Color-Dual EFT

MITP 2022

Based on ongoing work with
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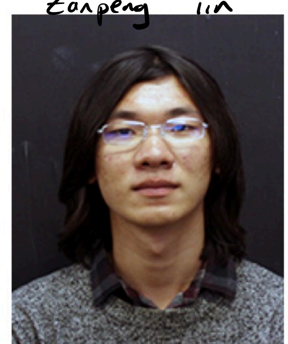
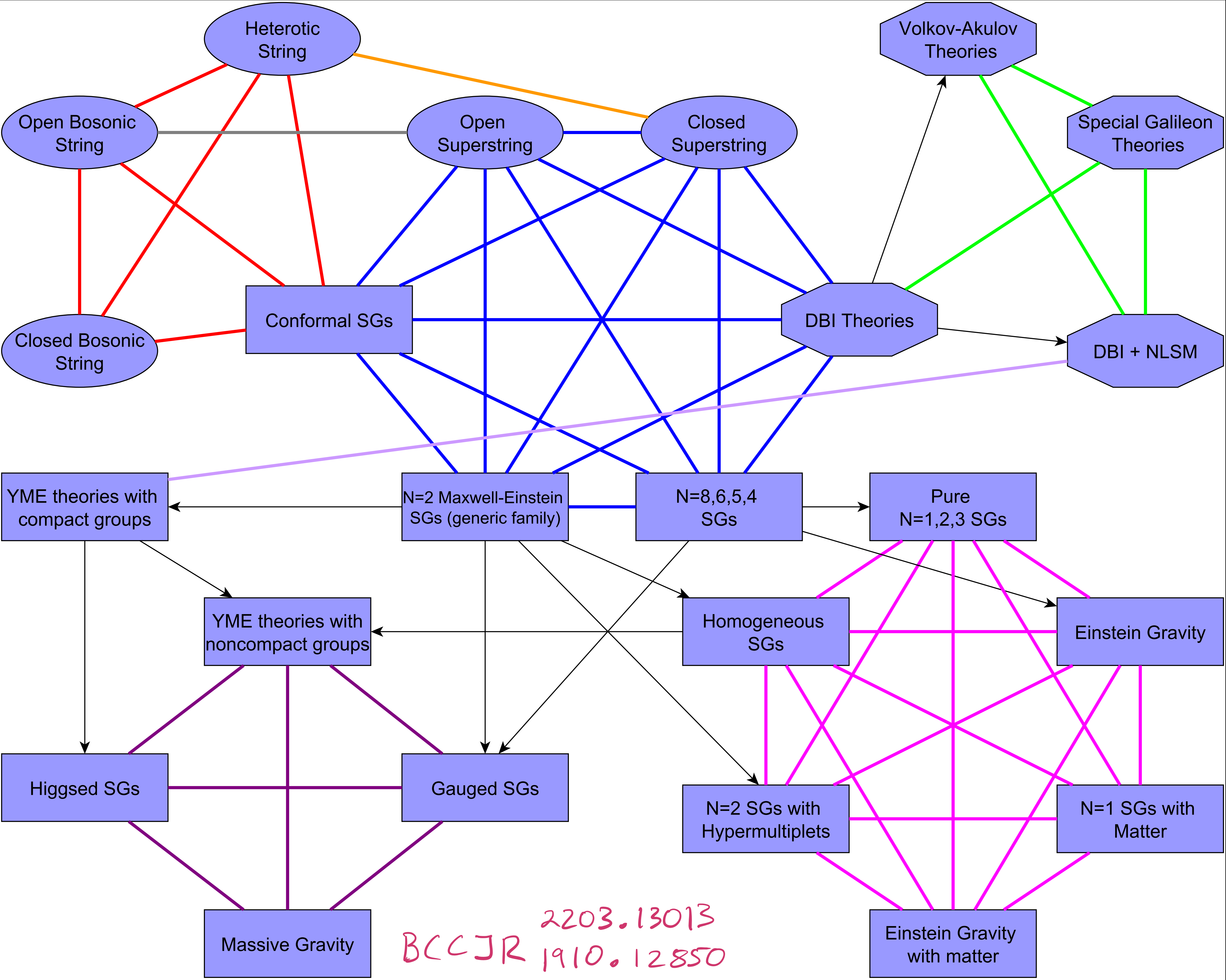



Heterotic String
Volkov-Akulov Theories
Open Bosonic String
Open Superstring
Closed Superstring
Special Galileon Theories
Conformal SGs
DBI Theories
Closed Bosonic String
DBI + NLSM
YME theories with compact groups
N=2 Maxwell-Einstein SGs (generic family)
N=8,6,5,4 SGs
Pure N=1,2,3 SGs
YME theories with noncompact groups
Homogeneous SGs
Einstein Gravity
Higgsed SGs
Gauged SGs
N=2 SGs with Hypermultiplets
N=1 SGs with Matter
Massive Gravity
Einstein Gravity with matter
BCCJR
2203.13013
1910.12850

I INTRODUCTION

double copy invites us to construct & combine our favorite theories using building blocks

$$\mathrm{SYM} = \text{color} \otimes \text{vector} + \text{susy}$$

$$\mathrm{NLSM} = \text{color} \otimes \overline{\pi}$$

$$\mathrm{SDBIVA} = \pi \otimes \text{vector} + \text{susy}$$

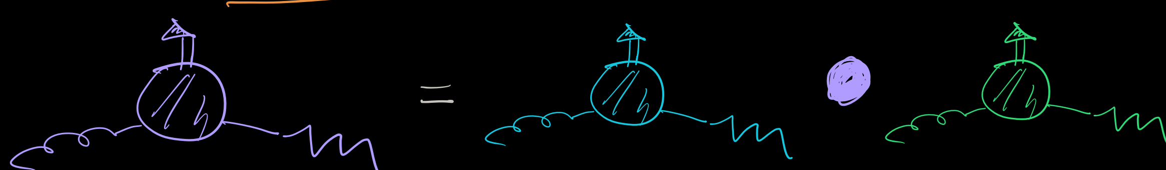
$$\mathrm{SG} = \text{vector} + \text{susy} \otimes \text{vector} + \text{susy}$$

$$\otimes \equiv \Big(\underset{\text{(tree level)}}{\mathrm{KLT} = \mathrm{BCJ}} \underset{\text{(tree level)}}{= \mathrm{CHY}} \Big)$$

