Bootstrapping the KLT Formula

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MITP Workshop 2022

Based on arXiv:2004.12948 and 2106.12600
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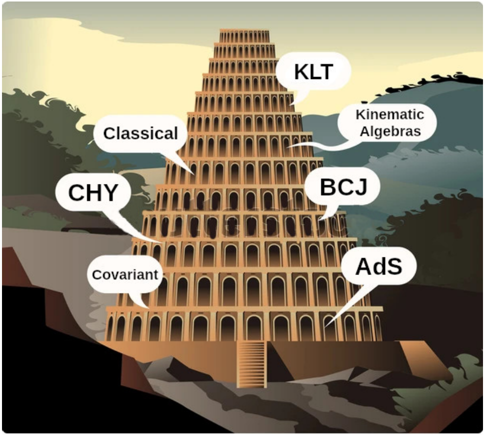
Rebuilding the Tower of Babel



In this talk...

We will study

- On-shell scattering amplitudes
- At tree-level
- In 4 dimensions
- Often with color-ordered external states

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- At tree-level
- In 4 dimensions
- Often with color-ordered external states

Because they are

- Indicators of structure on the space of QFTs
- Gauge-invariant
- Manifestly local and unitary based on their analytic structure

Double Copy

- Theory C = Theory A $\otimes$ B
- Works for tree-level amplitudes, loop integrands, classical observables, string amplitudes...

[Kawai, Lewellen, Tye][Bern, Carrasco, Johansson][Cachazo, He, Yuan]

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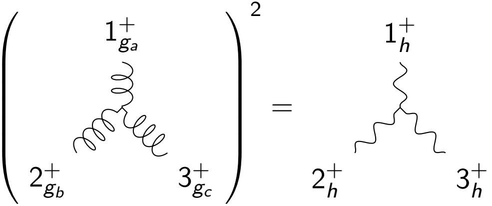
- Works for tree-level amplitudes, loop integrands, classical observables, string amplitudes...

[Kawai, Lewellen, Tye][Bern, Carrasco, Johansson][Cachazo, He, Yuan]

Original example : (Yang-Mills)^2 = Gravity + Dilaton + 2-form



The KLT Formula

At tree-level,

$$\text{Closed} = \text{Open} \ \times \ \text{Open}$$

More precisely,

$$\mathcal{M}_n = (-1)^{n-3} \sum_{\sigma,\gamma} S_{\alpha'}[\gamma|\sigma]_{k_1} \mathcal{A}_n[1,\sigma,n-1,n] \tilde{\mathcal{A}}_n[n-1,n,\gamma,1].$$

[Kawai, Lewellen, Tye]

where the KLT kernel is

$$S_{\alpha'}[\{i\}|\{j\}]_p \equiv \left(\frac{\pi\alpha'}{2}\right)^{-k} \prod_{t=1}^{k} \sin\left(\pi\alpha'\left(p\cdot k_{i_t} + \sum_{q>t}^{k} \theta(i_t,i_q) k_{it}\cdot k_{iq}\right)\right).$$

[Bjerrum-Bohr, Damgaard, Søndergaard]

to all orders in α'.

Field Theory KLT Formula

In the field theory limit,

$$\text{Open} \ \otimes_{\mathsf{KLT}} \ \text{Open} \ = \ \text{Closed} \ \xrightarrow{\alpha' \to 0} \ \text{YM} \ \otimes_{\mathsf{KLT}} \ \text{YM} \ = \ \text{GR} + \phi$$

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Benefits:

$$\mathcal{M}_n^{\text{C}=\text{A}\otimes\text{B}} = \sum_{\alpha,\beta} \mathcal{A}_n^{\text{A}}[\alpha] S_n^{\text{KLT}}[\alpha|\beta] \mathcal{A}_n^{\text{B}}[\beta]$$

- BCJ color-kinematic duality
- Works at loop-level and for some classical observables
- Commutes with supersymmetry
- CHY formula
- Web of double copies:

	ϕ^3	χPT	YM
ϕ^3	ϕ^3	χPT	YM
χPT	χPT	Special Galileon	Born-Infeld
YM	YM	Born-Infeld	Dilaton Gravity

[Cachazo, He, Yuan]

Field Theory KLT Formula

Drawbacks:

- External states $\xrightarrow{\alpha'\to 0}$ Massless
- All orders in α' $\xrightarrow{\alpha'\to 0}$ Leading order

String Theory	Field theory
Carefully tuned α' corrections	No higher-derivative corrections
Infinite tower of masses	No masses

1. Is there a more general KLT formula that interpolates between these two extremes?
2. How does one traverse the space of double copy maps?
3. What makes the string KLT relations special?

