The Quantumness of hadronisation

Pol B Gossiaux, SUBATECH (NANTES)

YOUNGST@RS (and old planets) - The Quantumness of Hard Probes

1. **Background and Motivation**
2. **Results with the Schroedinger-Langevin Approach**
3. **One specific Open Quantum System Scheme (Blaizot-Escobedo)**

With Joerg Aichelin, Denys Yen Arrebato Villar, Aoumeur Daddi Hammou, Stéphane Delorme, Thierry Gousset & Roland Katz

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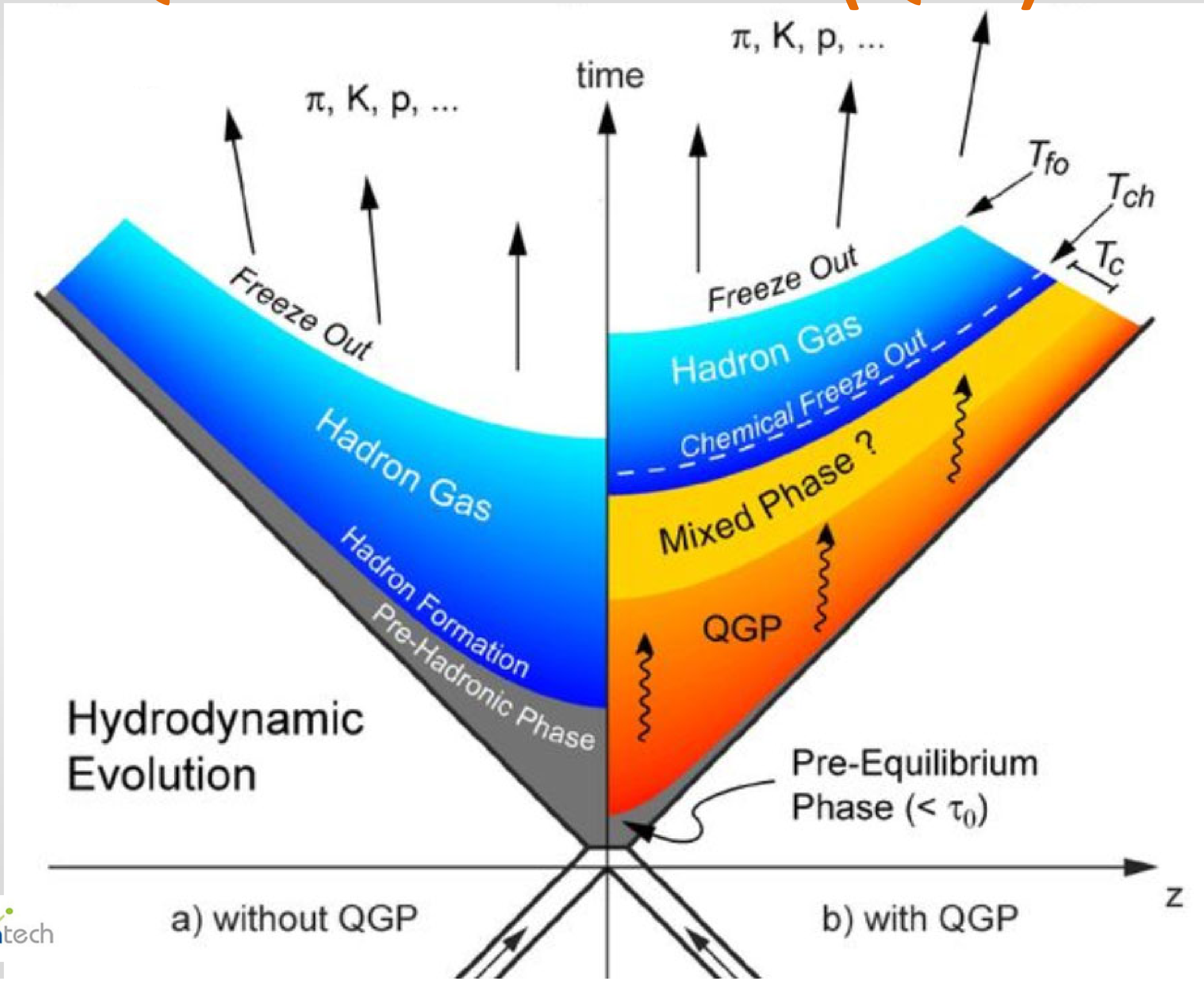
STRONG 2020

The Quark Gluon Plasma (QGP) in AA



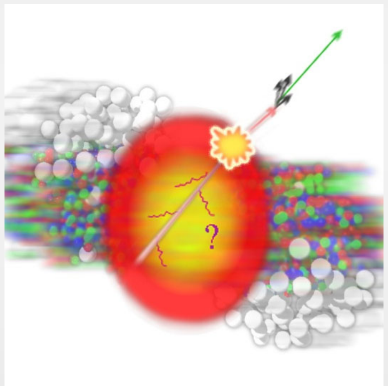
« Hard » probes

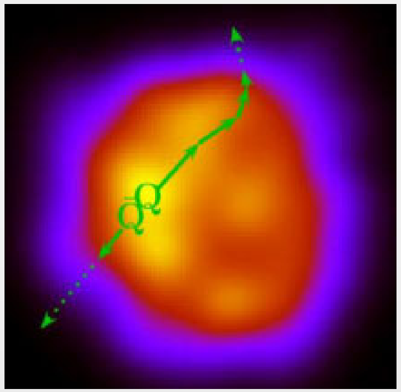
To study the medium properties before the freeze out «horizon»…

Deconfined ? Density and T ? Transport properties ? …

… one can analyse the « tomography » of the medium as seen by the hard probes (⇔ incomplete thermalisation)

High p_T partons quenching





Massive quarks diffusion

Why hard probes ?

- ✓ **Produced only in early pQCD processes before the QGP medium**
- ✓ Do not flow hydrodynamically but propagate/interact inside the medium via other processes sensitive to its properties
- ✓ Less sensitive to hadronic stages

Questions

Are charmonia genuine hard probes of the QGP ? At initial time, what is produced are c-cbar pairs

⇕

When are they formed / observed ? At freeze out only ?

⇕

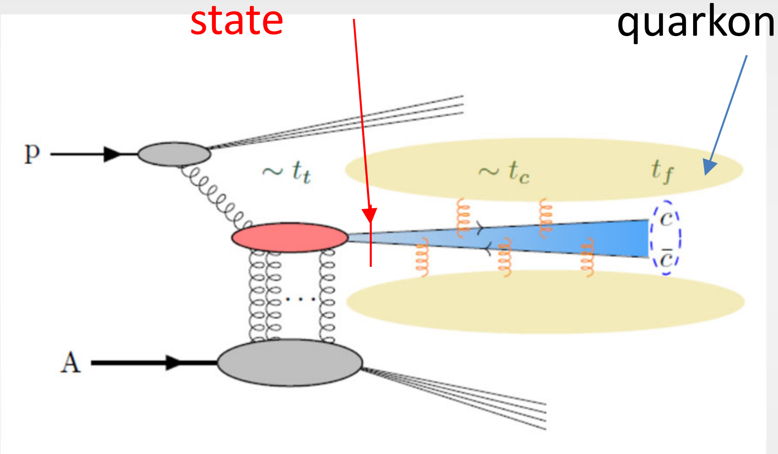
How much of Quantum Mechanics do we need to understand their formation ?

Quarkonia suppression

Suppression = less than expected in experimental data …. Not « formed and then destroyed »

Coherent quantum state

Final quarkonia



Evolution operator between both… can of course be evaluated in any eigenstate basis…

$$p_{\Phi_n}(t) = |\langle \Phi_n | U(t,0) | Q\bar{Q}(t=0)\rangle|^2$$

questionnable

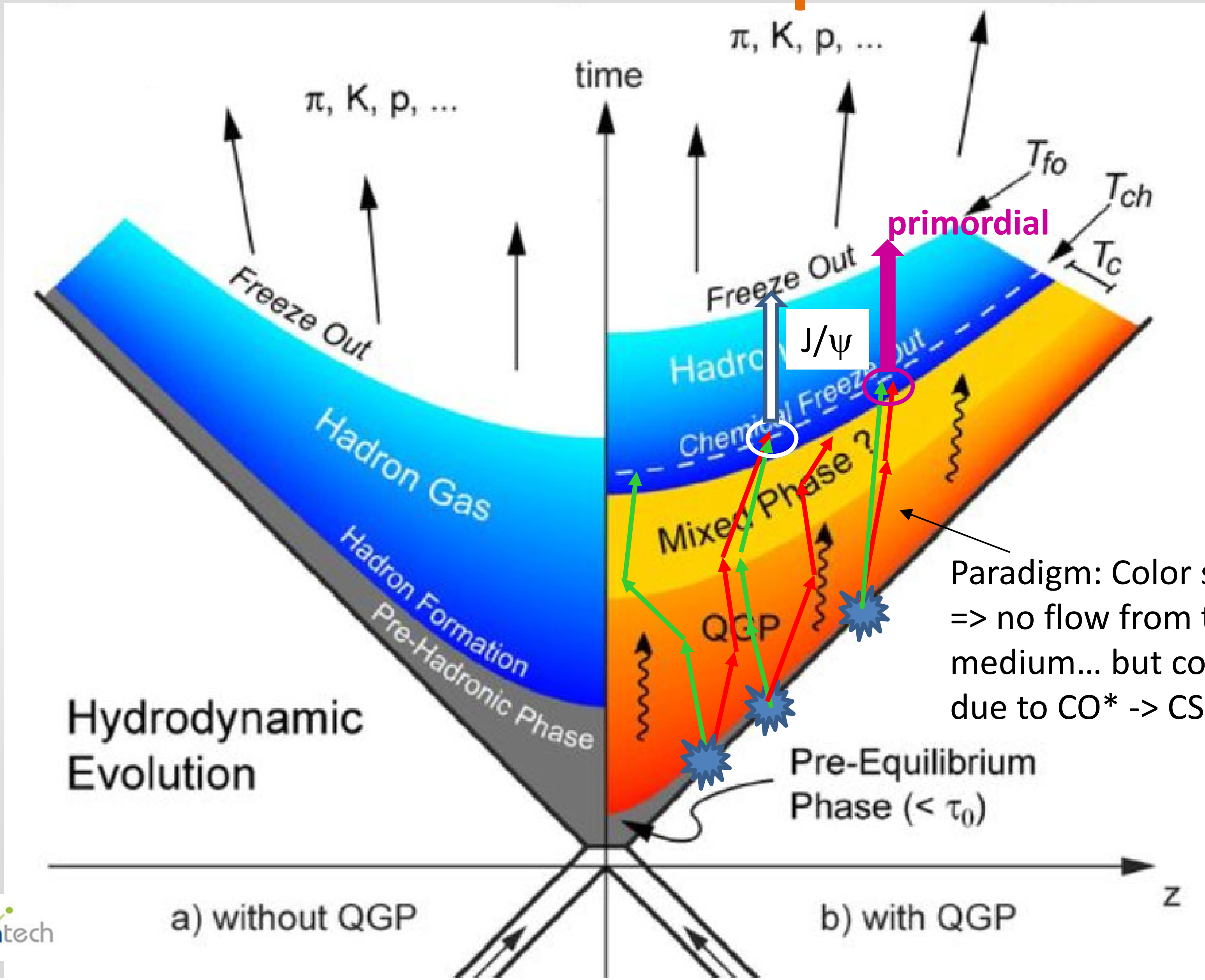
$$\frac{dp_n(t)}{dt} = \sum_i^{+\infty} w_{ni} p_i(t) \quad \text{With} \quad p_i(0)$$

Even more questionnable

$$\frac{dp_n(t)}{dt} = -\Gamma_n p_n(t)$$

… but maybe if we want to talk about «early production », we could be closer to a faithful description by taking eigenstates of the the Debye-like screened potential : « local basis »

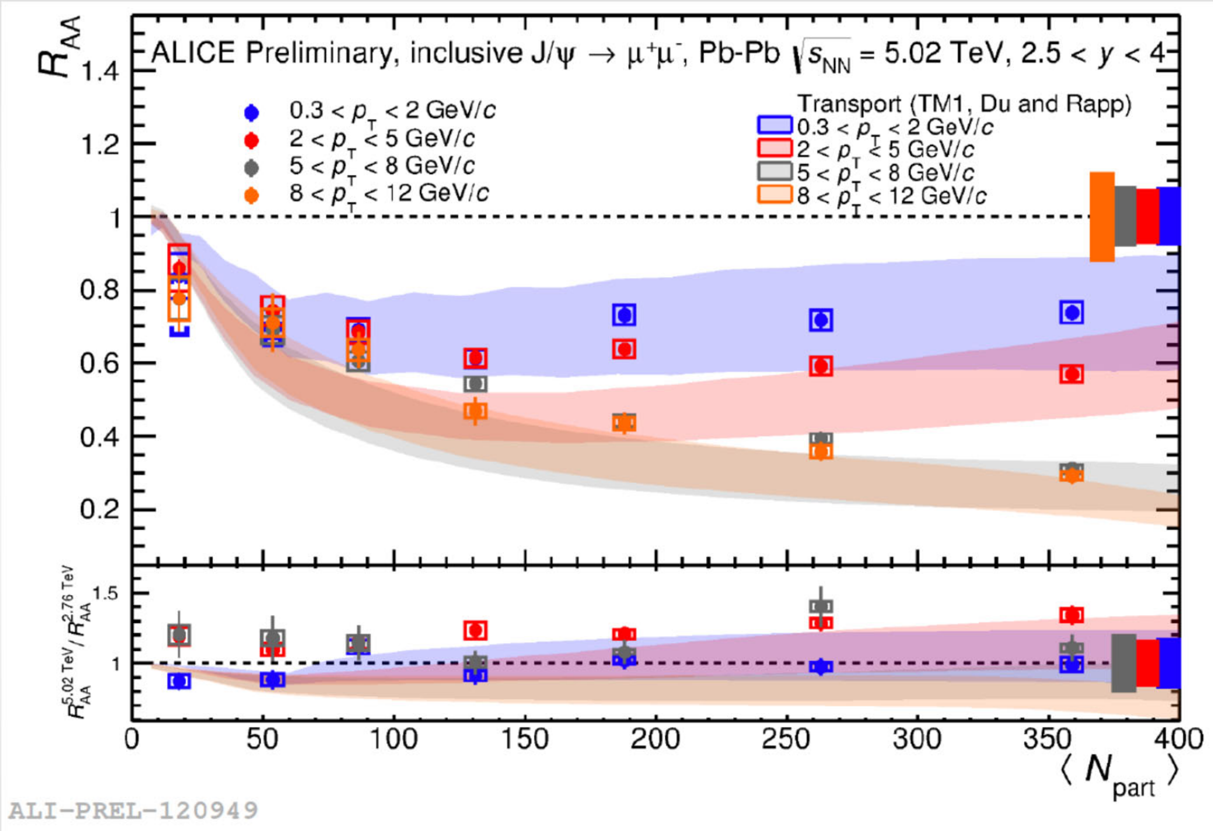
Charmonia in the transport models



Paradigm: Color singlet => no flow from the medium… but could be due to CO* -> CS

Looking at recent data

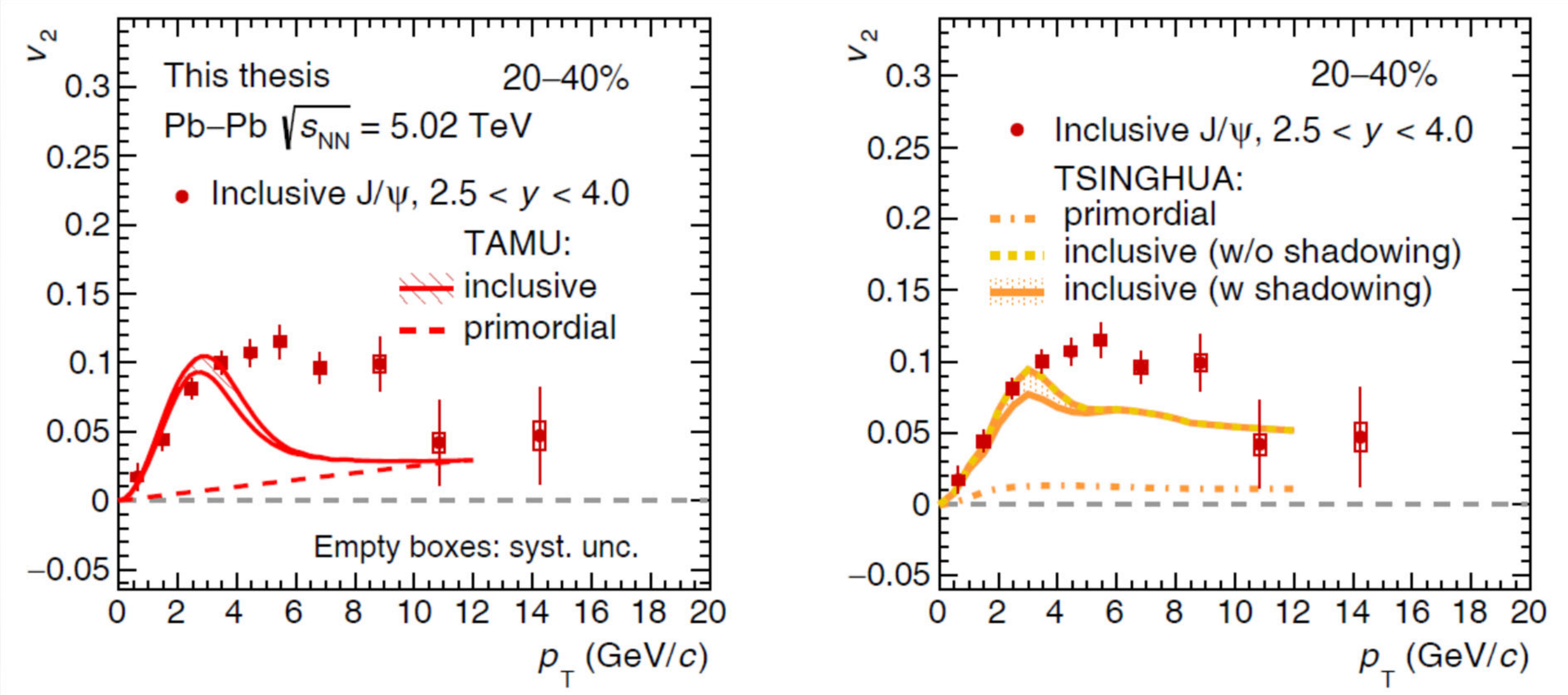
Transport theories



- In transport theory, primordial component is mandatory to reproduce the absolute production as a function of centrality & p_T class

Looking at recent data

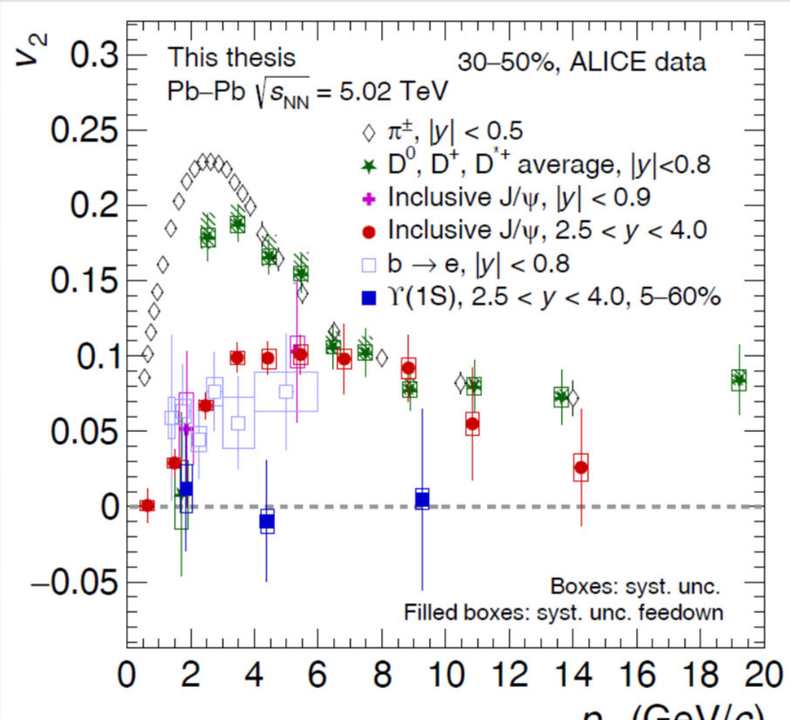
Transport theories



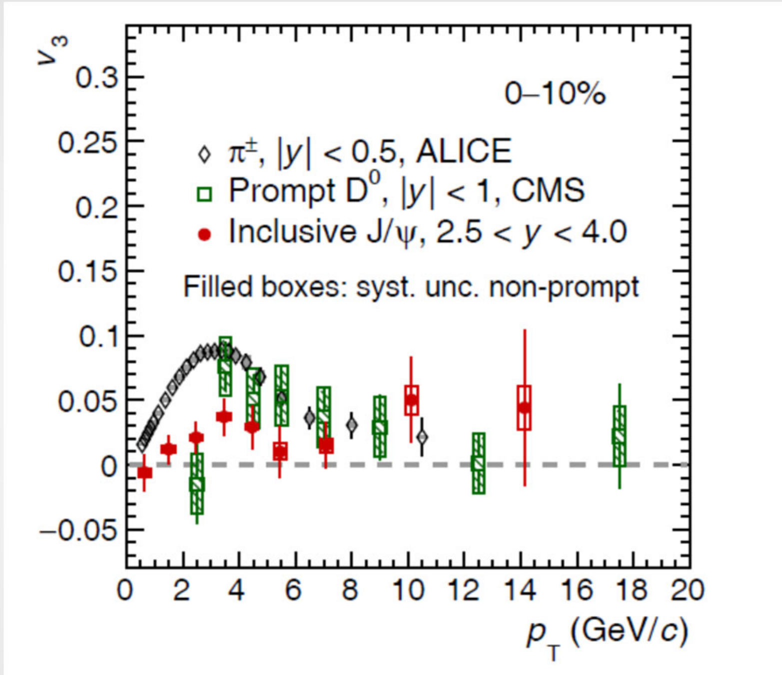
- Good agreement for low p_T, where J/ψ formation proceeds through recombination at FO
- Disagreement from intermediate p_T on, where primordial production start having a large weight (crucial for the $R_{AA}(p_T)$)

