INCLUSIVE
WHERE WE STAND

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MITP WORKSHOP 20/4/2015

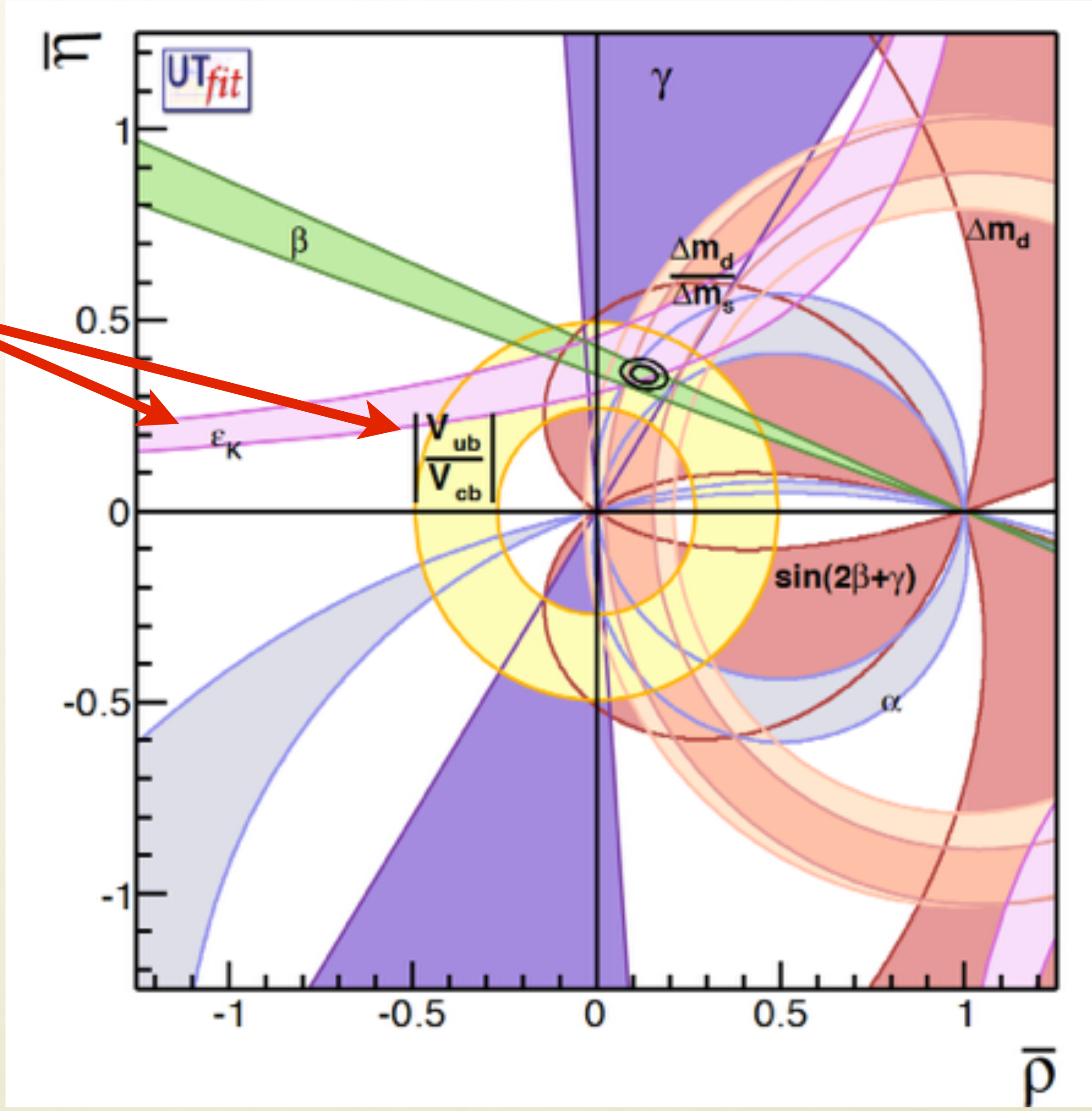
IMPORTANCE OF $|V_{cb}|$

V_{cb} plays an important role in the determination of UT

$$\varepsilon_K \approx x|V_{cb}|^4 + \dots$$

and in the prediction of FCNC:

$$\propto |V_{tb}V_{ts}|^2 \simeq |V_{cb}|^2\left[1 + O(\lambda^2)\right]$$



Since several years, exclusive decays prefer smaller $|V_{ub}|$ and $|V_{cb}|$

Inclusive decays: basics

- ***Simple idea:*** inclusive decays do not depend on final state, long distance dynamics of the B meson factorizes. An OPE allows to express it in terms of B meson matrix elements of local operators

- The Wilson coefficients are perturbative, matrix elements of local ops parameterize non-pert physics: ***double series in α_s, Λ/m_b***

- Lowest order: decay of a free b, linear Λ/m_b absent. Depends on $m_{b,c}$, 2 parameters at $O(1/m_b^2)$, 2 more at $O(1/m_b^3)$...

$$\mu_\pi^2(\mu) = \frac{1}{2M_B}\left\langle B\left|\bar{b}\left(i\vec{D}\right)^2 b\right|B\right\rangle_\mu \qquad \mu_G^2(\mu) = \frac{1}{2M_B}\left\langle B\left|\bar{b}\frac{i}{2}\sigma_{\mu\nu}G^{\mu\nu}b\right|B\right\rangle_\mu$$

OBSERVABLES IN THE OPE

$$M_i = M_i^{(0)} + \frac{\alpha_s(\mu)}{\pi} M_i^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 M_i^{(2)} + \left(M_i^{(\pi,0)} + \frac{\alpha_s(\mu)}{\pi} M_i^{(\pi,1)}\right)\frac{\mu_\pi^2}{m_b^2}$$
$$+ \left(M_i^{(G,0)} + \frac{\alpha_s(\mu)}{\pi} M_i^{(G,1)}\right)\frac{\mu_G^2}{m_b^2} + M_i^{(D)}\frac{\rho_D^3}{m_b^3} + M_i^{(LS)}\frac{\rho_{LS}^3}{m_b^3} + \ldots$$

NEW

OPE valid for inclusive enough measurements, away from perturbative singularities ➠ semileptonic width, moments

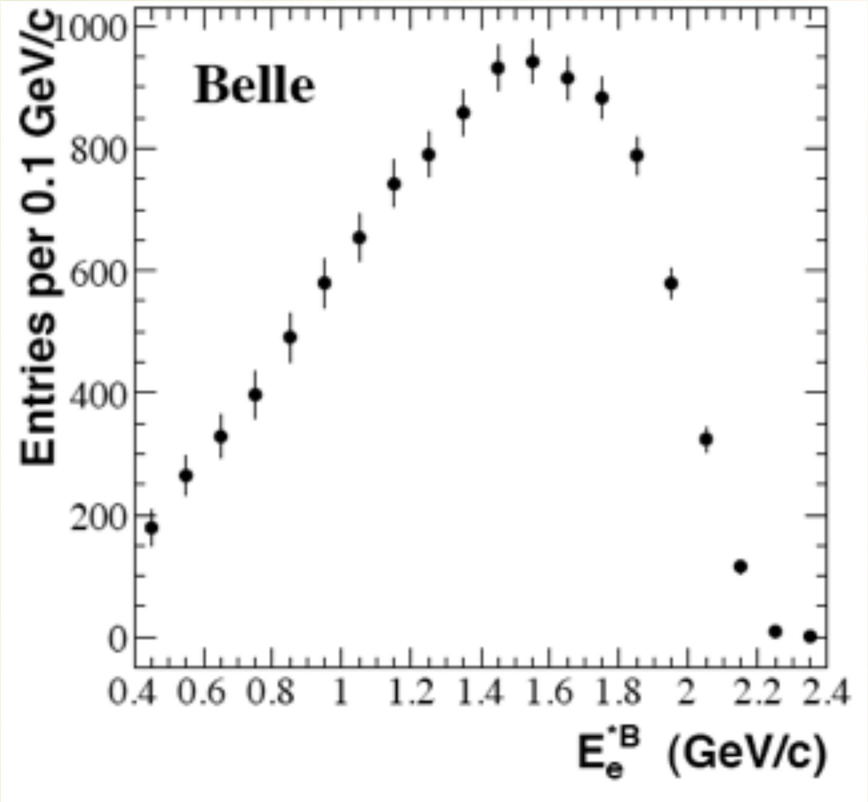
Current fits includes 6 non-pert parameters

$m_{b,c}$ $\quad$ $\mu^2{}_{\pi,G}$ $\quad$ $\rho^3{}_{D,LS}$

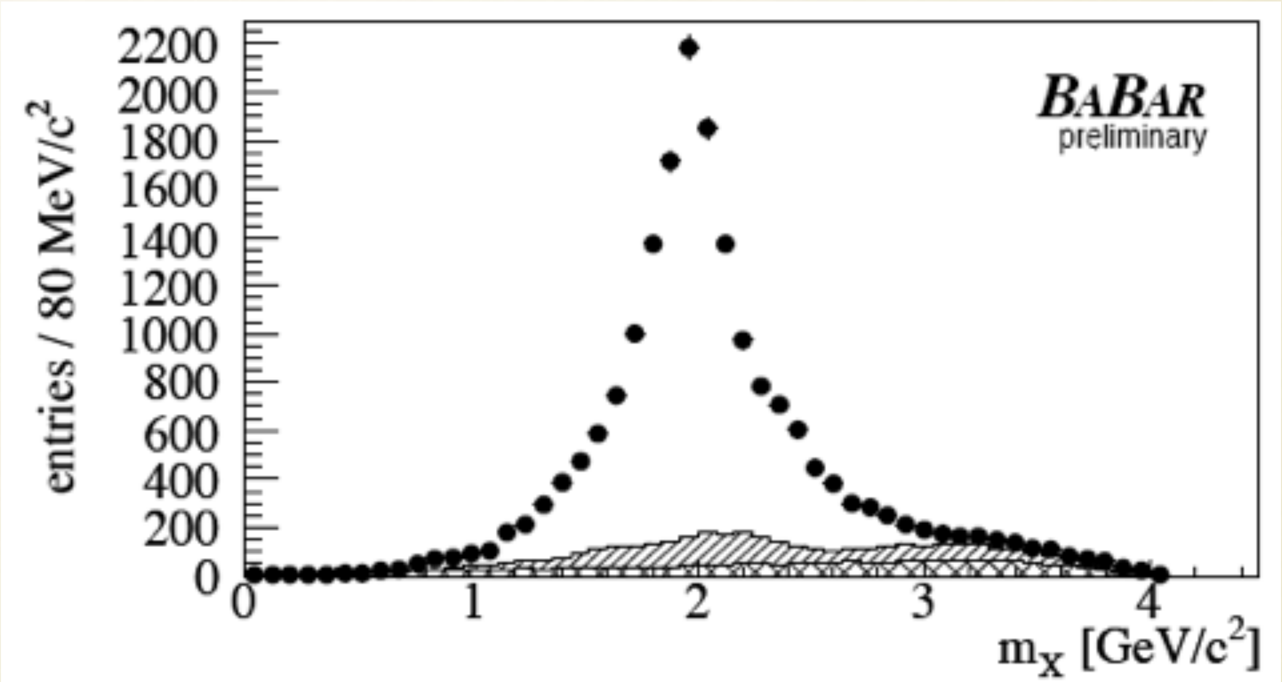
and all known corrections up to $O(\Lambda^3/m_b^3)$

Extraction of the OPE parameters

E_l spectrum



hadronic mass spectrum



*Global **shape** parameters (first moments of the distributions) tell us about m_b, m_c and the B structure, total **rate** about $|V_{cb}|$*

OPE parameters describe universal properties of the B meson and of the quarks → useful in many applications (rare decays, V_{ub},...)

Let's focus on:

1. Status of higher order corrections
2. Estimate of residual theoretical errors
3. How the fit is actually done
   (assumptions, additional inputs,…)
4. Electroweak corrections

HIGHER ORDER EFFECTS

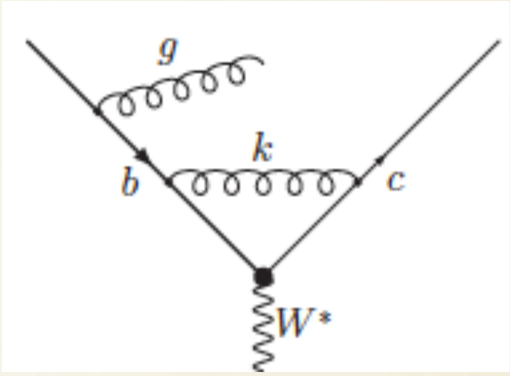
- Reliability of the method depends on our ability to control higher order effects. Quark-hadron duality violation would manifest as inconsistency in the fit.