Our Goals

- Develop a formalism to study the space of double copy relations
- Search for a field theory KLT formula that includes effective operators
- Understand leading order limits and locality in these novel double copies

- What Yang-Mills EFT double copies to which gravity EFT?
- Is trivalent graph decomposition necessary?

[Johansson, Ochirov, Naculich, Chiodaroli, Gunaydin, Roiban, Bautista, Guevara...]
[Dixon, Broedel, Mizera, Carrasco, Yin, Zekioglu]

The KLT Algebra

$$\mathcal{M}_n^{\mathsf{C}=\mathsf{A}\otimes\mathsf{B}} = \sum_{\alpha,\beta} \mathcal{A}_n^{\mathsf{A}}[\alpha] S_n^{\mathsf{KLT}}[\alpha|\beta] \mathcal{A}_n^{\mathsf{B}}[\beta]$$

[Kawai, Lewellen, Tye]

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[Cachazo, He, Yuan]

ϕ^3 = bi-adjoint scalar theory with interaction $f_{abc} f_{a'b'c'} \phi_{aa'} \phi_{bb'} \phi_{cc'}$

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ϕ^3 = bi-adjoint scalar theory with interaction $f_{abc} f_{a'b'c'} \phi_{aa'} \phi_{bb'} \phi_{cc'}$

Bi-adjoint scalar theory is special because it is the identity of the double copy.

Zeroth Copy ⇒ Double Copy

Starting assumption: All generalizations of the KLT double copy have an identity element.

$$1 \otimes A = A$$

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$$1 \otimes A = A$$

1. KLT formula sums over color-orderings. Since A has adjoint states, the identity element must be bi-adjoint.
2. To ensure the correct Lorentz covariance of external states of A, the identity must be scalar.

$$\mathcal{A}_n^{\mathrm{A}}[\alpha] = \sum_{\beta,\gamma} m_n^{\mathrm{id}}[\alpha|\beta] S_n^{\mathrm{KLT}}[\beta|\gamma] \mathcal{A}_n^{\mathrm{A}}[\gamma]$$

$$\{\text{Identity Elements}\} = \{\text{Bi-adjoint scalar theories}\} \stackrel{?}{=} \{\text{Double copies}\}$$

Zeroth Copy ⇒ Double Copy

Once we have an identity:

1. Inverse of any full-rank submatrix gives the double copy kernel.

$$\mathbb{1} = \mathbb{1} \otimes \mathbb{1}$$

$$m[\alpha|\beta] = \sum_{\gamma,\delta} m[\alpha|\gamma] S[\gamma|\delta] m[\delta|\beta]$$

$$\Rightarrow \sum_{\beta} m[\alpha|\beta] m[\beta|\sigma]^{-1} = \sum_{\gamma} m[\alpha|\gamma] S[\gamma|\sigma]$$

$$\Rightarrow S[\rho|\sigma] = m[\rho|\sigma]^{-1}$$

{Identity Elements} = {Bi-adjoint scalar theories} = {Double copies}

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Zeroth Copy ⇒ Double Copy

Once we have an identity:

1. Inverse of kernel: $S = m^{-1}$
2. Basis-independence gives us the left and right BCJ relations.

Right BCJ $\quad A = 1 \otimes A$

$$\mathcal{A}[\alpha] = \sum_{\gamma,\delta} m[\alpha|\gamma] m[\gamma|\delta]^{-1} \mathcal{A}[\delta]$$

Left BCJ $\quad A = A \otimes 1$

$$\mathcal{A}[\alpha] = \sum_{\gamma,\delta} \mathcal{A}[\gamma] m[\gamma|\delta]^{-1} m[\delta|\alpha]$$

Only theories that satisfy BCJ can be double-copied.

[Bern, Carrasco, Johansson]

Zeroth Copy $\Rightarrow$ Double Copy

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Zeroth Copy ⇒ Double Copy

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2. Basis-independence of $\mathrm{A} = 1 \otimes \mathrm{A}$ and $\mathrm{A} = \mathrm{A} \otimes 1$ gives us the left and right BCJ relations.
3. Self-consistency conditions arise from basis-independence of $1 \otimes 1 = 1$,

Bootstrap

$$1 = 1 \otimes 1$$

$$m[\alpha|\beta] = \sum_{\gamma,\delta} m[\alpha|\gamma] m[\gamma|\delta]^{-1} m[\delta|\beta]$$

[Bern, Carrasco, Johansson]

Summarizing...

