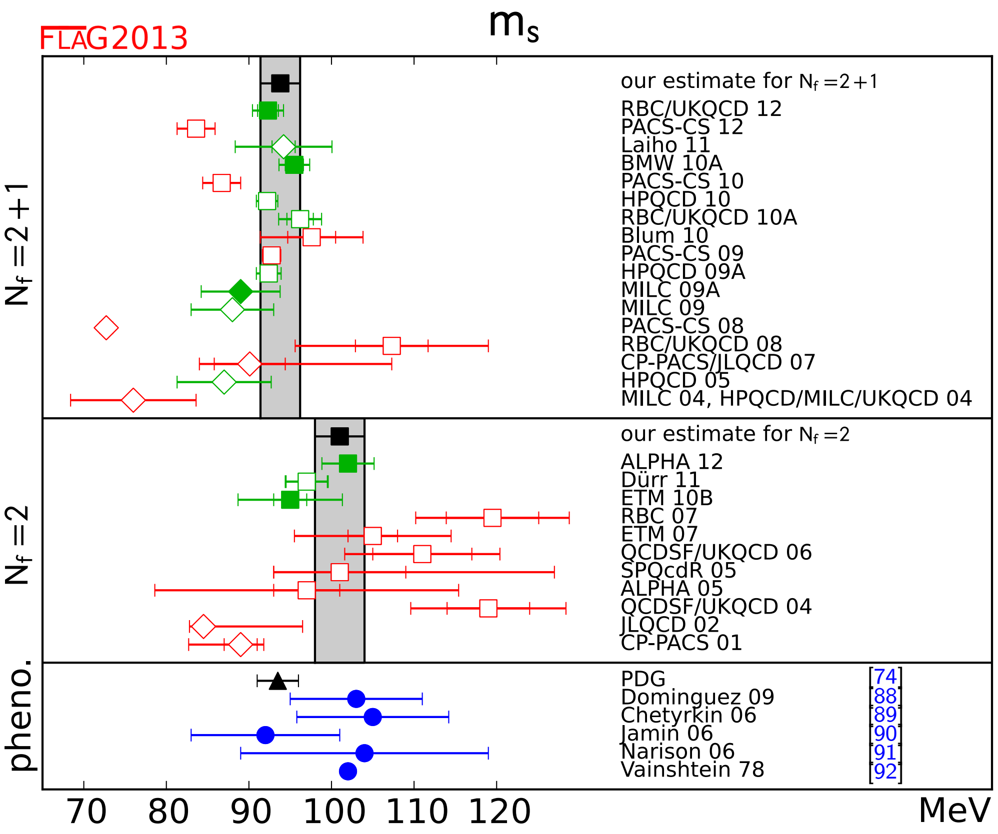
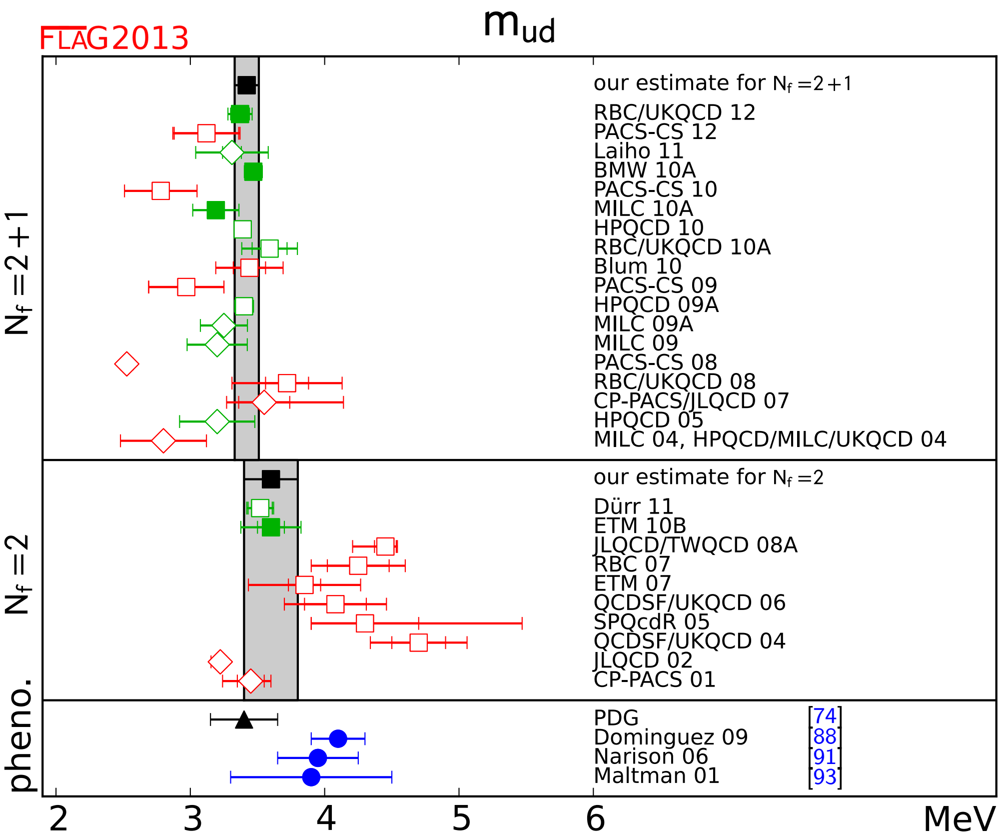
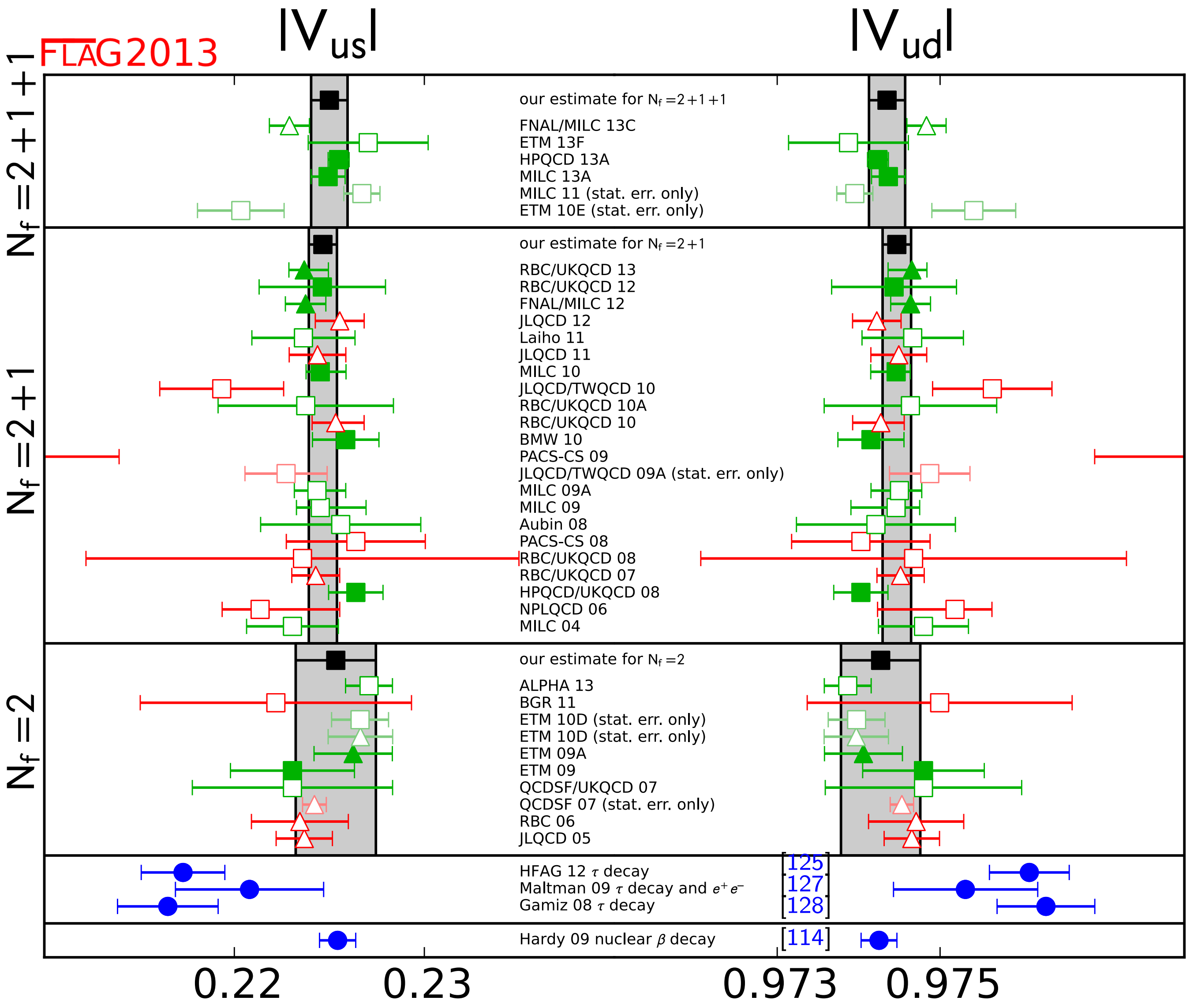
lattice QCD reach: a precision era





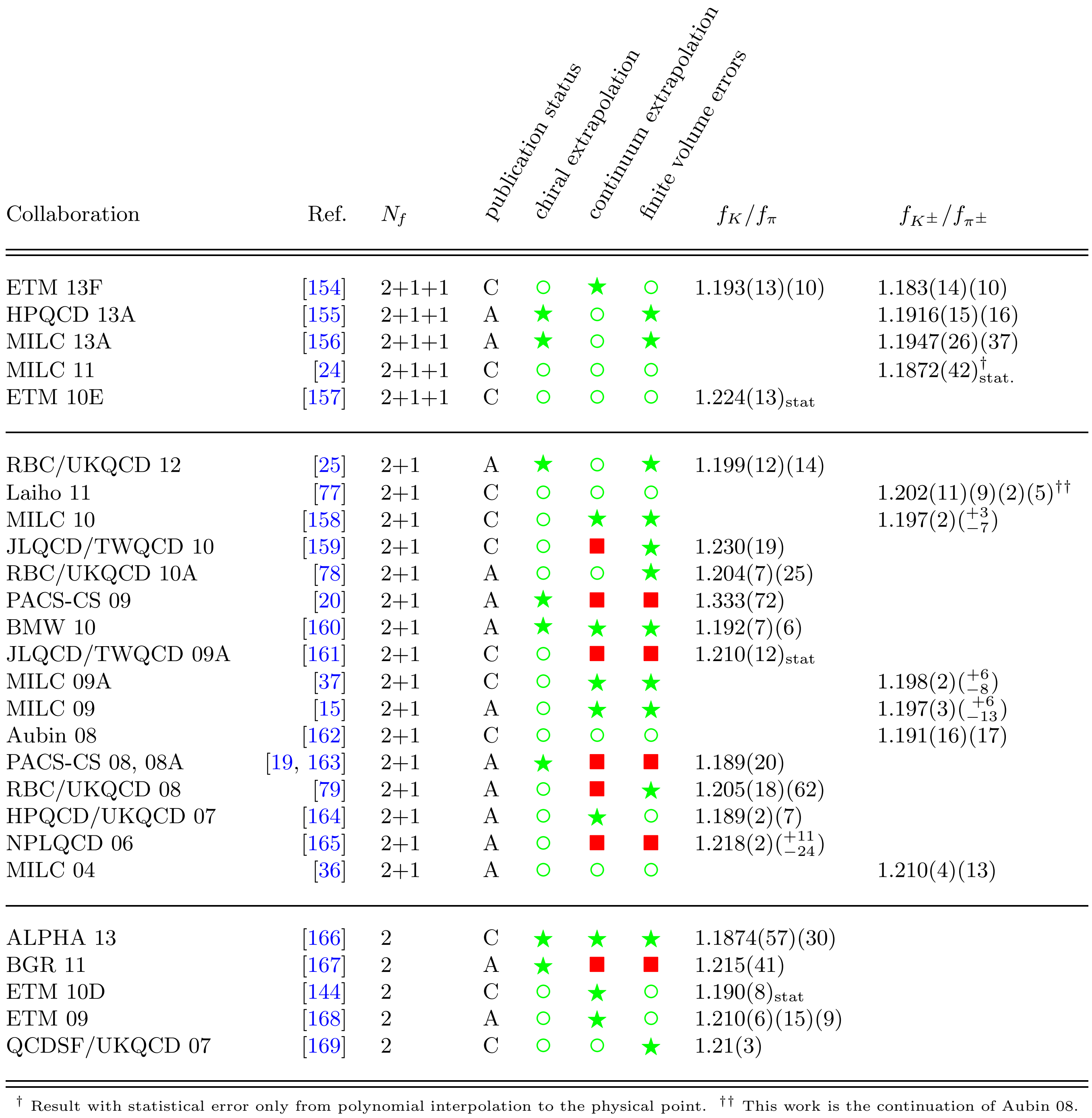
[FLAG 2013]

lattice QCD reach: a precision era



[FLAG 2013]

