Quark masses from lattice QCD

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Challenges in Semileptonic *B* Decays

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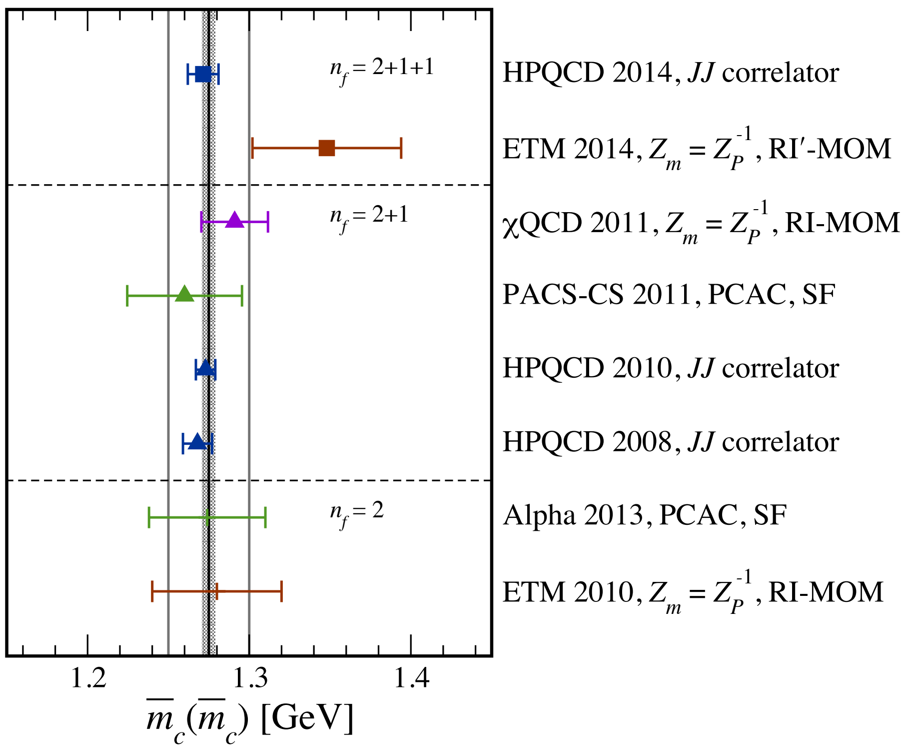
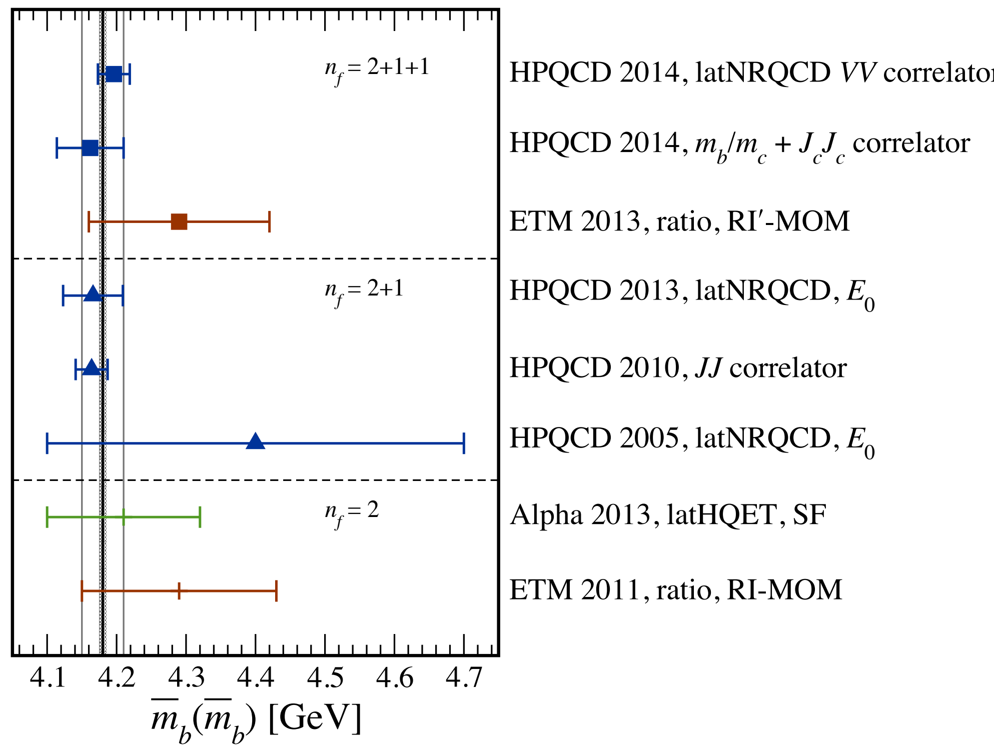
IAS TUM Institute for Advanced Study

