Future prospects for moments measurements

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THEORY RECAP: WHY DO WE MEASURE MOMENTS IN SEMILEPTONIC B DECAYS?

$|V_{cb}|$ from inclusive decays

$B \to Xl\nu$

$$\Gamma = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2\left(1 + \frac{c_5(\mu)\langle O_5\rangle(\mu)}{m_b^2} + \frac{c_6(\mu)\langle O_6\rangle(\mu)}{m_b^3} + \mathcal{O}(\frac{1}{m_b^4})\right)$$

- Based on the Operator Product Expansion (OPE)
- $<O_i>$: hadronic matrix elements (non-perturbative)
  c_i: coefficients (perturbative)
- Parton-hadron duality → the hadronic ME depend only on the initial state

	Kinetic scheme [JHEP 1109 (2011) 055]	1S scheme [PRD70, 094017 (2004)]
O(1)	m_b, m_c	m_b
$O(1/m^2_b)$	μ^2_π, μ^2_G	λ_1, λ_2
$O(1/m^3_b)$	ρ^3_D, ρ^3_{LS}	ρ_1, $\tau_{1\text{-}3}$

Moments of the E_l and M^2_X spectrum

Also other observables in B → Xlν can be expanded into an OPE with the same heavy quark parameters, e.g.,

- The n^th moment of the (truncated) lepton energy spectrum

$$R_n(E_{\rm cut},\mu) = \int_{E_{\rm cut}} (E_\ell - \mu)^n \frac{{\rm d}\Gamma}{{\rm d}E_\ell}\,{\rm d}E_\ell\,, \quad \langle E_\ell^n \rangle_{E_{\rm cut}} = \frac{R_n(E_{\rm cut},0)}{R_0(E_{\rm cut},0)}$$

- The n^th moment of the (truncated) M^2_X spectrum

$$\langle m_X^{2n} \rangle_{E_{\rm cut}} = \frac{\displaystyle\int_{E_{\rm cut}} (m_X^2)^n \frac{{\rm d}\Gamma}{{\rm d}m_X^2}\,{\rm d}m_X^2}{\displaystyle\int_{E_{\rm cut}} \frac{{\rm d}\Gamma}{{\rm d}m_X^2}\,{\rm d}m_X^2}$$

Master plan:

- Measure the quark masses and heavy quark parameters using moments
- Substitute them in the formula of the semileptonic width
- Determine $|V_{cb}|$ from the semileptonic branching fraction

THE CURRENT STATUS

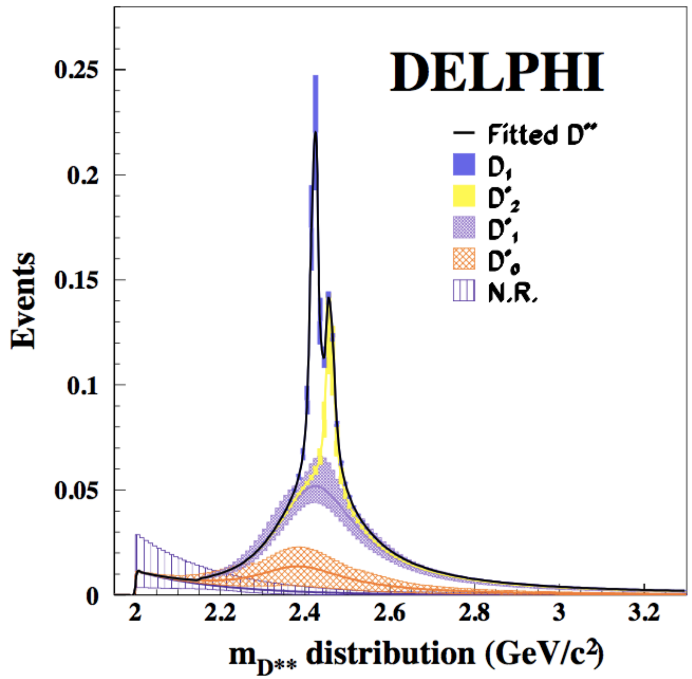
Moment measurements available

BaBar	$\langle E^n_l \rangle$: n=0,1,2,3 [PRD 69, 111104 (2004), PRD 81, 032003 (2010)] $\langle M^{2n}_X \rangle$: n=1,2, 3 [PRD 81, 032003 (2010)]
Belle	$\langle E^n_l \rangle$: n=0,1,2,3 [PRD 75, 032001 (2007)] $\langle M^{2n}_X \rangle$: n=1,2 [PRD 75, 032005 (2007)]
CDF	$\langle M^{2n}_X \rangle$: n=1,2 [PRD 71, 051103 (2005)]
CLEO	$\langle M^{2n}_X \rangle$: n=1,2 [PRD 70, 032002 (2004)]
DELPHI	$\langle E^n_l \rangle$: n=1,2,3 $\langle M^{2n}_X \rangle$: n=1,2,3 [EPJ C45, 35 (2006)]