Looking at recent data

Recently : More global view

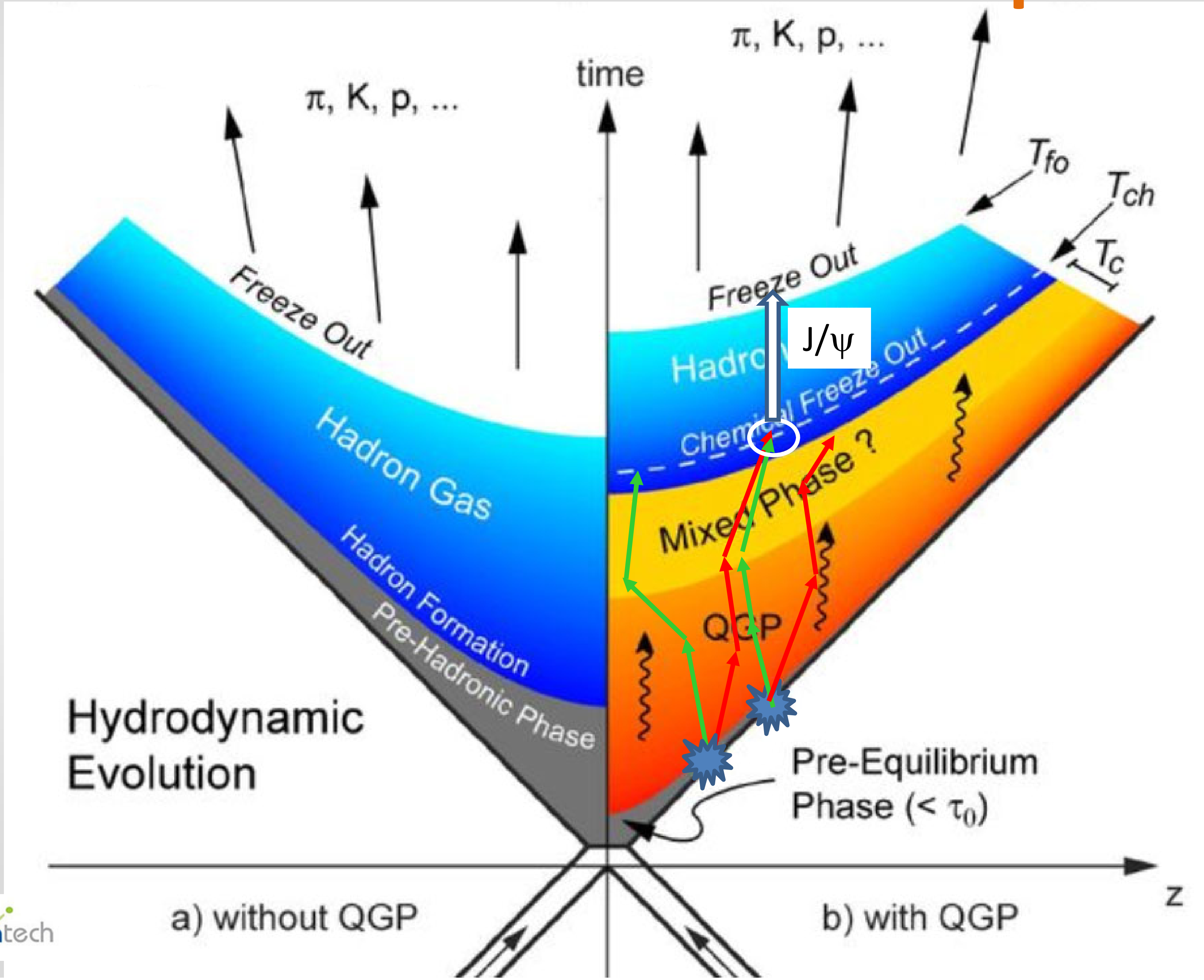


Thesis R. Caron (2021)



- v2 **and v3** analysis confirm that J/ψ flows
- Flow compatible with 0 for the upsilon 1S

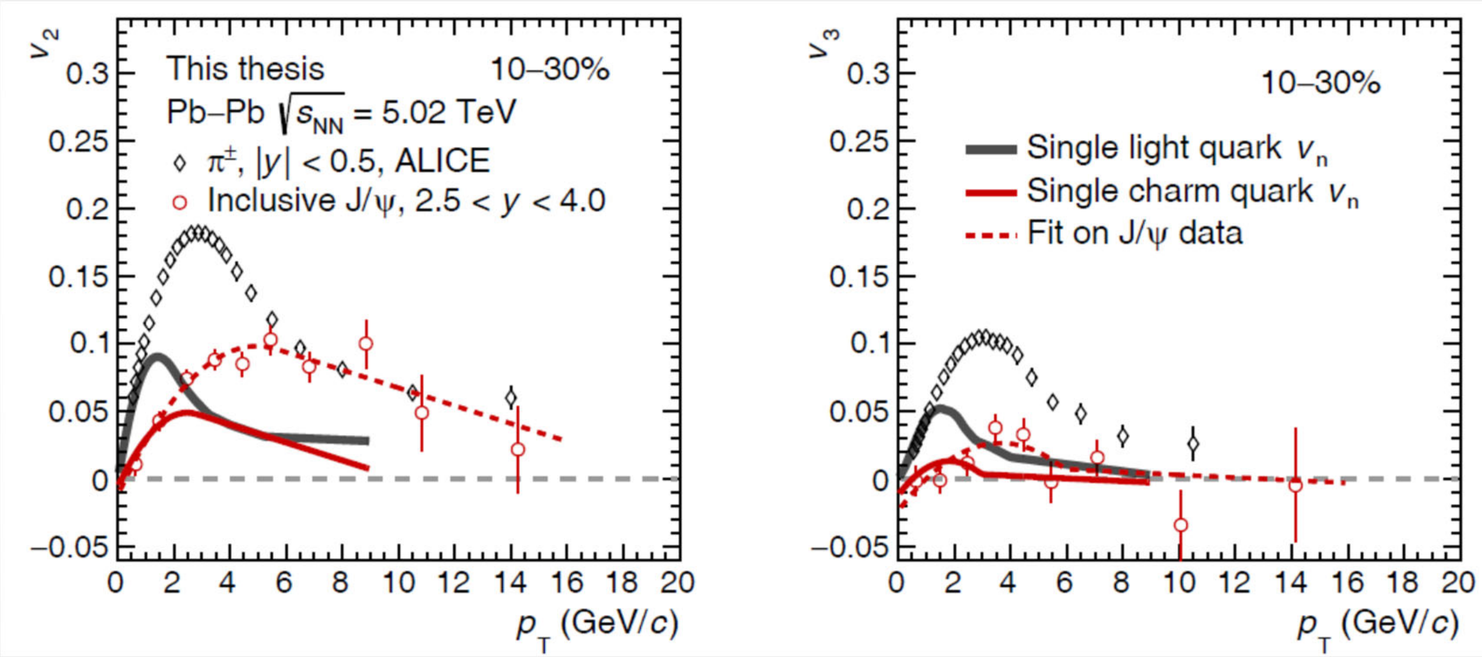
Charmonia in the coalescence picture



Looking at recent data

Coalescence explains it all ?

- v_2 & $v_3(\pi)$ => v_2 & $v_3(q)$ (reverse engineering)
- v_2 & $v_3(J/\psi$ fit) => v_2 & $v_3(c)$ (reverse engineering)

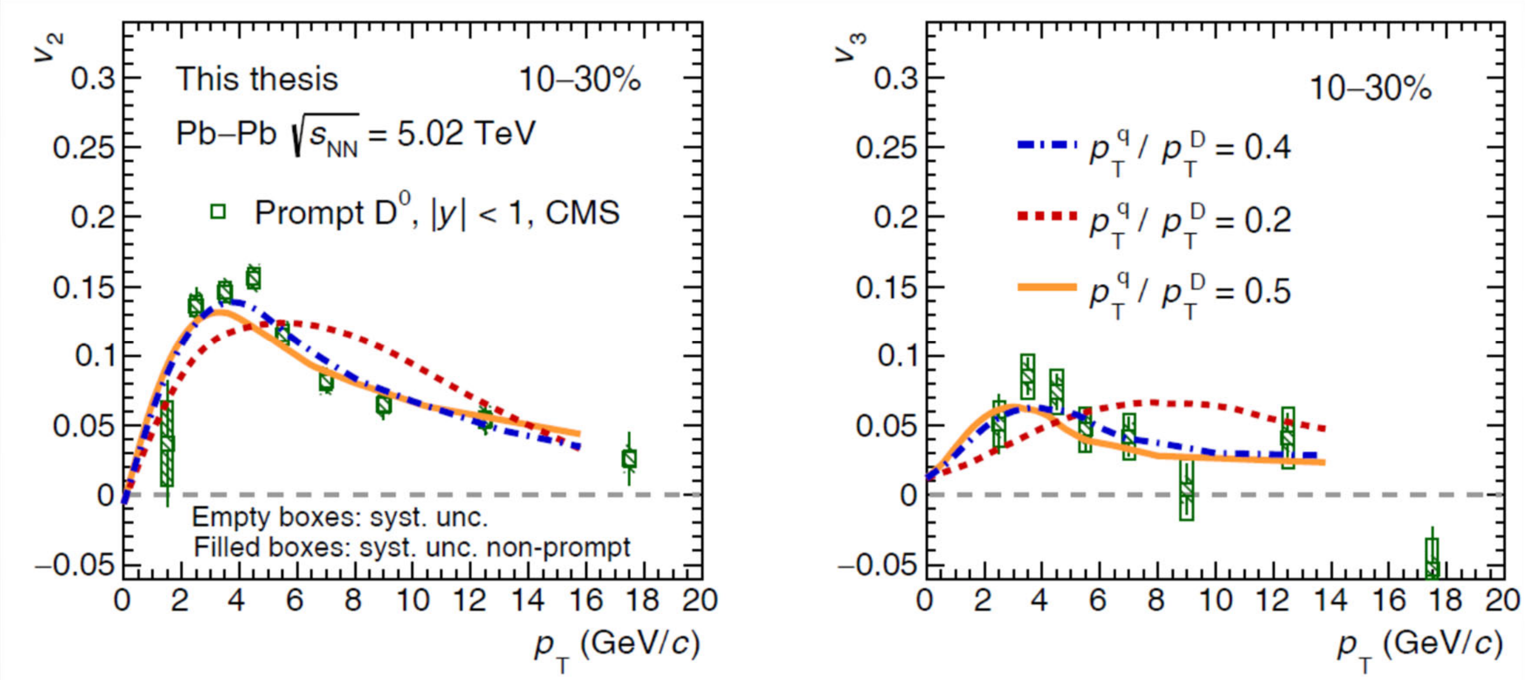


Shreyasi Acharya et al. (ALICE) JHEP, 10:141, 2020.

Looking at recent data

Coalescence explains it all ?

- $v_n(q)$ & $v_n(c)$ + relative weights of masses (momenta) => $v_n(D)$



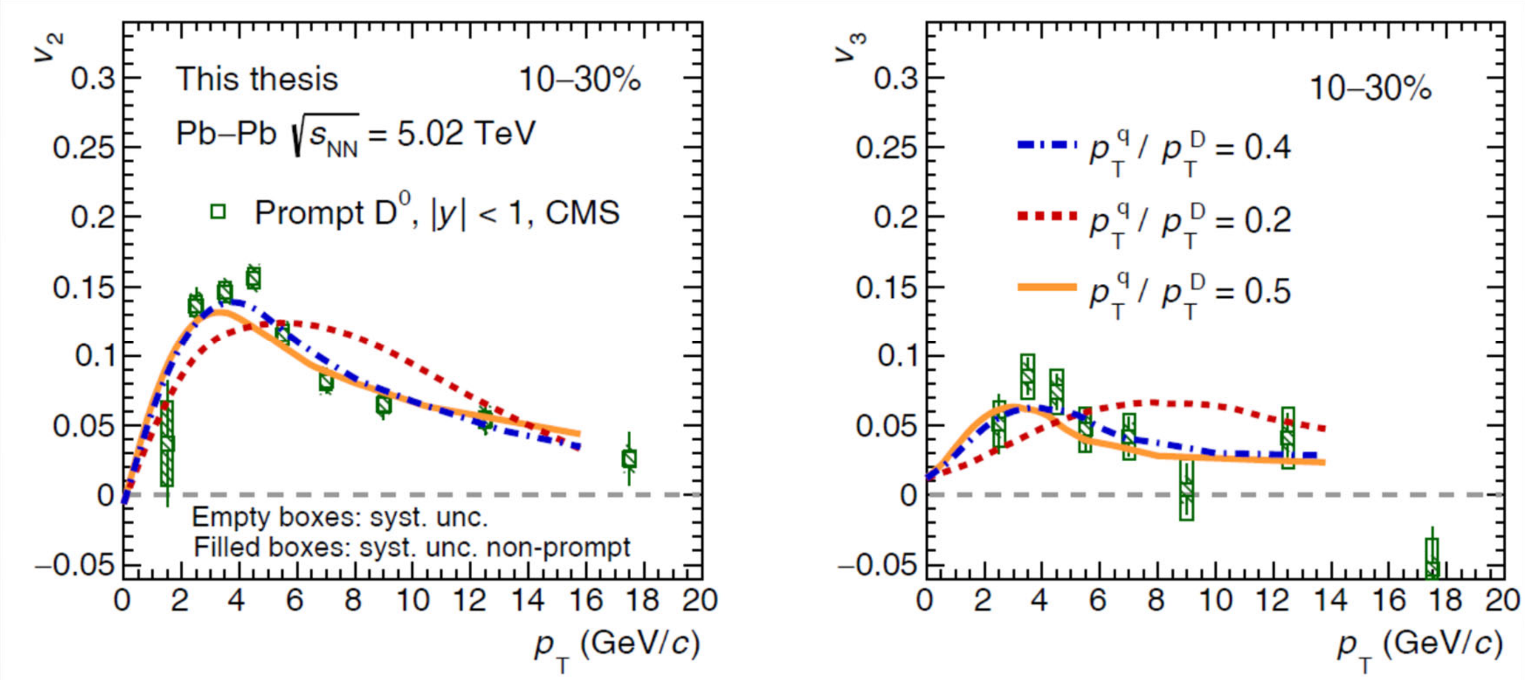
Shreyasi Acharya et al. (ALICE) JHEP, 10:141, 2020.

- Good global agreement for $p_T^q/p_T^D = 0.4 \Leftrightarrow m_q \approx 0.7 - 0.8$ GeV
- Either … you consider that this is way too high => discard the plausibility of coalescence approach

Looking at recent data

Coalescence explains it all ?

- $v_n(q)$ & $v_n(c)$ + relative weights of masses (momenta) => $v_n(D)$



Shreyasi Acharya et al. (ALICE) JHEP, 10:141, 2020.

- Good global agreement for $p_T^q/p_T^D = 0.4 \Leftrightarrow m_q \approx 0.7 - 0.8$ GeV
- Or you consider such light-quark masses are achievable close to T_c => coalescence is indeed a good scheme to understand both charmonia and D mesons flows…

However, no attempt to explain $R_{AA}(p_T)$

Disappearance of all $c - \bar{c}$ correlations before FO

Motivations

- Need to revisit how robustly we understand the survival of primordial component (possible role of singlet <-> octet transitions)
- Need to understand the « coalescence » in the late time
- J/ψ are *quantum* bound states => need for a formalism that preserves *quantum* properties… and continuous transitions between bound and unbound states
- Even with the tools and methods of the OQS, such prerequisite may be extremely difficult to achieve for the case of the recombination of many c-cbar pairs => resort to more phenomenological/effective approaches.

τ_E: environment correlation time

$$\tau_E \sim \frac{1}{T}$$

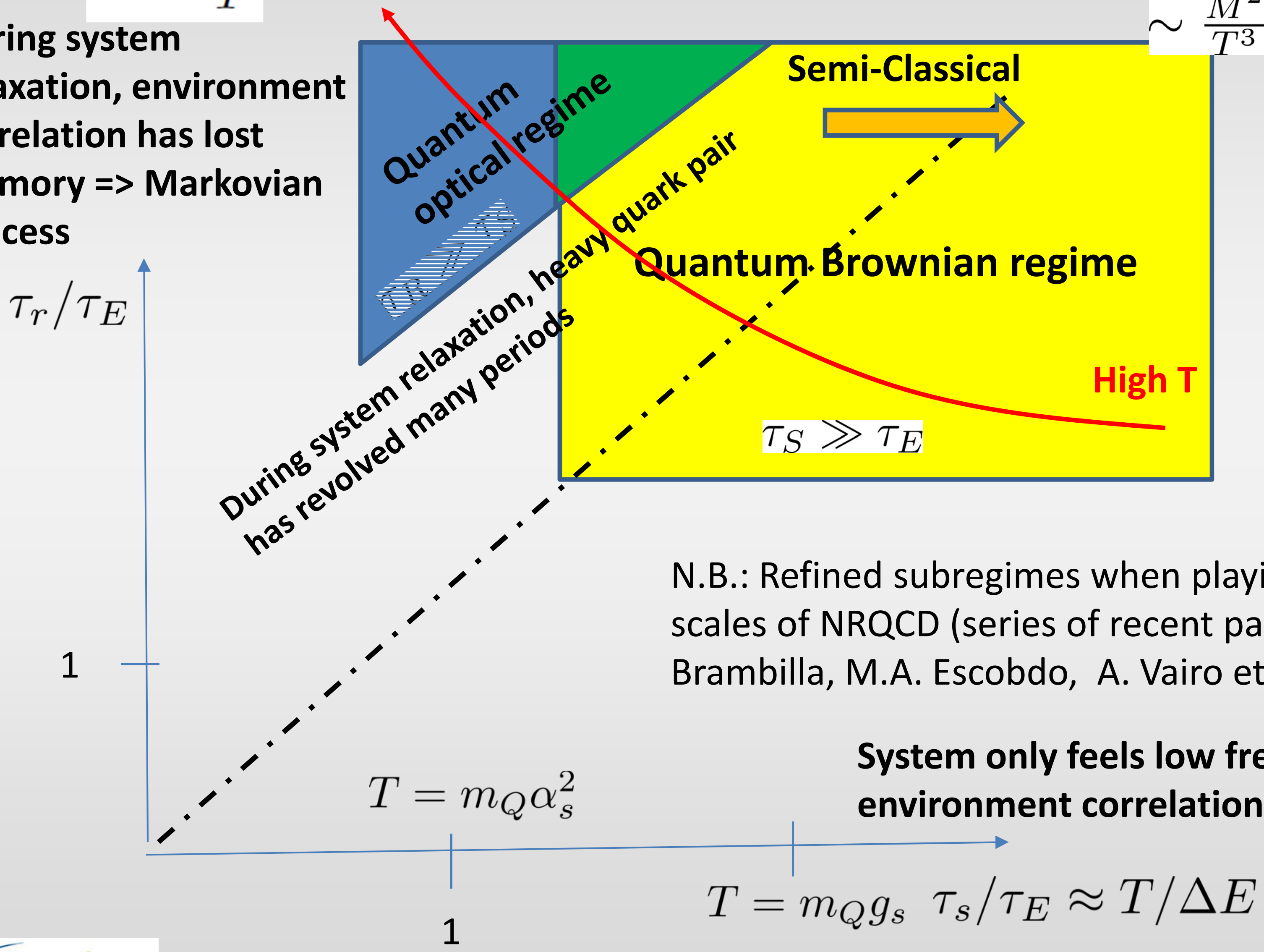
τ_S: system intrinsic time scale

$$\tau_S \approx \frac{1}{Mv^2} \approx \frac{1}{\Delta E}$$

τ_R: system relax time

$$\tau_r \sim \frac{1}{\Gamma} \approx \frac{1}{\alpha T (m_D^2 \langle r^2 \rangle)} \sim \frac{M^2}{T^3} \gg \tau_E$$

During system relaxation, environment correlation has lost memory => Markovian process



$$\frac{\tau_r}{\tau_E} \sim \frac{M^2}{T^2}$$

$$\tau_s/\tau_E \approx T/\Delta E$$

N.B.: Refined subregimes when playing with the scales of NRQCD (series of recent papers by N. Brambilla, M.A. Escobdo, A. Vairo et al)

