Precision Determination of Inclusive $|V_{ub}|$

Gil Paz

Department of Physics and Astronomy, Wayne State University

Outline

- Introduction
- Classic BLNP: 2004 - 2006
- Further developments: 2006 - 2010
- Future: 2015 -

Introduction

Motivation

- Determination of a fundamental parameter: $|V_{ub}|$
- Theoretically clean
- Theoretically interesting

Determination of fundamental parameters

- Inclusive semileptonic B decays

$\Rightarrow$ precision determination of $|V_{cb}|$ &$|V_{ub}|$

- PDG 2014:

  Inclusive $|V_{cb}| = 42.2 \pm 0.7 \times 10^{-3}$

  $\big($exclusive $|V_{cb}| = 39.5 \pm 0.8 \times 10^{-3}\big)$

  Inclusive $|V_{ub}| = 4.41 \pm 0.15\ {}_{\text{exp}}{}^{+0.15}_{-0.17}\ {}_{\text{theo}}$

  $\big($exclusive $|V_{ub}| = 3.28 \pm 0.29 \times 10^{-3}\big)$

- Unresolved tension for $|V_{cb}|$ &$|V_{ub}|$: Inclusive $>$ Exclusive

Theoretically Clean

Since $5\,\mathrm{GeV} \sim m_b \gg \Lambda_{\mathrm{QCD}} \sim 0.5\,\mathrm{GeV}$

Observables expanded as a power series in $\Lambda_{\mathrm{QCD}}/m_b \sim 0.1$

$$d\Gamma = \sum_n c_n \frac{\langle O_n \rangle}{m_b^n}$$

c_n perturbative, $\langle O_n \rangle$ non-perturbative

- Improvable:
- Calculate c_n to higher order in α_s
- Expand to higher orders in $\Lambda_{\mathrm{QCD}}/m_b$

Theoretically Interesting

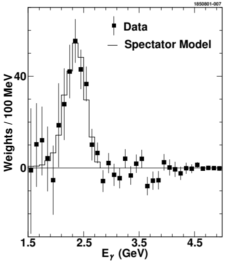
- Theoretically Interesting: test of basic QFT tools
- Factorization theorems
- Operator product expansion

Theoretically Interesting

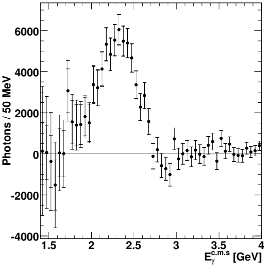
- Theoretically Interesting: test of basic QFT tools
- Factorization theorems
- Operator product expansion

- Theoretically Interesting: window to non-perturbative physics

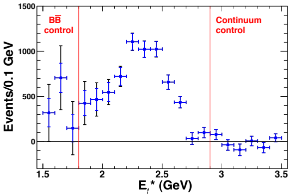
CLEO (2001)



Belle (2008)



BaBar (2012)



- At leading twist the photon spectrum is the B-meson pdf

Goal

Goal

- ~~Smallest error on $|V_{ub}|$ compared to other approaches~~

Goal

- ~~Smallest error on $|V_{ub}|$ compared to other approaches~~
- ~~Get Inclusive and exclusive $|V_{ub}|$ to match~~

Goal

- ~~Smallest error on $|V_{ub}|$ compared to other approaches~~
- ~~Get Inclusive and exclusive $|V_{ub}|$ to match~~
- Reliable extraction of $|V_{ub}|$ from inclusive B decays with conservative estimate of errors and non-perturbative effects

Take home message

- 1990's -2000's: Next to Leading Order (NLO) Era:

  c_0 at $\mathcal{O}(\alpha_s)$ + first power corrections at $\mathcal{O}(\alpha_s^0)$

- 2010's: Next to Next to Leading Order (NNLO) Era

  c_0 at $\mathcal{O}(\alpha_s^2)$ + first power corrections at $\mathcal{O}(\alpha_s)+\ldots$

Classic BLNP: 2004 - 2006

$\bar{B} \to X_u \, \ell \, \bar{\nu}$

- In principle local OPE describes $\bar{B} \to X_u \, \ell \, \bar{\nu}$ observables

  Assuming $M_X^2 \sim m_b^2 \Rightarrow$ local OPE

$\bar{B} \to X_u \, \ell \, \bar{\nu}$

- In principle local OPE describes $\bar{B} \to X_u \, \ell \, \bar{\nu}$ observables

  Assuming $M_X^2 \sim m_b^2 \Rightarrow$ local OPE

- In practice, to reject $\bar{B} \to X_c \, \ell \, \bar{\nu}$ background need cuts: $M_X^2 < M_D^2$

  $M_X^2 < M_D^2 \sim m_b \, \Lambda_{\mathrm{QCD}} \Rightarrow$ non-local OPE

$\bar{B} \to X_u \,\ell\, \bar{\nu}$

- In principle local OPE describes $\bar{B} \to X_u \,\ell\, \bar{\nu}$ observables