DELPHI hadronic moments

EPJ C45, 35 (2006)

- D^{**} mass distribution
  - Reconstructed decays: $b \to \{D^0\pi^+, D^+\pi^-, D^{*+}\pi^-\} l^+ \nu$
  - Fit to the $D^0\pi^+$, $D^+\pi^-$ and $D^{*+}\pi^-$ mass distributions in which the narrow D^{**} states (D_1, D^*_2) are constrained
- Used to calculate hadronic moments up to M^5_X (for $E_{cut} = 0$)



$$< m_H^n > = p_D \; m_D^n + p_{D^*} \; m_{D^*}^n + p_{D^{**}} \; < m_{D^{**}}^n >$$

$$p_{D_i} = \frac{\mathrm{BR}(\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}_i \ell^- \overline{\nu}_\ell)}{\mathrm{BR}(\overline{\mathrm{B}^0_\mathrm{d}} \to c \ell^- \overline{\nu}_\ell)}$$

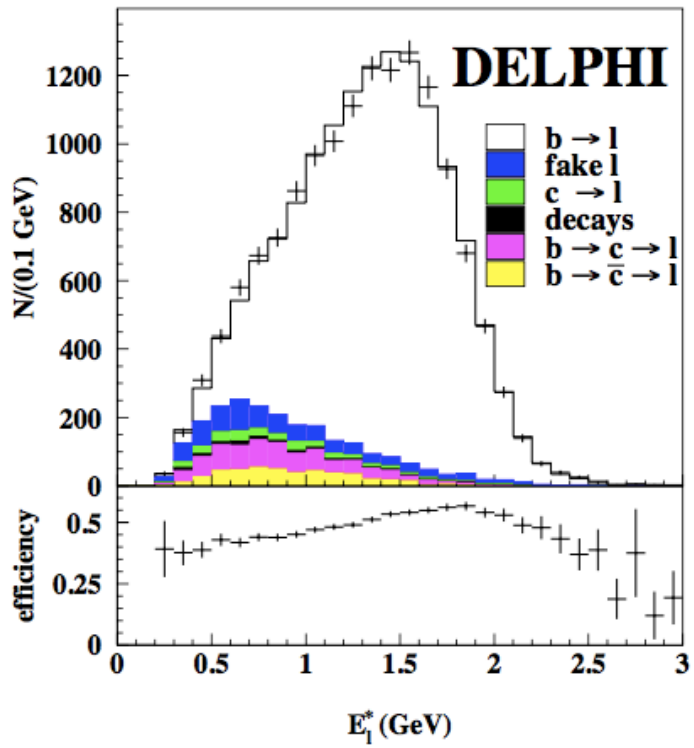
	M_1^H $(\mathrm{GeV}/c^2)^2$	M_2^H $(\mathrm{GeV}/c^2)^4$	M_3^H $(\mathrm{GeV}/c^2)^6$	M_4^H $(\mathrm{GeV}/c^2)^8$	M_5^H $(\mathrm{GeV}/c^2)^{10}$
value	0.647	1.98	7.4	35.7	205
stat. uncert.	±0.046	±0.23	±1.3	±7.9	±1080
Ext. BR	0.079	0.22	0.8	4.1	23.4
$b_{\pi\pi}$	±0.039	±0.15	±0.6	±2.8	±16.4
$m_{D_0^*}$	±0.015	±0.04	∓0.0	∓0.7	∓7.0
backg. param	0.007	0.04	0.2	1.2	8.0
$\overline{\mathrm{B}^0_s}$, Λ_b^0	±0.007	±0.03	±0.2	±0.9	±5.2
$d_\pm$ dist.	0.005	0.03	0.2	1.1	7.4
$\Gamma(D_2^*) = 40\mathrm{MeV}/c^2$	−0.004	−0.02	−0.1	−0.9	−5.8
Tot. syst.	0.090	0.27	1.1	5.4	32.3

DELPHI lepton moments

EPJ C45, 35 (2006)

- Lepton spectrum in inclusive semileptonic b-hadron decays
- The lepton energy moments are calculated from the background subtracted, unfolded spectrum