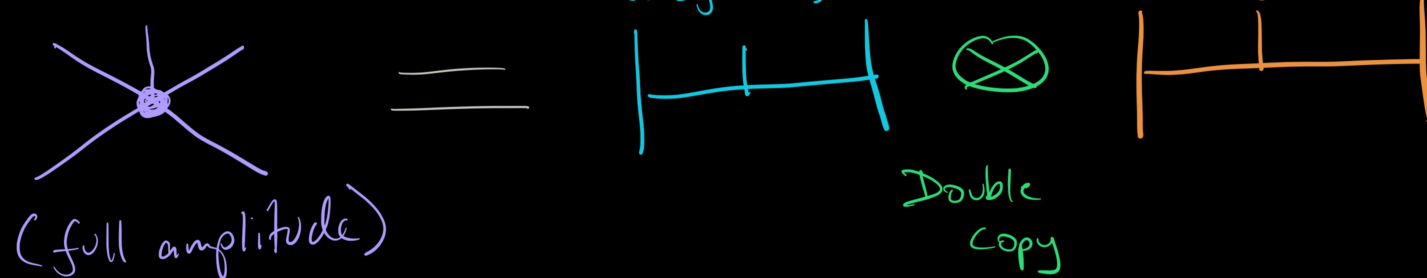
This talk will largely be at Tree Level
but borrowing insight & approaches from
Loop Level

Will refine language around double
copy via **Something New:** alg. composition

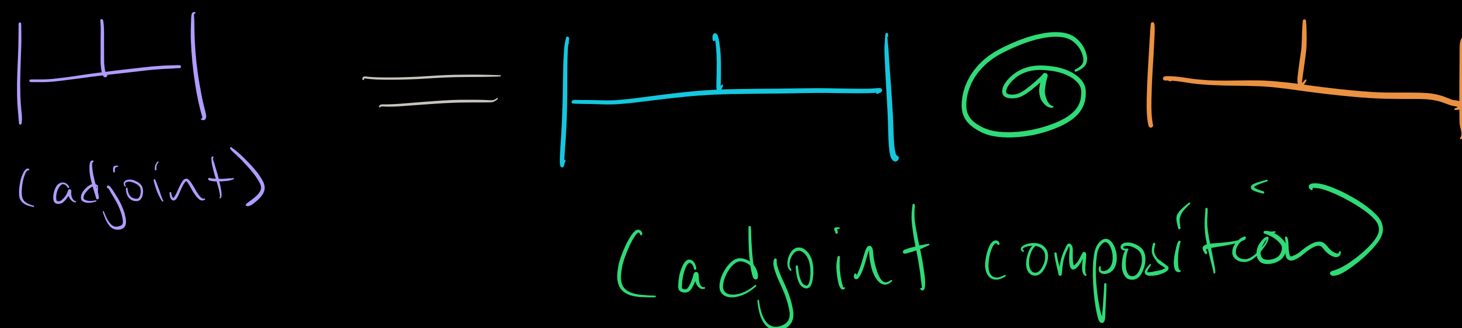
JJMC, ZR, ZY, SŽ
1910.12850



Composition $\neq$ Double copy



vs.



E.g.

$$\overline{LL} = (D\varphi)^2 \,ⓐ\, (D\varphi)^2$$

PIONS

covariantized

free scalar

Summary:

- composition lets us build & classify EFT operators for gauge & gravity theories with SMALL set of Building Blocks

1910.12850
2104.08370

- $\mathcal{N}=4$ SG may have a non-local color-dual fate:
  - Berkovits Witten CSG
  - Heterotic String

2203.03592

EFT

- Higher derivative operators (or counterterms)

let you parameterize your ignorance of new physics with available fields

$$\mathcal{L} = -\partial^2\varphi + g\varphi^3 + c_2\frac{(\partial^2\varphi^2)}{\Lambda} + c_4\frac{\partial^4\varphi^4}{\Lambda^2} + \cdots$$

Goal:

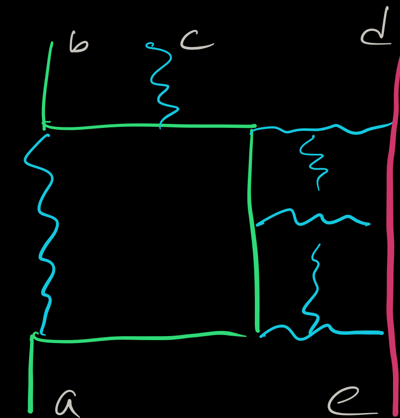
$$\boxed{\partial^n F^m, \ \partial^n R^m}$$

How we calculate at loops

- Not Feynman diagrams
    - off shell unweildy
    - factorial # of diagrams
- But Graph Organized
    - Unitarity methods efficient for VERIFICATION & Cut construction
    - Virtual integration wants local reps.

(Gauge) Loop Integrand Map

Mapping from labeled topology



to

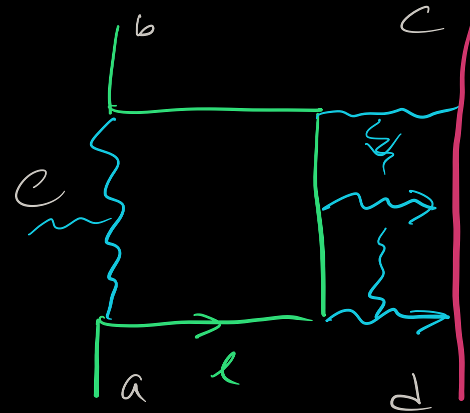
color weights $\left(f^{abc} \quad T^{a}_{ij} \right) C$

propagator weights $\left(\xrightarrow{\ell} = \frac{1}{\ell^2 - m^2} \right)$

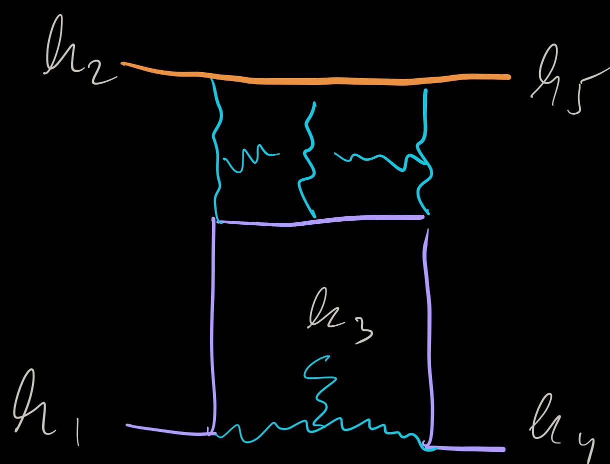
numerator weights $\left(k_a \cdots k_e,\ \varepsilon_c,\ \ell_1 \cdots \ell_4 \right)$

Weights should obey

symmetries of topology



no



?

$a \to k_1$ $\quad$ $a \to k_4$

$b \to k_4$ $\quad$ or $\quad$ $b \to k_1$

$e \to k_3$ $\quad$ $e \to k_3$

& unitarity cuts.

