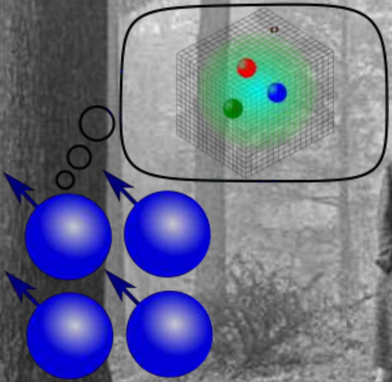
Quantum Algorithms for Quantum Field Theory

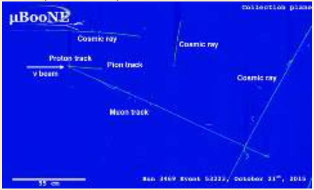
Hank Lamm

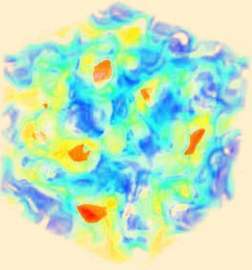


Fermilab

Formulating the problem of real-time dynamics

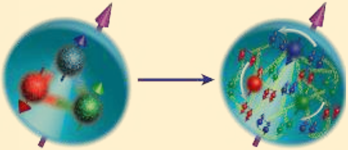
$\sigma(\nu N \to X) =$???





$\frac{\eta}{s}(T) =$???

$H^q(x,\xi,t) =$??? $E^q(x,\xi,t) =$???



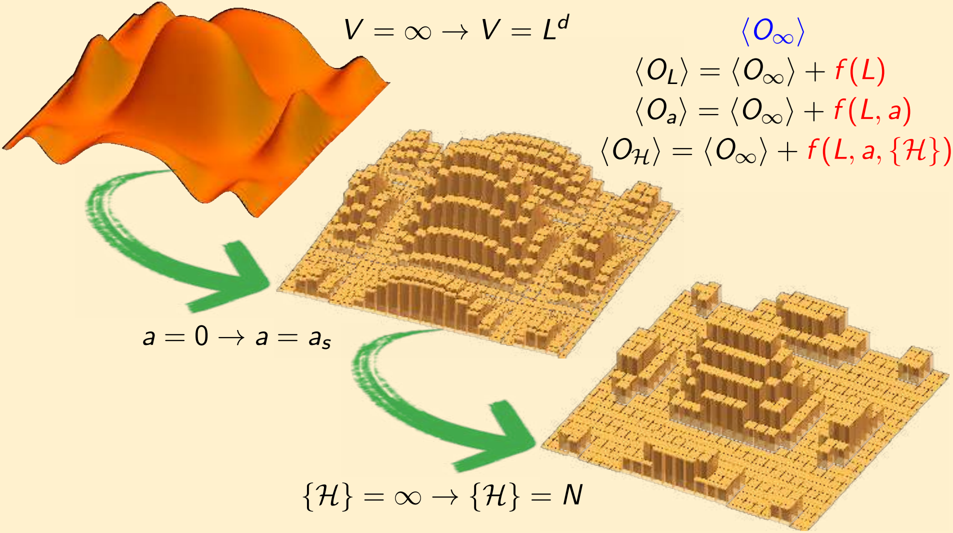
Examples: $\nu - N$ scattering, QGP Transport, Hadron Tomography[1]

$$\langle \prod_i \mathcal{O}_i(t_i) \rangle = \int_{\psi(0)}^{\psi(T)} \mathcal{D}\psi \prod_i \mathcal{O}_i(t_i) e^{-iS} = \langle \psi(T) | \prod_i \mathcal{O}_i(t_i) | \psi(0) \rangle$$

We are concerned with **nonperturbative** results

[1] Carena, M. et al. In: *Snowmass 2021 LOI* TF10-077 (2020).

QFT is about infinities and how to regulate them



$$\langle O_\infty \rangle$$
$$\langle O_L \rangle = \langle O_\infty \rangle + f(L)$$
$$\langle O_a \rangle = \langle O_\infty \rangle + f(L, a)$$
$$\langle O_{\mathcal{H}} \rangle = \langle O_\infty \rangle + f(L, a, \{\mathcal{H}\})$$

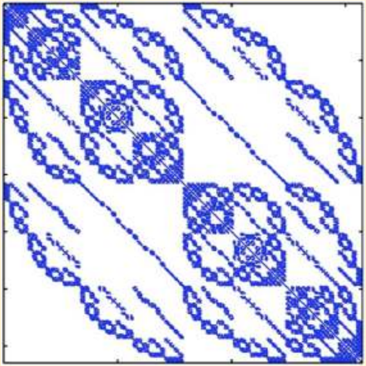
How can we attack these problems today?

- Paper and Pencil
- Deterministic Methods
- Monte Carlo Methods

What are deterministic methods?

Same input, same output

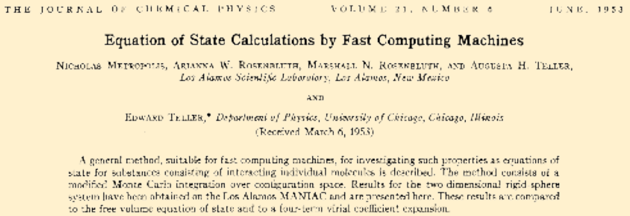
e.g. Exact Diagonalization, Lanczos, Runge-Kutta

$\mathcal{H} =$ 

For quantum field theory, H is a $\infty \times \infty$ matrix

Monte Carlo methods present a practical solution...