Summary Plots





- Narrow vertical bands are PDG “weighted averages” of continuum results w/ ETM $n_f = 2$ and HPQCD 2010; wider vertical lines show ± in PDG “evaluation”.

- Error bar widened because of QED effects; concerns of alia et Hoang et alia.

Questions arising from these plots

- Why are the HPQCD results have such smaller errors than the others?
- To what extent are QED effects taken into account?
- Do the criticisms of arXiv:1102.2264 apply to lattice QCD?
    - Sensitivity to higher-order corrections in current-current correlators!
    - Treatment of $e^+e^- \to c\bar{c}$ in regions with sparse or untagged data!
- How are the calculations actually done?
- What are the prospects for further improvement?

Basic Steps

- Objective: $\overline{\text{MS}}$ mass at some specified scale μ.
    - Objection: $\overline{\text{MS}}$ mass defined only in perturbation theory; overruled for now.
- First step: adjust bare lattice masses so that various hadron masses agree with experiment

$\frac{1}{2}(m_u+m_d)$	m_s	$m_d - m_u$	m_c	m_b	m_t	θ	α_s
M_π^2	M_K^2	$\Delta M_K^2 - \text{em}$	D_s or η_c	B_s or η_b	—	0	w_0 or r_1

- Cross check: rest of low-lying hadron masses agree with experiment.
- Remaining steps: compute other observable(s) nonperturbatively to build a bridge to the $\overline{\text{MS}}$ mass.

Error Budgets

HPQCD 2014, $m_c(3\text{ GeV})$, m_b/m_c

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TABLE IV. Error budget [31] for the c mass, QCD coupling, and the ratios of quark masses m_c/m_s and m_b/m_c from the $n_f = 4$ simulations described in this paper. Each uncertainty is given as a percentage of the final value. The different uncertainties are added in quadrature to give the total uncertainty. Only sources of uncertainty larger than 0.05% have been listed.

	$m_c(3)$	$\alpha_{\overline{\mathrm{MS}}}(M_Z)$	m_c/m_s	m_b/m_c
Perturbation theory	0.3	0.5	0.0	0.0
Statistical errors	0.2	0.2	0.3	0.3
$a^2 \to 0$	0.3	0.3	0.0	1.0
$\delta m_{uds}^{\mathrm{sea}} \to 0$	0.2	0.1	0.0	0.0
$\delta m_c^{\mathrm{sea}} \to 0$	0.3	0.1	0.0	0.0
$m_h \neq m_c$ [Eq. (15)]	0.1	0.1	0.0	0.0
Uncertainty in w_0, w_0/a	0.2	0.0	0.1	0.4
α_0 prior	0.0	0.1	0.0	0.0
Uncertainty in m_{η_s}	0.0	0.0	0.4	0.0
$m_h/m_c \to m_b/m_c$	0.0	0.0	0.0	0.4
δm_{η_c}: electromag., annih.	0.1	0.0	0.1	0.1
δm_{η_b}: electromag., annih.	0.0	0.0	0.0	0.1
Total:	0.64%	0.63%	0.55%	1.20%

- Statistical errors are tiny.
- Higher-order PT: fit extra terms
  - next 1 in β and γ running;
  - next 8–15 in moments;
  - offsets μ dependence.
- Extrapolation errors controlled by precision plus numerous (constrained) fit parameters.
- EM part of mass subtracted.