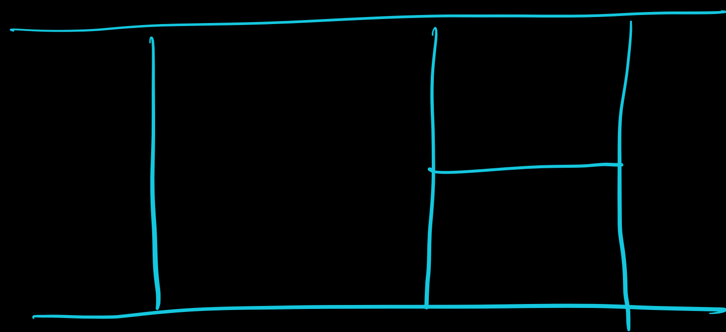
Recipe for Calculation

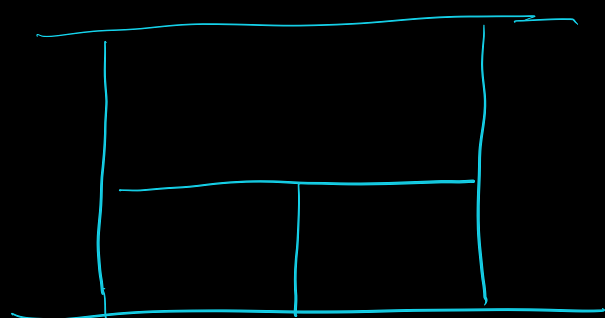
- Give Ansätze to topologies
- Fix on Symmetries & Cuts
- Duality between color & kinematics

  can help!

