Worldsheet description of Kerr interactions

based on 2012.11570
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the Various Languages of Color-Kinematics Duality",
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Introduction

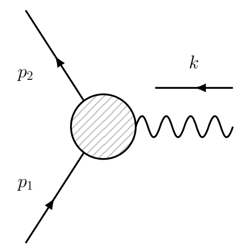
- Amazing activity in "Amplitudes $\to$ GR", compl. to class. methods

  Bern, Cheung, Kosmopoulos, Luna, Parra-Martinez, Roiban, Rothstein, Ruf, Shen, Solon, Zeng; Dlapa, Kälin, Liu, Porto, Yang; Bjerrum-Bohr, Damgaard, Festuccia, Planté, Vanhove; Levi, Mcleod, Mougiakakos, Teng, Vernizzi, Vieira, von Hippel; Edison, Kim, Yin; di Vecchia, Heissenberg, Russo, Veneziano, Alessio; Adamo, Cristofoli, Ilderton, Tourkine; Jakobsen, Loebbert, Mogull, Plefka, Shi, Steinhoff, Wang, ...

- Inclusion of class. spin $\leftarrow$ large quantum spin,
  Kerr $\leftarrow$ AHH "minimal-coupling" amplitudes Arkani-Hamed, Huang, Huang '17

$$\mathcal{M}_3^{(s,+)} = -\frac{\kappa}{2}\frac{\langle 12\rangle^{\odot 2s}}{m^{2s-2}}x^2 \;\to\; e^{-k\cdot a}\mathcal{M}_3^{(0,+)}$$

$$\mathcal{A}_3^{(s,+)} = \theta\frac{\langle 12\rangle^{\odot 2s}}{m^{2s-1}}x \to e^{-k\cdot a}\mathcal{A}_3^{(0,+)}$$



Guevara, AO, Vines '18; Chung, Huang, Kim, Lee '18

- $\sqrt{\text{Kerr}}$ in EM — excellent toy model Arkani-Hamed, Huang, O'Connell '19

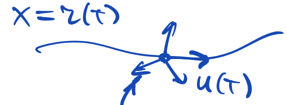
- **This talk:** purely classical-physics implications of above;
  double copy $(\sqrt{\text{Kerr}})^2 = \text{Kerr}$

Talk outline

- Introduce worldline action for $\sqrt{\text{Kerr}}$
- Rewrite via worldsheet
- Double-copy to Kerr
- Expand into familiar multipole form Porto, Rothstein '06, '08; Levi, Steinhoff '15
- Comment on spinor equations of motion
- Concluding remarks

NB! Minkowski signature $(1,3)$; see paper for detour to split signature $(2,2)$
also Monteiro, O'Connell, Peinador Veiga, Sergola '20; Guevara's talk

Worldline action for $\sqrt{\text{Kerr}}$



$\hat{e}^a_\mu(\tau)$ - body-fixed tetrad

$\Omega^{\mu\nu}(\tau) = \hat{e}^{a\mu}\,\dot{\hat{e}}^{\nu}_{a}$

$$S = \int d\tau \Big\{ -m\sqrt{u^2} - Q A_\mu u^\mu - \tfrac{1}{2} S_{\mu\nu}\Omega^{\mu\nu} \Big\} + S_{EFT}[z^\mu, a^\mu, F_{\mu\nu}(z)]$$

$$p_\mu := -\frac{\partial L}{\partial u^\mu} = \frac{m u_\mu}{\sqrt{u^2}} + O(Q)$$

$u^2 = 1 \Rightarrow p_\mu \simeq m u_\mu$

[reparam. gauge freedom

[spin-gauge freedom [Steinhoff '15] $\quad S_{\mu\nu}p^\nu = 0$

Def $\quad a^\mu(\tau) = \dfrac{1}{2p^2}\,\epsilon^{\mu\nu\rho\sigma} p_\nu S_{\rho\sigma}$

$$S_{EFT} = Q \sum_{n=1}^{\infty} \int d\tau\, u^\mu a^\nu \Big\{ \frac{B_n}{(2n-1)!}(a\cdot\partial)^{2n-2}\, {}^*F_{\mu\nu}(x) + \frac{C_n}{(2n)!}(a\cdot\partial)^{2n-1} F_{\mu\nu}(x) \Big\}\Big|_{x=z(\tau)}$$

$$\mathcal{A}_3^{\sqrt{\text{Kerr}}}(p, k^\pm \to p+k) = e^{\mp k\cdot a}\,\mathcal{A}_3^{\text{Coulomb}}(p, k^\pm \to p+k)$$

