Non-Lattice Determinations of the $B \to D(*)$ Form Factors

Thomas Mannel

Theoretische Physik I
Universität Siegen

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Contents

Introduction

- Over the last few years:
  Enormous refinement of Lattice Methods
- This does NOT make non-lattice methods obsolete:
  - In many cases complementary information
  - Important for cross checks of lattice results
- V_{cb} from exclusive decays:
  $B \to D$ and $B \to D^*$ at zero recoil ($v = v'$)

- Kinematic variable for a heavy quark: Four Velovity v
- Differential Rates

$$\frac{d\Gamma}{d\omega}(B \to D^* \ell \bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 m_{D^*}^3 (\omega^2 - 1)^{1/2} P(\omega)(\mathcal{F}(\omega))^2$$
$$\frac{d\Gamma}{d\omega}(B \to D \ell \bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 (m_B + m_D)^2 m_D^3 (\omega^2 - 1)^{3/2} (\mathcal{G}(\omega))^2$$

- with $\omega = vv'$ and
- $P(\omega)$: Calculable Phase space factor
- $\mathcal{F}$ and $\mathcal{G}$: Form Factors

- Normalization of the Form Factors is known at $vv' = 1$: (both initial and final meson at rest)
- Corrections can be calculated / estimated

$$\mathcal{F}(\omega) = \eta_{\mathrm{QED}} \eta_A \left[1 + \delta_{1/\mu^2} + \cdots\right] + (\omega - 1)\rho^2 + \mathcal{O}((\omega - 1)^2)$$
$$\mathcal{G}(1) = \eta_{\mathrm{QED}} \eta_V \left[1 + \mathcal{O}\left(\frac{m_B - m_D}{m_B + m_D}\right)\right]$$

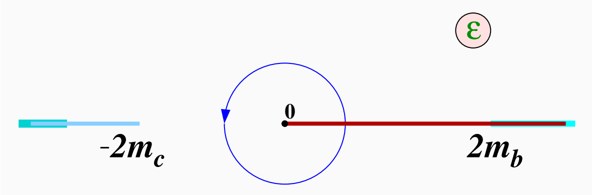
- Parameter of HQS breaking: $\frac{1}{\mu} = \frac{1}{m_c} - \frac{1}{m_b}$
- $\eta_A = 0.960 \pm 0.007$, $\eta_V = 1.022 \pm 0.004$, $\delta_{1/\mu^2} = -(8 \pm 4)\%$, $\eta_{\mathrm{QED}} = 1.007$

$B \to D^*$ from Zero Recoil Sum Rules

- Start from (ϵ: Excitation Energy)

$$T(\epsilon) = \frac{1}{3} \int d^4x \, e^{i\,(v\cdot x)[\epsilon-(M_B-M_{D^*})]}$$
$$\langle B(P)|\mathcal{T}\left\{\bar{b}_v(x)\gamma_\mu\gamma_5 c_v(x)\,\bar{c}_v(0)\gamma^\mu\gamma_5 b_v(0)\right\}|B(P)\rangle$$

- compute the contour integral $I_0(\mu) = -\frac{1}{2\pi i} \oint\limits_{|\varepsilon|=\mu} T(\varepsilon)\,\mathrm{d}\varepsilon$



- Insert a complete set of states:

$$I_0(\epsilon_M) = \frac{1}{3} \sum_n^{\epsilon_n < \epsilon_M} \langle B(P)|\bar{b}_v \gamma_\mu \gamma_5 c_v|n\rangle \langle n|\bar{c}_v \gamma^\mu \gamma_5 b_v|B(P)\rangle$$

- The lowest resonance state is the D^*

$$\langle B(v)|\bar{b}_v \gamma_\mu \gamma_5 c_v|D^*(v,\varepsilon)\rangle = \mathcal{F}(1)\, 2\sqrt{M_B M_{D^*}}\, \varepsilon_\mu$$

- Hence

$$I_0(\epsilon_M) = |\mathcal{F}(1)|^2$$
$$+ \text{ inelastic contributions with } \epsilon > 0\,,$$ non-negative!