System only feels low frequency part of environment correlation

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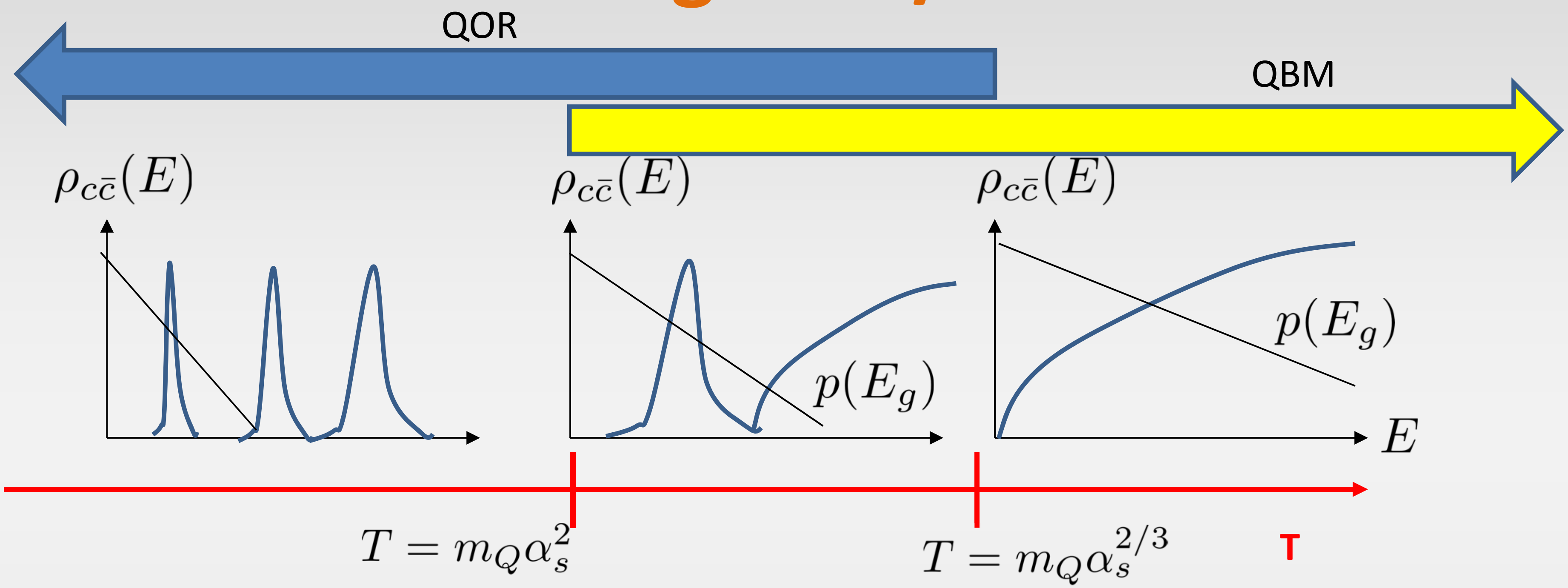
Not clear all states goes from one regime to the other at the same T

Several regimes / effects

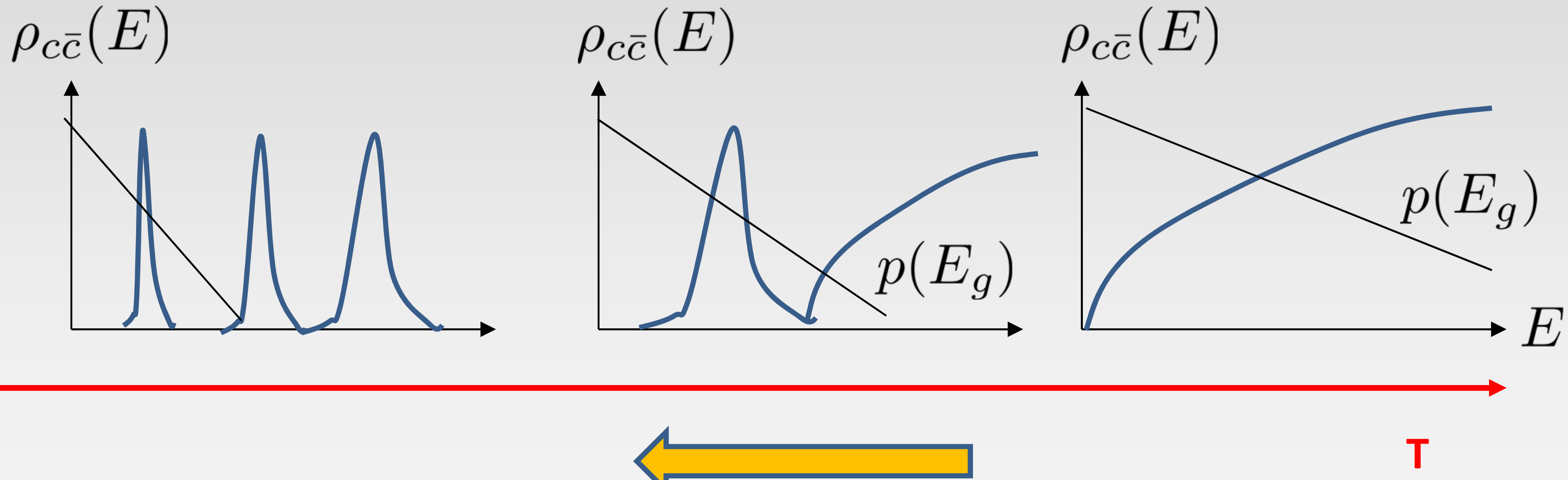


Gluo-dissociation of well identified levels by scarce "high-energy" gluons (dilute medium => cross section ok)

Multiple scattering on quasi free states

Well identified formalisms (Quantum Master Equation, Boltzmann transport, Stochastic equations,…) in well identified regimes, but continuous evolution and no unique framework continuously applicable (to my knowledge)

Several regimes / effects



Time (slow QGP cooling)

Linblad equations expressed on the basis of (vacuum eigenstates)

Rate equations on the probabilities

… as in N.Borghini

After decoherence Time

Linblad equations expressed on the basis of (x,x') ⇔ (p,X) after Wigner transform

$$p \approx \sqrt{MT}$$

Semi-classical treatment (Fokker Planck equations)… as in Shuryak and Young

How can we estimate the decoherence time ? Is it large ?

Our current investigations

- Schroedinger Langevin Equation

- Quantum Master Equation of Blaizot - Escobedo (with Stéphane Delorme, Roland Katz and Thierry Gousset)

Jean-Paul Blaizot and Miguel Angel Escobedo, JHEP06 (2018) 034

- Remler density matrix approach (with Denys Yen Arrebato Villar and Joerg Aichelin)

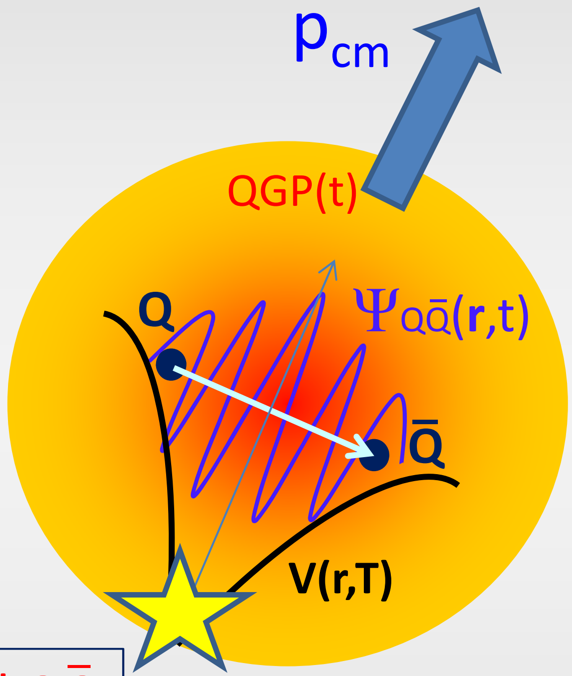
E.A. Remler, ANNALS OF PHYSICS 136, 293-316 (1981)

Both approaches able to deal with the dynamical recombination / hadronisation

Ingredients of our model

QGP temperature scenarios T(t,x)

Cooling QGP



Initial $Q\bar{Q}$ state

Mean field: color screened binding potential V(r,T)

polarization due to color charges

+

Thermalisation and diffusion

Direct interactions with the thermal bath

+ Schroedinger Langevin equation

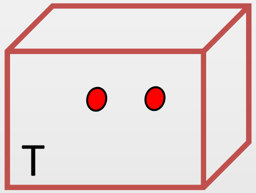
Inner dynamics: Schrödinger-Langevin (SL) equation

Derived from the Heisenberg-Langevin equation*, in Bohmian mechanics** …

$$i\hbar \frac{\partial \Psi_{Q\bar{Q}}(\mathbf{r},t)}{\partial t} = \left(\widehat{H}_{\mathrm{MF}}(\mathbf{r}) - \mathbf{F}_{\mathbf{R}}(t).\mathbf{r} + A\big(S(\mathbf{r},t) - \langle S(\mathbf{r},t)\rangle_{\mathbf{r}}\big) \right) \Psi_{Q\bar{Q}}(\mathbf{r},t)$$

Hamiltonian: Mean Field: T-dependent color screened potential
Taken from lattice-QCD or NRQCD theory

Static lQCD calculations (maximum heat exchange with the medium):



F : free energy
S : entropy

U=F+TS : internal energy
(no heat exchange)

- "Weak potential" F<Vweak<U => some heat exchange
- "Strong potential" V=U => adiabatic evolution
- …

* Kostin The J. of Chem. Phys. 57(9):3589–3590, (1972) ** Garashchuk et al. J. of Chem. Phys. 138, 054107 (2013)
Mócsy & Petreczky Phys.Rev.D77:014501,2008 ; Kaczmarek & Zantow arXiv:hep-lat/0512031v1

Inner dynamics: SL equation

Derived from the Heisenberg-Langevin equation, in Bohmian mechanics …

$$i\hbar\frac{\partial\Psi_{Q\bar{Q}}(\mathbf{r},t)}{\partial t} = \left(\widehat{H}_{\mathrm{MF}}(\mathbf{r}) - \mathbf{F_R}(t).\mathbf{r} + A\left(S(\mathbf{r},t) - \langle S(\mathbf{r},t)\rangle_{\mathbf{r}}\right)\right)\Psi_{Q\bar{Q}}(\mathbf{r},t)$$

Dissipation: thermal de-excitation

$$S(\mathbf{r},t) = \arg\left(\Psi_{Q\bar{Q}}(\mathbf{r},t)\right)$$

✓non-linearly dependent on $\Psi_{Q\bar{Q}}$

✓ real and ohmic

✓ brings the system to the lowest state at T=0

✓with *A*(T) α T^2 the Drag coefficient from a microscopic model (pQCD - HTL) by Gossiaux and Aichelin

P.B. Gossiaux and J. Aichelin, Phys.Rev. C78 (2008) 014904
R. Katz and P.B. Gossiaux, Annals Phys. 368 (2016) 267-295

Inner dynamics: SL equation

$$i\hbar\frac{\partial\Psi_{Q\bar{Q}}(\mathbf{r},t)}{\partial t} = \left(\widehat{H}(\mathbf{r}) - \mathbf{F}(t).\mathbf{r} + A\big(S(\mathbf{r},t) - \langle S(\mathbf{r},t)\rangle_{\mathbf{r}}\big)\right)\Psi_{Q\bar{Q}}(\mathbf{r},t)$$

dissipative non-linear potential
(wavefunction dependent)

where $S(\mathbf{r},t) = \arg\big(\Psi_{Q\bar{Q}}(\mathbf{r},t)\big)$

- ✓ Brings the QQ to the lowest state (0 node)
- ✓ Friction (assumed to be local in time)

- ➢ Solution for V=0 (free wave packet): $\psi(\vec{x},t) \propto e^{i\vec{p}_{\rm cl}(t)\cdot\vec{r}+i\alpha(t)(\vec{r}-\vec{r}_{\rm cl}(t))^2-i\varphi(t)}$

  where $\vec{p}_{\rm cl}(t)$ and $\vec{x}_{\rm cl}(t)$ satisfy the classical laws of motion

- ➢ $\vec{p}_{\rm cl}(t) = \vec{p}_{\rm cl}(0)e^{-At} \Rightarrow$ *A* is the drag coefficient (inverse relaxation time)

➡ *A* can be fixed through the modelling of single heavy quarks observables and comparison with the data OR using lattice QCD calculations

Subatech

Inner dynamics: SL equation

$$i\hbar\frac{\partial\Psi_{Q\bar{Q}}(\mathbf{r},t)}{\partial t} = \left(\widehat{H}(\mathbf{r}) - \mathbf{F}(t).\mathbf{r} + A\big(S(\mathbf{r},t) - \langle S(\mathbf{r},t)\rangle_{\mathbf{r}}\big)\right)\Psi_{Q\bar{Q}}(\mathbf{r},t)$$

dissipative non-linear potential (wavefunction dependent

where $S(\mathbf{r},t) = \arg\big(\Psi_{Q\bar{Q}}(\mathbf{r},t)\big)$

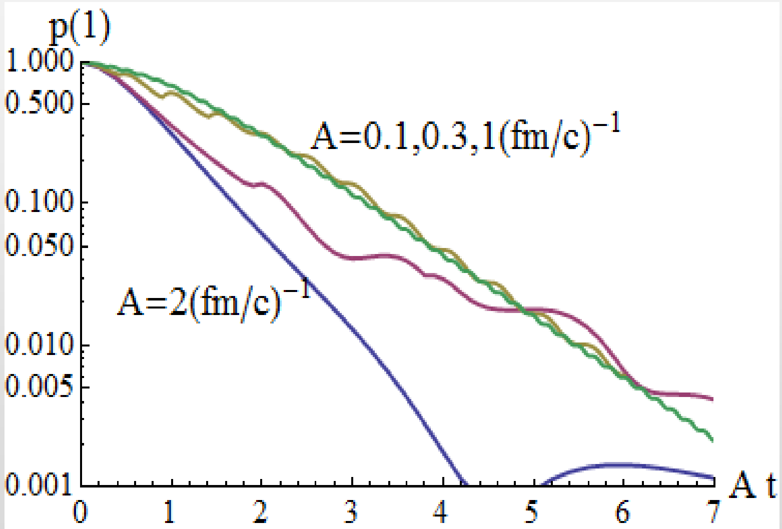
- ✓ Brings the QQ to the lowest state (0 node)
- ✓ Friction (assumed to be local in time)

➢ Solution for harmonic potential as well: $\psi(\vec{x},t) \propto e^{i\vec{p}_{\rm cl}(t)\cdot\vec{r}+i\alpha(t)(\vec{r}-\vec{r}_{\rm cl})^2-i\varphi(t)}$

Illustration: probability of finding the first excited state in a 1D-harmonic potential, as function of time, for various values of *A* …

Scaling relation found for A<ω

$\Leftrightarrow \tau_R \gg \tau_S$



Inner dynamics: SL equation

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Fluctuations: thermal excitation

Taken as a « classical » white stochastic force/noise scaled such as to obtain $T_{Q\bar{Q}} = T_{QGP}$ at equilibrium

The noise operator is assumed here to be a commutating c-number whereas it is a non-commutating q-number within the Heisenberg-Langevin framework.

I. R. Senitzky, Phys. Rev. 119, 670 (1960); 124}, 642 (1961).
G. W. Ford, M. Kac, and P. Mazur, J. Math. Phys. 6, 504 (1965).
R. Katz and P.B. Gossiaux, Annals Phys. 368 (2016) 267-295

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Fluctuations

taken as a « classical » stochastic force

White quantum noise *

$$\langle F_R(t)F_R(t+\tau)\rangle = 2mA\,E_0\left[\coth\left(\frac{E_0}{kT_{\mathrm{bath}}}\right) - 1\right]\delta(\tau)$$

Color quantum noise **

$$\langle N[F_R(t)F_R(t+\tau)]\rangle = \frac{2mA}{\pi}\int_0^\infty \frac{\hbar\omega}{\exp(\hbar\omega/kT_{\mathrm{bath}}) - 1}\cos(\omega\tau)\,d\omega.$$

Autocorrelation time $\Leftrightarrow \tau_E \ll \tau_R$

Subatech

* I. R. Senitzky, Phys. Rev. 119, 670 (1960); 124}, 642 (1961).
** G. W. Ford, M. Kac, and P. Mazur, J. Math. Phys. 6, 504 (1965).

Properties of the SL equation

- 2 parameters: A (Drag) and T (temperature)
- Norm conserving and Heisenberg principle satisfied at any T
- Non linear => Violation of the superposition principle (=> decoherence)
- A priori not univoquely related to a quantum master equation: effective treatment
- Gradual evolution from pure to mixed states (large statistics)
- Mixed state observables from statistics:

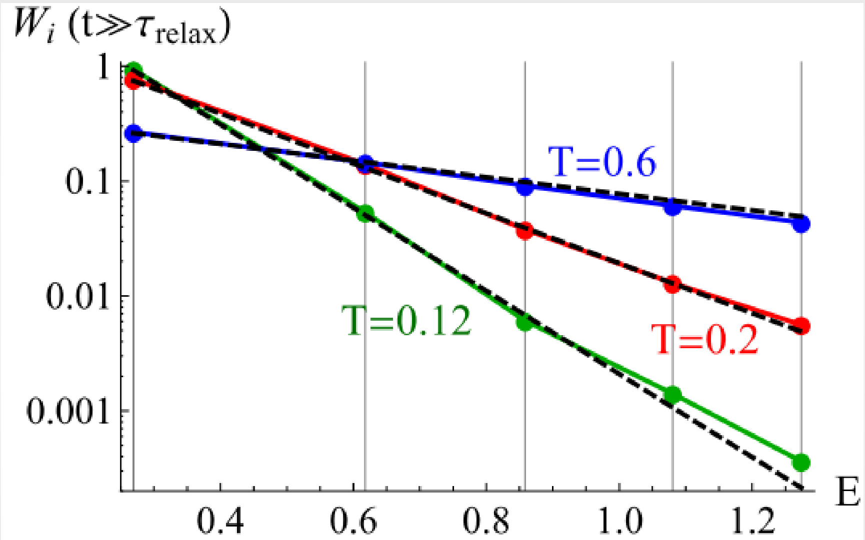
$$\Big\langle \langle\psi(t)|\hat{O}|\psi(t)\rangle \Big\rangle_{\rm stat} = \lim_{n_{\rm stat}\to\infty} \frac{1}{n_{\rm stat}} \sum_{r=1}^{n_{\rm stat}} \langle\psi^{(r)}(t)|\hat{O}|\psi^{(r)}(t)\rangle$$

- Easy to implement numerically (especially in Monte-Carlo generator)

R. Katz and P.B. Gossiaux, Annals Phys. 368 (2016) 267-295

Properties of the SL equation

➢ Leads to local « thermal » distributions: Boltzmann behaviour for at least the low lying states



(GeV)

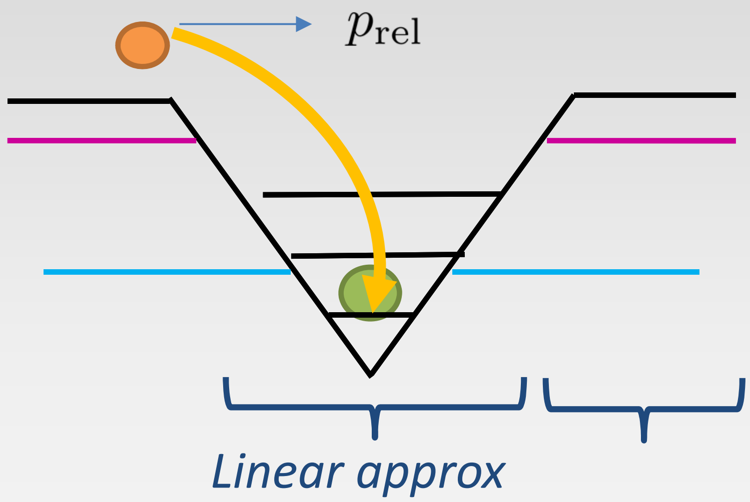
Populations $\propto \exp\left(\frac{-E_n}{k_B T}\right)$

(weak coupling limit: no shift and broadening of the energy levels assumed)

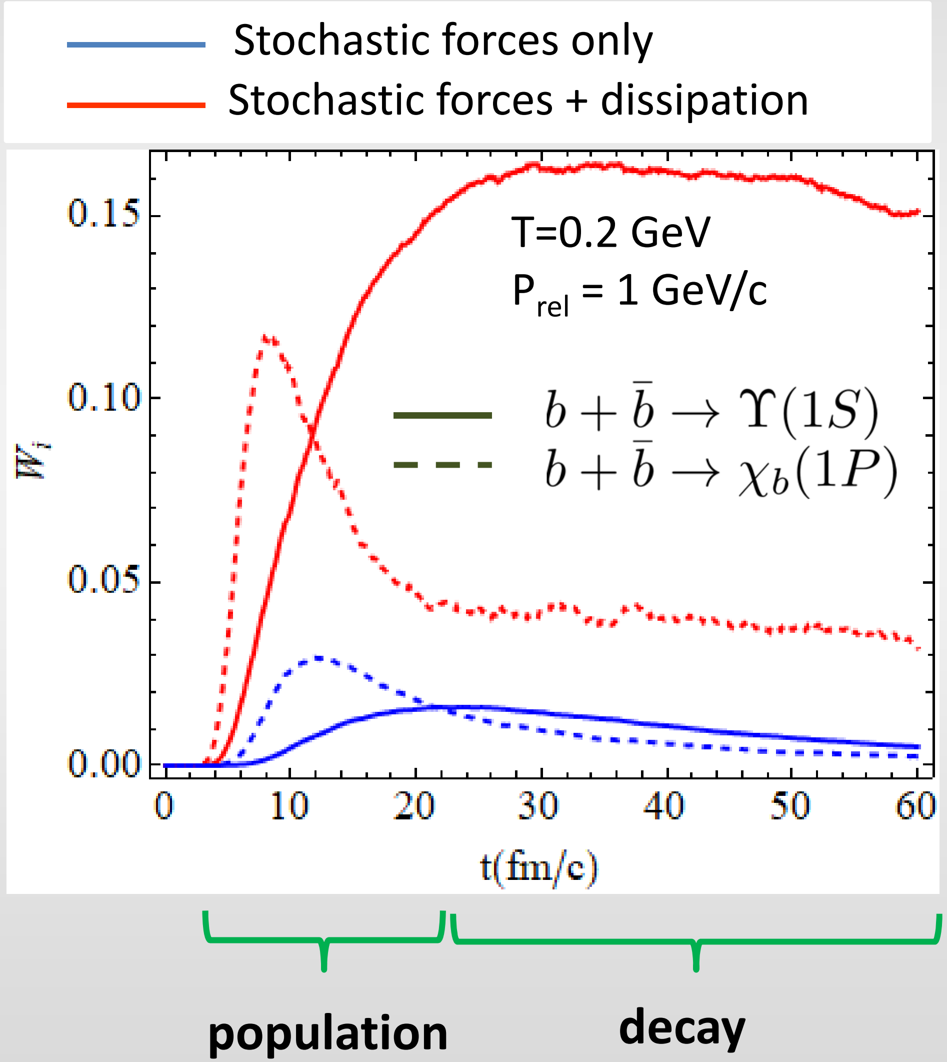
=> Fluctuation-dissipation mechanism compatible with quantum mechanics and effective !!