$A_\mu(k) = \hat{\delta}(k^2)\,\varepsilon^\pm_\mu(k)$

$$S_{int} = -Q\int d\tau\, A\cdot u + S_{EFT} = -\int \hat{d}^4k\, A_\mu(k)\, J^\mu(-k)$$

$$A_\mu(k)\,J^\mu(-k) \propto \mathcal{A}_3(p, k \to p+k) \quad \Rightarrow \quad B_n = C_n = (-1)^n$$

Worldsheet action for $\sqrt{\text{Kerr}}$

$$S_{EFT} = Q\int d\tau\, u^\mu a^\nu \,\mathrm{Re} \sum_{n=0}^{\infty} \frac{i^{n+1}}{(n+1)!}(a\cdot\partial)^n \Big\{ F_{\mu\nu} + i F^*_{\mu\nu} = F^+_{\mu\nu} \Big\}\Big|_{x=z(\tau)}$$

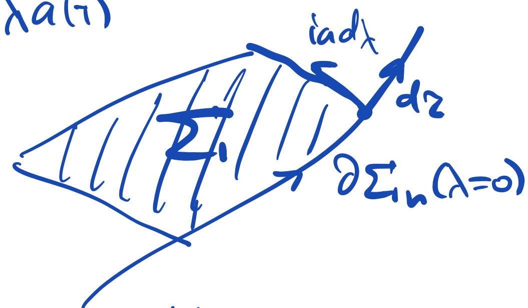
$$iT(ia\cdot\partial), \qquad T(x) := \sum_{n=0}^{\infty} \frac{x^n}{(n+1)!} = \frac{e^x - 1}{x} = \int_0^1 e^{\lambda x}\, d\lambda$$

$$= Q\int d\tau \int_0^1 d\lambda\, \mathrm{Re}\; i u^\mu a^\nu e^{\lambda(ia\cdot\partial)} F^+_{\mu\nu}(x)\Big|_{x=z(\tau)}$$

$$= Q\,\mathrm{Re}\int d\tau \int_0^1 d\lambda\; i u^\mu a^\nu F^+_{\mu\nu}(z+i\lambda a)$$ Newman-Janis shift

$$S_{int} = -Q\int_{\partial\Sigma_{in}} A + Q\,\mathrm{Re}\int_{\Sigma_i} \overset{dA^+}{F^+}$$

$$F^+ = \tfrac{1}{2}F^+_{\mu\nu}(z)\, dz^\mu \wedge dz^\nu, \qquad z^\mu(\tau,\lambda) = z(\tau) + i\lambda a(\tau)$$



$$= -Q\,\mathrm{Re}\int_{\partial\Sigma_{i,f}} A^+$$

$$dF^+ = \overset{=0}{dF} + i\,\overset{=0}{dF^*} = 0 \;\Rightarrow\; F^+ = dA^+$$

$$A^+\big|_{\lambda=0} = A$$

$$= -Q\,\mathrm{Re}\int d\tau\, u^\mu A^+_\mu(z+ia)$$ ← NJ shift

Worldsheet action for Kerr

$$S_{EFT} = \Theta \int d\tau \int_0^1 d\lambda \, \mathrm{Re} \; i u^\mu a^\nu e^{\lambda(ia\cdot\nabla)} F^+_{\mu\nu}(x)\Big|_{x=z(\tau)}$$

curved space

$$\frac{Du^\mu(\tau,\lambda)}{D\lambda} \overset{!}{=} 0$$

$\downarrow$ uplift to YM

$$S_{EFT} = \mathrm{Re} \int_{\Sigma_1} d\tau \int_0^1 d\lambda \; c^a(\tau,\lambda)\, u^\mu(\tau,\lambda)\, i a^\nu(\tau,\lambda) F^{a\,+}_{\mu\nu}(z(\tau,\lambda))$$

$$= \mathrm{Re} \int_{\Sigma_1} d\tau \int_0^1 d\lambda \; c^a(\tau)\, u^\mu(\tau)\, i a^\nu(\tau)\, e^{\lambda(ia\cdot\nabla)} F^{a+}_{\mu\nu}(x)\Big|_{x=z(\tau)}$$

$\downarrow$ DC

$$S_{EFT} = m \, \mathrm{Re} \int_{\Sigma_1} d\tau \int_0^1 d\lambda \; u_\mu(\tau)\, u^a(\tau)\, i a^b(\tau)\, e^{\lambda(ia\cdot\nabla)} \omega^{+\mu}{}_{ab}(x)\Big|_{x=z(\tau)}$$

$$\omega_\mu{}^{ab} = e^b_\nu \nabla_\mu e^{a\nu}$$

tetrad.

