B meson decay constant and $B \to D\tau\nu$ form factors from lattice QCD

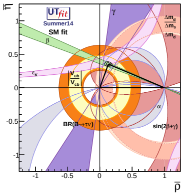
Gregorio Herdoíza

IFT, UAM/CSIC

Challenges in Semileptonic B decays, Mainz, April 24, 2015

CKM matrix & lattice QCD

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



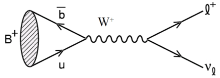
role of lattice determinations

CKM	process	lattice	precision (%)
$\lvert V_{us}\rvert$	$K \to \ell\nu$	f_K	$\sim$ 0.5
	$K \to \pi\ell\nu$	$f_+^{K\pi}(q^2=0)$	$\sim$ 0.5
$\lvert V_{us}\rvert/\lvert V_{ud}\rvert$	$K \to \mu\nu/\pi \to \mu\nu$	f_K/f_π	$\sim$ 0.5
$\lvert V_{cd}\rvert$	$D \to \ell\nu$	f_{D_s}/f_D	$\sim$ 1.0
	$D \to \pi\ell\nu$	$f_{+,0}^{D\pi}(0)$	$\sim$ 5.0
$\lvert V_{cs}\rvert$	$D_s \to \ell\nu$	f_{D_s}	$\sim$ 1.0
	$D \to K\ell\nu$	$f_+^{DK}(0)$	$\sim$ 3.0
$\lvert V_{ub}\rvert$	$B \to \tau\nu$	f_B	$\sim$ 2.5
		f_{B_s}/f_B	$\sim$ 2.0
	$B \to \pi\ell\nu$	$f_+^{B\pi}(q^2)$	$\sim$ 3.5
$\lvert V_{cb}\rvert$	$B \to D^{(*)}\ell\nu$	$\mathcal{F}_{B\to D^{(*)}}$	$\sim$ 1.5
$V_{tq}^* V_{tq'}$; $V_{cq}^* V_{cq'}$	ϵ_K	$\hat{B}_K$	$\sim$ 1.5
		$K \to \pi\pi$	$\to$ 30.0
$V_{tq}^* V_{tq'}$	Δm_d	$\hat{B}_{B_s}/\hat{B}_{B_d}$; ξ	$\leq$ 10.0
	Δm_s	$\hat{B}_{B_s}$	$\leq$ 5.0

- quark masses, BSM four-fermion operators, ...

τ lepton

- $(m_\tau/m_\mu)^2 \sim 300$ $(m_\tau/m_e)^2 \sim 10^7$
- Decay width : $B \to \ell\nu$



$$\Gamma(B \to \ell\nu) = \frac{G_\mu^2}{8\pi} \times |V_{ub}|^2 \times M_B^3 \left(\frac{m_\ell}{M_B}\right)^2 \left[1 - \left(\frac{m_\ell}{M_B}\right)^2\right] \times f_B^2$$

- alternative method to determine $|V_{ub}|$
- helicity suppression : $(m_\ell/M_B)^2$
- $B \to \tau\nu$

  BaBar, Belle : [$\sim$ 15%] $\rightsquigarrow$ Belle II : [$\sim$ 5%]

- $B \to \mu\nu$

  $\mathcal{B}(B^+ \to \mu^+\nu_\mu) < 2.7 \times 10^{-6}$ [Belle, 1406.6356] $\rightsquigarrow$ Belle II

  $\mathcal{B}(B^+ \to \mu^+\nu_\mu) = (3.4 \pm 0.3) \times 10^{-7}$ SM

B-meson decay constants

- definition of decay constants

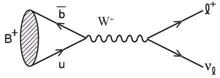
$$\langle 0|\, A^\mu \,|B_q(p)\rangle \;=\; p_B^\mu \, f_{B_q}$$

with $q = u, d, s$

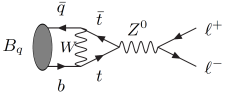
- f_B , f_{B_s} : $|V_{ub}|$, $\mathcal{B}(B \to \tau\nu)$, rare leptonic decays, $B - \bar{B}$ mixing, ...

- f_B : leptonic decays

$$\mathcal{B}(B \to \tau\nu) \;\propto\; G_F^2 \, f_B^2 \, |V_{ub}|^2$$



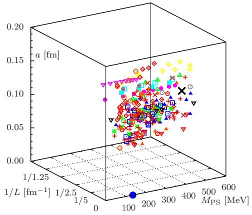
- f_{B_s} : rare leptonic decays



$$\mathcal{B}(B_s \to \mu^+\mu^-) \;\propto\; M_{B_s} \, f_{B_s}^2 \, |V_{tb}^* V_{ts}|^2$$

CMS, LHCb : [$\sim$ 25%]

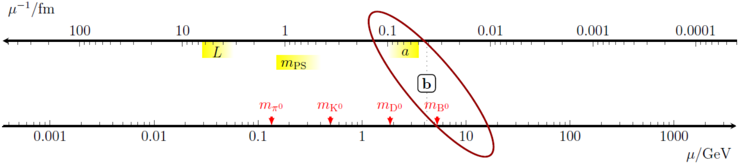
heavy quarks on the lattice



- lattice spacing : a
- lattice size : L
- pion masses : $M_{\rm PS}$
- number of flavours : $N_{\rm f}$

$$a \in [0.05,\ 0.15]\,\mathrm{fm} \quad \rightsquigarrow \quad 1/a \in [1.3,\ 4.0]\,\mathrm{GeV}$$

$$M_{\rm PS} L \geq 4$$



- ▶ to control discretisation effects for heavy quark masses: $m_h < 1/a$

$m_c \approx 1.3\,\mathrm{GeV}$ $\quad$ $m_b \approx 4.3\,\mathrm{GeV}$ $\quad$ $am_b > 1$

- ▶ for heavy quarks:

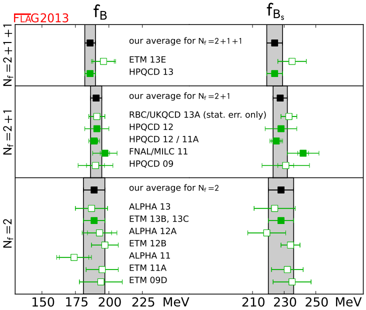
$$O[a] = O|_{\rm C.L.} + c \cdot (am_h)^2 + \ldots$$

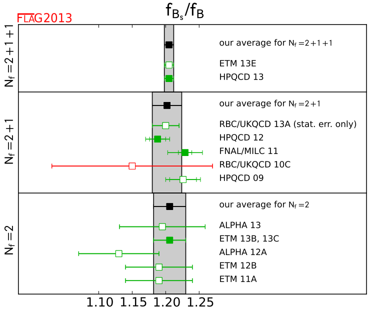
how to treat bottom quark on the lattice?

heavy quarks on the lattice

- how to treat bottom quark on the lattice? effective theories
  - heavy quark effective theory (HQET)
  - relativistic action with heavy-quark interpretation
  - relativistic charm quarks $+$ LO HQET
  - chain of ratios with $m_h \geq m_c$ $+$ scaling laws of HQET $\rightsquigarrow$ ratio method
  - NRQCD

FLAG : f_B, f_{B_s}, f_{B_s}/f_B





- $N_f = 2 + 1 + 1$

$$f_B = 186(4)\,\text{MeV}\ \ [2.2\%] \qquad f_{B_s} = 224(5)\,\text{MeV}\ \ [2.2\%] \qquad \frac{f_{B_s}}{f_B} = 1.205(7)\ \ [0.6\%]$$

- $N_f = 2 + 1$

$$f_B = 190.5(4.2)\,\text{MeV}\ \ [2.2\%] \qquad f_{B_s} = 227.7(4.5)\,\text{MeV}\ \ [2.0\%] \qquad \frac{f_{B_s}}{f_B} = 1.202(22)\ \ [1.8\%]$$