lattice QCD reach: a precision era



Collaboration	Ref.	N_f	publication status	chiral extrapolation	continuum extrapolation	finite volume errors	f_K/f_π	$f_{K^\pm}/f_{\pi^\pm}$
ETM 13F	[154]	2+1+1	C	○	★	○	1.193(13)(10)	1.183(14)(10)
HPQCD 13A	[155]	2+1+1	A	★	○	★		1.1916(15)(16)
MILC 13A	[156]	2+1+1	A	★	○	★		1.1947(26)(37)
MILC 11	[24]	2+1+1	C	○	○	○		$1.1872(42)^{\dagger}_{\mathrm{stat.}}$
ETM 10E	[157]	2+1+1	C	○	○	○	$1.224(13)_{\mathrm{stat}}$	
RBC/UKQCD 12	[25]	2+1	A	★	○	★	1.199(12)(14)	
Laiho 11	[77]	2+1	C	○	○	○		$1.202(11)(9)(2)(5)^{\dagger\dagger}$
MILC 10	[158]	2+1	C	○	★	★		$1.197(2)(^{+3}_{-7})$
JLQCD/TWQCD 10	[159]	2+1	C	○	■	★	1.230(19)	
RBC/UKQCD 10A	[78]	2+1	A	○	○	★	1.204(7)(25)	
PACS-CS 09	[20]	2+1	A	★	■	■	1.333(72)	
BMW 10	[160]	2+1	A	★	★	★	1.192(7)(6)	
JLQCD/TWQCD 09A	[161]	2+1	C	○	■	■	$1.210(12)_{\mathrm{stat}}$	
MILC 09A	[37]	2+1	C	○	★	★		$1.198(2)(^{+6}_{-8})$
MILC 09	[15]	2+1	A	○	★	★		$1.197(3)(^{+6}_{-13})$
Aubin 08	[162]	2+1	C	○	○	○		1.191(16)(17)
PACS-CS 08, 08A	[19, 163]	2+1	A	★	■	■	1.189(20)	
RBC/UKQCD 08	[79]	2+1	A	○	■	★	1.205(18)(62)	
HPQCD/UKQCD 07	[164]	2+1	A	○	★	○	1.189(2)(7)	
NPLQCD 06	[165]	2+1	A	○	■	■	$1.218(2)(^{+11}_{-24})$	
MILC 04	[36]	2+1	A	○	○	○		1.210(4)(13)
ALPHA 13	[166]	2	C	★	★	★	1.1874(57)(30)	
BGR 11	[167]	2	A	★	■	■	1.215(41)	
ETM 10D	[144]	2	C	○	★	○	$1.190(8)_{\mathrm{stat}}$	
ETM 09	[168]	2	A	○	★	○	1.210(6)(15)(9)	
QCDSF/UKQCD 07	[169]	2	C	○	○	★	1.21(3)	

† Result with statistical error only from polynomial interpolation to the physical point. †† This work is the continuation of Aubin 08.

[FLAG 2013]

lattice QCD reach: data analysis

significant differences in estimates of systematics by different collaborations

MILC: $f_{K^\pm}/f_{\pi^\pm}|_{N_\mathrm{f}=2+1+1} = 1.1947(26)(33)(17)(2)$

[MILC 2013]

HPQCD: $f_{K^\pm}/f_{\pi^\pm}|_{N_\mathrm{f}=2+1+1} = 1.1916(15)(12)(1)(10)$

[HPQCD 2013]

lattice QCD reach: data analysis

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MILC:

[MILC 2013]

$$f_{K^\pm}/f_{\pi^\pm}|_{N_{\rm f}=2+1+1} = 1.1947\overset{\text{stat}}{(26)}\overset{\text{CL}}{(33)}\overset{\text{FV}}{(17)}\overset{\text{e.m.}}{(2)}$$

HPQCD:

[HPQCD 2013]

$$f_{K^\pm}/f_{\pi^\pm}|_{N_{\rm f}=2+1+1} = 1.1916\underset{\text{stat}}{(15)}\underset{\text{CL}}{(12)}\underset{\text{FV}}{(1)}\underset{\text{(misc)}}{(10)}$$

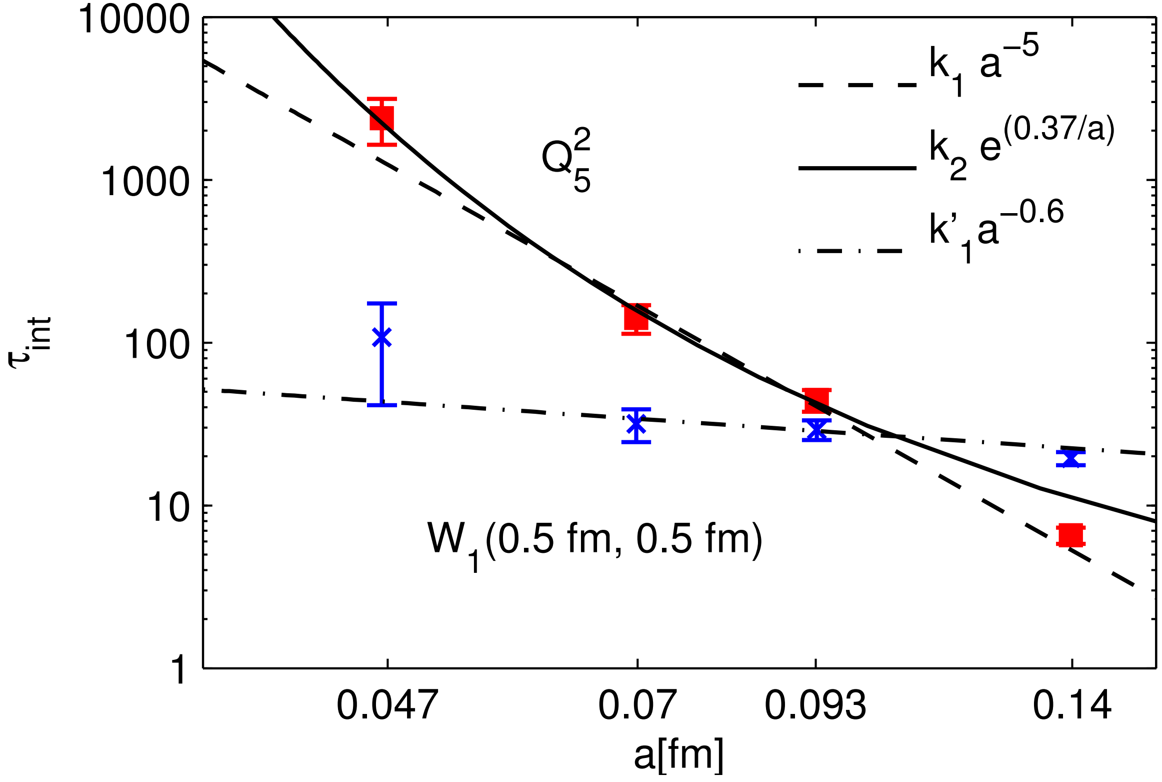
ensembles very similar (HPQCD uses MILC ensembles without finest lattice spacing, has some additional masses)

strong effect of data analysis / fitting strategies

lattice QCD reach: small lattice spacing

HMC algorithm efficiency degrades rapidly below lattice spacings ~0.05 fm ("topology freezing")

[Del Debbio, Panagopoulos, Vicari 2002]
[Schaefer, Sommer, Virotta 2010]



statistical uncertainties may be easily (and severely) underestimated for fine lattice spacings

lattice QCD reach: small lattice spacing

HMC algorithm efficiency degrades rapidly below lattice spacings ~0.05 fm ("topology freezing")

[Del Debbio, Panagopoulos, Vicari 2002]
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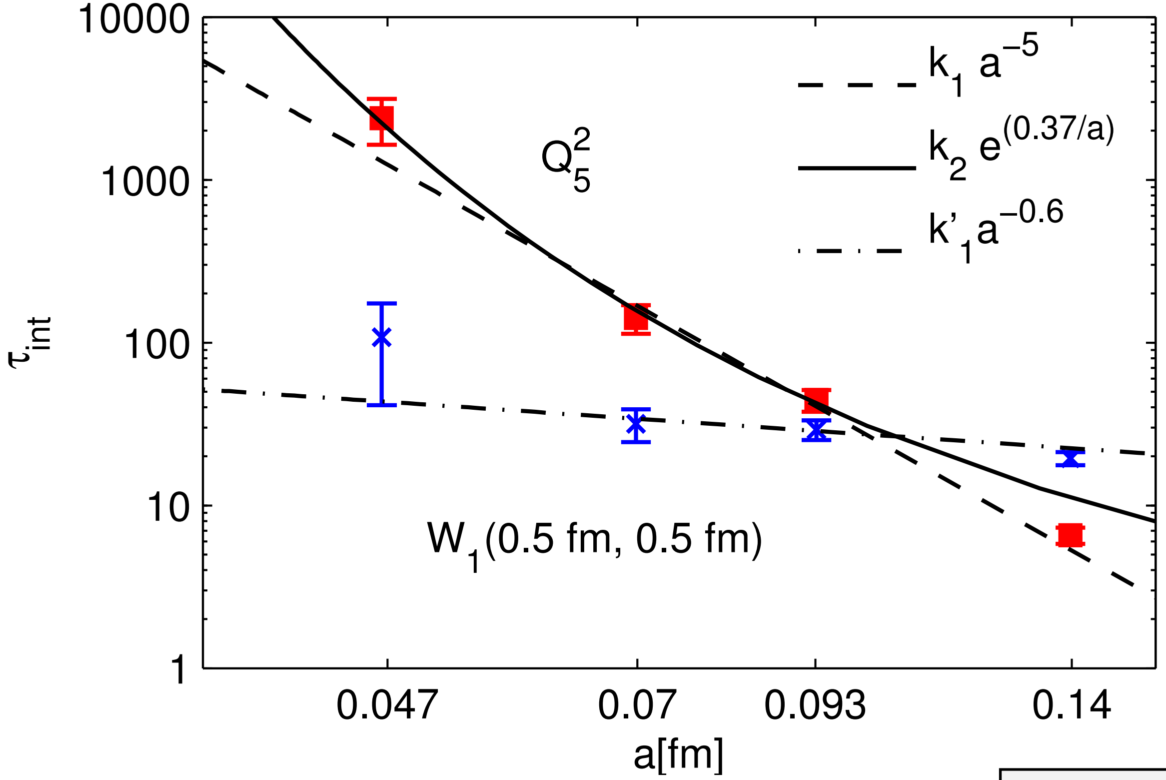


n.b.: $0.05\ \mathrm{fm}\ \times\ 4\ \mathrm{GeV} \approx 1$

work with open boundary conditions?

[Lüscher, Schaefer 2011, CLS N_f=2+1]

issues for B-physics: accessing the *b* scale

the fact that current lattice spacings are below or around the *b* scale means that some form of effective theory has to be (heavily) relied upon to perform B-physics computations on the lattice

issues for B-physics: accessing the *b* scale

- NRQCD: combined expansion in $v^2,\ \Lambda/m_b,\ a$, perturbative matching to QCD
  + easy to carry out to high orders, allows to work at large lattice spacing
  − only works in scaling window $a\Lambda \ll 1,\ m_{\rm h}a \gtrsim 1$, no continuum limit

  [HPQCD]

- npHQET: expansion in $m_{\rm h}^{-1}$, matched non-perturbatively to QCD (using small V)
  + continuum limit exists at any order in the expansion, systematic tool
  − difficult to go beyond $1/m_{\rm h}$ order (⇒ percent systematic uncertainties)

  [ALPHA]

- combined: (smartly) interpolate between charm region and static limit
  + well-controlled systematics in either end
  − systematics associated to true mass dependence not easy to control

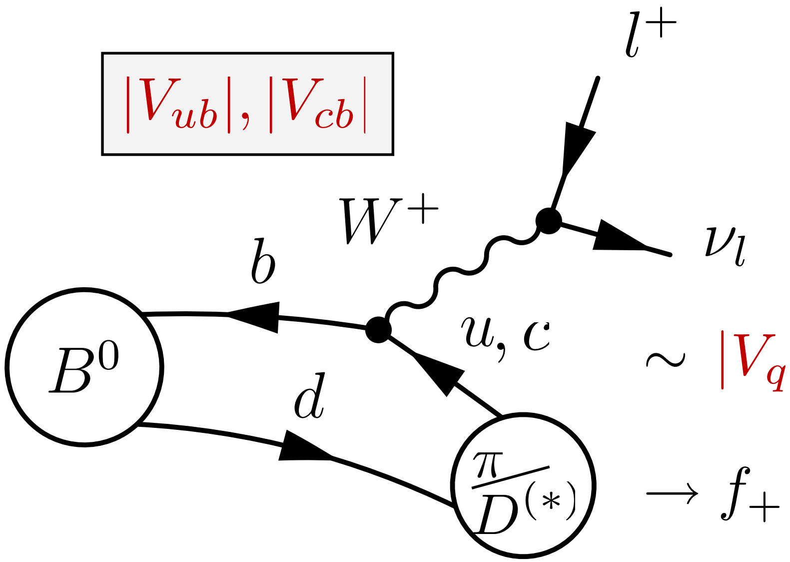
  [ETMC, ALPHA]

- relativistic b-quark: (HQET-inspired) tuning of counterterms to improve scaling
  + easy to carry out to high orders in the O(a) improvement philosophy
  − systematics difficult to test (perturbative matching, true mass dependence)