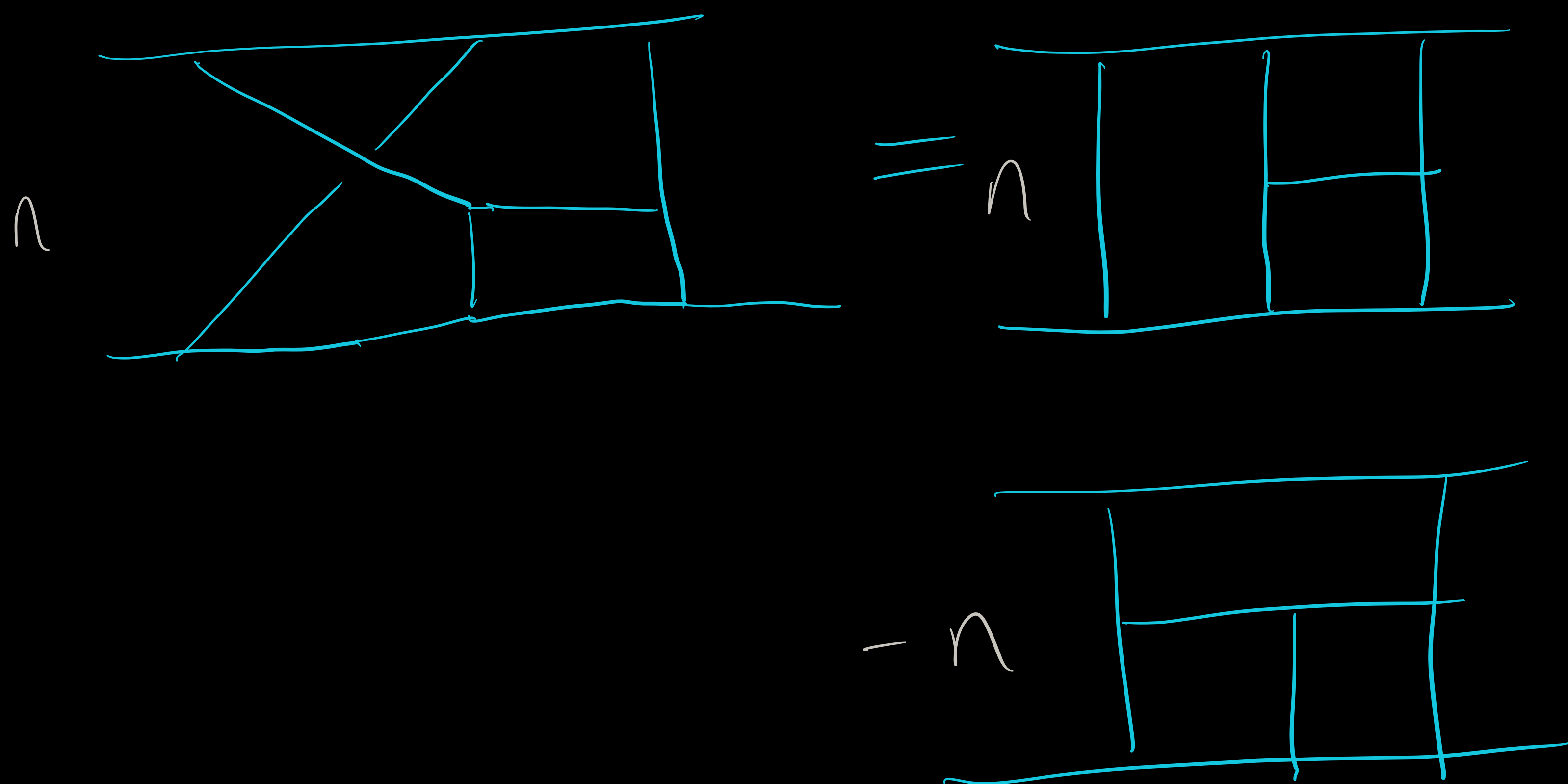
Adjoint-type color weights

f^{abc}

c  $=$

c  $+\ c$ 

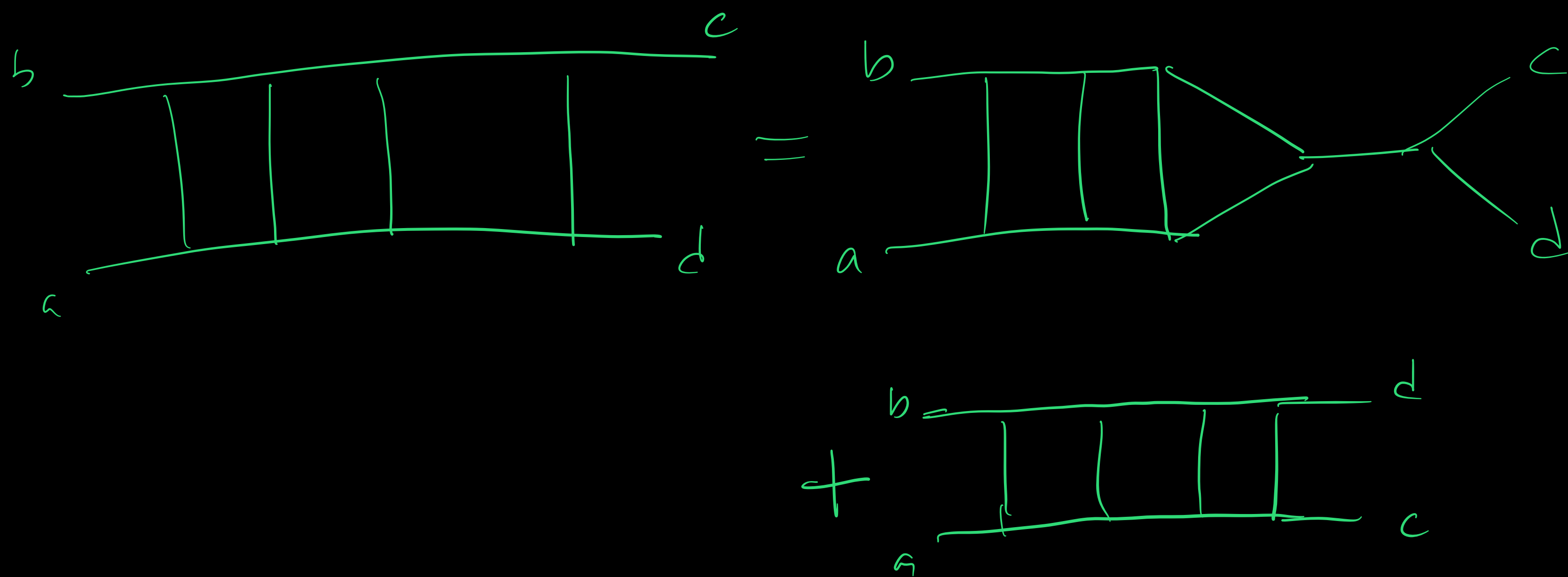
Demand this holds for n as well!



All of 3-loop 4pt $\mathcal{N}=4$ SYM in one topology!

Bern, Carrasco, Johansson '08 '10

Note feedback constraints:



Jacobi relations are functional at loop level

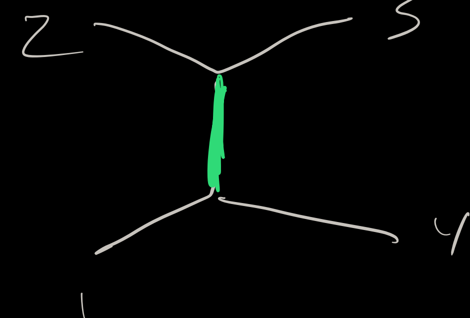
so Ansätze are natural. (but can get too expensive)

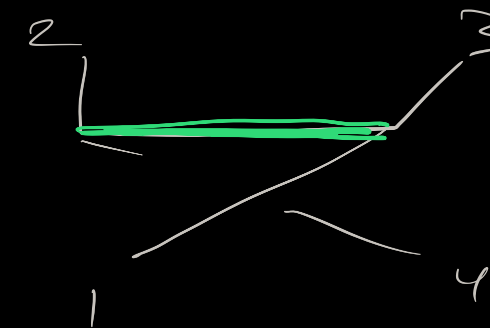
Higher Derivative Operators

- ∂_i in $\mathcal{L}$ $\longrightarrow$ k_i in $\mathcal{A}$
- Game is to figure out all distinct ways of sprinkling in some order in MD k into your scattering amplitude.

  w/out messing w/ symmetries.
- Easy: scalar permutation invariants

4 pt tree level adjoint type n(1−1)

