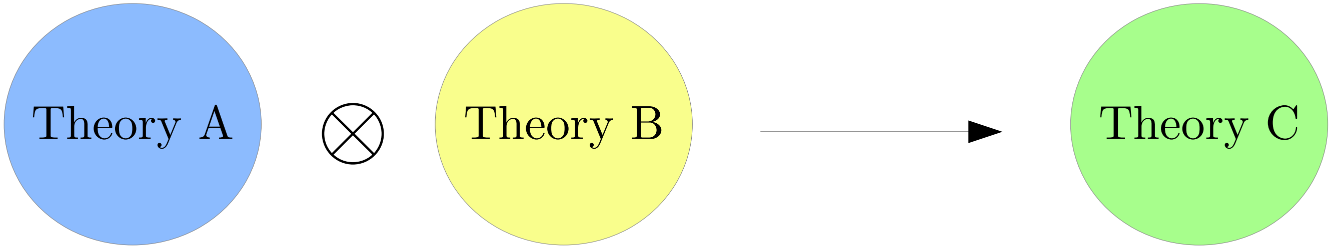
The seeds of EFT double copy

Q. Bonnefoy, G. Durieux, C. Grojean, C. Machado
Jasper Roosmale Nepveu

YOUNGST@RS MITP - Rebuilding the Tower of Babel: Bringing Together the Various Languages of Color-Kinematics Duality

[arXiv: 2112.11453] (accepted by JHEP)

What is the double copy?



- What is this map (tree level)? Which theories participate?

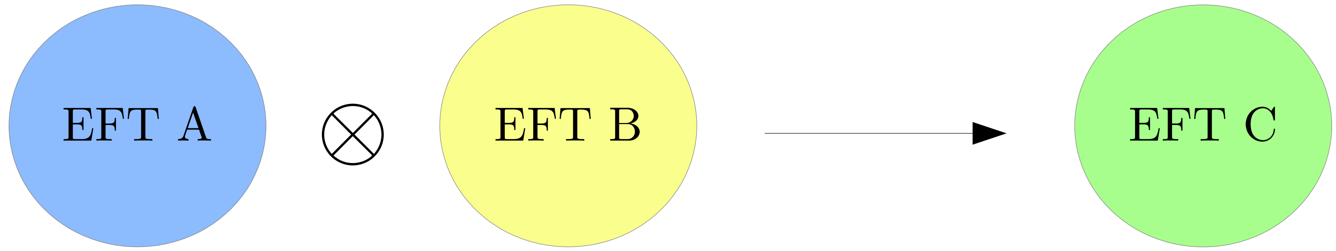
  → KLT: theories that satisfy the KK and BCJ relations

  → CK: theories that satisfy the color-kinematics duality

- These maps and the theories that participate are the same
- Web of connections between theories
- What happens for small perturbations, such as non-renormalizable operators?

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}^{(4)} + \frac{a}{\Lambda}\mathcal{O}$$

What is the double copy of EFTs?



$$\mathcal{L}_{\text{EFT}} = \mathcal{L}^{(4)} + \frac{a_i}{\Lambda}\mathcal{L}^{(5)} + \frac{b_i}{\Lambda^2}\mathcal{L}^{(6)} + \ldots$$

- Which theories as input? Which theories as output?
- Traditional double copy also works for EFTs, but can we generalize?
- Are $\otimes^{\text{KLT}}_{\text{EFT}}$ and $\otimes^{\text{CK}}_{\text{EFT}}$ still equivalent?

Double copy of EFTs

KLT double copy

- Color-ordered amplitudes: $\mathcal{A} = \mathrm{Tr}(\ldots) A(\ldots) + \ldots$
- Bi-adjoint scalar (BAS) matrix of doubly ordered amplitudes: $\mathcal{A}^{\mathrm{BAS}} = \mathrm{Tr}(\ldots) m(\ldots|\ldots) \tilde{\mathrm{Tr}}(\ldots) + \ldots$
- Defines KK&BCJ relations: $m \cdot \text{“}m^{-1}\text{”} \cdot A = A$
- Double copy: $\mathcal{M} = A \cdot \text{“}m^{-1}\text{”} \cdot \tilde{A}$

$\Longrightarrow$

“Generalized KLT”

- Include higher-derivative corrections to the BAS matrix

$$m \to m^{\mathrm{hd}}$$

- Allows for more EFT amplitudes

➔ [2106.12600] and Paranjape's talk tomorrow

Color-kinematics double copy

- Amplitudes from trivalent graphs: $\mathcal{A} = \sum_i \frac{c_i\, n_i}{d_i}$
- Color-kinematics duality: Kinematic numerators satisfy the same algebraic properties as color numerators
- Double copy: $\mathcal{M} = \sum_i \frac{n_i\, \tilde{n}_i}{d_i}$