- On the other side: Compute $T(\epsilon)$ in OPE
- ... and take this to compute I_0

$$I_0(\epsilon_M) = \xi^{\text{pert}}(\epsilon_M, \mu) - \Delta_{1/m^2}(\epsilon_M, \mu) - \Delta_{1/m^3}(\epsilon_M, \mu) + \cdots$$

- $\xi^{\text{pert}}(\epsilon_M, \mu)$: Perturbative Contribution, computed in Wilsonian Cut Off Scheme

$$\xi^{\text{pert}}(0, 0) = \eta_A$$

Perturbative Corrections

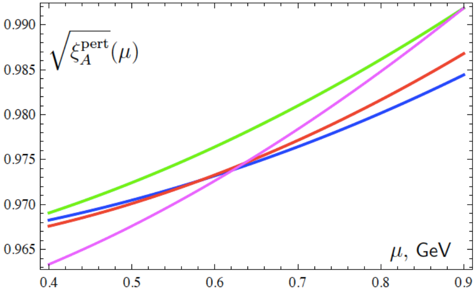


Figure 4: $\sqrt{\xi_A^{\rm pert}}(\mu)$ as a function of μ for $m_b = 4.6\,\mathrm{GeV}$, $m_c = 1.2\,\mathrm{GeV}$, $\alpha_s(m_b) = 0.22$. The curves represent the one-loop result evaluated with $\alpha_s = 0.3$ (blue), one-loop plus first order BLM (green), complete $\mathcal{O}(\alpha_s^2)$ (red), two-loop plus third-order BLM (maroon).

$$\sqrt{\xi^{\rm pert}(0.75\,\mathrm{GeV}, 0.75\,\mathrm{GeV})} = 0.98 \pm 0.01$$

Non-perturbative Corrections

$$\Delta_{1/m^2} = \frac{\mu_G^2}{3m_c^2} + \frac{\mu_\pi^2 - \mu_G^2}{4}\left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_b m_c}\right)$$

$$\Delta_{1/m^3} = \frac{3\rho_D^3 - \rho_{LS}^3}{12m_c^3} + \frac{\rho_D^3 + \rho_{LS}^3}{12m_b}\left(\frac{1}{m_c^2} + \frac{3}{m_b^2} + \frac{1}{m_b m_c}\right)$$

$$\Delta_{1/m^{4,5}} = \text{known}$$

Numerically:

$$\Delta_{1/m^2} + \Delta_{1/m^3} = 0.102 \pm 0.017$$

$$\Delta_{1/m^4} \sim -0.023 \qquad \Delta_{1/m^5} \sim -0.013$$

The Sum Rule

Combining everything

$$\mathcal{F}(1) \leq 0.925$$

- Saturation of the sum rule means no inelastic contributions
- To obtain a value of $\mathcal{F}(1)$ one needs to estimate the inelastic contribution

Estimate of the Inelastic Contribution

Starting point: First Moment

$$I_1(\mu) = -\frac{1}{2\pi i} \oint_{|\varepsilon|=\mu} \varepsilon T(\varepsilon)\, \mathrm{d}\varepsilon$$

- No contribution from the ground state $\epsilon = 0$
- Estimate of the inelastic contribution:

$$I_{\text{inel}} = \frac{I_1(\epsilon_M)}{\tilde{\epsilon}}, \qquad \tilde{\epsilon} = \text{average excitation energy}$$

- for moderate ϵ_M:
  I_1 is dominated by the lowest "radial" excitations, so

$$\tilde{\epsilon} \sim 700\,\mathrm{MeV}$$

- Compute I_1 in OPE:

$$I_1(\epsilon) = -\frac{\rho^3_{\pi G} + \rho^3_A}{3m_c^2} - \frac{2\rho^3_{\pi\pi} + \rho^3_{\pi G}}{3m_c m_b} + \frac{1}{4}(\rho^3_{\pi\pi} + \rho^3_{\pi G} + \rho^3_S + \rho^3_A)\left(\frac{1}{m_c^2} + \frac{2}{3m_c m_b} + \frac{1}{m_b^2}\right) + \cdots$$

in terms of ($'$ means ground state removed)

$$4M_B\rho^3_{\pi\pi} = i\int d^4x\, \langle B|T[\bar{b}(x)(iD_\perp)^2 b(x)\, \bar{b}(0)(iD_\perp)^2 b(0)]|B\rangle'$$

$$4M_B\rho^3_{\pi G} = i\int d^4x\, \langle B|T[\bar{b}(x)(iD_\perp)^2 b(x)\, \bar{b}(0)(\vec{\sigma}\cdot\vec{B}(0))b(0)]|B\rangle'$$

$$4M_B\left(\frac{1}{3}\delta_{ij}\delta_{kl}\rho^3_S + \frac{1}{6}(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk})\rho_A\right) = i\int d^4x\, \langle B|T[\bar{b}(x)\sigma_i B_k(x) b(x)\, \bar{b}(0)\sigma_j B_l(0) b(0)]|B\rangle'$$