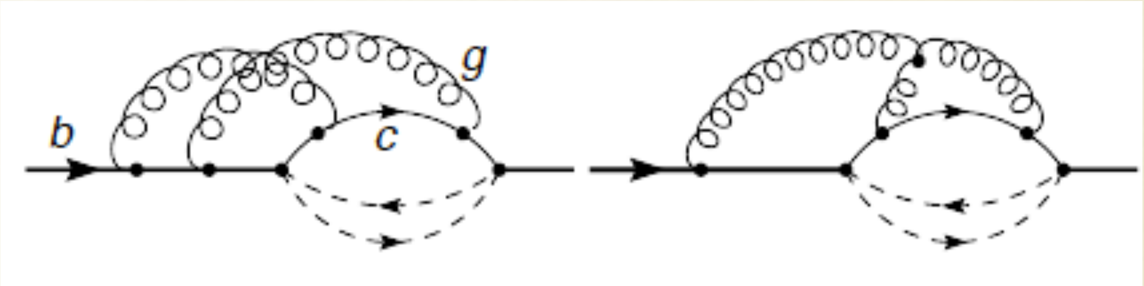
- **Purely perturbative corrections** complete at NNLO, small residual error (kin scheme) Melnikov, Biswas, Czarnecki, Pak, PG

- **Higher power corrections** $O(1/m_Q^{4,5})$ known
Mannel, Turczyk, Uraltsev 2010 See Sascha's talk

- **Mixed corrections** perturbative corrections to power suppressed coefficients completed at $O(\alpha_s/m_b^2)$
Becher, Boos, Lunghi, Alberti, Ewerth, Nandi, PG

NNLO perturbative corrections





- Complete 2loop corrections to width and moments with cuts now known, either in expansion m_c/m_b or numerically Biswas-Melnikov, Pak-Czarnecki

$$d\Gamma = \Gamma_0 \left[dF_0 + \frac{\alpha_s(m_b)}{\pi} dF_1 + (\frac{\alpha_s}{\pi})^2 (\beta_0 \, dF_{\rm BLM} + dF_2) + \ldots \right]$$

- Non-BLM effects ~15-30% of BLM ones when $\alpha_s(m_b)$ is used, residual th error on V_{cb} O(0.5%).
- Strong cancellations between different contributions make NNLO to lept moments small: non-accidental, numerical accuracy crucial PG 2011

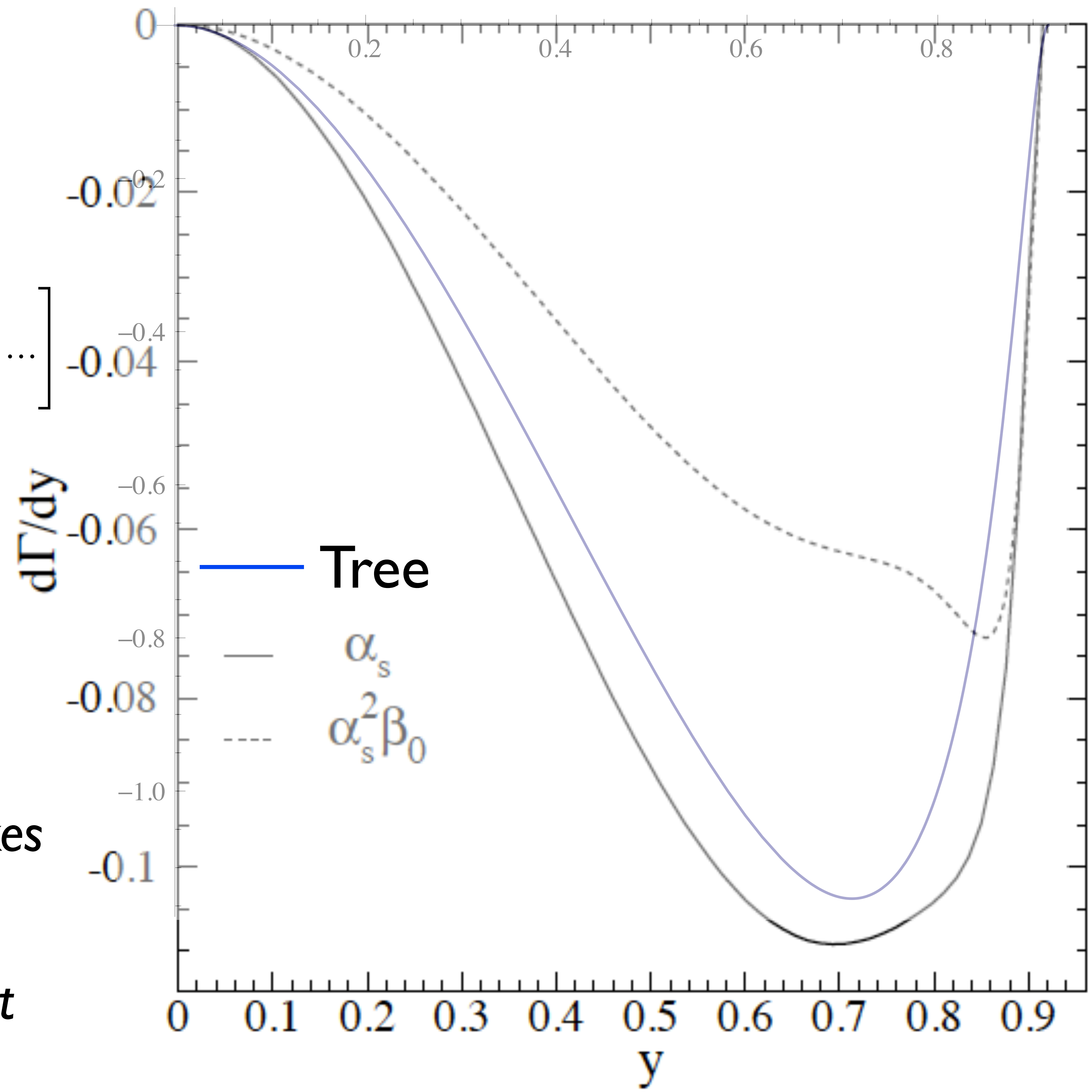
$$\langle E_l \rangle_{E_l > 1 \rm GeV} = 1.54\,{\rm GeV} \left[1 + (0.96_{den} - 0.93)\frac{\alpha_s}{\pi} + (0.48_{den} - 0.46)\,\beta_0 \left(\frac{\alpha_s}{\pi}\right)^2 \right.$$
$$\left. + [1.69(7) - 1.75(9)_{den}] \left(\frac{\alpha_s}{\pi}\right)^2 + O(1/m_b^2, \alpha_s^3) \right]$$

$$\ell_2 = \langle E_\ell^2 \rangle - \langle E_\ell \rangle^2 = (2.479 - 2.393)\,{\rm GeV}^2 = 0.087\,{\rm GeV}^2.$$

Lepton energy spectrum

$$\langle E_\ell^n \rangle = \frac{L_n^{(0)}}{L_0^{(0)}} \left[1 + \frac{\alpha_s(m_b)}{\pi} \eta_n^{(1)} + \right.$$
$$\left. + \left(\frac{\alpha_s}{\pi}\right)^2 \left(\beta_0\, \eta_n^{(\mathrm{BLM})} + \eta_n^{(2)} - \eta_n^{(1)} \frac{L_0^{(1)}}{L_0^{(0)}} \right) + \eta_n^{(pow)} + \ldots \right]$$

$$\boxed{\eta_i^{(a)} = \frac{L_i^{(a)}}{L_i^{(0)}} - \frac{L_0^{(a)}}{L_0^{(0)}}}$$



Suppression of hard gluon radiation makes spectrum similar at any pert order.

Correction η_i vanishes close to endpoint and for same spectrum

NNLO results

PG, JHEP 9(2011)055

	ℓ_1	ℓ_2	ℓ_3	R^*
	$\mu = 0$			
tree	1.5674	0.0864	-0.0027	0.8148
$1/m_b^3$	1.5426	0.0848	-0.0010	0.8003
$O(\alpha_s)$	1.5398	0.0835	-0.0010	0.8009
$O(\beta_0\alpha_s^2)$	1.5343	0.0818	-0.0009	0.7992
$O(\alpha_s^2)$	1.5357(2)	0.0821(6)	-0.0011(16)	0.7992(1)
	$\mu = 1\text{GeV}$			
$O(\alpha_s)$	1.5455	0.0858	-0.0003	0.8029
$O(\beta_0\alpha_s^2)$	1.5468	0.0868	0.0005	0.8035
$O(\alpha_s^2)$	1.5466(2)	0.0866(6)	0.0002(16)	0.8028(1)
$O(\alpha_s^2)^*$	–	0.0865	0.0004	–
tot error [6]	0.0113	0.0051	0.0022	

E$_{cut}$=1GeV, m_c/m_b=0.25

	$\mu = 1\text{GeV},\ m_c^{\overline{\rm MS}}(3\text{GeV})$			
	ℓ_1	ℓ_2	ℓ_3	R^*
tree	1.6021	0.0940	-0.0043	0.8296
$1/m_b^3$	1.5748	0.0922	-0.0020	0.8159
$O(\alpha_s)$	1.5613	0.0894	-0.0004	0.8118
$O(\beta_0\alpha_s^2)$	1.5629	0.0904	0.0004	0.8125
$O(\alpha_s^2)$	1.5571(4)	0.0890(9)	-0.0008(25)	0.8090(2)
$O(\alpha_s^2)^*$	–	0.0889	0.0006	–

Small corrections. Cancellations may be partially spoiled by choice of scheme

	$\mu = 0$			$\mu = 1\text{GeV}$		
	h_1	h_2	h_3	h_1	h_2	h_3
LO	4.345	0.198	-0.02	4.345	0.198	-0.02
$1/m_b^3$	4.452	0.515	4.90	4.452	0.515	4.90
$O(\alpha_s)$	4.563	0.814	5.96	4.426	0.723	4.50
$O(\beta_0\alpha_s^2)$	4.701	1.105	6.85	4.404	0.894	4.08
$O(\alpha_s^2)$	4.682(1)	1.066(3)	6.69(4)	4.411(1)	0.832(4)	4.08(4)
tot error [6]				0.149	0.501	1.20

$O(\alpha_s/m_b^2)$ EFFECTS

Boos,Becher,Lunghi 2007
Ewerth,Nandi, PG 2009
Alberti,Ewerth,Nandi,PG 2012
Alberti,Nandi,PG 2013

Hadronic tensor $$W^{\alpha\beta} = \frac{(2\pi)^3}{2m_B} \sum_{X_c} \delta^4(p_b - q - p_X) \langle \bar{B} | J_L^{\dagger\alpha} | X_c \rangle \langle X_c | J_L^\beta | \bar{B} \rangle$$

$$m_b W^{\alpha\beta} = -W_1 g^{\alpha\beta} + W_2 v^\alpha v^\beta + i W_3 \epsilon^{\alpha\beta\rho\sigma} v_\rho \hat{q}_\sigma + W_4 \hat{q}^\alpha \hat{q}^\beta + W_5 (v^\alpha \hat{q}^\beta + v^\beta \hat{q}^\beta)$$

$$W_i = W_i^{(0)} + \frac{\mu_\pi^2}{2m_b^2} W_i^{(\pi,0)} + \frac{\mu_G^2}{2m_b^2} W_i^{(G,0)} + \frac{C_F \alpha_s}{\pi} \left[W_i^{(1)} + \frac{\mu_\pi^2}{2m_b^2} W_i^{(\pi,1)} + \frac{\mu_G^2}{2m_b^2} W_i^{(G,1)} \right]$$

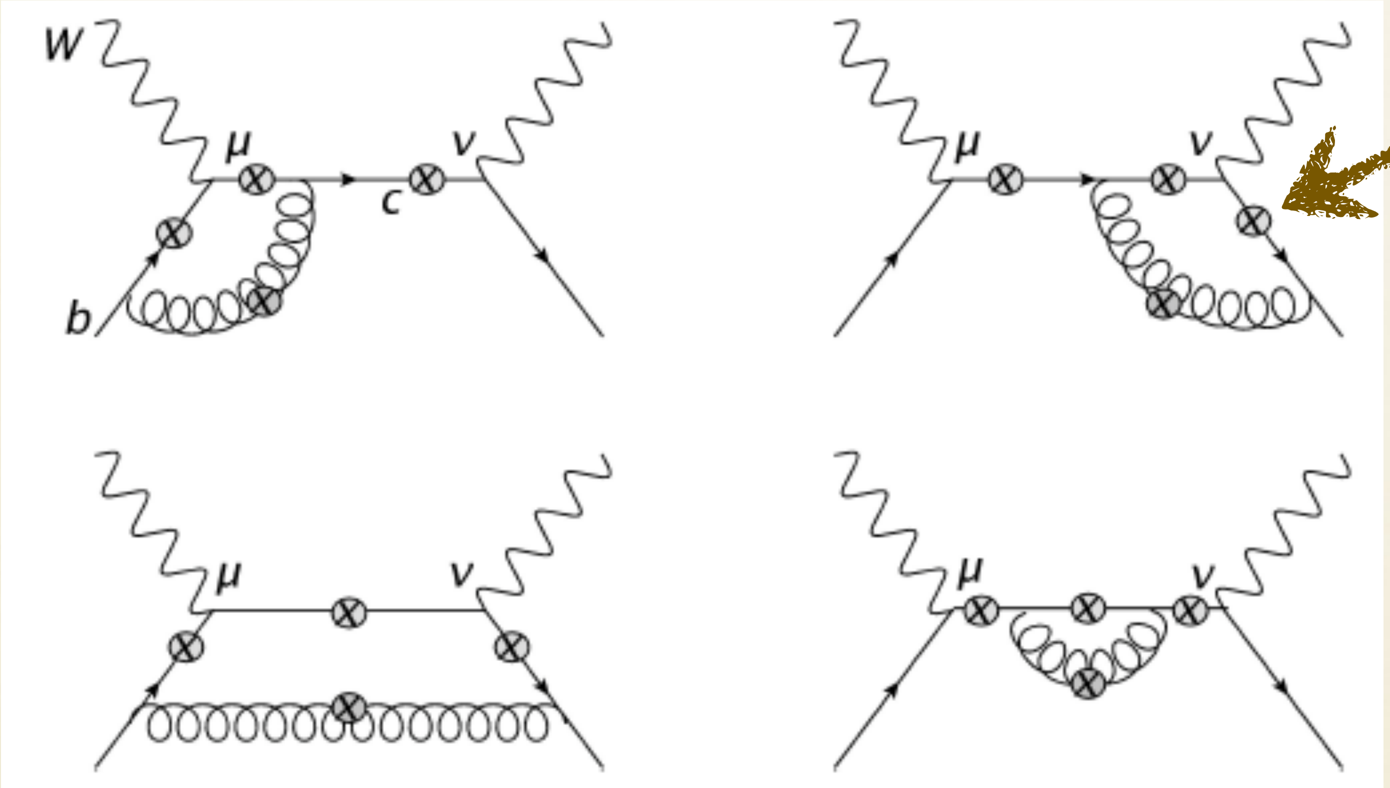
$W_i^{(\pi,n)}$ can be computed using **reparameterization invariance** which relates different orders in the HQET: e.g. for *i=3* at all orders

$$W_3^{(\pi,n)} = \frac{5}{3} \hat{q}_0 \frac{dW_3^{(n)}}{d\hat{q}_0} - \frac{\hat{q}^2 - \hat{q}_0^2}{3} \frac{d^2 W_3^{(n)}}{d\hat{q}_0^2}$$

Manohar 2010

Proliferation of power divergences, up to $1/u^3$,
and complex kinematics (q^2, q_0, m_c, m_b) $W_i^{(G,1)}$ requires proper matching.