  [FNAL/MILC, HPQCD]

issues for B-physics: parametrisation of form factors

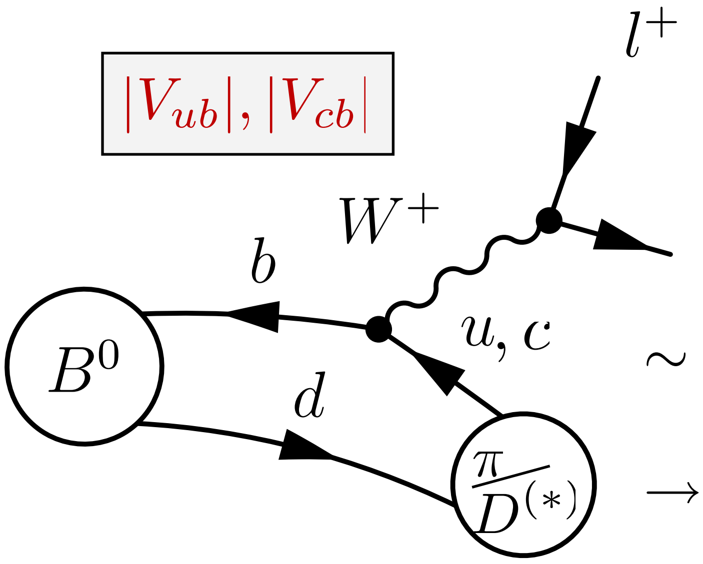
$\boxed{|V_{ub}|, |V_{cb}|}$



$$\sim \; |V_{qb}| \, \langle X | \bar{b} \gamma_\mu q | B^0 \rangle$$

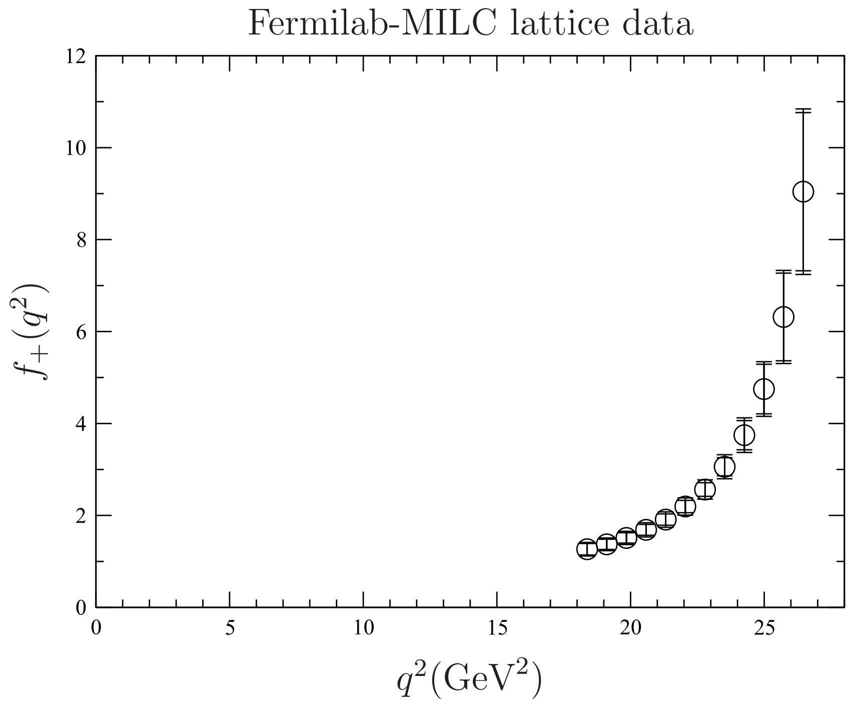
$$\rightarrow f_+(q^2), \;\; f_0(q^2)$$

issues for B-physics: parametrisation of form factors



$$\sim |V_{qb}|\, \langle X | \bar{b}\gamma_\mu q | B^0 \rangle$$

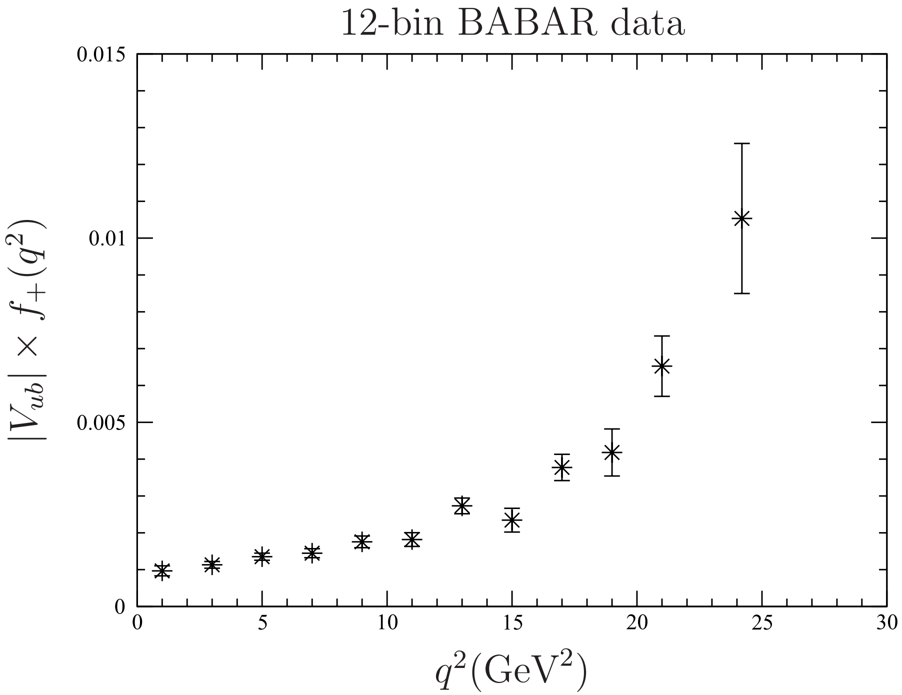
$$\rightarrow f_+(q^2),\ f_0(q^2)$$


Fermilab-MILC lattice data

issue #1

decay into π: lattice QCD limited by discretisation effects to $q^2 \gtrsim 15\ \mathrm{GeV}^2$

experiment most precise in low q^2 region


12-bin BABAR data

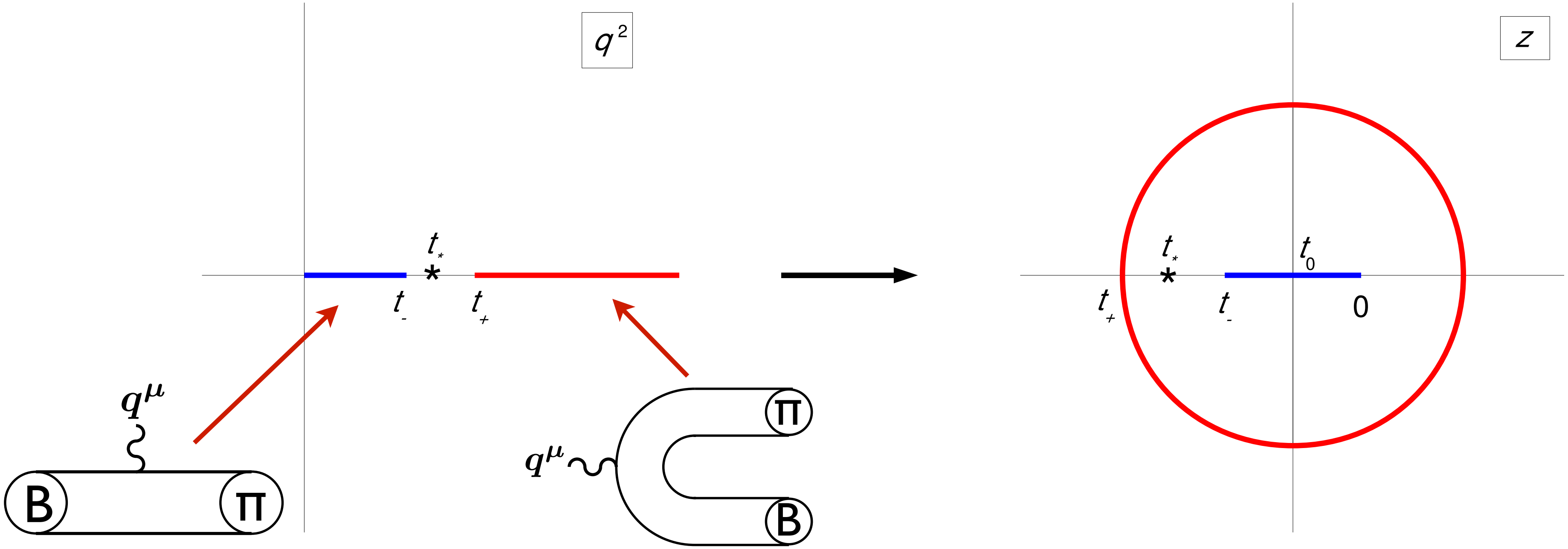
issue #2

lack of fully first-principles ansatz for momentum dependence

issues for B-physics: parametrisation of form factors

simultaneous solution: use dispersion relations, analyticity, unitarity to find well-behaved parametrisation

[Okubo et al. 71; Bourrely et al. 81]
[Boyd, Grinstein, Lebed 95; Bourrely, Caprini, Lellouch 09]
[several other contributions...]



$$z(q^2,t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}} \quad \rightarrow \quad f_+(q^2) = \frac{1}{B(q^2)\phi(q^2,t_0)} \sum_{n\geq 0} a_n(t_0)\, z(q^2,t_0)^n$$

technique adopted by B-factories, HFAG, FLAG

issues for B-physics: parametrisation of form factors

simultaneous solution: use dispersion relations, analyticity, unitarity to find well-behaved parametrisation

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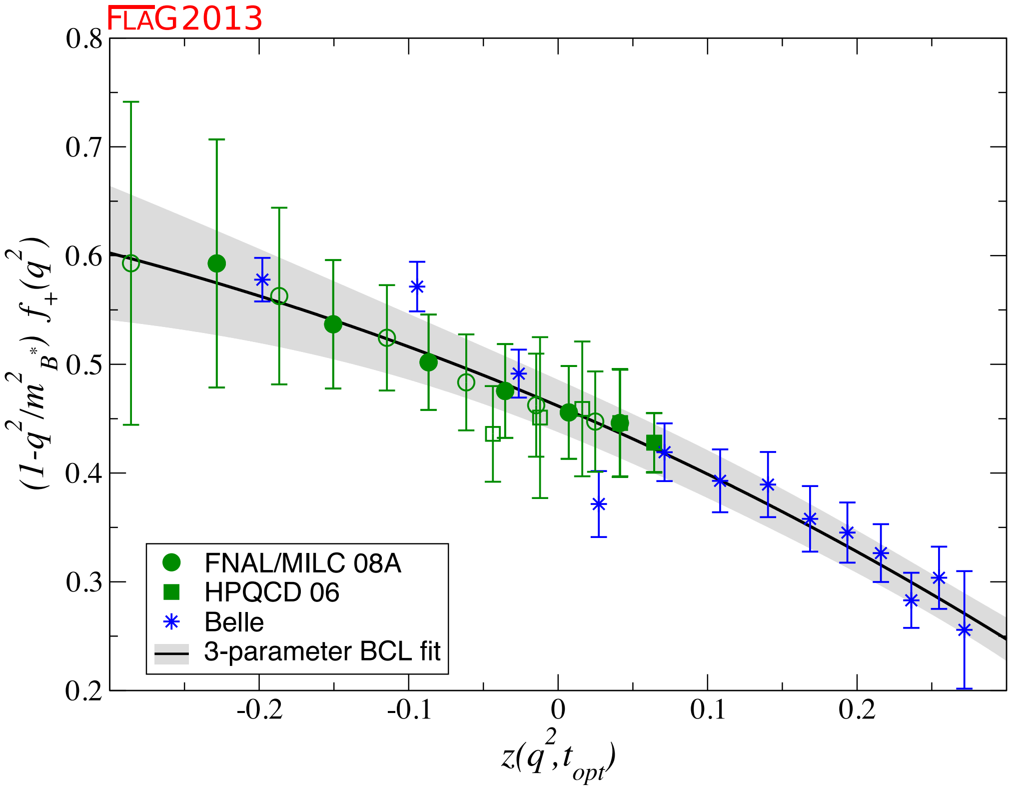