- $1 \otimes 1 = 1$
  determines the space of identities i.e. possible double copy relations
- $1 \otimes A = A$
  determines the types of theories we can double-copy
- $A \otimes B = C$
  determines what we can calculate using the double copy relations

Summarizing...

- $1 \otimes 1 = 1$
  determines the zeroth copies
- $1 \otimes A = A$
  determines the single copies
- $A \otimes B = C$
  determines the double copies

Using the Formalism in Practice

Construct rank R_n matrix of bi-adjoint scalar amplitudes

$\downarrow$

Take the inverse to construct a KLT kernel

$\downarrow$

Solve KLT bootstrap equations

$\downarrow$

Check that amplitudes $\mathcal{A}_n[\alpha]$ satisfy the BCJ relations

$\downarrow$

Use KLT formula to construct a double copy

Cubic Massless Bi-adjoint Scalar Theory

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + g\ f_{abc} f_{a'b'c'} \phi^{aa'} \phi^{bb'} \phi^{cc'}$$

- Doubly-color ordered tree amplitudes

$$\mathcal{A}_n(1^{a_1 a_1'} \cdots n^{a_n a_n'}) = \sum_{\sigma,\gamma} \mathcal{A}_n[1\gamma|1\sigma] \mathrm{Tr}\left[T^{a_1} T^{\gamma(a_2} \ldots T^{a_n)}\right] \times \mathrm{Tr}\left[T^{a_1'} T^{\sigma\left(a_2'\right.} \ldots T^{\left.a_n'\right)}\right]$$

- KLT bootstrap equation $1 \otimes 1 = 1$ gives
  1. Kleiss-Kuijf relations $\Rightarrow$ $(n-2)!$ distinct color-orderings
  2. BCJ relations $\Rightarrow$ $(n-2)! \times (n-2)!$ matrix of amplitudes has rank $(n-3)!$

Bi-adjoint Scalar Amplitudes

A simple 4-point example,

$$m[1234|1234] = \frac{1}{s} + \frac{1}{u} \qquad m[1234|1243] = -\frac{1}{s}$$
$$m[1243|1234] = -\frac{1}{s} \qquad m[1243|1243] = \frac{1}{s} + \frac{1}{t}$$

$m[\alpha|\beta]$ has rank 1 $\Rightarrow$ $S[1234|1243] = -s$.

Choosing different bases of orderings gives us consistency conditions or BCJ relations on the single-copy amplitudes,

$$\begin{aligned}\mathcal{A}_4[1234] =& m[1234|1234]S[1234|1243]\mathcal{A}_4[1243] \\ =& -\frac{t}{us} \times -s \times \mathcal{A}_4[1243] \overset{\text{BCJ}}{=\!=} \frac{t}{u}\mathcal{A}_4[1243] \\ \mathcal{M}_4 =& -\frac{us}{t}\mathcal{A}_4[1234]^2 \overset{\text{BCJ}}{=\!=} -s\mathcal{A}_4[1234]\mathcal{A}_4[1243]\end{aligned}$$

[Cachazo, He, Yuan][Mizera]

Locality

- Massless leading order:
  Locality is a carefully tuned consequence of the structure of bi-adjoint scalar theory.

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- Minimal rank conjecture:
  Any $m_n[\alpha|\beta]$ with rank $(n-3)!$ will lead to a local double copy.

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- Minimal rank conjecture:
  Any $m_n[\alpha|\beta]$ with rank $(n-3)!$ will lead to a local double copy.

- Motivation:
  Reduces to known double copy relations when $m \to 0$ or $\Lambda \to \infty$.

[Johnson, Jones, SP]

Adding Effective Operators

1. Leading order is massless cubic bi-adjoint scalar i.e. has rank $(n-3)!$.

$$m_n[\beta|\alpha] = m_n^{(0)}[\beta|\alpha] + m_n^{(1)}[\beta|\alpha] + \dots$$

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$$m_n[\beta|\alpha] = m_n^{(0)}[\beta|\alpha] + m_n^{(1)}[\beta|\alpha] + \dots$$

2. Bi-adjoint scalar EFT satisfies $1 \otimes 1 = 1$ i.e.

$$m[\alpha|\beta]m[\beta|\gamma]^{-1}m[\gamma|\delta] = m[\alpha|\delta]$$

for all choices of $(n-3)!$ bases.

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   for all choices of $(n-3)!$ bases.