$\Longrightarrow$

“Generalized color-kinematics”

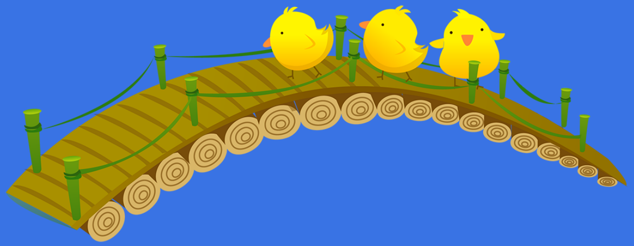
- Allow numerators to depend on both color and kinematics at the same time:

$$c_i \to c_i^{\mathrm{hd}}(color, kin.)$$

➔ [1910.12850, 2104.08370] and Carrasco's talk yesterday

The seeds of EFT double copy

Generalized
Color-kinematics



Generalized KLT

Seeds

- Idea: adjoint algebraic properties are complicated, but the adjoint numerators can be expressed in terms of simpler functions:

$$f^{abx}f^{xcd} = \mathrm{Tr}(T^aT^bT^cT^d) - \mathrm{Tr}(T^aT^bT^dT^c) + \mathrm{Tr}(T^aT^dT^cT^b) - \mathrm{Tr}(T^aT^cT^dT^b)$$

Or in matrix form:

$$\begin{pmatrix} f^{12x}f^{x34} \\ f^{13x}f^{x42} \\ f^{14x}f^{x23} \end{pmatrix} = \begin{pmatrix} 0 & 1 & -1 & 0 & 1 & -1 \\ -1 & 0 & 1 & -1 & 0 & 1 \\ 1 & -1 & 0 & 1 & -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} \mathrm{Tr}(1324) \\ \mathrm{Tr}(1234) \\ \mathrm{Tr}(1243) \\ \mathrm{Tr}(1423) \\ \mathrm{Tr}(1432) \\ \mathrm{Tr}(1342) \end{pmatrix}$$

Adjoint numerator c_i

Matrix that encodes the Jacobi identities

Linearly independent entries, determined by one cyclically invariant functional form

Seeds

- Similarly, generalized numerators can be written as

$$\begin{pmatrix} c^{\text{hd}}(1,2,3,4) \\ c^{\text{hd}}(1,3,4,2) \\ c^{\text{hd}}(1,4,2,3) \end{pmatrix} = \begin{pmatrix} 0 & 1 & -1 & 0 & 1 & -1 \\ -1 & 0 & 1 & -1 & 0 & 1 \\ 1 & -1 & 0 & 1 & -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} c_0^{\text{hd}}(1324) \\ c_0^{\text{hd}}(1234) \\ c_0^{\text{hd}}(1243) \\ c_0^{\text{hd}}(1423) \\ c_0^{\text{hd}}(1432) \\ c_0^{\text{hd}}(1342) \end{pmatrix}$$

$c_i^{\text{hd}}(\text{kinematics}, \text{color})$

Same matrix as before!
Encodes the complicated
adjoint algebraic properties

- The “seeds” $c_0^{\text{hd}}(abcd)$ are simple functions of kinematics and color which are cyclic in their arguments
- Traditional seeds (i.e. only kinematics) were considered before: e.g. [1103.0312], [1404.7141]

Bases of color structures (4-point)

Single traces

$$\begin{pmatrix} \mathrm{Tr}(1324) \\ \mathrm{Tr}(1234) \\ \mathrm{Tr}(1243) \\ \mathrm{Tr}(1423) \\ \mathrm{Tr}(1432) \\ \mathrm{Tr}(1342) \end{pmatrix}$$

Reversal-symmetric combinations

$$\frac{1}{2}\begin{pmatrix} \mathrm{Tr}(1324) + \mathrm{Tr}(1423) \\ \mathrm{Tr}(1234) + \mathrm{Tr}(1432) \\ \mathrm{Tr}(1243) + \mathrm{Tr}(1342) \end{pmatrix}$$

Reversal-antisymmetric combinations

$$\frac{1}{2}\begin{pmatrix} \mathrm{Tr}(1324) - \mathrm{Tr}(1423) \\ \mathrm{Tr}(1234) - \mathrm{Tr}(1432) \\ \mathrm{Tr}(1243) - \mathrm{Tr}(1342) \end{pmatrix}$$

Permutation invariant color factor

$$d^{1234}$$

$$\mathbf{J}_4 = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \end{pmatrix}$$

