Jet quenching and quantum applications

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The Quantumness of Hard Probes Workshop

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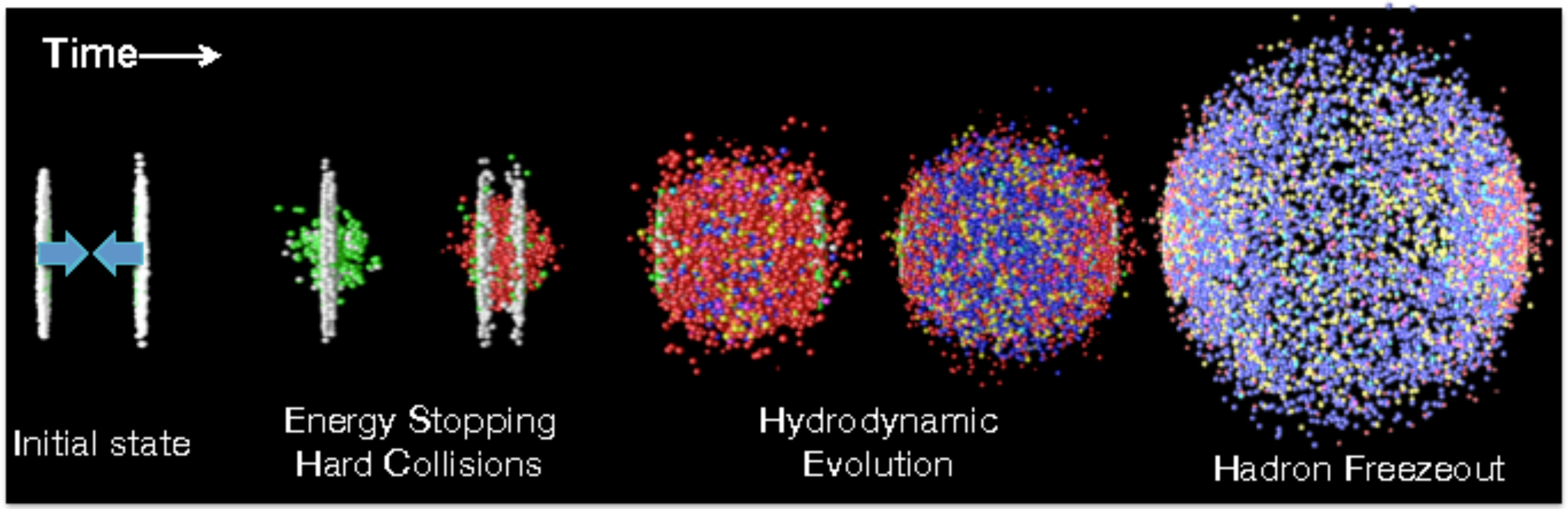
Outline

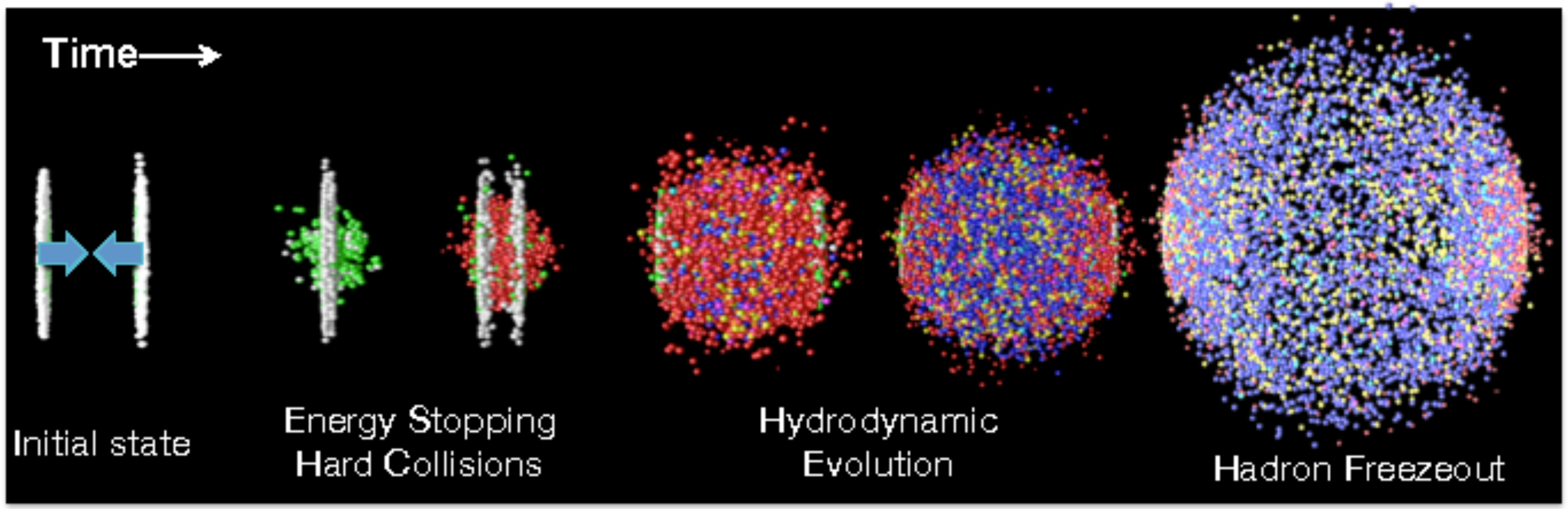
- Introduction to jet quenching
- non-relativistic (2+1 D) quantum mechanics formulation
- Application: medium-induced radiative spectrum
- Quantum entanglement of jets in the QGP (leading order)
- Jet quenching in the QIS era, final remarks

The little bang



- A short lived hot medium of deconfined matter (QGP) forms in ultra-relativistic heavy ion collisions (RHIC, LHC)
- Emergent phenomena from large number of d.o.f.: collectivity, turbulence, plasma instabilities, themalization, anomalous transport

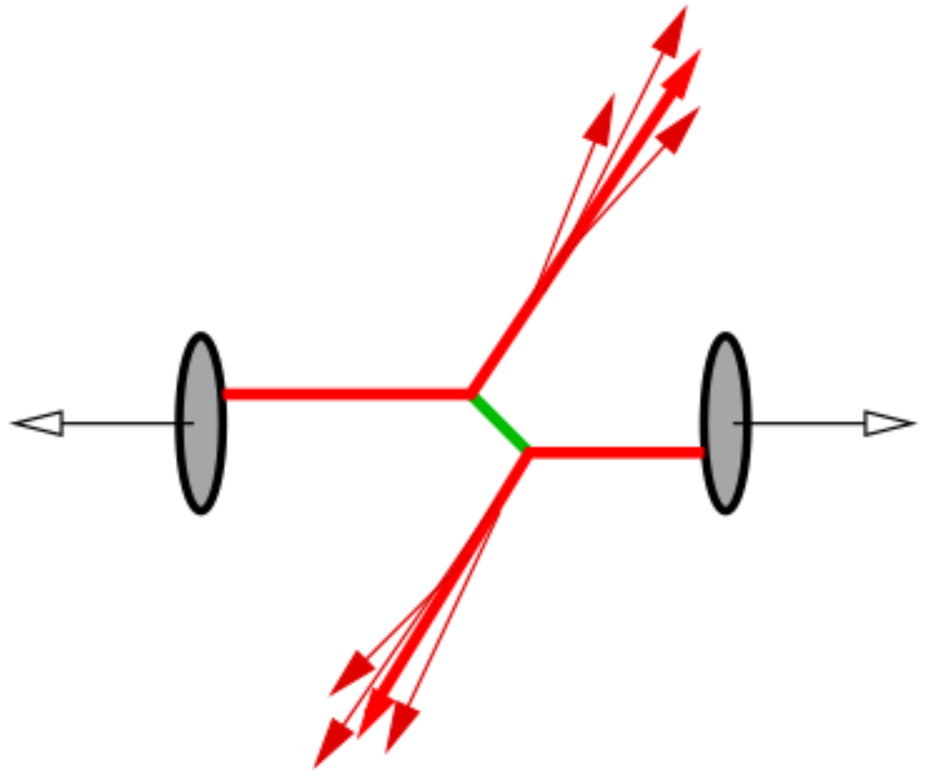
The little bang

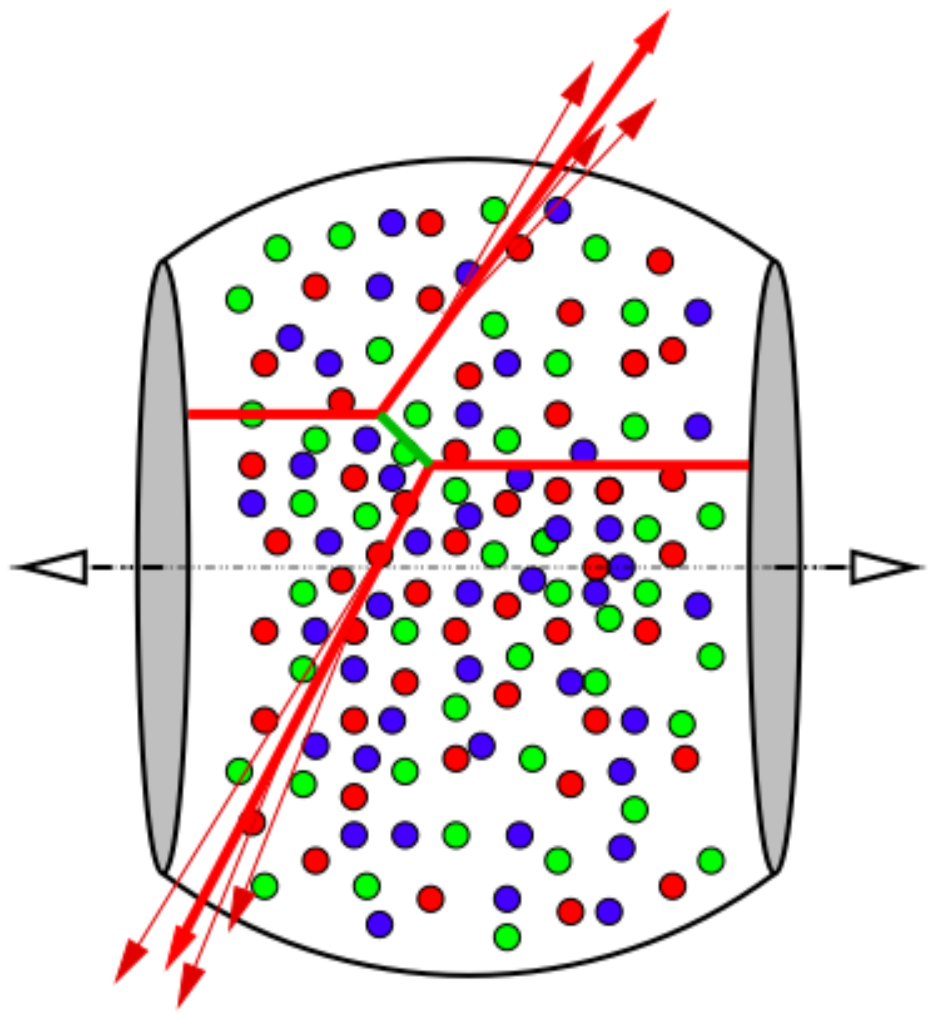


- Soft probes: collectivity at low momenta $p_t \lesssim 1-3$ GeV probed with flow harmonics v_n's
- Hard probes: quarkonia suppression (QGP thermometer), high pt hadrons and QCD jets