When the state space gets too big, to evaulate $\int \mathrm{d}x\, p(x)$, randomly sample values according to $p(x)$


THE JOURNAL OF CHEMICAL PHYSICS — VOLUME 21, NUMBER 6 — JUNE, 1953

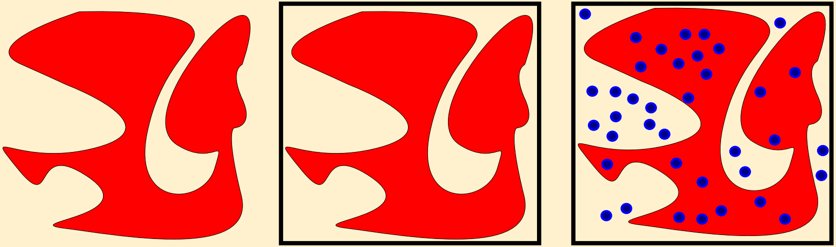
Equation of State Calculations by Fast Computing Machines

NICHOLAS METROPOLIS, ARIANNA W. ROSENBLUTH, MARSHALL N. ROSENBLUTH, AND AUGUSTA H. TELLER,
Los Alamos Scientific Laboratory, Los Alamos, New Mexico

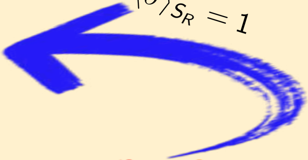
AND

EDWARD TELLER,* *Department of Physics, University of Chicago, Chicago, Illinois*
(Received March 6, 1953)

A general method, suitable for fast computing machines, for investigating such properties as equations of state for substances consisting of interacting individual molecules is described. The method consists of a modified Monte Carlo integration over configuration space. Results for the two dimensional rigid sphere system have been obtained on the Los Alamos MANIAC and are presented here. These results are compared to the free volume equation of state and to a four-term virial coefficient expansion.



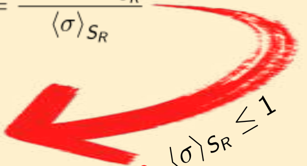
...but struggle with sign problems



$S_I = 0$, $\langle \sigma \rangle_{S_R} = 1$

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}\phi \; e^{-iS_I} \mathcal{O} e^{-S_R}}{\int \mathcal{D}\phi \; e^{-S_R}} \frac{\int \mathcal{D}\phi \; e^{-S_R}}{\int \mathcal{D}\phi \; e^{-S_R} e^{-iS_I}}$$

$$= \frac{\left\langle \mathcal{O} e^{-iS_I} \right\rangle_{S_R}}{\langle \sigma \rangle_{S_R}}$$



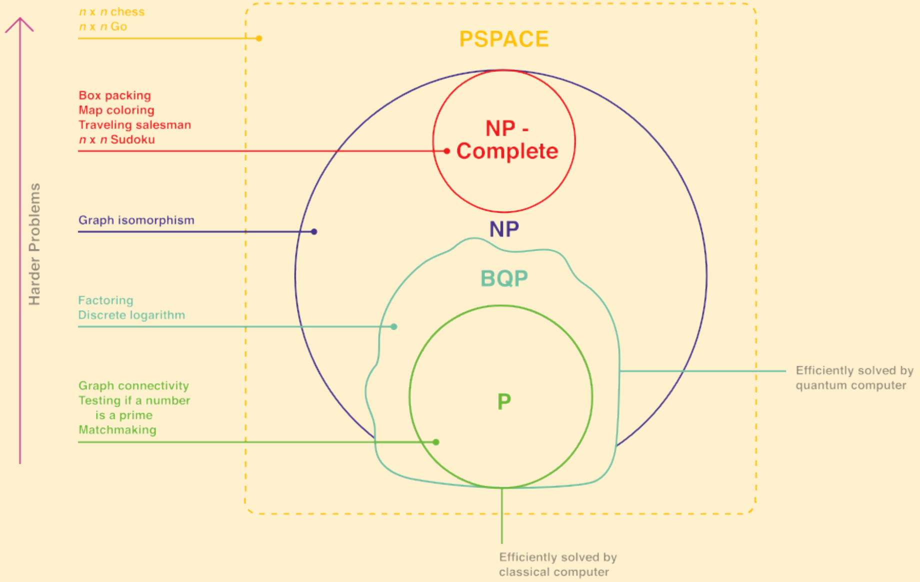
$S_I \neq 0$, $\langle \sigma \rangle_{S_R} \leq 1$

For **real-time** dynamics, $S_R = 0$!

Stated succinctly...

$|\psi\rangle$ is a **complex-valued** probability amplitude

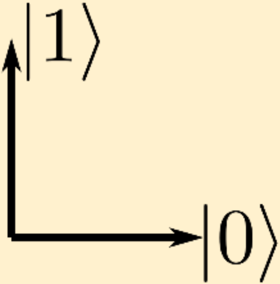
Fundamentally, physics needs quantum computers.

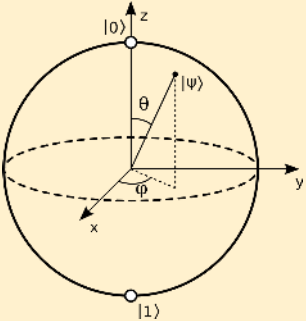


Credit: Scott Aaronson

What do I gain with a quantum computer?[2]

$$\langle\psi_f|U(t)|\psi_i\rangle = \langle\psi_f|e^{-iHt}|\psi_i\rangle = \int \mathcal{D}\phi e^{-S[\phi]}$$





QC can **efficently represent** superpositions and entanglement

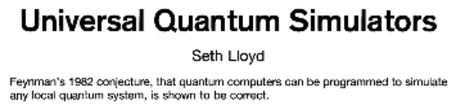
[2] Feynman, R. P. In: *Int.J.Theor.Phys.* 21 (1982).

What should physicists be aware of?

Solovay-Kitaev Theorem

Universal quantum computation can be performed provided one has a one-qubit gateset sufficiently dense in $SU(2)$ and a two-qubit gate.

Ex. of Universal Set: CNOT+H+S+T



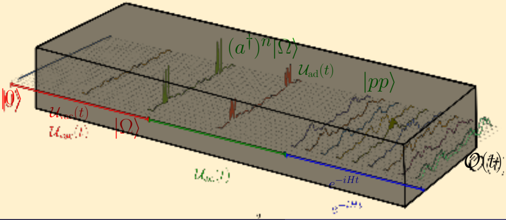
Universal Quantum Simulators

Seth Lloyd

Feynman's 1982 conjecture, that quantum computers can be programmed to simulate any local quantum system, is shown to be correct.