Matching at O(α_s)



possible gluon insertions

QCD

HQET

$$\frac{2i}{\pi}\int d^4x\, e^{-iq\cdot x}\, T[J_L^{\dagger\mu}(x)J_L^{\nu}(0)] = \sum_i c^{(i)\mu\nu}_{\{\alpha\}}(v,q)\, O_i^{\{\alpha\}}(0)$$

Taylor expansion around on-shell b quark matched onto HQET local operators. Analytic formulae. RPI relations reproduced. Unlike μ_π^2, μ_G^2 gets renormalized, therefore Wilson coefficients are scale-dependent.

Numerical results

In on-shell scheme (m_b=4.6GeV, m_c=1.15GeV) without cuts

$$\Gamma_{B\to X_c\ell\nu} = \Gamma_0 \left[\left(1 - 1.78\,\frac{\alpha_s}{\pi}\right)\left(1 - \frac{\mu_\pi^2}{2m_b^2}\right) - \left(1.94 + 2.42\,\frac{\alpha_s}{\pi}\right)\frac{\mu_G^2(m_b)}{m_b^2}\right]$$

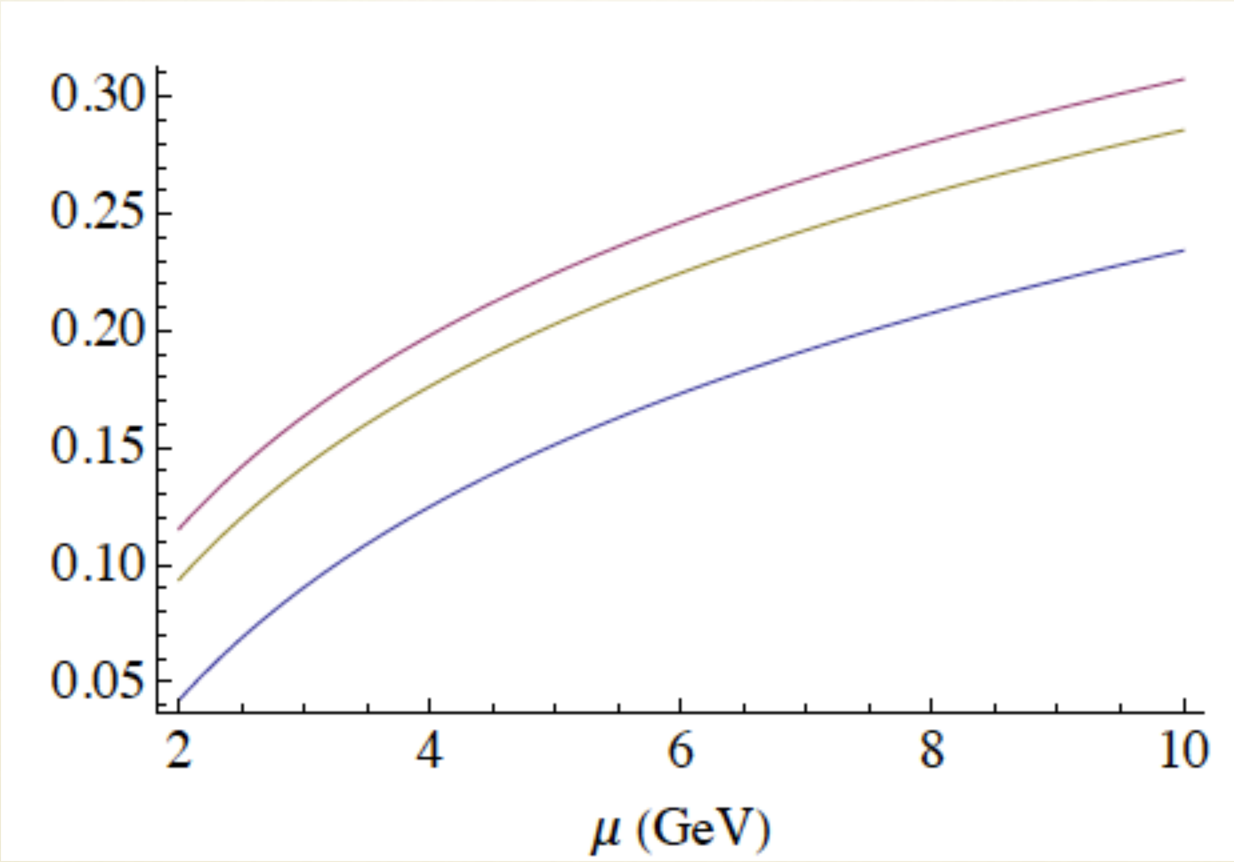
$$\langle E_\ell \rangle = 1.41\mathrm{GeV} \left[\left(1 - 0.02\,\frac{\alpha_s}{\pi}\right)\left(1 + \frac{\mu_\pi^2}{2m_b^2}\right) - \left(1.19 + 4.20\,\frac{\alpha_s}{\pi}\right)\frac{\mu_G^2(m_b)}{m_b^2}\right]$$

$$\ell_2 = 0.183\,\mathrm{GeV}^2 \left[1 - 0.16\,\frac{\alpha_s}{\pi} + \left(4.98 - 0.37\,\frac{\alpha_s}{\pi}\right)\frac{\mu_\pi^2}{m_b^2} - \left(2.89 + 8.44\,\frac{\alpha_s}{\pi}\right)\frac{\mu_G^2(m_b)}{m_b^2}\right]$$

Similar results in the kinetic scheme. NLO corrections generally *O(15-20%)* of tree level coefficients, **shifts in some cases larger than experimental error**. Impact on V_{cb} requires new fit of semileptonic moments.

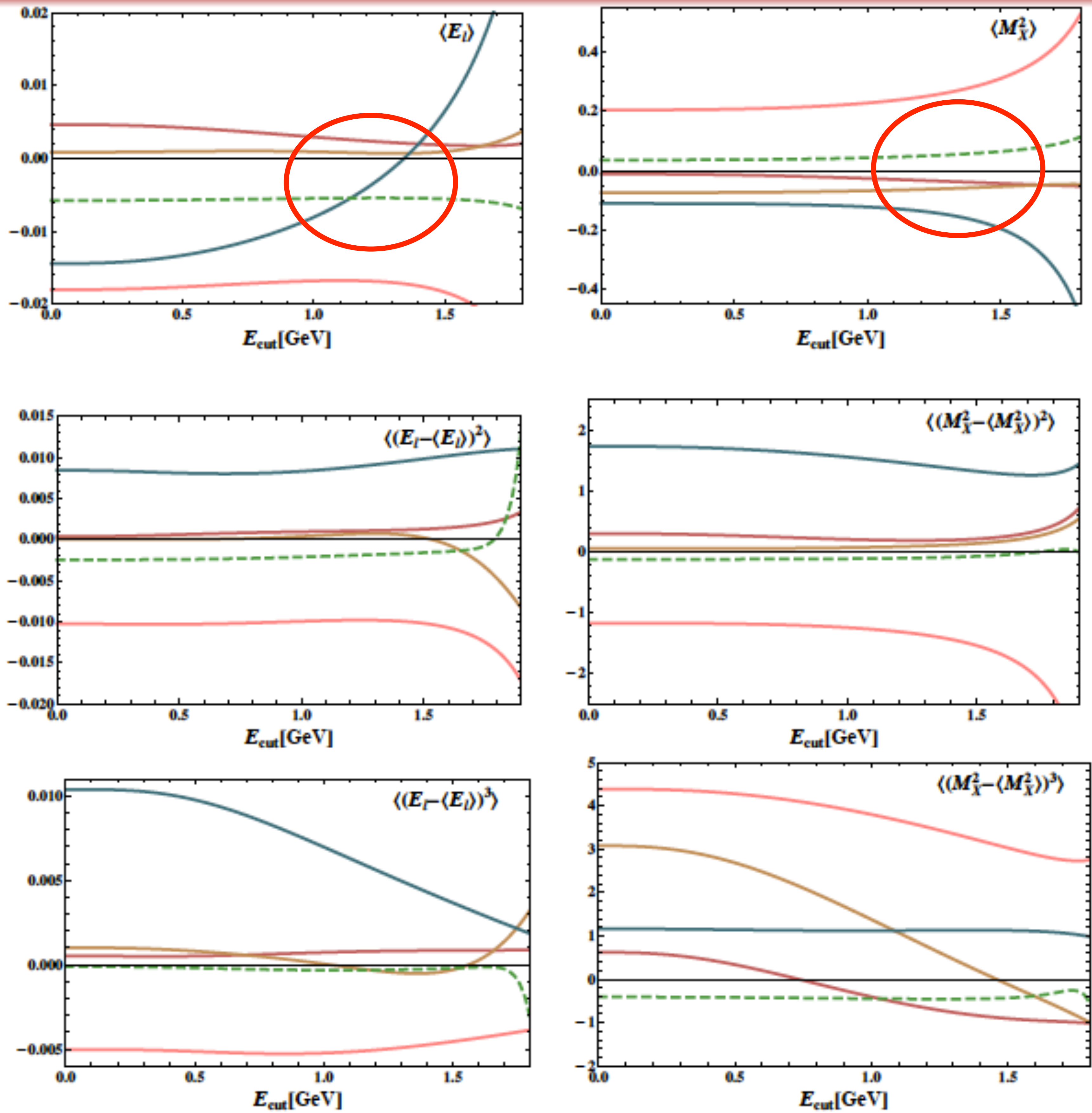
Mannel, Pivovarov, Rosenthal (1405.5072) have computed the μ_G^2 correction to the width in the limit m_c=0 and find compatible result. Analytic checks under way.

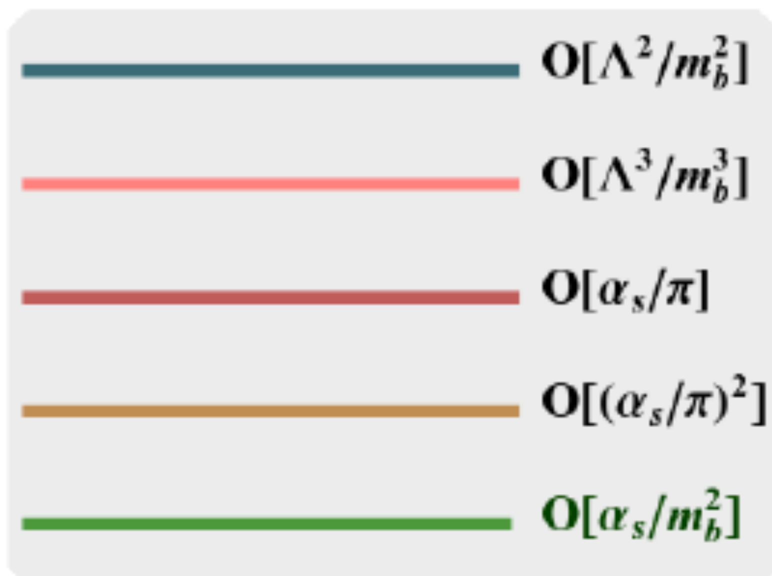
μ_G^2-SCALE DEPENDENCE



Relative NLO correction to the coefficients of μ_G^2 in the width (blue), first (red) and second central (yellow) leptonic moments as a function of the renormalization scale. Smaller corrections for smaller scale.

