AdS solutions in various dimensions from pure spinor methods

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based on
1506.05462 with A. Passias, A. Rota;
1502.06620 with F. Apruzzi, M. Fazzi, A. Passias; 1502.06622 with A. Rota
1309.2949 with F. Apruzzi, M. Fazzi, D. Rosa; 1404.0711 with D. Gaiotto
1407.6359 with M. del Zotto, J.Heckman, C.Vafa
+ work in progress with S. Cremonesi

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Introduction

Original motivation of this work: 6d CFTs

- Mothers of interesting theories in $d \leq 4$ [Gaiotto '09, Alday, Gaiotto, Tachikawa '09...]
- They might lead to progress on $(2,0)$ theory on M5 stacks

Holography: classification of AdS7 solutions?

- AdS$_7 \times M_4$ in 11d sugra: cone over M_4 should have reduced holonomy $\Rightarrow$ $M_4 = S^4/\Gamma_{\mathrm{ADE}}$
- In type II: pure spinor methods

This talk: emphasis on gravity aspects

Plan

- Pure spinor methods

- Classification of AdS_7 solutions in type II sugra

  - infinitely many; analytical

  - They generate anal. infinitely many AdS_5 and AdS_4 solutions

$$\begin{tikzcd} & \mathrm{AdS}_7 \arrow[dl, shift left] \arrow[dr, shift left] & \\ \mathrm{AdS}_4 \times \Sigma_3 \arrow[ur, shift left] \arrow[rr, shift left] & & \mathrm{AdS}_5 \times \Sigma_2 \arrow[ul, shift left] \arrow[ll, shift left] \end{tikzcd}$$

might also be useful
as flux compactifications...

- Their CFT_6 duals: NS5-D6-D8 brane constructions

Pure spinor methods

Working on $T \oplus T^*$

susy parameters $\epsilon_{1,2}$ define

- two ordinary G-structures

$\Downarrow$

many possible cases
depending on $G(\epsilon_1) \cap G(\epsilon_2)$

or

- one G-structure on $T \oplus T^*$

nicer equations; easier classifications

described by forms
obeying algebraic constraints:
often 'pure spinors'

original example

$\left.\begin{array}{l}\text{Mink}_4\\ \text{AdS}_4\end{array}\right\} \times M_6$

[Graña, Minasian, Petrini, AT '05]

$\mathrm{SU}(3)\times\mathrm{SU}(3)$ structure

nice differential equations

[AT '11]

any M_{10}:

$(\mathrm{Spin}(7) \ltimes \mathbb{R}^8)^2$ structure*

NS 3-form

defined by Φ

$$\boxed{(d + H\wedge)\Phi = (\iota_K + \tilde{K}\wedge)F}$$

total RR flux

+ extra equations, almost never important

system for $\mathrm{AdS}_7 \times M_3$

Id$\times$Id structure

[Apruzzi, Fazzi, Rosa, AT '13]

system for $\mathrm{AdS}_5 \times M_5$

Id$\times$Id structure

[Apruzzi, Fazzi, Passias, AT '13]

*simplifying the story a bit...

system for $\mathrm{AdS}_7 \times M_3$

Id$\times$Id structure

origin: 3d part $\chi_{1,2}$ of susy parameters $\epsilon_{1,2}$

both define a vielbein (= Id structure) for the internal metric

$$\psi^1 = \chi_1 \otimes \chi_2^\dagger$$
$$\psi^2 = \chi_1 \otimes \overline{\chi_2}$$

bispinors $\cong$ forms

$$\left[\begin{array}{c} \gamma^{i_1 \ldots i_k} \\ \wr\! \| \\ dx^{i_1} \wedge \ldots \wedge dx^{i_k} \end{array} \right]$$

better parameterization:

one vielbein $\{e_i\}$

and three angles: θ_1, θ_2, ψ

[sorry: don't confuse angle ψ with forms $\psi^{1,2}$!]

for example:

$$\psi^1_+ = e^{i\theta_1} \left[\cos(\psi) + e_1 \wedge (-i e_2 + \sin(\psi) e_3)\right]$$

[+ = even part]

system for $\mathrm{AdS}_7 \times M_3$

Id$\times$Id structure

The differential system reads*

$$d_H \mathrm{Im}\psi^1_\pm = -2\mathrm{Re}\psi^1_\mp$$
$$d_H \mathrm{Re}\psi^1_\pm = 4\mathrm{Im}\psi^1_\mp$$
$$d_H \psi^2_\pm = -4i\psi^2_\mp$$
$$\pm *_3 F = dA \wedge \mathrm{Im}\psi^1_\pm + \mathrm{Re}\psi^1_\mp$$
$$dA \wedge \mathrm{Re}\psi^1_\mp = 0$$

total
RR flux

$d_H \equiv d - H\wedge$

$A =$ warping

upper sign: IIA

lower sign: IIB

After some computations...