3. Assuming UV physics decouples when $\Lambda \to \infty$,

$$\lim_{\Lambda\to\infty} m_n[\beta|\alpha] = m_n^{(0)}[\beta|\alpha]$$

   $\Rightarrow$ Higher-derivative corrections can not change the leading order rank.

[Chi, Herderschee, Elvang, Jones, SP]

Allowing Different Color Structures

Only one color structure f^{abc} $\Rightarrow$ Kleiss-Kuijf relations,

$$m_n[1,\alpha,n,\beta|\gamma] = (-1)^{|\beta|} \sum_{\sigma\in\alpha \sqcup\!\sqcup \beta} m[1,\sigma,n|\gamma]$$

Higher-derivative corrections to bi-adjoint scalar theory may have different color structures e.g.

$$\begin{aligned} d^{abcd} &= \mathrm{Tr}\left[T^a T^{(b} T^c T^{d)}\right], \\ d^{abcde} &= \mathrm{Tr}\left[T^a T^{(b} T^c T^d T^{e)}\right], \ldots \end{aligned}$$

To account for different color structures, we start with $(n-1)!$ basis rather than an $(n-2)!$ KK-independent basis $\rightarrow$ and then we impose $(n-3)!$ minimal rank.

KLT Bootstrap at 4-Point

- Start with a 6×6 matrix $m[\alpha|\beta]$,

$$
\begin{aligned}
m_4[1234|1234] &= f_1(s,t) = f_1(-s-t,t)\,,\\
m_4[1234|1243] &= f_2(s,t)\,,\\
m_4[1234|1324] &= f_3(s,t) = f_2(-s-t,t)\,,\\
m_4[1234|1342] &= f_4(s,t) = f_2(s,t)\,,\\
m_4[1234|1423] &= f_5(s,t) = f_2(-s-t,t)\,,\\
m_4[1234|1432] &= f_6(s,t) = f_6(-s-t,t)\,,
\end{aligned}
$$

- Impose bootstrap condition $1 \otimes 1 = 1$ on the three independent functions,

$$
\boxed{
\begin{gathered}
f_6(s,t) = f_1(s,t)\\
f_1(s,t) = \frac{f_2(s,t) f_2(-s-t,s)}{f_2(t,s)}\\
f_2(s,t) f_2(-s-t,s) f_2(t,-s-t) = f_2(t,s) f_2(-s-t,t) f_2(s,-s-t)
\end{gathered}
}
$$

[Chi, Herderschee, Elvang, Jones, SP]

Comparing to String Theory

Can the bootstrap equations be solved?

Known solution: String theory

$$f_2(s,t) = \frac{1}{\sin(\alpha' \pi s)}$$
$$\Rightarrow m[1234|1243] = \frac{1}{\pi\alpha' s} + \frac{\pi\alpha' s}{6} + \frac{7}{360}\pi^3\alpha'^3 s^3 + O\left(\alpha'^4\right)$$
$$\Rightarrow S[1234|1243] = \sin(\alpha'\pi s) = \pi\alpha' s - \frac{1}{6}\alpha'^3\left(\pi^3 s^3\right) + O\left(\alpha'^4\right)$$

Another possible solution:

$$f_2(s,t) = \frac{1}{s}\frac{G_1(s)G_2(t)}{G_3(u)}$$

Not all solutions correspond to sensible field theories.

Perturbative Solution to Bootstrap Equation

- Begin with an ansatz for $f_2(s,t)$,

$$f_2(s,t) = -\frac{g^2\Lambda^2}{s} + \sum_{k=0}^{N} \sum_{r=0,k} \frac{a_{k,r}}{\Lambda^{2k}}\, s^r\, t^{k-r}$$

- Solve bootstrap equations,

$$\begin{aligned} f_2(s,t) = &-\frac{g^2\Lambda^2}{s} + \frac{1}{\Lambda^2}(a_{1,0}t + a_{1,1}s) + \frac{a_{2,0}}{\Lambda^4}t(s+t) \\ &+ \frac{1}{\Lambda^6}\left[a_{3,0}t^3 + a_{3,1}st^2 + a_{3,2}s^2t + a_{3,3}s^3\right] + \mathcal{O}\left(\frac{1}{\Lambda^8}\right) \end{aligned}$$

- To recover string theory result,

$$a_{k,i} = 0 \text{ for } k > i \qquad a_{1,1} = -\frac{1}{6}, \quad a_{3,3} = -\frac{7}{360}, \dots$$