	M_1^ℓ (GeV)	$M_2'^\ell$ (GeV)^2	$M_3'^\ell$ (GeV)^3
value	1.3782	0.1838	−0.0301
stat. uncert.	±0.0073	±0.0058	±0.0015
B species	±0.0027	∓0.0017	∓0.0005
$B \to D,\ D^*,\ D^{**}\ell\overline{\nu}_\ell$	±0.0010	∓0.0005	±0.0001
B fragmentation	±0.0027	∓0.0020	∓0.0007
$B \to X_u \ell\overline{\nu}_\ell$	±0.0008	±0.0003	∓0.0001
e.m. radiation	±0.0035	∓0.0001	∓0.0004
Bkg modelling	±0.0026	∓0.0011	∓0.0005
B direction reconstruction	±0.0027	∓0.0018	∓0.0006
B energy reconstruction	±0.0027	±0.0011	∓0.0003
B mass cut	±0.0051	∓0.0031	∓0.0017
Unfolding	±0.0031	∓0.0028	±0.0029
Tot. syst.	±0.0092	±0.0055	±0.0036



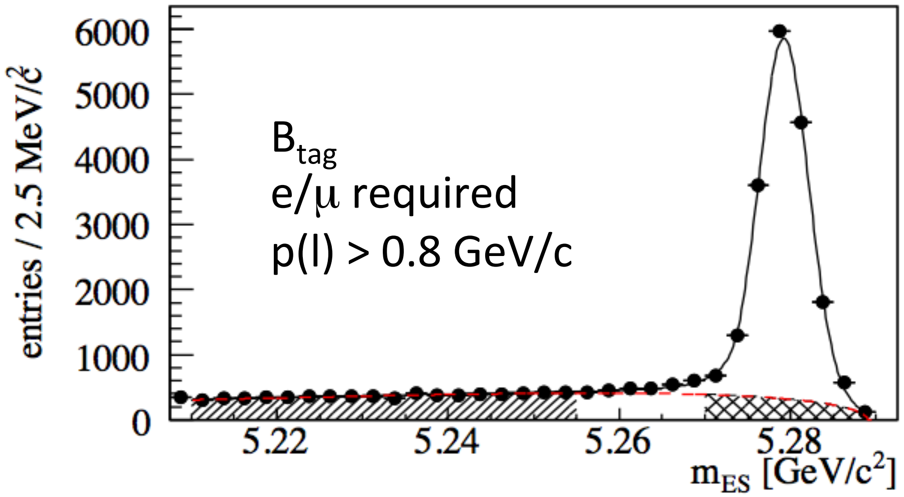
Again, moments are for E_{cut} = 0

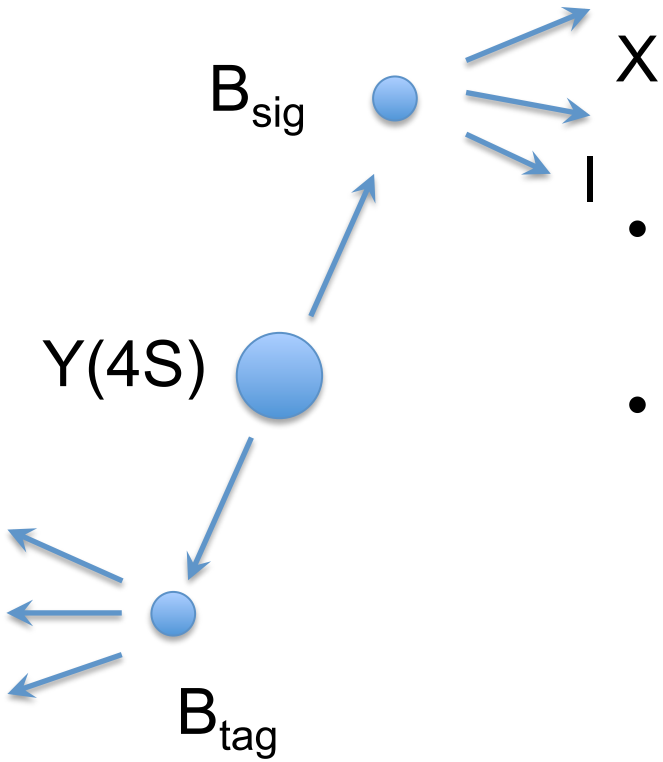
BaBar hadronic moments

232M BB

PRD 81, 032003 (2010)