R. Katz and P.B. Gossiaux, Annals Phys. 368 (2016) 267-295

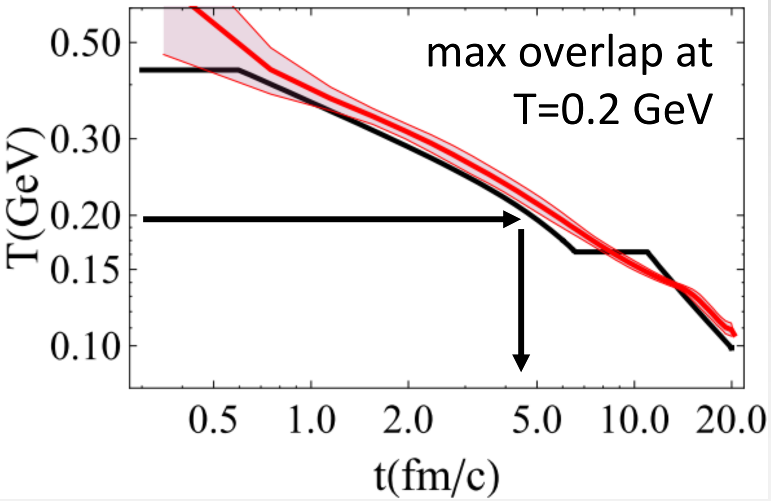
Results from init. Scattering state at cst T

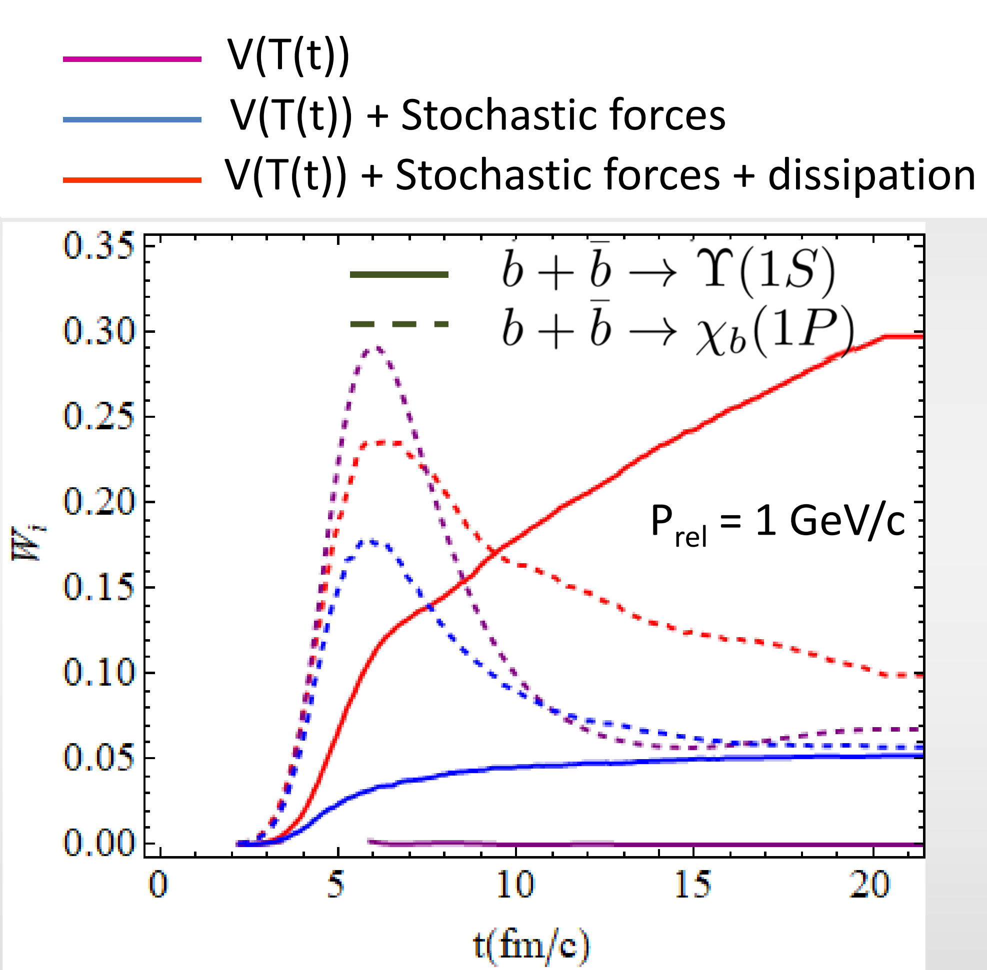


- ✓ some bound state creation through stochastic forces
- ✓ most substantial effects from dissipation
- ✓ relative weights after population compatible Boltzmann probability exp(-E_n/T)
- ✓ … but needs a long time.



Results with SLE in *evolving* medium



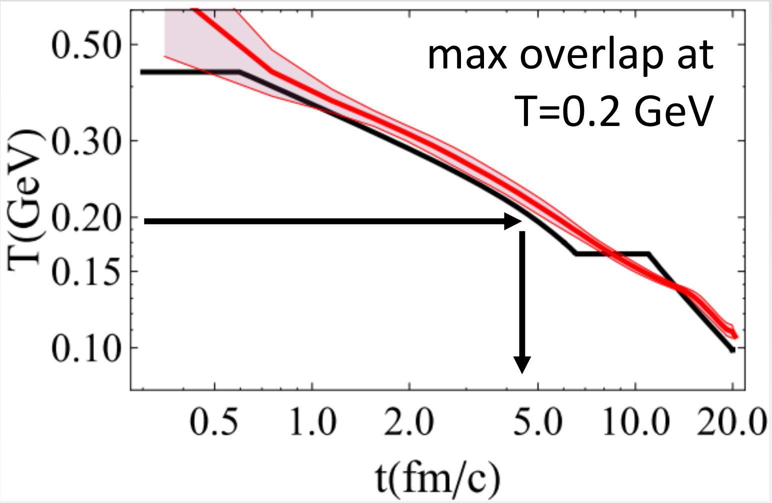


✓ Substantial recombination probability at the end of the evolution provided one includes dissipation

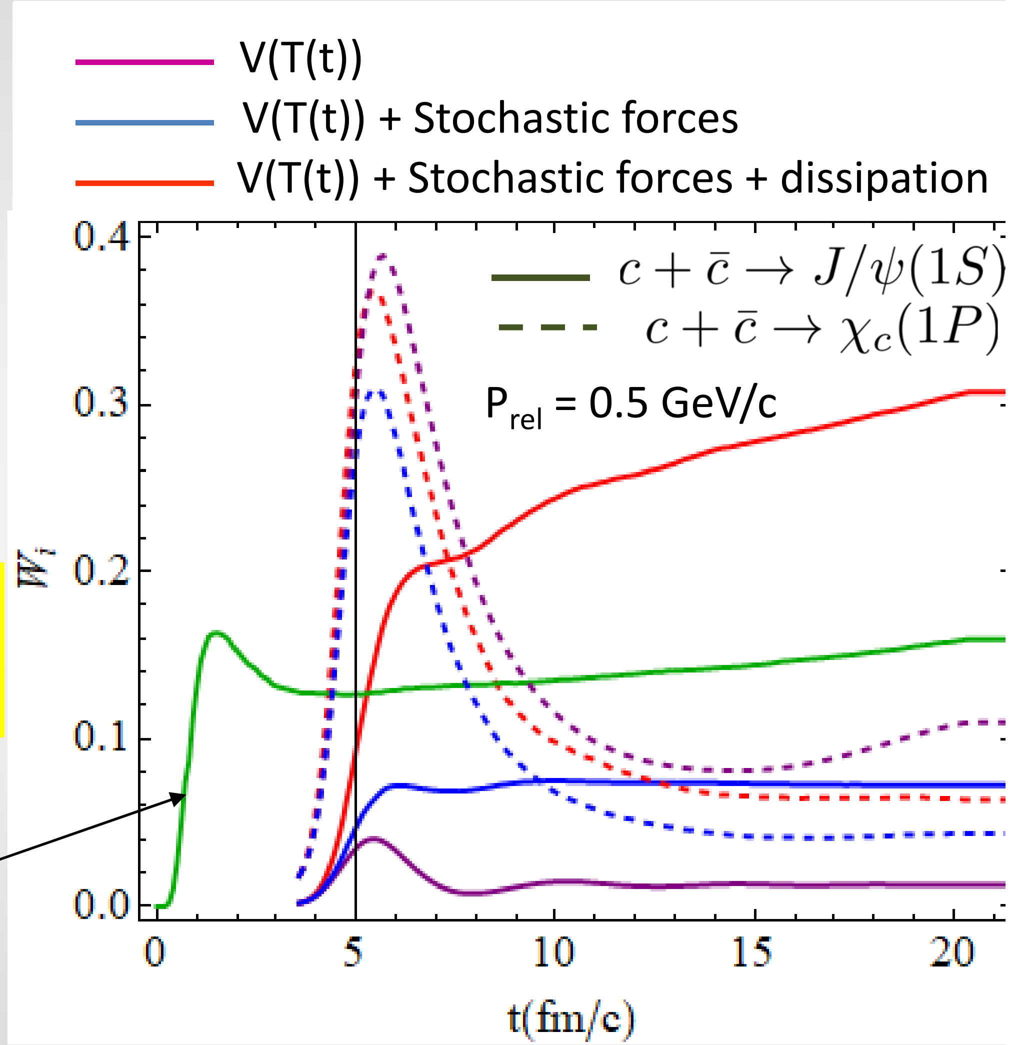
✓ Yet, no instantaneous thermalisation.

✓ Recombination probability tend to decrease for larger p_{rel}

Results with SLE in *evolving* medium



✓ Similar pattern seen for c+cbar -> charmonia family



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Both approaches able to deal with the dynamical recombination / hadronisation

Case of the B-E Quantum Master Equation

$$i\frac{d\mathcal{D}}{dt} = [\mathcal{H}, \mathcal{D}] \xrightarrow[\text{representation}]{\text{Interaction}} i\frac{d\mathcal{D}^I(t)}{dt} = [\mathcal{H}_1(t), \mathcal{D}^I(t)]$$

$$\mathcal{H} = \mathcal{H}_Q + \mathcal{H}_1 + \mathcal{H}_{pl}$$

Coulomb gauge

$$\mathcal{D}^I(t) = \mathcal{U}_I(t,t_0)\mathcal{D}(t_0)\mathcal{U}_I^\dagger(t,t_0)$$

Free Quark Hamiltonian

Plasma Hamiltonian

Quark-Plasma Interactions…

$$H_1 = -g\int_{\boldsymbol{r}} A_0^a(\boldsymbol{r})\,n^a(\boldsymbol{r})$$

No magnetic term (NR)

color charge density of the heavy particles

Average over plasma d.o.f + rapid environment hypothesis

Generic Linblad – like QME on $\mathcal{D}_Q$

… treated as a perturbation

$$\frac{\mathrm{d}\mathcal{D}_Q^I(t)}{\mathrm{d}t} = -\int_{t_0}^{t}\mathrm{d}t'\int_{\boldsymbol{x}\boldsymbol{x}'}\left([n^a(t,\boldsymbol{x}), n^a(t',\boldsymbol{x}')\mathcal{D}_Q^I(t_0)]\Delta^>(t-t',\boldsymbol{x}-\boldsymbol{x}')\right.$$
$$\left.+[\mathcal{D}_Q^I(t_0)n^a(t',\boldsymbol{x}'), n^a(t,\boldsymbol{x})]\Delta^<(t-t',\boldsymbol{x}-\boldsymbol{x}')\right)$$

$\Delta^>, \Delta^<$ Time ordered HTL gluon propagators

Case of the B-E Quantum Master Equation

$$\frac{\mathrm{d}\mathcal{D}_Q^I(t)}{\mathrm{d}t} = -\int_{t_0}^{t} \mathrm{d}t' \int_{\boldsymbol{x}\boldsymbol{x}'} \big([n^a(t,\boldsymbol{x}), n^a(t',\boldsymbol{x}')\mathcal{D}_Q^I(t_0)]\Delta^>(t-t',\boldsymbol{x}-\boldsymbol{x}') + [\mathcal{D}_Q^I(t_0)n^a(t',\boldsymbol{x}'), n^a(t,\boldsymbol{x})]\Delta^<(t-t',\boldsymbol{x}-\boldsymbol{x}')\big)$$

Not local in time

Next hypothesis/simplification : large relaxation time ($\tau_R >> \tau_E$) : Markov

=> Slow evolution of D_Q over QGP correlation time τ_E

$$\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} + i[H_Q, \mathcal{D}_Q(t)] = -\int_{\boldsymbol{x}\boldsymbol{x}'}\int_0^{t-t_0} \mathrm{d}\tau\, [n^a_{\boldsymbol{x}}, U_Q(\tau) n^a_{\boldsymbol{x}'} U_Q^\dagger(\tau)\mathcal{D}_Q(t)]\, \Delta^>(\tau;\boldsymbol{x}-\boldsymbol{x}')) - \int_{\boldsymbol{x}\boldsymbol{x}'}\int_0^{t-t_0} \mathrm{d}\tau\, [\mathcal{D}_Q(t) U_Q(\tau) n^a_{\boldsymbol{x}'} U_Q^\dagger(\tau), n^a_{\boldsymbol{x}}]\, \Delta^<(\tau;\boldsymbol{x}-\boldsymbol{x}').$$

Local in time !

Free quarks

Case of the B-E Quantum Master Equation

Local in time !

$$\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} + i[H_Q, \mathcal{D}_Q(t)] = -\int_{\boldsymbol{x}\boldsymbol{x}'} \int_0^{t-t_0} \mathrm{d}\tau \, [n^a_{\boldsymbol{x}}, U_Q(\tau) n^a_{\boldsymbol{x}'} U^\dagger_Q(\tau) \mathcal{D}_Q(t)] \, \Delta^>(\tau; \boldsymbol{x} - \boldsymbol{x}')) - \int_{\boldsymbol{x}\boldsymbol{x}'} \int_0^{t-t_0} \mathrm{d}\tau \, [\mathcal{D}_Q(t) U_Q(\tau) n^a_{\boldsymbol{x}'} U^\dagger_Q(\tau), n^a_{\boldsymbol{x}}] \, \Delta^<(\tau; \boldsymbol{x} - \boldsymbol{x}').$$

Further hypothesis/simplification : the response of the plasma to the perturbation caused by the heavy quarks is fast compared to the characteristic time scales of the heavy quark ($\tau_S >> \tau_E \Leftrightarrow T >> m_Q \alpha_s^2$).

High Temperature regime

=> Series expansion of U_Q around τ=0

$$\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} + i[H_Q, \mathcal{D}_Q(t)] \approx -\int_{\boldsymbol{x}\boldsymbol{x}'} [n^a_{\boldsymbol{x}}, n^a_{\boldsymbol{x}'} \mathcal{D}_Q] \int_0^\infty \mathrm{d}\tau \Delta^>(\tau; \boldsymbol{x} - \boldsymbol{x}')) - \int_{\boldsymbol{x}\boldsymbol{x}'} [\mathcal{D}_Q n^a_{\boldsymbol{x}'}, n^a_{\boldsymbol{x}}] \int_0^\infty \mathrm{d}\tau \, \Delta^<(\tau; \boldsymbol{x} - \boldsymbol{x}') + \int_{\boldsymbol{x}\boldsymbol{x}'} [n^a_{\boldsymbol{x}}, \dot{n}^a_{\boldsymbol{x}'} \mathcal{D}_Q] \int_0^\infty \mathrm{d}\tau \, \tau \, \Delta^>(\tau; \boldsymbol{x} - \boldsymbol{x}')) + \int_{\boldsymbol{x}\boldsymbol{x}'} [\mathcal{D}_Q \dot{n}^a_{\boldsymbol{x}'}, n^a_{\boldsymbol{x}}] \int_0^\infty \mathrm{d}\tau \, \tau \, \Delta^<(\tau; \boldsymbol{x} - \boldsymbol{x}').$$

B-E Quantum Master Equation

$$\begin{aligned}\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} + i[H_Q, \mathcal{D}_Q(t)] \approx & -\int_{\boldsymbol{x}\boldsymbol{x}'} [n^a_{\boldsymbol{x}}, n^a_{\boldsymbol{x}'}\mathcal{D}_Q] \int_0^\infty \mathrm{d}\tau \Delta^>(\tau; \boldsymbol{x}-\boldsymbol{x}')) \\ & - \int_{\boldsymbol{x}\boldsymbol{x}'} [\mathcal{D}_Q n^a_{\boldsymbol{x}'}, n^a_{\boldsymbol{x}}] \int_0^\infty \mathrm{d}\tau\, \Delta^<(\tau; \boldsymbol{x}-\boldsymbol{x}') \\ & + \int_{\boldsymbol{x}\boldsymbol{x}'} [n^a_{\boldsymbol{x}}, \dot{n}^a_{\boldsymbol{x}'}\mathcal{D}_Q] \int_0^\infty \mathrm{d}\tau\, \tau\, \Delta^>(\tau; \boldsymbol{x}-\boldsymbol{x}')) \\ & + \int_{\boldsymbol{x}\boldsymbol{x}'} [\mathcal{D}_Q \dot{n}^a_{\boldsymbol{x}'}, n^a_{\boldsymbol{x}}] \int_0^\infty \mathrm{d}\tau\, \tau\, \Delta^<(\tau; \boldsymbol{x}-\boldsymbol{x}').\end{aligned}$$

Time integrals involve only the 0-frequency part of the HTL propagators, i.e. the real and imaginary potentials, leading to :

$$\begin{aligned}\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} + i[H_Q, \mathcal{D}_Q(t)] \approx & -\frac{i}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} V(\boldsymbol{x}-\boldsymbol{x}')[n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q], \\ & + \frac{1}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left(\{n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q\} - 2 n^a_{\boldsymbol{x}} \mathcal{D}_Q n^a_{\boldsymbol{x}'}\right) \\ & + \frac{i}{4T}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left([n^a_{\boldsymbol{x}}, \dot{n}^a_{\boldsymbol{x}'}\mathcal{D}_Q] + [n^a_{\boldsymbol{x}}, \mathcal{D}_Q \dot{n}^a_{\boldsymbol{x}'}]\right)\end{aligned}$$

From there on, possibility to use lQCD potentials instead of HTL ones.

B-E Quantum Master Equation

Series expansion in τ_E/τ_S

Compact form: $\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} = \mathcal{L}\,\mathcal{D}_Q$ with $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \cdots$

$$\mathcal{L}_0\,\mathcal{D}_Q \equiv -i[H_Q, \mathcal{D}_Q],$$

$$\mathcal{L}_1\,\mathcal{D}_Q \equiv -\frac{i}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} V(\boldsymbol{x}-\boldsymbol{x}')[n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q],$$

Mean field hamiltonian

$$\mathcal{L}_2\,\mathcal{D}_Q \equiv \frac{1}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left(\{n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q\} - 2 n^a_{\boldsymbol{x}} \mathcal{D}_Q n^a_{\boldsymbol{x}'}\right),$$

Fluctuations, Linblad form

$$\mathcal{L}_3\,\mathcal{D}_Q \equiv \frac{i}{4T}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left([n^a_{\boldsymbol{x}}, \dot{n}^a_{\boldsymbol{x}'}\mathcal{D}_Q] + [n^a_{\boldsymbol{x}}, \mathcal{D}_Q \dot{n}^a_{\boldsymbol{x}'}]\right)$$

Friction

N.B. : Friction is NOT of the Linbladian form => the evolution breaks positivity.

Positivity and Linblad form can be restored at the price **of extra subleading terms*** :

$$\left\{\left(n^a_{\mathbf{x}} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}}\right)\left(n^a_{\mathbf{x}'} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}'}\right), \mathcal{D}_{Q\bar{Q}}\right\} - 2\left(n^a_{\mathbf{x}} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}}\right)\mathcal{D}_{Q\bar{Q}}\left(n^a_{\mathbf{x}'} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}'}\right)$$

$\mathcal{L}_2$

Subatech

* As well as another time discretization

B-E Quantum Master Equation

Series expansion in τ_E/τ_S

Compact form: $\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} = \mathcal{L}\,\mathcal{D}_Q$ with $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \cdots$

$$\mathcal{L}_0\,\mathcal{D}_Q \equiv -i[H_Q, \mathcal{D}_Q],$$

$$\mathcal{L}_1\,\mathcal{D}_Q \equiv -\frac{i}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} V(\boldsymbol{x}-\boldsymbol{x}')[n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q],$$

Mean field hamiltonian

$$\mathcal{L}_2\,\mathcal{D}_Q \equiv \frac{1}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left(\{n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q\} - 2n^a_{\boldsymbol{x}}\mathcal{D}_Q n^a_{\boldsymbol{x}'}\right),$$

Fluctuations, Linblad form

$$\mathcal{L}_3\,\mathcal{D}_Q \equiv \frac{i}{4T}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left([n^a_{\boldsymbol{x}}, \dot{n}^a_{\boldsymbol{x}'}\mathcal{D}_Q] + [n^a_{\boldsymbol{x}}, \mathcal{D}_Q\dot{n}^a_{\boldsymbol{x}'}]\right)$$

Friction

N.B. : Friction is NOT of the Linbladian form => the evolution breaks positivity.