Efficient simulation for **local** Hamiltonians...

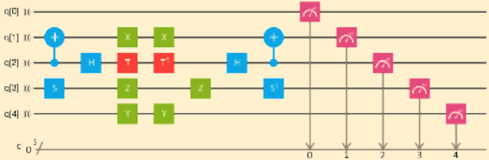
...but, its a **quantum** simulation!



Operator	Gate(s)		Matrix
Pauli-X (X)	X		$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Y (Y)	Y		$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli-Z (Z)	Z		$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard (H)	H		$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Phase (S, P)	S		$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
$\pi/8$ (T)	T		$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$
Controlled Not (CNOT, CX)			$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
Controlled Z (CZ)	Z		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
SWAP			$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Toffoli (CCNOT, CCX, TOFF)			$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

What should physicists really know?

Think **instructions=gates** and **bits=qubits**...
$+$**Noise** limits fideltiy of primative gates to 95 $-$ 99% today



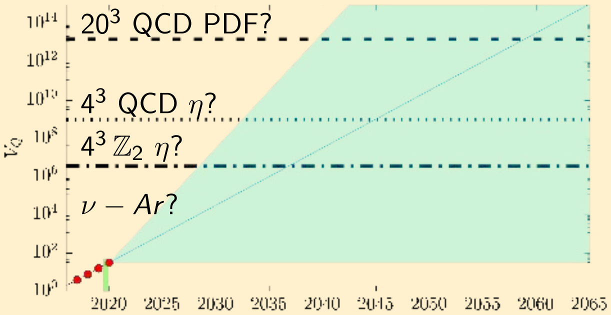
State of the art devices: $\mathcal{O}(10)$ **physical** qubits with $\mathcal{O}(10)$ gates

Error correction: Noiseless, **logical** qubits from set of physical ones[3].
Threshold theorems: QEC is possible if primitive gate errors are $\leq p_{\rm QEC}$

Laptop can do $\sim$ 17 **logical** qubits and $\mathcal{O}(10^3)$ gates in minutes

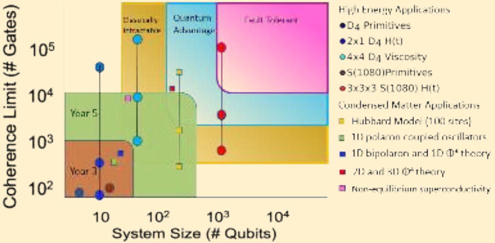
[3] Roffe, J. In: *Contemporary Physics* 60 (2019).

Balancin' on breaking branches



IBM & Honeywell QV

SQMS Center @ Fermilab

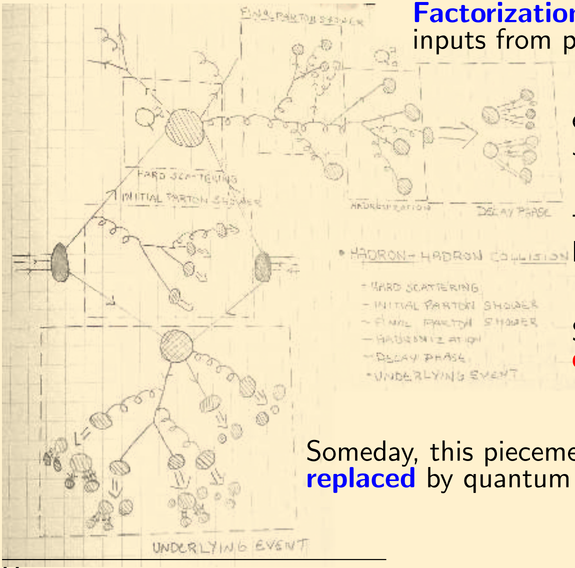


QCD η: $\mathcal{O}(10^5)$ qubits

$\mathcal{O}(10^{53})$ gates[4]

[4] Kan, A. and Y. Nam. In: (July 2021). arXiv: 2107.12769 [quant-ph].

Particle collisions require quantum advantage



Factorization separates non-perturbative inputs from perturbative calculations

e.g. PDF, TMD, Jet Functions[5][6]

This is an **assumption**! It gets $\mathcal{O}(\alpha)$ corrections

Showering based on **classical** models.

Someday, this piecemeal approach could be **replaced** by quantum simulation.

[5] Bauer, C. W., M. Freytsis, and B. Nachman. In: (Feb. 2021). arXiv: 2102.05044 [hep-ph].

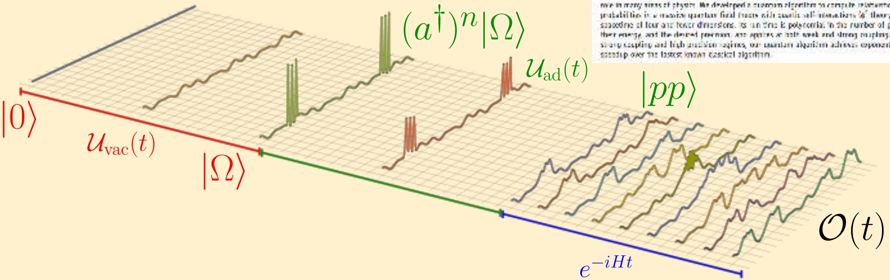
[6] Barata, J. a. and C. A. Salgado. In: (Apr. 2021). arXiv: 2104.04661 [hep-ph].

What might a galactic algorithm look like?


Quantum Algorithms for Quantum Field Theories

Stephen P. Jordan,[1*] Keith S. M. Lee,[2] John Preskill[1]

Quantum field theory reconciles quantum mechanics and special relativity, and plays a central role in many areas of physics. We developed a quantum algorithm to compute relativistic scattering probabilities in a massive quantum field theory with quartic self-interactions (ϕ^4 theory) in spacetime of four and fewer dimensions. Its run time is polynomial in the number of particles, their energy, and the desired precision, and applies at both weak and strong coupling. In the strong-coupling and high-precision regimes, our quantum algorithm achieves exponential speedup over the fastest known classical algorithm.



