Massive color-kinematics duality and well-defined double copies

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Based on work with: Liang, Trodden   Momeni, Rumbutis

YOUNGST@RS - Rebuilding the Tower of Babel: Bringing Together the Various Languages of Color-Kinematics Duality

Color-kinematics duality for massive mediators

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Momeni, Rumbutis, Tolley
Johnson-Engelbrecht, Jones, Paranjape

Define by taking $s_i \to s_i + m^2$

$$\mathcal{A}_{YM} = \sum_i \frac{c_i n_i}{s_i + m^2} = c^T D^{-1} n \qquad \mathcal{M}_G = \sum_i \frac{n_i n_i}{s_i + m^2} = n^T D^{-1} n$$

Color factors satisfy Jacobi relations: $Mc = 0$

Under generalized gauge transformations $n \to n + \Delta n$

$A_{YM} \to A_{YM}$
$(D^{-1}\Delta n = M^T v)$

$Mn \to$ $\neq 0$
$Mn + MDM^T v = 0$

CK duality for any theory!

Check locality and factorization of DC

Shifted numerators satisfying CK require:

$$MDM^T v = -Mn \xrightarrow{\text{non-zero}} A\, v = -U$$

need to invert matrix A to get double copy:

$$\mathcal{M}_G = n^T D^{-1} n - U^T A^{-1} U$$

SPURIOUS POLES can arise from inverting A.

- 4pt: $A = s + m^2 + t + m^2 + u + m^2 =$-$m^2$
- 5pt: $\det A \propto \mathcal{P}\left(s_{ij}, m^2\right)$ SPURIOUS POLES!

Avoiding spurious poles

$m=0 \xrightarrow{SSB} m\neq 0$

- SSB in Supergravities
  Chiodaroli, Gunaydin, Johansson, Roiban
- Kaluza-Klein theories
  Johnson-Engelbrecht, Jones, Paranjape
  Momeni, Rombouts, Tolley

$$\sum_{i=1}^{4} m_i^2 = m_s^2 + m_t^2 + m_u^2$$

$\rightarrow \det A^{SC} = 0$

1 Cubic Scalar EFT

MCG, Liang, Trodden

3D Kinematics

$$\det A \propto \det(p_i \cdot p_j), \quad i,j < 5$$

$\rightarrow \det A^{3D} = 0$

Only 1 5pt "BCJ" relation

No spurious poles + correct factorization

2 Topologically massive theories

MCG, Momeni, Rombouts

Burger, Emond, Moynihan

From Kaluza-Klein

- (massive pions)^2 = massive special galileons

+ other operators?

Construct EFT for massive "pions"

Avoid spurious poles assuming spectral condition

$\boxed{\text{2202.00620 MCG, Liang, Trodden} \quad \text{massive tower + cubic interactions}}$

Kaluza-Klein : $\phi^{5d} = \sum_{n=-\infty}^{\infty} \phi_n(x^\mu)\, e^{inmy}$ ← 5th direction

$\mathbb{R}^{4+1} = \mathbb{R}^{3+1} \times S^1$

$\phi_n(x^\mu)$ ↑ 4d

-

$$\rightarrow P_i^{5d} = (P_i^{4d}, m_i)$$

$$A^{KK}(\tilde{s}_{ij}^{4d} \equiv s_{ij} - m_{ij}^2) = A^{m=0}(s_{ij}^{5d} \to \tilde{s}_{ij}^{4d})$$

KK amplitudes automatically satisfy CK duality

★ SGal in 5d has new contribution but

$$A^{SG6}(s_{ij}^{5d} \to \tilde{s}_{ij}^{4d}) = 0$$

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Use freedom of infinite towers of massive states

$$-\mathcal{L} \supset F \, \mathrm{Tr}(T^a T^b T^c) \, V_3[n_1, n_2, n_3] \, \phi^a_{n_1} \phi^b_{n_2} \phi^c_{n_3}$$

cyclic, antisym.

4pt BCJ

$$V_3(1,2,12)V_3(3,4,12) - V_3(1,3,13)V_3(2,4,13) + V_3(1,4,14)V_3(2,3,14) = 0$$

5pt BCJ + 1 constraint

Always enough freedom to satisfy BCJ relations

No analogue with 4pt vertices

Topologically massive theories 2

2107.00611
MCG, Momeni, Rumbutis

DC @
3,4,5 pt

PARITY BROKEN

Topologically Massive Gravity 1 dof

$$S_{TMG} = \frac{1}{\kappa^2}\int d^3x\sqrt{-g}\left(-R - \frac{1}{2m}\varepsilon^{\mu\nu\rho}\left(\Gamma^{\alpha}_{\mu\sigma}\partial_\nu\Gamma^{\sigma}_{\alpha\rho} + \frac{2}{3}\Gamma^{\alpha}_{\mu\sigma}\Gamma^{\sigma}_{\nu\beta}\Gamma^{\beta}_{\rho\alpha}\right)\right)$$