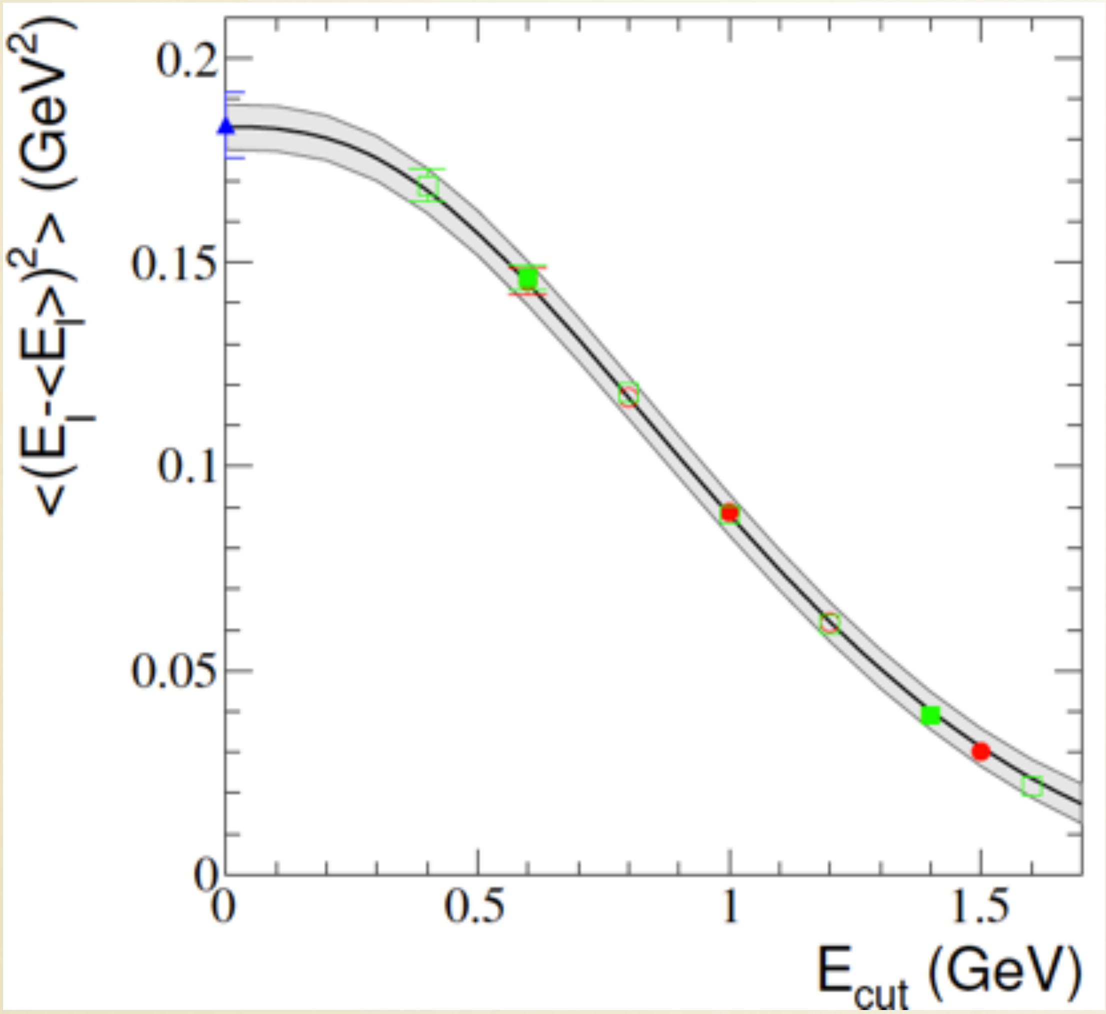
New Contributions $\mathcal{O}(\alpha_s/m_b^2)$: R

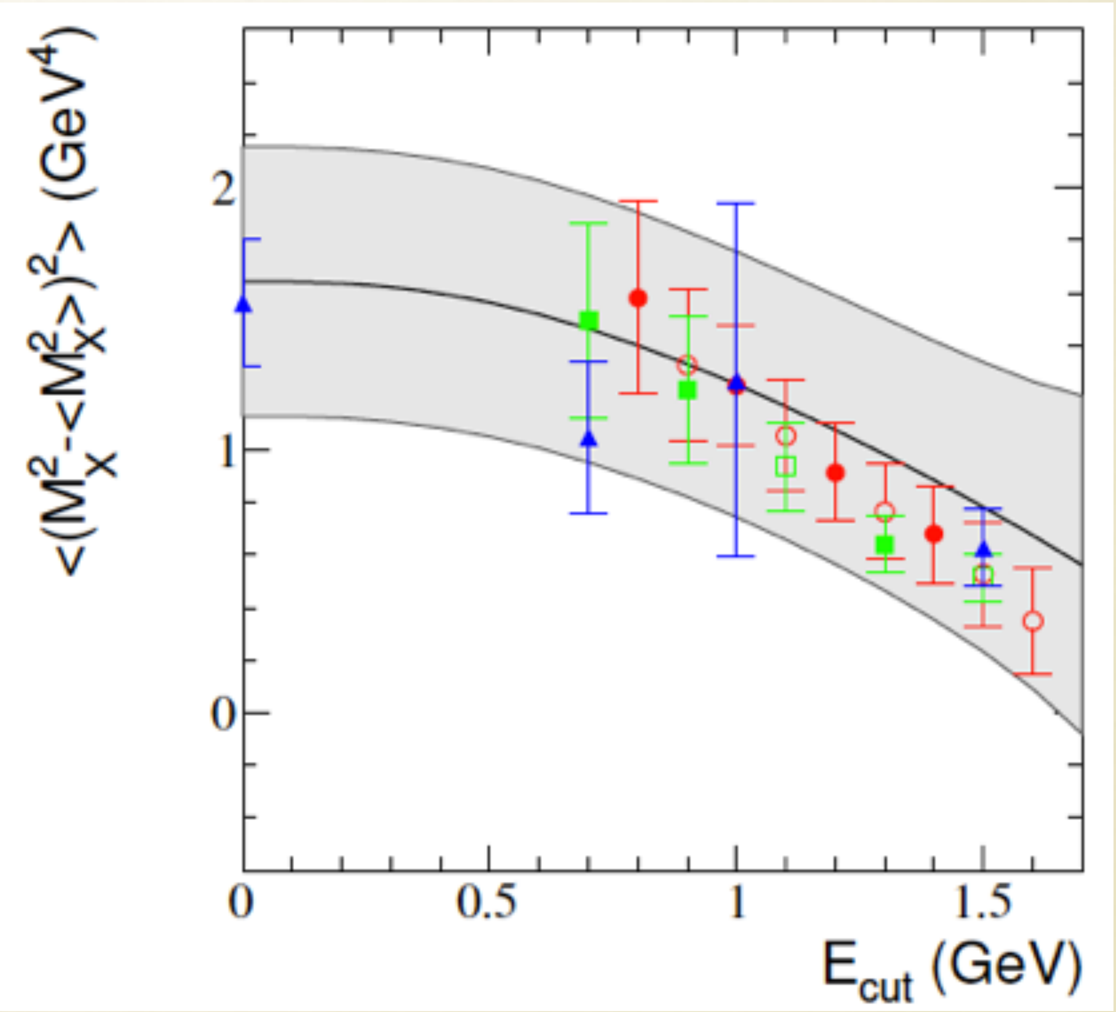




In the kinetic scheme μ=1GeV

Theoretical errors

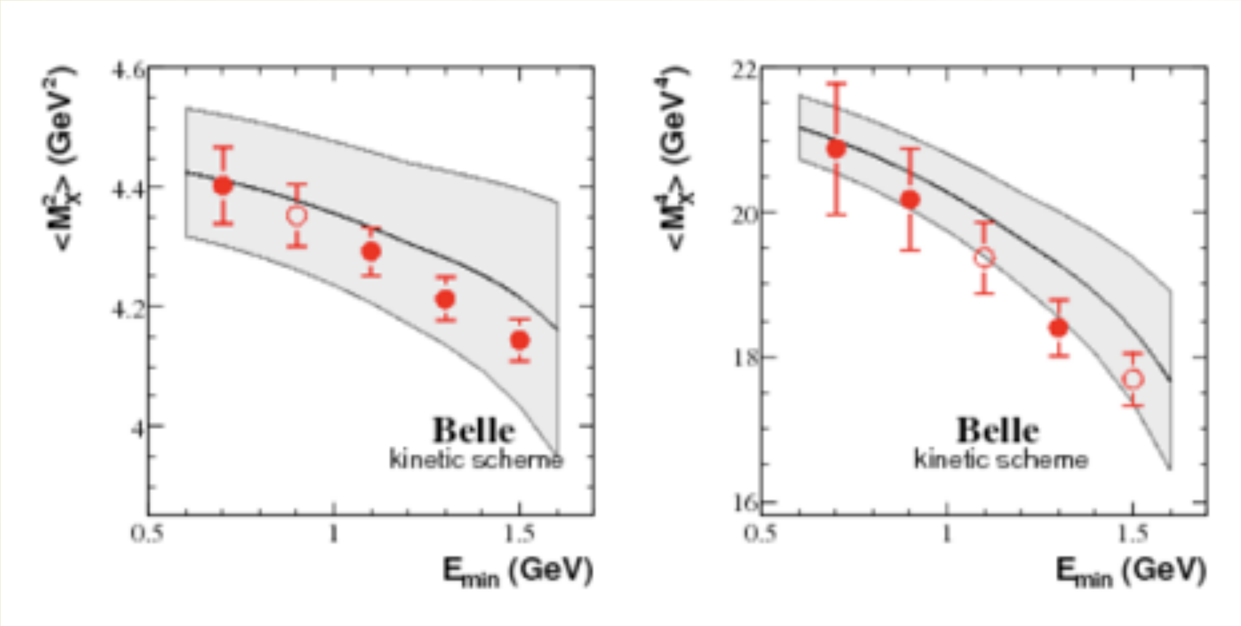


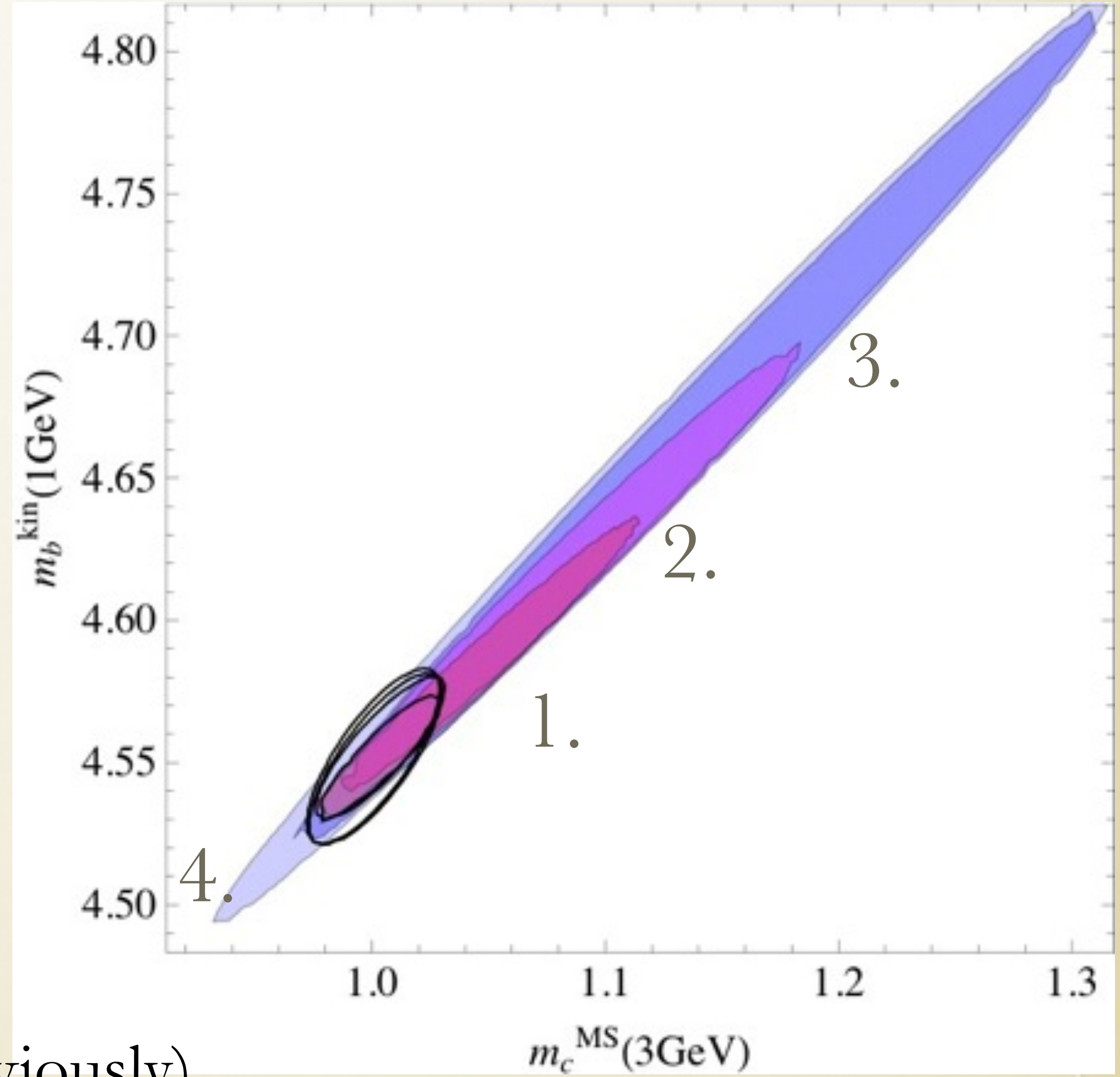


Theoretical errors are generally the **dominant** ones in the fits.
We estimate them in a **conservative** way, mimicking higher orders by varying the parameters by fixed amounts: $m_{c,b}$ 8MeV, $\alpha_s(m_b)$ 0.018, 7% in $1/m^2$ parameters, 30% in $1/m^3$ parameters

New corrections have been within theor. uncertainties so far.