ETM 2014, $m_c(2\text{ GeV})$

NPB **887** (2014) 19

Source	%	ETM/ HPQCD
Stats+fits	2.6	13
χPT	0.2	—
a^2	0.4	1.3
Matching	1.4	NA
PT	1.3	4.2
tuning	—	?
QED	—	?

- Statistics are a killer:
  - 60–150 configs (ETM) vs. 200–1020 (HPQCD←MILC).
- Staggered fermions are fast enough to allow
  - physical light quarks;
  - more lattice spacings.
- Precision brings PT under control.

Quantum Field Theory

Quark Masses

- In a QFT like QCD quark masses really mean one of
  - bare Lagrangian mass;
  - renormalized mass:
    - mass independent scheme, such as $\overline{\mathrm{MS}}$ or Schrödinger functional;
    - mass dependent scheme, such as RI-MOM in Landau gauge;
    - additive scheme, such as kinetic scheme, potential subtracted, 1S, ...
  - Hadron mass with binding energy subtracted off.

Perturbative Matching

- In perturbation theory, assert that lattice and $\overline{\mathrm{MS}}$ yield the same pole mass:

$$Z_{\bar{m}}\bar{m} = m_{\text{pole}} = Z_{m_0} m_0$$

Although m_{pole} has infrared problems,* it is IR finite & gauge independent, so the matching is well defined it PT.

*renormalons, instantons, confinement

- Drawbacks: lattice perturbation theory for Z_{m0} is difficult:
  - two loops at most;
  - sometimes (e.g., lattice HQET, Wilson or nonrelativistic quarks) additive renormalization is needed too: $m_{\text{pole}} = Z_{m_0}(m_0 - m_{\text{crit}})$
- Move to methods that are more convenient and, hence, more accurate.

Nonperturbative Matching I

- Multiplicative renormalization can be computed nonperturbatively from the scalar or pseudoscalar density:

$$Z_{m_0} = \frac{Z_V}{Z_S} = \frac{Z_A}{Z_P}$$

- Noether currents on the lattice are point-split and, hence, noisier.

- Local currents have $Z_{V,A} \neq 1$, but Z_V is easy to obtain from the flavor charge, and Z_A can be obtained from chiral Ward-Takahashi identities. Matching.

- The QFT mass, and Z_S & Z_P too, have an anomalous dimension: need to define a renormalization scheme, and introduce a renormalization scale.

Nonperturbative Matching II

- When the bare mass suffers from additive renormalization, one can define

$$m_c' - m_s' = (M_D - M_K)\frac{\langle K|\bar{s}\gamma^0 c|D\rangle}{\langle K|\bar{s}c|D\rangle} \qquad \text{PCVC}$$

$$m_c' + m_s' = M_{D_s}\frac{\langle 0|\bar{s}\gamma^0\gamma^5 c|D_s\rangle}{\langle 0|\bar{s}\gamma^5 c|D_s\rangle} \qquad \text{PCAC}$$