- $N_f = 2$

$$f_B = 189(8)\,\text{MeV}\ \ [4.2\%] \qquad f_{B_s} = 228(8)\,\text{MeV}\ \ [3.5\%] \qquad \frac{f_{B_s}}{f_B} = 1.206(24)\ \ [2.0\%]$$

[FLAG, 1310.8555]

Heavy Quark Effective Theory (HQET)

- $1/m$ expansion

$$\mathcal{L}_{\rm HQET} = \overline{\psi}_{\rm h}\Big[\underbrace{D_0 + \delta m}_{\text{static limit (LO)}} \underbrace{-\omega_{\rm kin}\,\boldsymbol{D}^2 - \omega_{\rm spin}\,\boldsymbol{\sigma}\mathbf{B}}_{\text{NLO, O(1/m)}}\Big]\psi_{\rm h} + \ldots\,, \qquad \left.\begin{matrix}\omega_{\rm kin}\\ \omega_{\rm spin}\end{matrix}\right\} \sim \frac{1}{2m}$$

- action

$$e^{-\mathcal{S}_{\rm HQET}} = \exp\Big\{-a^4\sum{}_x\mathcal{L}_{\rm stat}(x)\Big\}\times$$
$$\Big\{1-a^4\sum{}_x\mathcal{L}^{(1)}(x) + \tfrac{1}{2}\Big[a^4\sum{}_x\mathcal{L}^{(1)}(x)\Big]^2 - a^4\sum{}_x\mathcal{L}^{(2)}(x) + \ldots\Big\}$$

- $1/m$ terms : insertions on local operators $\rightsquigarrow$ renormalizability order by order
- observables

$$\langle\mathcal{O}\rangle = \mathcal{Z}^{-1}\int\mathcal{D}[\phi]\,e^{-\mathcal{S}_{\rm rel}-\mathcal{S}_{\rm stat}}\,\mathcal{O}\Big\{1-a^4\sum{}_x\mathcal{L}^{(1)}(x) + \ldots\Big\}$$

- HQET on the lattice

$$\langle\mathcal{O}\rangle = \langle\mathcal{O}\rangle_{\rm stat} + \omega_{\rm kin}\;a^4\sum_x\langle\mathcal{O}\mathcal{O}_{\rm kin}(x)\rangle_{\rm stat} + \omega_{\rm spin}\;a^4\sum_x\langle\mathcal{O}\mathcal{O}_{\rm spin}(x)\rangle_{\rm stat}$$

$$\mathcal{O}_{\rm kin} \equiv -\overline{\psi}_{\rm h}\boldsymbol{D}^2\psi_{\rm h}$$
$$\mathcal{O}_{\rm spin} \equiv -\overline{\psi}_{\rm h}\boldsymbol{\sigma}\mathbf{B}\psi_{\rm h}$$

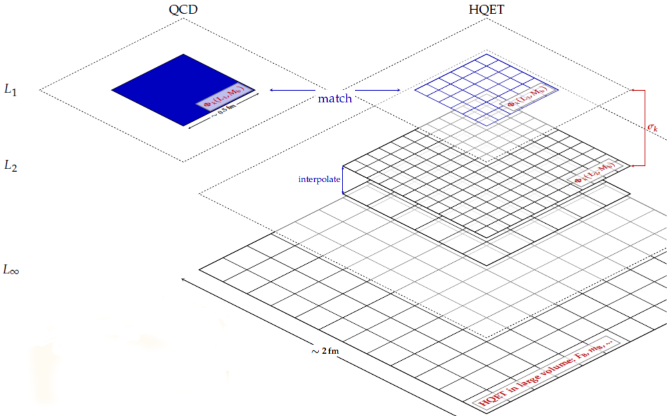
HQET on the lattice [ALPHA]

- mixing : power divergences e.g. $\overline{\psi}_h D_0 \psi$ and $\overline{\psi}_h D_0 \psi$

$$\rightsquigarrow \quad \delta m = c(g)/a \;\sim\; e^{1/(2b_0 g^2)}\,(c_1 g^2 + c_2 g^4 + \dots)$$

- uncontrolled truncation error in perturbation theory

- non-perturbative matching : determination of HQET parameters in small volume

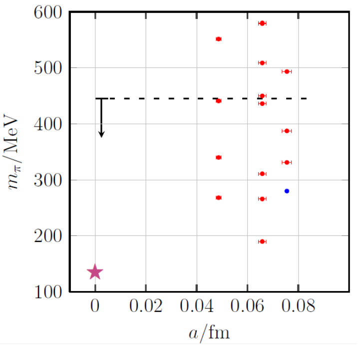


[P. Fritzsch]

B-meson decays constants [ALPHA]

- Wilson fermions : $O(a)$ improved with non-perturbative $c_{\rm SW}$

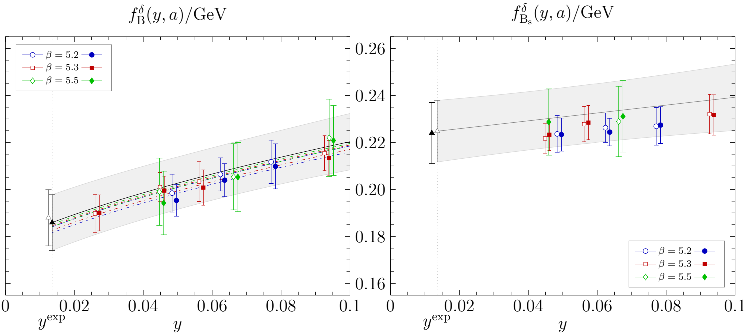
  [CLS; ALPHA, 1205.5380]
- $N_{\rm f} = 2$
- Wilson plaquette gauge action
- heavy quarks : non-perturbative HQET at $O(1/m_b)$
- $a = \{0.08,\ 0.07,\ 0.05\}$ fm
- $L = \{2.1,\ 3.2\}$ fm , $M_{\rm PS}L > 4$
- $M_\pi \in \{190,\ 630\}$ MeV
- scale setting: f_K



B-meson decays constants [ALPHA]

$$f_{\mathrm{B_i}} = \exp(\phi_i) \Big/ \sqrt{a^3 m_{\mathrm{B_i}}/2}$$

$$\phi_i = \ln(Z_{\mathrm{A}}^{\mathrm{HQET}}) + b_{\mathrm{A}}^{\mathrm{stat}} a m_{\mathrm{q},i} + \left[\ln(a^{3/2} p^{\mathrm{stat}}) + \omega_{\mathrm{kin}} p^{\mathrm{kin}} + \omega_{\mathrm{spin}} p^{\mathrm{spin}} + c_{\mathrm{A}}^{(1)} p^{\mathrm{A}^{(1)}} \right]_{m_{\mathrm{q},i}}$$



$f_B = 186(13)(2)_\chi$ MeV [7%] $\qquad f_{B_s} = 224(14)(2)_\chi$ MeV [6%] $\qquad \frac{f_{B_s}}{f_B} = 1.203(62)(19)_\chi$ [5%]

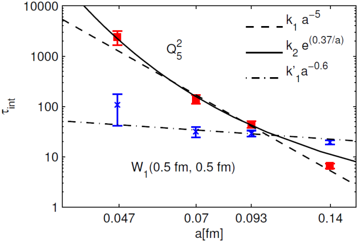
$f_B^{\mathrm{stat}} = 190(5)(2)_\chi$ MeV [3%] $\qquad f_{B_s}^{\mathrm{stat}} = 226(6)(9)_\chi$ MeV [5%] $\qquad \frac{f_{B_s}^{\mathrm{stat}}}{f_B^{\mathrm{stat}}} = 1.189(24)(30)_\chi$ [3%]