  Assuming $M_X^2 \sim m_b^2 \Rightarrow$ local OPE

- In practice, to reject $\bar{B} \to X_c \,\ell\, \bar{\nu}$ background need cuts: $M_X^2 < M_D^2$

  $M_X^2 < M_D^2 \sim m_b \,\Lambda_{\rm QCD} \Rightarrow$ non-local OPE

- Three scales in the problem

hard	$\mu_h \sim m_b$	~ 5 GeV
jet	$\mu_i \sim \sqrt{m_b \Lambda_{\rm QCD}}$	~ 1.5 GeV
soft	$\mu_0 \sim \Lambda_{\rm QCD}$	~ 0.5 GeV

Classic BLNP: Leading Power Factorization

- At the end point region, spectra of

- $\bar{B} \to X_u \, l \, \bar{\nu}$
- $Q_{7\gamma} - Q_{7\gamma}$ contribution to $\bar{B} \to X_s \gamma$

obey a **leading power** factorization formula

[Korchemsky, Sterman '94; Bauer, Pirjol, Stewart '01]

$$\begin{aligned} \bar{B} \to X_s \gamma : &\qquad d\Gamma_s^{77} \sim H_s \cdot J \otimes S + ... \\ \bar{B} \to X_u \, l \, \bar{\nu} : &\qquad d\Gamma_u \sim H_u \cdot J \otimes S + ... \end{aligned}$$

- S (leading) shape function, non-perturbative
- H_i and J calculable using PT in α_s

Classic BLNP: Leading Power Factorization at $\mathcal{O}(\alpha_s)$

- Leading power factorization formula

  [Korchemsky, Sterman '94; Bauer, Pirjol, Stewart '01]

$$
\begin{aligned}
\bar{B} \to X_s \gamma : &\qquad d\Gamma_s^{77} \sim H_s \cdot J \otimes S + \ldots \\
\bar{B} \to X_u \, l \, \bar{\nu} : &\qquad d\Gamma_u \sim H_u \cdot J \otimes S + \ldots
\end{aligned}
$$

Classic BLNP: Leading Power Factorization at $\mathcal{O}(\alpha_s)$

- Leading power factorization formula

  [Korchemsky, Sterman '94; Bauer, Pirjol, Stewart '01]

$$\begin{aligned} \bar{B} \to X_s \gamma : \qquad & d\Gamma_s^{77} \sim H_s \cdot J \otimes S + \ldots \\ \bar{B} \to X_u \, l \, \bar{\nu} : \qquad & d\Gamma_u \sim H_u \cdot J \otimes S + \ldots \end{aligned}$$

- H_u and J calculated at $\mathcal{O}(\alpha_s)$

  [Bauer, Manohar '03; Bosch, Lange, Neubert, GP '04]
- H_s calculated at $\mathcal{O}(\alpha_s)$ [Neubert '04]

Classic BLNP: Leading Power Factorization at $\mathcal{O}(\alpha_s)$

- Leading power factorization formula

  [Korchemsky, Sterman '94; Bauer, Pirjol, Stewart '01]

$$\begin{aligned} \bar{B} \to X_s\gamma: &\qquad d\Gamma_s^{77} \sim H_s \cdot J \otimes S + \ldots \\ \bar{B} \to X_u l\bar{\nu}: &\qquad d\Gamma_u \sim H_u \cdot J \otimes S + \ldots \end{aligned}$$

- H_u and J calculated at $\mathcal{O}(\alpha_s)$

  [Bauer, Manohar '03; Bosch, Lange, Neubert, GP '04]
- H_s calculated at $\mathcal{O}(\alpha_s)$ [Neubert '04]

- H evolved from μ_h down to μ_i resumming Sudakov double logs

Classic BLNP: Subleading Power Factorization at $\mathcal{O}(\alpha_s^0)$

- *One* type of $1/m_b$ power corrections:
  subleading shape functions (subleading "twist")

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + \ldots$$

- SCET: [K.S.M. Lee, Stewart '04; Bosch, Neubert, GP '04; Beneke, Campanario, Mannel, Pecjak '04]
- Non SCET: [Bauer, Luke, Mannel '01, '02; Leibovich, Ligeti, Wise '02; Neubert '02; Burrell, Luke, Williamson '03; Tackmann '05]
  See GP talk at Vub/SF Workshop SLAC 2004

- s_i are non-perturbative, appear at tree level

Classic BLNP: Other Contributions

- In absence of proper factorization for α_s/m_b terms "kinematical corrections" are incorporated as Kinematical $=$ (Partonic - LO corrections) $\otimes$ LO SF Partonic at $\mathcal{O}(\alpha_s)$ from [De-Fazio, Neubert '99]