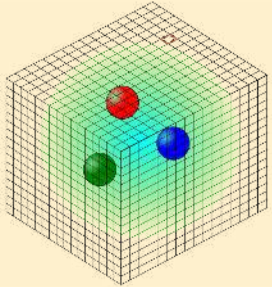
Vacuum Prep+**Adiabatic evolution**+**Trotterization**+Measurements[7]

Example: $|\langle p\bar{p}|U(t)|\pi\pi\pi\pi\rangle|^2$ needs $\mathcal{O}(10^7)$ **logical qubits**

$$\approx \left(\frac{3\text{ fm}}{0.05\text{ fm}}\right)^3 \times (3\text{ links} \times 11\text{ qubits} + 3\text{ colors} \times 2\text{ flavors} \times 2\text{ spins} \times 1\text{ qubit})$$

[7] Jordan, S. P., K. S. M. Lee, and J. Preskill. In: *Science* 336 (2012). arXiv: 1111.3633 [quant-ph].

What will it take for practical quantum advantage?



$$N_q \propto N_{dof} \left(\frac{L}{a}\right)^d \text{ \& } N_g \propto N_{\mathcal{U}} \left(\frac{T}{a_t}\right)$$

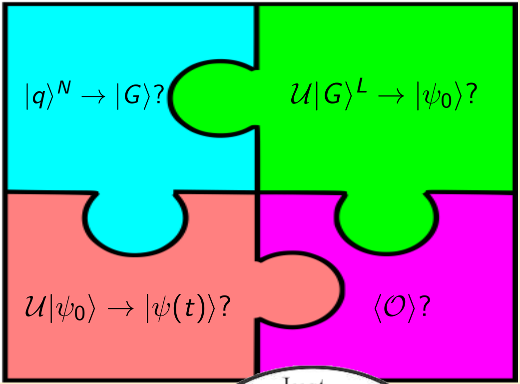
- **Hadron scattering**: $L, T = O(10)$ **fm**, $a, a_t = O(0.1)$ **fm**
- **PDFs**[8]: $L = O(1)$ **fm**, $T = O(10)$ fm, $a, a_t = O(0.1)$ fm
- **Transport coefficients**[9]: $L, T = O(1)$ **fm**, $a, a_t = O(1)$ **fm**

[8] **Lamm, H.**, S. Lawrence, and Y. Yamauchi. In: *Phys. Rev. Res.* 2 (2020). arXiv: 1908.10439 [hep-lat].

[9] Cohen, T. D., **H. Lamm**, S. Lawrence, and Y. Yamauchi. In: (Apr. 2021). arXiv: 2104.02024 [hep-lat].

What "champagne problems" need to be solved?

- **Digitize**: How are bosons represented as registers?
  - Discrete Subgroups[10]
- **Initalize**: How can registers be set to a state?
  - Stochastically?[11]
- **Propagate**: How can gates evolve states?[12]
- **Evaluate**: How can observables be computed?[13]



[10] Alexandru, A. et al. In: *Phys.Rev.D* 100 (2019). arXiv: 1906.11213 [hep-lat].
[11] Gustafson, E. J. and **H. Lamm**. In: *Phys. Rev. D* 103 (2021). arXiv: 2011.11677 [hep-lat].
[12] **Lamm, H.**, S. Lawrence, and Y. Yamauchi. In: *Phys. Rev.* D100 (2019). arXiv: 1903.08807 [hep-lat].
[13] **Lamm, H.**, S. Lawrence, and Y. Yamauchi. In: *Phys. Rev. Res.* 2 (2020). arXiv: 1908.10439 [hep-lat].

So ahead of the curve, the curve becomes a sphere

(1970s) Formulate th…

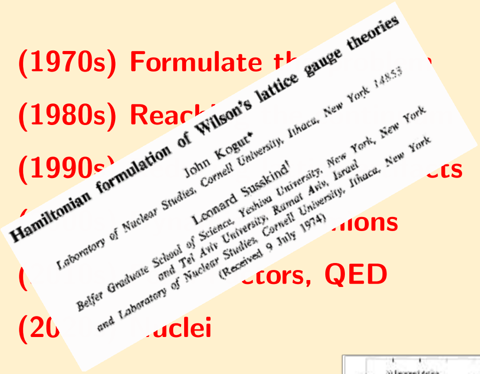
(1980s) Reac…

(1990s) … …cts

… …ions

(… …ctors, QED

(20… …uclei



Hamiltonian formulation of Wilson's lattice gauge theories

John Kogut*

Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

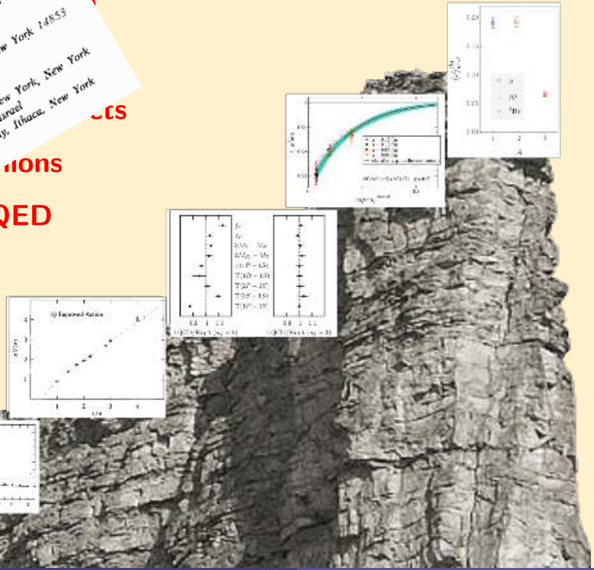
Leonard Susskind†

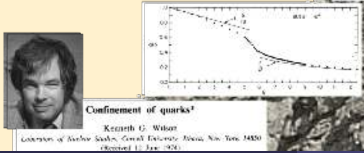
Belfer Graduate School of Science, Yeshiva University, New York, New York

and Tel Aviv University, Ramat Aviv, Israel

and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York

(Received 9 July 1974)





Confinement of quarks*

Kenneth G. Wilson

How do I digitize a gluon?