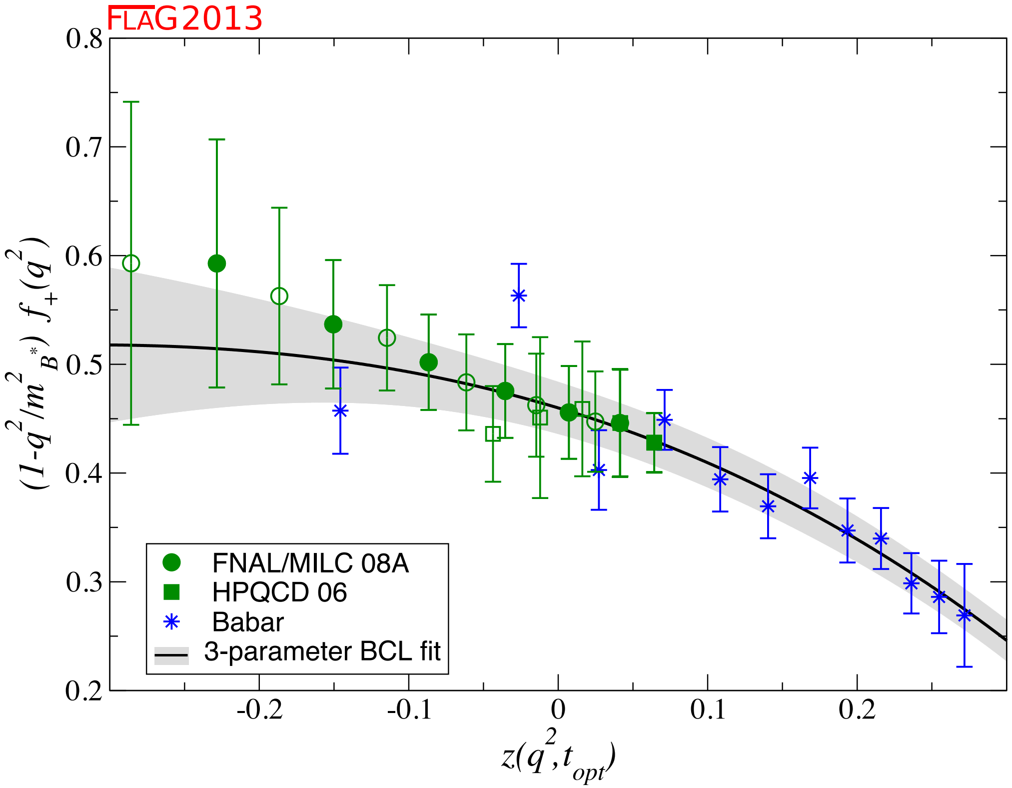

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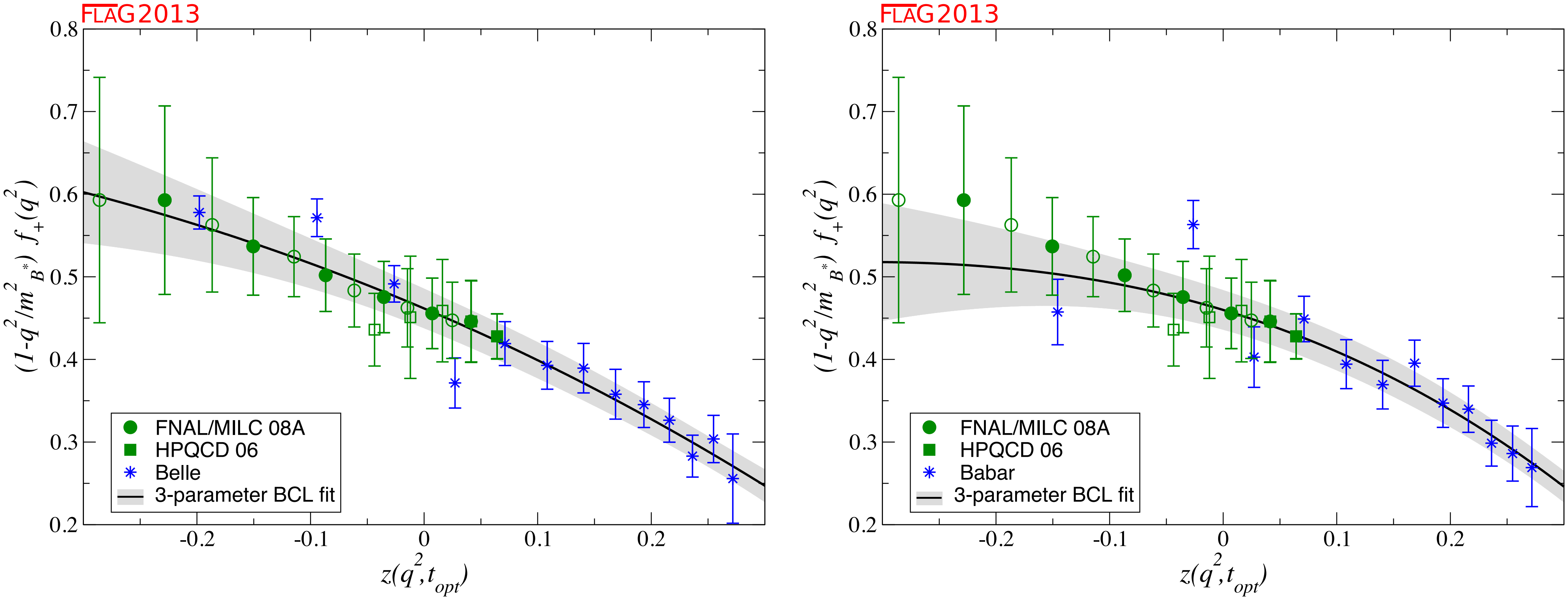

some issues still remain, active discussion

[Bećirević et al, arXiv:1407.1019]

outline

- relevant channels, form factors

- lattice QCD in the precision era
  - reach of lattice simulations
  - summaries of lattice results: FLAG
  - issues for B-physics

- status
  - pre-2014: form factors at zero recoil
  - recent developments: improved precision, q^2 dependence, baryon channels

- outlook

results at zero recoil

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	heavy-quark treatment	form factor	
FNAL/MILC 13B	[446]	2+1	C^{∇}	★	○	★	○	✓	$\mathcal{F}^{B\to D^*}(1)$	0.906(4)(12)
FNAL/MILC 10	[443]	2+1	$C^{\S}$	★	○	★	○	✓	$\mathcal{F}^{B\to D^*}(1)$	0.9017(51)(87)(83)(89)(30)(33)
FNAL/MILC 08	[444]	2+1	A	★	○	★	○	✓	$\mathcal{F}^{B\to D^*}(1)$	0.921(13)(8)(8)(14)(6)(3)(4)
FNAL/MILC 13B	[446]	2+1	C	★	○	★	○	✓	$\mathcal{G}^{B\to D}(1)$	1.081(25)
FNAL/MILC 04A	[445]	2+1	C	■	■	○*	○†	✓	$\mathcal{G}^{B\to D}(1)$	1.074(18)(16)
FNAL/MILC 12A	[452]	2+1	A	○	○	★	○	✓	$R(D)$	0.316(12)(7)
Atoui 13	[448]	2	P	★	★	★	—	✓	$\mathcal{G}^{B\to D}(1)$	1.033(95)
Atoui 13	[448]	2	P	★	★	★	—	✓	$\mathcal{G}^{B_s\to D_s}(1)$	1.052(46)

FNAL/MILC 13B: proceedings, full B→D* published in Bailey et al, PRD 89 (2014) 114504
FNAL/MILC 12A: PRL 109 (2012) 071802
Atoui 13: Eur.Phys.J. C74 (2014) 5, 2861

results at zero recoil

	Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	heavy-quark treatment	form factor	
∗	FNAL/MILC 13B	[446]	2+1	C^{∇}	★	○	★	○	✓	$\mathcal{F}^{B\to D^*}(1)$	0.906(4)(12)
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	FNAL/MILC 04A	[445]	2+1	C	■	■	$○^*$	$○^\dagger$	✓	$\mathcal{G}^{B\to D}(1)$	1.074(18)(16)
	FNAL/MILC 12A	[452]	2+1	A	○	○	★	○	✓	$R(D)$	0.316(12)(7)
	Atoui 13	[448]	2	P	★	★	★	—	✓	$\mathcal{G}^{B\to D}(1)$	1.033(95)
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FNAL/MILC 13B: proceedings, full B→D* published in Bailey et al, PRD 89 (2014) 114504
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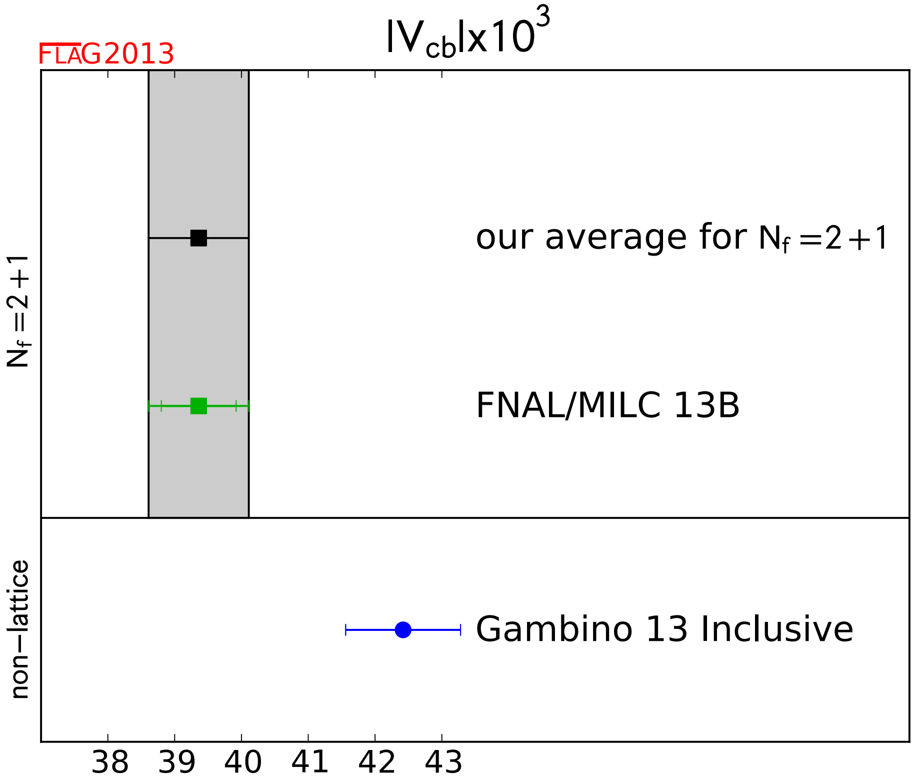
Atoui 13: Eur.Phys.J. C74 (2014) 5, 2861

results at zero recoil

	Ref.	from	$\lvert V_{cb}\rvert \times 10^3$
our average for $N_f = 2+1$	[443]	$B \to D^*\ell\nu$	39.36(56)(50)
Inclusive (Gambino 13)	[465]	$B \to X_c\ell\nu$	42.42(86)



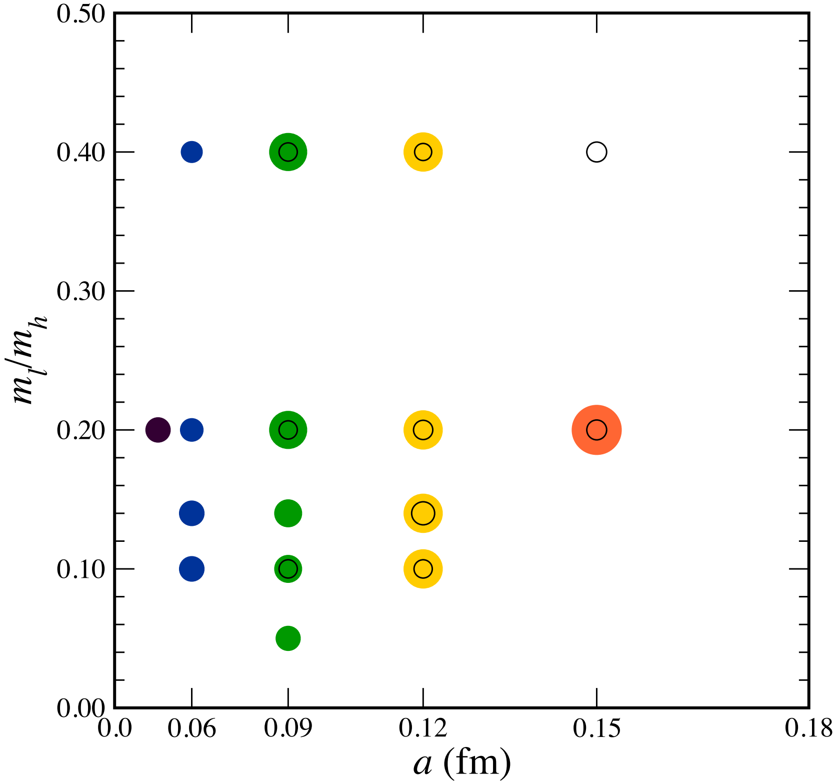
FNAL/MILC 13B: proceedings, full B→D* published in Bailey et al, PRD 89 (2014) 114504
FNAL/MILC 12A: PRL 109 (2012) 071802
Atoui 13: Eur.Phys.J. C74 (2014) 5, 2861

FNAL/MILC results for B→D*

[Bailey et al, PRD 89 (2014) 114504]

$$\boxed{\mathcal{F}^{B\to D^*}(1) = 0.906(4)_{\text{stat}}(12)_{\text{sys}}}$$

- N_f=2+1 rooted staggered sea quarks
- Fermilab heavy quarks
- several lattice spacings (finest 0.045 fm) and (not very) light masses
- mostly non-perturbative renormalisation



FNAL/MILC results for B→D*

[Bailey et al, PRD 89 (2014) 114504]

$$\boxed{\mathcal{F}^{B\to D^*}(1) = 0.906(4)_{\mathrm{stat}}(12)_{\mathrm{sys}}}$$

Uncertainty	$h_{A_1}(1)$
Statistics	0.4%
Scale (r_1) error	0.1%
χPT fits	0.5%
$g_{D^*D\pi}$	0.3%
Discretization errors	1.0%
Perturbation theory	0.4%
Isospin	0.1%
Total	1.4%

- N_f=2+1 rooted staggered sea quarks
- Fermilab heavy quarks
- several lattice spacings (finest 0.045 fm) and (not very) light masses
- mostly non-perturbative renormalisation