*up to factors of dilaton and warping

AdS7 classification

- IIB: no solutions!

- IIA: internal M_3 is locally S^2-fibration over interval

[no Ansatz necessary]

$$ds^2 \sim e^{2A(r)} ds^2_{\mathrm{AdS}_7} + dr^2 + v^2(r) ds^2_{S^2}$$

This S^2 realizes
the $\mathrm{SU}(2)$ R-symmetry
of a $(1,0)$ 6d theory.

Fluxes: $F_0, F_2 \sim \mathrm{vol}_{S^2}, H \sim dr \wedge \mathrm{vol}_{S^2}$

$A(r)$, $\phi(r)$, $v(r)$ determined by ODEs

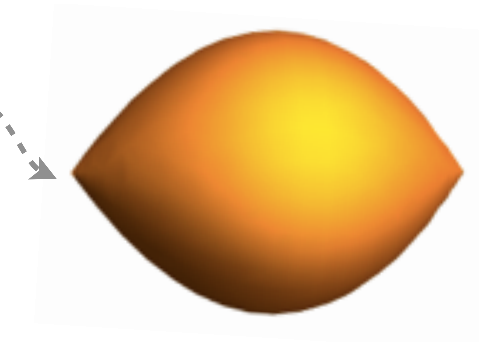
solved at first numerically [Apruzzi, Fazzi, Rosa, AT '13]

then analytically with the help of AdS4 and AdS5
[Rota, AT '15] [Apruzzi, Fazzi, Passias, AT '15]

- $F_0 = 0$: only one solution

reduction of
$\mathrm{AdS}_7 \times S^4/\mathbb{Z}_k$

k D6s



k $\overline{\mathrm{D6}}$s

- $F_0 \neq 0$: many new solutions

$$ds^2_{M_3} = \frac{n_{\mathrm{D6}}}{F_0}\left(\frac{dy^2}{4\sqrt{y+2}(1-y)} + \frac{1}{3}\frac{(1-y)(y+2)^{3/2}}{8-4y-y^2}ds^2_{S^2}\right) .$$

we can make one
of the poles regular:

D6 stack

reg. point

local solutions also in [Blåbäck, Danielsson, Junghans, Van Riet, Wrase, Zagermann '11]
susy-breaking? in [Junghans, Schmidt, Zagermann '14]

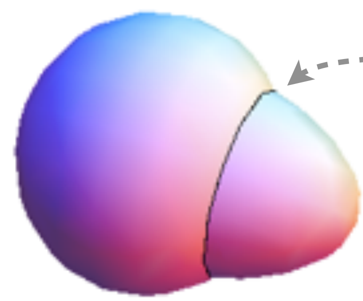
more generally we can have
two unequal D6 stacks

or also an O6 and a D6 stack

these solutions are also analytic, but a bit more complicated.

we can also
include D8's:

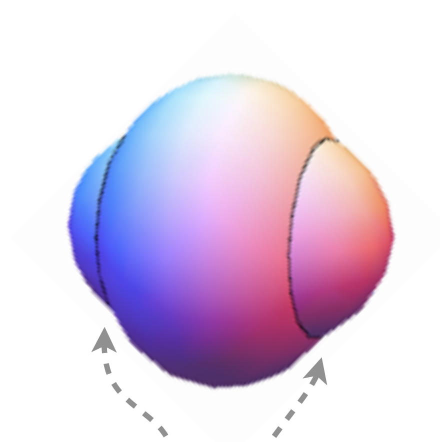
D8–D6 stack



actually, 'magnetized' D8's
||
D8–D6 bound states

metric: gluing of two pieces of earlier metric

intuitively: D8's don't slip off
because of electric attraction



stacks with opposite D6 charge

metric: gluing of two pieces of metric in prev. slide
+ central region from two slides ago

If you're curious about the analytic expressions:

- All is determined by a single function $\beta(y)$ where $\left(\frac{y^2\beta}{\beta'^2}\right)' = \frac{F_0}{72}$

$$ds^2 = \frac{4}{9}\sqrt{-\frac{\beta'}{y}}\left[ds^2_{\mathrm{AdS}_7} - \frac{1}{16}\frac{\beta'}{y\beta}dy^2 + \frac{\beta/4}{4\beta - y\beta'}ds^2_{S^2}\right]$$