All things considered...

Exploring Digitizations of Quantum Fields for Quantum Devices

Erik Gustafson,[1] Hiroki Kawai,[2,*] Henry Lamm,[3,†] Indrakshi Raychowdhury,[4,‡] Hersh Singh,[5,6,§] Jesse Stryker,[4,6,¶] and Judah Unmuth-Yockey[7]

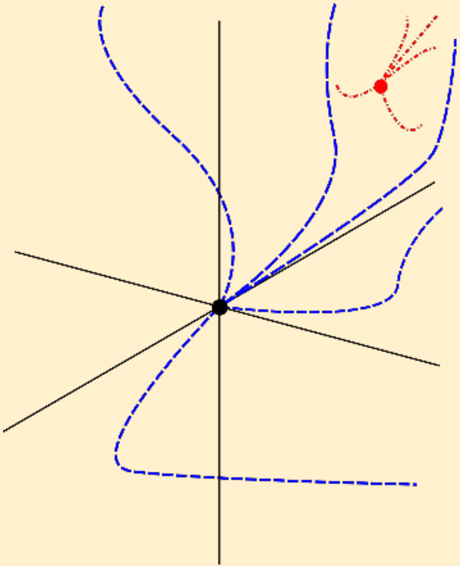
[1]University of Iowa, Iowa City, Iowa, 52242[**]
[2]Department of Physics, Boston University, 590 Commonwealth Avenue, Boston, MA 02215, USA
[3]Fermi National Accelerator Laboratory, Batavia, Illinois, 60510, USA
[4]Maryland Center for Fundamental Physics and Department of Physics, University of Maryland, College Park, MD 20742, USA
[5]Department of Physics, Box 90305, Duke University, Durham, North Carolina 27708, USA
[6]Institute for Nuclear Theory, University of Washington, Seattle, WA 98195, USA
[7]Syracuse University, Syracuse NY[††]

In this LOI we undertake to enumerate promising digitization schemes for quantum fields that could allow near-term calculations on quantum devices. Further we discuss the outstanding questions that must be resolved in evaluating their potential, providing potential benchmarking on the way to practical quantum advantage in high energy physics.

What qualities make a GOOD scheme?

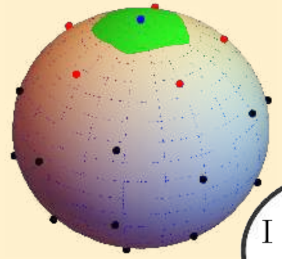
- What **quantum resources** are required to get physical point?
- What symmetries are being **broken** in digitization?
- Can the scheme be simulated **classically**?

This is not a triviality!



Discrete subgroups allow plug-and-play[14][15][16]

Replace $G \to H$ in $e^{-S}, e^{-i\mathcal{H}}$

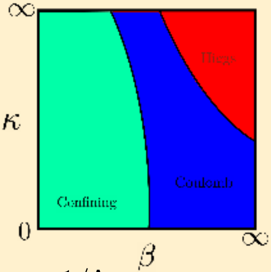


- $SU(3) \to \mathbb{V}$ reduces qubits by $O(10^2)$
- I **believe** endgame will be **3x3** matrices

[14] Bhanot, G. In: *Phys. Lett.* 108B (1982), Hackett, D. C. et al. In: *Phys. Rev.* A99 (2019).
[15] Bender, J., E. Zohar, A. Farace, and J. I. Cirac. In: *New J. Phys.* 20 (2018). arXiv: 1804.02082 [quant-ph]
[16] Haase, J. F. et al. In: (June 2020). arXiv: 2006.14160 [quant-ph].

Discrete groups are continuous groups+Higgs[18][19]

- Starting from G **coupled to ϕ**
- The rep of ϕ **determines** the breaking $G \to H$
- **Higher** rep (larger H) $\to$ **larger** Λ_{SB} $\to$ **smaller** $a > 1/\Lambda_{SB}$
- On-going work to understand how Higgs couples to **Nonabelian** G[17]

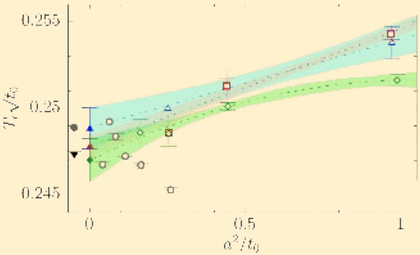


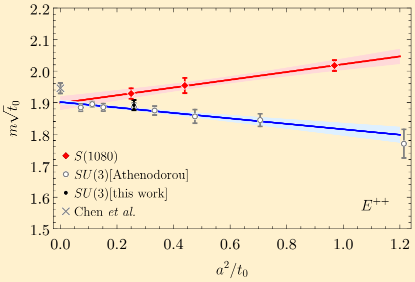
[17] Das, S. and A. Hook. In: *JHEP* 10 (2020). arXiv: 2006.10767 [hep-ph].
[18] Fradkin, E. H. and S. H. Shenker. In: *Phys. Rev. D* 19 (1979).
[19] Labastida, J. M. F., E. Sanchez-Velasco, R. E. Shrock, and P. Wills. In: *Phys. Rev. D* 34 (1986).