Mode	$10^3\lvert V_{cb}\rvert\lvert\bar{\eta}_{\mathrm{EW}}\rvert\mathcal{F}(1)$	Ref.	$\lvert\bar{\eta}_{\mathrm{EW}}\rvert$	$10^3\lvert V_{cb}\rvert$
B^0	35.60 ± 0.57	[81]	1.0182 ± 0.0016	$38.59 \pm 0.62_{\mathrm{expt}} \pm 0.52_{\mathrm{QCD}} \pm 0.06_{\mathrm{QED}}$
$B^\pm$	35.14 ± 1.45	BaBar [82]	1.0066 ± 0.0016	$38.53 \pm 1.60_{\mathrm{expt}} \pm 0.52_{\mathrm{QCD}} \pm 0.06_{\mathrm{QED}}$
Both	40.00 ± 2.04	CLEO [83]	1.0124 ± 0.0058	$43.61 \pm 2.22_{\mathrm{expt}} \pm 0.59_{\mathrm{QCD}} \pm 0.25_{\mathrm{QED}}$
Both	35.83 ± 1.12	BaBar [84]	1.0124 ± 0.0058	$39.06 \pm 1.22_{\mathrm{expt}} \pm 0.53_{\mathrm{QCD}} \pm 0.22_{\mathrm{QED}}$
Both	35.90 ± 0.45	HFAG [76]	1.015 ± 0.005	$39.04 \pm 0.49_{\mathrm{expt}} \pm 0.53_{\mathrm{QCD}} \pm 0.19_{\mathrm{QED}}$

Atoui et al. results for $B_{(s)} \to D_{(s)}$

[Atoui, Bećirević, Morénas, Sanfilippo, Eur.Phys.J. C74 (2014) 5, 2861]

- N_f=2 maximally tmQCD sea quarks
- ETMC-like ratio method for heavy sector
- four lattice spacings, various masses
- no renormalisation required
- cover up to *w*=1.062
- precision not competitive, mostly due to small statistics

$$\mathcal{G}^{B\to D}(1) = 1.033(95)$$

$$\mathcal{G}^{B_s\to D_s}(1) = 1.052(46)$$

N.B. 1: relative error size (mostly due to chiral extrapolations: $m_\pi \gtrsim 270\ \mathrm{MeV}$, which are comparable to those of FNAL/MILC)

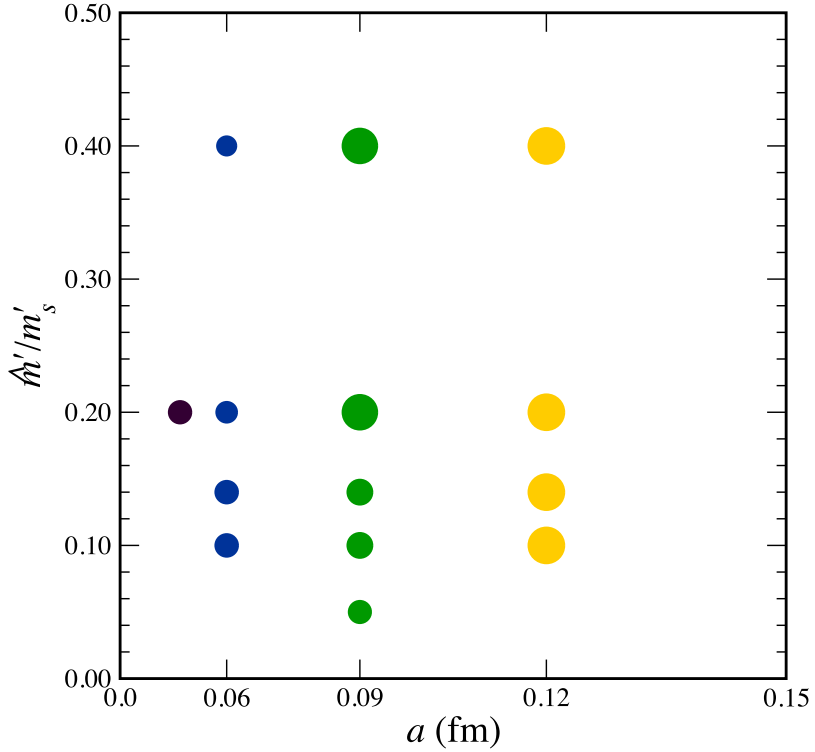
N.B. 2: related ETMC study has explored feasibility of $B \to D^{**}$ computation

[ETMC, Atoui et al, arXiv:1312.2914]

FNAL/MILC results for B→D

[Bailey et al, arXiv:1503.07237]

- N_f=2+1 rooted staggered sea quarks
- Fermilab heavy quarks
- several lattice spacings (finest 0.045 fm) and (not very) light masses
- mostly non-perturbative renormalisation
- explore non-zero recoil and fit FF



$$\frac{\langle D(p_D)|\mathcal{V}^\mu|B(p_B)\rangle}{\sqrt{M_B M_D}} = h_+(w)(v+v')^\mu + h_-(w)(v-v')^\mu$$

$$f_+(q^2) = \frac{1}{2\sqrt{r}}\left[(1+r)h_+(w) - (1-r)h_-(w)\right]$$

$$f_0(q^2) = \sqrt{r}\left[\frac{w+1}{1+r}h_+(w) - \frac{w-1}{1-r}h_-(w)\right]$$

$$\mathcal{G}(w) = h_+(w) - \left(\frac{1-r}{1+r}\right)h_-(w)$$

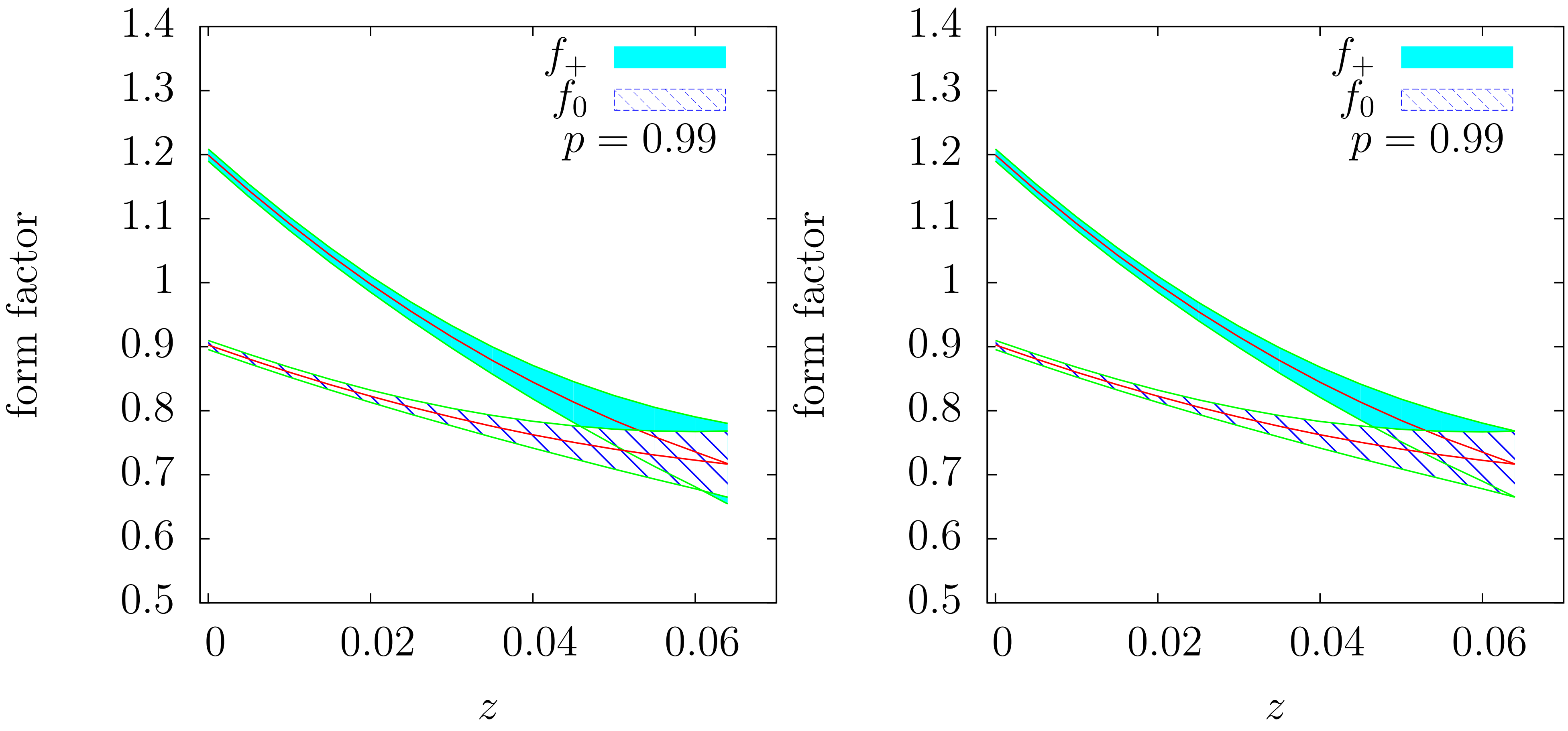
$$r = M_D/M_B = 0.354$$

FNAL/MILC results for B→D

[Bailey et al, arXiv:1503.07237]

cubic fits to BGL parametrisation $$z(w) = \frac{\sqrt{1+w}-\sqrt{2}}{\sqrt{1+w}+\sqrt{2}}$$

$$w_{\mathrm{latt}} \in [1.00, 1.16]\,, \qquad w_{\mathrm{exp}} \in [1.00, 1.58]$$

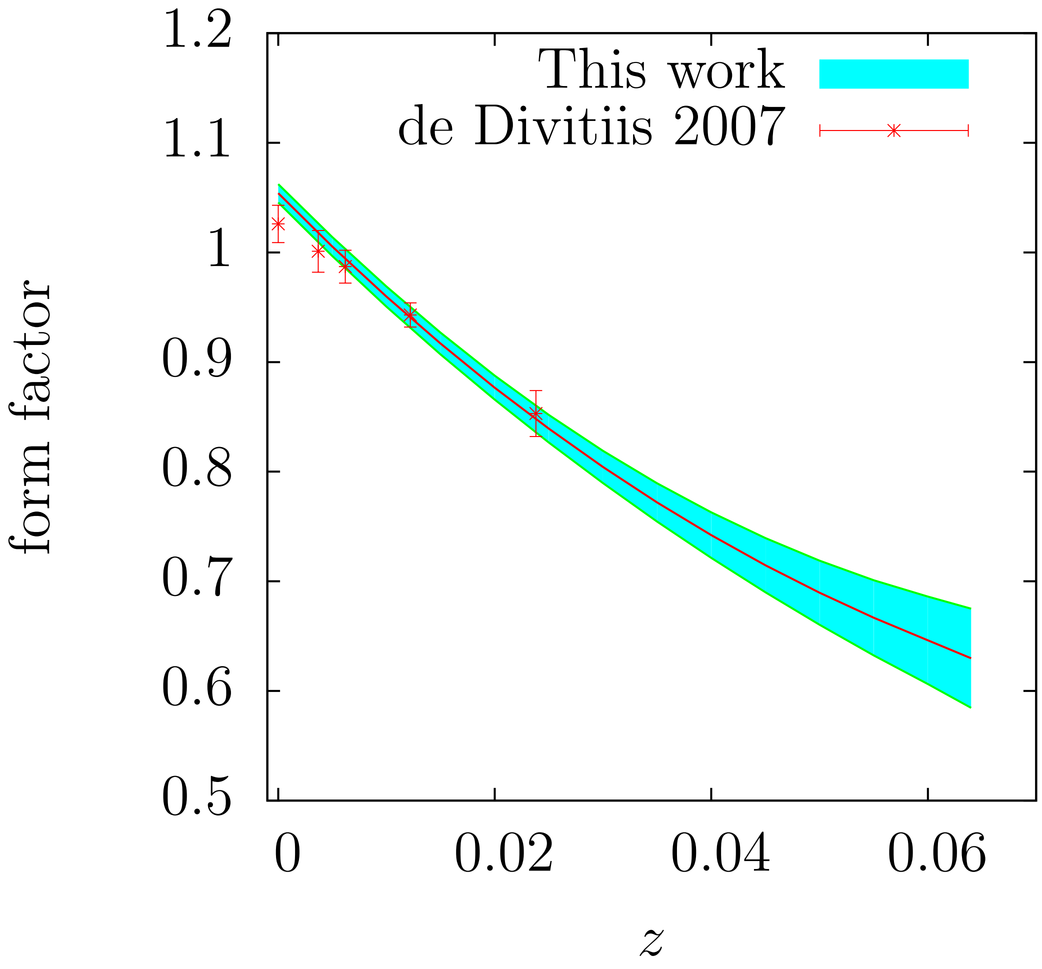


FNAL/MILC results for B→D

[Bailey et al, arXiv:1503.07237]

comparison to quenched results for $\mathcal{G}(w)$

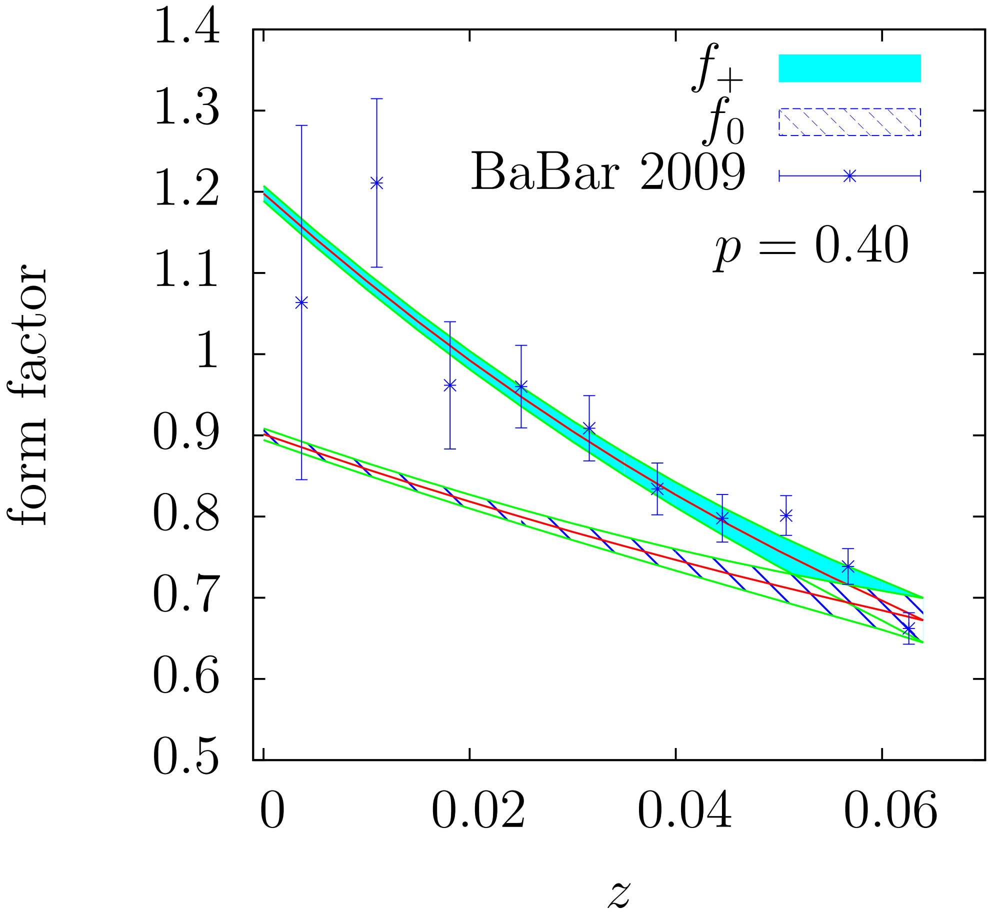
[De Divitiis, Molinaro, Petronzio, Tantalo, PLB 655 (2007) 45]