Positivity and Linblad form can be restored at the price **of extra subleading terms*** :

$$\left\{\left(n^a_{\mathbf{x}} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}}\right)\left(n^a_{\mathbf{x}'} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}'}\right), \mathcal{D}_{Q\bar{Q}}\right\} - 2\left(n^a_{\mathbf{x}} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}}\right)\mathcal{D}_{Q\bar{Q}}\left(n^a_{\mathbf{x}'} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}'}\right)$$

$\mathcal{L}_3$

Subatech

* As well as another time discretization

B-E Quantum Master Equation

Series expansion in τ_E/τ_S

Compact form: $\frac{d\mathcal{D}_Q}{dt} = \mathcal{L}\,\mathcal{D}_Q$ with $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \cdots$

$$\mathcal{L}_0\,\mathcal{D}_Q \equiv -i[H_Q, \mathcal{D}_Q],$$

$$\mathcal{L}_1\,\mathcal{D}_Q \equiv -\frac{i}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} V(\boldsymbol{x}-\boldsymbol{x}')[n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q],$$

} **Mean field hamiltonian**

$$\mathcal{L}_2\,\mathcal{D}_Q \equiv \frac{1}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left(\{n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q\} - 2 n^a_{\boldsymbol{x}} \mathcal{D}_Q n^a_{\boldsymbol{x}'}\right),$$

} **Fluctuations, Linblad form**

$$\mathcal{L}_3\,\mathcal{D}_Q \equiv \frac{i}{4T}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left([n^a_{\boldsymbol{x}}, \dot{n}^a_{\boldsymbol{x}'}\mathcal{D}_Q] + [n^a_{\boldsymbol{x}}, \mathcal{D}_Q \dot{n}^a_{\boldsymbol{x}'}]\right)$$

} **Friction**

N.B. : Friction is NOT of the Linbladian form => the evolution breaks positivity.

Positivity and Linblad form can be restored at the price **of extra subleading terms** :

$$\{(n^a_{\mathbf{x}} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}})(n^a_{\mathbf{x}'} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}'}), \mathcal{D}_{Q\bar{Q}}\} - 2(n^a_{\mathbf{x}} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}})\mathcal{D}_{Q\bar{Q}}(n^a_{\mathbf{x}'} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}'})$$

$\mathcal{L}_4$

Application to QED and QCD for both cases of 1 body and 2 body densities…

B-E Quantum Master Equation: QED case

- For the relative motion (2 body):

$$\left.\begin{aligned}\vec{s} &= \vec{x}_1 - \vec{x}_2 \\ \vec{s}\,' &= \vec{x}_1' - \vec{x}_2'\end{aligned}\right\} \qquad \vec{r} = \frac{\vec{s}+\vec{s}\,'}{2} \quad \text{and} \quad \vec{y} = \vec{s} - \vec{s}\,'$$

- Near thermal equilibrium, Density operator is nearly diagonal => **semi-classical expansion** (power series in *y* up to 2nd order)

... However, we know from open heavy flavor analysis that it takes some finite relaxation time to reach this state

$$\frac{d}{dt}\mathcal{D}(r,y) = \mathcal{L}\mathcal{D}(r,y)$$

$$\begin{cases}\mathcal{L}_0 = \frac{2i\nabla_y\cdot\nabla_r}{M} \\ \mathcal{L}_1 = i\vec{y}\cdot\nabla V(r) \\ \mathcal{L}_2 = -\frac{1}{4}\vec{y}\cdot(\mathcal{H}(\vec{r})+\mathcal{H}(0))\cdot\vec{y} \\ \mathcal{L}_3 = -\frac{1}{2MT}\vec{y}\cdot(\mathcal{H}(\vec{r})+\mathcal{H}(0))\cdot\nabla_{\vec{y}}\end{cases}$$

$\mathcal{H}(\vec{r})$: Hessian matrix of im. pot. W

$$W(\vec{y}) = W(\vec{0}) + \frac{1}{2}\vec{y}\cdot\mathcal{H}(0)\cdot\vec{y}$$

- Wigner transform -> $\mathcal{D}(\vec{r},\vec{p})$ => $\{\vec{y},\nabla_y\} \to \{\nabla_p,\vec{p}\}$ Usual Fokker Planck eq.
- Easy MC implementation + generalization for N body system

B-E Quantum Master Equation: QCD case

$$\frac{d}{dt}\begin{pmatrix}\mathcal{D}_s \\ \mathcal{D}_o\end{pmatrix} = \mathcal{L}\begin{pmatrix}\mathcal{D}_s(\mathbf{r}_{rel}, \mathbf{r}'_{rel}, t) \\ \mathcal{D}_o(\mathbf{r}_{rel}, \mathbf{r}'_{rel}, t)\end{pmatrix}$$

singlet density matrix

octet density matrix

$$\mathcal{L} = \begin{pmatrix}\mathcal{L}_{ss} & \mathcal{L}_{so} \\ \mathcal{L}_{os} & \mathcal{L}_{oo}\end{pmatrix}$$

singlet-octet transitions

2 coupled color representations (singlet octet)

Alternate choice : $\begin{pmatrix}\mathcal{D}_0 \\ \mathcal{D}_8\end{pmatrix}$ Off color-equilibrium component

With (infinite mass limit)

$$\mathcal{D}_8(r,t) \sim \mathcal{D}_8(r,0)e^{-N_c\Gamma(r)t} \to 0$$

Color equilibration

Still semi-classical approximation (power series in *y*).

Example of the $\mathcal{D}_s$ evolution (after SC expansion)

$$\begin{aligned}(D_s|\mathcal{L}|\mathcal{D}) = &\left(2i\frac{\nabla_r\cdot\nabla_y}{M} + i\frac{\nabla_R\cdot\nabla_Y}{2M} + iC_F\boldsymbol{y}\cdot\boldsymbol{\nabla}V(\boldsymbol{r})\right)D_s \\ &-2C_F\Gamma(\boldsymbol{r})(D_s - D_o) \\ &-\frac{C_F}{4}\left(\boldsymbol{y}\cdot\mathcal{H}(\boldsymbol{r})\cdot\boldsymbol{y}\,D_s + \boldsymbol{y}\cdot\mathcal{H}(0)\cdot\boldsymbol{y}\,D_o\right) \\ &-C_F\boldsymbol{Y}\cdot[\mathcal{H}(0)-\mathcal{H}(\boldsymbol{r})]\cdot\boldsymbol{Y}\,D_o \\ &+\frac{C_F}{2MT}\left[\nabla^2W(0)-\nabla^2W(\boldsymbol{r})-\boldsymbol{\nabla}W(\boldsymbol{r})\cdot\boldsymbol{\nabla}_r\right](D_s-D_o) \\ &-\frac{C_F}{2MT}\left(\boldsymbol{y}\cdot\mathcal{H}(\boldsymbol{r})\cdot\boldsymbol{\nabla}_y D_s + \boldsymbol{y}\cdot\mathcal{H}(0)\cdot\boldsymbol{\nabla}_y D_o\right) \\ &-\frac{C_F}{2MT}\boldsymbol{Y}\cdot[\mathcal{H}(0)-\mathcal{H}(\boldsymbol{r})]\cdot\boldsymbol{\nabla}_Y D_o.\end{aligned}$$

Subatech

Our current project

Our Goal:

- Explicitly restore the Linbladian form and the positivity of BE equations => term $\mathcal{L}_4$
- Gain insight on the quarkonium dynamics inside the QGP by **solving exactly the B-E equations** for a single $c\bar{c}$ pair without performing the Semi-Classical approximation:
  - Evolution of the density matrix
  - Evolution of states probabilities over time
  - Singlet-octet transitions
  - Color relaxation time
  - …
- Comparison with the semi-classical approach for a various range of QGP temperatures (should be fine at large temperature… but down to ?)
- Possibly design improved algorithm for intermediate temperatures

Our current project

<u>Adopted method:</u>

- Trace out the global center of mass motion R (Heavy quarks => should not be deflected much) => equations on the relative coordinates only:

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\vec{s}|\mathcal{D}_{c\bar{c}}|\vec{s}\,'\rangle = \int d^3R d^3R'\delta^{(3)}(\vec{R}-\vec{R}\,')\langle\vec{R}\vec{s}|\mathcal{D}_{c\bar{c}}|\vec{R}\,'\vec{s}\,'\rangle$$

$$\stackrel{?}{=} \mathcal{L}'[\langle\vec{s}|\mathcal{D}_{c\bar{c}}|\vec{s}\,'\rangle]$$

Both $\mathcal{L}_2$ and $\mathcal{L}_3$ can be reduced to $\mathcal{L}'_2$ and $\mathcal{L}'_3$. For $\mathcal{L}_4$ some terms can only be reduced at the price of assuming a state with a specific total momentum p_{tot}:

$$\langle\vec{R}\vec{s}|\mathcal{D}_{c\bar{c}}|\vec{R}\,'\vec{s}\,'\rangle = \langle\vec{s}|\mathcal{D}_{c\bar{c}}|\vec{s}\,'\rangle \times e^{i\vec{p}_{\mathrm{tot}}(\vec{R}-\vec{R}\,')}$$

➡ $\mathcal{L}'_{4,\vec{p}_{\mathrm{tot}}}$ Which might be good for phenomenology

- In a first approach, perform the study for a 1D reduced problem => reduced computational cost, although sufficient to gain insight

- Brute numerics for the residual equations (checking basic important properties + benchmarking on known solutions)

Positivity

- Equations for the QED-like plasma in 1D :

$$\frac{1}{\hbar}\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{D} = \frac{i}{M}(\hbar c)^2(\partial_s^2 - \partial_{s'}^2)\mathcal{D} - i[V(s) - V(s')]\mathcal{D}$$
$$+\left[2W(0) - W(s) - W(s') - 2W\left(\tfrac{s-s'}{2}\right) + 2W\left(\tfrac{s+s'}{2}\right)\right]\mathcal{D}$$
$$+\frac{(\hbar c)^2}{4MT}\left[2W''(0) - W''(s) - W''(s') - 2W''\left(\tfrac{s-s'}{2}\right) + 2W''\left(\tfrac{s+s'}{2}\right)\right]\mathcal{D}$$
$$-\frac{(\hbar c)^2}{4MT}\left[2W'(s)\partial_s + 2W'(s')\partial_{s'} + 2W'\left(\tfrac{s-s'}{2}\right)(\partial_s - \partial_{s'}) - 2W'\left(\tfrac{s+s'}{2}\right)(\partial_s + \partial_{s'})\right]\mathcal{D}$$
$$+\frac{(\hbar c)^4}{64M^2T^2}\left[2W''''(0) + W''''(s) + W''''(s') - 2W''''\left(\tfrac{s-s'}{2}\right) + 2W''''\left(\tfrac{s+s'}{2}\right)\right]\mathcal{D}$$
$$+\frac{(\hbar c)^4}{64M^2T^2}\left[4W'''(s)\partial_s + 4W'''(s')\partial_{s'} - 4W'''\left(\tfrac{s-s'}{2}\right)(\partial_s - \partial_{s'}) + 4W'''\left(\tfrac{s+s'}{2}\right)(\partial_s + \partial_{s'})\right]\mathcal{D}$$
$$+\frac{(\hbar c)^4}{64M^2T^2}\left[4W''(0)(\partial_s^2 + \partial_{s'}^2) + 4W''(s)\partial_s^2 + 4W''(s')\partial_{s'}^2 + 8W''\left(\tfrac{s-s'}{2}\right)\partial_s\partial_{s'} + 8W''\left(\tfrac{s+s'}{2}\right)\partial_s\partial_{s'}\right]\mathcal{D}$$
$$+\frac{(\hbar c)^4}{64M^2T^2}p_{\mathrm{tot}}^2\left[-2W''(0) + W''(s) + W''(s') + 2W''\left(\tfrac{s-s'}{2}\right) - 2W''\left(\tfrac{s+s'}{2}\right)\right]\mathcal{D}$$

$\mathcal{L}_4'$

- Indeed subleading in 1/T expansion
- No higher derivatives on D than the 2nd one => still a FP equation in the semi-classical limit.
- Higher derivatives of the imaginary potential W => possible UV divergences => need for some regularization.

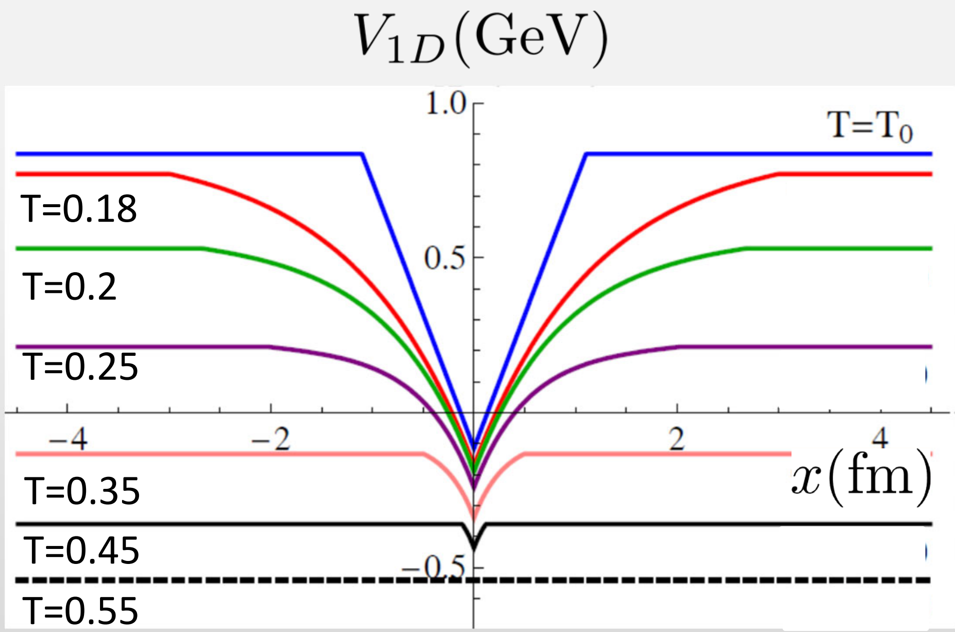
Further implementation features

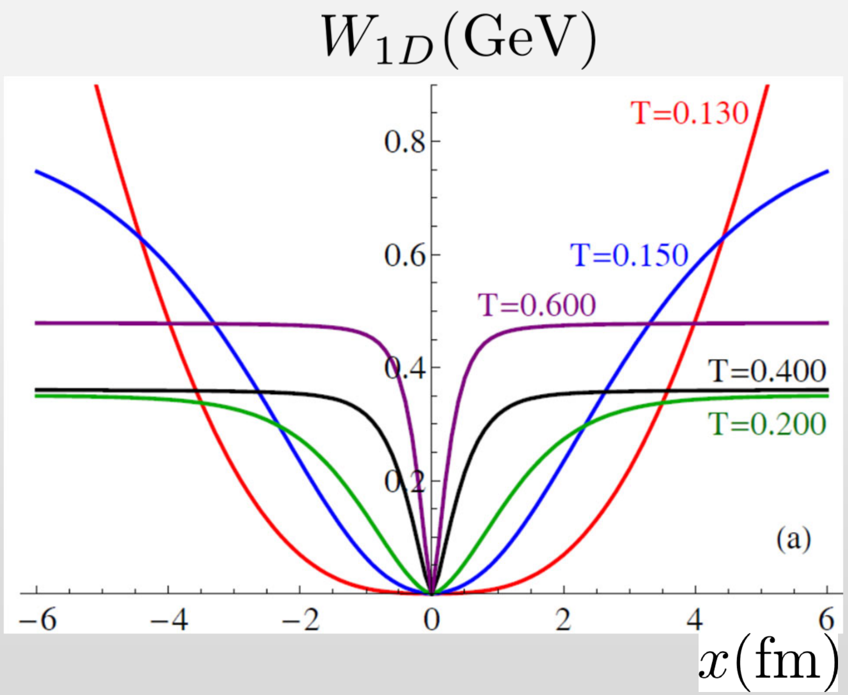
- 1D grid for both $s \in [-s_{\max}, +s_{\max}]$ and $s' \in [-s_{\max}, +s_{\max}]$

!!! Not the radial decomposition of $\mathcal{D}_{c\bar{c}}(\vec{s}, \vec{s}\,')$ which is more cumbersome

Even states will be considered as « S like » while odd states will be considered as « P like » states

Need to design a realistic 1D bona fide potential V + i W (based on 3d lQCD results, tuned to reproduce 3D mass spectra and decay widths)

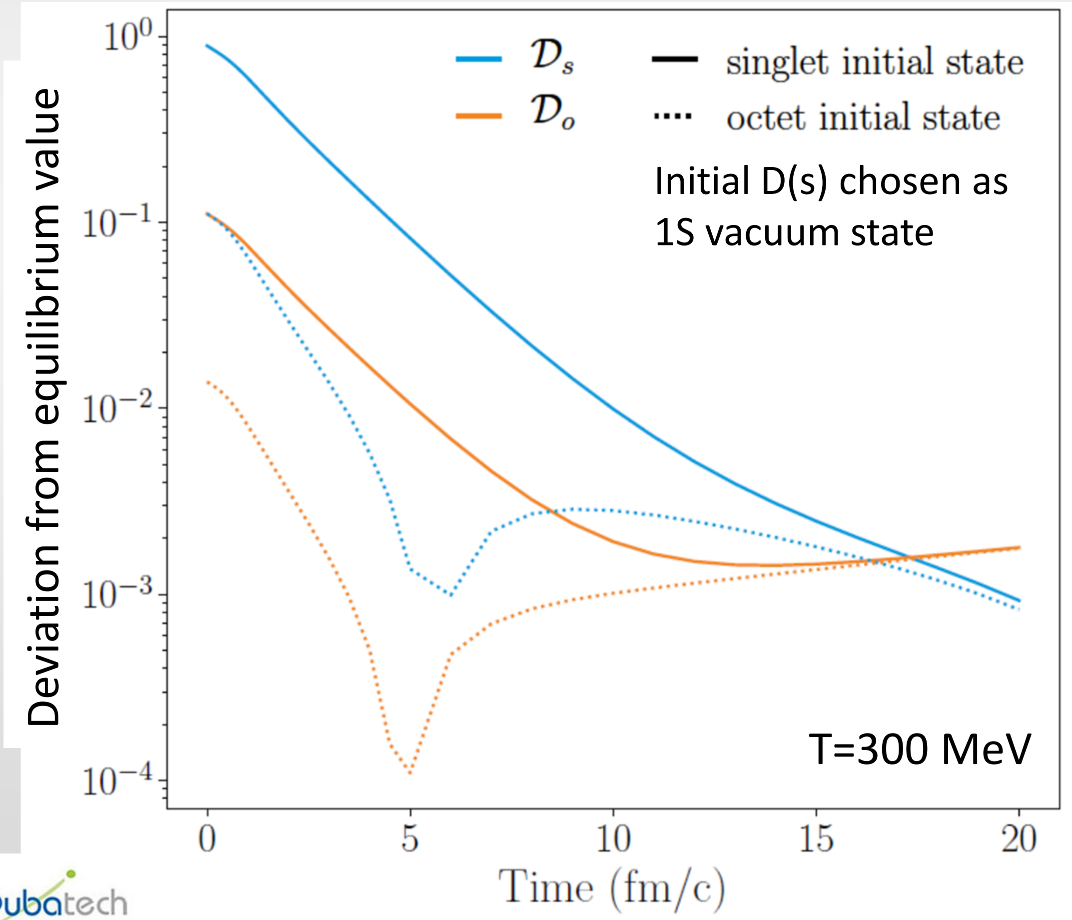




Results for c-cbar system

Color Dynamics : Singlet – octet probabilities:

- Starting from singlet (———) or octets (- - - - -) states, one expects some equilibration / thermalisation -> asymptotic values : $D_s^{\rm eq} = D_o^{\rm eq} = \frac{1}{9} \quad (1+8) \times \frac{1}{9}$
- Study the deviations $|D_s - D_s^{\rm eq}|$ and $|D_o - D_o^{\rm eq}|$

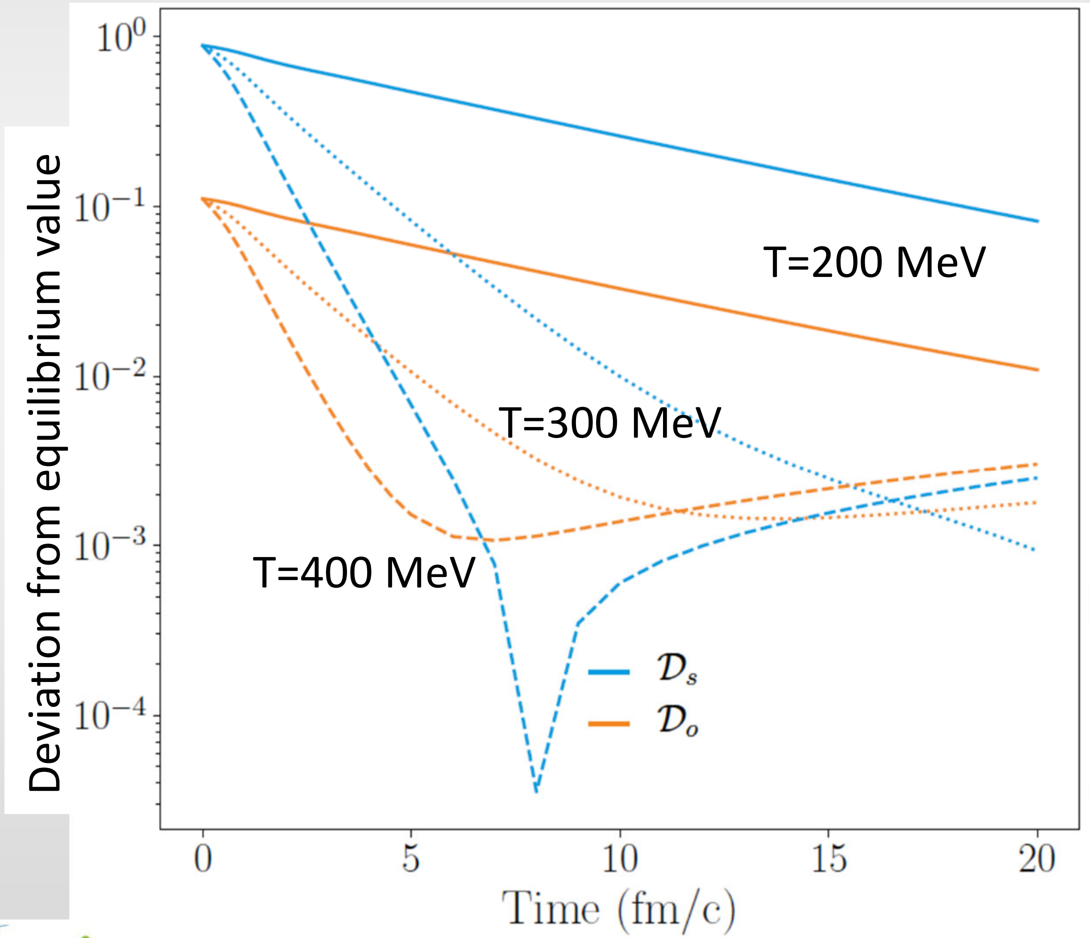


- At early times : Quasi exponential behaviour exp(-t/τ), with thermalisation time $\tau_o < \tau_s \approx 2$ fm/c
- At later time : Saturation possibly due to the grid size.
- Color appears to thermalize on time scales < QGP life time, but not instantaneoulsy.