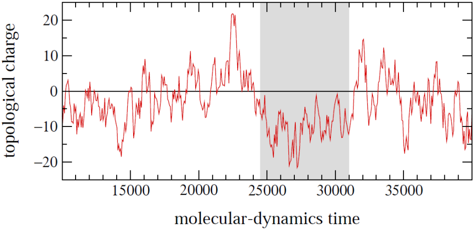
[ALPHA, 1404.3590]

statistical errors

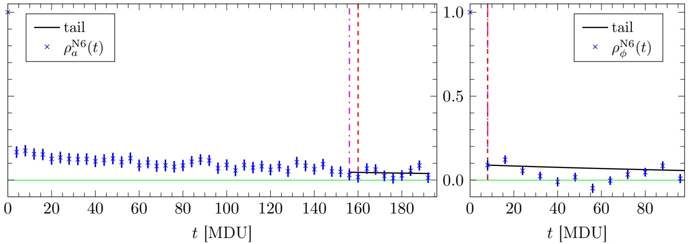
- autocorrelations :

quenched, $64^3 \times 32$, $a = 0.07$ fm, HMC





- autocorrelation function : lattice spacing and ϕ



Wilson clover action with relativistic heavy-quark interpretation

based on the Fermilab approach

- rest frame heavy-light meson : $|\vec{p}_h| \sim \Lambda_{QCD} \ll 1/a$
- expansion in powers of $a|\vec{p}_h|$ while keeping all orders in am_h and D_0
- Symanzik improvement : coef. depend on am_h

- discretization effects are parametrically similar to those of light quarks

- Relativistic Heavy Quark (RHQ) action : non-perturbative tuning of parameters of the clover action
- tuning using B_s-meson: spin-averaged mass and hyperfine splitting ; equal rest and kinetic masses
- improvement of the current : $Z_\Phi = \rho_A^{bl} \sqrt{Z_V^{ll} Z_V^{bb}}$

  ρ_A^{bl} from PT : 1-loop tadpole improved

B-meson decays constants [RBC-UKQCD]

- domain-wall (Shamir) fermions [1011.0892]
- Iwasaki gauge action

- $N_\mathrm{f} = 2 + 1$
- $a = \{0.08,\ 0.11\}$ fm
- $L = 2.6$ fm , $M_\mathrm{PS} L > 3.7$
- $M_\pi \in \{290,\ 420\}$ MeV

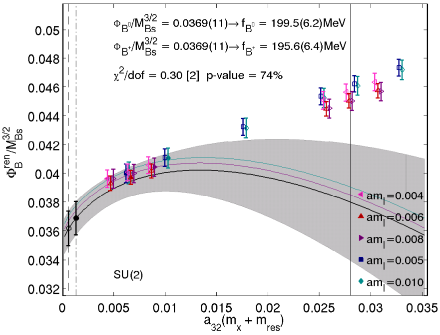
- scale setting: M_Ω

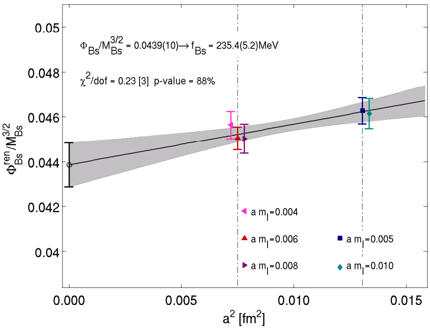
L	a(fm)	m_l	m_s	M_π(MeV)
24	≈ 0.11	0.005	0.040	329
24	≈ 0.11	0.010	0.040	422
32	≈ 0.08	0.004	0.030	289
32	≈ 0.08	0.006	0.030	345
32	≈ 0.08	0.008	0.030	394

[RBC-UKQCD, 1404.4670]

B-meson decays constants [RBC-UKQCD]

- SU(2) HMχPT : unitary points





[RBC-UKQCD, 1404.4670]

B-meson decays constants [RBC-UKQCD]

- Relative contributions to the error

	f_{B^+}(%)	f_{B_s}(%)
statistics	3.1	3.3
chiral-continuum extrapolation	4.4	5.9
lattice-scale uncertainty	1.5	1.5
light- and strange-quark mass uncertainty	0.1	0.1
RHQ parameter tuning	1.2	1.2
HQ discretization errors	1.7	1.7
LQ and gluon discretization errors	1.1	1.1
renormalization factor	1.7	1.7
finite volume	0.4	0.5
isospin-breaking and EM	0.7	0.7
total	6.3	7.6

$$f_{B^0} = 199.5(12.6)\,\mathrm{MeV}, \qquad f_{B^+} = 195.6(14.9)\,\mathrm{MeV} \qquad f_{B_s} = 235.4(12.2)\,\mathrm{MeV}$$

$$f_{B_s}/f_{B^0} = 1.197(50)\ [4\%] \qquad f_{B_s}/f_{B^+} = 1.223(71)\ [6\%]$$

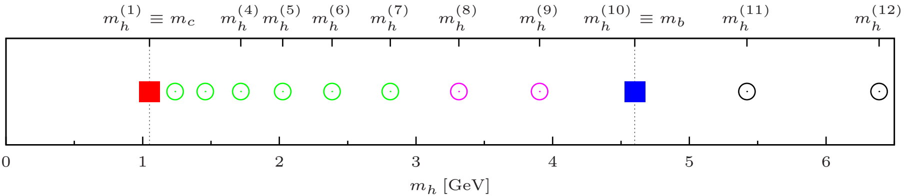
[RBC-UKQCD, 1404.4670]

Ongoing : physical point results (Möbius DWF), AMA

see also [RBC-UKQCD, Aoki et al., 1406.6192] : f_B^{stat}

ratio method [ETMC]

- series of quark masses : $m_h^{(i+1)} = \lambda\, m_h^{(i)}$ $\qquad m_h^{(1)} \approx m_c$ $\qquad \lambda \approx 1.18$



- chain equation for observable O :

$$O(m_b) \equiv O(m_h^{(10)}) = O(m_h^{(1)}) \times \frac{O(m_h^{(2)})}{O(m_h^{(1)})} \times \ldots \times \frac{O(m_h^{(10)})}{O(m_h^{(9)})}$$

- ratios z :

$$z(m_h^{(i)}) \equiv \frac{O(m_h^{(i)})}{O(m_h^{(i-1)})}$$

ratio method [ETMC]

$\lambda \approx 1.18$

- cut-off effects in ratios z :

$$z(m_h^{(i)}) \equiv \frac{O(m_h^{(i)})}{O(m_h^{(i-1)})}$$

$$\approx z(m_h^{(i)})\Big|_{\mathrm{C.L.}} + c \cdot (\lambda^2 - 1) \cdot \left(am_h^{(i)}\right)^2 + \ldots$$

- choice of O : scaling law of HQET e.g. $O = \Phi = f_H \sqrt{M_H}$

$$O(m_h) = O_{\mathrm{stat}} + \frac{d}{m_h} + \ldots$$

- scaling law of ratio z

$$z(m_h) = 1 + \frac{\tilde{d} \cdot (\lambda - 1)}{m_h} + \ldots \xrightarrow[m_h \to \infty]{} 1$$

ratio method: implementation

chain equation for observable O :

$$O(m_b) \equiv O(m_h^{(10)}) = O(m_h^{(1)}) \times \frac{O(m_h^{(2)})}{O(m_h^{(1)})} \times \ldots \times \frac{O(m_h^{(10)})}{O(m_h^{(9)})}$$

1. determine $O(m_h^{(1)})$ in the charm region
2. determine $z(m_h^{(i)})$, $i = 2, \ldots, 7$, for $\lambda \approx 1.18$
3. use $z(m_h) \xrightarrow[m_h \to \infty]{} 1$ to reach m_b
4. apply chain eq. to get $O(m_b)$