- As expected of bottom-up approach, this solution is more general

[Chi, Herderschee, Elvang, Jones, SP]

Zeroth Copy Lagrangian

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + f^{abc} f^{a'b'c'} \phi^{aa'} \phi^{bb'} \phi^{cc'}$$
$$+ \frac{a_L + a_R}{2\Lambda^4} f^{abx} f^{cdx} f^{a'b'x'} f^{c'd'x'} (\partial_\mu \phi^{aa'})(\partial^\mu \phi^{bb'}) \phi^{cc'} \phi^{dd'}$$
$$+ \frac{a_R}{\Lambda^4} f^{abx} f^{cdx} d^{a'b'x'} d^{c'd'x'} (\partial_\mu \phi^{aa'}) \phi^{bb'} (\partial^\mu \phi^{cc'}) \phi^{dd'}$$
$$+ \frac{a_L}{\Lambda^4} d^{abx} d^{cdx} f^{a'b'x'} f^{c'd'x'} (\partial_\mu \phi^{aa'}) \phi^{bb'} (\partial^\mu \phi^{cc'}) \phi^{dd'} + \mathcal{O}\left(\frac{1}{\Lambda^6}\right)$$

Zeroth Copy Lagrangian

$$\begin{aligned}\mathcal{L} =& -\frac{1}{2}(\partial\phi)^2 + f^{abc}f^{a'b'c'}\phi^{aa'}\phi^{bb'}\phi^{cc'} \\ &+ \frac{a_L + a_R}{2\Lambda^4} f^{abx}f^{cdx}f^{a'b'x'}f^{c'd'x'}(\partial_\mu\phi^{aa'})(\partial^\mu\phi^{bb'})\phi^{cc'}\phi^{dd'} \\ &+ \frac{a_R}{\Lambda^4} f^{abx}f^{cdx}d^{a'b'x'}d^{c'd'x'}(\partial_\mu\phi^{aa'})\phi^{bb'}(\partial^\mu\phi^{cc'})\phi^{dd'} \\ &+ \frac{a_L}{\Lambda^4} d^{abx}d^{cdx}f^{a'b'x'}f^{c'd'x'}(\partial_\mu\phi^{aa'})\phi^{bb'}(\partial^\mu\phi^{cc'})\phi^{dd'} + \mathcal{O}\left(\frac{1}{\Lambda^6}\right)\end{aligned}$$

What can we double copy with this?

Solving BCJ Relations at 3-Point

What higher derivative corrections can we add to YM so that
$(\text{YM} + \text{hd}) \otimes_{\text{KLT}} (\text{YM} + \text{hd}) = (\text{GR} + \text{hd})$?

$\Rightarrow$ Corrections must satisfy the BCJ relations i.e. basis-independence of the KLT formula $\text{A} = 1 \otimes \text{A} = \text{A} \otimes 1$.

At 3-point this amounts to anti-symmetry of identical states, so the possible interactions are

$$\mathcal{A}_3[1^-, 2^-, 3^+] = g_{\text{YM}} \frac{\langle 12 \rangle^3}{\langle 23 \rangle \langle 31 \rangle}$$
$$\mathcal{A}_3[1^-, 2^-, 3^-] = \frac{g_{F^3}}{\Lambda^2} \langle 12 \rangle \langle 23 \rangle \langle 31 \rangle$$

Solving BCJ Relations at 4-Point

A local ansatz for the 4-point Yang-Mills amplitude is

$$\mathcal{A}_4[1^+2^+3^-4^-] = \frac{[12]^2\langle 34\rangle^2}{su}\left[\left(g_{\text{YM}}^2 - \frac{1}{\Lambda^4}g_{F^3}^2 ut\right) + \sum_{k=2}^{N}\sum_{r=1}^{k-1}\Lambda^{-2k} e_{k,r} s^r u^{k-r}\right]$$

$$\mathcal{A}_4[1^+2^-3^+4^-] = \frac{[13]^2\langle 24\rangle^2}{su}\left[g_{\text{YM}}^2 + [13]^2\langle 24\rangle^2 \sum_{k=2}^{N}\sum_{r=1}^{k-1}\Lambda^{-2k} h_{k,r} s^r u^{k-r}\right]$$