Results

<u>Color Dynamics : Singlet – octet probabilities:</u>

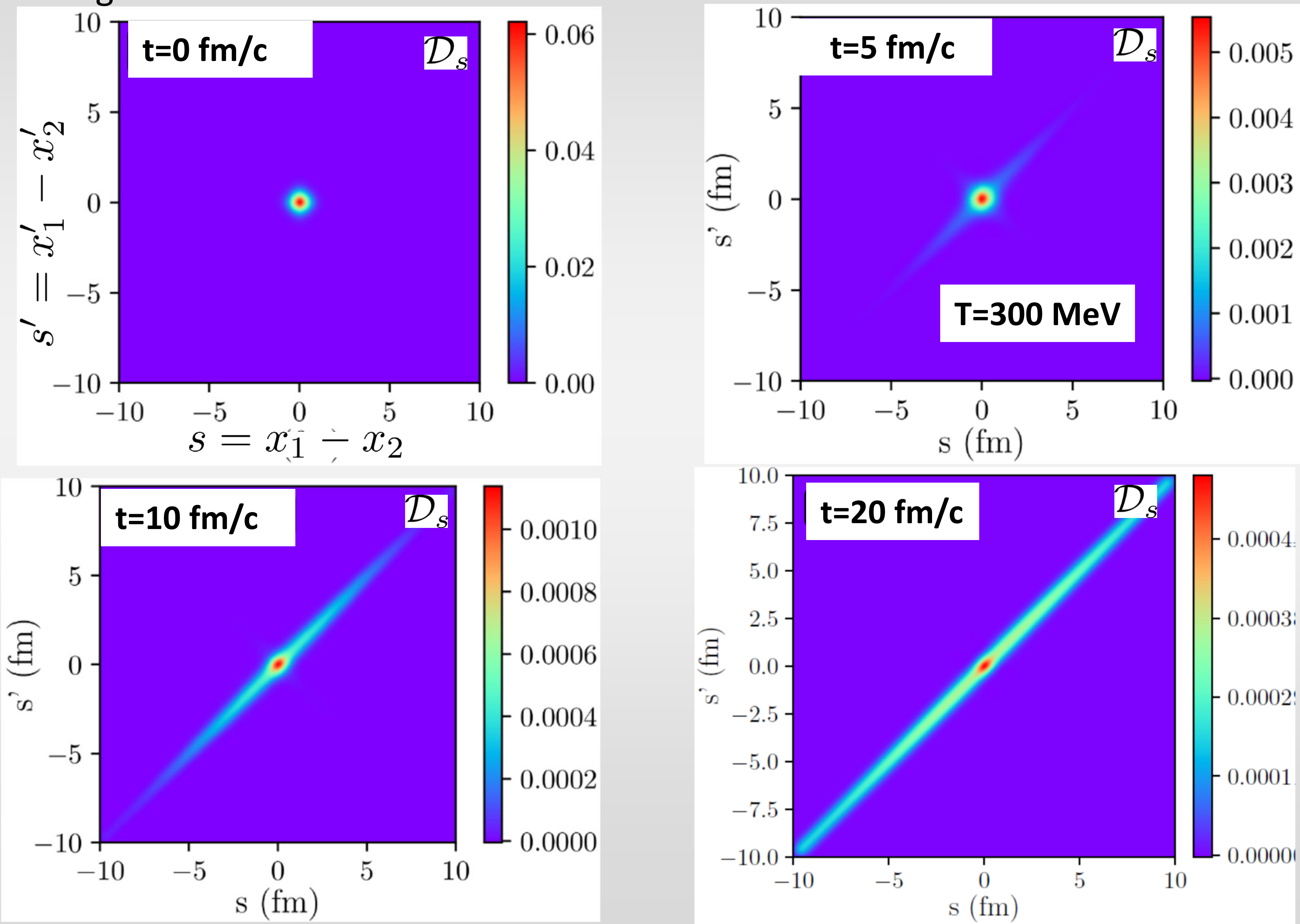
- Starting from singlet states with different QGP temperatures
- Study the deviations $|D_s - D_s^{\mathrm{eq}}|$ and $|D_o - D_o^{\mathrm{eq}}|$



- As expected, thermalisation time τ_{singlet} decreases for higher temperature.
- In concrete scenarios, might justify the « fast color equilibration » which later survive at smaller temperature.

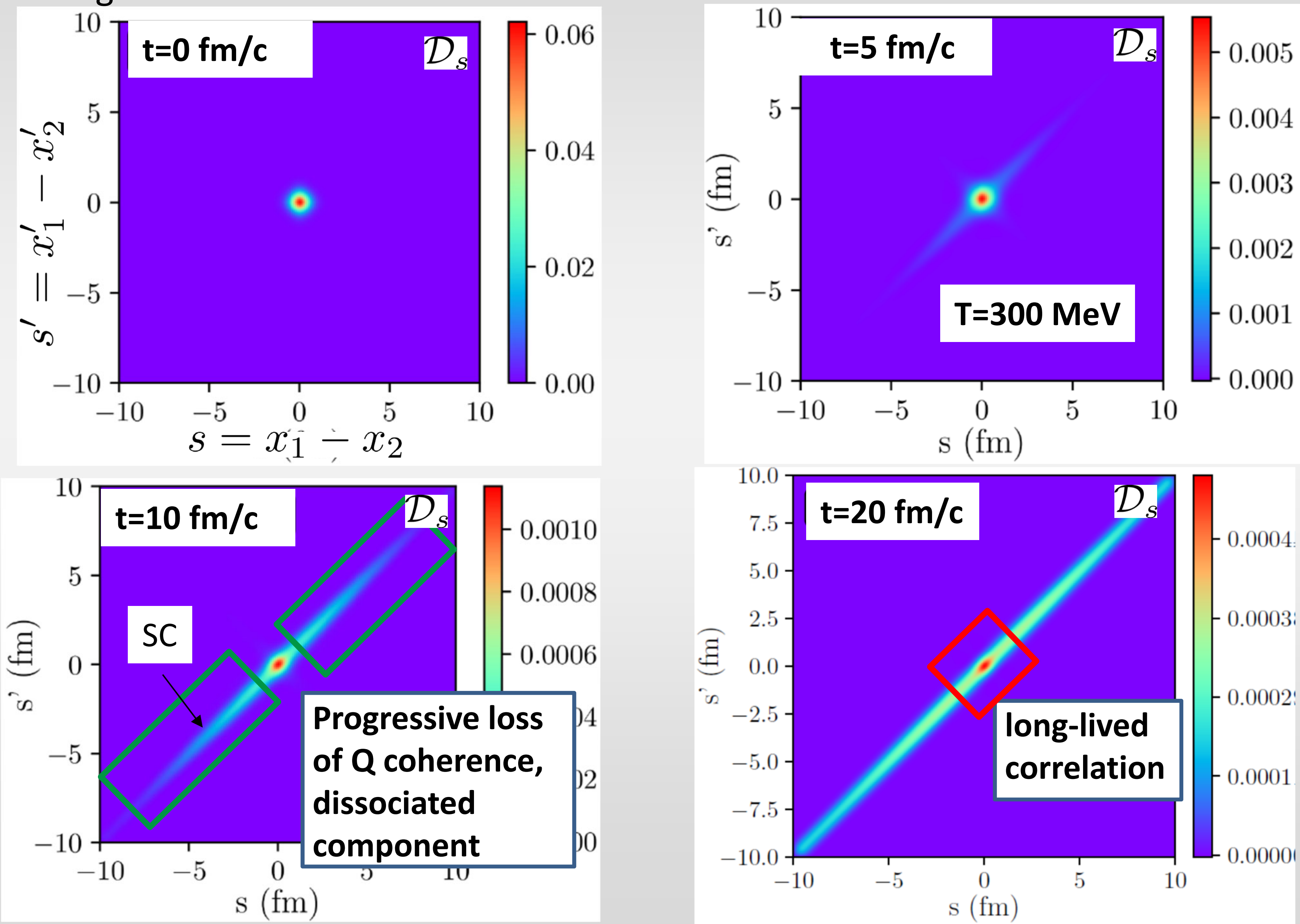
Results for Density matrix

1S singlet initial state:



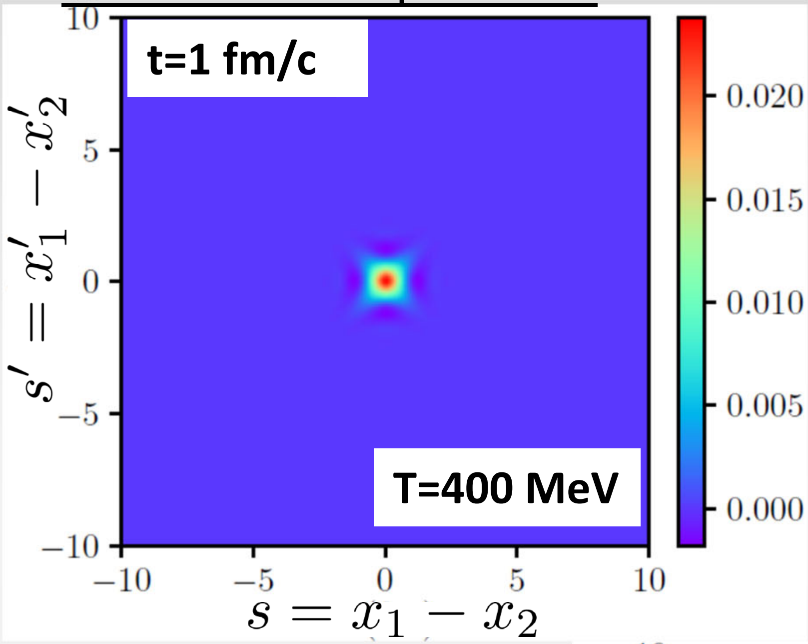
Results for Density matrix

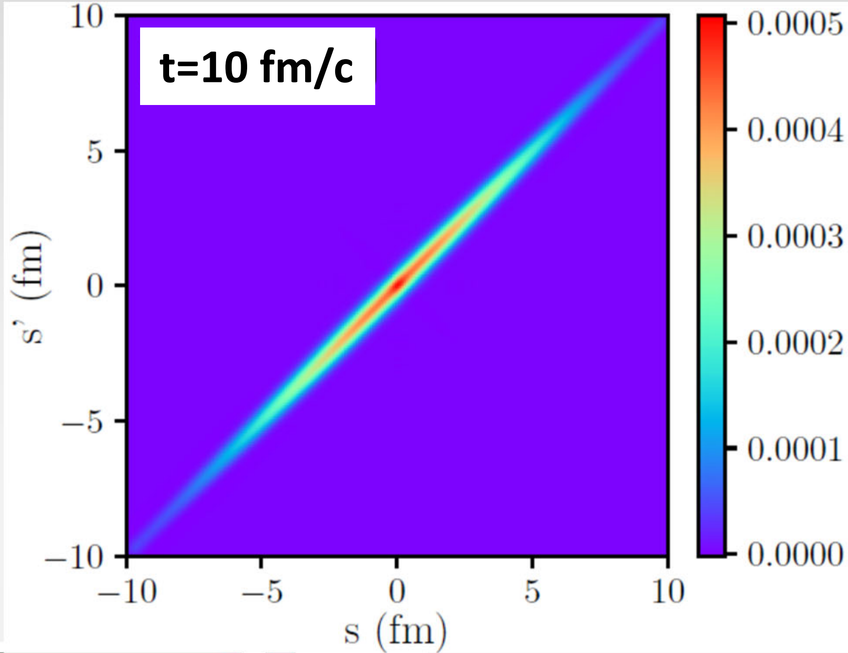
1S singlet initial state:

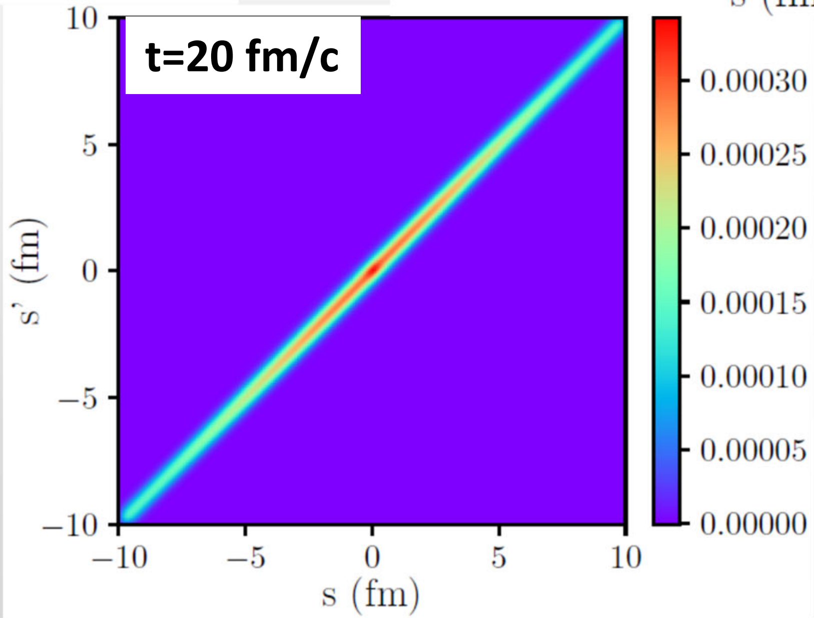


Results for Density matrix

Role of the temperature:



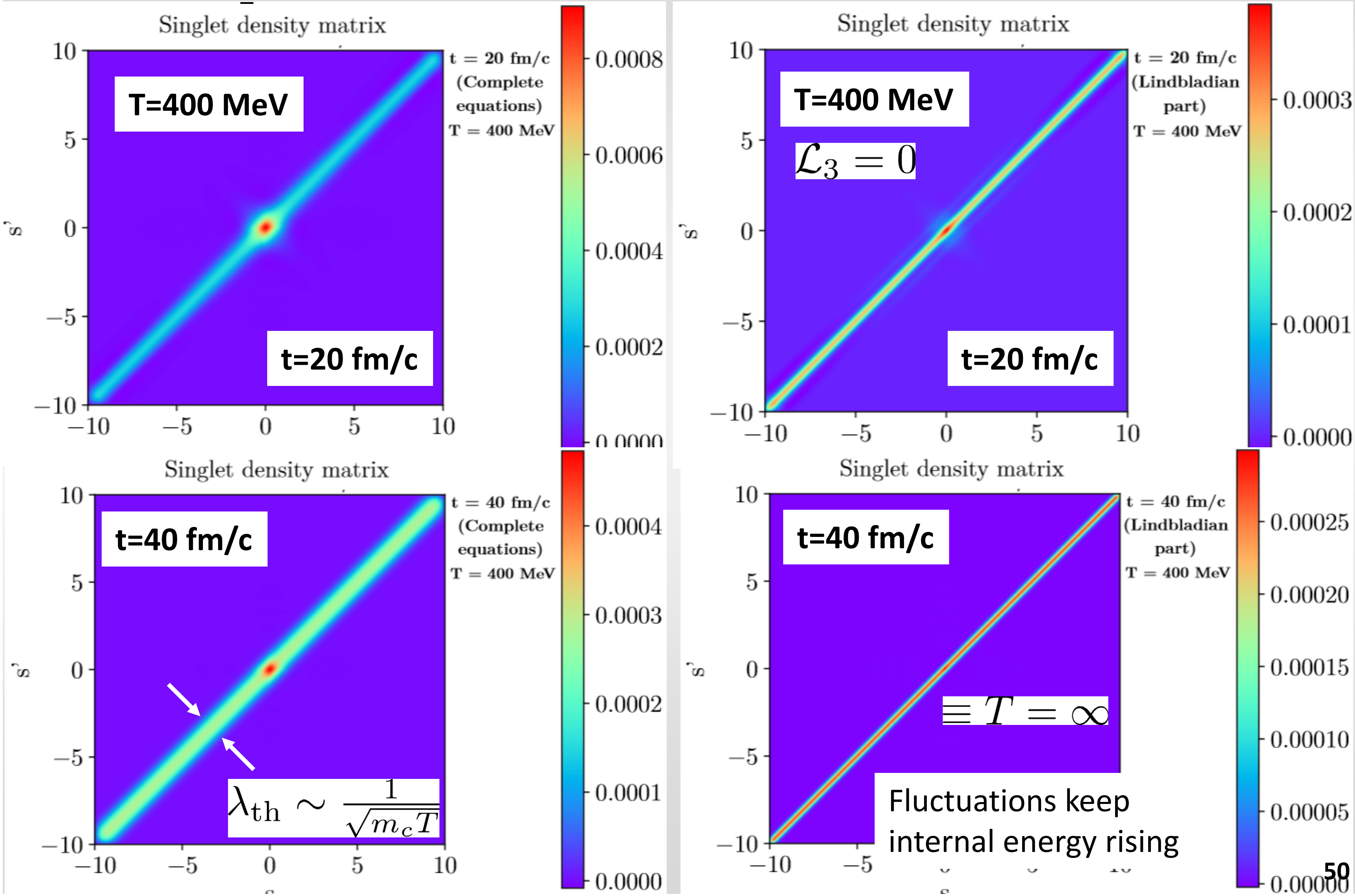




Larger T, faster decoherence

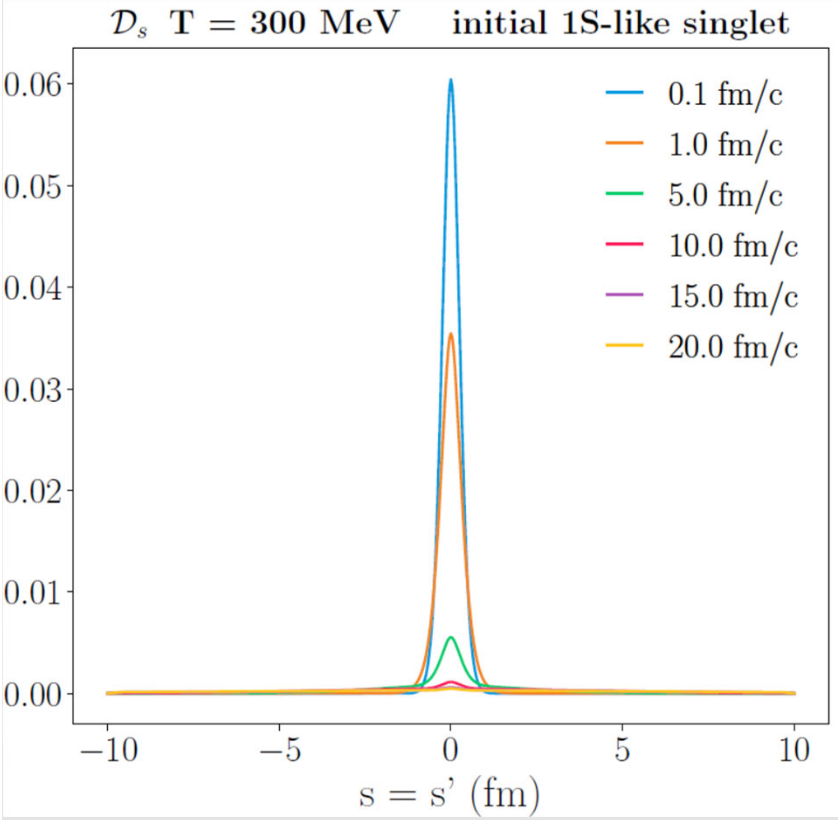
Results for Density matrix

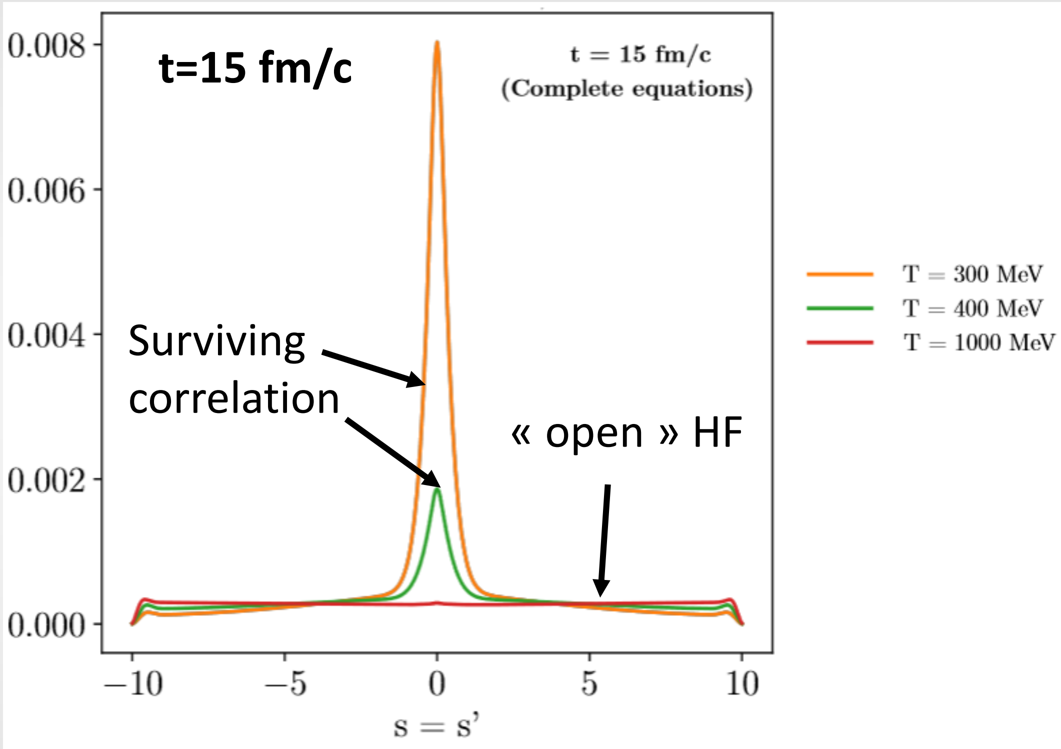
Role of the $\mathcal{L}_3$ dissipation-term:



Results for Density

$$\rho_s(s) = D_s(s, s' = s)$$



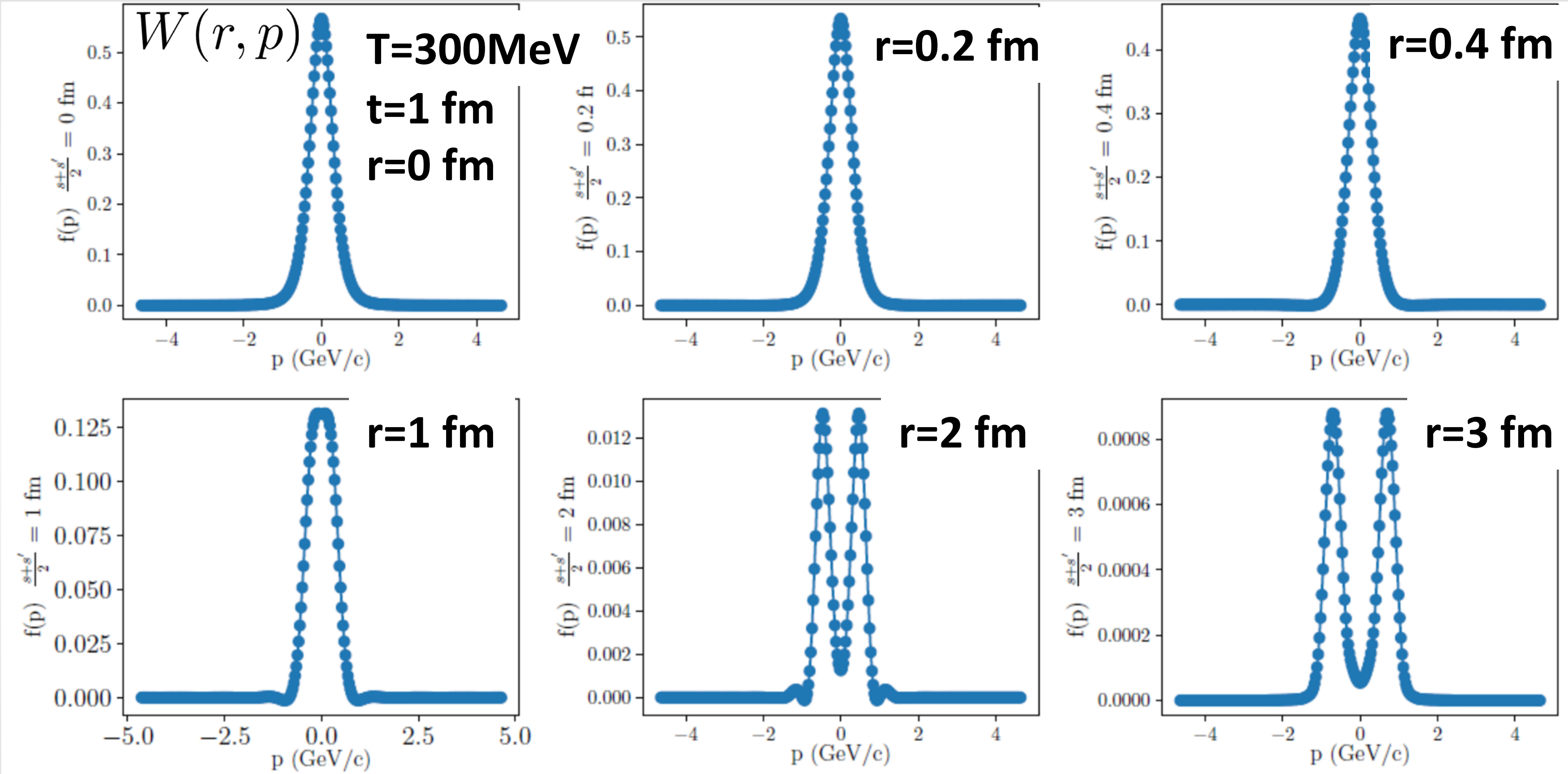


At a given T, increasing delocalisation with time

At a given time t, increasing delocalisation with T

Results for Density

Semi-classical analysis: computation of the discretized Wigner transform W(r,p) of D_s for different values of $r = \frac{s+s'}{2}$ (≡ position in a semi-classical approach)

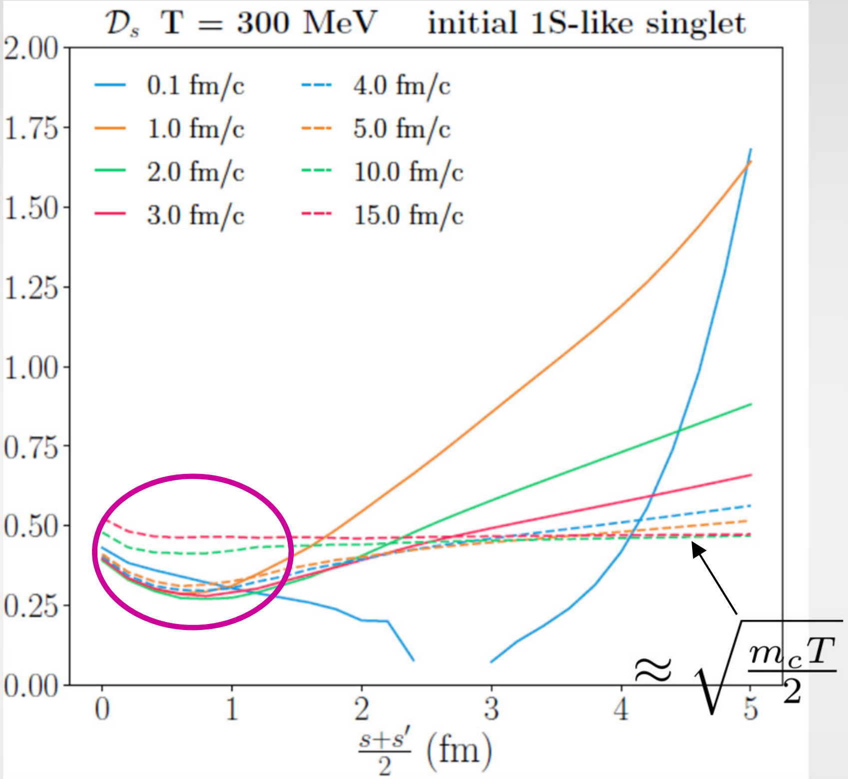


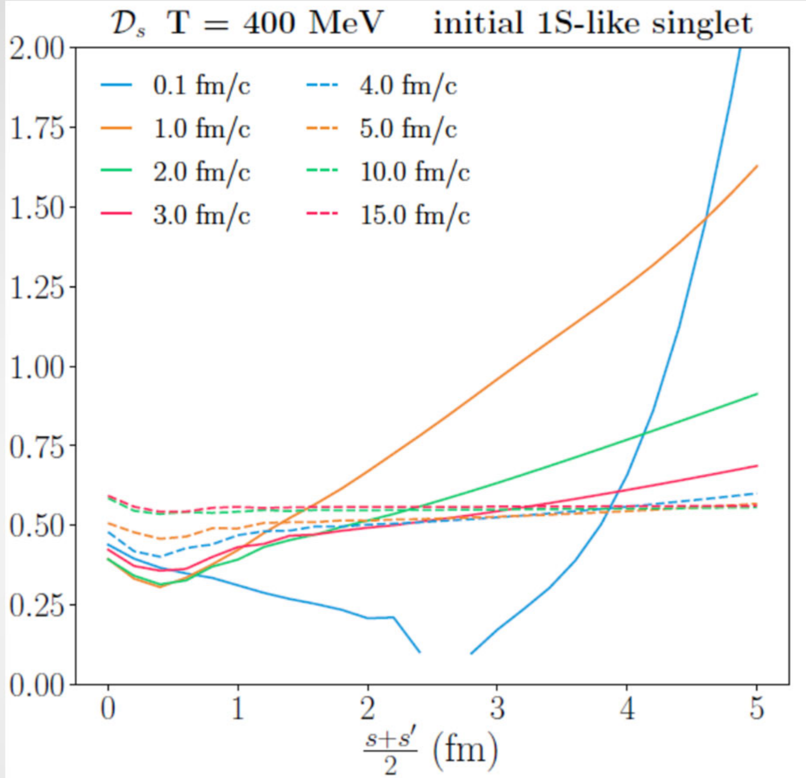
For a wide set of (t,r) : positive defined, Gaussian-like… However, some non Gaussian shapes observed as well

For large r – however supra luminous – some negative shoulders are observed.

Results for Density

Semi-classical analysis: Next compute the r.m.s. p : $\sqrt{\langle p^2 \rangle_W}$



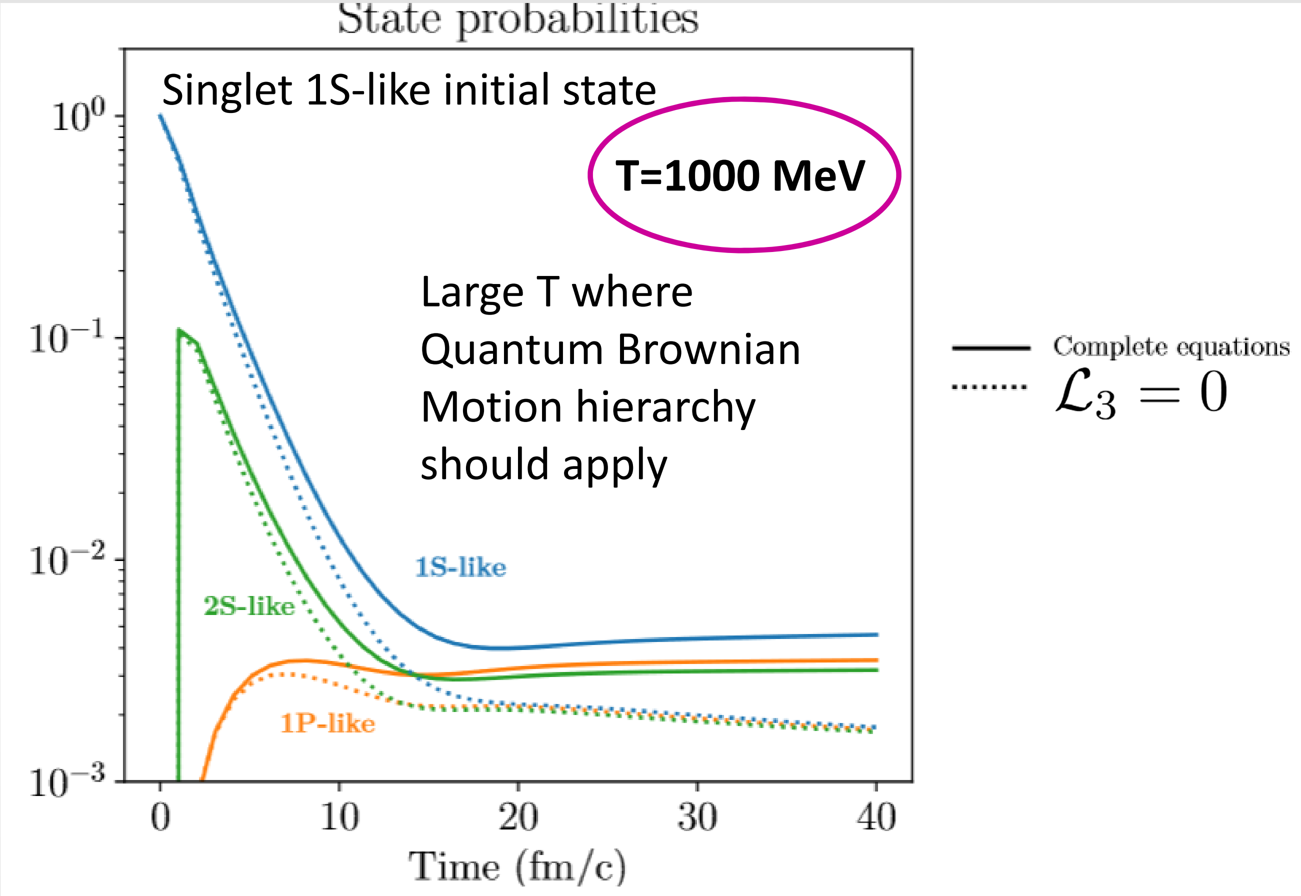


- At asymptotic times : convergence -> thermal value whatever $c\bar{c}$ distance
- At early times : some undefined $\sqrt{\langle p^2 \rangle_W}$ due to the negative shoulders. Genuine quantum effect, however at supra luminous separations
- For intermediate times : survival of the $c\bar{c}$ correlation at small distance, with r.m.s. p < thermal value (cold state need some time to heat up)… How realistic is it described by SC equations ? Under investigation.

Results for projection on vacuum states

!!! Vacuum states ≠ eigenstates at local T

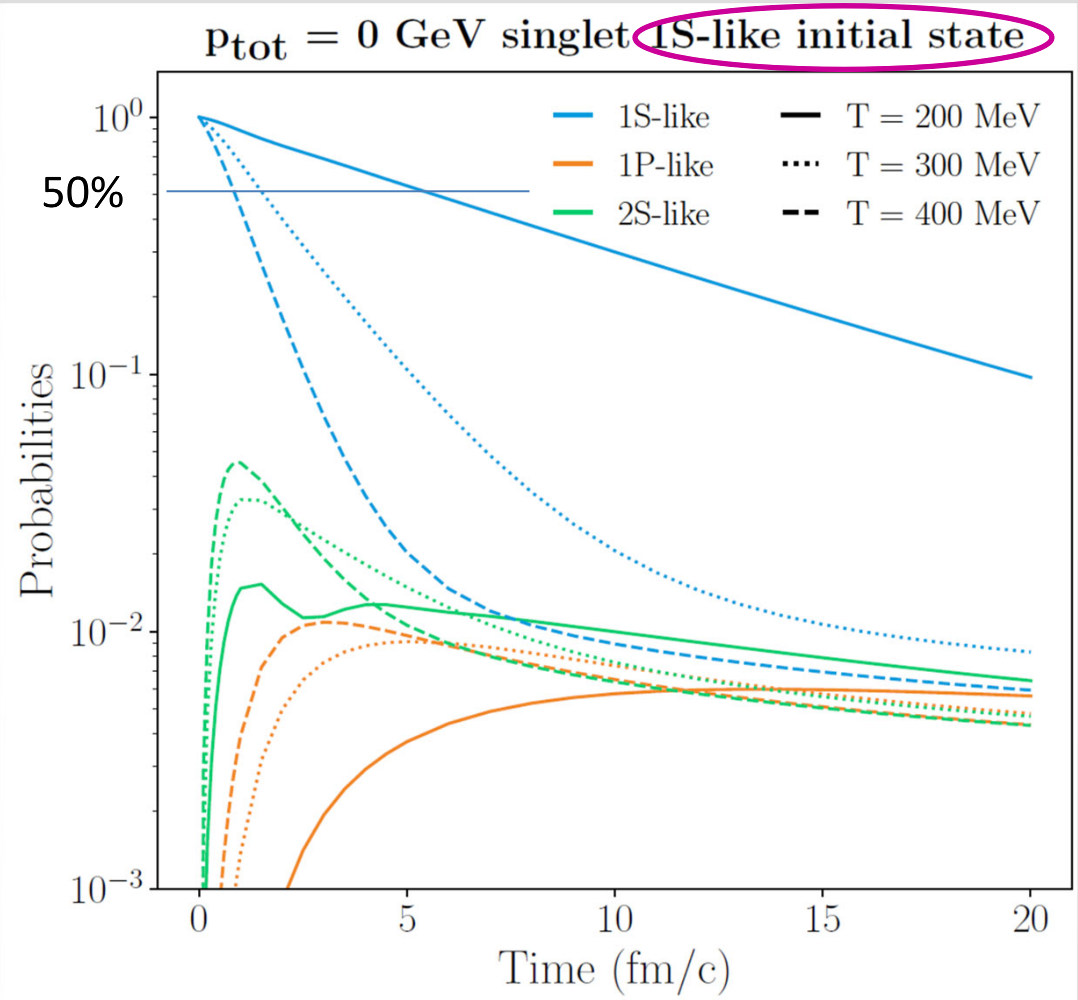
Gedanken experiment : instantaneous cooling down -> T=0 after t in QGP



- At small times, $\mathcal{L}_3 \ll \mathcal{L}_2$ fluctuations dominate… higher state repopulation
- At late times, $\mathcal{L}_3 \sim \mathcal{L}_2$ leading to asymptotic distribution of states. If $\mathcal{L}_3 = 0$, no dissipation => internal energy keeps rising.

Results for projection on vacuum states

For more « realistic » temperatures



Pretty complex interplay between binding, diffusion and transitions between states

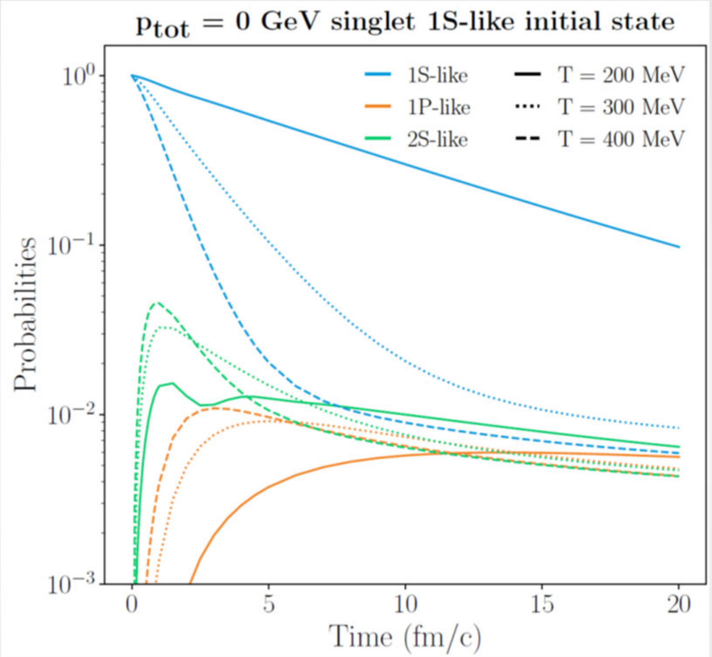
- Faster (and larger) suppression for larger QGP temperature
- Transient phase up to 5 fm/c : re-equilibration
- Common evolution (decrease) of all states at large times for T=300 and 400 MeV

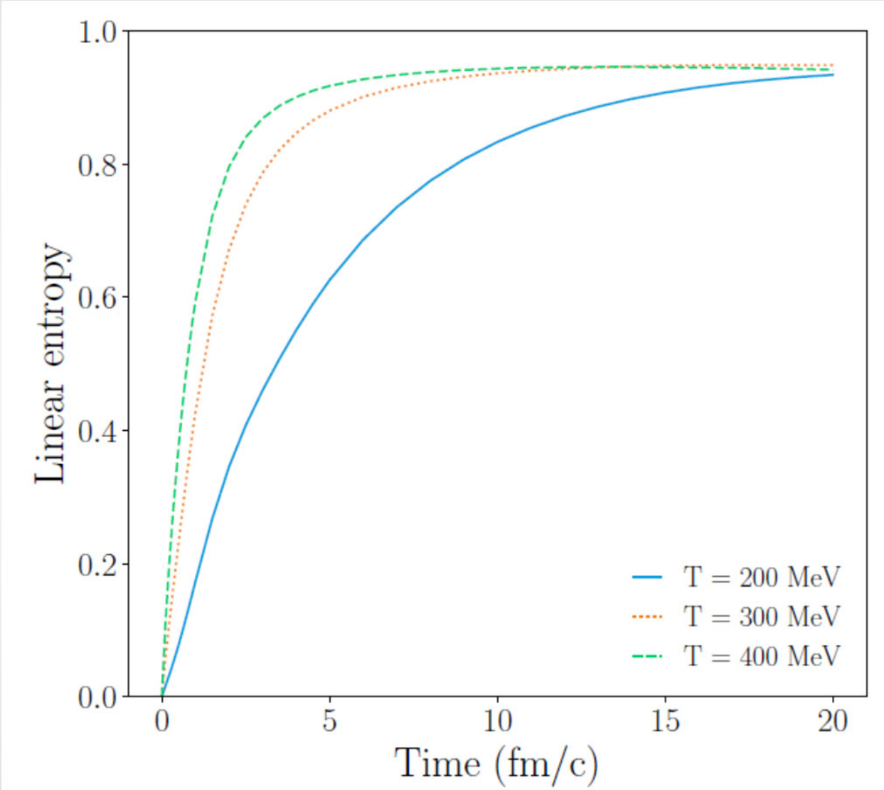
Results for Linear quantum entropy

$$S_L = \mathrm{Tr}\hat{\rho} - \mathrm{Tr}\hat{\rho}^2 = 1 - \mathrm{Tr}\hat{\rho}^2$$