FNAL/MILC results for B→D

[Bailey et al, arXiv:1503.07237]

joint fit with BaBar data

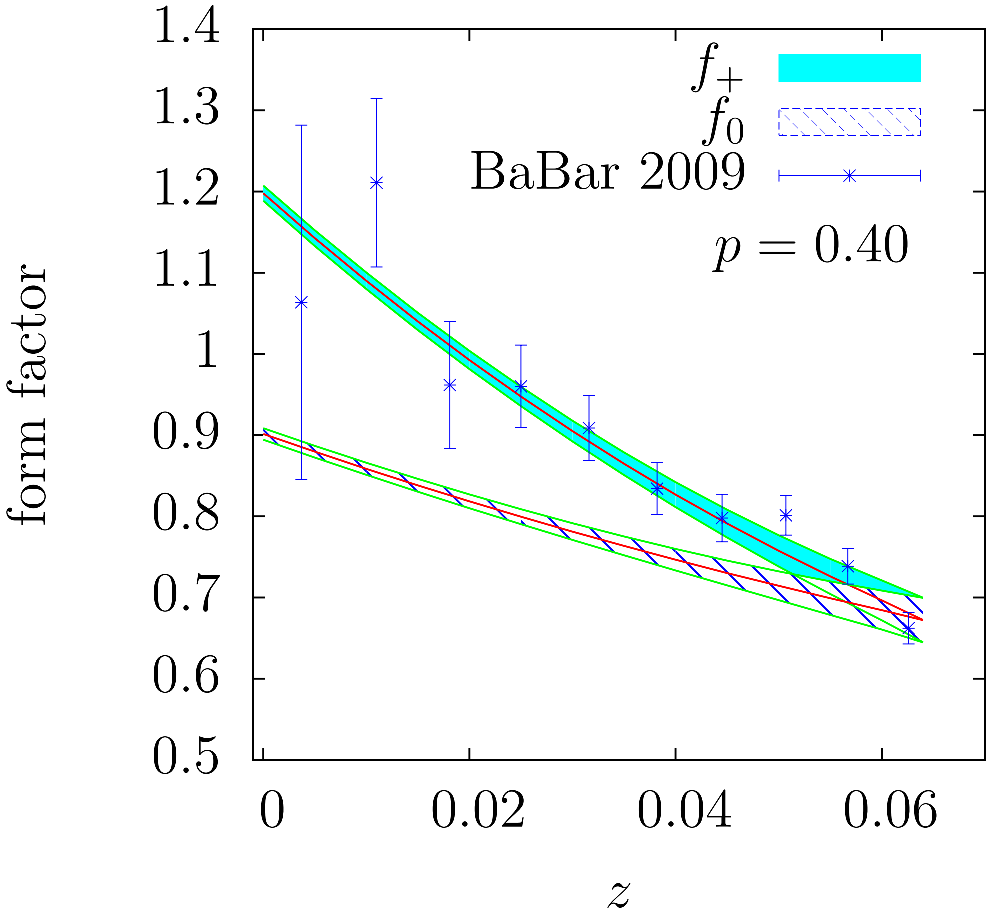


$$\Rightarrow \quad \boxed{|V_{cb}| = (39.6 \pm 1.7_{\text{QCD+exp}} \pm 0.2_{\text{QED}}) \times 10^{-3}}$$

FNAL/MILC results for B→D

[Bailey et al, arXiv:1503.07237]

joint fit with BaBar data



bonus:

$$\boxed{R(D) = \frac{\mathcal{B}(B \to D\tau\nu)}{\mathcal{B}(B \to D\ell\nu)} = 0.299(11)}$$

HPQCD results for B→D

[unpublished, preliminary results provided by Heechang Na]

- N_f=2+1 rooted staggered sea quarks
- NRQCD bottom, HISQ charm and light valence quarks
- two lattice spacings (0.12 fm and 0.09 fm)
- one-loop matching of currents
- explore non-zero recoil and fit FF

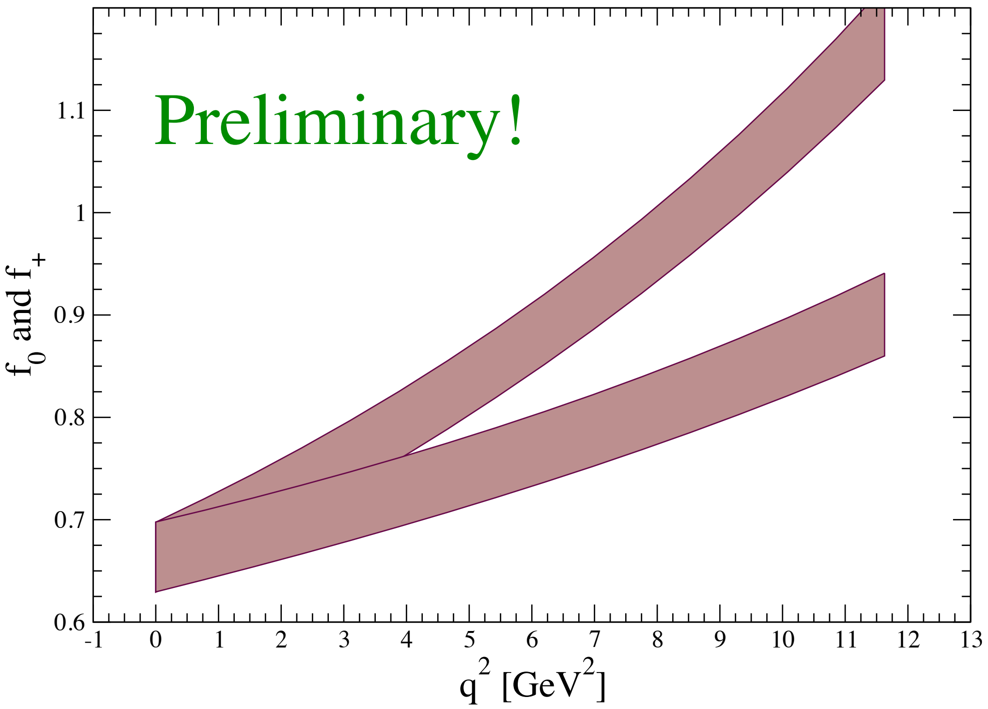
$$\boxed{|V_{cb}| = 40.2(1.7)_{\rm lat}(1.3)_{\rm exp} \times 10^{-3}}$$

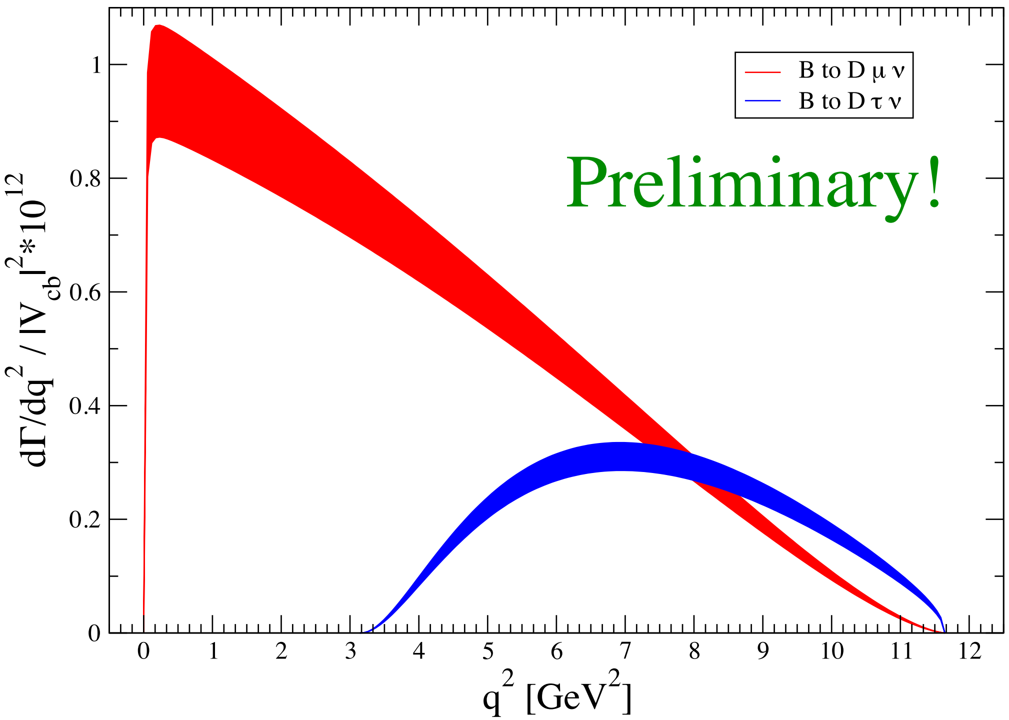
Preliminary!