- What do we know about the (non-local) matrix elements?

The BPS Limit (Uraltsev (2003)

- Observation: Numerically we have $\mu_\pi^2 - \mu_G^2 \leq \mu_\pi^2$
- Consider a limit where $\mu_\pi^2 = \mu_G^2$
- Generalization: (... looks like the BPS relation known from string theory)

$$i\not{D}_\perp b_v|B\rangle = 0\,, \quad D_\perp^\mu = D_\mu - v_\mu(v \cdot D)$$

- This implies

$$0 = \langle B|\bar{b}_v(i\not{D}_\perp)^2 b_v|B\rangle = 2M_B(\mu_\pi^2 - \mu_G^2)$$

- ... and also

$$\rho_D^3 = -\rho_{LS}^3 \quad \rho_{\pi G}^3 = -2\rho_{\pi\pi}^3 \quad \rho_{\pi G}^3 + \rho_A^3 = -(\rho_{\pi\pi}^3 + \rho_S^3)$$

- In the BPS limit: $I_1 = -\frac{\rho_{\pi G}^3 + \rho_A^3}{3m_c^2}$

What else do we know?

Hyperfine Splitting of the Ground State Mesons:

$$M_{B^*} - M_B = \frac{2}{3}\frac{\mu_G^2}{m_b} + \frac{\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3}{3m_b^2} + \cdots$$

- Constraints on μ_G and ρ_{LS} from heavy quarks sum rules (Uraltsev)

$$\rho_{\pi G}^3 + \rho_A^3 \sim 0.45\,\mathrm{GeV}^3 \quad I_{inel}(\epsilon_M = 0.75) \sim \frac{0.45\,\mathrm{GeV}^3}{3m_c\tilde{\epsilon}}$$

- Numerically $I_{inel} = 0.13 \pm 0.04$ (30% uncertainity estimated)

Remarks

- Estimate of the continuum $D\pi$ contribution yields + (3 ... 5) %
- Other estimates yield a similar value for I_{inel}
- from this we obtain

$$\mathcal{F}(1) \le 0.90 \quad \mathcal{F}(1) = 0.86 \pm 0.04$$

- this depends a bit on the value of μ_π^2
- This is lower than the lattice values!

$B \to D$ from BPS Limit

- For the form factor $\mathcal{G}$ we have

$$\mathcal{G}(w) = h_+(w) - \frac{m_B - m_D}{m_B + m_D} h_-(w)$$

- The presence of $h_-(w)$ was considered a problem for a precise V_{cb} determination
- In the BPS limit we have have to all orders in $1/m$

$$M_B = m_b + \bar{\Lambda} \quad M_D = m_c + \bar{\Lambda} \quad M_B - M_D = m_b - m_c$$

- With this and from the equation of motion

$$i\partial^\mu(\bar{b}\gamma_\mu c) = (m_b - m_c)(\bar{b}c) =$$

we get $h_-(w) = 0$

- Furthermore, in the BPS limit: $h_+(1) = 1$ to all orders in $1/m$, hence

$$\mathcal{G}(1) = \xi_V(\mu) \sim 1.03$$

- $\xi_V(\mu)$ are QCD corrections to the vector current
- Corrections to the BPS limit: Expansion in

$$\beta^2 = \frac{\mu_\pi^2 - \mu_G^2}{\mu_\pi^2}$$

- For the form factor:

$$\mathcal{G}(1) = \xi_V(\mu) + \beta^2 \frac{\mu_\pi^2}{3\tilde{\epsilon}} \left(\frac{1}{m_c} - \frac{1}{m_b} \right) \frac{(M_B - M_D)}{(M_B + M_D)} + \mathcal{O}(1/m_Q^2)$$