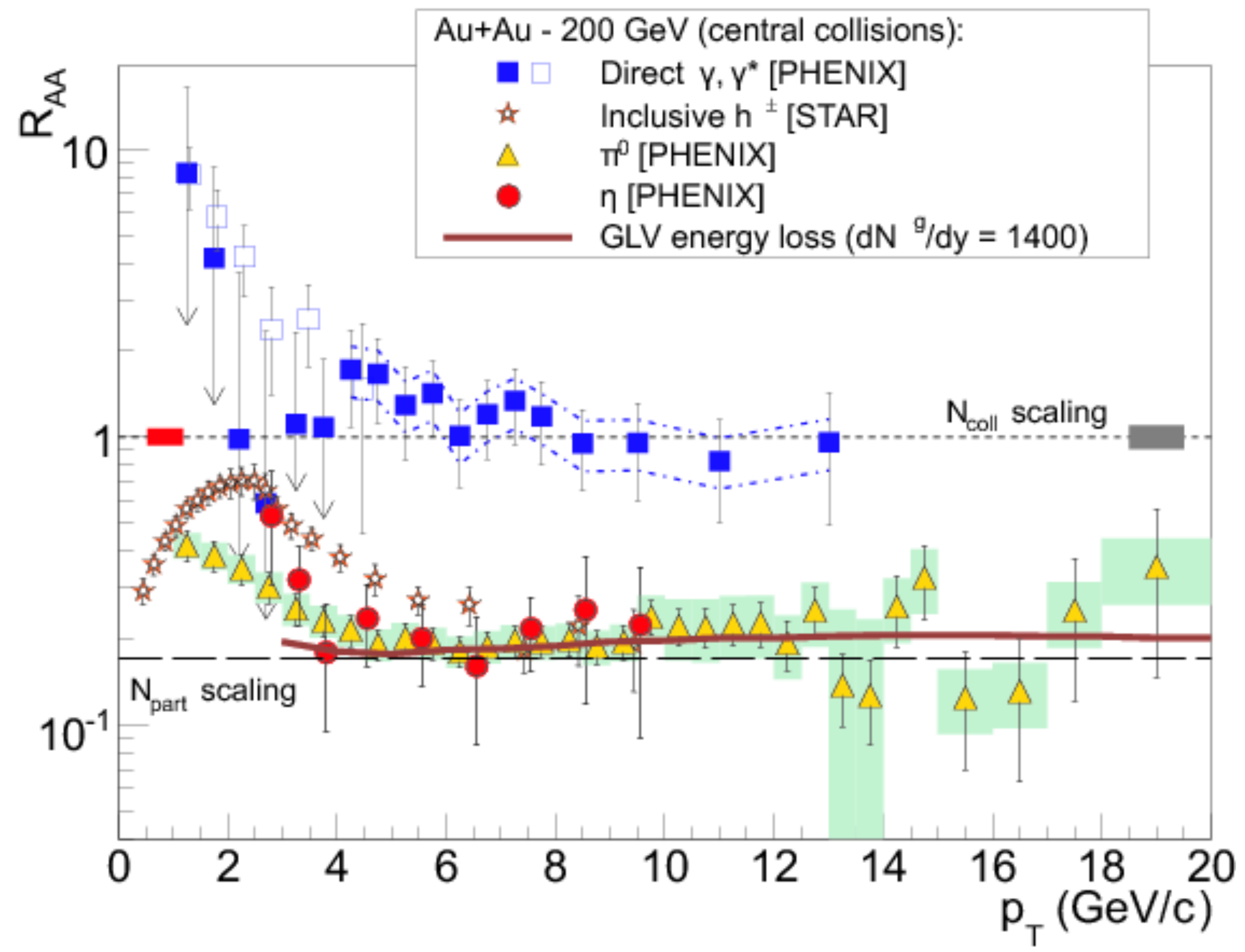
The discovery of the QGP

proton-proton nucleus-nucleus

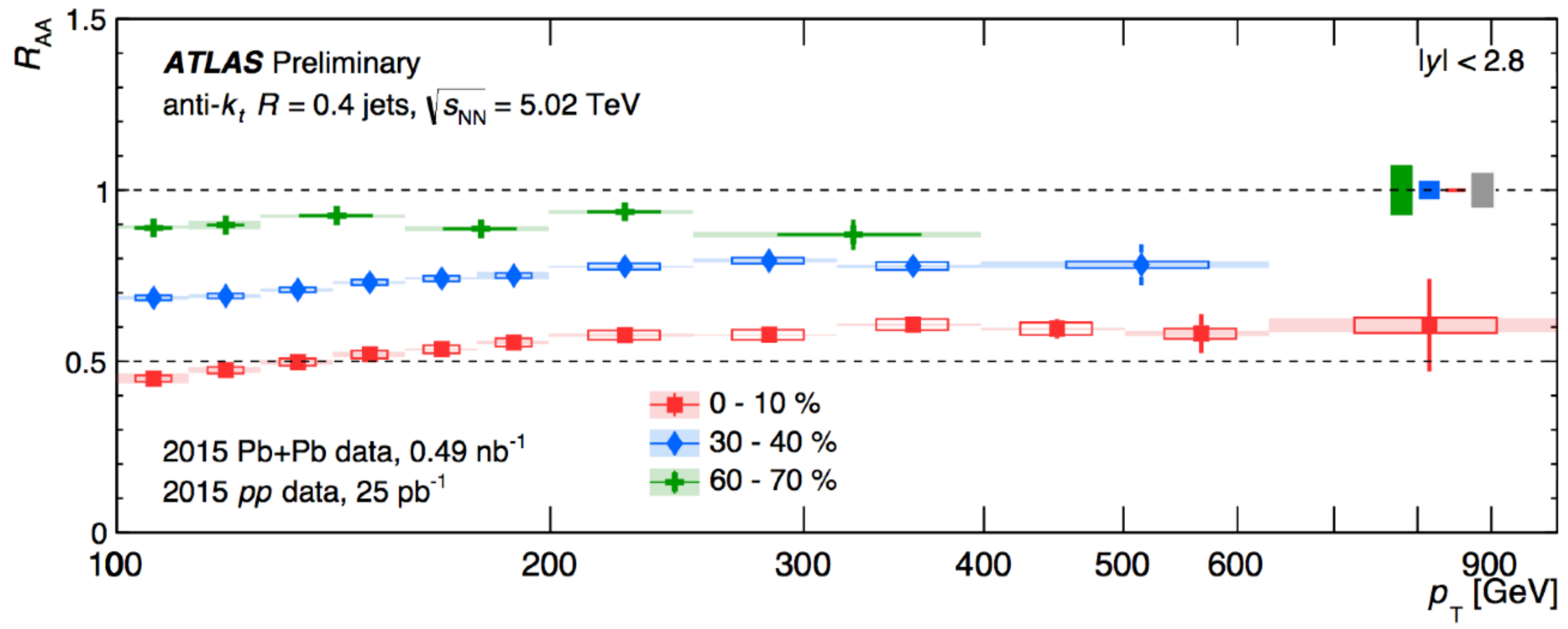




- Substantial final state interactions: jets lose energy to the QGP constituents
- Strong suppression and modification of jets observed at RHIC and LHC



RHIC 2000'