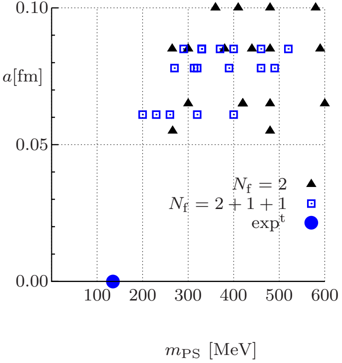
properties:

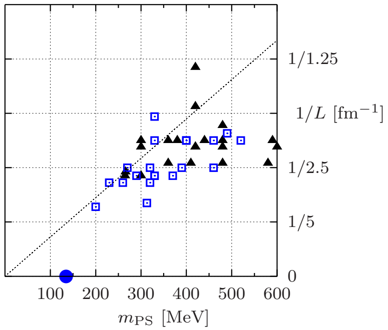
- continuum limit taken at each step : regularisation independent
- does not require explicit computation in an effective theory
- not very computationally costly [multi-mass solver]

ETMC ensembles

- fermionic lattice action : Wilson twisted-mass
- $N_f = 2$: u, d
- $N_f = 2 + 1 + 1$: u, d, s, c

- physical input : M_π , M_K , f_π





- $m_h \in [m_c,\ 3m_c]$
- to isolate ground state : smearing $\rightsquigarrow$ GEVP

[1308.1851, 1311.2837]

ratio method : f_B , f_{B_s}

chain equation for observable O :

$$O(m_b) \equiv O(m_h^{(10)}) = O(m_h^{(1)}) \times \frac{O(m_h^{(2)})}{O(m_h^{(1)})} \times \ldots \times \frac{O(m_h^{(10)})}{O(m_h^{(9)})}$$

observables O

- $f_{hs}\sqrt{M_{hs}} \rightsquigarrow f_{B_s}$
- $f_{hs}\sqrt{m_h} \rightsquigarrow f_{B_s}$
- $f_{hs}/f_{hl} \rightsquigarrow f_{B_s}/f_B$
- $[(f_{hs}/f_{hl})/(f_{sl}/f_{ll})] \times (f_K/f_\pi) \rightsquigarrow f_{B_s}/f_B$

in practice, for $O = f_{hs}\sqrt{M_{hs}}$:

$$\tilde{z}_s(\overline{\mu}_h, \lambda) = \frac{f_{hs}(\overline{\mu}_h)\sqrt{M_{hs}(\overline{\mu}_h)}}{f_{hs}(\overline{\mu}_h/\lambda)\sqrt{M_{hs}(\overline{\mu}_h/\lambda)}} \cdot \frac{C_A^{\mathrm{stat}}(\mu^*, \overline{\mu}_h/\lambda)}{C_A^{\mathrm{stat}}(\mu^*, \overline{\mu}_h)}$$

chain equation :

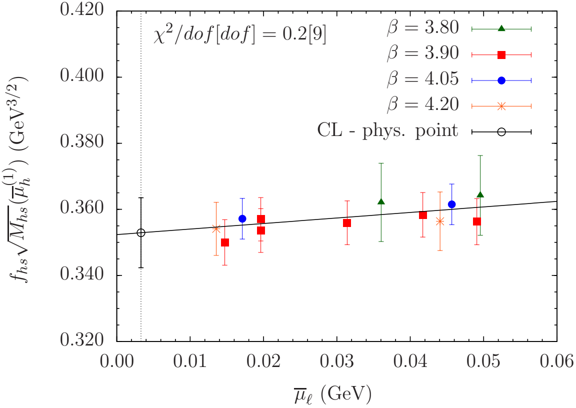
$$\tilde{z}_s(\overline{\mu}_h^{(2)}) \times \tilde{z}_s(\overline{\mu}_h^{(3)}) \times \ldots \times \tilde{z}_s(\overline{\mu}_h^{(10)}) = \frac{f_{hs}(\overline{\mu}_h^{(10)})\sqrt{M_{hs}(\overline{\mu}_h^{(10)})}}{f_{hs}(\overline{\mu}_h^{(1)})\sqrt{M_{hs}(\overline{\mu}_h^{(1)})}} \cdot \left[\frac{C_A^{\mathrm{stat}}(\mu^*, \overline{\mu}_h^{(1)})}{C_A^{\mathrm{stat}}(\mu^*, \overline{\mu}_h^{(10)})}\right]$$

f_{B_s} $N_f = 2$

chain equation :

$$\tilde{z}_s(\overline{\mu}_h^{(2)}) \times \tilde{z}_s(\overline{\mu}_h^{(3)}) \times \ldots \times \tilde{z}_s(\overline{\mu}_h^{(10)}) = \frac{f_{hs}(\overline{\mu}_h^{(10)}) \sqrt{M_{hs}(\overline{\mu}_h^{(10)})}}{f_{hs}(\overline{\mu}_h^{(1)}) \sqrt{M_{hs}(\overline{\mu}_h^{(1)})}} \cdot \left[\frac{C_A^{\rm stat}(\mu^*, \overline{\mu}_h^{(1)})}{C_A^{\rm stat}(\mu^*, \overline{\mu}_h^{(10)})}\right]$$

1. triggering point : $f_{hs}(\overline{\mu}_h^{(1)}) \sqrt{M_{hs}(\overline{\mu}_h^{(1)})}$



$\overline{\mu}_h^{(1)} = \overline{\mu}_c$

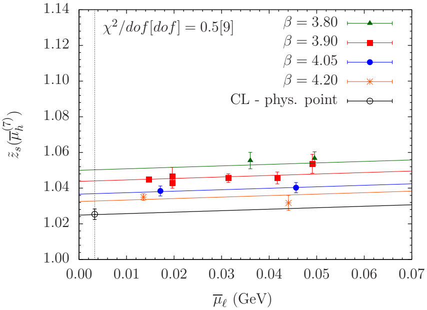
f_{B_s} $\qquad N_f = 2$

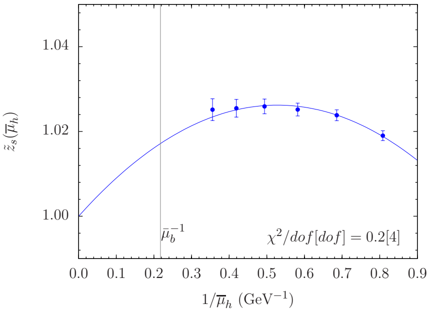
chain equation :

$$\tilde{z}_s(\overline{\mu}_h^{(2)}) \times \tilde{z}_s(\overline{\mu}_h^{(3)}) \times \ldots \times \tilde{z}_s(\overline{\mu}_h^{(10)}) = \frac{f_{hs}(\overline{\mu}_h^{(10)})\sqrt{M_{hs}(\overline{\mu}_h^{(10)})}}{f_{hs}(\overline{\mu}_h^{(1)})\sqrt{M_{hs}(\overline{\mu}_h^{(1)})}} \cdot \left[\frac{C_A^{\rm stat}(\mu^*, \overline{\mu}_h^{(1)})}{C_A^{\rm stat}(\mu^*, \overline{\mu}_h^{(10)})}\right]$$

2. ratios : $\tilde{z}_s(\overline{\mu}_h^{(i)})$, $\quad i = 2, \ldots, 7$, for $\lambda = 1.1784$

3. $\tilde{z}_s(\overline{\mu}_h) = 1 + \frac{\eta_1}{\overline{\mu}_h} + \frac{\eta_2}{\overline{\mu}_h^2}$