- Numerically

$$\mathcal{G}(1) = 1.04 \pm 0.02$$

Zero Recoil Sum Rule for $\Lambda_b \to \Lambda_c$

- Start from

$$T(\epsilon) = \frac{1}{3} \int d^4x \, e^{i\,(v\cdot x)[\epsilon-(M_{\Lambda_b}-M_{\Lambda_c})]}$$
$$\langle \Lambda_b(P) | \mathcal{T}\left\{ \bar{b}_v(x) \gamma_\mu \gamma_5 c_v(x) \, \bar{c}_v(0) \gamma^\mu \gamma_5 b_v(0) \right\} | \Lambda_b(P) \rangle$$

- and perform the same steps as for $B \to D^*$

- Inserting a complete set of states:
  Lowest state is Λ_c
- Form factor definition: (T. Feldmann , M. Yip (2011))

$$\langle \Lambda_c(v',s')|\bar{c}\gamma_5\gamma_\mu b|\Lambda_b(v,s)\rangle = \bar{u}_{\Lambda_c}(v',s')\gamma_5 \left[g_0(w)(M_{\Lambda_b}+M_{\Lambda_c})\frac{q^\mu}{q^2} \right.$$
$$+g_+(w)\frac{M_{\Lambda_b}-M_{\Lambda_c}}{s_-}\left(M_{\Lambda_b}v_\mu + M_{\Lambda_c}v'_\mu - (M^2_{\Lambda_b}-M^2_{\Lambda_c})\frac{q^\mu}{q^2}\right)$$
$$\left. +g_\perp(w)\left(\gamma_\mu - \frac{2M_{\Lambda_c}M_{\Lambda_b}}{s_+}(v_\mu - v'_\mu)\right)\right] u_{\Lambda_b}(v,s)$$

- Zero Recoil Sum Rule:

$$I_0(\epsilon_M) = \frac{1}{3}\left[2|g_\perp(1)|^2 + |g_+(1)|^2\right] + \text{ inelastic}$$
$$= \xi^{\text{pert}}(\epsilon_M,\mu) - \Delta_{1/m^2}(\epsilon_M,\mu) - \Delta_{1/m^3}(\epsilon_M,\mu) + \cdots$$

- Perturbative Contribution as before:

$$\xi^{\rm pert}(\epsilon_M = \mu = 0.75\,\mathrm{GeV}) = 0.970 \pm 0.02$$

- Nonperturbative Contributions

$$\Delta_{1/m^2} = \frac{\mu_\pi^2(\Lambda_b)}{4}\left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_b m_c}\right)$$
$$\Delta_{1/m^3} = \frac{\rho_D^3(\Lambda_b)}{4m_c^3} + \frac{\rho_D^3(\Lambda_b)}{12m_b}\left(\frac{1}{m_c^2} + \frac{3}{m_b^2} + \frac{1}{m_b m_c}\right)$$

- Note $\mu_G = \rho_{LS} = 0$ and $\mu_\pi^2(\Lambda_b) \sim \mu_\pi^2(B)$

- ... or as an inequality

$$\frac{1}{3}\left[2|g_\perp(1)|^2 + |g_+(1)|^2\right] \leq \xi^{\rm pert}(\epsilon_M,\mu) - \Delta_{1/m^2}(\epsilon_M,\mu) - \Delta_{1/m^3}(\epsilon_M,\mu) + \cdots$$

- Numerically we have (Preliminary)

$$\frac{1}{3}\left[2|g_\perp(1)|^2 + |g_+(1)|^2\right] \leq 0.86$$

- ... to be compared to the lattice number (W. Detmold, C. Lehner, S. Meinel (2015))

$$\frac{1}{3}\left[2|g_\perp(1)|^2 + |g_+(1)|^2\right] = 0.824 \pm 0.020$$

Outlook

- There is a persisting tension between the lattice determinations of the form factors and the ones form "continuum methods"
- $\to$ further scrutiny needed: "Hyperfine constraint"
- This seems also to be the case for the $\Lambda_b \to \Lambda_c$ form factors.
- Inclusive V_{cb} may give a hint,
- ... or there is new physics? (see V_{ub})