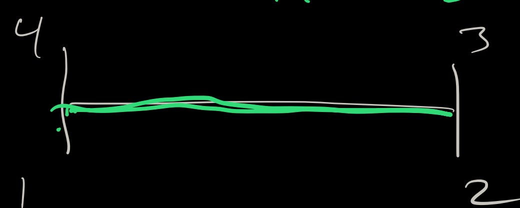


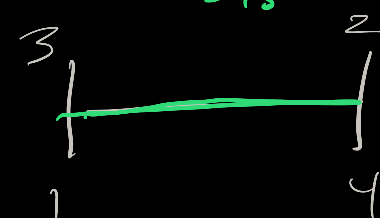


$s=(k_1+k_2)^2 = s_{12} = s_{34}$

$t=(k_1+k_4)^2 = s_{14} = s_{23}$

$u=(k_1+k_3)^2 = s_{13} = s_{24}$





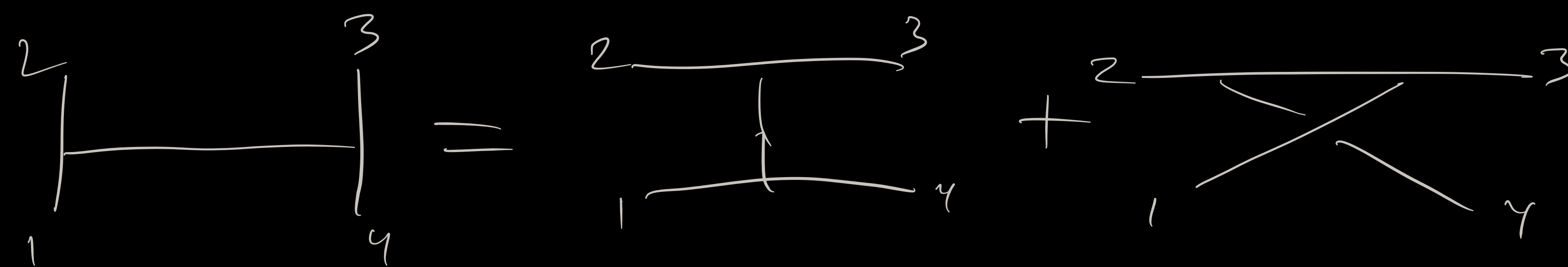
$g(s,t,u)$

$g(t,s,u)$

$g(u,t,s)$

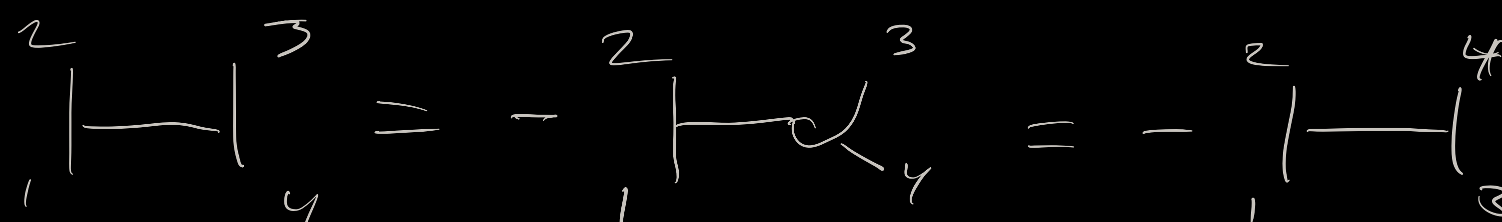
$= -g(s,u,t)$

Adjoint conditions



$$n_a(s,t,u) = n_a(t,s,u) + n_a(u,t,s)$$

&

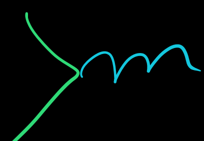


$$n_a(s,t,u) = -\,n_a(s,u,t)$$

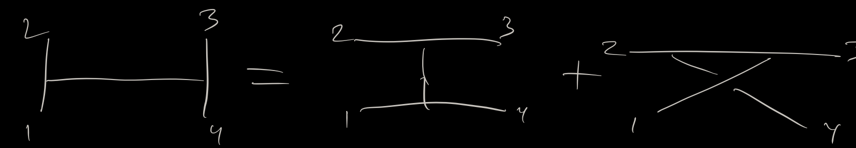
Cov free scalar

$$(\partial\phi)^2 \sim$$

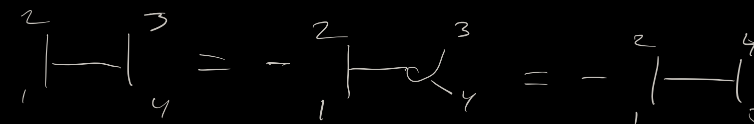
$$(\partial_\mu \phi_a) A_b^\mu \phi_c f^{abc}$$



Adjoint type!



$$n_a(s,t,u) = n_a(t,s,u) + n_a(u,t,s)$$



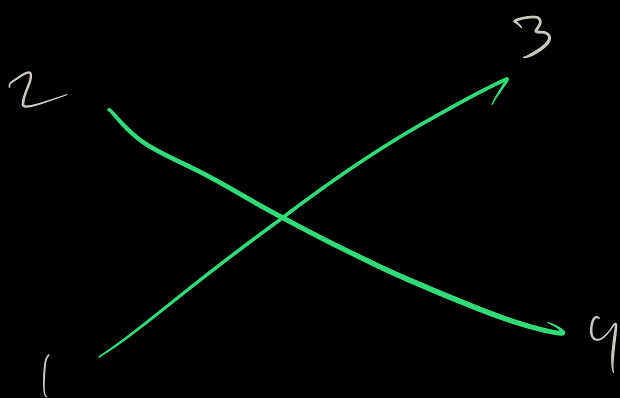
$$n_a(s,t,u) = -n_a(s,u,t)$$

$$n^{ss}(a,b,c) = b-c$$

$$(t-u)=(s-u)+(t-s)$$

$$t-u = -(u-t)$$

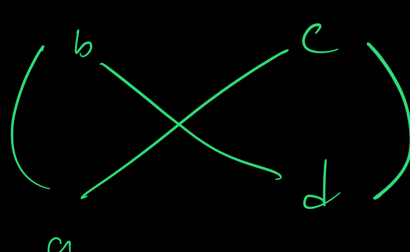
Permutation Invt Conditions



$$s\,t\,A^{\mu\nu}(s,t)$$

P_4 doesn't care about leg order

inv under S_4.

$C\Big($ 

$\Big) = d^{abcd}$

Note, no liner perm invt for massless

$$s+t+u=0$$

But can compose adjoint g_a to get perm invt.

$$n^{adj} \circledast \tilde{n}^{adj} = n_s\tilde{n}_s + n_t\tilde{n}_t + n_u\tilde{n}_u$$

$$P_2 = n^{ss} \circledast n^{ss} = s^2+t^2+u^2 = \sigma_2$$

Note also, trivially

$$n_s = \tilde{n}^{adj} \, a_s \, p = p \, a_s \, \tilde{n}^{adj} = p \, \tilde{n}_s^{adj}$$

Adjoint Comp at 4pt !

$$n_s = \tilde{n}^{adj} @_s \check{n}^{adj} = \tilde{n}_t \check{n}_t - \tilde{n}_u \check{n}_u$$

n_t, & n_u just come functionally by relabeling.

$$\Pi_s = n^{ss} @_s n^{ss} \propto s(t-u)$$

$$P_3 = \underbrace{n^{ss}}_{1} \wp \underbrace{\overline{\Pi}}_{2} \propto stu = \sigma_3$$

$$P_4 = \Pi \wp \overline{\Pi} \propto \sigma_2 \wp \sigma_2 = (s^2+t^2+u^2)^2$$

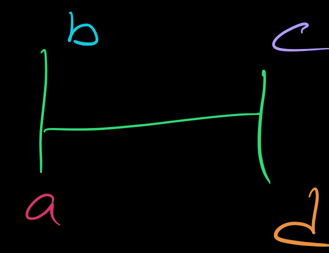
Simple Covariantized Scalar

spans all scalar adjoint type BB under composition.

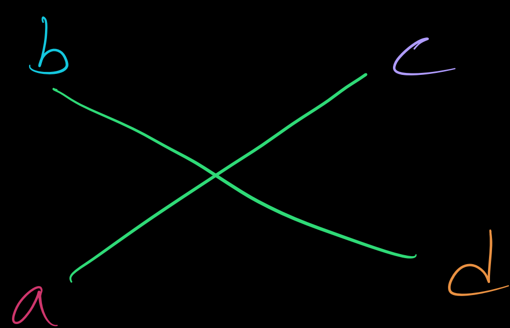
- Climbs a ladder MD by MD
- Closes after 2nd rung under perm invariants.

$$n_s^{MD} = \sum_{X,Y}^{3Y+2X=MD-1} C_{XY}^{ss}\, 6_2^{X}\, 6_3^{Y}\, n_s^{ss} + \sum_{X,Y}^{3Y+2X=MD-2} C_{XY}^{\pi}\, 6_2^{X}\, 6_3^{Y}\, \overline{\pi}_s$$

Can add color: $c_s^{adj} = f^{abe} f^{ecd}$



$\varepsilon_i \quad d^{abcd}$



to n's using composition in relevant ways.

eg:

$$\Pi_s^{abcd} = d^{abcd}\, \Pi_s$$

Note: $A^{\mathrm{tr}(F^4)} = d^{abcd}\, st\, A^{YM}(1234)$

$= T^{abcd} \otimes \text{vector}$

It's ordered amplitudes don't satisfy (n-3)! BCJ amplitude relations (they're perm inv't!)

Rather it is a double copy

C.f. Shruti Paranjape's TALK!