Classic BLNP: Other Contributions

- In absence of proper factorization for α_s/m_b terms
  "kinematical corrections" are incorporated as
  Kinematical = (Partonic - LO corrections) $\otimes$ LO SF
  Partonic at $\mathcal{O}(\alpha_s)$ from [De-Fazio, Neubert '99]

- $1/m_b^2$ hadronic corrections from OPE Calculation
  [Blok, Koyrakh, Shifman '93, Vainshtein; Manohar, Wise '93]
  Extract and express as convolution with LO SF

Classic BLNP: Other Contributions

- In absence of proper factorization for α_s/m_b terms "kinematical corrections" are incorporated as Kinematical = (Partonic - LO corrections) $\otimes$ LO SF Partonic at $\mathcal{O}(\alpha_s)$ from [De-Fazio, Neubert '99]

- $1/m_b^2$ hadronic corrections from OPE Calculation [Blok, Koyrakh, Shifman '93, Vainshtein; Manohar, Wise '93] Extract and express as convolution with LO SF

- Example: for $E_X - |\vec{P}_X| = P_+ \leq M_D^2/M_B$

$$
\begin{array}{lllllll}
 & \Gamma_u^{(0)} & + (\Gamma_u^{\mathrm{kin}(1)} & + \Gamma_u^{\mathrm{hadr}(1)}) & + (\Gamma_u^{\mathrm{kin}(2)} & + \Gamma_u^{\mathrm{hadr}(2)}) & \\
= [& 53.225 & + (4.646 & - 11.746) & + (0.328 & - 0.469)] & |V_{ub}|^2\,\mathrm{ps}^{-1}
\end{array}
$$

[Lange, Neubert, GP '05]

Classic BLNP: Summary

- Based on

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + ...$$

- Leading power $H \cdot J \otimes S$ at $\mathcal{O}(\alpha_s)$
- Subleading shape functions: $H \cdot J \otimes s_i$ at $\mathcal{O}(\alpha_s^0)$
- α_s/m_b, α_s/m_b^2 and $1/m_b^2$ as above
- S extracted from $\bar{B} \to X_s\gamma$, s_i modeled ($\sim$ 700 models)
- Smoothly merges to local OPE when integrated over phase space

Classic BLNP: Summary

- Based on

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + ...$$

- Leading power $H \cdot J \otimes S$ at $\mathcal{O}(\alpha_s)$
- Subleading shape functions: $H \cdot J \otimes s_i$ at $\mathcal{O}(\alpha_s^0)$
- α_s/m_b, α_s/m_b^2 and $1/m_b^2$ as above
- S extracted from $\bar{B} \to X_s\gamma$, s_i modeled ($\sim$ 700 models)
- Smoothly merges to local OPE when integrated over phase space

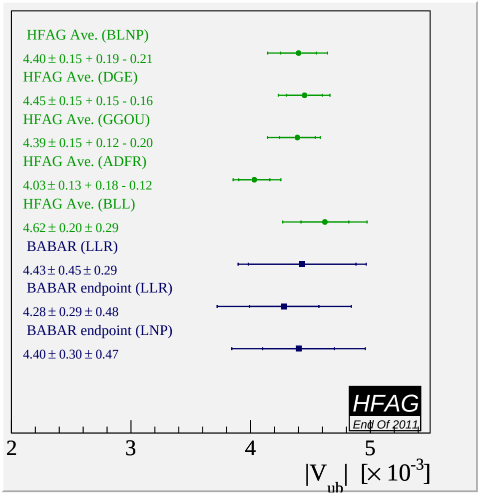
- Precision determination of $|V_{ub}|$ ("NLO")

  [Lange, Neubert, GP '05]

  Error on $|V_{ub}|$: **18%** (PDG 2004) $\Rightarrow$ **8%** (PDG 2006)

$\bar{B} \to X_u \ell \bar{\nu}$: Present

- Consistent extractions based on various theoretical approaches



- PDG 2014: Inclusive $|V_{ub}| = 4.41 \pm 0.15\ \text{exp}\,{}^{+0.15}_{-0.17}\ \text{theo}$

  exclusive $|V_{ub}| \quad = 3.28 \pm 0.29 \times 10^{-3}$

Further developments: 2006 - 2010

Leading Power Factorization at $\mathcal{O}(\alpha_s^2)$

- Progress in the perturbative calculation:

$$\begin{aligned} \bar{B} \to X_s \gamma : &\qquad d\Gamma_s^{77} \sim H_s \cdot J \otimes S + \ldots \\ \bar{B} \to X_u \, l \, \bar{\nu} : &\qquad d\Gamma_u \sim H_u \cdot J \otimes S + \ldots \end{aligned}$$

Leading Power Factorization at $\mathcal{O}(\alpha_s^2)$

- Progress in the perturbative calculation:

$$\begin{aligned} \bar{B} \to X_s \gamma: \quad & d\Gamma_s^{77} \sim H_s \cdot J \otimes S + \ldots \\ \bar{B} \to X_u\, l\, \bar{\nu}: \quad & d\Gamma_u \sim H_u \cdot J \otimes S + \ldots \end{aligned}$$

- 2005: H_s calculated at $\mathcal{O}(\alpha_s^2)$
  [Melnikov, Mitov '05, ...]