De Boni, J. High Energ. Phys. (2017) 2017: 64

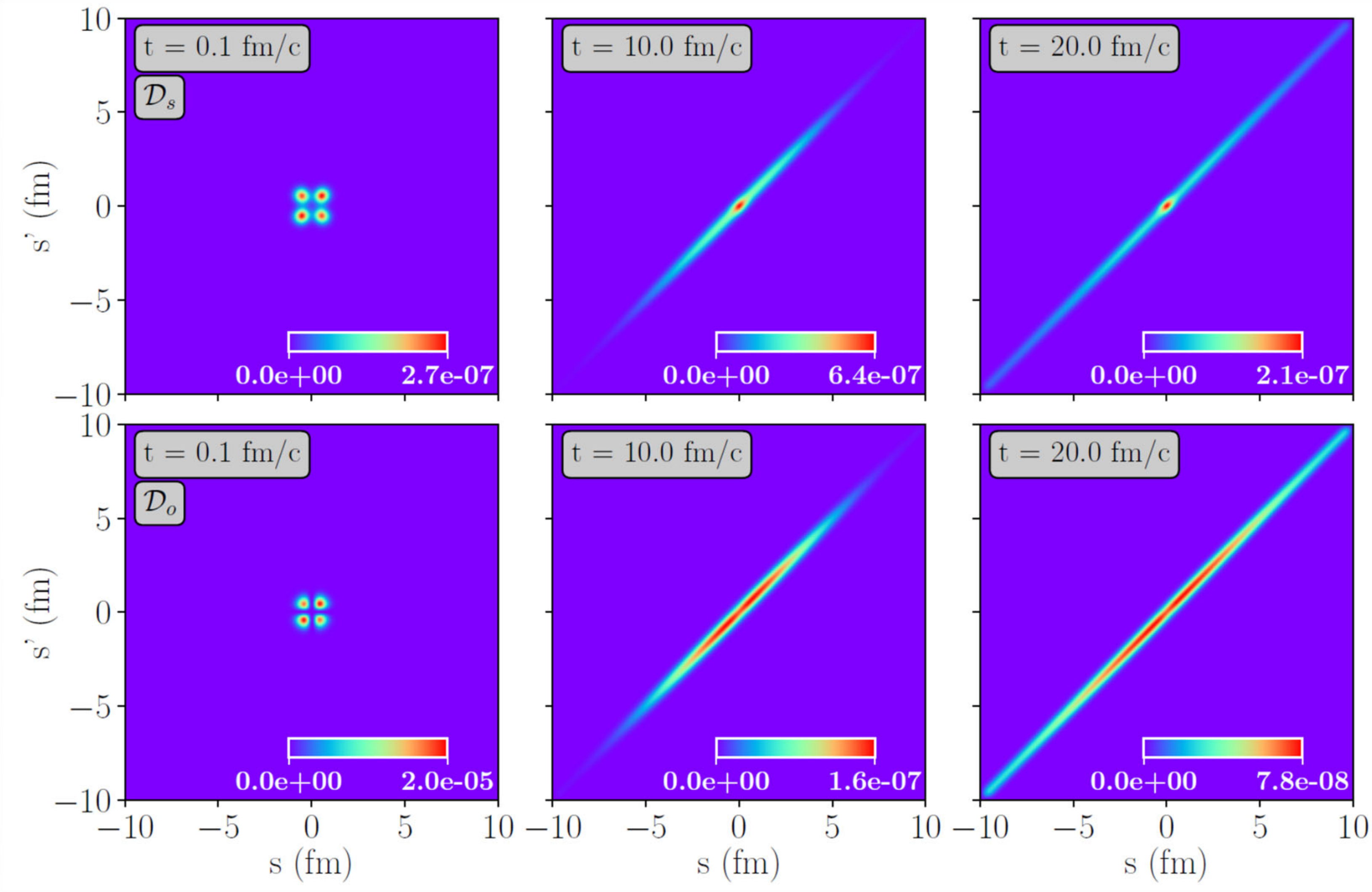
(results for QED like evolution)

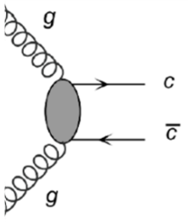




- Suppression and decoherence appear to happen on the same time scale…
- … does not seem in favour of applying classical rate equations (to be investigated further)

Results for more realistic octet Initial State

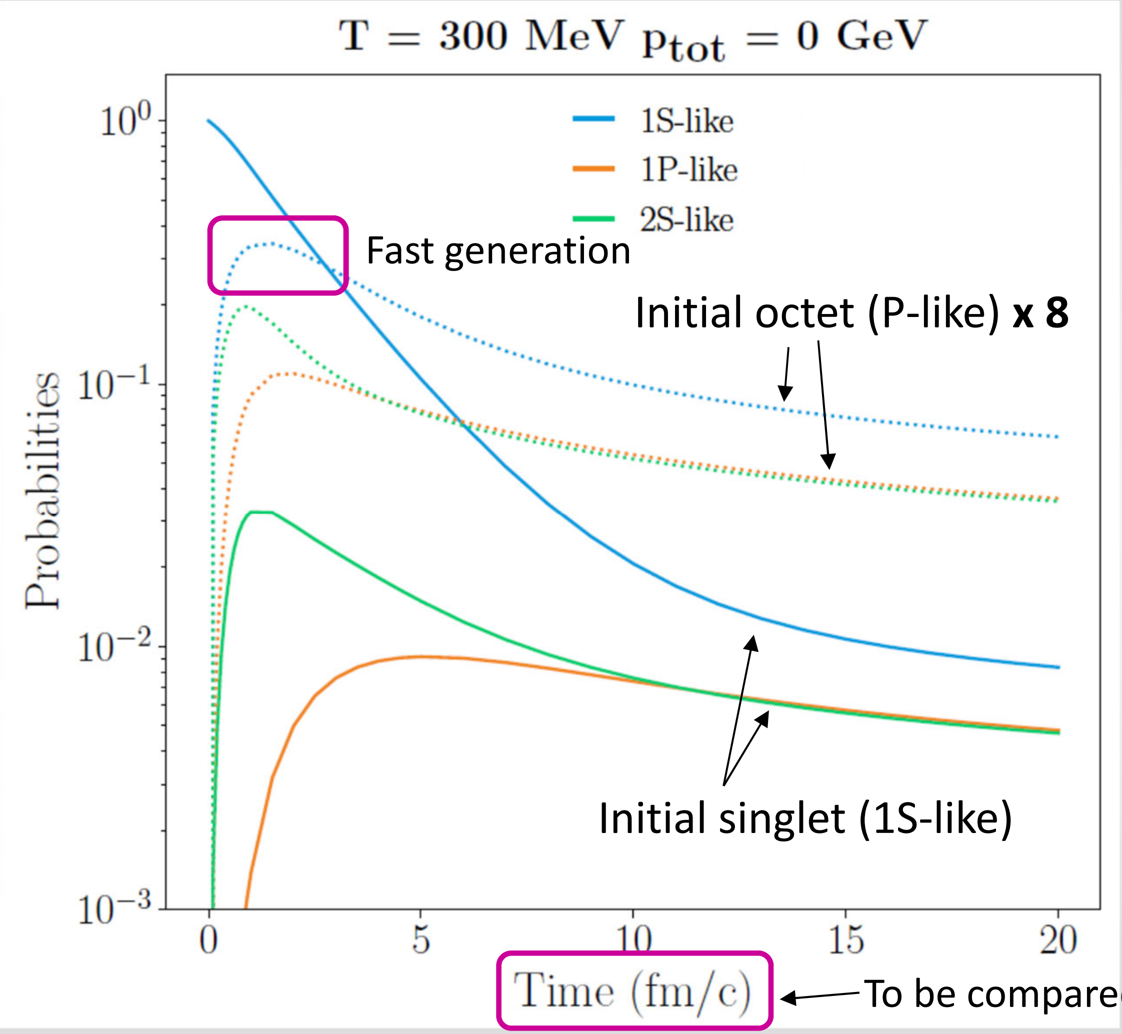


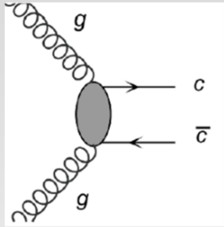


- Initial population of Ds shows a node at x=0 due to dipolar transitions… However, similar asymptotic behavior as for the singlet initial state.
- Delocalization of initial state along s = s′ axis (especially in the octet channel)

Results for more realistic octet Initial State

Projection on vacuum states:





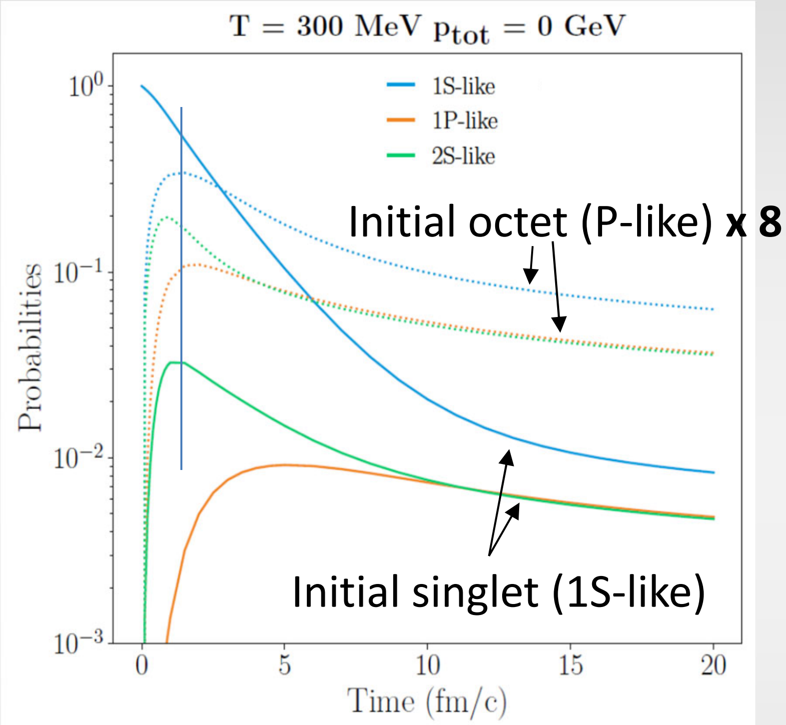
To be compared with QGP lifetime or path length

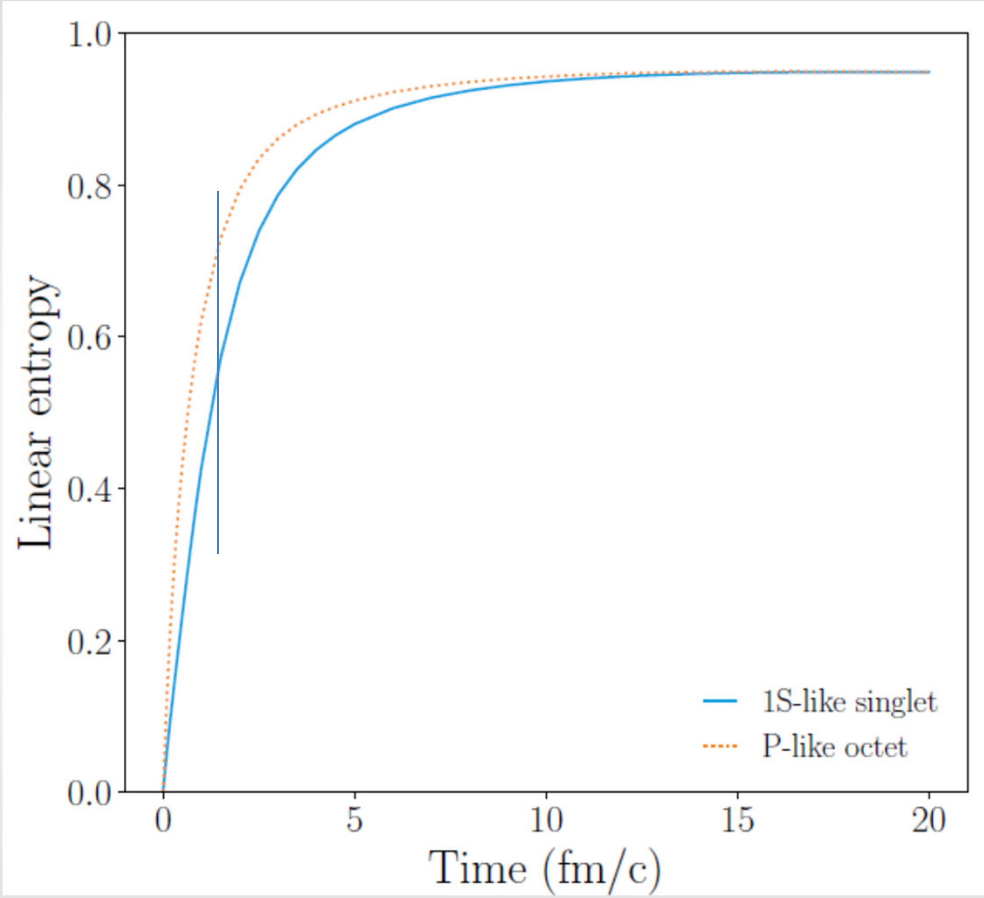
No effective exp(-Γt) decrease of the 1S probability

- Rather fast initial population of singlet 1S due to color transitions induced by QGP degrees of freedom.
- Similar late time asymptotics (memory loss of the initial color state)

Results for more realistic octet Initial State

Linear entropy:

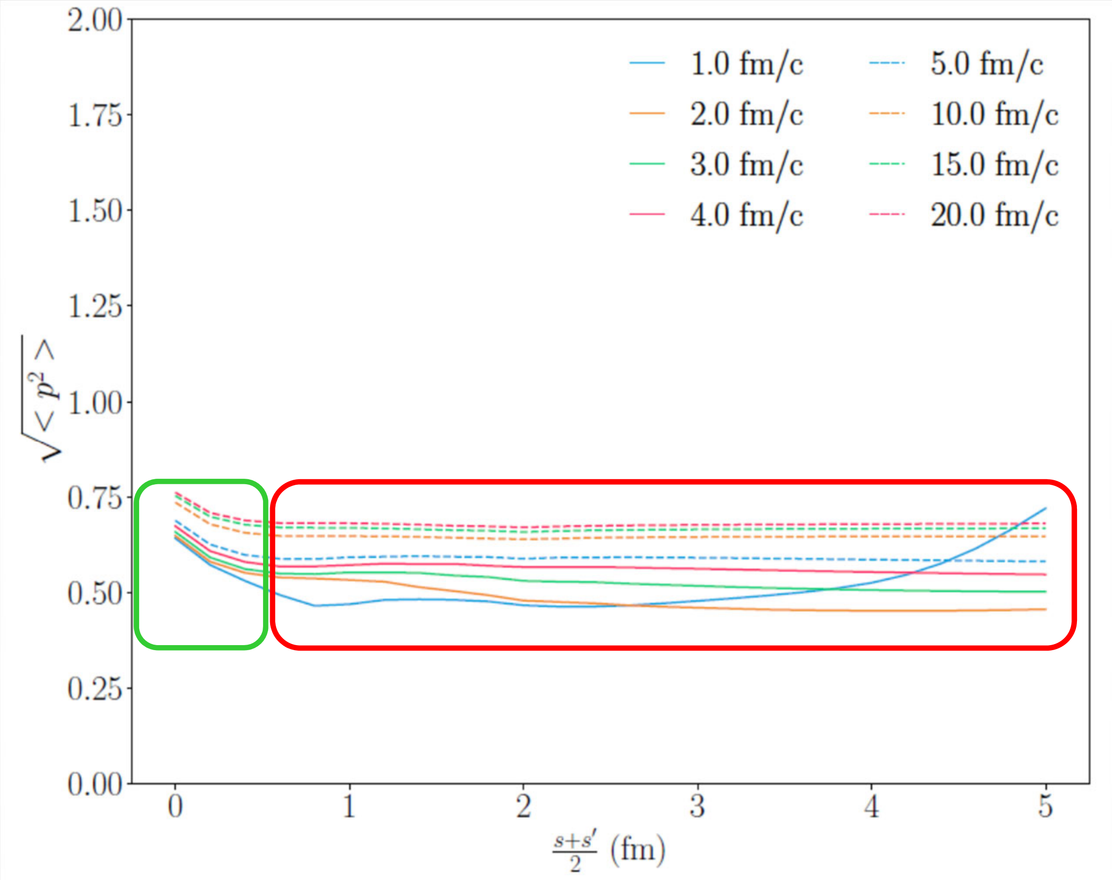




- Faster decoherence from initial P-like octet
- Generation of the 1S-singlet when quantum entropy has already increased -> saturation… but it may be misleading to conclude that quantum features are negligible

Results for more realistic octet Initial State

Wigner transform:

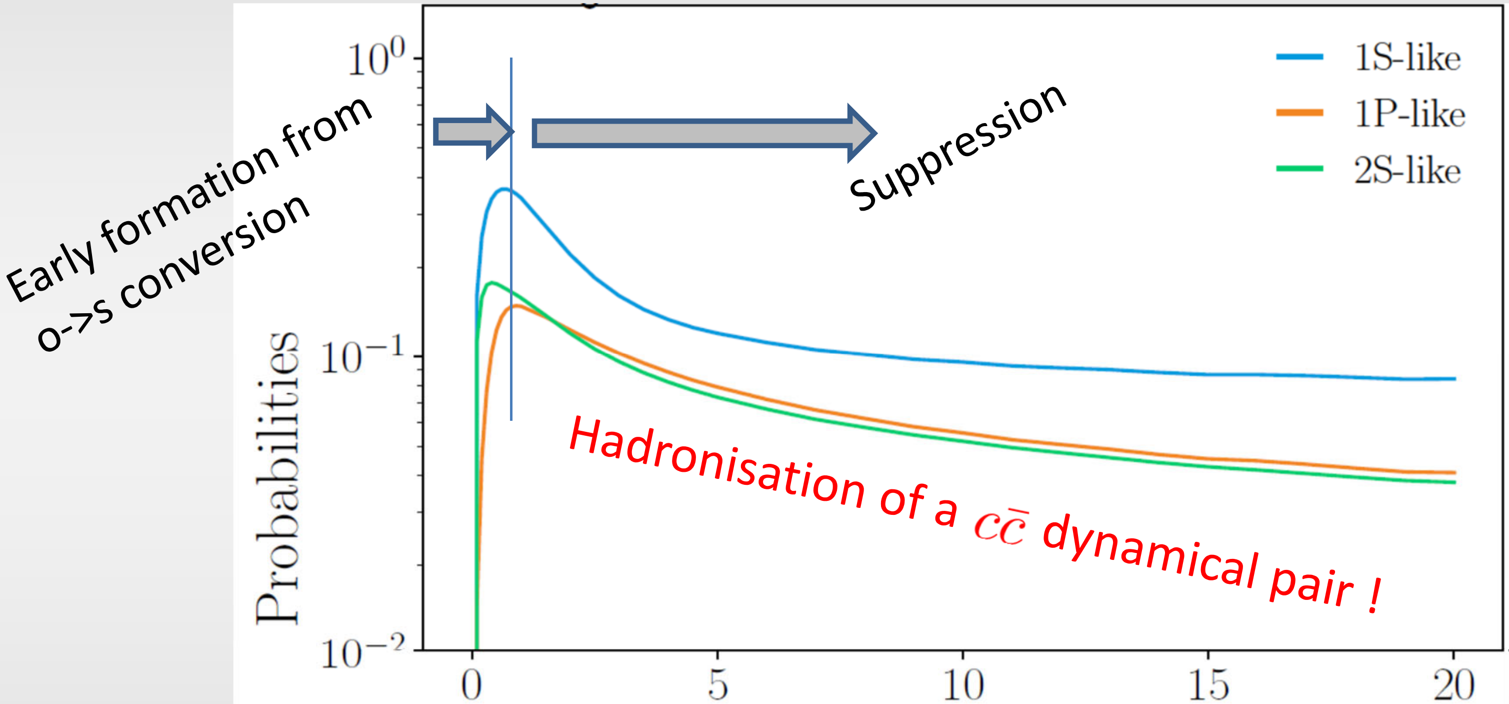


- Convergence -> thermal value for most « large » relative distance
- Deviation at small relative distance, footprint of the long lasting correlation

Results for more realistic octet Initial State

Now with cooling medium:

- Bjorken-like evolution of the temperature : $T(t) = T_0 \times \left(\frac{\tau_0}{t+\tau_0}\right)^{\frac{1}{3}}$ $T_0 = 600\,\mathrm{MeV}$ $\tau_0 = 1\,\mathrm{fm}$



- Bound state formation at **early times**… (rather opposite to the statistical hadronization picture… however not “exogeneous” pair => to be taken with a grain of salt)
- Even moderate *repopulation* of the ground state at late times… can be understood as the cooling of the level distribution.

Conclusions and Perspectives

- 35-40 years after the concept of « quarkonia as hard probe », the field is now evolving in the direction of imbedding quantum features in the theoretical treatment ! Still many challenges to solve and thus fantastic field for the youngstars (and not so young) generation

- Some « historical assumptions » (exponential decay, instantaneous coalescence, adiabaticity) do not seem to be supported by modern ab initio microscopic calculations…

- … In particular, the « re »combination process seems to require extended time to bloom fully.

- Semi-classical approximation has a limited range of applicability and needs to be better investigated… Need for more benchmark solutions

Subatech

Thank you !