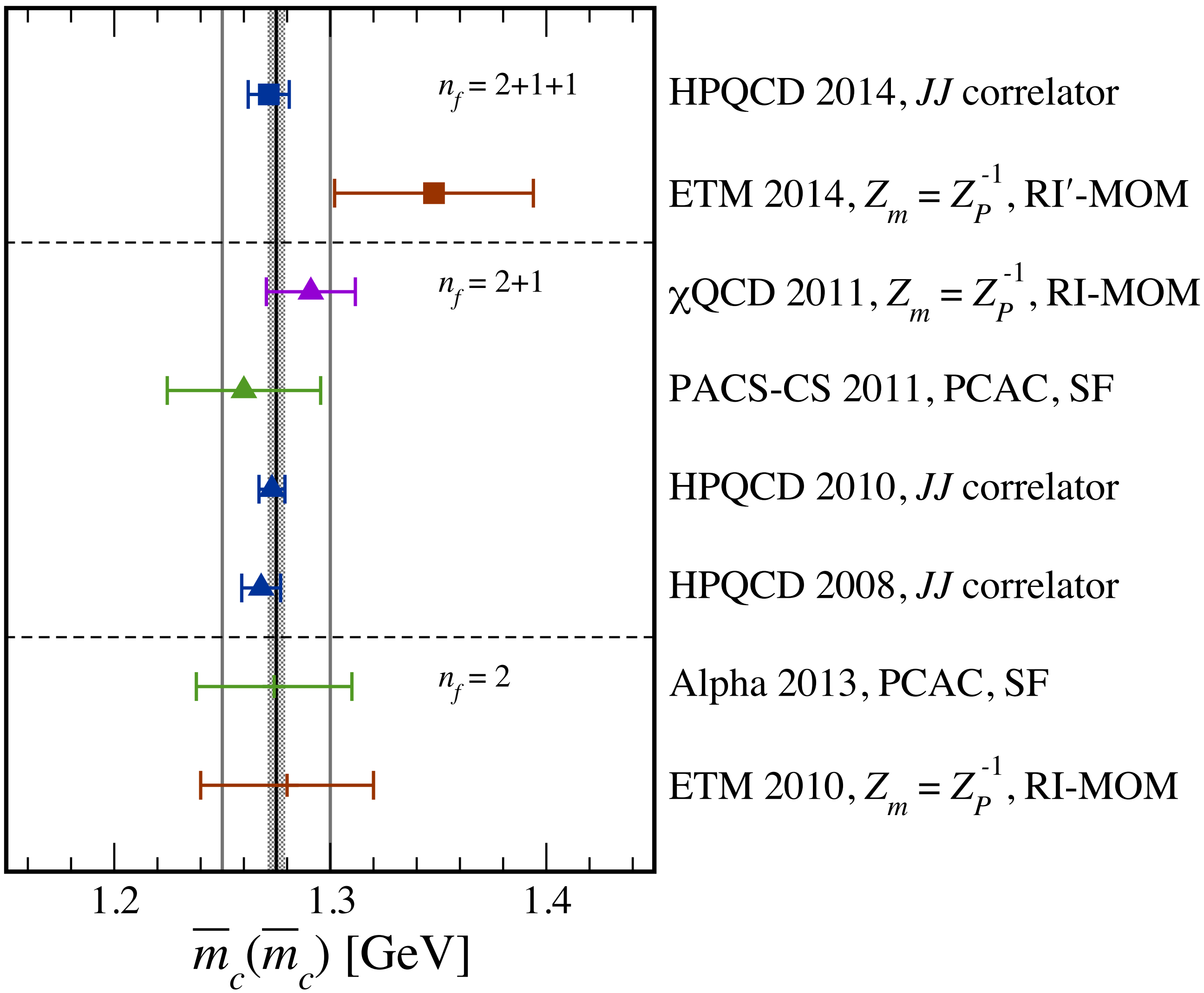
using flavor nonsinglet PCAC to avoid a term from the axial anomaly.

- States chosen here are for illustration: in SF method use finite-volume states.

- As before, the renormalization scheme of m' is inherited from Z_V/Z_S & Z_A/Z_P.

- Can use fictitious mesons with equal-mass flavors to avoid PCVC step.

Summary Plot for Charm



- No additive renorm:
  - twisted-mass (ETM);
  - overlap (χQCD);
  - domain-wall (RBC);
  - staggered (MILC).
- Additive renorm:
  - Wilson/♣ (Alpha).