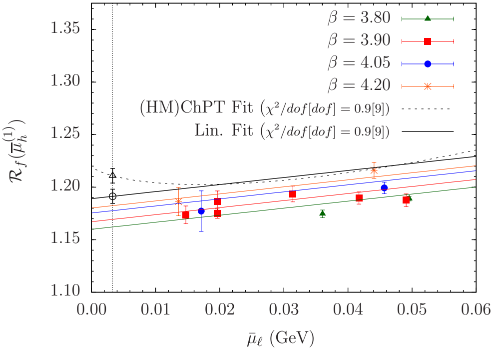


$\overline{\mu}_h^{(7)} \approx 2.7\,\overline{\mu}_c$

4. chain equation : $f_{B_s} = 228(5)(6)$ MeV

f_{B_s}/f_B $\qquad N_f = 2$

$$O = \mathcal{R}_f \equiv [(f_{hs}/f_{hl})/(f_{sl}/f_{ll})] \times (f_K/f_\pi)$$



$$\mathcal{R}_f = a_h^{(1)} + b_h^{(1)}\overline{\mu}_\ell + D_h^{(1)}a^2$$

$$\mathcal{R}_f = a_h^{(2)}\left[1 + b_h^{(2)}\overline{\mu}_\ell + \left[\frac{3(1+3\hat{g}^2)}{4} - \frac{5}{4}\right]\frac{2B_0\overline{\mu}_\ell}{(4\pi f_0)^2}\log\left(\frac{2B_0\overline{\mu}_\ell}{(4\pi f_0)^2}\right)\right] + D_h^{(2)}a^2$$

$$\rightsquigarrow \quad \frac{f_{B_s}}{f_B} = 1.206(10)(22)$$

statistical and systematic uncertainties

$N_f = 2$

source of uncertainty [%]	f_{B_s}	f_{B_s}/f_B	f_B
stat. + fit (C.L. and chiral)	2.2	0.8	2.1
lat. scale	2.0	-	2.0
discr. effects	1.3	0.4	1.7
$1/\mu_h$	1.0	0.1	1.1
chiral extr. trig. point	-	1.7	1.7
total	3.4	2.0	4.0

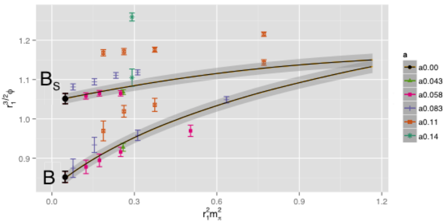
- $N_f = 2$

$$f_B = 189(8)\ \text{MeV} \quad , \quad f_{B_s} = 228(8)\ \text{MeV} \quad , \quad \frac{f_{B_s}}{f_B} = 1.206(24)$$

- $N_f = 2+1+1$: [PRELIMINARY]

$$f_B = 196(9)\ \text{MeV} \quad , \quad f_{B_s} = 235(9)\ \text{MeV} \quad , \quad \frac{f_{B_s}}{f_B} = 1.201(25)$$

B-meson decays constants [Fermilab-MILC]



- MILC ensembles : rooted asqtad staggered fermions
- $N_f = 2 + 1$
- tadpole-improved gauge action
- Fermilab action for *b* quark
- $a = \{0.045, 0.15\}$ fm
- $L = [2.4, 5, 5]$ fm , $M_{PS} L > 3.8$
- $M_\pi^{min} \approx 174\,\text{MeV}$
- scale setting: f_π

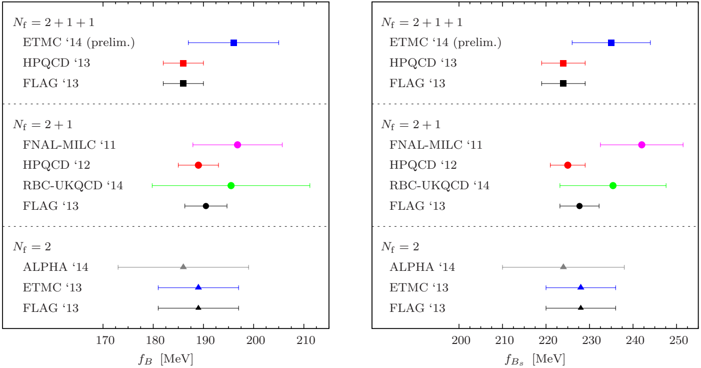
Source	f_{B_s}/f_B	f_B/f_D
Statistics	0.4%	0.6%
Heavy-quark discretization	0.6%	0.8%
Light-quark discretization	0.1%	0.3%
Chiral extrapolation	0.6%	1.0%
Heavy-quark tuning	0.1%	1.7%
$Z_{V^4_{qq}}$	0.0%	—
$Z_{V^4_{QQ}}$	—	0.2%
Finite volume	0.2%	0.3%
Higher-order $\rho_{A^4_{Qq}}$	0.1%	4.1%
Total projected error	0.9%	4.7%

[PRELIMINARY, Fermilab-MILC, 1501.01991]

$f_{B_s}/f_{B^+} = 1.229(26)$ [2%]

[Fermilab-MILC, 1112.3051]

comparison : f_B, f_{B_s}



Static limit of HQET:

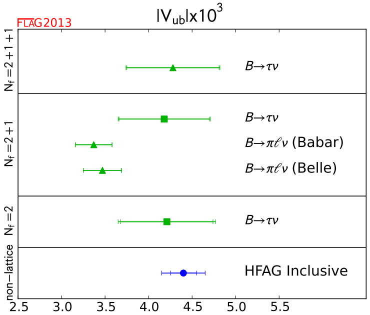
[Aoki et al., 1406.6192]

$$f_B^{\rm stat} = 219(17)\,\text{MeV}\;\;[8\%] \qquad f_{B_s}^{\rm stat} = 264(19)\,\text{MeV}\;\;[7\%] \qquad \frac{f_{B_s}^{\rm stat}}{f_B^{\rm stat}} = 1.193(41)\;\;[3\%]$$

[ALPHA, 1404.3590]

$$f_B^{\rm stat} = 190(5)(2)_\chi\,\text{MeV}\;\;[3\%] \qquad f_{B_s}^{\rm stat} = 226(6)(9)_\chi\,\text{MeV}\;\;[5\%] \qquad \frac{f_{B_s}^{\rm stat}}{f_B^{\rm stat}} = 1.189(24)(30)_\chi\;\;[3\%]$$

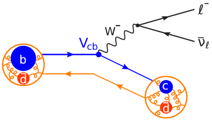
FLAG : $|V_{ub}|$



	from	$|V_{ub}| \times 10^3$
FLAG $N_f = 2$	$B \to \tau\nu$	4.21(53)(18)
FLAG $N_f = 2+1$	$B \to \tau\nu$	4.18(52)(9)
FLAG $N_f = 2+1+1$	$B \to \tau\nu$	4.28(53)(9)
FLAG $N_f = 2+1$	$B \to \pi\ell\nu$ (Babar)	3.37(21)
FLAG $N_f = 2+1$	$B \to \pi\ell\nu$ (Belle)	3.47(22)
HFAG inclusive average	$B \to X_u\ell\nu$	4.40(15)(20)

[FLAG, 1310.8555]

$\bar{B} \to D\ell\bar{\nu}_\ell$: τ lepton



- $(m_\tau/m_\mu)^2 \sim 300$ $\qquad$ $(m_\tau/m_e)^2 \sim 10^7$

- matrix element

$$\langle D(p')|\bar{c}\gamma_\mu b|\bar{B}(p)\rangle = \left(p_\mu + p'_\mu - \frac{m_B^2 - m_D^2}{q^2} q_\mu\right) f_+(q^2) + \frac{m_B^2 - m_D^2}{q^2} q_\mu f_0(q^2)$$