- Fully reconstruct the hadronic decay of one B in Y(4S) → BB (efficiency ~0.4%, purity ~80%)



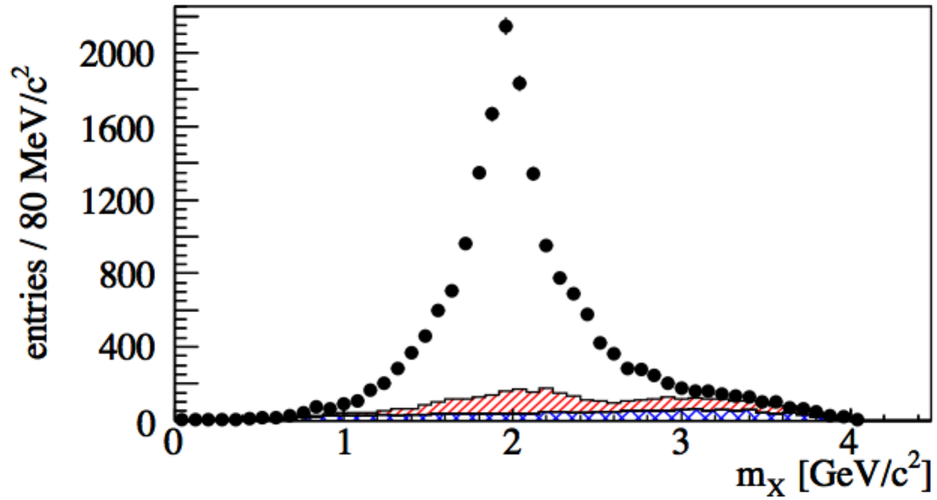


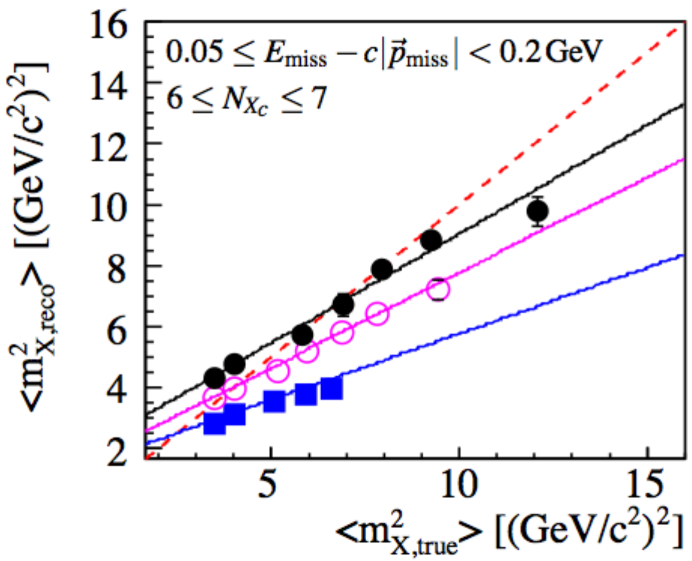
- Require one identified lepton amongst the signal-side particles (p > 0.8 GeV/c)
- Combine all remaining particles to the X system and do a kinematic fit
  - 4-momentum conservation
  - Missing mass consistent with zero mass neutrino

Moment measurement

PRD 81, 032003 (2010)

- Hadronic mass spectrum after kinematic fit





- Linear correction of the measured moments in bins of X multiplicity, E_{miss}-cp_{miss} and lepton momentum