[it's easy to solve]

- β has single zero $\Rightarrow$ regular point; double zero $\Rightarrow$ D6 stack

examples:

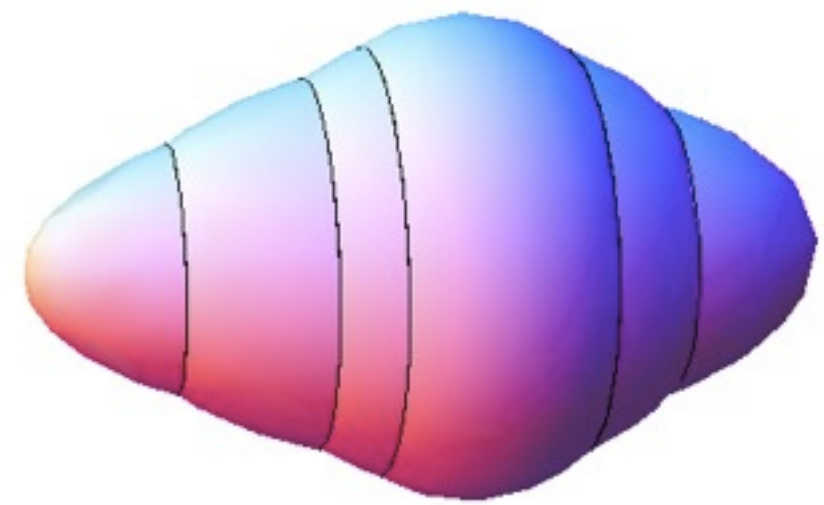
$F_0 = 0$, two D6 stacks $\beta \propto (y^2 - y_0^2)^2$

$F_0 \neq 0$, one D6 stack $\beta_{\mathrm{D6}} \propto (y - y_0)(y + 2y_0)^2$

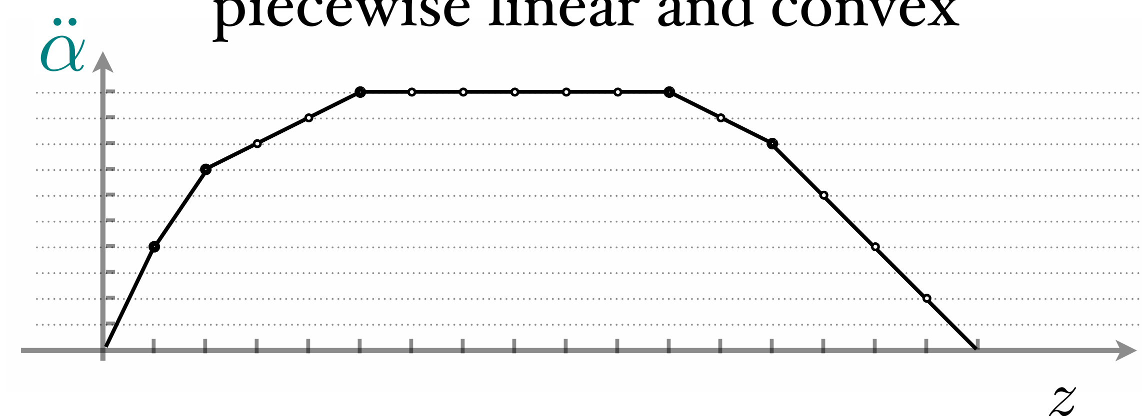
$F_0 \neq 0$, most general: $\beta \propto (\beta_{\mathrm{D6}} + \mathrm{const})^2$

More generally:

$$ds^2 = 8\sqrt{-\frac{\ddot\alpha}{\alpha}}\,ds^2_{\mathrm{AdS}_7} + \sqrt{-\frac{\alpha}{\ddot\alpha}}\,dz^2 + \frac{\alpha^{3/2}(-\ddot\alpha)^{1/2}}{\sqrt{2\alpha\ddot\alpha - \dot\alpha^2}}\,ds^2_{S^2}$$



piecewise linear and convex



AdS5, AdS4

To any of our solutions

$$e^{2A} ds^2_{\mathrm{AdS}_7} + dr^2 + v^2 ds^2_{S^2}$$

[Apruzzi, Fazzi Passias, AT '15]

$$\frac{3}{4} e^{2A} (ds^2_{\mathrm{AdS}_5 \times \Sigma_2}) + dr^2 + \frac{v^2}{1-4v^2} e^{2A} ds^2_{S^2}$$

dual to $\mathrm{CFT}_4 \cong \mathrm{CFT}_6/\Sigma_2$

[twisted compactification]