$T_c\sqrt{t_0}$ suggests $a \approx 0.07$ fm ≈ 2 GeV^{-1} possible[20][21]

$$S = \sum \frac{\beta_0}{3} \mathrm{Re}\,\mathrm{Tr}\, U + \beta_1 f(U) \text{ with } f(U) = \{\mathrm{Tr}^2\, U + \mathrm{Tr}\, U^2, |TrU|^2\}$$

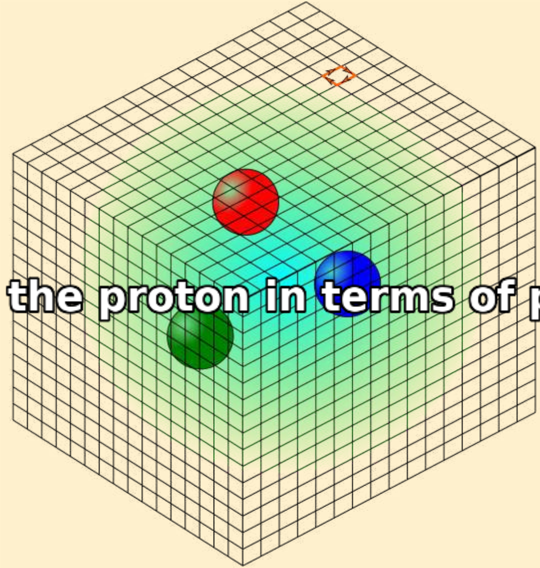
Compare to SU(3)





[20] Alexandru, A. et al. In: *Phys.Rev.D* 100 (2019). arXiv: 1906.11213 [hep-lat].

[21] Alexandru, A., P. F. Bedaque, R. Brett, and **H. Lamm**. In: (Dec. 2021). arXiv: 2112.08482 [hep-lat].



What is the proton in terms of partons?

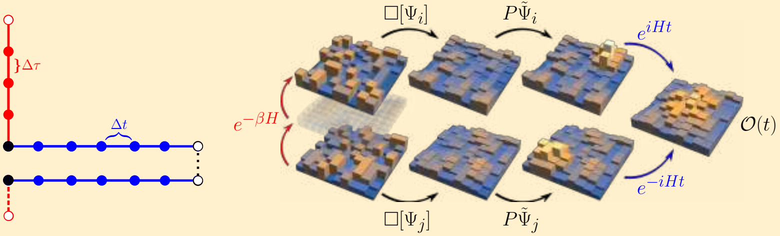
How can we prepare $|\psi\rangle$?

- Adiabatic evolution
- Quantum Heat Pumps
- VQE
- Classical Stochastic Methods
- Quantum Stochastic Methods
- ...

$E\rho OQ$: A hybrid technique for real-time LQCD[22]

Preping $|\psi_0\rangle$ is expensive! Use classical resources via Schwinger-Keldysh

$$\langle \mathcal{O}(t) \rangle = \frac{\mathrm{Tr}\, P^\dagger e^{-\beta H} P e^{-iHt} \mathcal{O} e^{iHt}}{\mathrm{Tr}\, e^{-\beta H}}$$



- **Classical: open-BC LQCD yields $(e^{-\beta H})_{ij}$ then project with P**
- **Quantum: Time-evolve elements of simpler $(P^\dagger e^{-\beta H} P)_{ij}$**

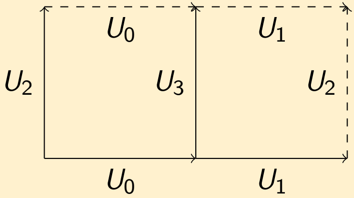
[22] Harmalkar, S., **H. Lamm**, and S. Lawrence. In: (). arXiv: 2001.11490 [hep-lat], Gustafson, E. J. and **H. Lamm**. In: *Phys. Rev. D* 103 (2021). arXiv: 2011.11677 [hep-lat].

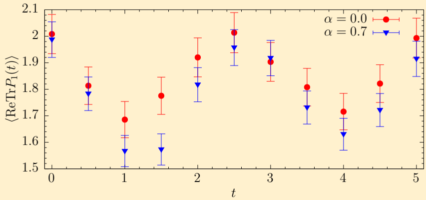
Simulator Results for 2+1D D_4 & $\mathbb{Z}_2$ gauge theories

14q D_4 gauge theory (NuQS)

✓ **Nonabelian** group with $N = 8$

✓ **Thermal** $|\psi\rangle$ with **Smearing**

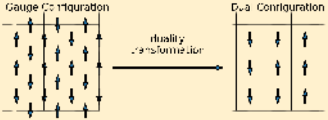


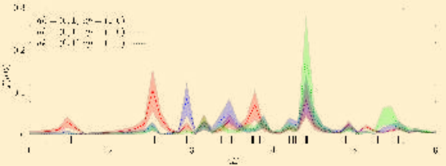


18q $\mathbb{Z}_2$ gauge theory (w/ Gustafson)

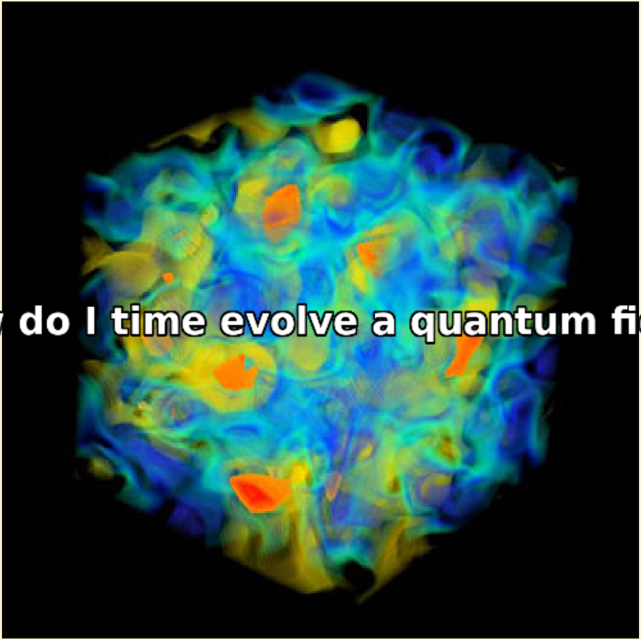
✓ **Abelian** group with $N = 2$

✓ ***n*-particle** $|\psi\rangle$





Proposals for these on **Rigetti & Google** on $\lesssim$ 5 year scale



How do I time evolve a quantum field?

How can we approximate $\mathcal{U}(t)$?

- Trotterization
- QDRIFT[23]
- Variational approaches[24]
- Taylor series[25]
- Qubitization[26]

[23] Campbell, E. In: *Phys. Rev. Lett.* 123 (7 2019).