Topologically Massive Yang-Mills 1 dof

$$S_{TMYM} = \int d^3x\left(-\frac{1}{4}F^{a\mu\nu}F_{a\mu\nu} + \varepsilon_{\mu\nu\rho}\frac{m}{12}\left(6A^{a\mu}\partial^\nu A^\rho_a + g\sqrt{2}f_{abc}A^{a\mu}A^{b\nu}A^{c\rho}\right)\right)$$

$1 \otimes 1 = 1$ ✓

Avoiding spurious poles in $D = 3$

$$\boxed{\mathcal{M}_G = n^T D^{-1} n - U^T A^{-1} U}$$

Let's examine the matrix A (non-zero: MDM^T)

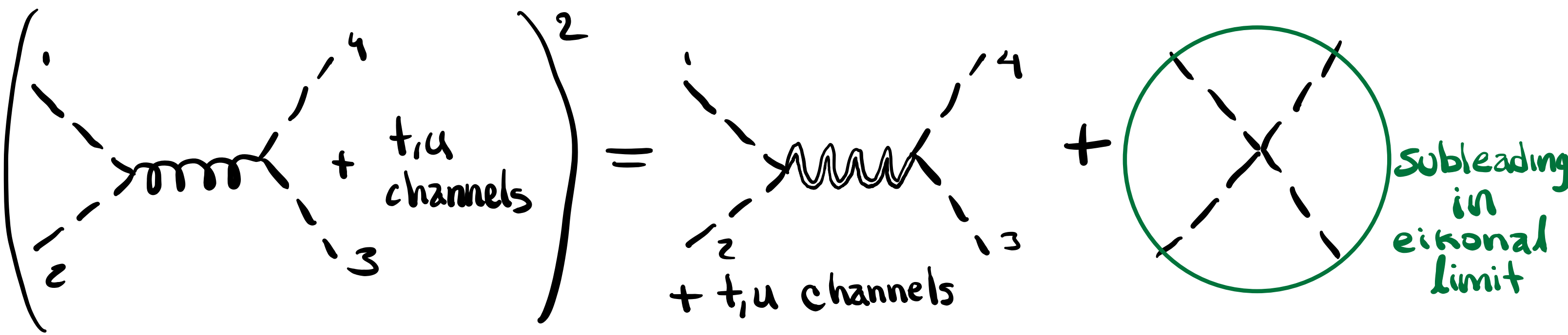
- $\det A \propto \det(p_i \cdot p_j), \quad i, j < 5 \quad \rightarrow \quad \det A^{D=3} = 0$
- A has null vector e_0 which implies "BCJ" relation: $e_0 \cdot U = 0$
  U (non-zero: Mn) measures the violation of the CK algebra
- Factorizes correctly when taking $s_{ij} \to 0$.
- Residue of the spurious pole is zero.

Including matter

Non-trivial Double Copy

$(\mathrm{TMYM})^2 = \mathrm{TMG} + \cancel{?}$

for matter with $T_\mu{}^\mu = 0$



2112.08401
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DC in eikonal limit

Also observed in:
Burger, Emond, Moynihan

$t << s \sim -u$
$m^2 << s$

In 4d $c \to s$ takes $A_n^{YM} \longrightarrow \mathcal{M}_n^{Gravity}$

$$i\mathcal{M}_{eik} = 2s \int db\, e^{-ibq} \left(e^{i\delta} - 1 \right)$$

$$\delta = \frac{1}{2s} \int \frac{dk_y}{2\pi} \mathcal{M}^{tree}(s, t = -k_y^2)\, e^{-ibk_y}$$

Here $A_{TME}^{n-1} = -\frac{Q^{2n} s^n \left(1 - i\frac{m}{\sqrt{-t}}\right)^n}{(2s)^{n-1}(t - m^2)^n} \xrightarrow[Q^2 \to s()]{} \neq \mathcal{M}_{TMG}^{n-1}$

Massive D.C. artifact

$$-i\mathcal{M}^{tree} \xrightarrow[eik.]{} \frac{n_t^2}{t - m^2} - \frac{(n_s + n_t + n_u)^2}{m^2}$$

Need info. Outside of eikonal lim.

Point-particle propagating in background:

$$ds^2 = -2dudv + \kappa f(y)\delta(u)du^2 + dy^2$$

$$\mathcal{M}^{P.P.}_{eik.} = \int \frac{dy}{2\pi} \, e^{-ik_y y - \frac{is}{4}\kappa f(y)} = \delta(q) + \frac{\mathcal{M}_{eik}}{2\pi s}$$

Boundary conditions on $g_{\mu\nu}$ = Choice of $i\epsilon$ prescription in phase shift

Which b.c. / $i\epsilon$ prescription → D.C. in coordinate space?

- Useful for time delay computation ($k_y \to k_y - i\epsilon$)

Edelstein, Giribert, Gomez, Kiricarslan, Leoni, Tekin; 2016

Kerr-Schild metric

$$h_{\mu\nu} = k_\mu k_\nu \phi \qquad k_\mu dx^\mu = du$$

★ Special choice of b.c.