This allows us to recover

Open & Closed Superstrings

at Tree Level as

Field Theory Double Copies

from scalar perm invariants (σ_2 , σ_3)

scalar adjoint numerators (n^{ss} , Π)

single trace color weights (f^{abc} , d^{abcd})

& sym numerator ($n^{(s)ym}$)

$$OSS = \Sigma \otimes sYM$$

$$CSS = sYM \otimes sYM^{(sv)}$$

$$\Sigma_s = \left(c^{adj} \text{@}\, n^{ss}\right)_s \sigma_2^x \sigma_3^y C^{ss}_{x,y}$$

$$+ \left(c^{adj} \text{@}\, \Pi\right)_s \sigma_2^x \sigma_3^y C^{\overline{\pi}}_{x,y}$$

$$+ \left(d^{abcd}\, \Pi\right)_s \sigma_2^x \sigma_3^y C^{d}_{x,y}$$

$$sYM^{(sv)}_s = n^{ym}_s \sigma_2^x \sigma_3^y C^{sv}_{x,y}$$

Composition Very Powerful

- Could verify up to insanely high MD $(\alpha')^{40} \sim (s_{ij})^{40}$ before finding the right way

  to express this in terms of OSS

  directly in closed $\Gamma(\ )$ form at 4pt.

- Check out PRL for example

  & proofs.

Why no $(C^{ss} @ n^{YM})$ or

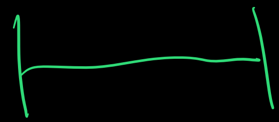
$(C^{\pi} @ n^{YM})$?

Resulting numerators satisfy adjoint constraints but <u>NOT</u> Gauge Inv.

What about eg other tensor structures?

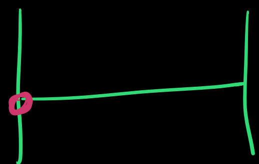
turns out $\exists$ 8 spanning building blocks at 4pts.

Vector Building Blocks

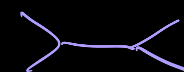


YM

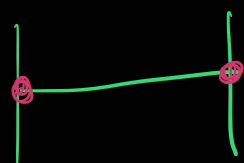
$\mathrm{tr}F^3$



α' above YM + 2 more

( $/ G_3$)

$((DF)^2)^2$



$2\ \mathrm{tr}F^3 + \mathrm{tr}(F^4)$ α'^2 above YM

+ 2 more

$\partial^2 F^4$

All other local color dual vectors spanned by these & perm invts.

Eg Bosonic String : $Z \otimes (DF)^2 + YM$

$$n^{(DF)^2+YM} = n_{ym} + \frac{\left[\alpha'\left(n^{F^3} + \alpha'\left(n^{2F^3+F^4} + \alpha'\left(n^{\partial^2 F^4} + \alpha' n^{(DF)^4}\right)\right)\right)\right]}{1 - \alpha'^2 \zeta_2 - \alpha'^3 \zeta_3}$$

Fine

But can these be algebraically constructed?

= STAY TUNED =

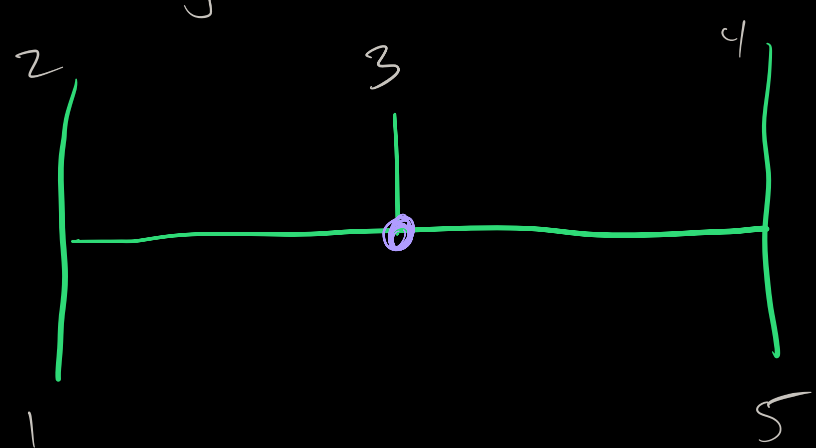
JJMC,
LR, SŽ

Is this just a feature of 4pt?

No! 5pt even richer &

a fascinating surprise

No adjoint linear building block

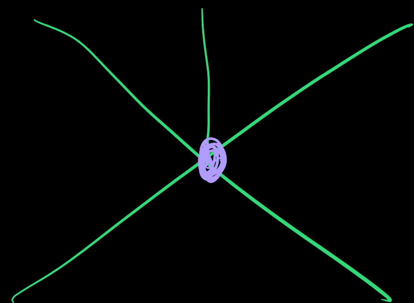
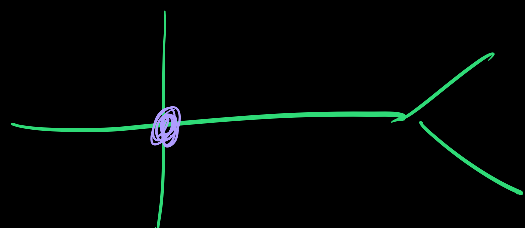
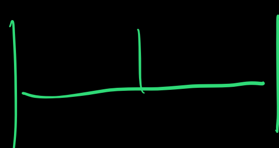
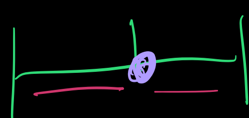
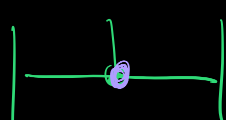


<u>Relaxed Instead</u>

$$(k_1-k_2)\cdot(k_5-k_4)$$

JJMC, LR, SŽ
2104.08370

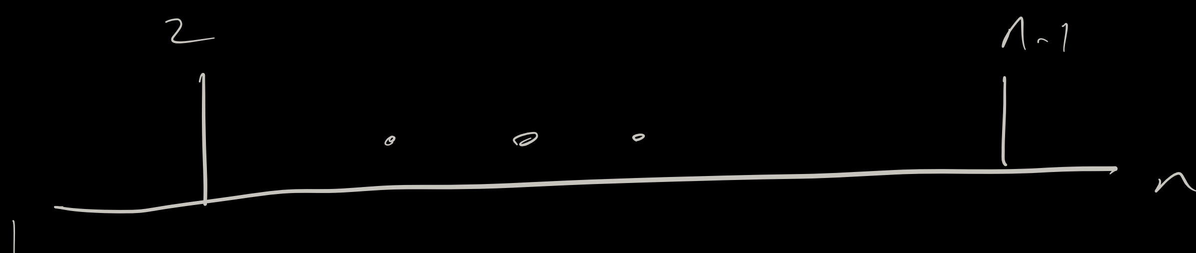
5 pt Algebraic Structures

	P_5	d^5	p
	Hybrid	$d^4 f^3$	h
	Adj	$f^3 f^3 f^3$	a
	Relaxed Adj		r
	Sandwhich	$f^3 d^3 f^3$	s

Lesson from five points.