Current-current Correlator

Moments of the Charmonium Correlator

- The non-lattice heavy-quark masses with the smallest error bars come from moments of the charmonium correlator:

$$\tilde{G}_n = \left(\tfrac{2}{3}\right)^2 \frac{12\pi^2}{n!} \left.\frac{d^n\Pi(s)}{ds^n}\right|_{s=0}$$

- Experiments measure $\Pi(s)$ in $e^+e^- \to c\bar{c}$ hadrons.

- The same idea can be exploited at spacelike momentum transfer

- Then $\Pi(s)$, $s < 0$, is the Fourier transform of the current-current correlation function.

$$G_n = \sum_t (t/a)^n \sum_{\boldsymbol{x}} \langle J(\boldsymbol{x},t) J(\boldsymbol{0},0)\rangle$$

- Calculable in lattice QCD [Bochkarev & de Forcrand, hep-lat/9505025]:

- In the correlator,

$$G_n = \sum_t (t/a)^n \sum_{\boldsymbol{x}} \langle J(\boldsymbol{x},t) J(\boldsymbol{0},0) \rangle$$

J is mP or V^0, for example.

- Take continuum limit of these moments & use continuum $\overline{\text{MS}}$ perturbation theory for several G_n ($n \leq 22$) to extract α_s and m_Q.

- Complication: lattice has finite time extent with, say, periodic boundary conditions:

  - at large t, the correlator saturates to the lowest-lying state (η_c or J/ψ);

  - HPQCD replaces correlator at these large t with this state.

- Study of several moments allows for cross checks.

Summary

Perspective

- Any lattice-QCD average a la FLAG would be dominated by HPQCD results.
- Higher statistics allows a much richer set of tests, cross-checks, and modeling of higher-order PT.
    - Fermilab/MILC, who use the same ensembles, should (and has started) a similar study of quarkonium correlators. (We do have m_c/m_s.)
- In my view, the QED effects and issues of the perturbative series have been addressed (in the current-current correlator analyses).
- Thus, the PDG "**OUR EVALUATION**" error bar is too large (*i.e.*, out of date).
- A thorough, professional (a la HFAG, FLAG, PDG!) averaging is called for.

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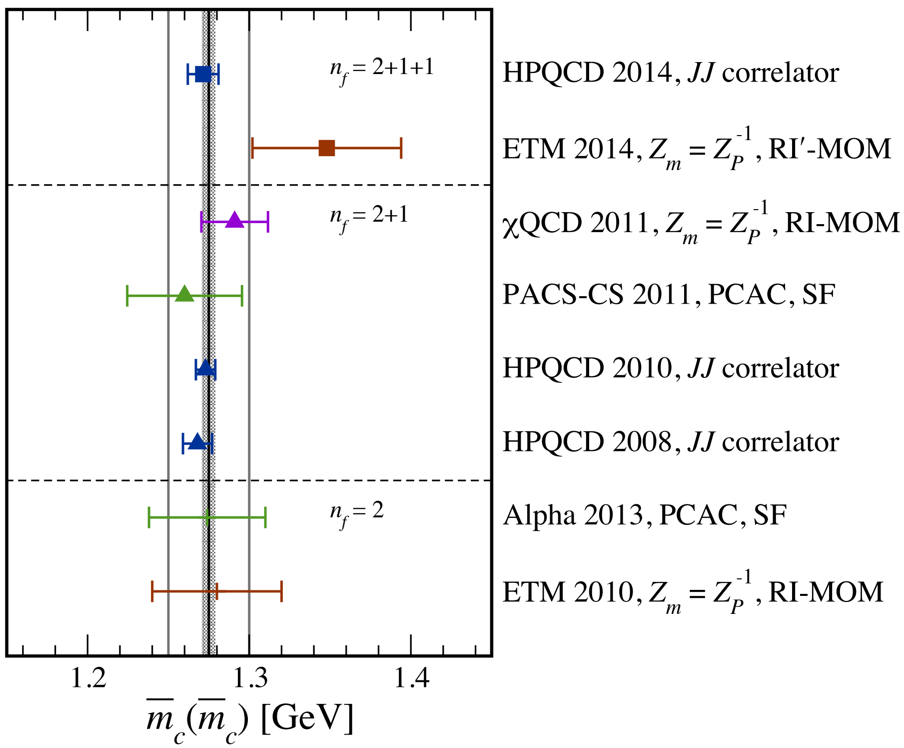