Solving the BCJ relations, the final answer is

$$\mathcal{A}_4^L[1^+2^+3^-4^-] = [12]^2\langle 34\rangle^2\left[\frac{(g_{\text{YM}}^L)^2}{su} + \frac{1}{\Lambda^4}\left(\frac{(g_{\text{YM}}^L)^2}{g^2}(a_{1,1} - a_{1,0}) - (g_{F^3}^L)^2\frac{t}{s}\right) - \frac{e_{3,1}^L}{\Lambda^6}t + \mathcal{O}\left(\frac{1}{\Lambda^8}\right)\right].$$

[Chi, Herderschee, Elvang, Jones, SP]

Higher-Derivative Corrections to Gravity

Double-copying the YM EFT amplitudes with themselves give us a gravity EFT amplitude,

$$\begin{aligned}\mathcal{M}_4(1^+2^+3^-4^-) =& [12]^4\langle 34\rangle^4 \\ &\times\Bigg[-\frac{(g^L_{\rm YM})^2(g^R_{\rm YM})^2}{g^2\Lambda^2}\frac{1}{stu} + \frac{((g^L_{\rm YM})^2(g^R_{F^3})^2+(g^R_{\rm YM})^2(g^L_{F^3})^2)}{g^2\Lambda^6}\frac{1}{s} \\ &+\frac{1}{\Lambda^8}\left(\frac{(g^L_{\rm YM})^2(g^R_{\rm YM})^2}{g^4}a_{2,0}+\frac{1}{g^2}((g^R_{\rm YM})^2e^L_{3,1}+(g^L_{\rm YM})^2e^R_{3,1})\right) \\ &+\mathcal{O}\left(\frac{1}{\Lambda^{10}}\right)\Bigg]\end{aligned}$$

4-point results imply:

- Single-copy (YM) : Larger class of models can be double-copied using the new KLT kernel
- Double-copy (GR) : Same corrections are produced regardless of which KLT kernel is used

[Chi, Herderschee, Elvang, Jones, SP]

Summary of 4-Point Results

Schematic Operator	Total	Generalized	String	Cubic BAS
$\mathrm{Tr}[F^4]$	1	1	0	$\times$
$\mathrm{Tr}[D^2F^4]$	2	1	1	1
$\mathrm{Tr}[D^4F^4]$	3	3	1	1
$\mathrm{Tr}[D^6F^4]$	4	3	2	2

Schematic Operator	Total	Generalized	Cubic BAS
R^4	1	1	1
∇^2R^4	1	1	1
∇^4R^4	2	2	2

5-Point

Use cyclic symmetry and momentum relabeling to write the $(n-1)! = 4! = 24$ doubly color-ordered amplitudes in terms of 8 functions:

$$
\begin{aligned}
m_5[12345|12345] &= g_1[12345]\,, & m_5[12345|13254] &= g_5[12345]\,,\\
m_5[12345|12354] &= g_2[12345]\,, & m_5[12345|13524] &= g_6[12345]\,,\\
m_5[12345|12453] &= g_3[12345]\,, & m_5[12345|14253] &= g_7[12345]\,,\\
m_5[12345|12543] &= g_4[12345]\,, & m_5[12345|15432] &= g_8[12345]\,.
\end{aligned}
$$

Construct local ansatz for each function such that

$$
\begin{aligned}
s_{12}\, g_1[12345]\Big|_{s_{12}=0} &= s_{12}\, m[12345|12345]\Big|_{s_{12}=0}\\
&= g^3\left(\frac{1}{s_{34}} + \frac{1}{s_{45}}\right) + m_3[12P|12P]\, \tilde{m}_4[345P|345P]
\end{aligned}
$$

KLT Bootstrap at 5-Point

Demand that $1 \otimes 1 = 1$ for all basis choices.

$$
\begin{aligned}
g_1[12345] =& m^{\phi^3}[12345|12345] \\
&+ \frac{g}{\Lambda^4}(a_{1,0} - 2a_{1,1})\left(\frac{s_{35}}{s_{12}} + \frac{s_{41}}{s_{23}} + \frac{s_{13}}{s_{45}} + \frac{s_{24}}{s_{51}} + \frac{s_{52}}{s_{34}}\right) \\
&+ 2\frac{g}{\Lambda^4}(2a_{1,1} - a_{1,0}) \\
&- \frac{g\, a_{2,0}}{\Lambda^6}\left(\frac{s_{35}^2}{s_{12}} + \frac{s_{41}^2}{s_{23}} + \frac{s_{13}^2}{s_{45}} + \frac{s_{24}^2}{s_{51}} + \frac{s_{52}^2}{s_{34}}\right) \\
&- \frac{g}{\Lambda^6} a_{2,0}(s_{12} + s_{23} + s_{34} + s_{45} + s_{51}) + \dots
\end{aligned}
$$

At higher orders: Independent 5-point coefficients

Now we can find higher-derivative corrections to YM and double copy them.