[Rota, AT '15]

$$\frac{5}{8} e^{2A} (ds^2_{\mathrm{AdS}_4 \times \Sigma_3}) + dr^2 + \frac{v^2}{1-6v^2} e^{2A} ds^2_{S^2}$$

dual to $\mathrm{CFT}_3 \cong \mathrm{CFT}_6/\Sigma_3$ [twisted compactification]

interesting flux compactification:
AdS4 solution with localized O6s and D6s

we came to suspect that there was a more general story:

To any of our solutions

$$e^{2A} ds^2_{\mathrm{AdS}_7} + dr^2 + v^2 ds^2_{S^2}$$

$$e^{2A} ds^2_7 + dr^2 + \frac{v^2}{1+16(X^5-1)v^2} e^{2A} ds^2_{S^2}$$

this is in fact an Ansatz for a consistent truncation!

For any AdS7 solution in IIA
there is a consistent truncation to
'minimal gauged 7d sugra'

[Passias, Rota, AT '15]

fields: $g^{(7)}_{\mu\nu}$, A^i_μ, X

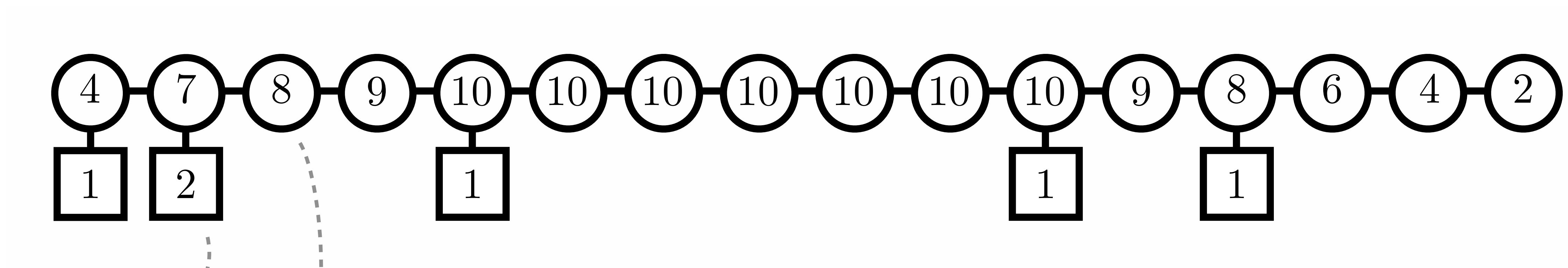
One can use it to establish

- RG flows from AdS_7 to $\mathrm{AdS}_5 \times \Sigma_2$ and $\mathrm{AdS}_4 \times \Sigma_3$
- $\mathrm{AdS}_3 \times \Sigma_4$ solutions
- non-susy AdS_7 solution

Holographic duals

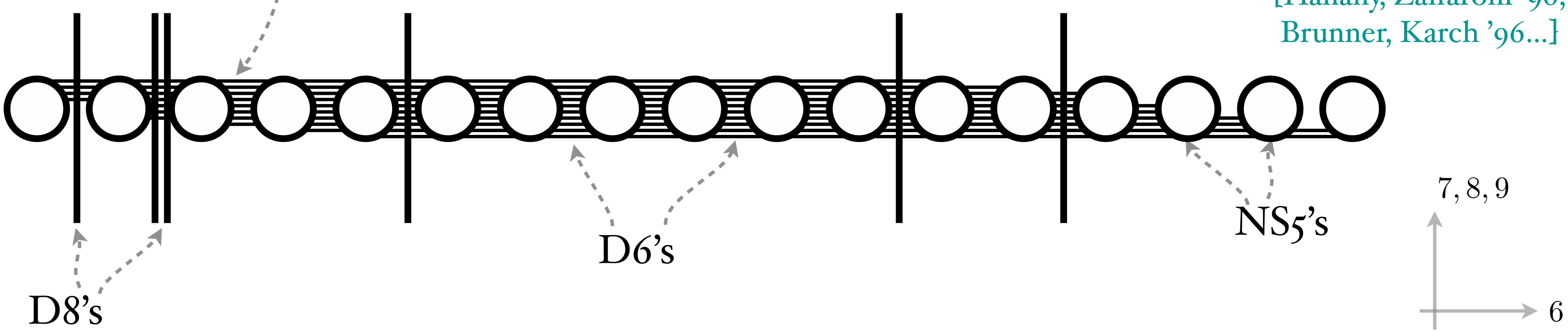
Natural class: linear quivers

At each node, $n_F = 2n_c$



D-brane engineering:

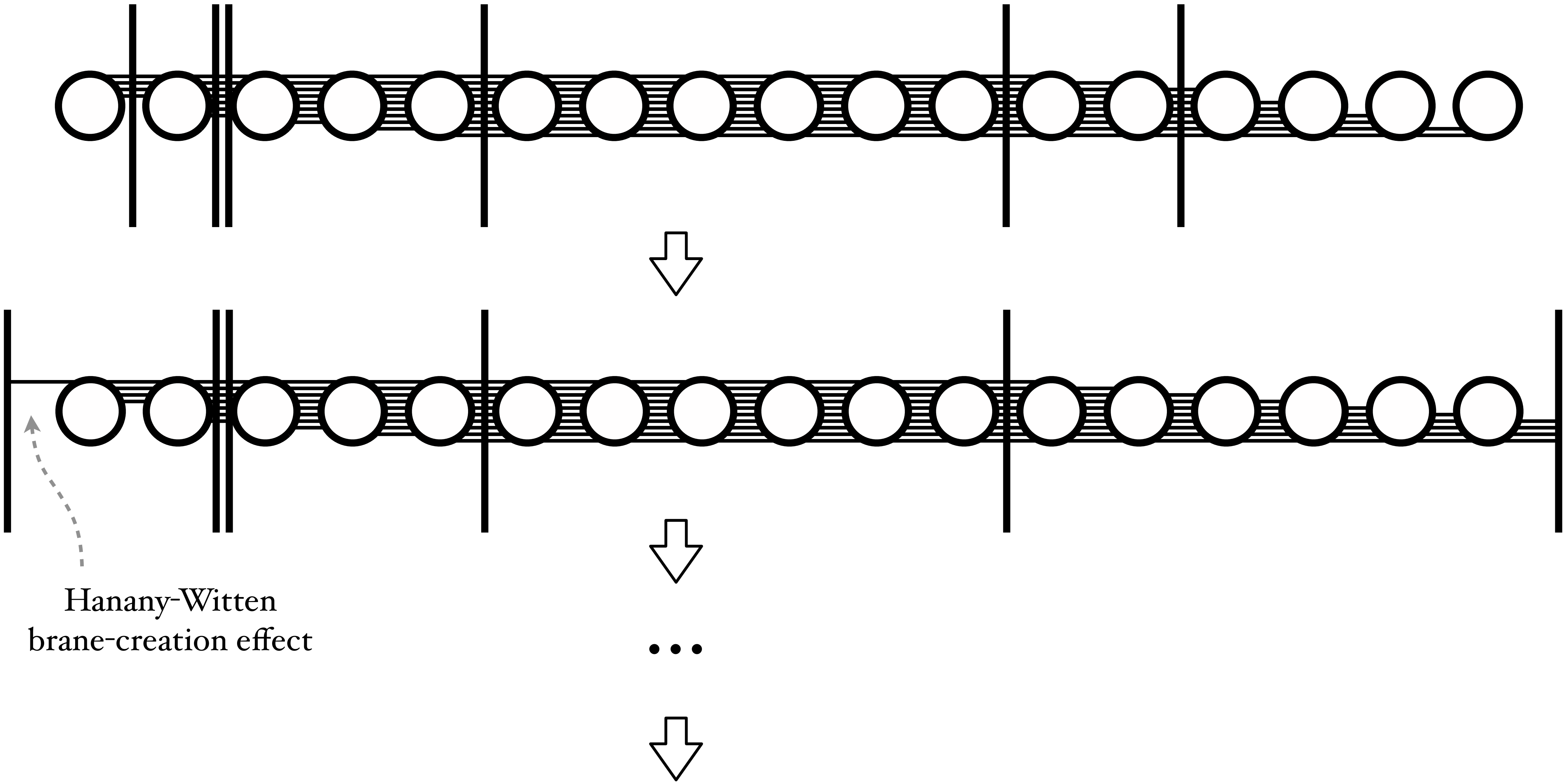
[Hanany, Zaffaroni '96, Brunner, Karch '96...]