Leading Power Factorization at $\mathcal{O}(\alpha_s^2)$

- Progress in the perturbative calculation:

$$\begin{aligned} \bar{B} \to X_s \gamma : \qquad & d\Gamma_s^{77} \sim H_s \cdot J \otimes S + ... \\ \bar{B} \to X_u \, l \, \bar{\nu} : \qquad & d\Gamma_u \sim H_u \cdot J \otimes S + ... \end{aligned}$$

- 2005: H_s calculated at $\mathcal{O}(\alpha_s^2)$
  [Melnikov, Mitov '05, ...]

- 2006: J calculated at $\mathcal{O}(\alpha_s^2)$
  [Becher, Neubert '06]

Leading Power Factorization at $\mathcal{O}(\alpha_s^2)$

- Progress in the perturbative calculation:

$$\begin{aligned} \bar{B} \to X_s \gamma : &\qquad d\Gamma_s^{77} \sim H_s \cdot J \otimes S + \ldots \\ \bar{B} \to X_u \, l \, \bar{\nu} : &\qquad d\Gamma_u \sim H_u \cdot J \otimes S + \ldots \end{aligned}$$

- 2005: H_s calculated at $\mathcal{O}(\alpha_s^2)$
  [Melnikov, Mitov '05, ...]

- 2006: J calculated at $\mathcal{O}(\alpha_s^2)$
  [Becher, Neubert '06]

- 2008: H_u calculated at $\mathcal{O}(\alpha_s^2)$
  [Bonciani, Ferroglia '08; Asatrian, Greub, Pecjak '08; Beneke, Huber, Li '08; Bell '08]

Leading Power Factorization at $\mathcal{O}(\alpha_s^2)$: Implications

- Study of implication of $\mathcal{O}(\alpha_s^2)$ on $|V_{ub}|$
  [Greub, Neubert, Pecjak '09]
  See Pecjak's talk at V_{xb} 2009

Leading Power Factorization at $\mathcal{O}(\alpha_s^2)$: Implications

- Study of implication of $\mathcal{O}(\alpha_s^2)$ on $|V_{ub}|$
  [Greub, Neubert, Pecjak '09]
  See Pecjak's talk at V_{xb} 2009

- "For the default choice $\mu_i = 1.5$ GeV used in ... BLNP framework, the NNLO corrections induce significant downward shifts"

Leading Power Factorization at $\mathcal{O}(\alpha_s^2)$: Implications

- Study of implication of $\mathcal{O}(\alpha_s^2)$ on $|V_{ub}|$
  [Greub, Neubert, Pecjak '09]
  See Pecjak's talk at V_{xb} 2009

- "For the default choice $\mu_i = 1.5$ GeV used in ... BLNP framework, the NNLO corrections induce significant downward shifts"

- "perturbative uncertainties are reduced, especially those at the jet scale, which are the dominant ones at next-to-leading order (NLO). "

Leading Power Factorization at $\mathcal{O}(\alpha_s^2)$: Implications

- Study of implication of $\mathcal{O}(\alpha_s^2)$ on $|V_{ub}|$
  [Greub, Neubert, Pecjak '09]
  See Pecjak's talk at V_{xb} 2009

- "For the default choice $\mu_i = 1.5$ GeV used in ... BLNP framework, the NNLO corrections induce significant downward shifts"

- "perturbative uncertainties are reduced, especially those at the jet scale, which are the dominant ones at next-to-leading order (NLO). "

- "For parameter and scale choices ... within the BLNP framework ... NNLO corrections raise the value of $|Vub|$ by slightly less than 10% compared to NLO"

Leading Power Factorization at $\mathcal{O}(\alpha_s^2)$: Implications

- Study of implication of $\mathcal{O}(\alpha_s^2)$ on $|V_{ub}|$
  [Greub, Neubert, Pecjak '09]
  See Pecjak's talk at V_{xb} 2009

- "For the default choice $\mu_i = 1.5$ GeV used in ... BLNP framework, the NNLO corrections induce significant downward shifts"

- "perturbative uncertainties are reduced, especially those at the jet scale, which are the dominant ones at next-to-leading order (NLO). "

- "For parameter and scale choices ... within the BLNP framework ... NNLO corrections raise the value of $|Vub|$ by slightly less than 10% compared to NLO"

- "factorization ... perturbative coefficient...into jet and hard functions is not strictly necessary: using ... fixed-order... does not lead to large scale uncertainties ... nor to a poor convergence ..."