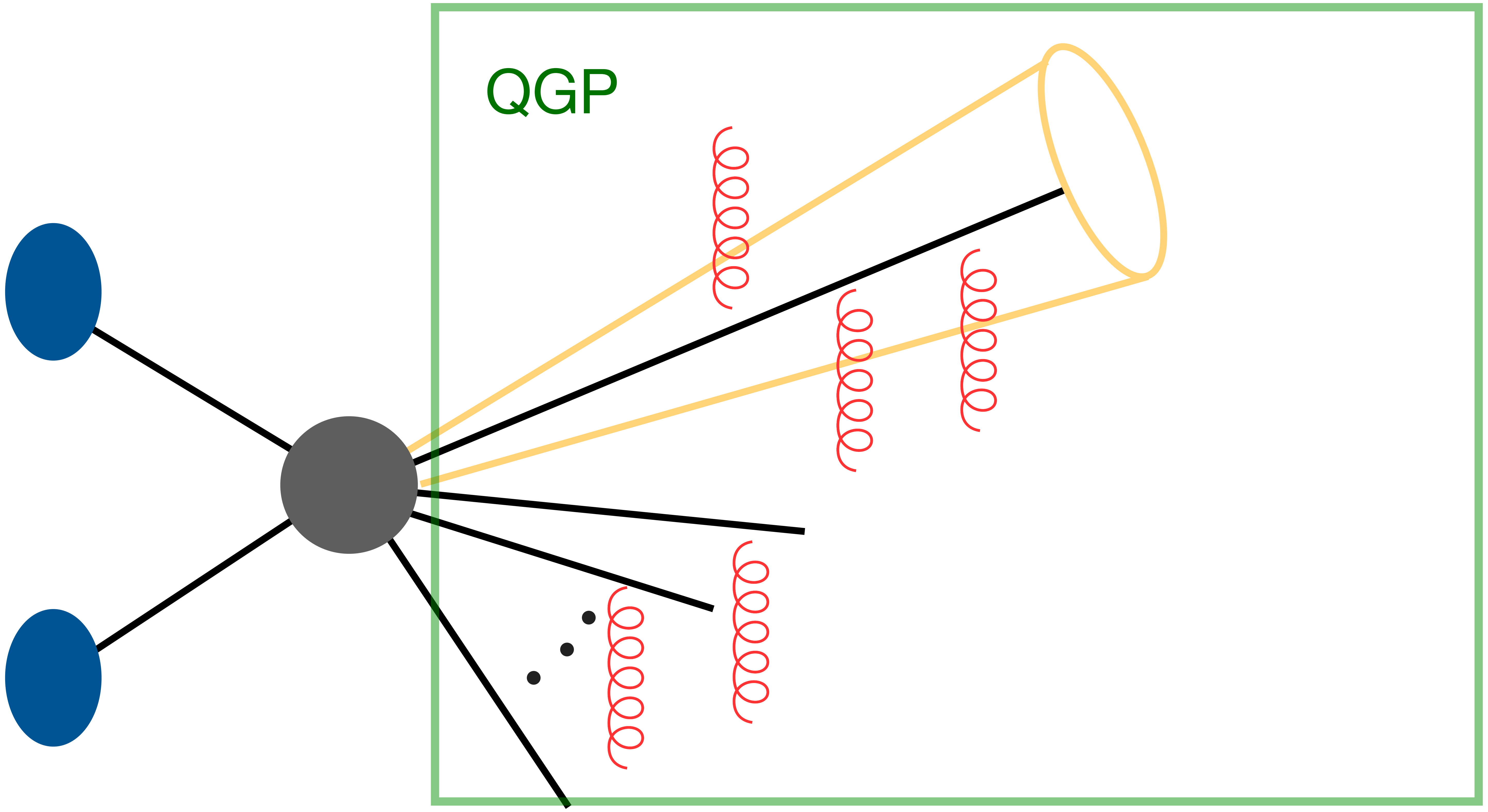


LHC 2010'

Nuclear modification factor

$$R_{AA} \equiv \frac{\text{Yield in AA}}{N_{\text{bin}}\text{Yield in AA}}$$

- Standard factorization approaches based on twist expansion not sufficient: MPI, underlying event, color randomization (decoherence)
- Strong final state interactions need resummations of all twists
- Perhaps the situation not dire in the thermodynamic limit



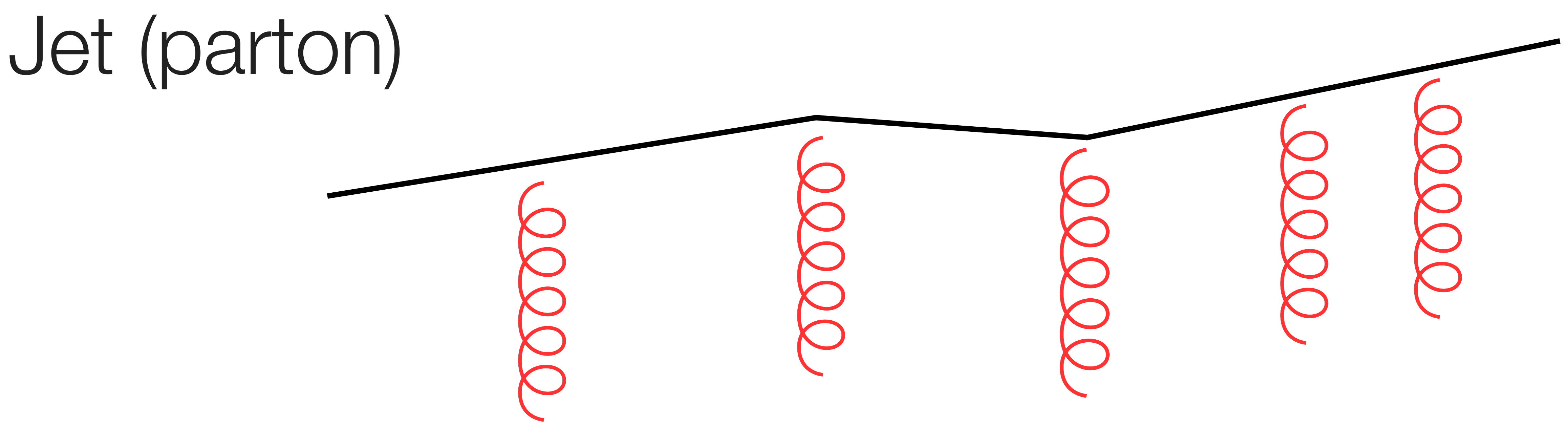
- What to expect: large number of degrees of freedom, coarse graining, unmeasured subsystems

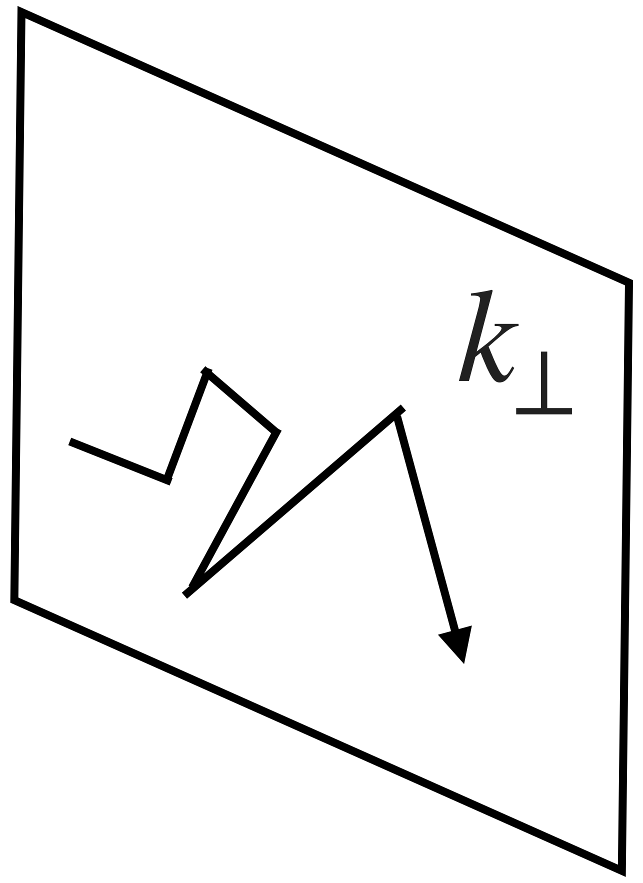
pure states ⟶ mixed states

- How do jets as quantum systems evolve in the presence of a QGP: quantum decoherence, entanglement, quantum/classical thermalization, color randomization…
- Need to define tools and observables

Underlying processes (elastic collisions)

- A fast parton (jet) accumulates a transverse momentum $k_\perp \ll E$ from multiple scatterings in the plasma



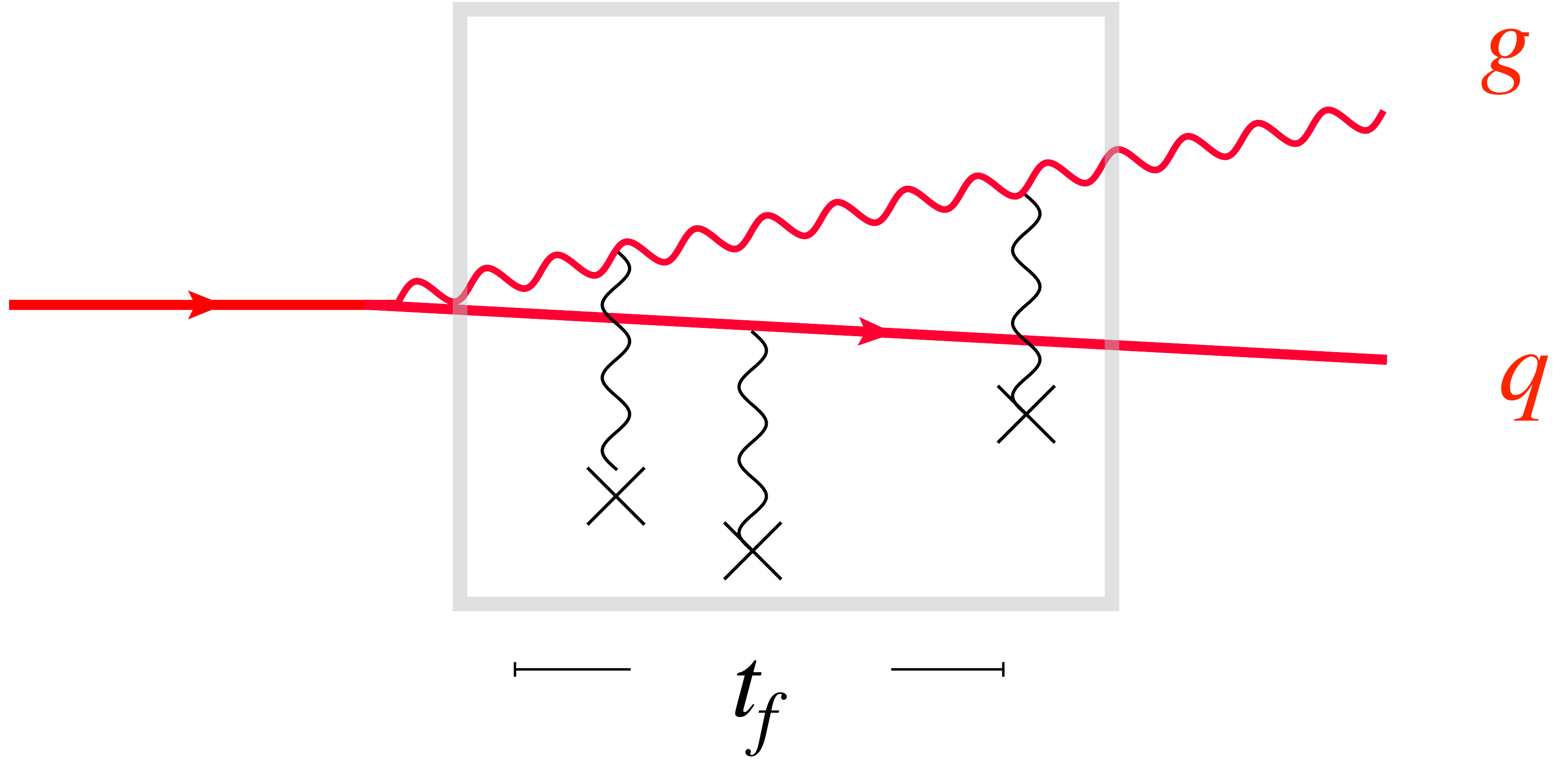


- Can be described by a Fokker-Planck equation

$$\frac{\partial}{\partial t} P(k_\perp) = \frac{1}{4}\,\hat{q}\,\nabla_\perp^2 P(k_\perp) \quad \Rightarrow \quad \boxed{\langle k_\perp^2 \rangle_{\text{typ}} \propto t}$$