[24] Cirstoiu, C. et al. In: *npj Quantum Information* 6 (2020), Gibbs, J. et al. *Long-time simulations with high fidelity on quantum hardware*. 2021. arXiv: 2102.04313 [quant-ph], Yao, Y.-X. et al. In: *arXiv preprint arXiv:2011.00622* (2020).

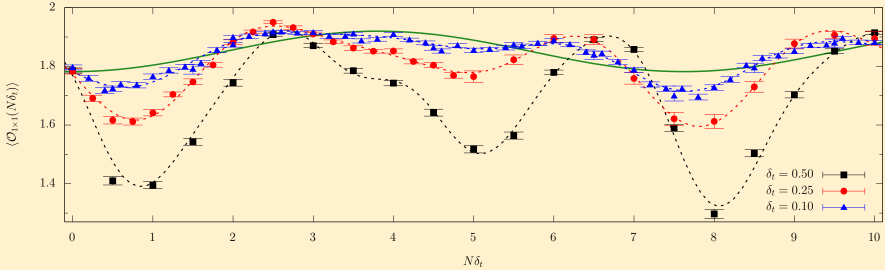
[25] Berry, D. W., A. M. Childs, R. Cleve, R. Kothari, and R. D. Somma. In: *Phys. Rev. Lett.* 114 (9 2015).

[26] Low, G. H. and I. L. Chuang. In: *Quantum* 3 (July 2019).

What is trotterization?

$$\mathcal{U}(t) = e^{-iHt} \approx \left(e^{-i\delta t\frac{H_V}{2}} e^{-i\delta t H_K} e^{-i\delta t\frac{H_V}{2}}\right)^{\frac{t}{\delta t}}$$

$$\approx \exp\left\{-it\left(H_K + H_V + \frac{\delta t^2}{24}(2[H_K,[H_K,H_V]] - [H_V,[H_V,H_K]])\right)\right\}$$

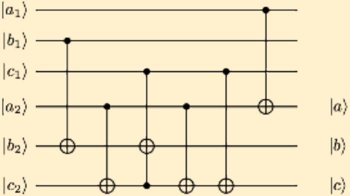


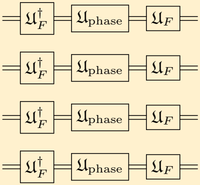
- Introduces **higher dimension operators**

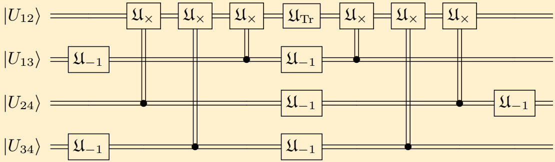
What low-level primatives are required for LGT?[27]

How do we build $U_K = e^{iH_K}$ **and** $U_V = e^{iH_V}$?

- Inversion gate: $\mathfrak{U}_{-1}\left|g\right\rangle = \left|g^{-1}\right\rangle$
- Multiplication gate: $\mathfrak{U}_{\times}\left|g\right\rangle\left|h\right\rangle = \left|g\right\rangle\left|gh\right\rangle$
- Trace gate $\mathfrak{U}_{\mathrm{Tr}}(\theta)\left|g\right\rangle = e^{i\theta\,\mathrm{Re\,Tr}\,g}\left|g\right\rangle$
- Fourier Transform gate: $\mathfrak{U}_F \sum_{g\in G} f(g)\left|g\right\rangle = \sum_{\rho\in\hat{G}} \hat{f}(\rho)_{ij}\left|\rho, i, j\right\rangle$

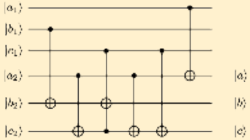


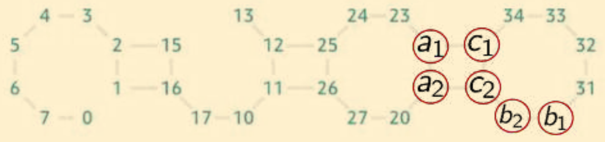




[27] **Lamm, H.**, S. Lawrence, and Y. Yamauchi. In: *Phys. Rev.* D100 (2019). arXiv: 1903.08807 [hep-lat].

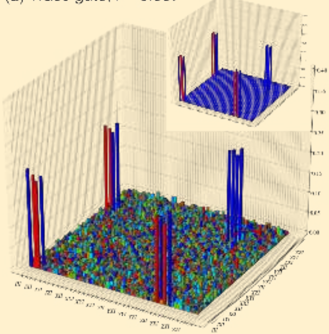
Primative gates on Rigetti[28]



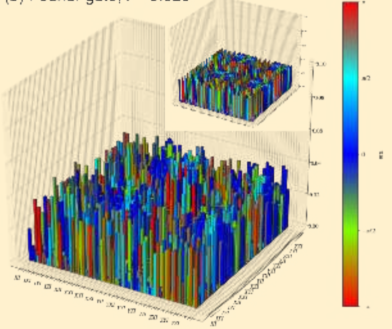


Primative gates for D_4 have $\geq 80\%$ fidelity

(a) Trace gate, $f = 0.857$

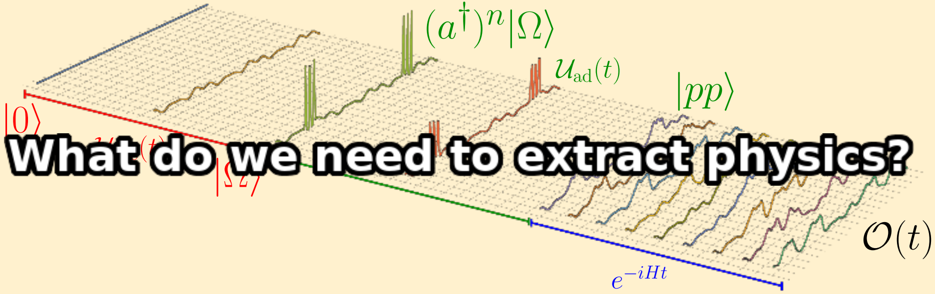


(b) Fourier gate, $f = 0.920$



[28] Alam, M. S., S. Hadfield, **H. Lamm**, and A. C. Y. Li. In: (Aug. 2021). arXiv: 2108.13305 [quant-ph].