Subleading Power Factorization at $\mathcal{O}(\alpha_s)$: Subleading jet functions

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b}\sum_i H \cdot J \otimes s_i + \frac{1}{m_b}\sum_i H \cdot j_i \otimes S + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{m_b^2}\right)$$

- Subleading jet functions, j_i, for $d\Gamma_s^{77}$ and $d\Gamma_u$ calculated at $\mathcal{O}(\alpha_s)$ [GP '09]

Subleading Power Factorization at $\mathcal{O}(\alpha_s)$: Subleading jet functions

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + \frac{1}{m_b} \sum_i H \cdot j_i \otimes S + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{m_b^2}\right)$$

- Subleading jet functions, j_i, for $d\Gamma_s^{77}$ and $d\Gamma_u$ calculated at $\mathcal{O}(\alpha_s)$ [GP '09]
- j_i are perturbative, arise at α_s/m_b, appear in convolution with LO SF i.e. do *not* introduce new hadronic uncertainties

Subleading Power Factorization at $\mathcal{O}(\alpha_s)$: Subleading jet functions

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b}\sum_i H \cdot J \otimes s_i + \frac{1}{m_b}\sum_i H \cdot j_i \otimes S + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{m_b^2}\right)$$

- Subleading jet functions, j_i, for $d\Gamma_s^{77}$ and $d\Gamma_u$ calculated at $\mathcal{O}(\alpha_s)$ [GP '09]

- j_i are perturbative, arise at α_s/m_b, appear in convolution with LO SF i.e. do *not* introduce new hadronic uncertainties

- As experimental cuts are relaxed
- s_i remain power suppressed
- j_i become less power suppressed

Subleading Power Factorization at $\mathcal{O}(\alpha_s)$: Implications

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + \frac{1}{m_b} \sum_i H \cdot j_i \otimes S + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{m_b^2}\right)$$

Subleading Power Factorization at $\mathcal{O}(\alpha_s)$: Implications

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + \frac{1}{m_b} \sum_i H \cdot j_i \otimes S + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{m_b^2}\right)$$

- To implement j_i, e.g. for $d\Gamma_s^{77}$, replace the α_s/m_b "kinematic"

$$W^{\rm Kin.} = \frac{1}{m_b} \frac{C_F\, \alpha_s(\mu)}{4\pi} \int d\omega\, \theta(\omega + n\cdot p) \left[32 \ln \frac{\omega + n\cdot p}{m_b} + 30\right] S(\omega) + \mathcal{O}(\alpha_s^2)$$

by

$$W^{\rm SJF} = \frac{1}{m_b} \frac{C_F \alpha_s(\mu)}{4\pi} \int d\omega\ \theta(\omega + n\cdot p) \left[32 \ln \frac{\mu^2}{m_b(\omega + n\cdot p)} - 18\right] S(\omega) + \mathcal{O}(\alpha_s^2)$$

Subleading Power Factorization at $\mathcal{O}(\alpha_s)$: Implications

$$d\Gamma \sim H\cdot J\otimes S+\frac{1}{m_b}\sum_i H\cdot J\otimes s_i+\frac{1}{m_b}\sum_i H\cdot j_i\otimes S+\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{m_b^2}\right)$$

- To implement j_i, e.g. for $d\Gamma_s^{77}$, replace the α_s/m_b "kinematic"

$$W^{\rm Kin.}=\frac{1}{m_b}\frac{C_F\,\alpha_s(\mu)}{4\pi}\int d\omega\,\theta(\omega+n\cdot p)\left[32\ln\frac{\omega+n\cdot p}{m_b}+30\right]S(\omega)+\mathcal{O}(\alpha_s^2)$$

  by

$$W^{\,\rm SJF}=\frac{1}{m_b}\frac{C_F\alpha_s(\mu)}{4\pi}\int d\omega\;\theta(\omega+n\cdot p)\left[32\ln\frac{\mu^2}{m_b(\omega+n\cdot p)}-18\right]S(\omega)+\mathcal{O}(\alpha_s^2)$$

- No change for s_i term, just s_i modeling to account for their non-zero one loop contribution

Subleading Power Factorization at $\mathcal{O}(\alpha_s)$: Implications

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + \frac{1}{m_b} \sum_i H \cdot j_i \otimes S + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{m_b^2}\right)$$

- To implement j_i, e.g. for $d\Gamma_s^{77}$, replace the α_s/m_b "kinematic"

$$W^{\rm Kin.} = \frac{1}{m_b} \frac{C_F\, \alpha_s(\mu)}{4\pi} \int d\omega\, \theta(\omega + n\cdot p) \left[32 \ln \frac{\omega + n\cdot p}{m_b} + 30 \right] S(\omega) + \mathcal{O}(\alpha_s^2)$$

  by

$$W^{\rm SJF} = \frac{1}{m_b} \frac{C_F \alpha_s(\mu)}{4\pi} \int d\omega\ \theta(\omega + n\cdot p) \left[32 \ln \frac{\mu^2}{m_b(\omega + n\cdot p)} - 18 \right] S(\omega) + \mathcal{O}(\alpha_s^2)$$