bonus: $\boxed{R(D) = 0.300(8)}$

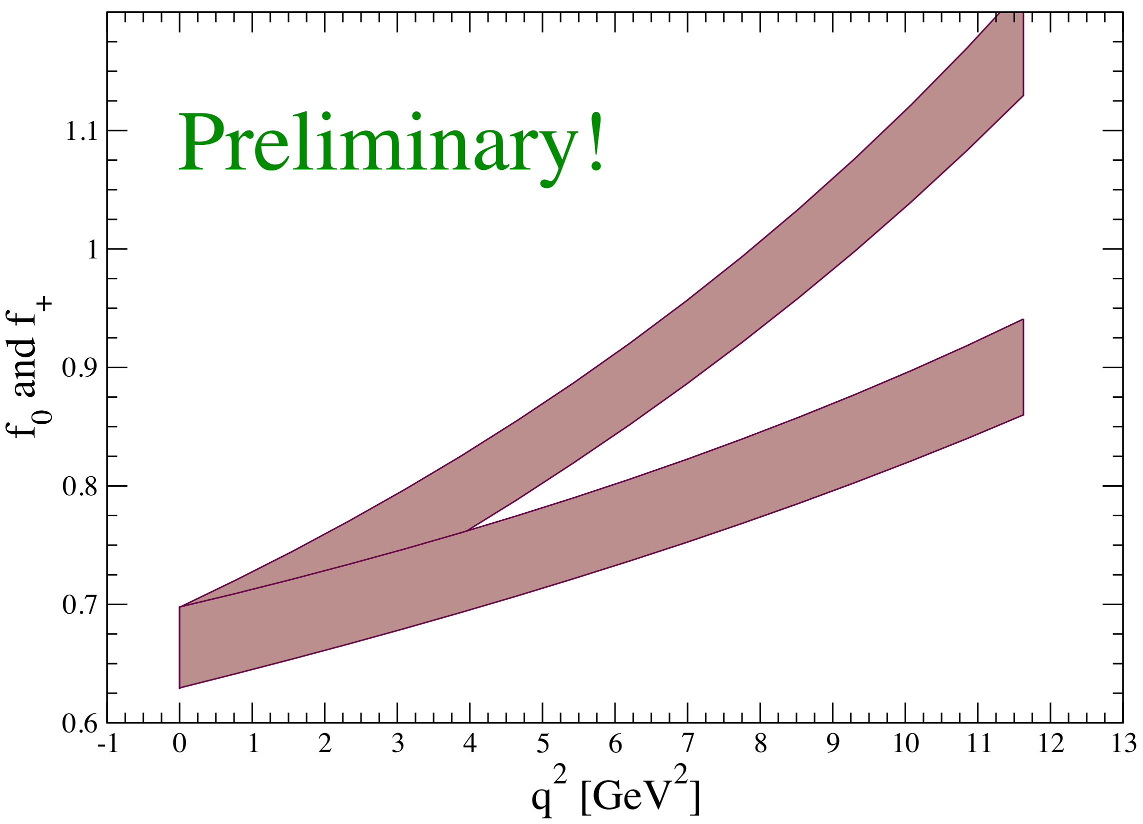
HPQCD results for B→D

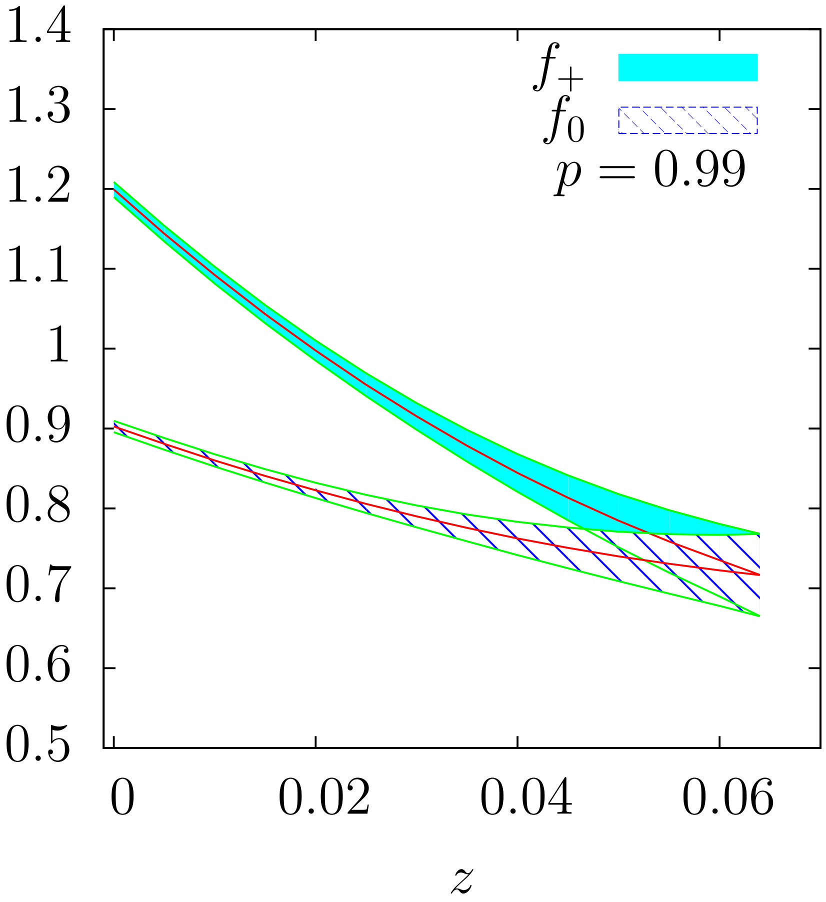
[unpublished, preliminary results provided by Heechang Na]





FFs for B→D: HPQCD vs FNAL/MILC

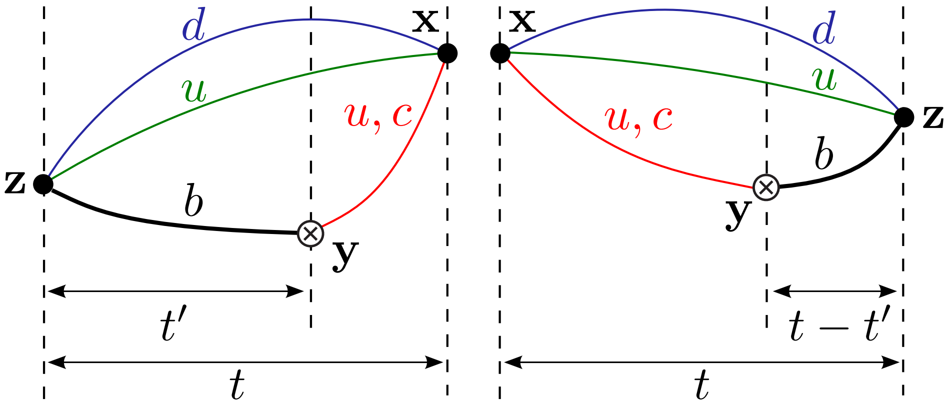


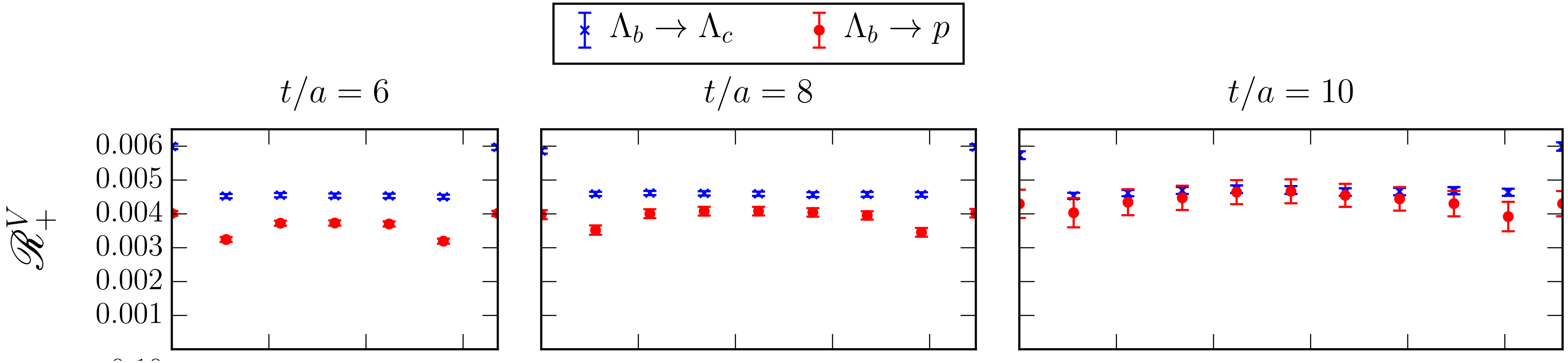


Λ_b decays

[Detmold, Lehner, Meinel, arXiv:1503.01421]

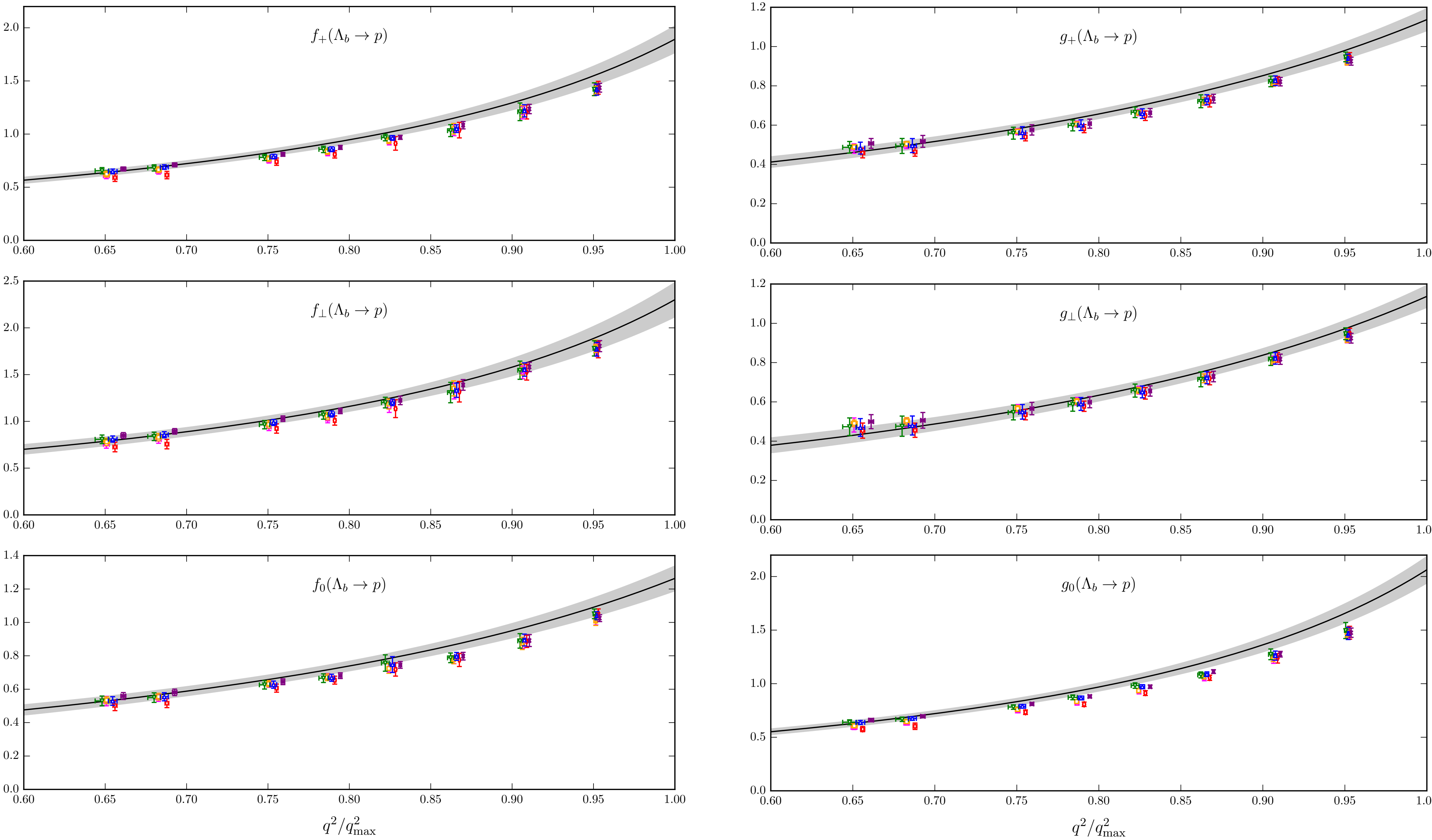
- N_f=2+1 domain-wall fermion sea
- anisotropic clover action for heavy quarks
- two lattice spacings (0.11 fm and 0.09 fm), few light masses
- mostly non-perturbative renormalisation
- explore wide region in momentum transfer; fit to one-pole *z*-parametrisation à la BCL, allowing for simulation parameter dependence in fit parameters





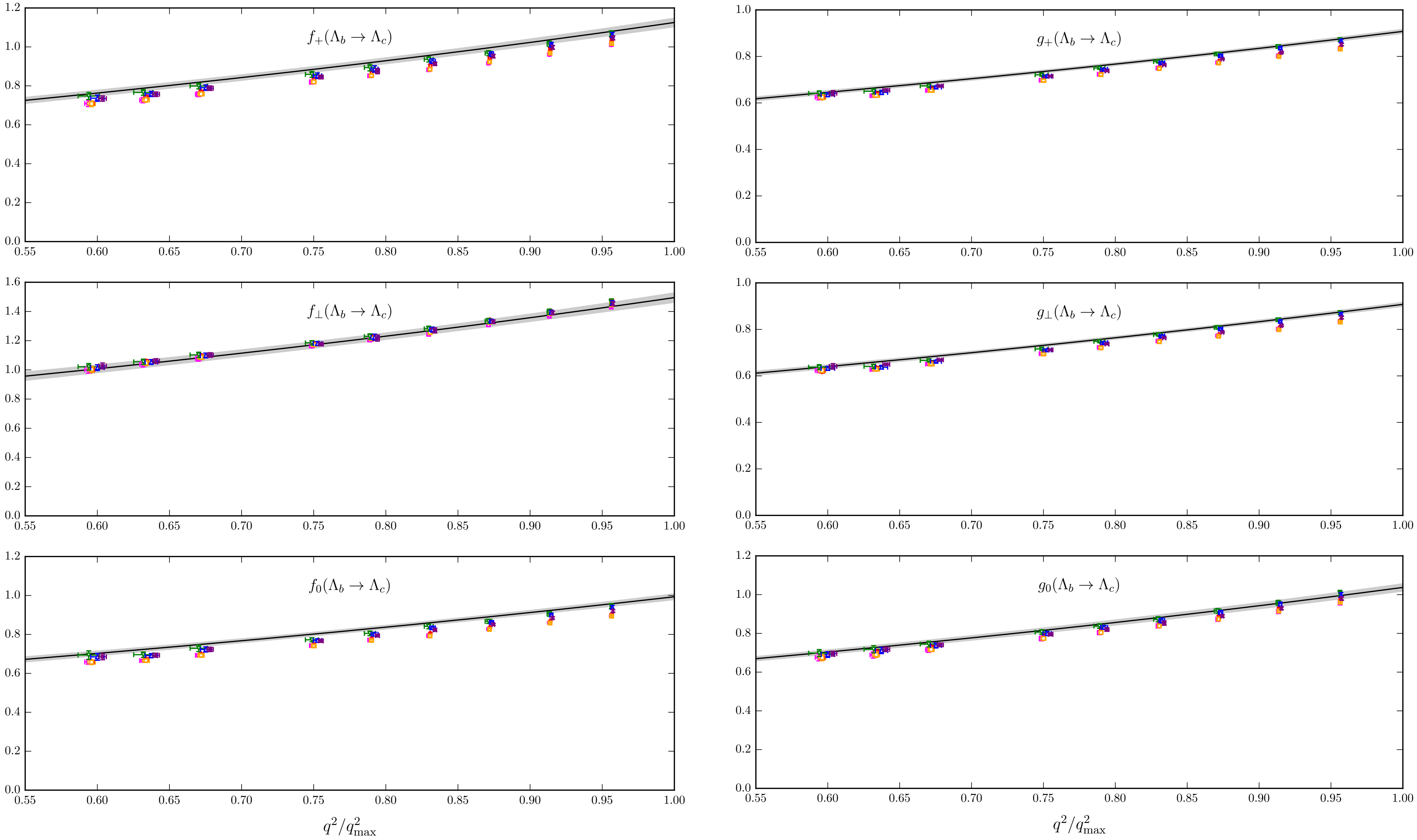
Λ_b decays

[Detmold, Lehner, Meinel, arXiv:1503.01421]