- differential decay rate

$$\frac{d\mathcal{B}(\bar{B} \to D\ell\bar{\nu}_\ell)}{dq^2} = \tau_{B^0} \frac{G_F^2 |V_{cb}|^2}{192\pi^3 m_B^3} \left[c_+^\ell(q^2)|f_+(q^2)|^2 + c_0^\ell(q^2)|f_0(q^2)|^2\right]$$

where

$$c_+^\ell(q^2) = \lambda^{3/2}(q^2, m_B^2, m_D^2)\left[1 - \frac{3}{2}\frac{m_\ell^2}{q^2} + \frac{1}{2}\left(\frac{m_\ell^2}{q^2}\right)^3\right]$$

$$c_0^\ell(q^2) = m_\ell^2\, \lambda^{1/2}(q^2, m_B^2, m_D^2)\frac{3}{2}\frac{m_B^4}{q^2}\left(1 - \frac{m_\ell^2}{q^2}\right)^2\left(1 - \frac{m_D^2}{m_B^2}\right)^2$$

$$\lambda(q^2, m_B^2, m_D^2) = [q^2 - (m_B + m_D)^2][q^2 - (m_B - m_D)^2]$$

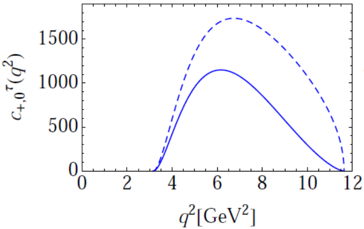
- τ lepton : coupling to a scalar non-SM particle can be probed

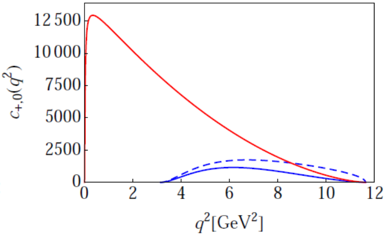
$\bar{B} \to D\ell\bar{\nu}_\ell$: τ lepton

$$m_\ell^2 \leq q^2 \leq (m_B - m_D)^2 = 11.63\ \text{GeV}^2\,.$$

$f_+(0) = f_0(0)$

- phase space:





dashed is scalar and solid is vector

blue is τ and red is μ

[Becirevic et al., 1206.4977]

- ratio:

$$R(D) = \frac{\mathcal{B}(\bar{B} \to D\tau\bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \to D\mu\bar{\nu}_\mu)}$$

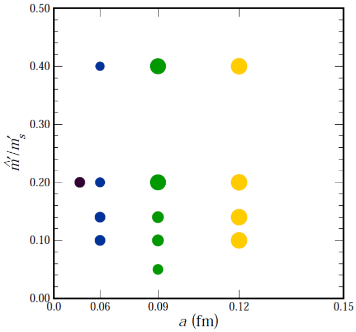
depends on $f_0(q^2)/f_+(q^2)$

$B \to D\ell\nu$ [Fermilab-MILC]

- MILC ensembles : rooted asqtad staggered fermions
- $N_\mathrm{f} = 2 + 1$
- tadpole-improved gauge action
- Fermilab action for b and c quarks

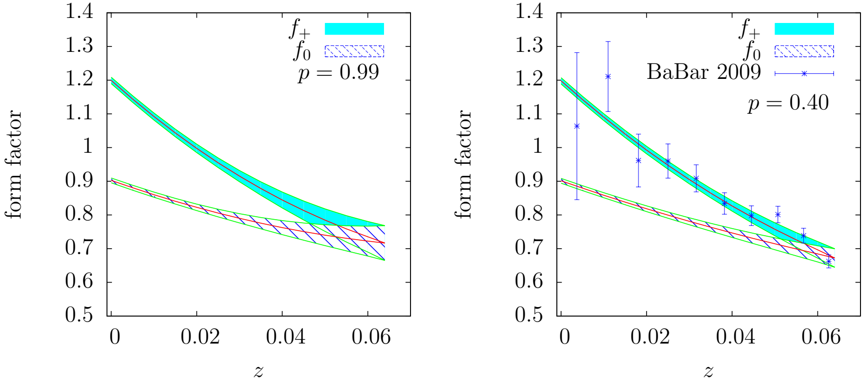
- $a = \{0.045,\ 0.15\}$ fm
- $L = [2.4, 5, 5]$ fm , $M_\mathrm{PS} L > 3.8$
- $M_\pi^\mathrm{min} \approx 174\,\mathrm{MeV}$

- scale setting: f_π



[Fermilab-MILC, 1503.07237]

$B \to D\ell\nu$ [Fermilab-MILC]



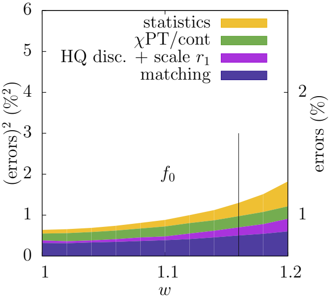
BGL parametrisation

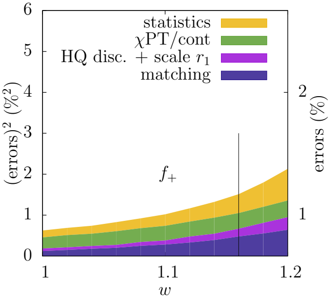
[Fermilab-MILC, 1501.0199]

$B \to D\ell\nu$ [Fermilab-MILC]

Source	f_+(%)	f_0(%)
Statistics+matching+χPT cont. extrap.	1.2	1.1
(Statistics)	(0.7)	(0.7)
(Matching)	(0.7)	(0.7)
(χPT/cont. extrap.)	(0.6)	(0.5)
Heavy-quark discretization	0.4	0.4
Lattice scale r_1	0.2	0.2
Total error	1.2	1.1

$w = 1.16$





[Fermilab-MILC, 1501.0199]

Ongoing : reduce heavy quark discretisation effects $\rightsquigarrow$ Oktay-Kronfeld action [Fermilab-MILC; SWME]

$R(D)$ and $R(D^*)$: comparison

$$R(D) = \frac{\mathcal{B}(\bar{B} \to D\tau\bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \to D\mu\bar{\nu}_\mu)}$$

$R(D) = 0.299(11)$ Fermilab-MILC (2015) + BaBar B-tagged (2009)

$R(D) = 0.316(12)(07)$ Fermilab-MILC (2012)

$R(D) = 0.300(08)$ [PRELIMINARY] HPQCD (2015) more details in [talk by Carlos Pena]

$R(D) = 0.440(58)(42)$ BaBar (2012) i.e. $2\,\sigma$ from Fermilab-MILC (2015)

$$R(D^*) = \frac{\mathcal{B}(\bar{B} \to D^*\tau\bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \to D^*\mu\bar{\nu}_\mu)}$$

in addition to $h_{A_1}(w)$, it requires the other form factors Belle II

$R(D^*) = 0.252(3)$ Fajfer et al. (2012)

$R(D^*) = 0.332(24)(18)$ BaBar (2012) i.e. $2.7\,\sigma$

$\bar{B} \to D\ell\bar{\nu}_\ell$: coupling to tensor operator

- matrix element

$$\langle D(p')|\bar{c}\sigma_{\mu\nu}b|\bar{B}(p)\rangle = -i\left(p_\mu p'_\nu - p'_\mu p_\nu\right)\frac{2\,f_T(q^2,\mu)}{m_B + m_D}$$