- No change for s_i term, just s_i modeling to account for their non-zero one loop contribution

- Although α_s and $1/m_b$ suppressed, effect can be non-negligible
  e.g. constant change from $+30$ to -18
  See GP talk at V_{xb} 2009

Subleading Power Factorization: Resolved Photons

- Several ops. contribute to $\bar{B} \to X_s \gamma$, most important: $Q_{7\gamma}, Q_{8g}, Q_1$
  At leading power only $Q_{7\gamma} - Q_{7\gamma}$, at higher power $Q_i - Q_j$

Subleading Power Factorization: Resolved Photons

- Several ops. contribute to $\bar{B} \to X_s\,\gamma$, most important: $Q_{7\gamma}, Q_{8g}, Q_1$
  At leading power only $Q_{7\gamma} - Q_{7\gamma}$, at higher power $Q_i - Q_j$

- Common lore: like semileptonic, no $1/m_b$ corrections for $\Gamma(\bar{B} \to X_s\,\gamma)$

Subleading Power Factorization: Resolved Photons

- Several ops. contribute to $\bar{B} \to X_s\,\gamma$, most important: $Q_{7\gamma}, Q_{8g}, Q_1$
  At leading power only $Q_{7\gamma} - Q_{7\gamma}$, at higher power $Q_i - Q_j$

- Common lore: like semileptonic, no $1/m_b$ corrections for $\Gamma(\bar{B} \to X_s\,\gamma)$
  Hints that not all is well

- $Q_{8g} - Q_{8g}$ [Ali, Greub '95; Kapustin, Ligeti, Politzer '95]
- $Q_1 - Q_{7\gamma}$ [Voloshin '96; Ligeti, Randall, Wise '97; Grant, Morgan, Nussinov, Peccei '97; Buchalla, Isidori, Rey '97]
- No local OPE for $\Gamma(\bar{B} \to X_s\gamma)$ [Ligeti, Randall, Wise '97]

Subleading Power Factorization: Resolved Photons

- Several ops. contribute to $\bar{B} \to X_s \gamma$, most important: $Q_{7\gamma}, Q_{8g}, Q_1$
  At leading power only $Q_{7\gamma} - Q_{7\gamma}$, at higher power $Q_i - Q_j$

- Common lore: like semileptonic, no $1/m_b$ corrections for $\Gamma(\bar{B} \to X_s \gamma)$
  Hints that not all is well

- $Q_{8g} - Q_{8g}$ [Ali, Greub '95; Kapustin, Ligeti, Politzer '95]
- $Q_1 - Q_{7\gamma}$ [Voloshin '96; Ligeti, Randall, Wise '97; Grant, Morgan, Nussinov, Peccei '97; Buchalla, Isidori, Rey '97]
- No local OPE for $\Gamma(\bar{B} \to X_s\gamma)$ [Ligeti, Randall, Wise '97]

- Effects thought under control or small ...

Subleading Power Factorization: Resolved Photons

- Several ops. contribute to $\bar{B} \to X_s\,\gamma$, most important: $Q_{7\gamma}, Q_{8g}, Q_1$
  At leading power only $Q_{7\gamma} - Q_{7\gamma}$, at higher power $Q_i - Q_j$

- Common lore: like semileptonic, no $1/m_b$ corrections for $\Gamma(\bar{B} \to X_s\,\gamma)$
  Hints that not all is well
  - $Q_{8g} - Q_{8g}$ [Ali, Greub '95; Kapustin, Ligeti, Politzer '95]
  - $Q_1 - Q_{7\gamma}$ [Voloshin '96; Ligeti, Randall, Wise '97; Grant, Morgan, Nussinov, Peccei '97; Buchalla, Isidori, Rey '97]
  - No local OPE for $\Gamma(\bar{B} \to X_s\gamma)$ [Ligeti, Randall, Wise '97]

- Effects thought under control or small ... *never* a systematic study

Subleading Power Factorization: Resolved Photons

- Several ops. contribute to $\bar{B} \to X_s \gamma$, most important: $Q_{7\gamma}, Q_{8g}, Q_1$
  At leading power only $Q_{7\gamma} - Q_{7\gamma}$, at higher power $Q_i - Q_j$