- Odd multiplicity EFT linear building block: Relaxed

- Even multiplicity: Adjoint



$(k_1 - k_2)\cdot$
$(k_n - k_{n-1})$

# of vector adjoint color-dual BB?

Open Question

- even 5pt exhausting via ansatz, motivating construction approach!

STAY TUNED!

Color-dual Fate of $\mathcal{N}=4$ SG

JJMC, ML, NP 2203.03592

- Diverges in 4D at 4-loops

  Bern, Davies, Dennen, Smirnov, Smirnov '13

- Seems tied to anomalous behavior at 1-loop

  JJMC, Kallosh, Roiban, Tseytlin '13

- Counterterm related to $\mathrm{tr}\,F^3$

  - does resolve 1-loop anomalous bhvr.
  - doesn't mess up UV at 1 & 2 loops.

'17 Bern, Edison, Kosower, Parra-Martinez

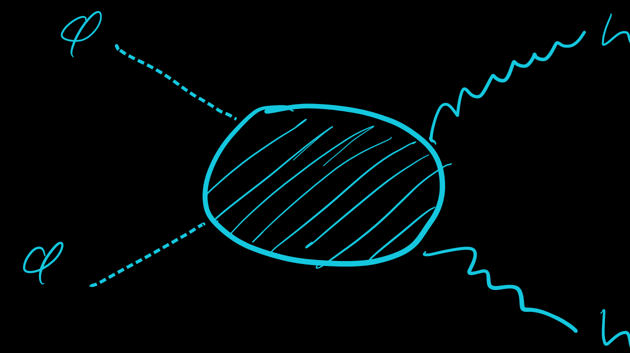
'17 Bern, Parra-Martinez, Roiban

'19 Bern, Kosower, Parra-Martinez

Anomalous Behavior

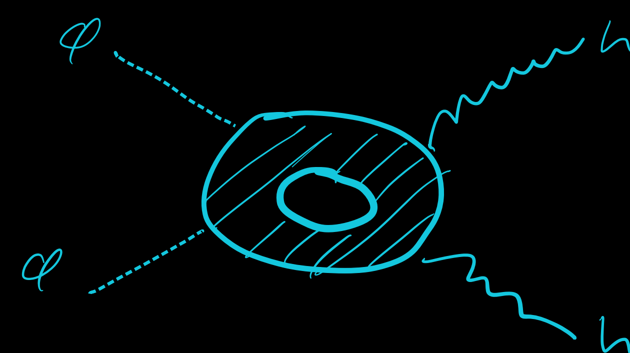
Marcus '85

$$A^{SG}_{\text{tree}}(\varphi\varphi h^+ h^+) = 0$$



$\mathcal{N}=4$

$$A^{SG}_{\text{loop}}(\varphi\varphi h^+ h^+) \neq 0$$



Can understand behavior via double-copy

half max SG = maximal SYM ⊗ pure YM

Anomalous Behavior

$$g^+ \otimes g^+ = h^+$$
$$g^+ \otimes g^- = \varphi$$
$$g^- \otimes g^- = h^-$$

$$A^{SG}(\varphi\varphi h^+ h^+) = \underbrace{A^{SYM}(g^- g^- g^+ g^+)}_{\text{B/C SUSY: ONLY VALID HEL STRUCTURE TREE OR LOOP}} \otimes \underbrace{A^{YM}(g^+ g^+ g^+ g^+)}_{\substack{\text{ZERO at tree level (secretly SUSY at tree)} \\ \text{NON-ZERO AT 1 LOOP!}}}$$

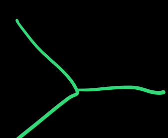
To cancel anomaly need COUNTERTERM that admits 4 g^+ even at tree level $\underbrace{tr(F^3)}$

So is $\mathrm{tr}F^2 + \alpha' \, \mathrm{tr}F^3$

Double Copy Consistent?

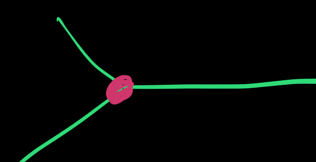
(ie color dual & factorizing for all multiplicity at tree level)

YM



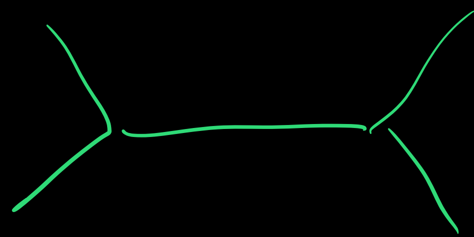
$+ \alpha'$

F^3



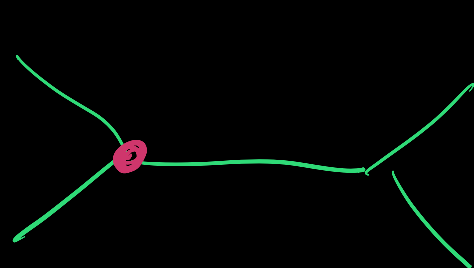
3pt sure!

4pt



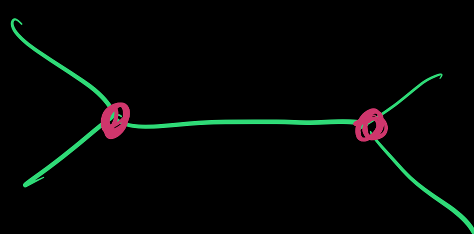
(YM) ✓

α'