Adjoint algebraic structure

$$\begin{pmatrix} f^{12x} f^{x34} \\ f^{13x} f^{x42} \end{pmatrix}$$

$(\ldots)$

Bases of kinematic functions (4-point)

Trace-like functions

$$f(1234) = f(2341)$$
$$\implies f(s,u) = f(u,s)$$

$$\begin{pmatrix} f(t,u) \\ f(s,u) \\ f(s,t) \\ f(t,u) \\ f(s,u) \\ f(s,t) \end{pmatrix}$$

$$H = \frac{1}{(1-q)(1-q^2)} = 1 + q + 2q^2 + 2q^3 + \ldots$$

Number of independent polynomials per degree

Reversal-symmetric functions

$$\begin{pmatrix} f(t,u) \\ f(s,u) \\ f(s,t) \end{pmatrix}$$

Reversal anti-symmetric

$$(\ldots)$$

Permutation invariant

$$d(s,t)$$

$$\mathbf{J}_4 = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \end{pmatrix}$$

Adjoint structures

$$\begin{pmatrix} f_{adj}(s,t) \\ f_{adj}(t,u) \end{pmatrix}$$

Bases of kinematic functions (4-point)

Trace-like functions

$$f(1234) = f(2341)$$
$$\implies f(s,u) = f(u,s)$$

$$\begin{pmatrix} f(t,u) \\ f(s,u) \\ f(s,t) \\ f(t,u) \\ f(s,u) \\ f(s,t) \end{pmatrix}$$

$$H = \frac{1}{(1-q)(1-q^2)} = 1 + q + 2q^2 + 2q^3 + \ldots$$

Reversal-symmetric functions

$$\begin{pmatrix} f(t,u) \\ f(s,u) \\ f(s,t) \end{pmatrix}$$

$$H = \frac{1}{(1-q)(1-q^2)} = 1 + q + 2q^2 + 2q^3 + \ldots$$

Reversal anti-symmetric

$$H = 0$$

$$(\ldots)$$

Permutation invariant

$$d(s,t)$$

$$H = \frac{1}{(1-q^2)(1-q^3)} = 1 + q^2 + q^3 + q^4 + \ldots$$

$$\mathbf{J}_4 = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \end{pmatrix}$$

Adjoint structures

$$\begin{pmatrix} f_{adj}(s,t) \\ f_{adj}(t,u) \end{pmatrix}$$

$$H = \frac{q}{(1-q)(1-q^3)} = q + q^2 + q^3 + \ldots$$

Bases of kinematic functions (5-point)

Useful choice of Mandelstam basis: $f(1,2,3,4,5) = \tilde{f}(s_{12}, s_{23}, s_{34}, s_{45}, s_{51})$

Cyclically invariant functions

$f(1,2,3,4,5) = f(2,3,4,5,1)$

Reversal symmetric

$f(1,2,3,4,5) = f(5,4,3,2,1)$

Permutation invariant

Relaxed structures

Sandwich structures

[2104.08370]

Adjoint structures

Reversal antisymmetric

$f(1,2,3,4,5) = -f(5,4,3,2,1)$

$\mathbf{J}_5$

Hybrid structures

Bases of kinematic functions (5-point)

Useful choice of Mandelstam basis: $f(1,2,3,4,5) = \tilde{f}(s_{12}, s_{23}, s_{34}, s_{45}, s_{51})$

Cyclically invariant functions

$f(1,2,3,4,5) = f(2,3,4,5,1)$

$$H = \frac{1+4q^3+3q^4+3q^5+4q^6+q^9}{(1-q)(1-q^2)^2(1-q^4)(1-q^5)}$$
$$= 1 + q + 3q^2 + 7q^3 + 14q^4 + \ldots$$

Reversal symmetric

$f(1,2,3,4,5) = f(5,4,3,2,1)$

$$H = \frac{1+q^3+2\,q^4+q^5+q^8}{(1-q)(1-q^2)^2(1-q^3)(1-q^5)}$$
$$= 1 + q + 3q^2 + 5q^3 + 10q^4 + \ldots$$

Reversal antisymmetric

$f(1,2,3,4,5) = -f(5,4,3,2,1)$

$$H = \frac{2\,q^3}{(1-q)^2(1-q^2)^2(1-q^5)}$$
$$= 2q^3 + 4q^4 + 10q^5 + \ldots$$

Permutation invariant

Relaxed structures

Sandwich structures

[2104.08370]

$\mathbf{J}_5$

Adjoint structures

$$H = \frac{q^3}{(1-q)^2(1-q^2)^2(1-q^5)}$$
$$= q^3 + 2q^4 + 5q^5 + \ldots$$

Hybrid structures

$$H = \frac{q^3}{(1-q)^2(1-q^2)^2(1-q^5)}$$

Double copy of EFTs

- The generalized numerators construct EFT amplitudes that satisfy the color-kinematics duality:

YM → YM+h.d.

$$\sum_i \frac{c_i\, n_i^{\mathrm{YM}}}{d_i} \qquad \sum_i \frac{c_i^{\mathrm{hd}}\, n_i^{\mathrm{YM}}}{d_i}$$

BAS → BAS+h.d.

$$\sum_i \frac{c_i\, \tilde{c}_i}{d_i} \qquad \sum_i \frac{c_i^{\mathrm{hd}}\, \tilde{c}_i^{\mathrm{hd}}}{d_i}$$

The bi-adjoint scalar takes a central role in the generalized KLT approach!

From seeds to KLT

- Use numerator seeds to construct KLT kernels:

$$\mathcal{A}^{\text{h.d.}}_{\text{BAS}} = \sum_i \frac{\tilde{c}_i^{\text{hd}}\, c_i^{\text{hd}}}{d_i}$$

$$= \tilde{c}^{\text{hd}} \cdot \mathbf{P} \cdot c^{\text{hd}}$$

$$= \tilde{c}_0^{\text{hd}} \cdot \mathbf{J}^{\mathrm{T}} \cdot \mathbf{P} \cdot \mathbf{J} \cdot c_0^{\text{hd}}$$

$$= \tilde{c}_0 \cdot H_{\text{L}}^{\text{hd}} \cdot \mathbf{J}^{\mathrm{T}} \cdot \mathbf{P} \cdot \mathbf{J} \cdot H_{\text{R}}^{\text{hd}} \cdot c_0$$

$$\mathbf{P} = \begin{pmatrix} 1/s & & \\ & 1/t & \\ & & 1/u \end{pmatrix}$$

Usual BAS matrix of ordered amplitudes

$$m(\alpha|\beta)$$

BAS+h.d. matrix of ordered amplitudes

$$m^{\text{hd}}(\alpha|\beta)$$

→ would be constructed by the bootstrap of [2106.12600]

From seeds to KLT

- Generalized numerators $c(\text{color}, \text{kin.}) = \mathbf{J} \cdot H(\text{kin.}) \cdot c_0(\text{color})$ generate BAS+h.d. matrices of ordered amplitudes,

$$m^{\text{hd}} = H_{\text{L}}^{\text{hd}} \cdot \mathbf{J}^{\text{T}} \cdot \mathbf{P} \cdot \mathbf{J} \cdot H_{\text{R}}^{\text{hd}}$$

- Defines generalized KKBCJ relations allowing for amplitudes

$$A_{\text{L}}^{\text{hd}} = n_0^{(\text{L})} \cdot \tilde{H}_{\text{L}}^{\text{hd}} \cdot \mathbf{J}^{\text{T}} \cdot \mathbf{P} \cdot \mathbf{J} \cdot H_{\text{R}}^{\text{hd}}$$
$$A_{\text{R}}^{\text{hd}} = H_{\text{L}}^{\text{hd}} \cdot \mathbf{J}^{\text{T}} \cdot \mathbf{P} \cdot \mathbf{J} \cdot \tilde{H}_{\text{R}}^{\text{hd}} \cdot n_0^{(\text{R})}$$

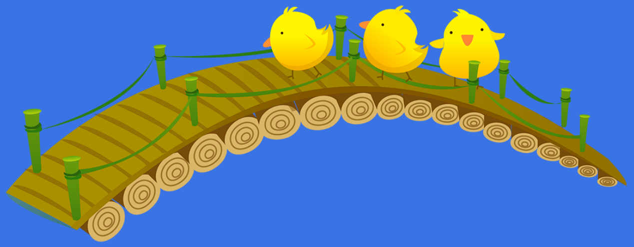
- And double-copy amplitudes $\mathcal{M} = n_0^{(\text{L})} \cdot \tilde{H}_{\text{L}}^{\text{hd}} \cdot \mathbf{J}^{\text{T}} \cdot \mathbf{P} \cdot \mathbf{J} \cdot \tilde{H}_{\text{R}}^{\text{hd}} \cdot n_0^{(\text{R})}$

Conclusions from seeds to KLT

- Seeds are straightforwardly generated and construct adjoint numerators and KLT kernels
- Via seeds, we can map between color-kinematics and KLT representations
- All KLT kernels can be generated from generalized seeds
  - **Subtlety:** it is unclear what locality properties the required seeds have. Do they have poles?
  - At 4-point, polynomial seeds generate all kernels
  - At 5-point, indications in the same direction

Thank you!

Generalized
Color-kinematics



Generalized KLT