Radiative processes and the LPM effect

- QCD analog of the Landau-Pomeranchuk-Migdal effect (1953): multiple scatterers act coherently as a single effective scattering center to induced gluon radiation during the quantum mechanical time $t_f(\omega) = \omega/k_\perp^2$



[Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996) Arnold, Moore, Yaffe (2001-2002)]

- Quantum interferences suppress the spectrum at large gluon frequencies

$$k_\perp^2 \sim \hat{q}\, t_f \quad \Rightarrow \quad t_f(\omega) = \sqrt{\frac{\omega}{\hat{q}}}$$

and

$$\omega \frac{\mathrm{d}I^{LPM}}{\mathrm{d}\omega} \sim \alpha_s N_{\mathrm{eff}} = \alpha_s \frac{L}{t_f(\omega)} \sim \frac{1}{\sqrt{\omega}}$$

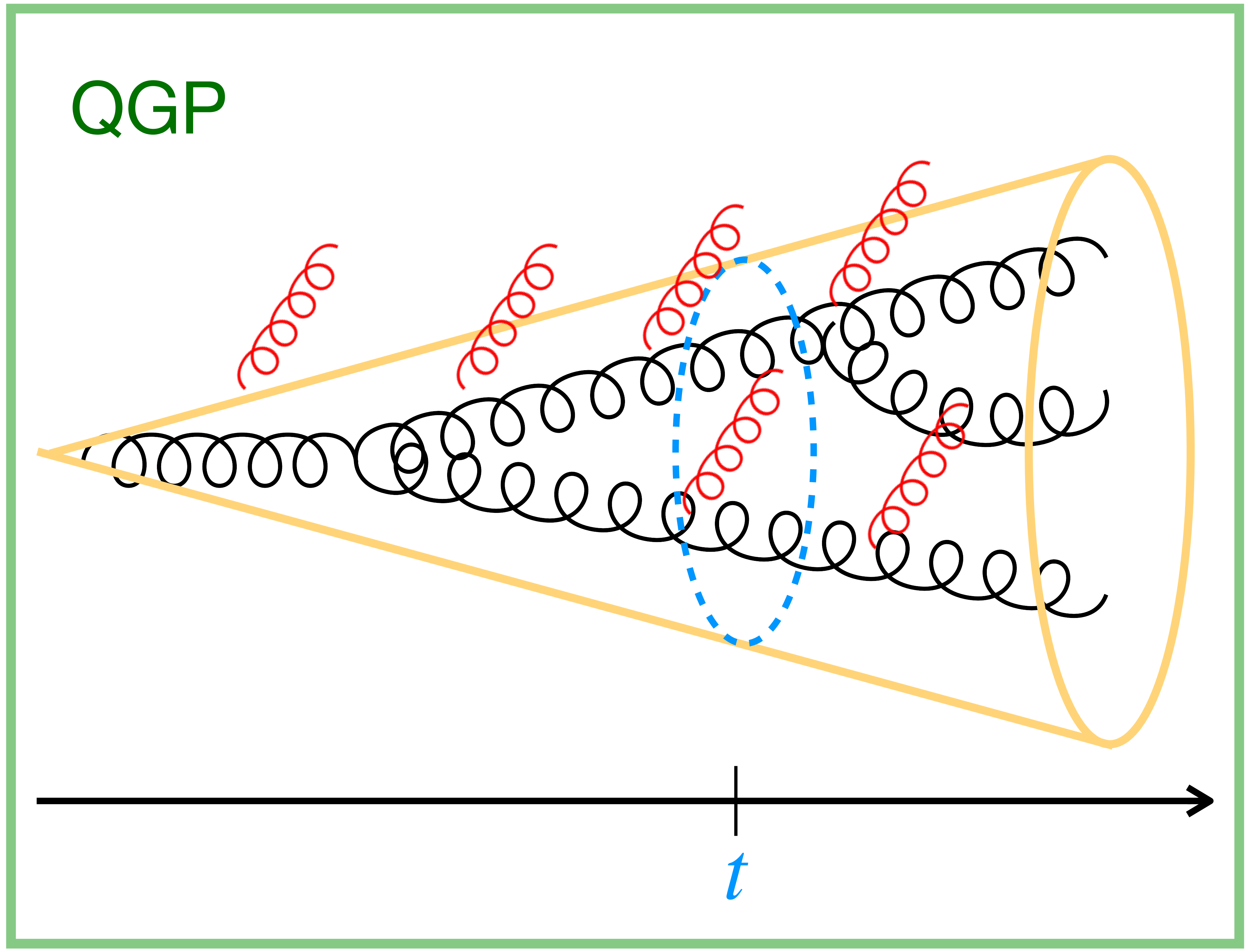
Light front formulation

S-matrix - factorization
(covariant-momentum space description)

→

2+1 D non-relativistic quantum mechanics

Transverse cross-section at time t



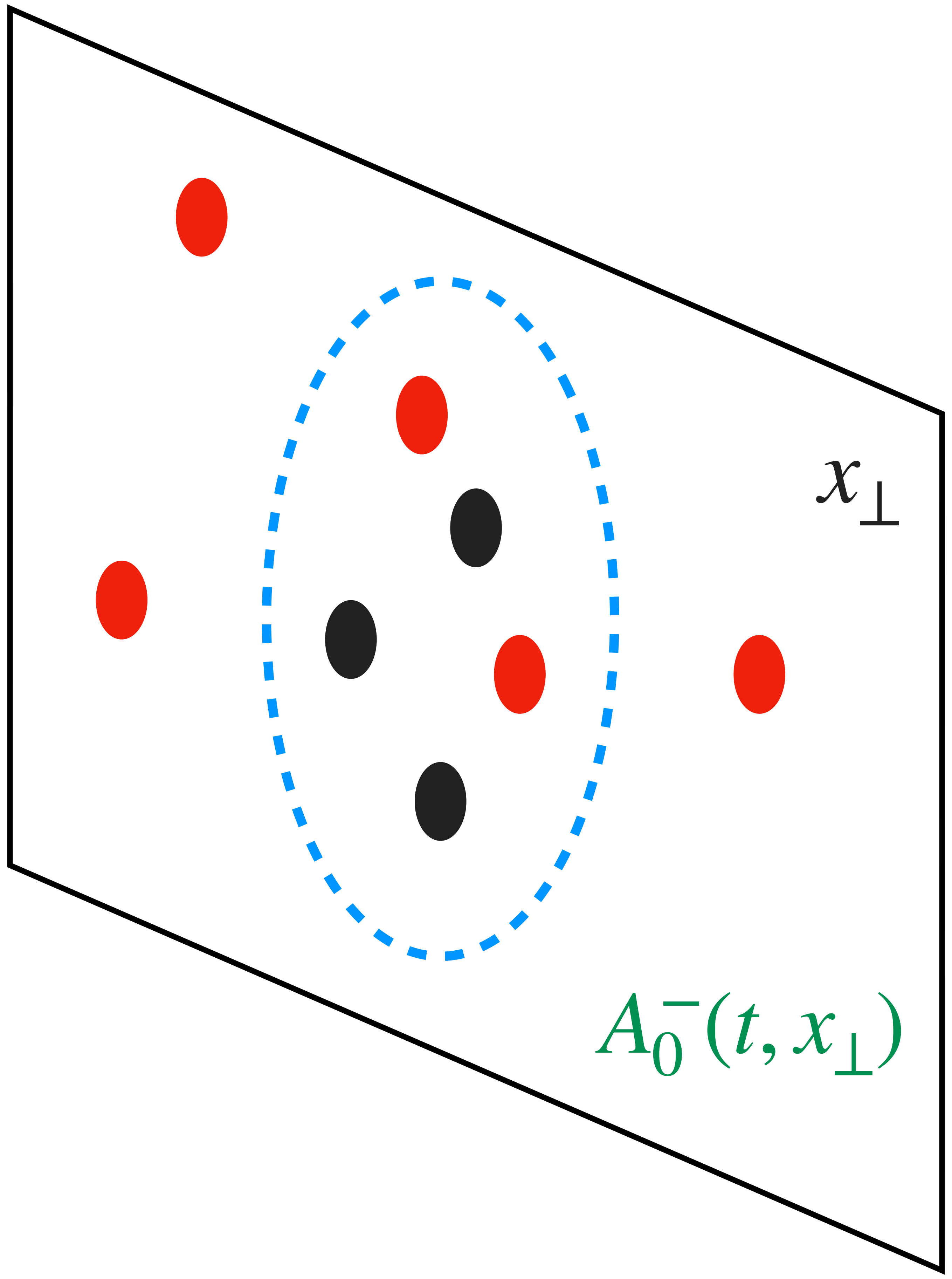
$p^+ \sim p_0 + p_z \sim p_z$

$z \sim t$

$\Rightarrow$

Light front Hamiltonian

$$P^- = \frac{\hat{P}_\perp^2}{2p^+}$$



The time evolution of fast (collinear partons) in the plasma can be viewed as that of non-relativistic color charges in andom background field in 2+1 D