- differential decay rate in generic New Physics scenario:

$$\frac{d\mathcal{B}(\bar{B}\to D\ell\bar{\nu}_\ell)}{dq^2} = |V_{cb}|^2\mathcal{B}_0|f_+(q^2)|^2\left\{|1+g_V|^2 c_+^\ell(q^2) + |g_T(\mu)|^2 c_T^\ell(q^2)\left|\frac{f_T(q^2,\mu)}{f_+(q^2)}\right|^2\right.$$
$$+\, c_{TV}^\ell(q^2)\,\mathrm{Re}\left[(1+g_V)g_T^*(\mu)\,\frac{f_T(q^2,\mu)}{f_+(q^2)}\right]$$
$$\left.+\left|(1+g_V) - \frac{q^2}{m_\ell}\,\frac{g_S(\mu)}{m_b(\mu)-m_c(\mu)}\right|^2 c_0^\ell(q^2)\left|\frac{f_0(q^2)}{f_+(q^2)}\right|^2\right\}$$

- $B_s \to D_s\ell\nu$:

$f_T(q_0^2, m_b)/f_+(q_0^2) = 1.08(7)\,;$ $\qquad f_0(q_0^2)/f_+(q_0^2) = 0.77(2)$ $\qquad$ for $q_0^2 = 11.5\ \mathrm{GeV}^2$

$N_f = 2$ Wilson twisted mass fermions with ratio method [Atoui et al., 1310.5238]

conclusion

- currently, lattice QCD can determine B-meson decay constants with a few % accuracy

- no significant effect of s and c sea quarks with current level of accuracy for the decay constants

- scalar form factor is available for various semileptonic decays : $B \to \pi \ell \nu$, $\Lambda_b \to \Lambda_c \ell \nu$, ...

- $B \to D^* \ell \nu$: extend computation to other form factors $\rightsquigarrow$ $R(D^*)$

- challenges : simulations at very fine lattice spacings, control of statistical error, isospin and QED effects, unstable hadrons, ...

f_B : $N_f = 2+1+1$ [FLAG]

[FLAG, 1310.8555]

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization/matching	heavy quark treatment	f_{B_s}/f_{B^+}	f_{B_s}/f_{B^0}	f_{B_s}/f_B
ETM 13E	[398]	2+1+1	C	★	○	○	○	✓	–	–	1.201(25)
HPQCD 13	[399]	2+1+1	A	★	★	★	○	✓	1.217(8)	1.194(7)	1.205(7)
RBC/UKQCD 13A	[400]	2+1	C	○	○	★	○	✓	–	–	$1.20(2)^{\diamond}_{\text{stat}}$
HPQCD 12	[401]	2+1	A	○	○	★	○	✓	–	–	1.188(18)
FNAL/MILC 11	[331]	2+1	A	○	○	★	○	✓	1.229(26)	–	–
RBC/UKQCD 10C	[405]	2+1	A	■	■	★	○	✓	–	–	1.15(12)
HPQCD 09	[402]	2+1	A	○	○	★	○	✓	–	–	1.226(26)
ALPHA 13	[403]	2	C	★	★	★	★	✓	–	–	1.195(61)(20)
ETM 13B, 13C	[334, 404]	2	P†	★	○	★	○	✓	–	–	1.206(24)
ALPHA 12A	[369]	2	C	★	★	★	★	✓	–	–	1.13(6)
ETM 12B	[392]	2	C	★	○	★	○	✓	–	–	1.19(5)
ETM 11A	[335]	2	A	○	○	★	○	✓	–	–	1.19(5)

$^{\diamond}$Statistical errors only.
†Update of ETM 11A and 12B.

f_B : $N_f = 2+1+1$ [FLAG]

[FLAG, 1310.8555]

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization/matching	heavy quark treatment	f_{B^+}	f_{B^0}	f_B	f_{B_s}
ETM 13E	[398]	2+1+1	C	○	○	○	○	✓	–	–	196(9)	235(9)
HPQCD 13	[399]	2+1+1	A	★	★	★	○	✓	184(4)	188(4)	186(4)	224(5)
RBC/UKQCD 13A	[400]	2+1	C	○	○	★	○	✓	–	–	$191(6)^{\diamond}_{\rm stat}$	$233(5)^{\diamond}_{\rm stat}$
HPQCD 12	[401]	2+1	A	○	○	★	○	✓	–	–	191(9)	228(10)
HPQCD 12	[401]	2+1	A	○	○	★	○	✓	–	–	$189(4)^{\triangle}$	–
HPQCD 11A	[365]	2+1	A	★	○	★	★	✓	–	–	–	$225(4)^{\nabla}$
FNAL/MILC 11	[331]	2+1	A	○	○	★	○	✓	197(9)	–	–	242(10)
HPQCD 09	[402]	2+1	A	○	○	★	○	✓	–	–	$190(13)^{\bullet}$	$231(15)^{\bullet}$
ALPHA 13	[403]	2	C	★	★	★	★	✓	–	–	187(12)(2)	224(13)
ETM 13B, 13C	[334, 404]	2	P†	★	○	★	○	✓	–	–	189(8)	228(8)
ALPHA 12A	[369]	2	C	★	★	★	★	✓	–	–	193(9)(4)	219(12)
ETM 12B	[392]	2	C	★	○	★	○	✓	–	–	197(10)	234(6)
ALPHA 11	[364]	2	C	★	○	★	★	✓	–	–	174(11)(2)	–
ETM 11A	[335]	2	A	○	○	★	○	✓	–	–	195(12)	232(10)
ETM 09D	[391]	2	A	○	○	○	○	✓	–	–	194(16)	235(12)

$^{\diamond}$Statistical errors only.
$^{\triangle}$Obtained by combining f_{B_s} from HPQCD 11A with f_{B_s}/f_B calculated in this work.
$^{\nabla}$This result uses one ensemble per lattice spacing with light to strange sea-quark mass ratio $m_\ell/m_s \approx 0.2$.
$^{\bullet}$This result uses an old determination of $r_1 = 0.321(5)$ fm from Ref. [379] that has since been superseded.
†Update of ETM 11A and 12B.

$f_B: \quad N_f = 2+1$ [HPQCD]

[HPQCD, 1202.4914]

- fermionic lattice action :HISQ (val) / asqtad (sea)
- $N_f = 2+1: \ u, d, s$
- combination of NRQCD (f_{B_s}/f_B) and HISQ (f_B) results

Source	f_{B_s} (%)	f_B (%)	f_{B_s}/f_B (%)
statistical	0.6	1.2	1.0
scale $r_1^{3/2}$	1.1	1.1	—
discret. corrections	0.9	0.9	0.9
chiral extrap. & $g_{B^*B\pi}$	0.2	0.5	0.6
mass tuning	0.2	0.1	0.2
finite volume	0.1	0.3	0.3
relativistic correct.	1.0	1.0	0.0
operator matching	4.1	4.1	0.1
Total	4.4	4.6	1.5

$f_B = 0.191(9)\text{GeV}, \qquad f_{B_s} = 0.228(10)\text{GeV}, \qquad f_{B_s}/f_B = 1.188(18)$

$$\left[\frac{f_{B_s}}{f_B}\right]^{-1}_{NRQCD} \times f_{B_s}^{(HISQ)} \equiv f_B = 0.189(4)\text{GeV} \ [2\%]$$

f_B : $N_\mathrm{f} = 2 + 1 + 1$ [HPQCD]

[HPQCD, 1302.2644]

- fermionic lattice action :HISQ
- $N_\mathrm{f} = 2 + 1 + 1$: u, d, s, c
- NRQCD
- ensembles at the physical point

Error %	Φ_{B_s}/Φ_B	$M_{B_s} - M_B$	Φ_{B_s}	Φ_B
EM:	0.0	1.2	0.0	0.0
a dependence:	0.01	0.9	0.7	0.7
chiral:	0.01	0.2	0.05	0.05
g:	0.01	0.1	0.0	0.0
stat/scale:	0.30	1.2	1.1	1.1
operator:	0.0	0.0	1.4	1.4
relativistic:	0.5	0.5	1.0	1.0
total:	0.6	2.0	2.0	2.1

$$f_B = 0.186(4) \text{ GeV}, \qquad f_{B_s} = 0.224(5) \text{ GeV}, \qquad f_{B_s}/f_B = 1.205(7)$$

$f_B^{\rm stat}$ static [Aoki et al.]