- Common lore: like semileptonic, no $1/m_b$ corrections for $\Gamma(\bar{B} \to X_s \gamma)$
  Hints that not all is well
  - $Q_{8g} - Q_{8g}$ [Ali, Greub '95; Kapustin, Ligeti, Politzer '95]
  - $Q_1 - Q_{7\gamma}$ [Voloshin '96; Ligeti, Randall, Wise '97; Grant, Morgan, Nussinov, Peccei '97; Buchalla, Isidori, Rey '97]
  - No local OPE for $\Gamma(\bar{B} \to X_s \gamma)$ [Ligeti, Randall, Wise '97]

- Effects thought under control or small ... *never* a systematic study
  Uncertainty from $Q_{7\gamma} - Q_{8g}$ was *missed* (tree level diagram...)
  [Lee, Neubert, GP '06]

Subleading Power Factorization: Resolved Photons

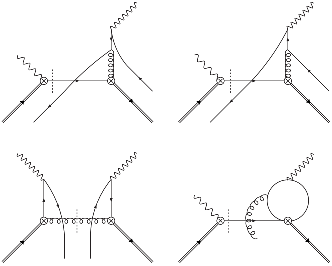
- Several ops. contribute to $\bar{B} \to X_s\,\gamma$, most important: $Q_{7\gamma}, Q_{8g}, Q_1$
  At leading power only $Q_{7\gamma} - Q_{7\gamma}$, at higher power $Q_i - Q_j$

- Common lore: like semileptonic, no $1/m_b$ corrections for $\Gamma(\bar{B} \to X_s\,\gamma)$
  Hints that not all is well

- $Q_{8g} - Q_{8g}$ [Ali, Greub '95; Kapustin, Ligeti, Politzer '95]
- $Q_1 - Q_{7\gamma}$ [Voloshin '96; Ligeti, Randall, Wise '97; Grant, Morgan, Nussinov, Peccei '97; Buchalla, Isidori, Rey '97]
- No local OPE for $\Gamma(\bar{B} \to X_s\gamma)$ [Ligeti, Randall, Wise '97]

- Effects thought under control or small ... *never* a systematic study
  Uncertainty from $Q_{7\gamma} - Q_{8g}$ was *missed* (tree level diagram...)
  [Lee, Neubert, GP '06]
  Non perturbative effects in $\Gamma(\bar{B} \to X_s\,\gamma)$ arise at $1/m_b$
  Effects on the integrated rate $\sim 5\%$ [Benzke, Lee, Neubert, GP '10]

Resolved Photons: Implications

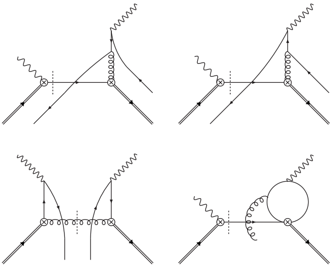


Top line:	$Q_{7\gamma} - Q_{8g}$
Bottom left:	$Q_{8g} - Q_{8g}$
Bottom right:	$Q_1 - Q_{7\gamma}$

- For the $\bar{B} \to X_s\,\gamma$ spectrum:
  Soft functions contributions unique to $B \to X_s\,\gamma$, e.g.

$$h_{17}(\omega, \omega_1) \quad \text{F.T. of} \quad \langle \bar{B}|\bar{b}(tn) \cdots G(tn + r\bar{n}) \cdots b(0)|\bar{B}\rangle$$

Resolved Photons: Implications



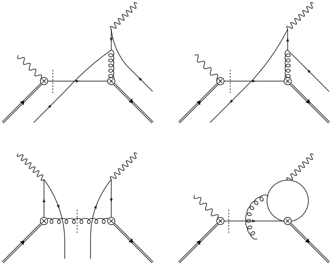
Top line:	$Q_{7\gamma} - Q_{8g}$
Bottom left:	$Q_{8g} - Q_{8g}$
Bottom right:	$Q_1 - Q_{7\gamma}$

- For the $\bar{B} \to X_s\,\gamma$ spectrum:
  Soft functions contributions unique to $B \to X_s\,\gamma$, e.g.

$$h_{17}(\omega,\omega_1) \quad \text{F.T. of} \quad \langle \bar{B}|\bar{b}(tn)\cdots G(tn+r\bar{n})\cdots b(0)|\bar{B}\rangle$$

- Very hard to model:
  Normalization: $2\lambda_2$, 1^{st} moment in $\omega : -\rho_2$, 1^{st} moment in ω_1: zero

Resolved Photons: Implications



Top line:	$Q_{7\gamma} - Q_{8g}$
Bottom left:	$Q_{8g} - Q_{8g}$
Bottom right:	$Q_1 - Q_{7\gamma}$

- For the $\bar{B} \to X_s\,\gamma$ spectrum:
  Soft functions contributions unique to $B \to X_s\,\gamma$, e.g.

$$h_{17}(\omega, \omega_1) \qquad \text{F.T. of} \quad \langle \bar{B}|\bar{b}(tn)\cdots G(tn + r\bar{n})\cdots b(0)|\bar{B}\rangle$$