Kinematics

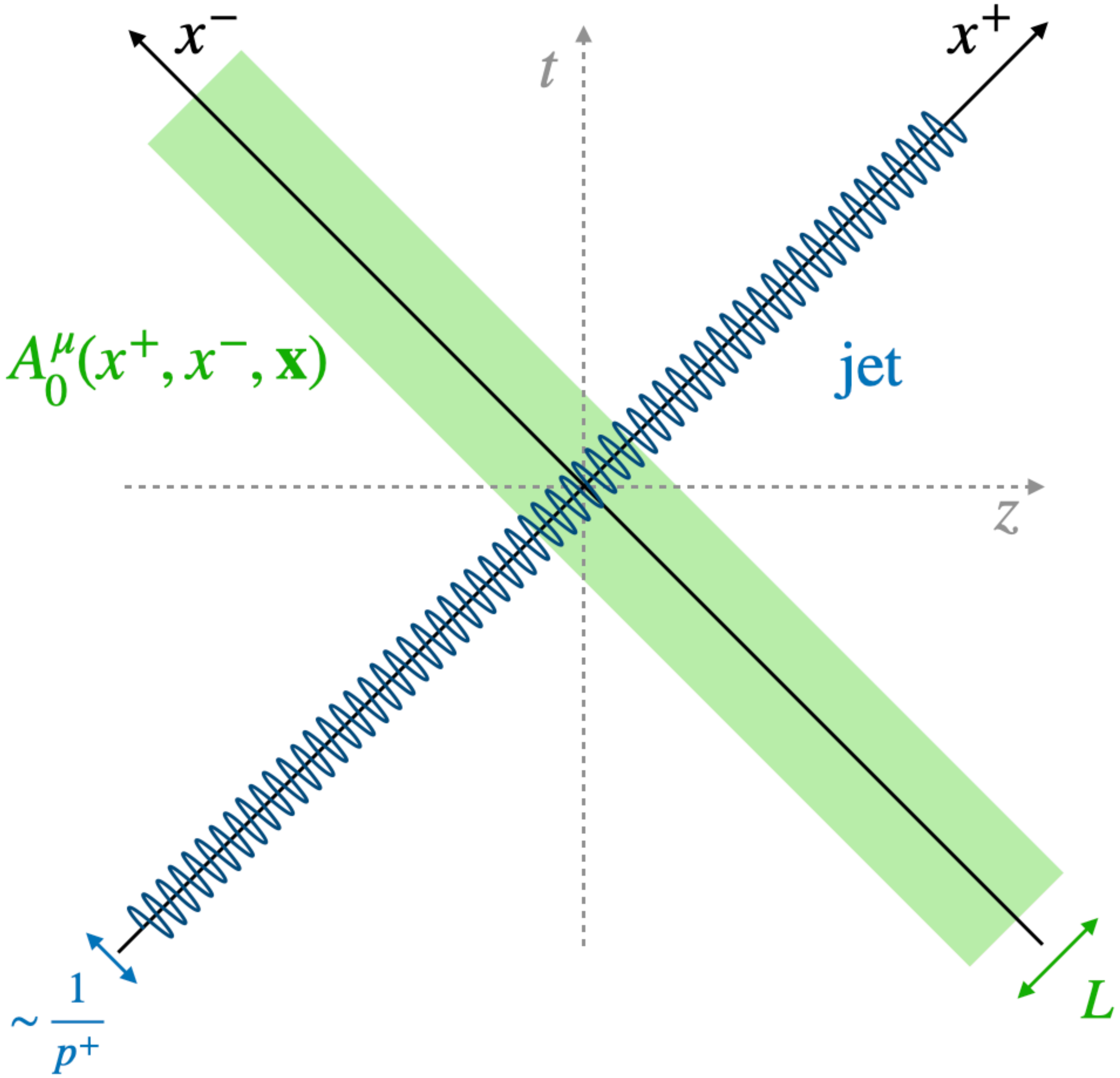
- At leading order a jet is a fast parton assumed to be moving in the $+z$ direction (Initially $p \equiv (p^+, 0^-, 0_\perp)$)
- The interaction with a medium background field is eikonal ($k^+ \ll p^+$)

$$-g\bar{u}(p-k)\not{A}(k)u(p) \simeq gp^+ A^-(k) \quad \sim \delta(k^+)$$

- Smaller momentum transfer $k_\perp^2 \sim \hat{q}\,L$

$$p \equiv (p^+, \frac{k_\perp^2}{2p^+}, k_\perp) \sim p^+(1, \lambda^2, \lambda)$$

Light-cone



$$x^+ = \frac{1}{\sqrt{2}}(t+z) \qquad x^- = \frac{1}{\sqrt{2}}(t-z)$$

Non-eikonal propagation

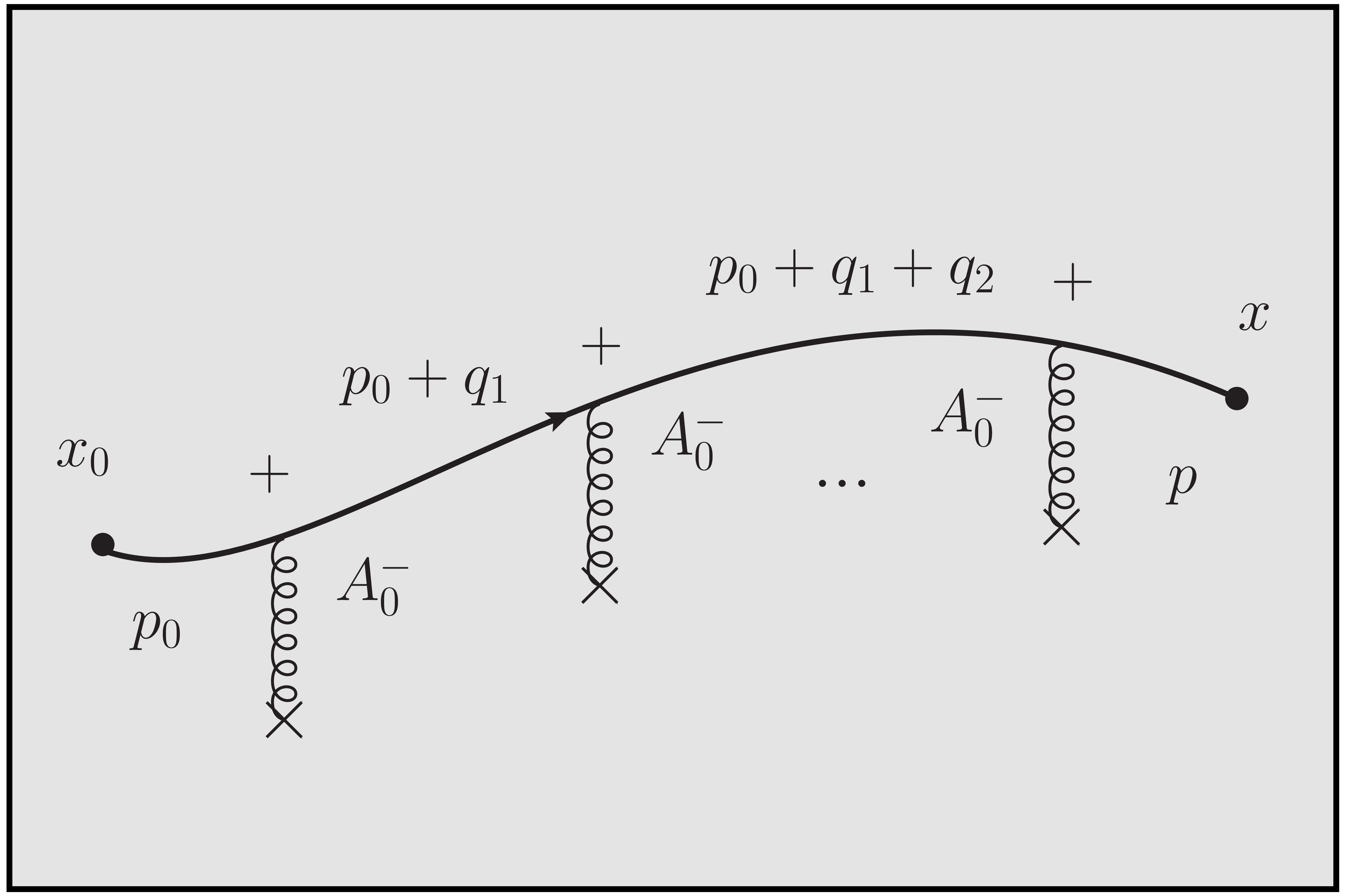
- Dirac propagator - spin/helicity and p^+ conservation

$$D(p,p_0) = i\frac{\gamma^+}{2p^+}\delta(p-p_0) + \frac{\not{p}\gamma^+\not{p}_0}{2p^+}G_{\rm scal}(p,p_0)$$

- Dynamics encoded in "scalar" propagator

$$\left[\Box - 2ig\partial^+ A_0^-(x)\cdot t\right] G_{\rm scal}(x,x') = \delta(x-x')$$

- Background field is a weak function of $x^- \sim 0$



Non-relativistic 2+1D quantum mechanics

- The propagator is invariant under translation in x^- . Hence

$$(\boldsymbol{x}|\mathcal{G}(t,t_0)|\boldsymbol{x}_0) = 2iE\int \mathrm{d}x^-\, \mathrm{e}^{iE(x-x_0)^-}\, G_{\rm scal}(x-x_0)$$

- The propagator $\mathscr{G}$ (evolution operator) obeys a 2+1D Schrödinger equation

$$\left[i\frac{\partial}{\partial t}+\frac{\partial_\perp^2}{2E}+gA_0(t,\boldsymbol{x})\right](\boldsymbol{x}|\mathcal{G}(t,t_0)|\boldsymbol{x}_0) = i\delta(t-t_0)\delta(\boldsymbol{x}-\boldsymbol{x}_0)$$