$f_B^{\rm stat}$: [RBC−UKQCD, Aoki et al., 1406.6192]

- fermionic lattice action : domain-wall (Shamir) fermions
- $N_{\rm f} = 2 + 1$: u, d, s
- static quarks
- $a = \{0.08,\ 0.11\}$ fm
- $L = 2.6$ fm , $M_{\rm PS} L > 3.7$
- $M_\pi \in \{290,\ 420\}$ MeV

	f_B	f_{B_s}	f_{B_s}/f_B
Statistics	2.99	1.81	1.65
Chiral/continuum extrapolation	3.54	1.98	2.66
Finite volume effect	0.82	0.0	1.00
Discretization	1.0	1.0	0.2
One-loop renormalization	6.0	6.0	0.0
$g_{B^* B\pi}$	0.24	0.00	0.00
Scale	1.82	1.85	0.04
Physical quark mass	0.05	0.01	0.06
Off-physical sea s quark mass	0.84	0.69	0.79
Fit-range	0.44	2.31	0.26
Total systematic error	7.38	7.09	2.97
Total error (incl. statistical)	7.96	7.32	3.40

$f_B^{\rm stat} = 219(17)\,\mathrm{MeV}$ [8%] $\quad f_{B_s}^{\rm stat} = 264(19)\,\mathrm{MeV}$ [7%] $\quad \dfrac{f_{B_s}^{\rm stat}}{f_B^{\rm stat}} = 1.193(41)$ [3%]

HQET on the lattice [ALPHA]

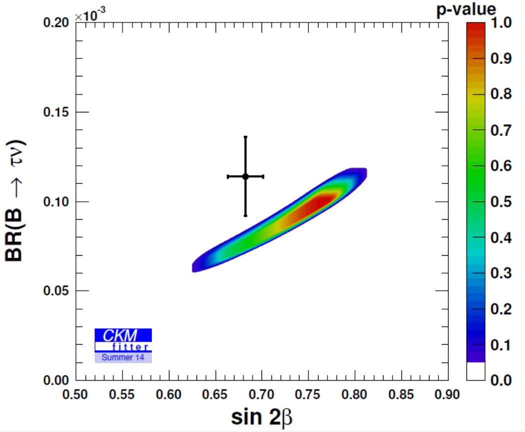
Source	f_{B_s}	f_B	f_{B_s}/f_B	$f_{B_s}^{\rm stat}$	$f_B^{\rm stat}$	$f_{B_s}^{\rm stat}/f_B^{\rm stat}$
A3	0.20 %	0.19 %	0.00 %	1.22 %	1.10 %	0.00 %
A4	5.94 %	9.36 %	14.27 %	8.06 %	2.76 %	14.36 %
A5	1.17 %	6.51 %	7.37 %	2.01 %	0.91 %	3.10 %
B6	3.32 %	2.99 %	0.00 %	2.70 %	1.44 %	0.26 %
E5	1.15 %	1.28 %	0.21 %	1.00 %	0.95 %	0.01 %
F6	1.70 %	2.21 %	6.44 %	1.85 %	2.62 %	9.65 %
F7	15.41 %	5.79 %	37.01 %	14.89 %	3.02 %	40.32 %
G8	13.96 %	12.81 %	0.00 %	15.36 %	13.26 %	0.00 %
N5	5.91 %	5.43 %	0.00 %	9.17 %	7.94 %	0.00 %
N6	19.42 %	13.78 %	29.87 %	8.35 %	24.10 %	28.61 %
O7	16.03 %	25.46 %	4.80 %	19.91 %	27.66 %	3.58 %
ω	14.02 %	12.72 %	0.01 %	8.35 %	7.21 %	0.00 %
$Z_{\rm A}$	1.77 %	1.46 %	0.01 %	7.13 %	7.04 %	0.09 %

$f_B = 186(13)(2)_\chi\ \mathrm{MeV}\ \ [7\%]$ $\quad f_{B_s} = 224(14)(2)_\chi\ \mathrm{MeV}\ \ [6\%]$ $\quad \frac{f_{B_s}}{f_B} = 1.203(62)(19)_\chi\ \ [5\%]$

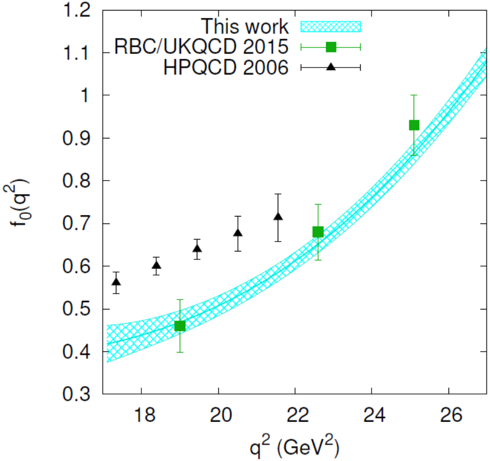
$f_B^{\rm stat} = 190(5)(2)_\chi\ \mathrm{MeV}\ \ [3\%]$ $\quad f_{B_s}^{\rm stat} = 226(6)(9)_\chi\ \mathrm{MeV}\ \ [5\%]$ $\quad \frac{f_{B_s}^{\rm stat}}{f_B^{\rm stat}} = 1.189(24)(30)_\chi\ \ [3\%]$

[ALPHA, 1404.3590]

correlation between sin(2β) and $\mathcal{B}(B \to \tau\nu)$



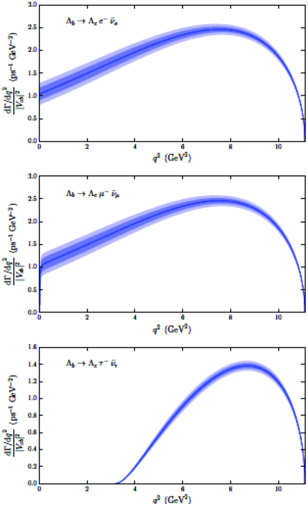
$B \to \pi \ell \nu$



for more details see [talk by Daping Du]

Λ_b decays

[Detmold, Lehner, Meinel, arXiv:1503.01421]



stat sys

$$\Gamma(\Lambda_b \to \Lambda_c\, e^-\bar{\nu}_e)/|V_{cb}|^2 = (21.1 \pm 0.8 \pm 1.4)\ \mathrm{ps}^{-1},$$
$$\Gamma(\Lambda_b \to \Lambda_c\, \mu^-\bar{\nu}_\mu)/|V_{cb}|^2 = (21.1 \pm 0.8 \pm 1.4)\ \mathrm{ps}^{-1},$$
$$\Gamma(\Lambda_b \to \Lambda_c\, \tau^-\bar{\nu}_\mu)/|V_{cb}|^2 = (7.13 \pm 0.17 \pm 0.29)\ \mathrm{ps}^{-1}.$$

$$\frac{\Gamma(\Lambda_b \to \Lambda_c\, \tau^-\bar{\nu}_\mu)}{\Gamma(\Lambda_b \to \Lambda_c\, e^-\bar{\nu}_\mu)} = 0.3378 \pm 0.0079 \pm 0.0085,$$
$$\frac{\Gamma(\Lambda_b \to \Lambda_c\, \tau^-\bar{\nu}_\mu)}{\Gamma(\Lambda_b \to \Lambda_c\, \mu^-\bar{\nu}_\mu)} = 0.3388 \pm 0.0078 \pm 0.0085.$$

(e.m. effects neglected)

for more details see [talk by Carlos Pena]