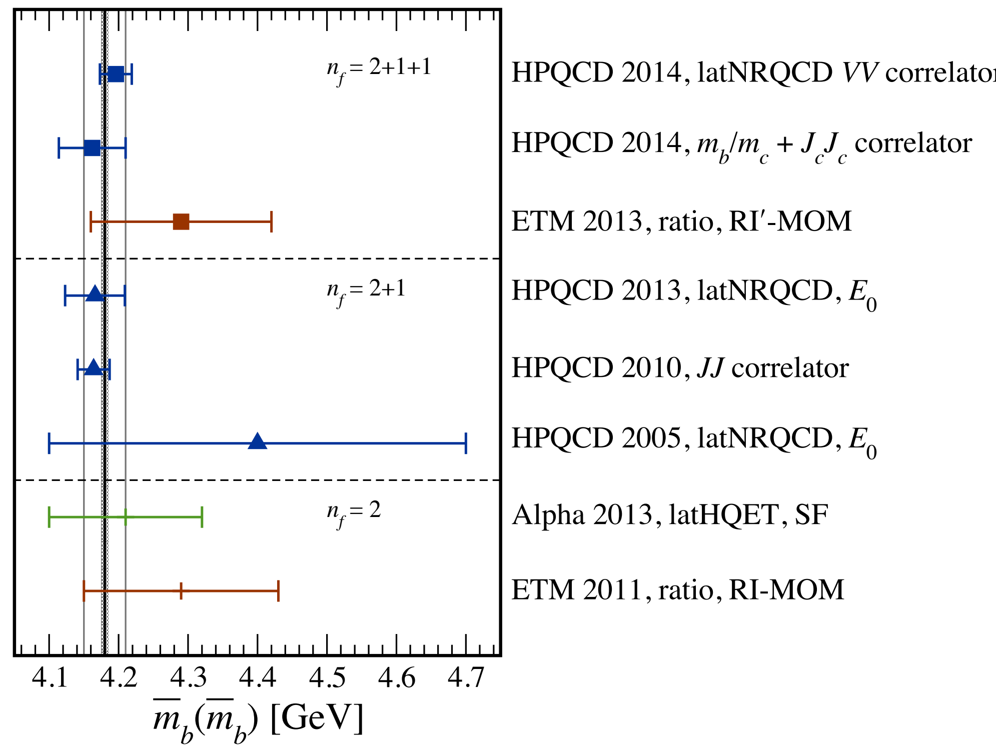

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Projections

Lepage, Mackenzie, Peskin, arXiv:1404.0319 [hep-ph]

	$\delta m_b(10)$	$\delta \alpha_s(m_Z)$	$\delta m_c(3)$	δ_b	δ_c	δ_g
current errors [10]	0.70	0.63	0.61	0.77	0.89	0.78
+ PT	0.69	0.40	0.34	0.74	0.57	0.49
+ 0.03 fm	0.30	0.53	0.53	0.38	0.74	0.65
+ 0.023 fm	0.14	0.35	0.53	0.20	0.65	0.43
+ PT + 0.03 fm	0.28	0.17	0.21	0.30	0.27	0.21
+ PT + 0.023 fm	0.12	0.14	0.20	0.13	0.24	0.17
+ PT + 0.023 fm + Stat100	0.09	0.08	0.20	0.10	0.22	0.09
ILC goal				0.30	0.70	0.60