Theoretical correlations





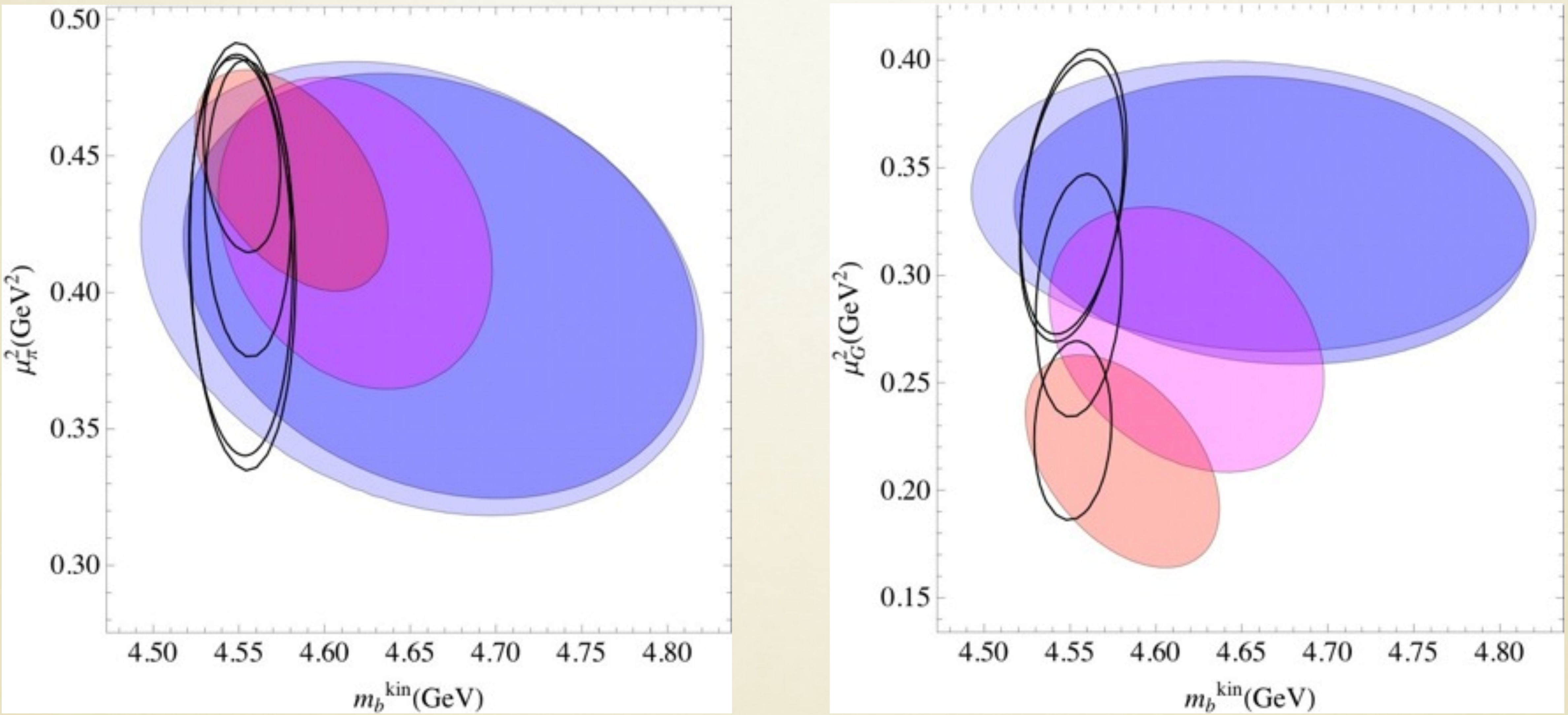
Correlations between theory errors of moments with different cuts difficult to estimate

1. 100% correlations (unrealistic but used previously)
2. corr. computed from low-order expressions
3. constant factor $0<\xi<1$ for 100MeV step
4. same as 3. but larger for larger cuts

always assume different central moments uncorrelated

Schwanda, PG 2013

Theoretical correlations



Schwanda, PG 2013

Therefore: 1) use a constraint on charm mass
2) reduce theory errors

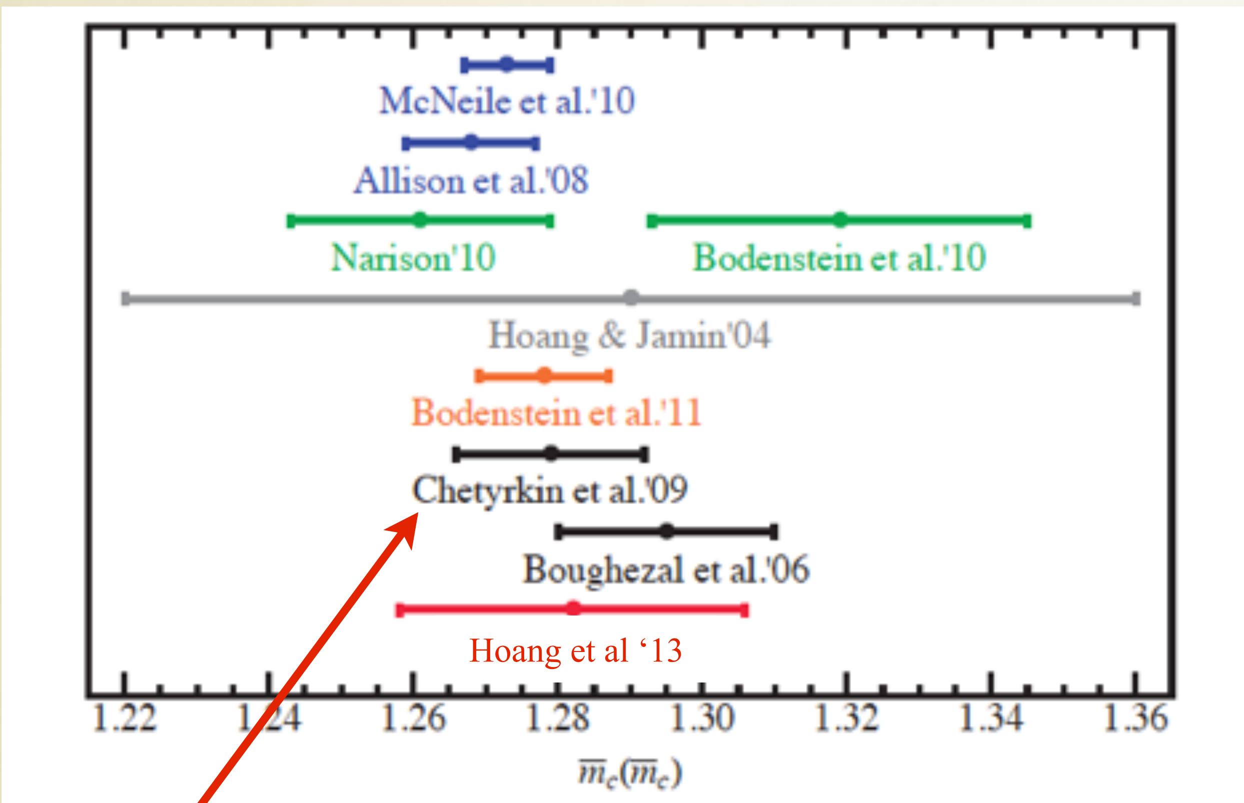
Latest semileptonic fit

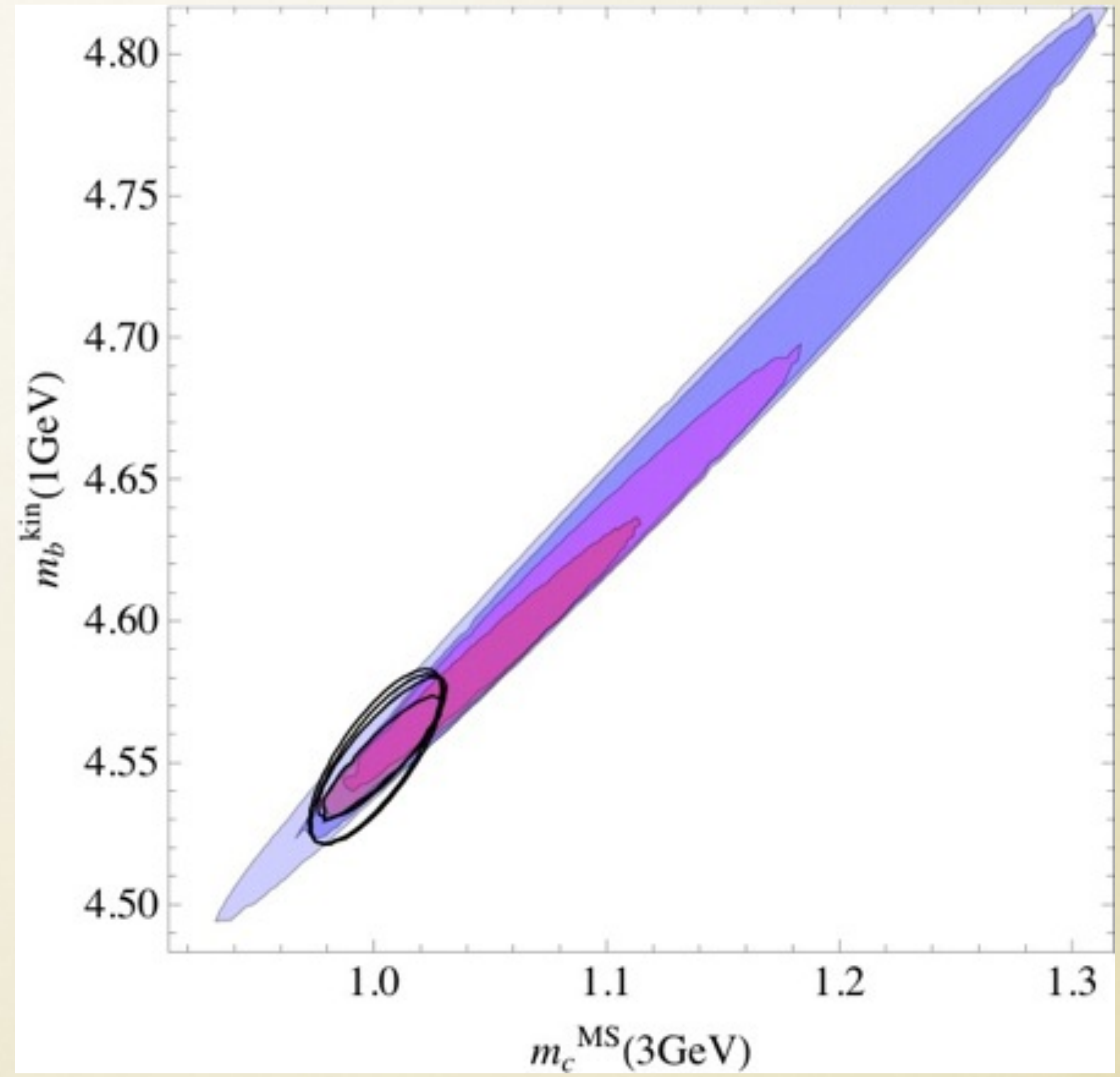
Alberti, Healey, Nandi, PG 1411.6560

- **updates** the fit in Schwanda, PG, 1307.4551
- kinetic scheme calculation based on 1107.3100; hep-ph/0401063
- includes all $O(\alpha_s^2)$ and $O(\alpha_s/m_b^2)$ corrections
- reassessment of theoretical errors, realistic correlations
- **external constraints**: precise heavy quark mass determinations, plus mild constraints on μ^2_G from hyperfine splitting and ρ^3_{LS} from sum rules

Previous fits: Buchmuller, Flaecher hep-ph/0507253,
Bauer et al, hep-ph/0408002 (1S scheme)

CHARM MASS DETERMINATIONS





our default choice

sum rules studies of $\sigma(e^+e^- \to \text{hadrons})$

almost all at NNNLO

Remarkable improvement in recent years.

m_c can be used as precise input to fix m_b instead of radiative moments

FIT RESULTS

1411.6560

m_b^{kin}	$\overline{m}_c(3\,\mathrm{GeV})$	μ_π^2	ρ_D^3	μ_G^2	ρ_{LS}^3	$\mathrm{BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.553	0.987	0.465	0.170	0.332	-0.150	10.65	42.21
0.020	0.013	0.068	0.038	0.062	0.096	0.16	0.78