Summary of 5-Point Results

Schematic Operator	Total	Bootstrapped	String	FT
$\mathrm{Tr}[F^5]$	2	$\times$	$\times$	$\times$
$\mathrm{Tr}[D^2F^5]$	5	$\times$	$\times$	$\times$
$\mathrm{Tr}[D^4F^5]$	14	1	1	1
$\mathrm{Tr}[D^6F^5]$	28	4	2	2

- Single-copy (YM) : Larger class of models can be double-copied using the new KLT kernel
- Double-copy (GR) : Same corrections are produced regardless of which KLT kernel is used

Web of Double Copies

	ϕ^3	χPT	YM
ϕ^3	ϕ^3	χPT	YM
χPT	χPT	Special Galileon	Born-Infeld
YM	YM	Born-Infeld	Dilaton Gravity

$$\mathcal{M}_4^{\text{BI}}(1^+2^+3^+4^+) = \frac{s^2 t u^2}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}\left[-\frac{2g^{\text{R}}_{F^3}g^{\text{R}}_{\text{YM}}}{\left(f^{\text{L}}_\pi\right)^2 g^2\Lambda^2} + \frac{\left(d_1^{\text{R}}g^2 - 4a_{1,1}g^{\text{R}}_{F^3}g^{\text{R}}_{\text{YM}}\right)\left(s^2+t^2+u^2\right)}{4\left(f^{\text{L}}_\pi\right)^2 g^4\Lambda^6}\right]$$

Local counterterms necessary to restore EM duality symmetry are not produced by the double-copy.

[Elvang, Hadjiantonis, Jones, SP]

Did We Need Minimal Rank?

All of these results are based on the KLT bootstrap equations, e.g. at 4-point

$$\boxed{\begin{aligned} f_6(s,t) &= f_1(s,t) \\ f_1(s,t) &= \frac{f_2(s,t)f_2(-s-t,s)}{f_2(t,s)} \\ f_2(s,t)f_2(-s-t,s)f_2(t,-s-t) &= f_2(t,s)f_2(-s-t,t)f_2(s,-s-t) \end{aligned}}$$

which needed minimal rank $(n-3)!$ to be preserved at higher orders.

What happens when we violate this?

- Fail to preserve $(n-3)!$ at subleading order
- Do not use a rank $(n-3)!$ leading order theory

Changing Rank at Subleading Order

Consider a deformation

$$\mathcal{L} = g\, f^{abc}\, \tilde{f}^{a'b'c'}\, \phi^{aa'}\, \phi^{bb'}\, \phi^{cc'} + \lambda\, d^{abcd}\, \tilde{d}^{a'b'c'd'}\, \phi^{aa'}\, \phi^{bb'}\, \phi^{cc'}\, \phi^{dd'}$$

where

$$d^{abcd} \equiv \text{Tr}\left[T^a\, T^{(b}\, T^c\, T^{d)}\right], \qquad \tilde{d}^{a'b'c'd'} \equiv \text{Tr}\left[\tilde{T}^{a'}\, \tilde{T}^{(b'}\, \tilde{T}^{c'}\, \tilde{T}^{d')}\right]$$

Turning this on modifies the amplitudes as

$$m_4\,[1234|1234] = m_4\,[1234|1432] = -\frac{g^2 t}{su} + \lambda, \qquad m_4\,[1234|1243] = -\frac{g^2}{s} + \lambda$$

Changing Rank at Subleading Order

Singularities of the determinant can lead to spurious poles when double-copying generic amplitudes,

$$S_{\mathrm{KLT}}[\alpha|\beta] = m[\alpha|\beta]^{-1} = \frac{1}{\det m}(\text{matrix of cofactors})$$

Changing Rank at Subleading Order

Singularities of the determinant can lead to spurious poles when double-copying generic amplitudes,

$$S_{\mathsf{KLT}}[\alpha|\beta] = m[\alpha|\beta]^{-1} = \frac{1}{\det m}(\text{matrix of cofactors})$$

In this case, the deformation does results in non-physical singularities in

$$\det \begin{pmatrix} m_4\,[1234|1234] & m_4\,[1234|1243] \\ m_4\,[1243|1234] & m_4\,[1243|1243] \end{pmatrix} = -\frac{g^2\lambda(s+2t)^2}{stu}$$

Non-Minimal Rank at Leading Order

We can use different zeroth copies at leading order.