- Very hard to model:
  Normalization: $2\lambda_2$, 1^{st} moment in $\omega : -\rho_2$, 1^{st} moment in ω_1: zero
  Effects on the *spectrum* not modeled yet
  $\Rightarrow$ Extra uncertainty on S as input to $\bar{B} \to X_u\,\ell\,\bar{\nu}$

$\bar{B} \to X_u \ell \bar{\nu}$: Further developments

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + \frac{1}{m_b} \sum_i H \cdot j_i \otimes S + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{m_b^2}\right)$$

- More recently
- J calculated at $\mathcal{O}(\alpha_s^2)$ [Becher, Neubert '06]
- H calculated at $\mathcal{O}(\alpha_s^2)$ [Bonciani, Ferroglia '08; Asatrian, Greub, Pecjak '08; Beneke, Huber, Li '08; Bell '08]
- j_i calculated at $\mathcal{O}(\alpha_s)$ [GP '09]
- New resolved photon contribution to $B \to X_s \gamma$ [Benzke, Lee, Neubert, GP '10]

- Calculations not fully combined yet

$\bar{B} \to X_u \ell \bar{\nu}$: Further developments

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + \frac{1}{m_b} \sum_i H \cdot j_i \otimes S + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{m_b^2}\right)$$

- More recently
  - J calculated at $\mathcal{O}(\alpha_s^2)$ [Becher, Neubert '06]
  - H calculated at $\mathcal{O}(\alpha_s^2)$ [Bonciani, Ferroglia '08; Asatrian, Greub, Pecjak '08; Beneke, Huber, Li '08; Bell '08]
  - j_i calculated at $\mathcal{O}(\alpha_s)$ [GP '09]
  - New resolved photon contribution to $B \to X_s \gamma$ [Benzke, Lee, Neubert, GP '10]

- Calculations not fully combined yet

  Once combined: NNLO Era!

Future: 2015 -

$\bar{B} \to X_u \, \ell \, \bar{\nu}$: Future

- New calculations should be combined

$\bar{B} \to X_u \, \ell \, \bar{\nu}$: Future

- New calculations should be combined
- Really extract S from $B \to X_s \, \gamma$, not just use m_b and μ_π^2

$\bar{B} \to X_u \, \ell \, \bar{\nu}$: Future

- New calculations should be combined
- Really extract S from $B \to X_s \, \gamma$, not just use m_b and μ_π^2
- What if we could relax the cuts?

  E.g. Belle's $p_\ell^{*B} > 1.0$ GeV [Belle, Urquijo et al. '10]

  Relaxing the cuts makes the measurement more inclusive

$\bar{B} \to X_u \, \ell \, \bar{\nu}$: Future

- New calculations should be combined
- Really extract S from $B \to X_s \, \gamma$, not just use m_b and μ_π^2
- What if we could relax the cuts?

  E.g. Belle's $p_\ell^{*B} > 1.0$ GeV [Belle, Urquijo et al. '10]

  Relaxing the cuts makes the measurement more inclusive

- Three options:

1) Use the same calculations as the end point region

   e.g. BLNP smoothly merges to local OPE

2) Use local OPE

   Recently free quark $d\Gamma(b \to u \, \ell \, \bar{\nu})$ was calculated at $\mathcal{O}(\alpha_s^2)$ [Burcherseifer, Caola, Melnikov '13]

3) Multi Scale OPE [Neubert '05]

   interpolating between local and non-local OPE

$\bar{B} \to X_u \, \ell \, \bar{\nu}$: Future

- New calculations should be combined
- Really extract S from $B \to X_s \, \gamma$, not just use m_b and μ_π^2
- What if we could relax the cuts?

  E.g. Belle's $p_\ell^{*B} > 1.0$ GeV [Belle, Urquijo et al. '10]

  Relaxing the cuts makes the measurement more inclusive

\- Three options:

1) Use the same calculations as the end point region

   e.g. BLNP smoothly merges to local OPE

2) Use local OPE

   Recently free quark $d\Gamma(b \to u \, \ell \, \bar{\nu})$ was calculated at $\mathcal{O}(\alpha_s^2)$ [Burcherseifer, Caola, Melnikov '13]

3) Multi Scale OPE [Neubert '05]

   interpolating between local and non-local OPE

\- My personal preference: try a variety of approaches

  Data with different *cuts* will allow to test these options

Take home message

- 1990's -2000's: Next to Leading Order (NLO) Era:

  c_0 at $\mathcal{O}(\alpha_s)$ + first power corrections at $\mathcal{O}(\alpha_s^0)$

- 2010's: Next to Next to Leading Order (NNLO) Era

  c_0 at $\mathcal{O}(\alpha_s^2)$ + first power corrections at $\mathcal{O}(\alpha_s)+\ldots$