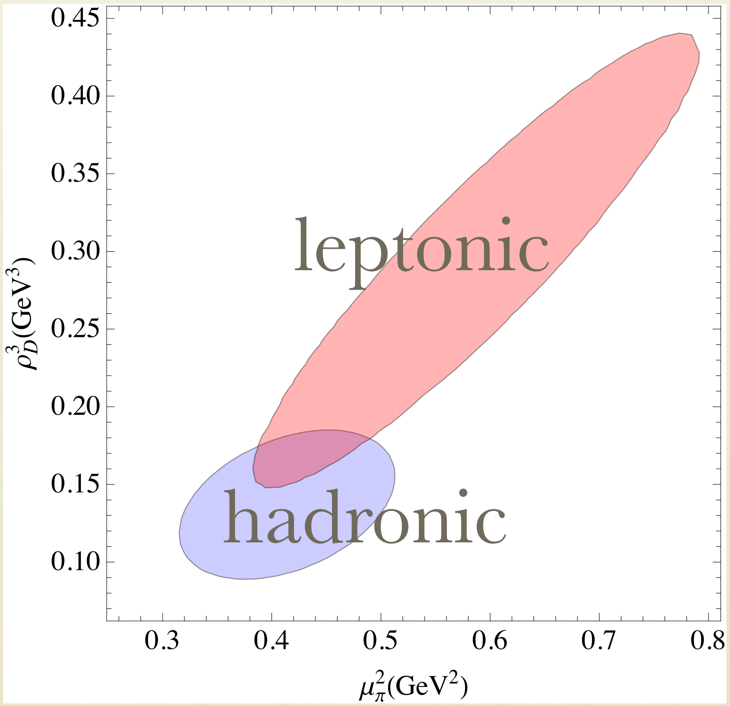
Schwanda PG 2013

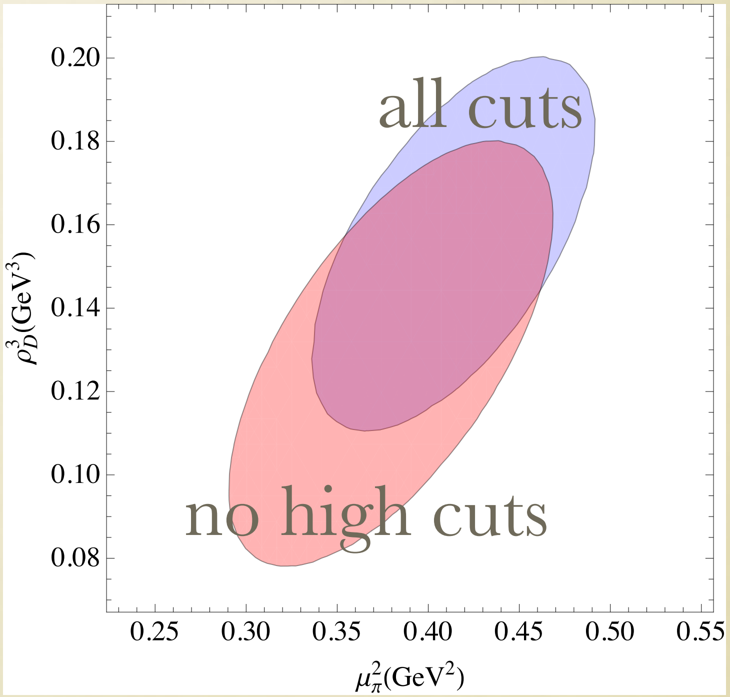
m_b^{kin}	$m_c^{(3\mathrm{GeV})}$	μ_π^2	ρ_D^3	μ_G^2	ρ_{LS}^3	$\mathrm{BR}_{c\ell\nu}(\%)$	$10^3\lvert V_{cb}\rvert$
4.541	0.987	0.414	0.154	0.340	-0.147	10.65	42.42
0.023	0.013	0.078	0.045	0.066	0.098	0.16	0.86

WITHOUT MASS CONSTRAINTS

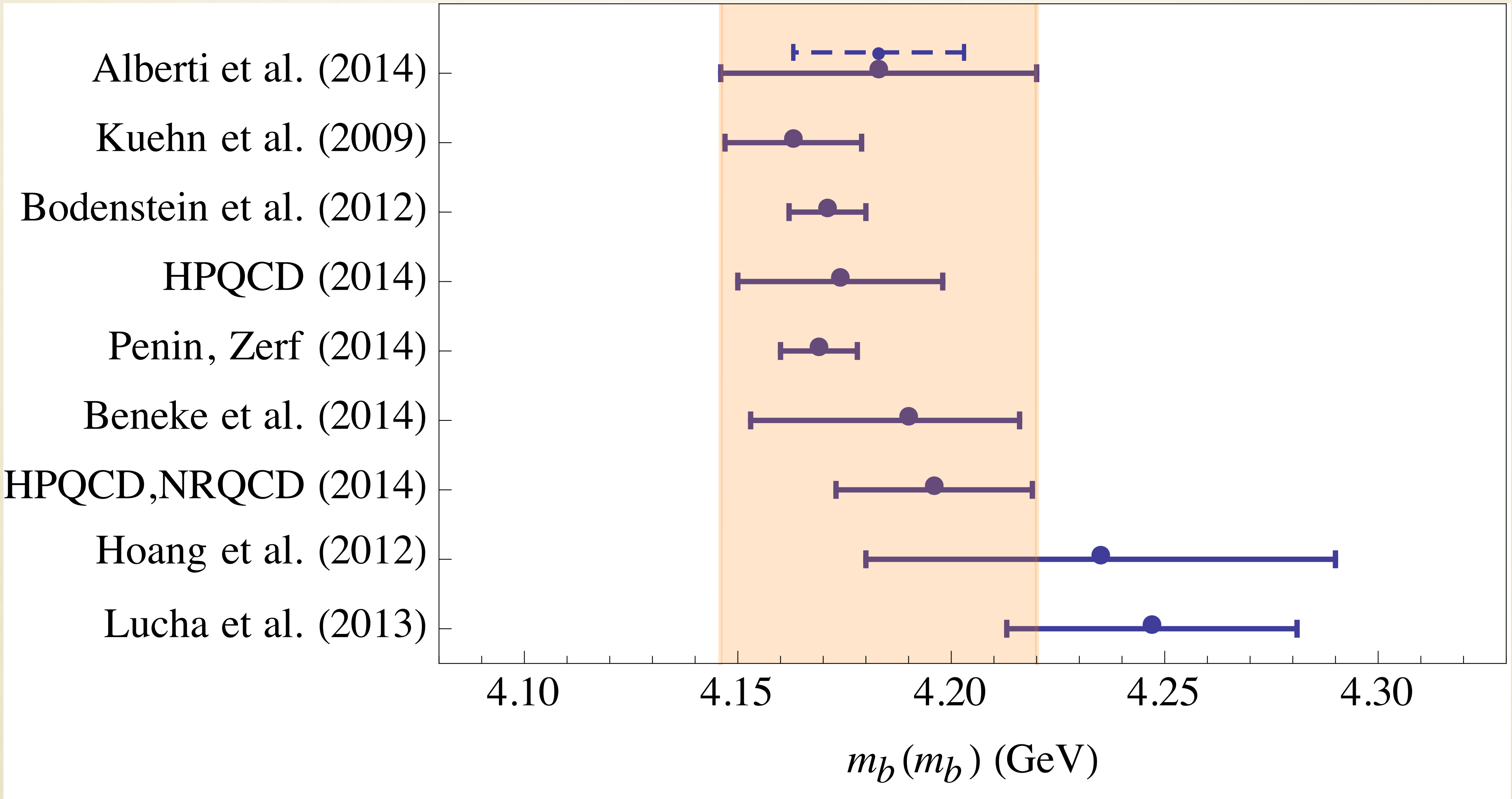
$$m_b^{kin}(1\mathrm{GeV}) - 0.85\,\overline{m}_c(3\mathrm{GeV}) = 3.714 \pm 0.018\,\mathrm{GeV}$$

- results depend little on assumption for correlations and choice of inputs, 1.8% determination of V_{cb}
- 20-30% determination of the OPE parameters





Results: bottom mass

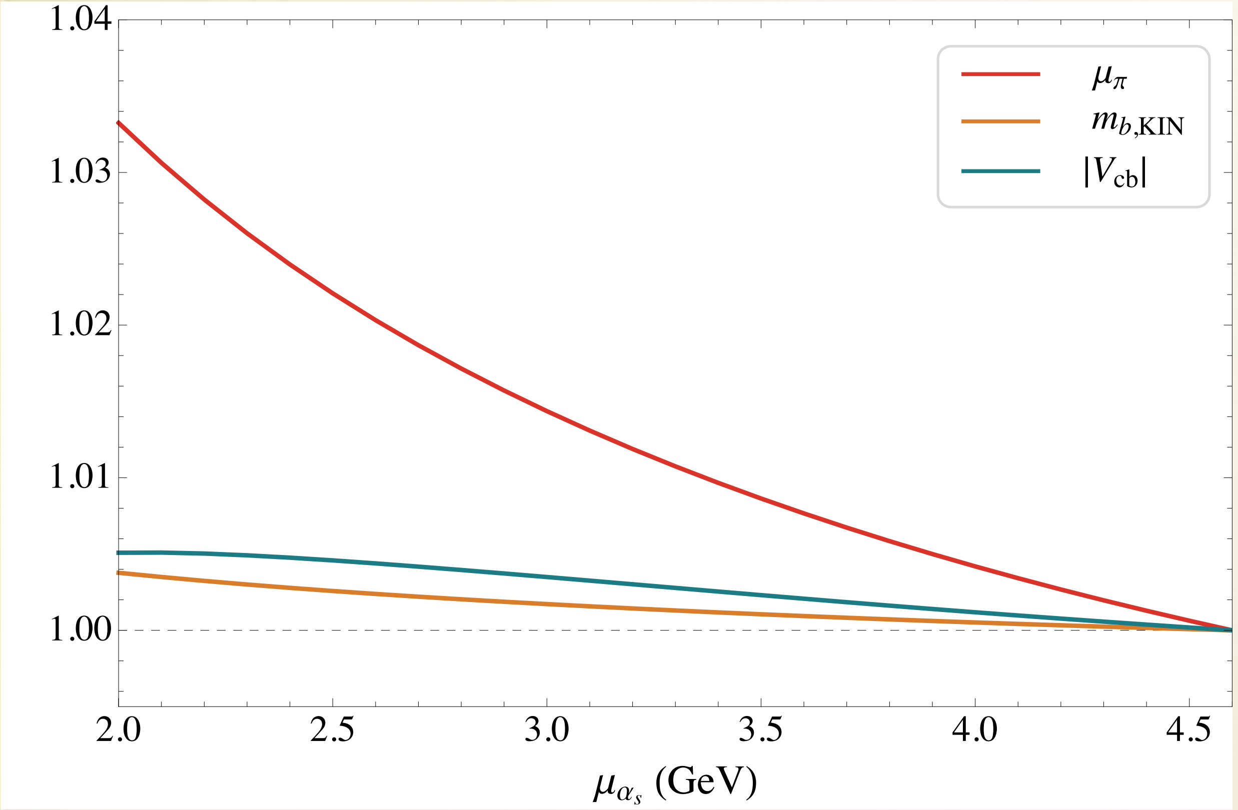


The fit gives $\boldsymbol{m_b^{kin}}$**(1GeV)=4.553(20)GeV**

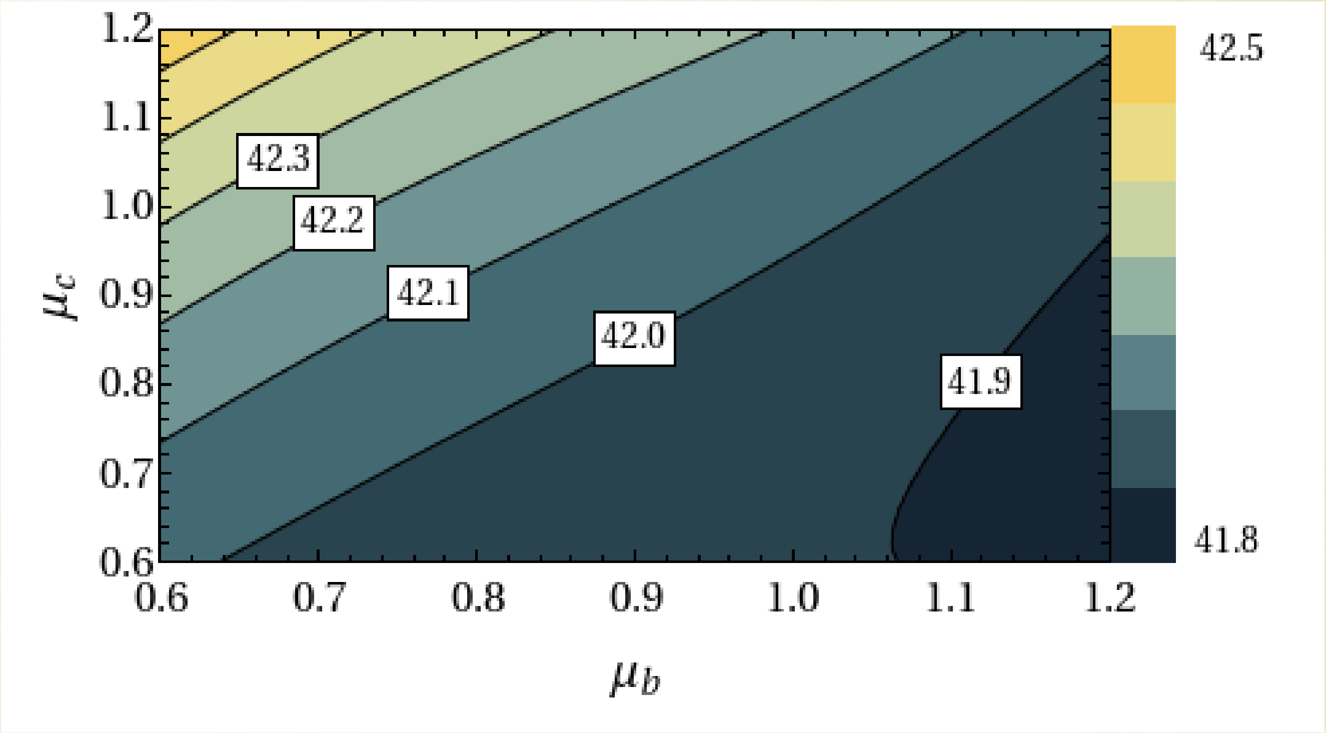
scheme translation error $m_b^{kin}(1\text{GeV}) = m_b(m_b) + 0.37(3)\text{GeV}$