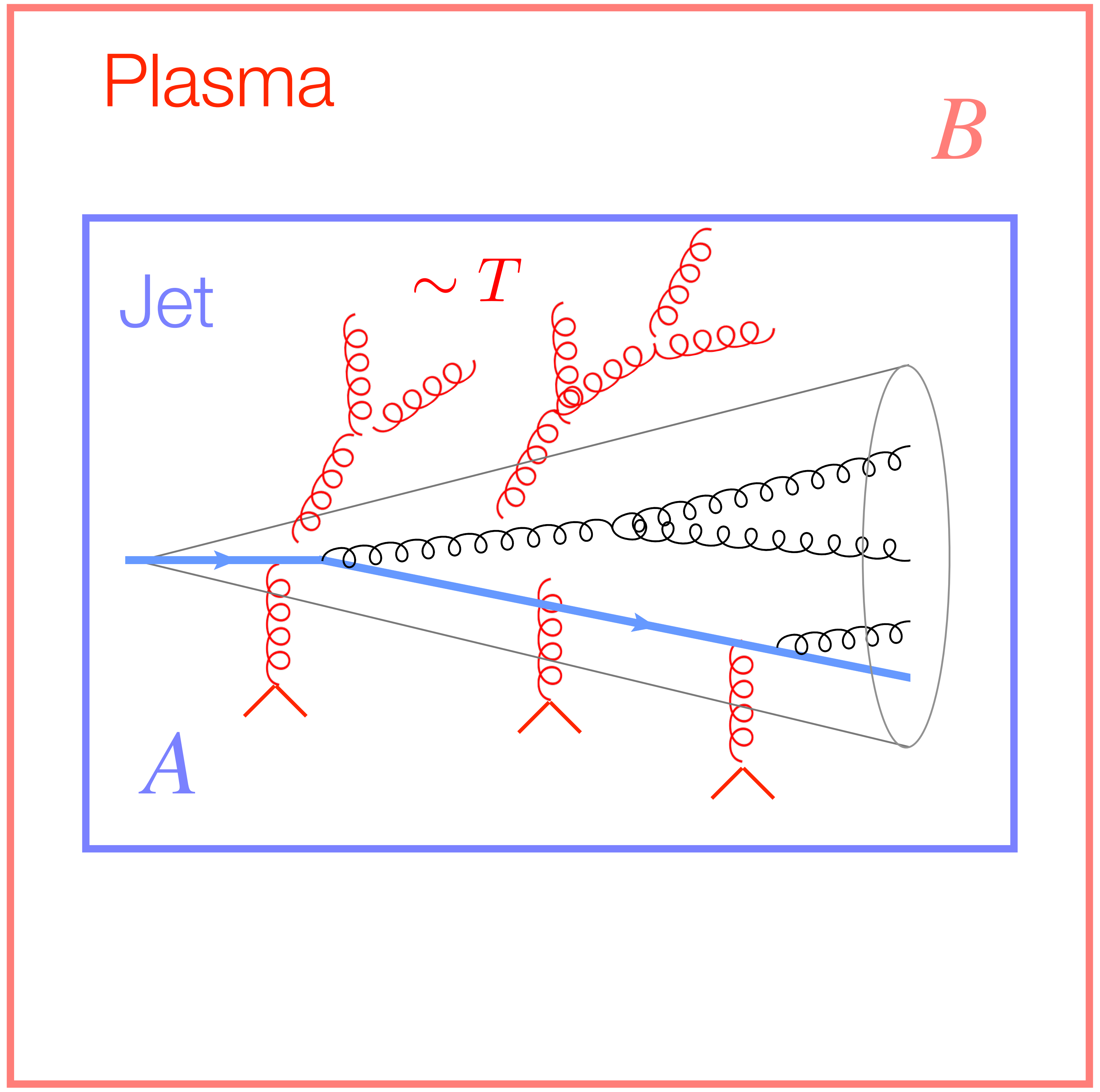
- The LO Hamiltonian reads

$$H = H_0 + H_I \equiv \frac{\hat{p}_\perp^2}{2p^+} + A_0^-(\hat{x}_\perp, t)$$

Computing observables

1. Compute observable for a given configuration of the background field $O[A_0]$ (unitary evolution)

2. Perform ensemble average: $\langle\, O[A_0]\,\rangle \Rightarrow$ (non-unitary)



- Open quantum system: alternative approach based on SCET discussed in Vaidya, Yao 2004.11403 [hep-ph] (see Vaidya's talk on Thursday)

Computing observables (medium average)

- Difficult in general but reduces to a Gaussian white noise for a weakly coupled plasmas ($g \ll 1$)

$$L \gg \frac{1}{g^2 T} \gg \frac{1}{gT}$$

mean-free-patrh   screening length

$$\langle A_0^{a,-}(t, q_\perp) A_0^{b,-}(t', q'_\perp) \rangle = \delta^{ab}\, \gamma(q_\perp)\, \delta(q_\perp - q'_\perp)\, \delta(t - t')$$

$$\langle A_0^- \rangle = 0$$

- For thermal medium and for $q_\perp \ll T$

$$\gamma(q_\perp) \sim \frac{\mathrm{d}\sigma_{\mathrm{el}}}{\mathrm{d}^2\boldsymbol{q}} = \frac{g^2 m_D^2 T}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$$

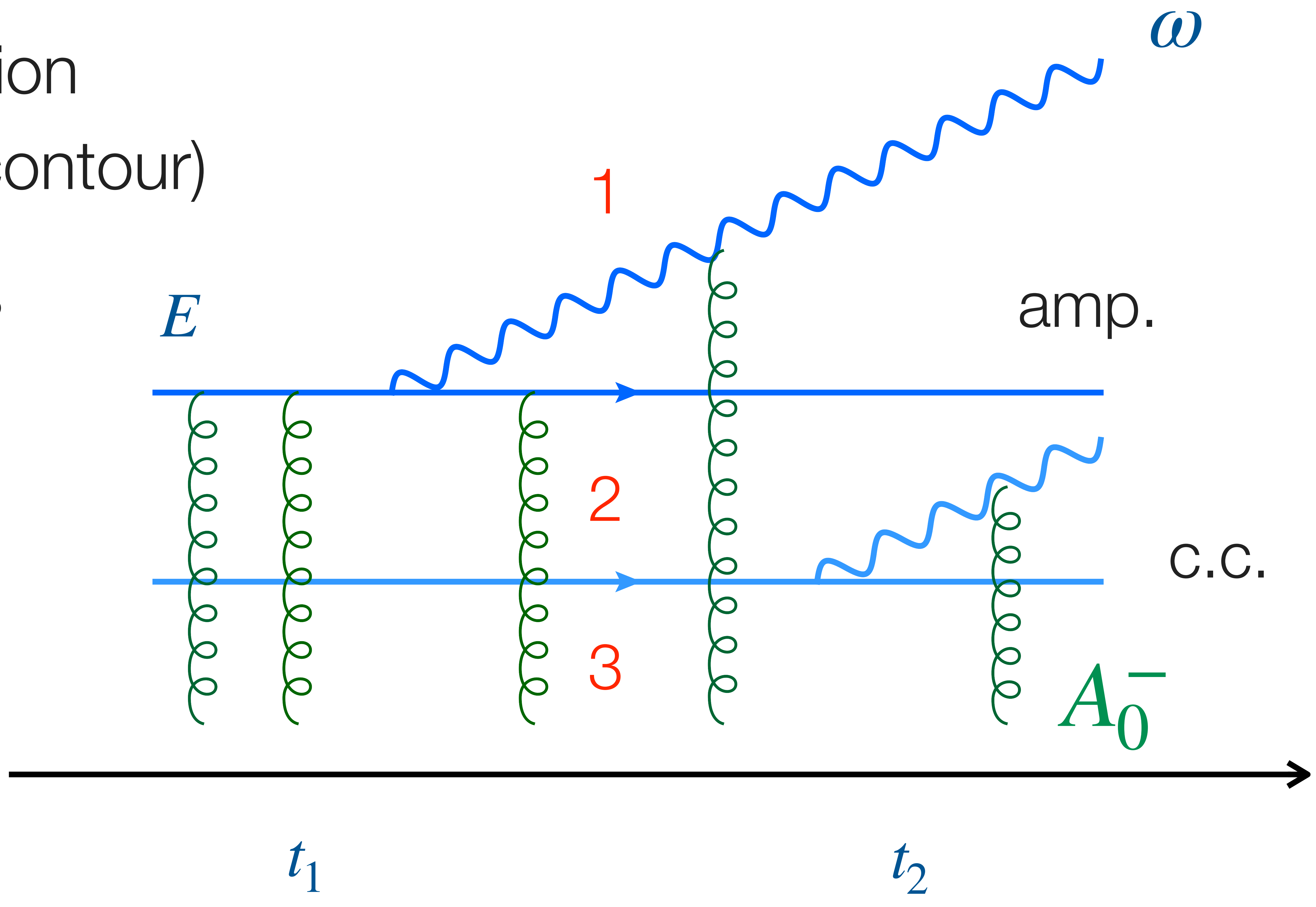
Quantum mechanics of the LPM effect

- Coherent medium induced gluon radiation between t_1 and t_2 (Schwinger-Keldish contour)
- 3-body system: color structure reduces

$$\mathcal{G}^{ab}(t_2,t_1)\mathrm{tr}\left(t^a U(t_2,t_1) t^b U^\dagger(t_2,t_1)\right)$$

$$\downarrow$$

$$\mathcal{K}(t_2,t_1) \sim \mathrm{Tr}(\mathcal{G} U_A)$$



- Where U_A is a path ordering exponential (Wilson line)

$$U_A(t_2,t_1) \equiv P \exp\left[ig\int_{t_1}^{t_2} \mathrm{d}t\, A(x_\perp, t)\right]$$

- Averaging over plasma configurations, medium induced spectrum yields

$$\omega\frac{\mathrm{d}I}{\mathrm{d}\omega} = \frac{\alpha_s C_R}{\omega^2}\,2\mathrm{Re}\int_0^\infty \mathrm{d}t_2 \int_0^{t_2} \mathrm{d}t_1\, \boldsymbol{\partial}_x \cdot \boldsymbol{\partial}_y \left[\mathcal{K}(\boldsymbol{x},t_2|\boldsymbol{y},t_1) - \mathcal{K}_0(\boldsymbol{x},t_2|\boldsymbol{y},t_1)\right]_{\boldsymbol{x}=\boldsymbol{y}=\boldsymbol{0}}$$

- The Green's function obeys the evolution equation (can also be expressed a a path integral)

$$\left[i\frac{\partial}{\partial t} + \frac{\boldsymbol{\partial}^2}{2\omega} + iv(\boldsymbol{x})\right]\mathcal{K}(\boldsymbol{x},t|\boldsymbol{y},t_1) = i\delta(\boldsymbol{x}-\boldsymbol{y})\delta(t-t_1)$$