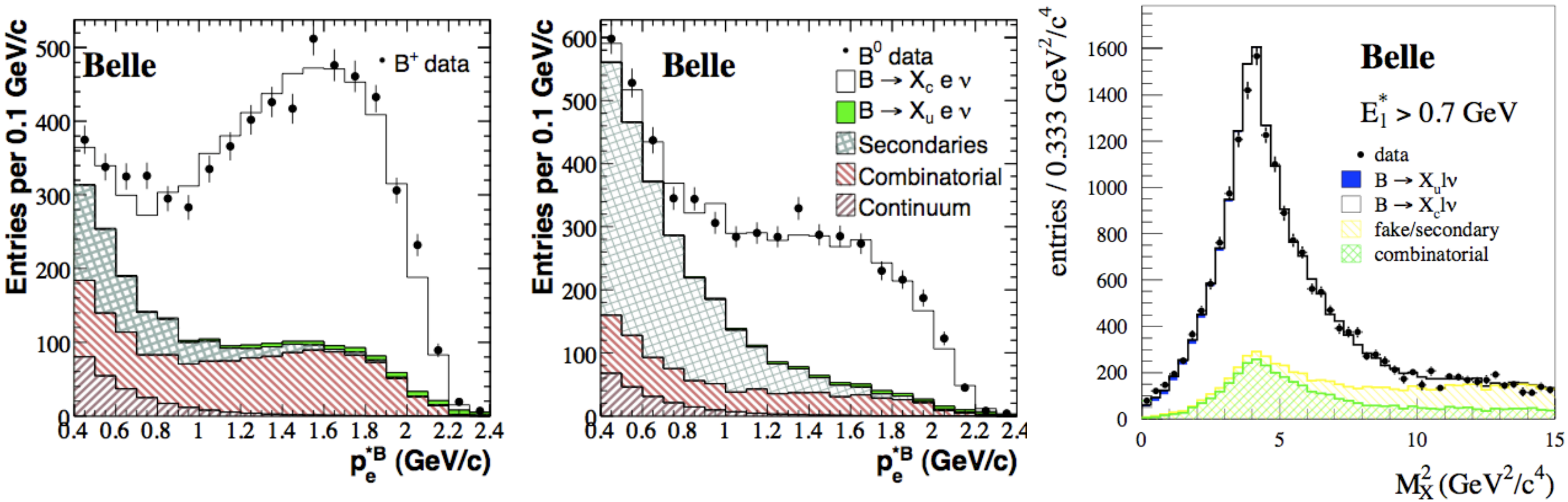
- Moments of the hadronic mass spectrum up to M^6_X for E_{cut} between 0.8 and 1.9 GeV (in B rest frame) are measured
- Also mixed mass-energy moments are determined and the electron energy moments from [PRD69, 111104] are re-evaluated