$\boldsymbol{\overline{m}_b(\overline{m}_b)}$**=4.183(37)GeV**

FURTHER CHECKS



Dependence of fit results on strong coupling scale



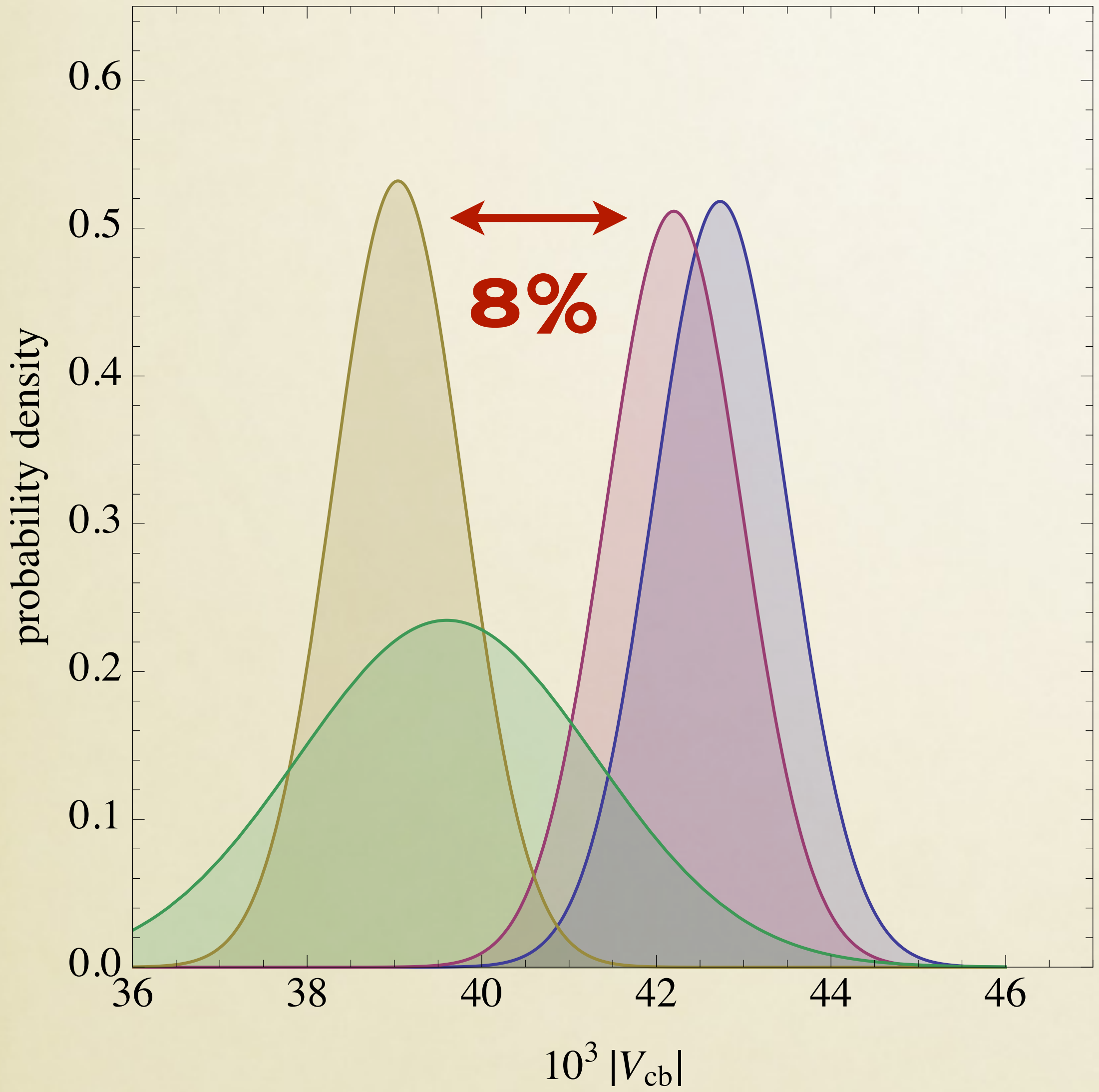
Dependence on kinetic cutoffs of bottom and charm masses

ELECTROWEAK CORRECTIONS

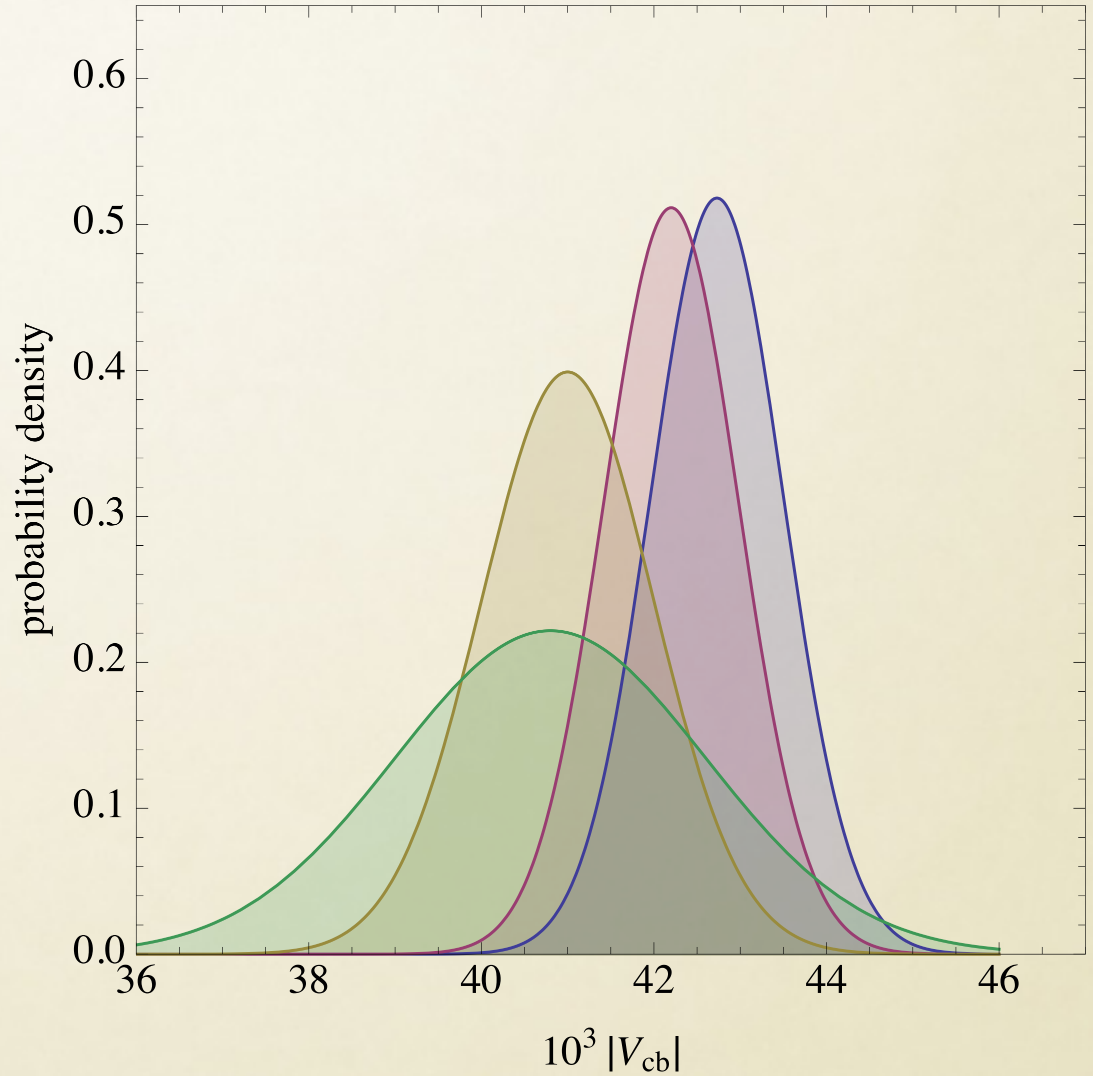
- Short-distance log (Sirlin 1982) included, ~0.7%
- Short-distance remainder (finite contribution to Wilson coefficient of 4f operator) tiny if G_μ is used to normalise decay
- QED soft and collinear radiation (and possibly some other stuff…) subtracted by experiments using PHOTOS
- QED hard radiation missing, calculation almost finished
- for B_0 only: static Coulomb interaction $1+\pi\alpha$ (Atwood, Marciano, Ginsberg) for mixture 37% B_0 this brings a 0.5% suppression of V_{cb} Should be included, but does it cancel in the moments?

V_{cb} VISUAL SUMMARY

Exclusive B→D*
Exclusive B→D
UTfit SM prediction:
(42.73±0.77) 10^{-3}
Inclusive



Latest lattice results for exclusives (FNAL/MILC)



HQSR,HQE for exclusives

Mannel, Uraltsev, PG

New physics?

The difference with FNAL/MILC is **quite large**: 3σ or about 8%. The perturbative corrections to inclusive V_{cb} total 5%, the power corrections about 4%.

Right Handed currents disfavored since

$$|V_{cb}|_{incl} \simeq |V_{cb}|\left(1 + \frac{1}{2}|\delta|^2\right)$$

$$|V_{cb}|_{B\to D^*} \simeq |V_{cb}|\left(1 - \delta\right)$$

$$|V_{cb}|_{B\to D} \simeq |V_{cb}|\left(1 + \delta\right)$$

Chen,Nam,Crivellin,Buras,Gemmler,Isidori,Pokorski...

$$\delta = \epsilon_R \frac{\tilde{V}_{cb}}{V_{cb}} \approx 0.08$$

Most general SU(2) invariant dim 6 NP (without RH neutrino) can explain results, but it is incompatible with $Z\to b\bar{b}$ data

Crivellin, Pokorski 1407.1320
see also Mannel, Turczyk et al

Summary

- Improvements of OPE approach to semileptonic decays continue. All effects $O(\alpha_s^2, \alpha_s \Lambda^2/m_b^2)$ implemented. **No sign of inconsistency in this approach so far**, **1.8% determination of V_{cb}**, **competitive m_b determination**.

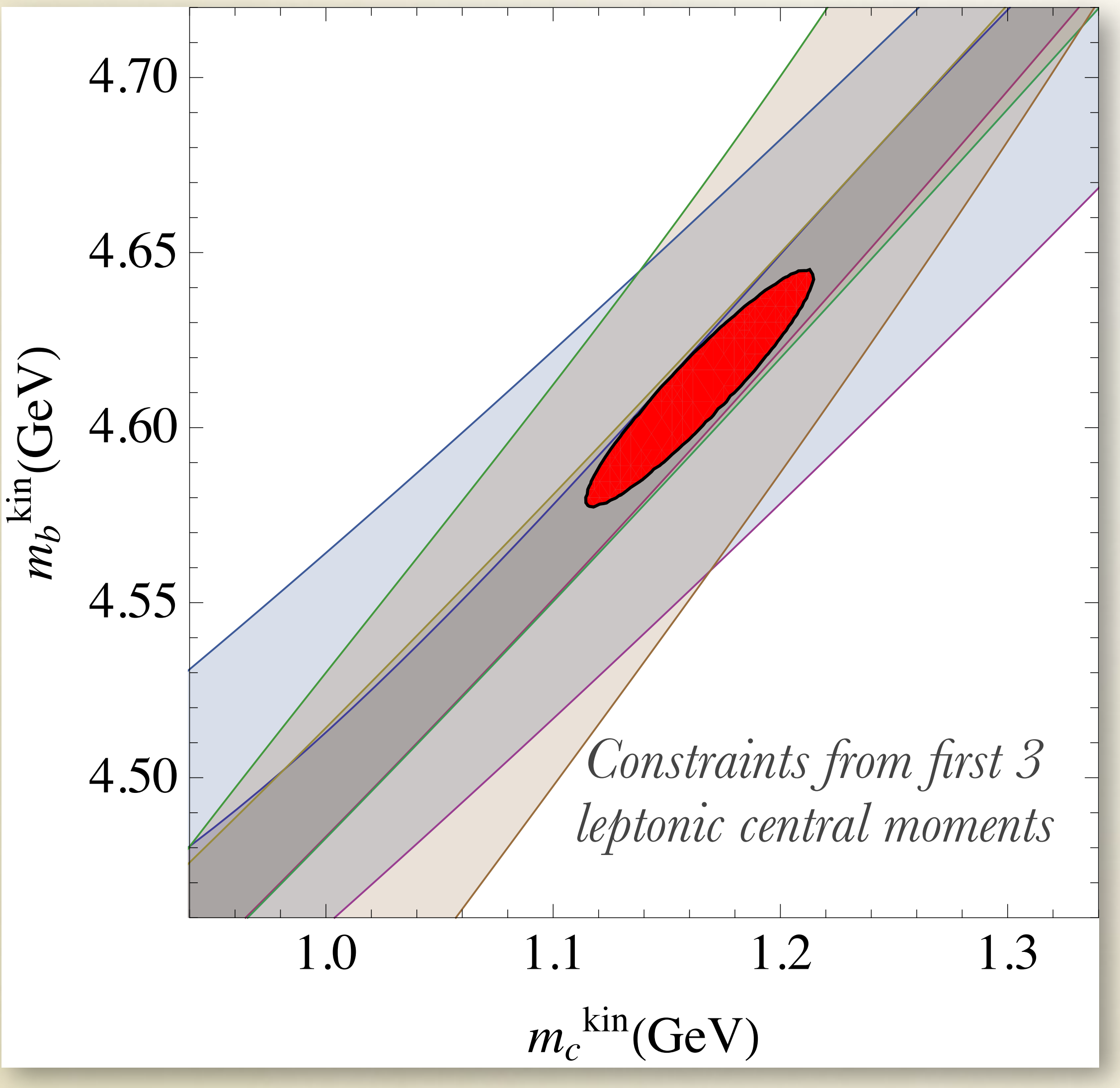
- Calculation of $O(\alpha_s \Lambda^3/m_b^3)$ effects, work on higher power corrections (see Sascha's talk) ongoing. QED corrections need to be reconsidered.