- The imaginary potential is related to the dipole cross-section and $\hat{q}$

$$v(\boldsymbol{x},t) \equiv \frac{1}{4}\hat{q}(\boldsymbol{x}^2,t)\,\boldsymbol{x}^2 \simeq \frac{g^4}{16\pi}N_c\,n(t)\,\boldsymbol{x}^2\,\ln\frac{1}{\mu^2\,\boldsymbol{x}^2}$$

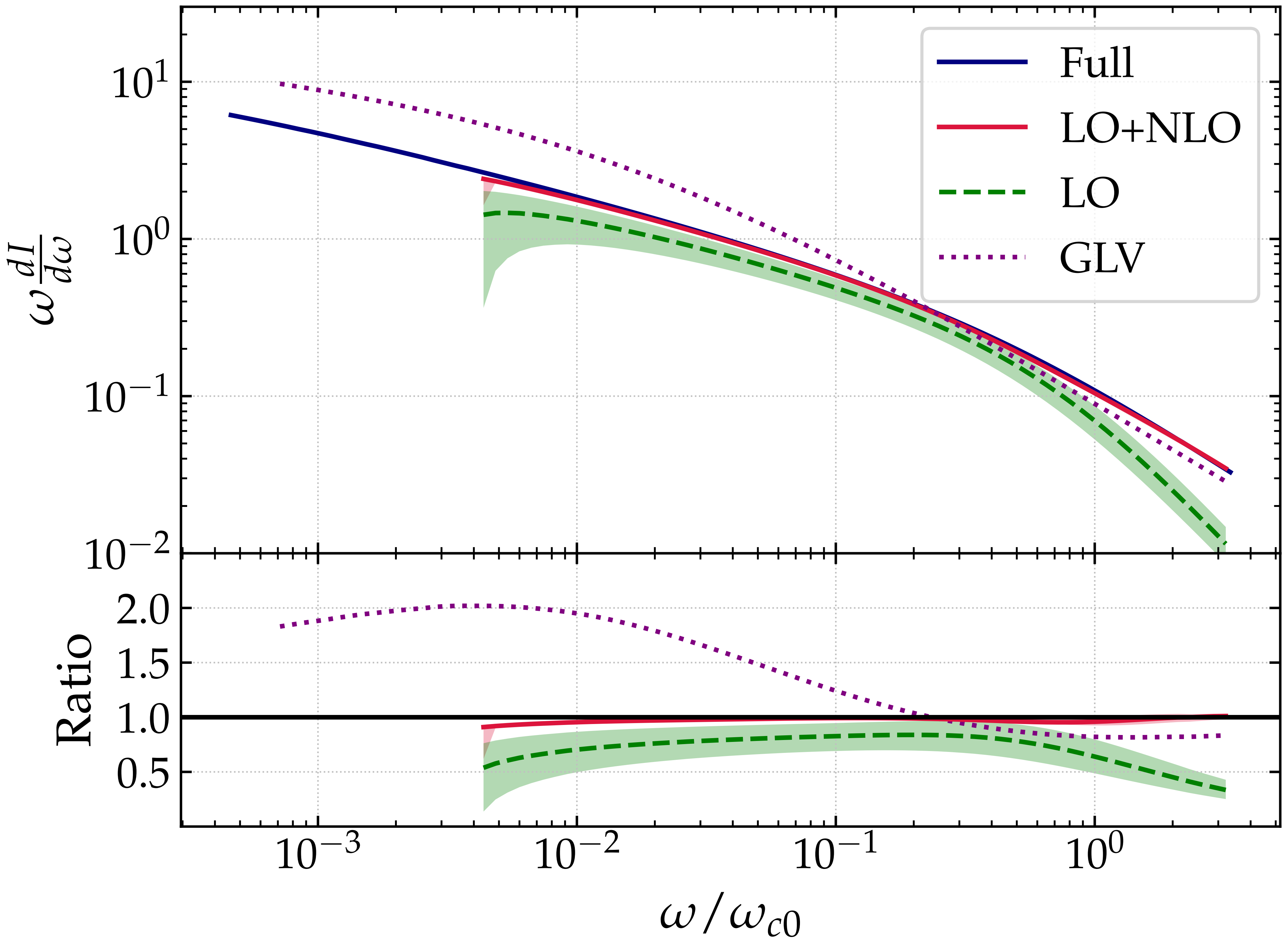
- Two solutions to the problem:

1. Opacity expansion: treat the imaginary potential as a perturbation (valid at high frequencies)

2. Improved opacity expansion: expand around the harmonic oscillaor

$$\sigma(\boldsymbol{x}) = N_c\,\boldsymbol{x}^2 \left(\ln \frac{Q^2}{\mu^2} + \ln \frac{1}{\boldsymbol{x}^2 Q^2} \right)$$

harmonic oscillator + perturbation



Barata, MT, Tywoniuk, Soto-Ontoso (2019-2021)

$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega} \simeq 2\,\bar{\alpha}\,\ln|\cos(\Omega L)| + \frac{1}{2}\,\bar{\alpha}\,\hat{q}_0\,\mathrm{Re}\int_0^L \mathrm{d}s\,\frac{1}{k^2(s)}\left[\ln\frac{k^2(s)}{Q^2} + \gamma\right]$$

Full: exact numerical results from Apolinario, et al 2002.01517, 2011.06522

Quantum entanglement of jets in the QGP

- It is natural to view the problem of jets in HIC as that of an open quantum system (jet) in interaction with an unobserved environment
- What is the density matrix of jets in this medium?
- What is the entanglement entropy of the subsystem?
- Reduced density matrix: trace out the environment (the QGP)

$$\rho_A = \mathrm{Tr}_B \, \rho_{AB} \approx \langle \rho_A[A_0] \rangle$$

system B: medium
(not necessarily thermal)

system A: jet

- Simple case: leading order (in the absence of gluon radiation)

Quantum entanglement in jet quenching

J.-P. Blaizot and YMT (work in progress)

- Initial density matrix (pure state): $\rho_{\text{jet}}(t_0) = |\psi_0\rangle\langle\psi_0|$ where $\psi_0(k_\perp)$ assumed to be peaked around $k^2_{\perp,0} \sim \mu^2 \ll \hat{q}L$
- Evolution of the pure state for a given configuration of the classical background field

$$\rho_{\text{jet}}[A] = U(t,t_0)\,|\psi_0\rangle\langle\psi_0|\,U^\dagger(t,t_0)$$

- At leading order $U(t,t_0) = \mathcal{G}(t,t_0)$

$$\rho_{\text{jet}} = \langle \rho_{\text{jet}}[A]\rangle_A = \int_{\boldsymbol{k}_f,\bar{\boldsymbol{k}}_f,\boldsymbol{k}_i,\bar{\boldsymbol{k}}_i} \Big\langle \langle \boldsymbol{k}_f|\,\mathcal{G}(t,t_0)\,|\boldsymbol{k}_i\rangle\,\langle\bar{\boldsymbol{k}}_i\,|\,\mathcal{G}(t,t_0)\,|\,\bar{\boldsymbol{k}}_i\rangle \Big\rangle_A \langle \bar{\boldsymbol{k}}_i\,|\,\rho_{\text{jet}}(t_0)\,|\,\boldsymbol{k}_i\rangle\,|\boldsymbol{k}_f\rangle\langle\bar{\boldsymbol{k}}_f|$$

Quantum entanglement in jet quenching

- At late times we expect quantum decoherence and the emergence of classical behavior, i.e., the diagonalization of the reduced density matrix

$$\Big\langle \langle \boldsymbol{k}_f | \rho^{ab}_{\rm jet}(t_f) | \bar{\boldsymbol{k}}_f \rangle \Big\rangle_A \to \delta^{ab} \delta(\boldsymbol{k}_f - \bar{\boldsymbol{k}}_f) \, P(\boldsymbol{k}_f, t_f)$$

- Under this assumption it is straightforward to show that

$$\rho_{\rm jet}(t) \to \int_{\boldsymbol{k}} P(\boldsymbol{k}, t) \, | \boldsymbol{k} \rangle \langle \boldsymbol{k} |$$

- $P(\boldsymbol{k}, t)$ is the broadening probability density that obeys a master equation

$$\frac{\partial}{\partial t} P(\boldsymbol{k}, t) = \int_{\boldsymbol{q}} \gamma(\boldsymbol{q}) \big[P(\boldsymbol{k} - \boldsymbol{q}, t) - P(\boldsymbol{k}, t) \big] \approx \frac{1}{4} \hat{q} \, \nabla^2_q P(\boldsymbol{k}, t)$$

Entanglement entropy (late times)

- The Von Neumann entropy tends to the differential (Gibbs) entropy

$$S \equiv -\,\mathrm{Tr}\,\rho_{\rm jet} \ln \rho_{\rm jet} \quad \longrightarrow \quad -\int_{\boldsymbol{k}} P(\boldsymbol{k}) \ln(\mu^2 P(\boldsymbol{k}))$$

- In the diffusive approximation $P(\boldsymbol{k}, t) = 4\pi/(\hat{q}t)\, \exp[-\boldsymbol{k}^2/(\hat{q}t)]$ one finds