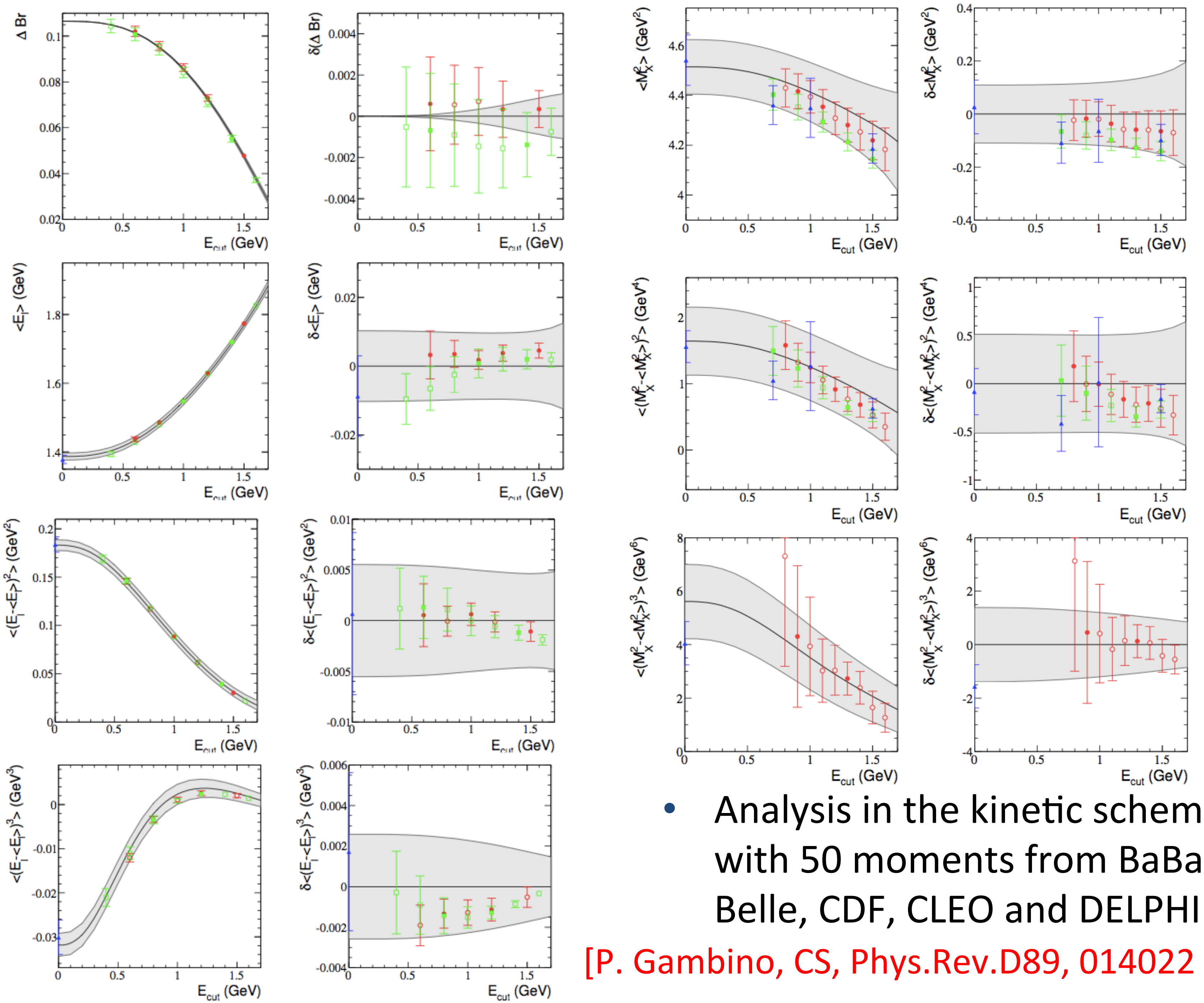
Belle E_l and M^2_X moments

PRD 75, 032001 (2007)
PRD 75, 032005 (2007)

152M BB

- For both the E_l and M^2_X measurements, similar experimental method using fully reconstructed events
- The finite detector resolution is unfolded with SVD algorithm [NIM A372, 469 (1996)]
- $\langle E^n_e \rangle$ measured for n=0,…,4 and E_{cut}=0.4-2.0 GeV
- $\langle M^{2n}_X \rangle$ measured for n=1,2 and E_{cut}=0.7-1.9 GeV



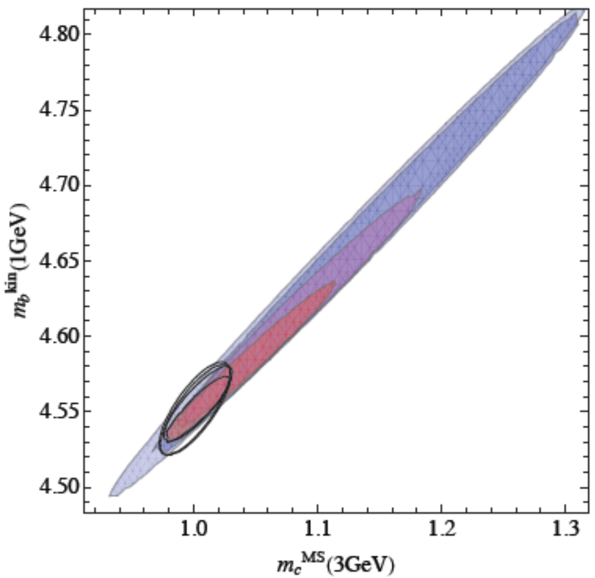


- Analysis in the kinetic scheme with 50 moments from BaBar, Belle, CDF, CLEO and DELPHI

[P. Gambino, CS, Phys.Rev.D89, 014022 (2014)]

Global fit (kinetic scheme)

[Phys.Rev.D89, 014022 (2014)]



th. corr. scenario	m_b^{kin}	m_c	μ_π^2	ρ_D^3	μ_G^2	ρ_{LS}^3	BR$_{c\ell\nu}$(%)	$10^3\,\lvert V_{cb}\rvert$
D [11]	4.541	0.987	0.414	0.154	0.340	-0.147	10.65	42.42
$\overline{m}_c$(3GeV)	0.023	0.013	0.078	0.045	0.066	0.098	0.16	0.86
A [11]	4.540	0.987	0.454	0.167	0.234	-0.078	10.45	41.85
$\overline{m}_c$(3GeV)	0.014	0.013	0.035	0.022	0.040	0.085	0.13	0.74
B [11]	4.542	0.987	0.457	0.184	0.290	-0.135	10.51	42.15
$\overline{m}_c$(3GeV)	0.017	0.013	0.056	0.035	0.056	0.095	0.14	0.77
C [11]	4.539	0.987	0.415	0.155	0.336	-0.147	10.65	42.45
$\overline{m}_c$(3GeV)	0.022	0.013	0.073	0.043	0.066	0.098	0.16	0.86
D [11]	4.538	0.986	0.415	0.153	0.336	-0.145	10.65	42.46
$\overline{m}_c$(3GeV), m_b	0.018	0.012	0.078	0.045	0.064	0.098	0.16	0.84
D [13]	4.549	0.996	0.413	0.154	0.339	-0.146	10.65	42.40
$\overline{m}_c$(3GeV)	0.029	0.026	0.078	0.045	0.066	0.098	0.16	0.87
D [11]	4.548	1.092	0.428	0.158	0.344	-0.146	10.66	42.24
m_c^{kin}	0.023	0.020	0.079	0.045	0.066	0.098	0.16	0.85
D [11]	4.553	1.088	0.428	0.155	0.328	-0.139	10.67	42.42
$\overline{m}_c$(2GeV), m_b	0.018	0.013	0.079	0.045	0.064	0.098	0.16	0.83