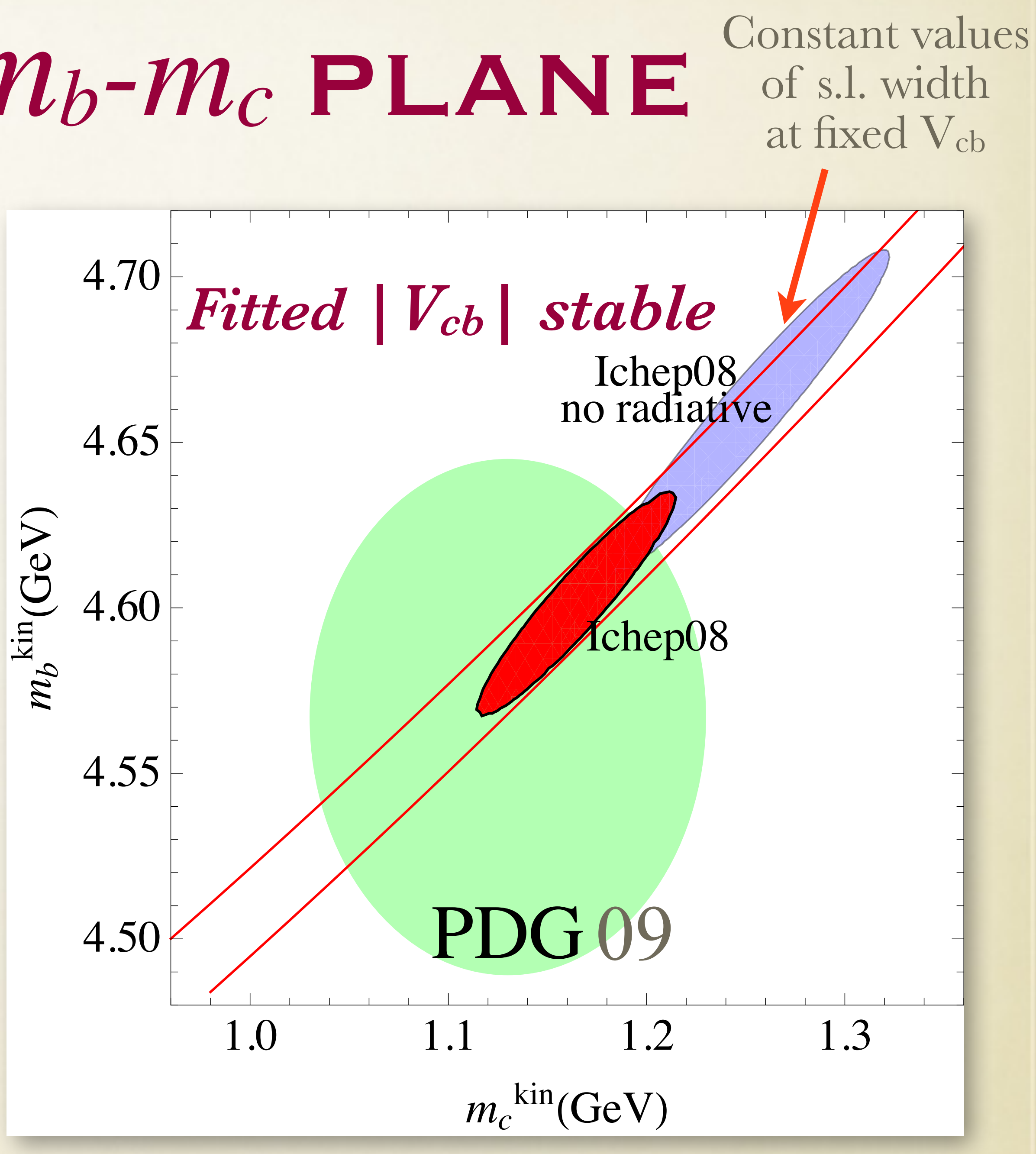
- Exclusive/inclusive tension in V_{cb} remains **large** (3σ, 8%). It cannot be explained by right-handed current and in general by SU(2)-invariant new physics.

- Belle-II will improve on exp precision. We need new ways to check and improve inclusive approach (new observables, lattice measurements of matrix elements or current correlators,…)

BACK-UP SLIDES

A STRIP IN THE m_b-m_c PLANE





Semileptonic moments do not measure m_b well. They rather identify a strip in (m_b,m_c) plane along which the χ^2 profile is shallow.

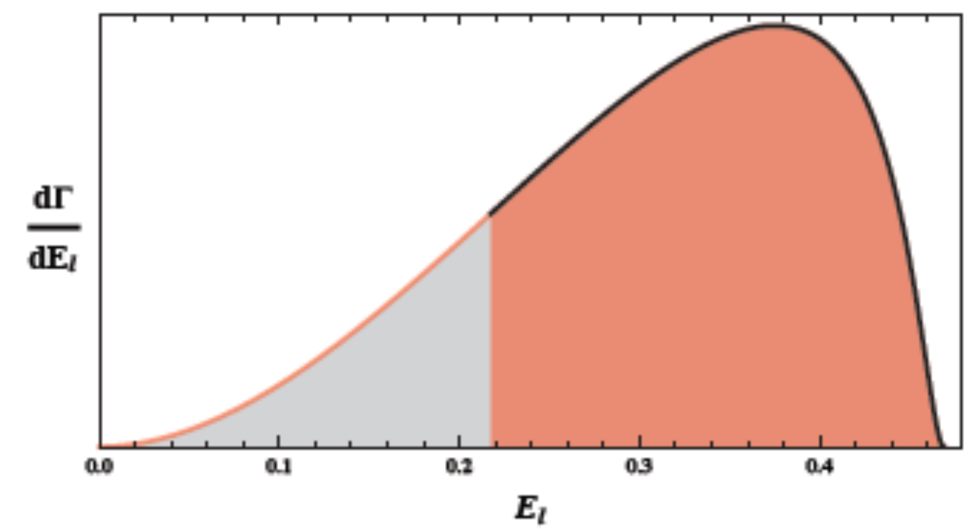
Relevant Observables

Leptonic Moments

$$\langle E_\ell^n \rangle_{E_\ell > E_{cut}} = \frac{\int_{E_{cut}}^{E_{max}} E_\ell^n \frac{d\Gamma}{dE_\ell} dE_\ell}{\int_{E_{cut}}^{E_{max}} dE_\ell \frac{d\Gamma}{dE_\ell}}$$

$$R^*(E_{cut}) = \frac{\int_{E_{cut}}^{E_{max}} \frac{d\Gamma}{dE_\ell} dE_\ell}{\int_0^{E_{max}} \frac{d\Gamma}{dE_\ell} dE_\ell}$$

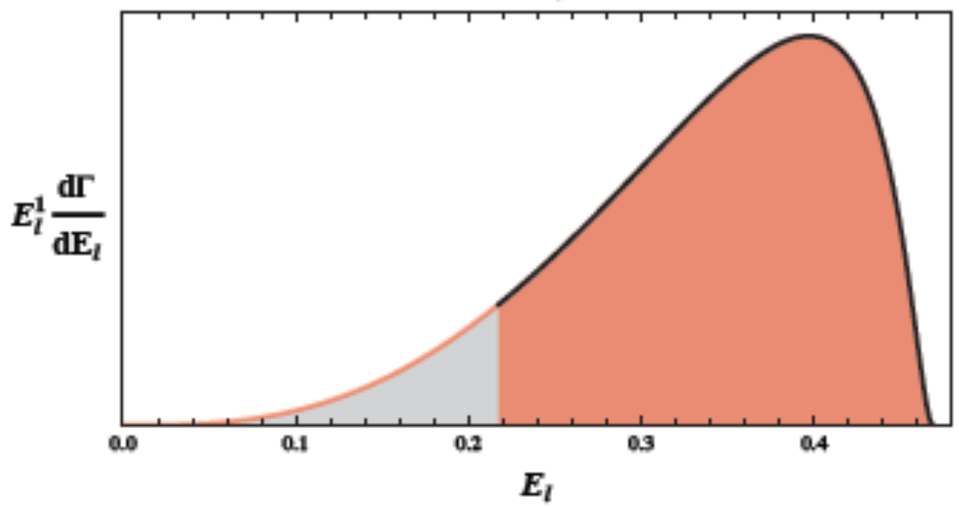
$\langle E_\ell^1 \rangle, \langle E_\ell^2 \rangle, \langle E_\ell^3 \rangle$ Highly Correlated



Central Leptonic Moments

$$\ell_1(E_{cut}) = \langle E_\ell \rangle_{E_\ell > E_{cut}}$$

$$\ell_{2,3}(E_{cut}) = \langle (E_\ell - \langle E_\ell \rangle)^{2,3} \rangle_{E_\ell > E_{cut}}$$



Hadronic Moments

$$\langle (M_X^2)^n \rangle_{E_\ell > E_{cut}} = \frac{\int_{E_{cut}}^{E_{max}} (M_X^2)^n \frac{d\Gamma}{dM_X^2} dM_x^2}{\int_{E_{cut}}^{E_{max}} \frac{d\Gamma}{dM_X^2} dM_X^2}$$

$$h_1(E_{cut}) = \langle M_X^2 \rangle_{E_\ell > E_{cut}}$$

$$h_{2,3}(E_{cut}) = \langle (M_X^2 - \langle M_X^2 \rangle)^{2,3} \rangle_{E_\ell > E_{cut}}$$

Experimental Observables

- Observables as $F_x(E_{cut}, m_c^2/m_b^2)$
- Express $m_b, \mu_{\pi,G}, \rho_{D,LS}$ in the "kinetic scheme" with a cutoff $\mu_{kin} = 1GeV$
- Additionally employ both kinetic and $\overline{\mathrm{MS}}$ definitions for m_c
- $\alpha_s(m_b = 4.6GeV) = 0.22$
  $\alpha_s \pm 0.005 \to \delta m_b < 1MeV$
- Additional Constraints:

Hyperfine Splitting

$$M_{B^*} - M_B = \frac{2}{3}\frac{\mu_G^2}{m_b} + O\left(\frac{\alpha_s \mu_G^2}{m_b}, \frac{1}{m_b^2}\right)$$

$$\mu_G^2 = (0.35 \pm 0.07)\,\mathrm{GeV}^2$$

Heavy Quark Sum Rules

$$\rho_{LS}^3 = (-0.15 \pm 0.10)\,\mathrm{GeV}^3$$

	Experiment	Values of $E_{cut}(\mathrm{GeV})$
R^*	BaBar	0.6, 1.2, 1.5
ℓ_1	BaBar	0.6, 0.8, 1, 1.2, 1.5
ℓ_2	BaBar	0.6, 1, 1.5
ℓ_3	BaBar	0.8, 1.2
h_1	BaBar	0.9, 1.1, 1.3, 1.5
h_2	BaBar	0.8, 1, 1.2, 1.4
h_3	BaBar	0.9, 1.3
R^*	Belle	0.6, 1.4
ℓ_1	Belle	1, 1.4
ℓ_2	Belle	0.6, 1.4
ℓ_3	Belle	0.8, 1.2
h_1	Belle	0.7, 1.1, 1.3, 1.5
h_2	Belle	0.7, 0.9, 1.3
$h_{1,2}$	CDF	0.7
$h_{1,2}$	CLEO	1, 1.5
$\ell_{1,2,3}$	DELPHI	0
$h_{1,2,3}$	DELPHI	0

Higher power corrections

Mannel,Turczyk,Uraltsev **1009.4622**

Proliferation of non-pert parameters and powers of $1/m_c$ starting $1/m^5$. At $1/m_b^4$

$$2M_B m_1 = \langle ((\vec{p})^2)^2 \rangle$$
$$2M_B m_2 = g^2 \langle \vec{E}^2 \rangle$$
$$2M_B m_3 = g^2 \langle \vec{B}^2 \rangle$$
$$2M_B m_4 = g \langle \vec{p} \cdot \mathrm{rot}\, \vec{B} \rangle$$

$$2M_B m_5 = g^2 \langle \vec{S} \cdot (\vec{E} \times \vec{E}) \rangle$$
$$2M_B m_6 = g^2 \langle \vec{S} \cdot (\vec{B} \times \vec{B}) \rangle$$
$$2M_B m_7 = g \langle (\vec{S} \cdot \vec{p})(\vec{p} \cdot \vec{B}) \rangle$$
$$2M_B m_8 = g \langle (\vec{S} \cdot \vec{B})(\vec{p})^2 \rangle$$
$$2M_B m_9 = g \langle \Delta(\vec{\sigma} \cdot \vec{B}) \rangle$$

can be estimated by **Lowest Lying State Saturation** approx by truncating

$$\langle B|O_1 O_2|B\rangle = \sum_n \langle B|O_1|n\rangle\langle n|O_2|B\rangle$$

In LLSA ***good convergence*** of the HQE. First fit with $1/m^{4,5}$:

$$\frac{\delta V_{cb}}{V_{cb}} \simeq -0.35\%$$

Turczyk,PG preliminary

Heinonen,Mannel 1407.4384 have more systematic approach

LLSA might set the scale of effect, not yet clear *how much it depends on assumptions on expectation values.* Large corrections to LLSA have been found.

Mannel, Uraltsev, PG, 2012

Allowing 80% gaussian deviations from LLSA seem to leave V_{cb} unaffected.