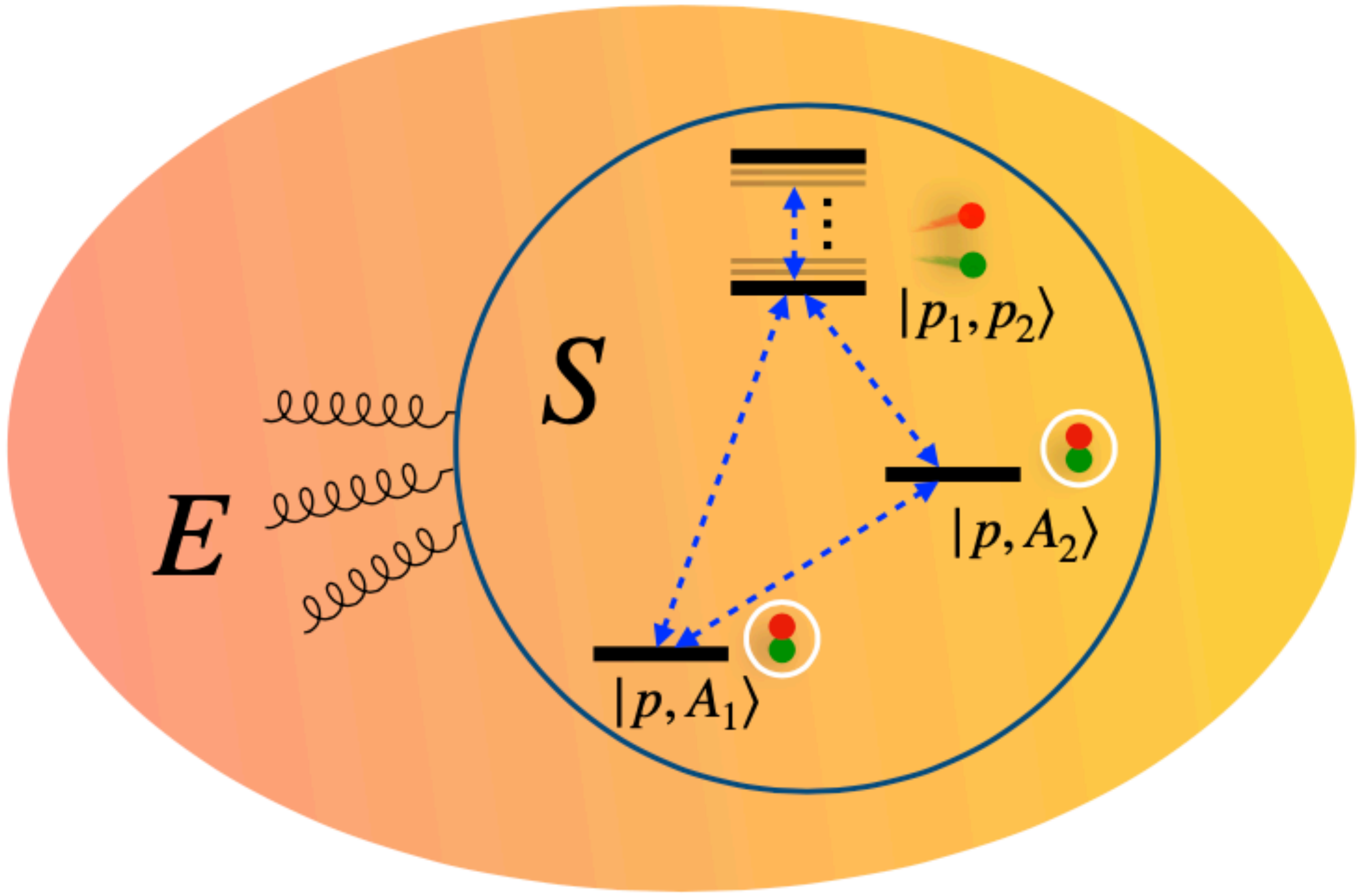
$$S(t) \approx \ln \frac{\hat{q}\, t}{4\pi\mu^2} + \frac{1}{2} \sim \ln t$$

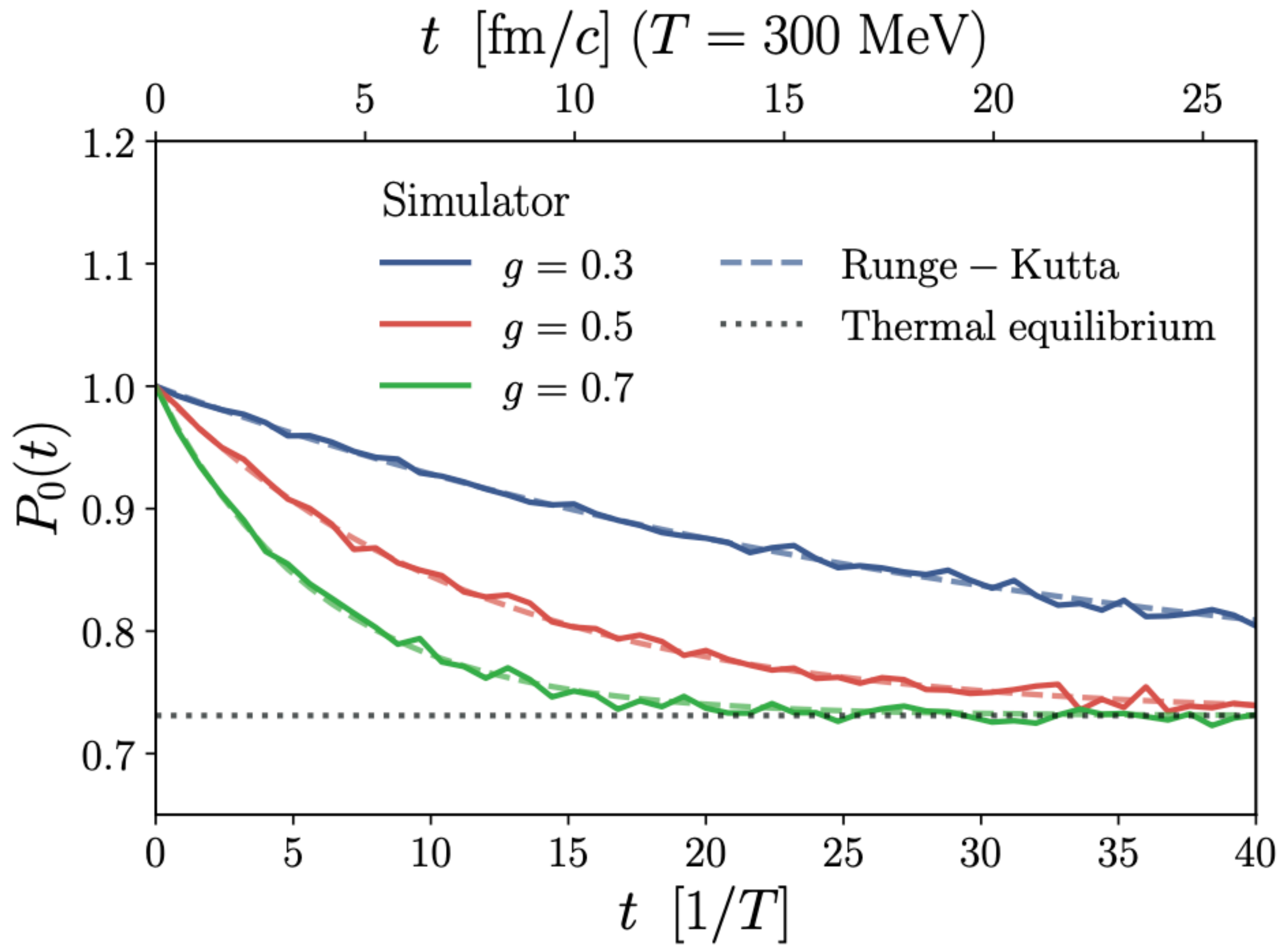
- The entropy increases logarithmically with time (system size)

Jet quenching on a quantum computer

- Recent effort to tackle this problem:

1. Simulating Lindblad equation for hard probes in thermal bath (QGP). Proof of concept: toy model for a 2 level system (De Jong, Metcalf, Mulligan, Polskon, Ringer, Yao 2010.03571 [hep-ph])



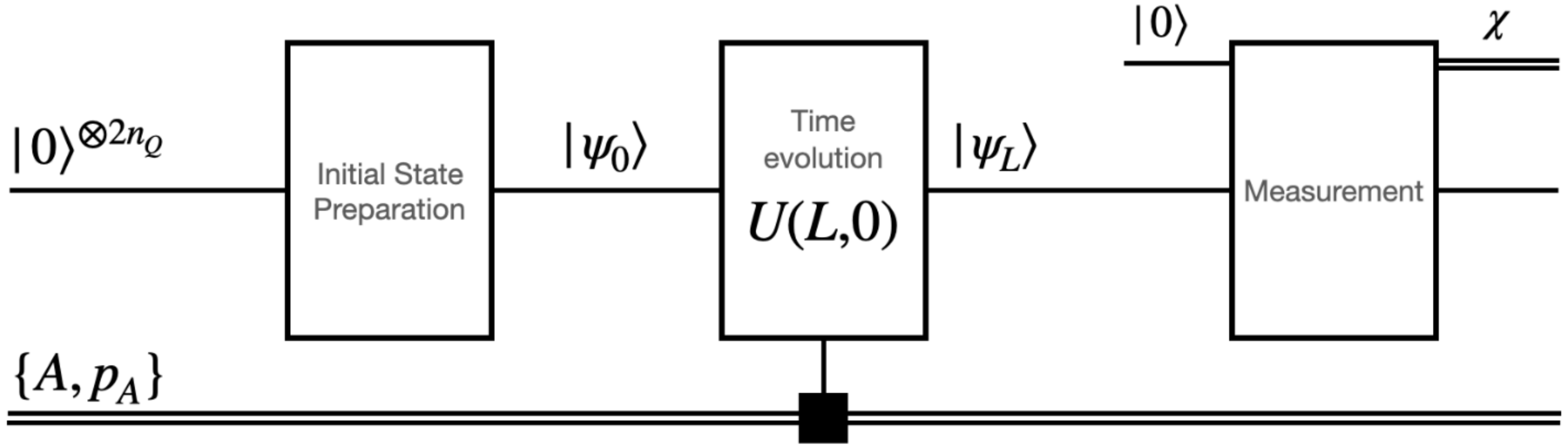


(see Felix's talk)

Jet quenching on a quantum computer

- Recent effort to tackle this problem:

 2. Momentum broadening: Barata , Salgado 2104.04661 [hep-ph] : hybrid strategy for simulating the propagator (time evolution operator), $\mathcal{G}[A]$ on quantum computer (n_Q qubits spanning the finite Hilbert space) while stochastic medium charge configurations on a classical computer



(see Felix's talk)

Roadmap and open questions

- Simulating momentum broadening on a Quantum computer: leading order process
- Inclusion of higher order corrections (parton branching) (Bauer et al arXiv:1904.03196, Bepari et al, arXiv:2010.00046), challenges: multi-parton system, gauge invariance…
- towards an amplitude level event generator that fully accounts for quantum interferences (beyond leading log accuracy)? Relevant for both pp and HIC
- Investigate quantum information in jet observables at higher orders (on-set of color and quantum decoherence). Entanglement entropy for resolved and unresolved partons studied for jets in vacuum in Neill, Waalewijn 1811.01021 [hep-ph]

Thank you!