$\overline{m}_c$(3 GeV)	m_b^{kin}(1 GeV)	$\overline{m}_b(\overline{m}_b)$
0.986(13) [11]	4.541(23)	4.171(38)
0.986(6) [12]	4.540(20)	4.170(36)
0.994(26) [13]	4.549(29)	4.179(42)

[11] K. G. Chetyrkin, J. H. Kuhn, A. Maier, P. Maierhofer, P. Marquard, M. Steinhauser and C. Sturm, Phys. Rev. D **80** (2009) 074010 [arXiv:0907.2110 [hep-ph]].

[12] I. Allison *et al.* [HPQCD Collaboration], Phys. Rev. **D78**, 054513 (2008) [arXiv:0805.2999 [hep-lat]]; C. McNeile, C. T. H. Davies, E. Follana, K. Hornbostel and G. P. Lepage,[HPQCD Collaboration] Phys. Rev. D **82** (2010) 034512 [arXiv:1004.4285 [hep-lat]].

[13] B. Dehnadi, A. H. Hoang, V. Mateu and S. M. Zebarjad, JHEP **1309** (2013) 103 [arXiv:1102.2264 [hep-ph]].

A FEW THOUGHTS ABOUT FUTURE MEASUREMENTS

Looking at the Belle hadron moments

$E^*_{\min}$ (GeV)	$\langle M_X^2 \rangle$ (GeV$^2/c^4$)	$\langle (M_X^2 - \langle M_X^2 \rangle)^2 \rangle$ (GeV$^4/c^8$)	$\langle M_X^4 \rangle$ (GeV$^4/c^8$)
0.7	$4.403 \pm 0.036 \pm 0.052$	$1.494 \pm 0.173 \pm 0.327$	$20.88 \pm 0.48 \pm 0.77$
0.9	$4.353 \pm 0.032 \pm 0.041$	$1.229 \pm 0.138 \pm 0.244$	$20.18 \pm 0.40 \pm 0.58$
1.1	$4.293 \pm 0.028 \pm 0.029$	$0.940 \pm 0.098 \pm 0.137$	$19.37 \pm 0.33 \pm 0.36$
1.3	$4.213 \pm 0.027 \pm 0.024$	$0.641 \pm 0.071 \pm 0.080$	$18.40 \pm 0.29 \pm 0.26$
1.5	$4.144 \pm 0.028 \pm 0.022$	$0.515 \pm 0.061 \pm 0.064$	$17.69 \pm 0.28 \pm 0.23$
1.7	$4.056 \pm 0.033 \pm 0.022$	$0.322 \pm 0.058 \pm 0.040$	$16.77 \pm 0.32 \pm 0.21$
1.9	$3.996 \pm 0.041 \pm 0.021$	$0.143 \pm 0.056 \pm 0.038$	$16.11 \pm 0.38 \pm 0.20$

- Systematic uncertainties dominate already with a dataset of 140/fb -- main contributions:
  - Background (secondary leptons) and signal model (D^{**}) at low E_{cut} values
  - Unfolding method at high E_{cut} values