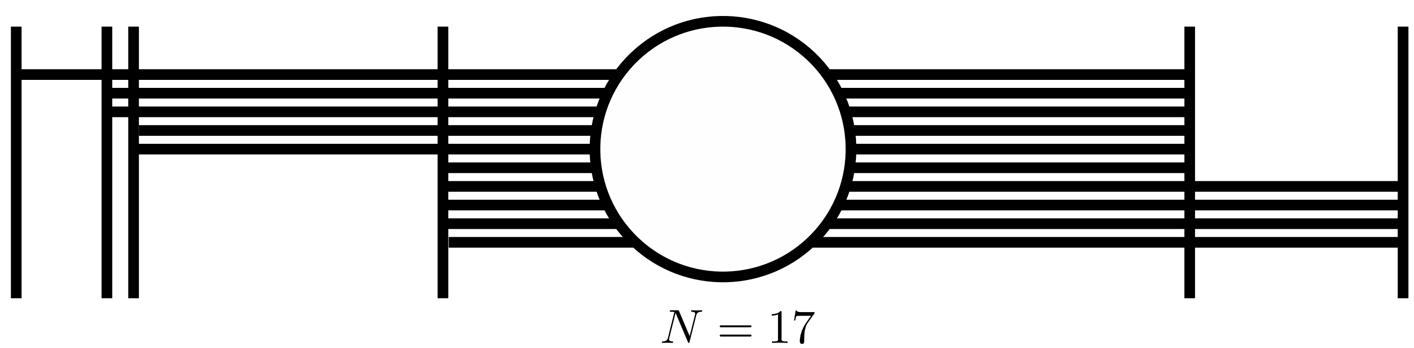
$\mathcal{L} \supset (\phi_{i+1} - \phi_i)\mathrm{Tr}F^2$ $\qquad$ $\phi_i = x^6$ positions of NS5's

coincident NS5s = strong coupling point; CFT?

the branes can also be arranged differently...

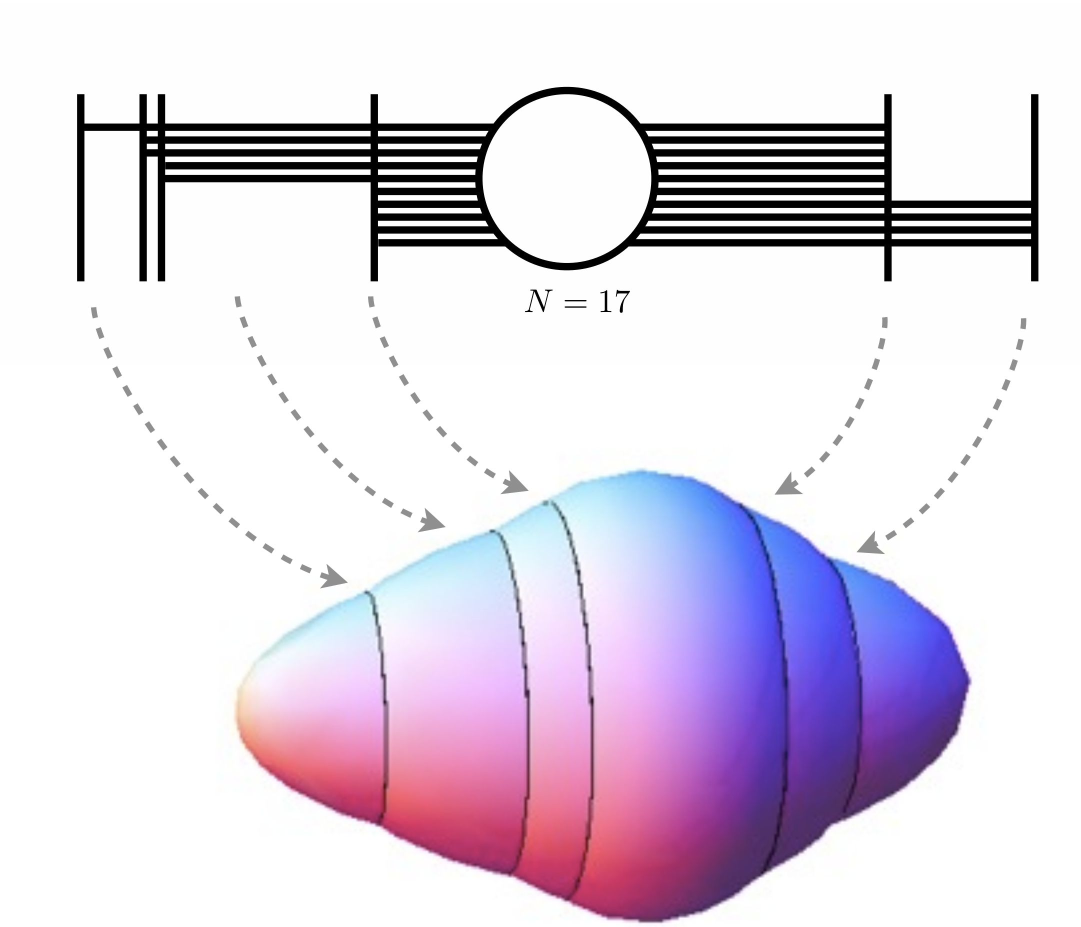


Until



brane supergravity solution not known, but...