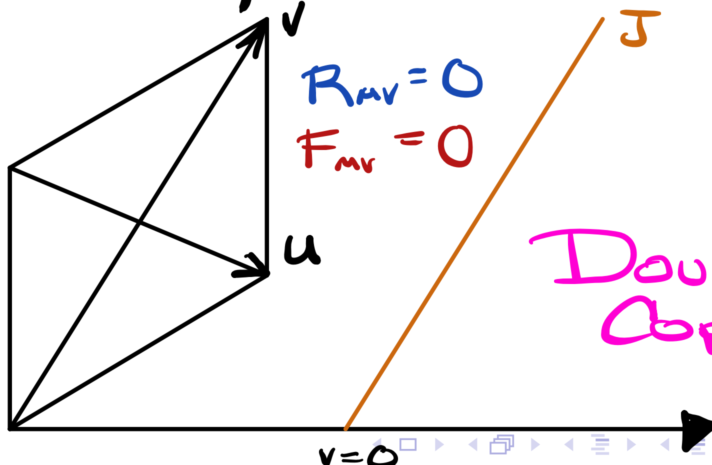
$$\phi = \frac{\kappa}{2}\frac{2E}{m}\left(e^{-my}\Theta(y) + (1-my)\Theta(-y)\right)$$

$$A^a_\mu = c^a k_\mu \phi$$

$$\phi = g\frac{Q}{m}\left(e^{-my}\Theta(y) + \Theta(-y)\right)$$

$$\phi^{a\tilde{a}} = c^a c^{\tilde{a}} \phi$$

$$\phi = \frac{\lambda}{2m} e^{-m|y|}$$



$R_{\mu\nu} = 0$

$F_{\mu\nu} = 0$

$\phi \to 0$

$\phi \to 0$

$\xrightarrow[y\to\infty]{}$

Double Copy

Shockwaves satisfy: $C_{uu} = \frac{m}{2}\,\frac{{}^*F_u\,{}^*F_u}{\phi}$ Why?

Look at linearized e.o.m plane waves

$$\varepsilon_{\mu\alpha\beta}\nabla^\alpha\left(R_\nu{}^\beta - \frac{1}{4}g_\nu{}^\beta R\right) \overset{\text{lin. eom}}{=} \frac{m}{2}\varepsilon_{(\nu\alpha\beta}F^{\alpha\beta}$$

$$\|$$

$$C^{lin}_{\mu\nu} = -\frac{1}{4}\,\frac{\overbrace{\nabla^\lambda F^{lin}_{\lambda(\nu}}\,\varepsilon_{\mu)\rho\gamma}F^{lin\,\rho\gamma}}{e^{ip\cdot x}}$$

Weyl D.C.

$$SO(1,3) \simeq \frac{SL(2,\mathbb{C})}{\mathbb{Z}_2}$$

$$\Psi_{ABCD} = \frac{f_{(AB} f_{CD)}}{S}$$

$$\nabla^{A\dot{A}} \Psi_{ABCD} = 0$$

$$\nabla^{A\dot{A}} f_{AB} = 0$$

$$(\Box - R/6) S = 0$$

Cotton(-York) D.C.

$$SO(1,2) \simeq \frac{SL(2,\mathbb{R})}{\mathbb{Z}_2}$$

$$C_{ABCD} = \frac{m}{2} \frac{f_{(AB} f_{CD)}}{S}$$

$$\nabla^{AE} C_{ABCD} - \frac{m}{\sqrt{2}} C^{E}{}_{BCD} = 0$$

$$\nabla^{AE} f_{AB} - \frac{m}{\sqrt{2}} f^{E}{}_{B} = 0$$

$$(\Box - R/6 - m^2) S = 0$$

Cotton DC for waves

$$C_{ABCD} = \Psi_4\, o_A o_B o_C o_D$$

2202.10476 Cotton DC
MCG, Momeni, Rumbutis

$$f_{AB} = \Phi_2\, o_A o_B$$

$$\Psi_4 = \frac{m}{2}\frac{\Phi_2^2}{\phi}$$

Non-zero at $u=0$ 1-side of SW.

	Ψ_4	Φ_2	ϕ
Shockwaves	$E m^2 e^{-my}$	$Q e^{-my}$	$\frac{\lambda}{m} e^{-my}$
Gyratons	$(E+\sigma m) m^2 e^{-my}$	$(Q+mQ') e^{-my}$	$\frac{\lambda+\lambda' m}{m} e^{-my}$
AdS-Shockwaves	$E m^2 \left(\frac{z}{z_0}\right)^{-1-Lm}$	$Q\left(\frac{z}{z_0}\right)^{-Lm}$	$\frac{\lambda}{m}\left(\frac{z}{z_0}\right)^{1-Lm}$

Conclusions and Future Directions

CK duality satisfied for all massive theories

but not all DC are local theories

Well-defined DC

- Kaluza-Klein theories
- Special cubic scalar theories
- Topologically massive theories

Symmetries of cubic interactions?

+ matter states?

Twistor origin?