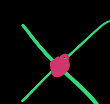


(single insertion) ✓

α'^2



(double insertion) ✗



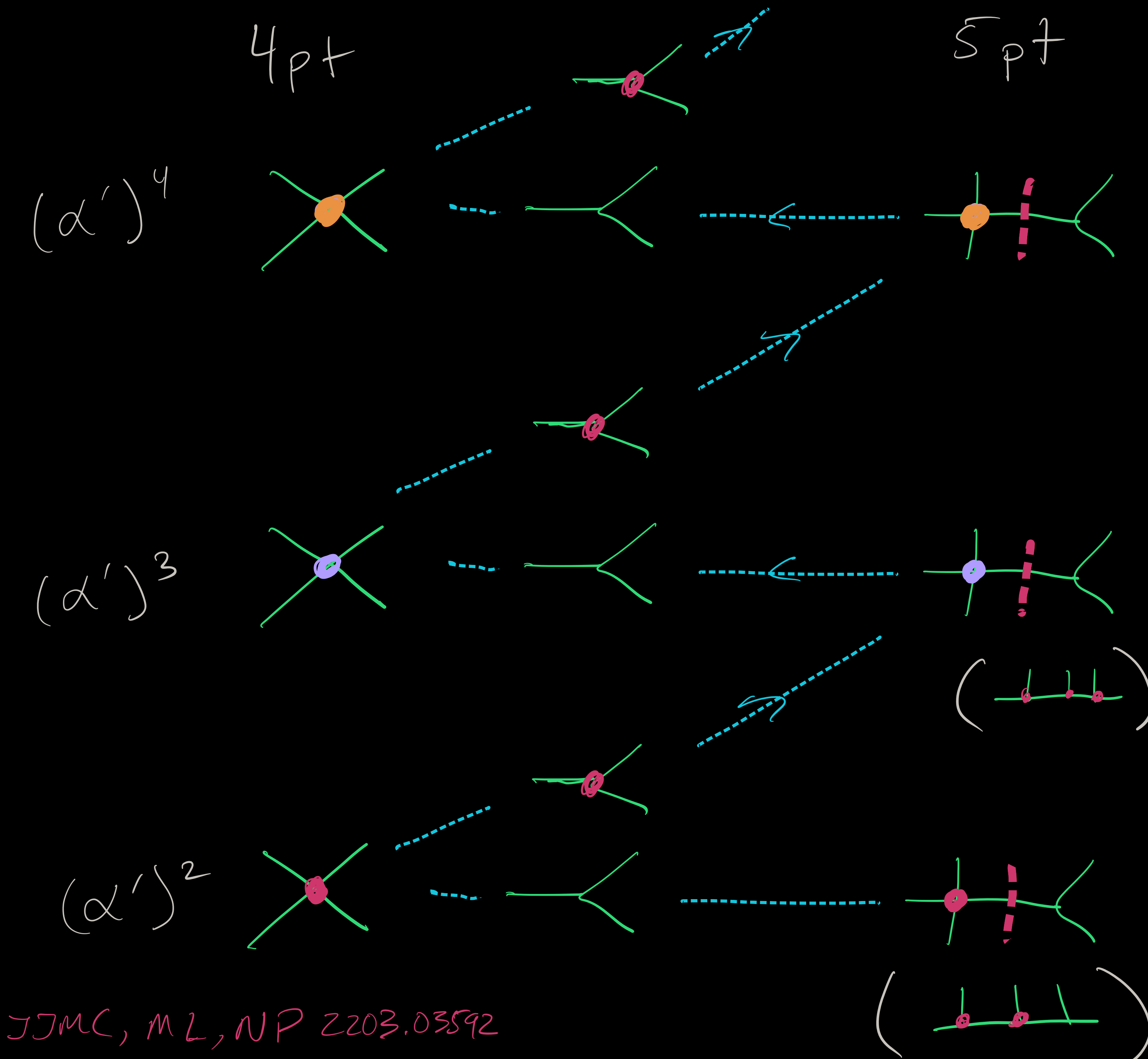
Needs 4pt $\mathrm{tr}(F^4)$ ct. OK

So is $\mathrm{tr}F^2 + \alpha' \,\mathrm{tr}F^3 + \dots$

Double Copy Consistent

with a finite # of CT?

Apparently not


4pt
5pt
$(\alpha')^4$
$(\alpha')^3$
$(\alpha')^2$
JJMC, ML, NP 2203.03592

Requiring C/K &

Factorization even

Between 4 & 5 point

INDUCES A TOWER

ALL THE WAY TO THE

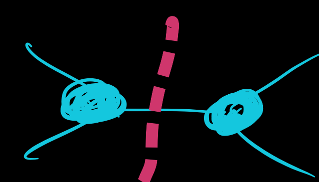
UV

How can you tell?

4pt Ansatz:

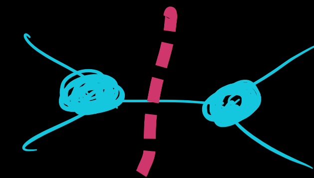
$$n^{4pt} = \sum_{i \in I}^{8} \sum_{x,y}^{2x+3y < MAX} c^i_{xy} \, \sigma_2^x \sigma_3^y \, n_i$$

fix on cuts to 3 points



Everything is either precluded, set to α' or 1 (YM coupling), or free (could be set to $\emptyset$)

$$A = A^{YM} + \alpha' A^{YM+F^3} + \alpha'^2 A^{(F^3)^2+F^4} +$$



$$\alpha'^3\Big(c_3 \big(A^{D^2F^4} + \zeta_2 A^{YM+F^3}\big) + \tilde{c}_3 \big(\zeta_3 A^{YM}\big)\Big) +$$

$$\alpha'^4 \Big[c_4 \big(A^{((DF)^2)^2_1} + \zeta_2 A^{(F^3)^2+F^4}\big) + \checkmark\, c_4\, A^{(DF^2)^2_2} + \tilde{c}_4\, c_3\, A^{YM+F^3} \Big]$$

c unconstrained – could be zero.

Unless ladder forces particular values.

which it does. (feedback on 5 pt c/k & factorization)

$$C_3 = 1$$

$$\tilde{C}_4 = 1 + \tilde{C}_3$$

$$C_4 = \check{C}_4 = 1$$

$$\Longrightarrow$$

$$n = n^{(DF)^2 + 4m} + \mathcal{O}(\alpha'^5)$$
$$+ \tilde{C}_3\, \alpha'^3 G_3 \left(n^{4m} + \alpha' n^{F^3} + \cdots \right)$$

$$\mathcal{N}=4\ SG + c.t. = \mathcal{N}=4\ sYM \otimes \left[\left(YM + tr(F^3) + \cdots \right) \equiv (DF^2 + YM) + \cdots \right]$$

$=$ Berkovitz Witten SG

or Heterotic String

Calculated through α'^4

Proof & Loop level results await!

(A few words about DF^2)

$$DF^2 \otimes sYM = CSG$$

Johanson, Nohle '17

$$(DF^2 + YM) \otimes sYM = \text{Weyl-Eistein}$$

Johansson, Mogull, Teng '18

$$(DF^2 + YM) \otimes Z =^{*} \text{Bosonic String}$$

$$(DF^2 + YM)^{sV} \otimes sYM =^{*} \text{gravitons in Heterotic String}$$

$$(X)^{sV}_{c} = X_a \otimes^{ab} sV(Z_{bc})$$

* Azvedo, Chiodaroli, Johansson, Schlotterer '18

Next Steps

- Composition to build vector BB?
- Proof that tr F^3 closes to $(DF)^2 + YM$
- Effect of tower on explicit loop-level calculation in $\mathcal{N}=4$ SG + ...
- Loop-level composition?

⋮ Lots to Do!

Summary

- composition lets us build & classify EFT operators for gauge & gravity theories with SMALL set of Building Blocks

- $\mathcal{N}=4$ SG may have a non-local color-dual fate:
  - Berkovits-Witten Conformal SG
  - Heterotic String