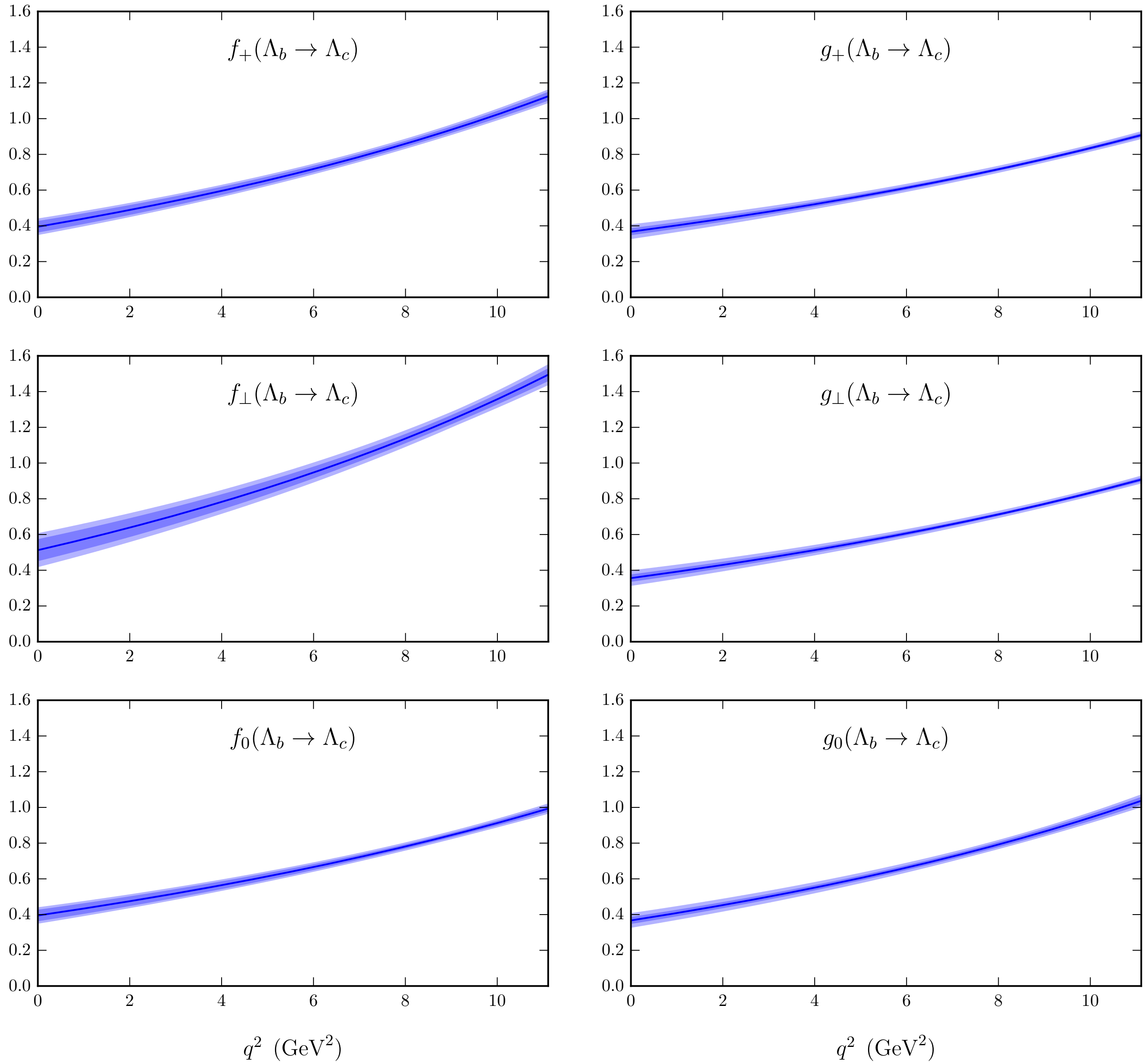
Λ_b decays

[Detmold, Lehner, Meinel, arXiv:1503.01421]



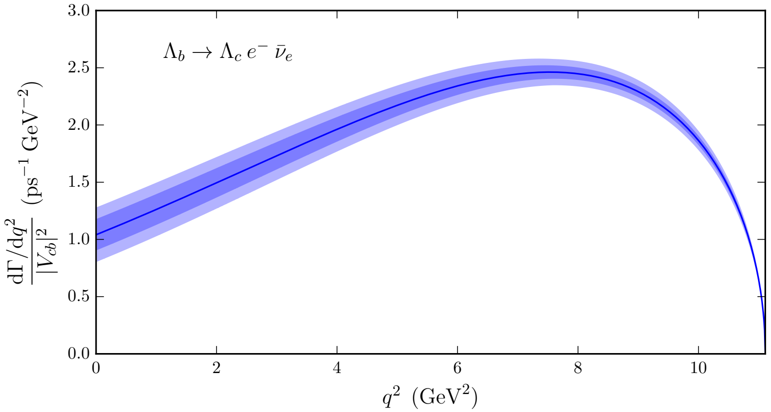
Λ_b decays

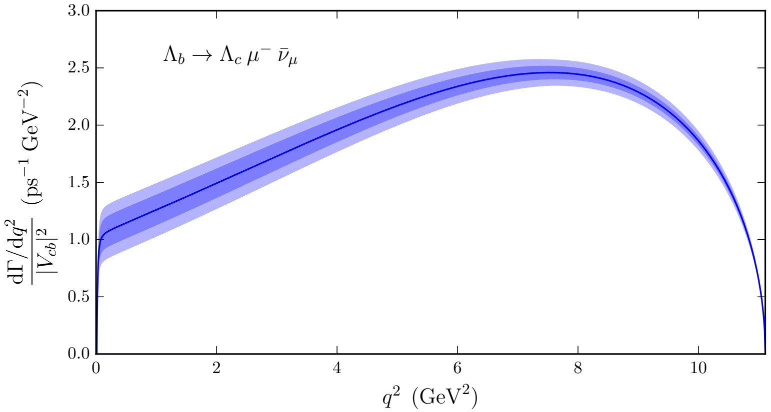
[Detmold, Lehner, Meinel, arXiv:1503.01421]

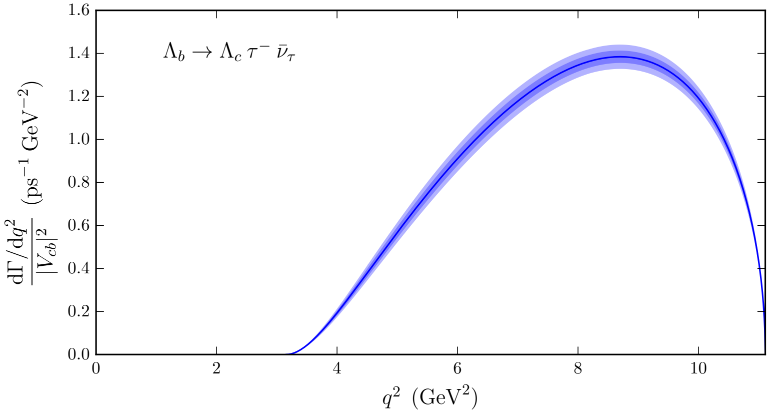


Λ_b decays

[Detmold, Lehner, Meinel, arXiv:1503.01421]







stat sys

$$\Gamma(\Lambda_b \to \Lambda_c\, e^- \bar{\nu}_e)/|V_{cb}|^2 = (21.1 \pm 0.8 \pm 1.4)\ \text{ps}^{-1},$$
$$\Gamma(\Lambda_b \to \Lambda_c\, \mu^- \bar{\nu}_\mu)/|V_{cb}|^2 = (21.1 \pm 0.8 \pm 1.4)\ \text{ps}^{-1},$$
$$\Gamma(\Lambda_b \to \Lambda_c\, \tau^- \bar{\nu}_\mu)/|V_{cb}|^2 = (7.13 \pm 0.17 \pm 0.29)\ \text{ps}^{-1}.$$

$$\frac{\Gamma(\Lambda_b \to \Lambda_c\, \tau^- \bar{\nu}_\mu)}{\Gamma(\Lambda_b \to \Lambda_c\, e^- \bar{\nu}_\mu)} = 0.3378 \pm 0.0079 \pm 0.0085,$$
$$\frac{\Gamma(\Lambda_b \to \Lambda_c\, \tau^- \bar{\nu}_\mu)}{\Gamma(\Lambda_b \to \Lambda_c\, \mu^- \bar{\nu}_\mu)} = 0.3388 \pm 0.0078 \pm 0.0085.$$

(e.m. effects neglected)

summary and outlook

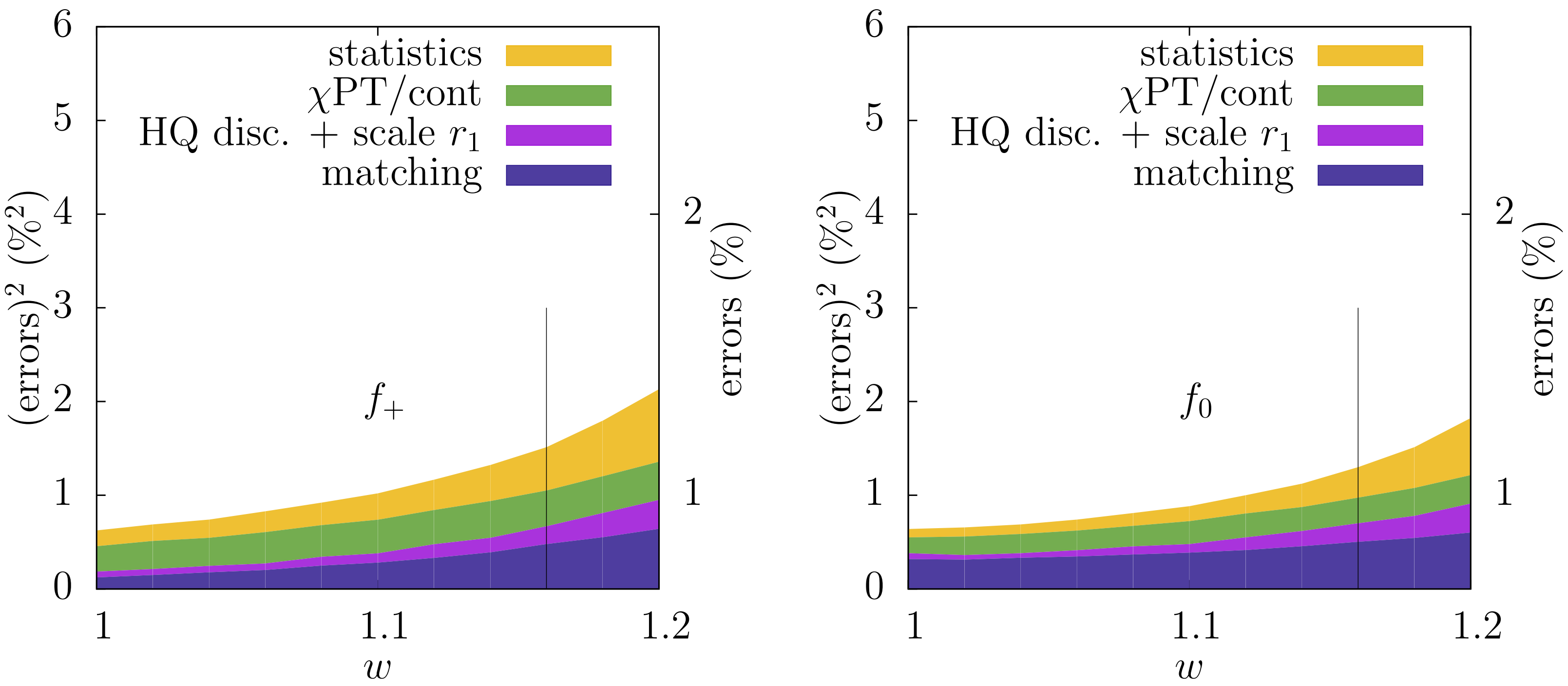
- significant recent advance in SL form factors
  - few % precision in FFs, error in $|V_{cb}|$ from meson channels dominated by exp

- immediate future
  - only results (meson channels) based on MILC ensembles: need crosscheck with other regularisations (and heavy quark treatments)
  - add *existing* ensembles with finer lattice spacings and lighter sea masses
  - fully understand analysis details (e.g. FF parametrisation, entanglement with chiral fits)

- what could be useful for the experiment???

backup: error budget in FNAL/MILC results for B→D

Bailey et al, arXiv:1503.07237



Source	$f_+(\%)$	$f_0(\%)$
Statistics+matching+χPT cont. extrap.	1.2	1.1
(Statistics)	(0.7)	(0.7)
(Matching)	(0.7)	(0.7)
(χPT/cont. extrap.)	(0.6)	(0.5)
Heavy-quark discretization	0.4	0.4
Lattice scale r_1	0.2	0.2
Total error	1.2	1.1

$w = 1.16$

backup: error budget in baryon FF