Most general cubic bi-adjoint theory also includes an interaction $\lambda_3\, d^{abc}\, \tilde{d}^{a'b'c'}\, \phi^{aa'}\, \phi^{bb'}\, \phi^{cc'}$,

$$m_3[123|123] = g + \lambda_3\,, \qquad m_3[123|132] = -g + \lambda_3\,,$$

5-point

Couplings	Matrix Rank	Spurious Poles in Det?
$g \neq 0,\ \lambda_3 \neq 0$	24	Yes
$g \neq 0,\ \lambda_3 = \sqrt{3}g$	21	Yes
$g = 0,\ \lambda_3 \neq 0$	11	Yes
$g \neq 0,\ \lambda_3 = 0$	2	No

Evading Bad Poles

Consider a single-color scalar theories,

$$d^{abc}\phi^a\phi^b\phi^c$$

This still produces local double copy amplitudes of ϕ^3.

A similar phenomena occurs for zeroth copy

$$\lambda d^{abcd}\tilde{d}^{a'b'c'd'}\phi^{aa'}\phi^{bb'}\phi^{cc'}\phi^{dd'}$$

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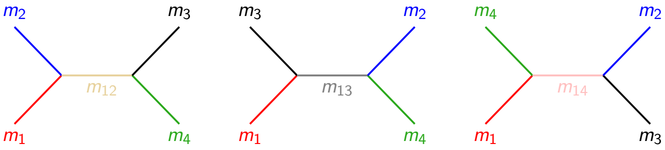
$$\text{Minimal rank} \Rightarrow \text{A local double copy}$$

$$\text{A local double copy} \overset{?}{\Longrightarrow} \text{Minimal rank}$$

[Chi, Herderschee, Elvang, Jones, SP]

KLT bootstrap ⇒ Massive Double Copy

Start with the most general massive deformation of bi-adjoint scalar amplitudes,



Achieved via propagator replacements

$$/.\{\mathtt{s_{ij}} \to \mathtt{s_{ij}} + \mathtt{m_{ij}^2}\}$$

This gives us a matrix of massive bi-adjoint scalar amplitudes:

$$m_4[\alpha|\beta] = \begin{bmatrix} \frac{1}{s+m_{12}^2} + \frac{1}{u+m_{14}^2} & -\frac{1}{s+m_{12}^2} \\ -\frac{1}{s+m_{12}^2} & \frac{1}{s+m_{12}^2} + \frac{1}{t+m_{13}^2} \end{bmatrix}$$

[Johnson, Jones, SP]

KLT Bootstrap ⇒ Massive Double Copy

At 4-point, rank of the matrix $m[\alpha|\beta]$ is $2 = (n-2)!$ i.e. it is full-rank.

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$$\det m[\alpha|\beta] = \frac{\sum_i m_i^2 - m_{12}^2 - m_{13}^2 - m_{14}^2}{(s+m_{12}^2)(t+m_{12}^2)(u+m_{12}^2)} = 0$$
$$\Rightarrow m_1^2 + m_2^2 + m_3^2 + m_4^2 - m_{12}^2 - m_{13}^2 - m_{14}^2 = 0$$

With associated massive BCJ relations,

$$\mathcal{A}_4[1234] \overset{\text{mBCJ}}{=\!=} \frac{t+m_{13}^2}{u+m_{14}^2}\mathcal{A}_4[1243]$$

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Kaluza-Klein reductions satisfy the spectral conditions.

[Johnson, Jones, SP][Momeni, Rumbutis, Tolley]

Higher-Derivative KLT: Outlook and Open Questions

- Landscape exploration: What other discontinuous deformations satisfy the minimal rank conditions?
- BCJ and CHY: How is our prescription related to color-kinematics duality and CHY?

  [Carrasco, Rodina, Yin, Zekioglu][Cachazo, He, Yuan]

- Other ranks: Fundamental matter couplings and d^{abcd} color structures

  [Brown, Naculich][Huang, Johansson, Lee][Johansson, Mogull, Teng][Huang, Johansson]

- Multiple masses: Generalize to case of multiple masses exchanged on each channel, string kernel is a special case
- Supersymmetry: Are there massive supersymmetric theories that double-copy?

$\vdots$

Summary

- Local extensions of the double copy can be formulated via deformations of bi-adjoint scalar theory that satisfy the minimal rank condition
- For effective field theories, the KLT bootstrap equations may be solved to derive general BCJ-compatible corrections to YM

Thank you for listening!