What do we need to extract physics?



How do I compute $\langle\Psi|\prod_n \mathcal{O}(t_n)|\Psi\rangle$?[29]

Want to measure $\langle O(t)\rangle$? **Measure** the qubits, or **phase estimation**

Acting on a quantum state $|\Psi\rangle$ with the first Hermitian $\mathcal{O}(t_0)$ leads to...

WAVEFUNCTION
COLLAPSE

So what is to be done?

Perturb $H \to H + \epsilon\mathcal{O}\delta(t)$, and take derivatives:

$$\langle\Psi|\mathcal{O}(t)\mathcal{O}(0)|\Psi\rangle = \frac{\partial}{\partial\epsilon_t}\frac{\partial}{\partial\epsilon_0}\langle\Psi|e^{-iHt}e^{-i\mathcal{O}\epsilon_t}e^{iHt}e^{i\mathcal{O}\epsilon_0}|\Psi\rangle$$

[29] Pedernales, J. S., R. Di Candia, I. L. Egusquiza, J. Casanova, and E. Solano. In: *Phys. Rev. Lett.* 113 (2 2014).

Deriving Lattice Hamiltonian Operators[30]

$$\eta = \frac{V}{T}\int_0^\infty \mathrm{d}t \langle T_{12}(t)\, T_{12}(0)\rangle$$

We construct a lattice Hamiltonian version of $T_{\mu\nu}$ that depends on $F_{\mu\nu}$

TABLE I. Gauge-invariant lattice operators in the Hamiltonian formalism in $3+1d$ dimensions: naive operators with $O(a)$ errors and improved operators with errors that are $O(a^2)$. Components of the energy-momentum tensor $T_{\mu\nu}$ are constructed as linear combinations of these operators according to Eq. (8). The plaquette $\hat{P}$ and clover $\hat{C}$ are defined in Eq. (10) and Eq. (15), respectively. Spatial indices are $i \neq j \neq k$.

Operator	$O(a)$	$O(a^2)$
$\mathrm{Tr} F_{0i}F_{0i}(n)$	$\frac{g_s^2}{a^4}\mathrm{Tr}\left[\pi^2_{n,i}\right]$	$\sum_{x=0,1}\frac{g_s^2}{2a^4}\mathrm{Tr}\left[\pi^2_{n-x\hat{i},i}\right]$
$\mathrm{Tr} F_{0i}F_{0j}(n)$	$\frac{g_s^2}{a^4}\mathrm{Tr}\left[\pi_{n,i}\pi_{n,j}\right]$	$\frac{g_s^2}{4a^4}\Big(\mathrm{Tr}\left[\hat{\pi}_{n,i}\hat{\pi}_{n,j}\right]+\mathrm{Tr}\left[\hat{\pi}_{n,i}\hat{U}^\dagger_{n-\hat{j},j}\hat{\pi}_{n-\hat{j},j}\hat{U}_{n-\hat{j},j}\right]+\mathrm{Tr}\left[\hat{U}^\dagger_{n-\hat{i},i}\hat{\pi}_{n-\hat{i}}\hat{U}_{n-\hat{i},i}\hat{\pi}_{n,j}\right]$ $+\mathrm{Tr}\left[\hat{U}^\dagger_{n-\hat{i},i}\hat{\pi}_{n-\hat{i},i}\hat{U}_{n-\hat{i},i}\hat{U}^\dagger_{n-\hat{j},j}\hat{\pi}_{n-\hat{j},j}\hat{U}_{n-\hat{j},j}\right]\Big)$
$\mathrm{Tr} F_{0j}F_{ij}(n)$	$-\frac{1}{a^4}\mathrm{Tr}\left[\hat{\pi}_{n,j}\mathrm{Im}\hat{P}_{ij}(n)\right]$	$-\frac{1}{2a^4}\left(\mathrm{Tr}\left[\hat{\pi}_{n,j}\mathrm{Im}\hat{C}_{ij}(n)\right]+\mathrm{Tr}\left[\hat{U}^\dagger_{n-\hat{j},j}\hat{\pi}_{n-\hat{j},j}\hat{U}_{n-\hat{j},j}\mathrm{Im}\hat{C}_{ij}(n)\right]\right)$
$\mathrm{Tr} F_{ij}F_{ij}(n)$	$\frac{2}{g_s^2 a^4}\mathrm{Re}\mathrm{Tr}\left[1-\hat{P}_{ij}(n)\right]$	$\sum_{x=0,1}\sum_{y=0,1}\frac{1}{2g_s^2a^4}\mathrm{Re}\mathrm{Tr}\left[1-\hat{P}_{ij}(n-x\hat{i}-y\hat{j})\right]$
$\mathrm{Tr} F_{ij}F_{kj}(n)$	$\mathrm{Tr}[\hat{F}^N_{ij}(n)\hat{F}^N_{kj}(n)]$	$\mathrm{Tr}[\hat{F}^C_{ij}(n)\hat{F}^C_{kj}(n)]$

[30] Cohen, T. D., **H. Lamm**, S. Lawrence, and Y. Yamauchi. In: (Apr. 2021). arXiv: 2104.02024 [hep-lat].

It’s time to go

So many things to do!...and lots can be done before the machine exists

- Digitizing SU(3)
    - **Spectroscopy** for $\mathbb{V}$
    - $\mathbb{V}$ **circuits**
- Reducing the errors
    - e.g. Finite volume, finite a, a_t, decimation errors, fidelity to obtain **realisitic** resource estimates
- Algorithms for **state prep, smearing**
- Investigate desirable properties
    - **PDF?, Viscosity?, Cosmology?**
- **Actual** simulations of toy models
    - $\mathbb{Z}_2$ & $\mathbb{D}_4$