Conjecture: near-horizon limit gives our AdS7 solutions

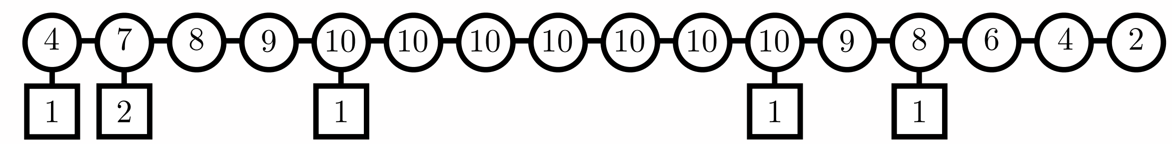


$N = \#$ NS5's ⟷ flux integer $\int_{M_3} H$

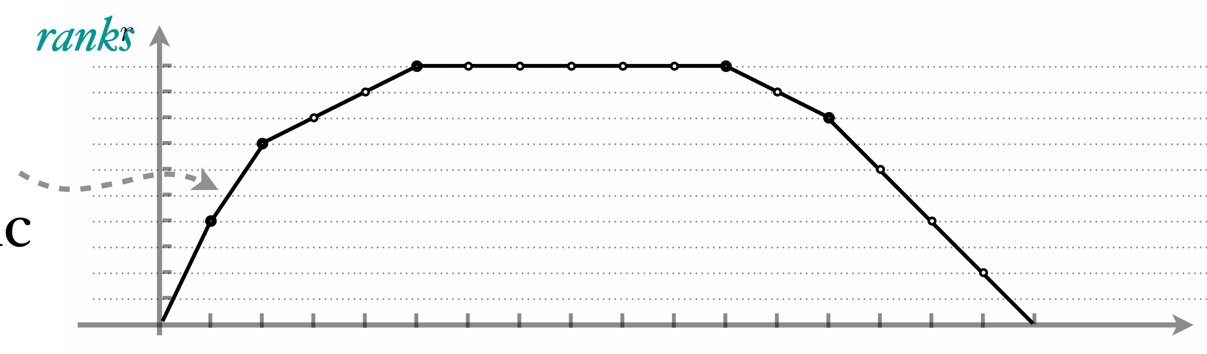
D6's ending on a D8 ⟷ D6 charge of the D8

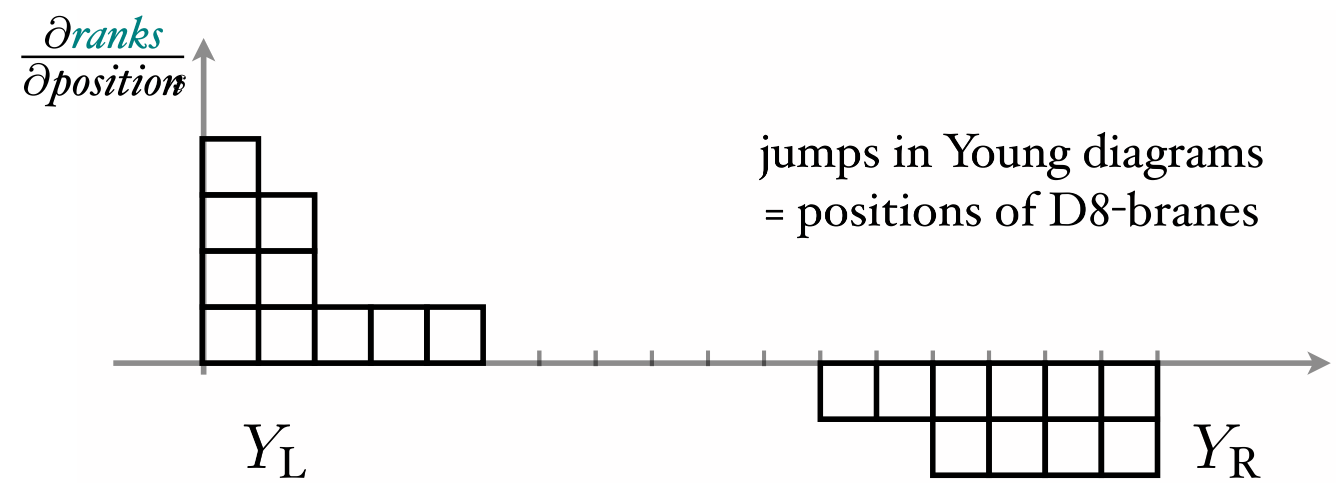
These theories can be labeled by two Young diagrams