Looking at the Belle electron moments

E_{cut}[GeV]	M_1 [MeV]	M_2 [10^{-3}GeV^2]	M_3 [10^{-3}GeV^3]	M_4 [10^{-3}GeV^4]
0.4	1393.92 ± 6.73 ± 3.02	168.77 ± 3.68 ± 1.53	-21.04 ± 1.93 ± 0.66	64.153 ± 1.813 ± 0.935
0.6	1427.82 ± 5.82 ± 2.55	146.15 ± 2.88 ± 1.08	-11.04 ± 1.35 ± 0.49	45.366 ± 1.108 ± 0.548
0.8	1480.04 ± 4.81 ± 2.13	117.97 ± 2.05 ± 0.55	-3.45 ± 0.83 ± 0.30	28.701 ± 0.585 ± 0.247
1.0	1547.76 ± 3.96 ± 1.45	88.17 ± 1.42 ± 0.36	0.83 ± 0.49 ± 0.20	15.962 ± 0.302 ± 0.142
1.2	1627.79 ± 3.26 ± 1.08	61.36 ± 1.02 ± 0.36	2.40 ± 0.30 ± 0.11	7.876 ± 0.162 ± 0.106
1.4	1719.96 ± 2.58 ± 1.10	38.99 ± 0.71 ± 0.24	2.33 ± 0.16 ± 0.07	3.314 ± 0.080 ± 0.055
1.6	1826.15 ± 1.80 ± 1.03	21.75 ± 0.47 ± 0.22	1.45 ± 0.08 ± 0.05	1.129 ± 0.033 ± 0.032
1.8	1943.18 ± 0.93 ± 1.16	10.14 ± 0.28 ± 0.18	0.68 ± 0.03 ± 0.04	0.283 ± 0.010 ± 0.017
2.0	2077.59 ± 0.21 ± 1.23	3.47 ± 0.13 ± 0.19	0.19 ± 0.01 ± 0.03	0.047 ± 0.002 ± 0.007

- Here, systematic uncertainties are still subdominant at 140/fb
- The main systematic error contribution is the signal model (D^{**}, $D^{(*)}$ form factors)

What can Belle II do?

- Repeating the lepton and hadron moment analyses on Belle II data samples will lead to some improvement in the accuracy (mostly for lepton moments)
- However, does this lead to an improvement in $|V_{cb}|$ and/or in the OPE parameters?

Does it help?

Current default fit:

```
FCN=   13.46754    FROM MINOS     STATUS=SUCCESSFUL    698 CALLS       959 TOTAL
                   EDM=   0.64E-06     STRATEGY= 1      ERROR MATRIX ACCURATE

 EXT PARAMETER                        PARABOLIC          MINOS ERRORS
 NO.   NAME         VALUE              ERROR        NEGATIVE        POSITIVE
  1    br          10.645             0.16027      -0.16025         0.16030
  2    mb           4.5510            0.28713E-01  -0.30340E-01     0.30309E-01
  3    mc           1.0014            0.27359E-01  -0.28595E-01     0.28609E-01
  4    mu2pi       0.41203            0.73521E-01  -0.73623E-01     0.73471E-01
  5    rhoD        0.15359            0.43028E-01  -0.42973E-01     0.43142E-01
  6    mu2G        0.33686            0.65477E-01  -0.65541E-01     0.65605E-01
  7    rhoLS      -0.14623            0.97875E-01  -0.97913E-01     0.97871E-01
  8    vcb         0.42416E-01        0.84703E-03  -0.83975E-03     0.86070E-03
```

Divide all Belle errors by a factor 2:

```
FCN=   21.85681    FROM MINOS     STATUS=SUCCESSFUL    662 CALLS       900 TOTAL
                   EDM=   0.16E-05     STRATEGY= 1      ERROR MATRIX ACCURATE

 EXT PARAMETER                        PARABOLIC          MINOS ERRORS
 NO.   NAME         VALUE              ERROR        NEGATIVE        POSITIVE
  1    br          10.674             0.11090      -0.11090         0.11090
  2    mb           4.5475            0.30988E-01  -0.30183E-01     0.30148E-01
  3    mc           1.0027            0.28923E-01  -0.28578E-01     0.28584E-01
  4    mu2pi       0.43533            0.71915E-01  -0.72010E-01     0.71814E-01
  5    rhoD        0.15989            0.42780E-01  -0.42668E-01     0.42799E-01
  6    mu2G        0.32749            0.65620E-01  -0.65305E-01     0.65384E-01
  7    rhoLS      -0.14599            0.97893E-01  -0.97868E-01     0.97817E-01
  8    vcb         0.42623E-01        0.80460E-03  -0.79087E-03     0.81180E-03
```

- Error on $|V_{cb}|$ improves only from 2.0% to 1.9%

Summary and conclusions

- A number of moment measurements are available from different experiments
  - The data is consistent and consistent with theory
  - They allow to determine $|V_{cb}|$ at the level of 2%
- Some improvement in the moment measurements is possible if these analyses are repeated on the Belle II data set but this will have little impact on $|V_{cb}|$ and the OPE parameters
- Clearly, we need new ideas to improve $|V_{cb}|$ inclusive
  - Can the theory errors be reduced?
  - Which observables (not used in the analysis so far) should be measured at Belle II to better constrain the OPE?
  - Can LHCb measure moments at $E_{cut} = 0$?

BACKUP