Spin multipoles of Kerr

$$S_{EFT} = m\,\mathrm{Re}\int_{\Sigma_1} d\tau \int_0^1 d\lambda\; u^\mu(\tau)\, u^a(\tau)\, i a^b(\tau)\; e^{\lambda(ia\cdot\nabla)}\, \omega^{+}{}_{\mu ab}(x)\big|_{x=z(\tau)}$$

$$= \sum_{n=0}^{\infty} \frac{1}{(n+1)!}\, \mathrm{Re}\int d\tau\, (ia\cdot\nabla)^n\, u^\mu(\tau)\, \omega^{+}{}_{\mu ab}(x)\, u^a(\tau)\, i a^b(\tau)$$

$$\downarrow \qquad \tilde{h} \sim h$$

$$C_{ES^{2k}} = 1$$

$$S_{EFT} = m\int d\tau \Big\{ \sum_{n=0}^{\infty} \frac{(-1)^k}{(2k)!} (a\cdot\nabla)^{2k-2}\, h_{\alpha\beta\mu\nu}(x)\, u^\alpha u^\mu a^\beta a^\nu$$

$$+ \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(2k+1)!} (a\cdot\nabla)^{2k-1}\, {}^{*}h_{\alpha\beta\mu\nu}(x)\, u^\alpha u^\mu a^\beta a^\nu \Big\} + \ldots$$

$$C_{BS^{2k}} = 1$$

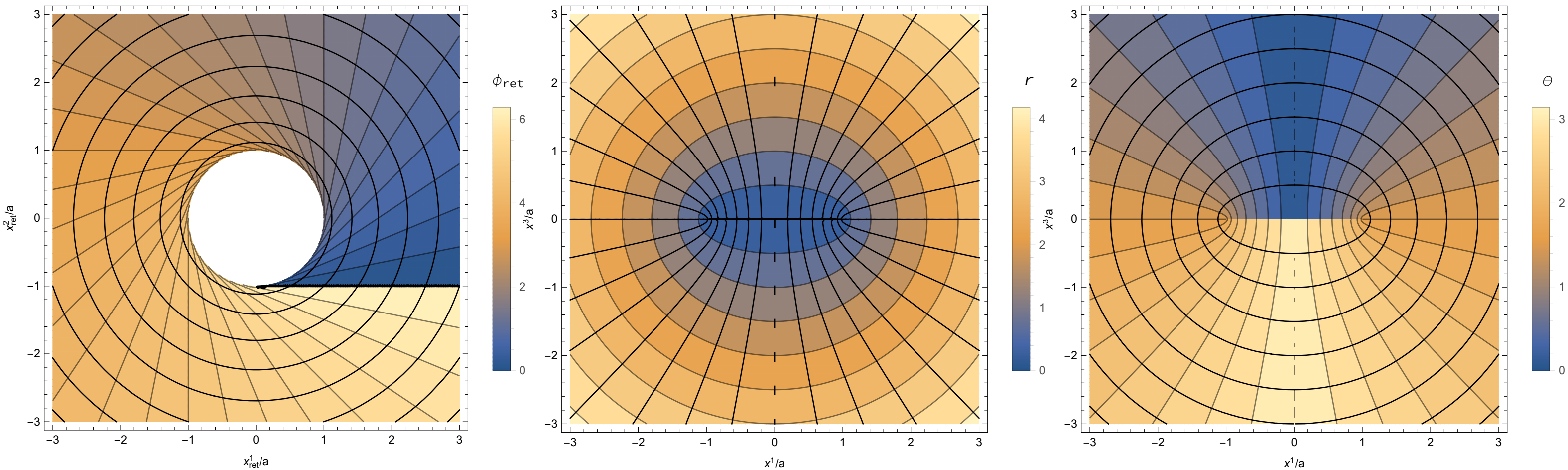
Eff Kerr worldline action

Levi, Steinhoff '15 ✓

Spinor equations of motion

Concluding remarks

- Worldsheet structure at LO in coupling
- Corresponds to Kerr ring singularity
  (e.g. seen in Kerr-Schild twisted oblate spheroidal coordinates)
- More detailed dynamics and validity to be investigated



- Spinor EoMs encode both linear and angular momenta
- Correspond to body-fixed Newman-Penrose tetrad
- Inspired by but complementary to ampl. methods
- Perfect for dealing with chiral EoMs