[combinatorics well-known
in other dimensions]

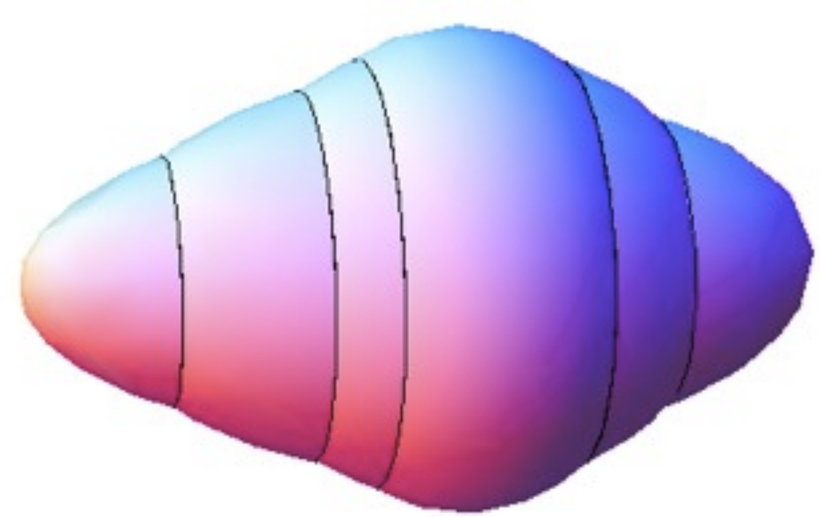


same function $\ddot{\alpha}(z)$
appearing in the metric

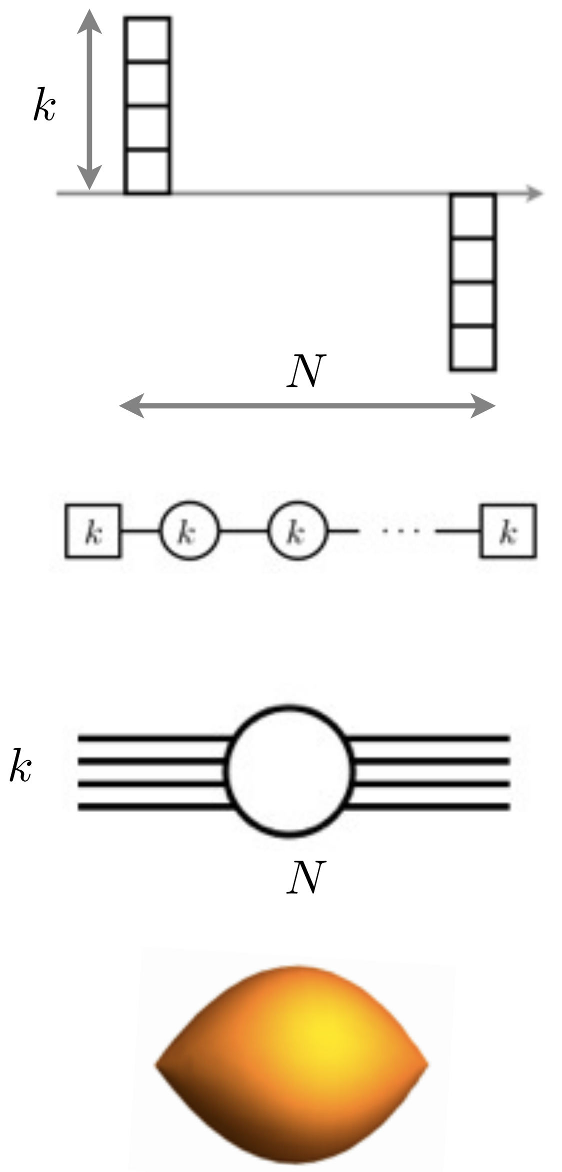




$$ds^2 = 8\sqrt{-\frac{\ddot{\alpha}}{\alpha}}ds^2_{\mathrm{AdS}_7} + \sqrt{-\frac{\alpha}{\ddot{\alpha}}}dz^2 + \frac{\alpha^{3/2}(-\ddot{\alpha})^{1/2}}{\sqrt{2\alpha\ddot{\alpha}-\dot{\alpha}^2}}ds^2_{S^2}$$

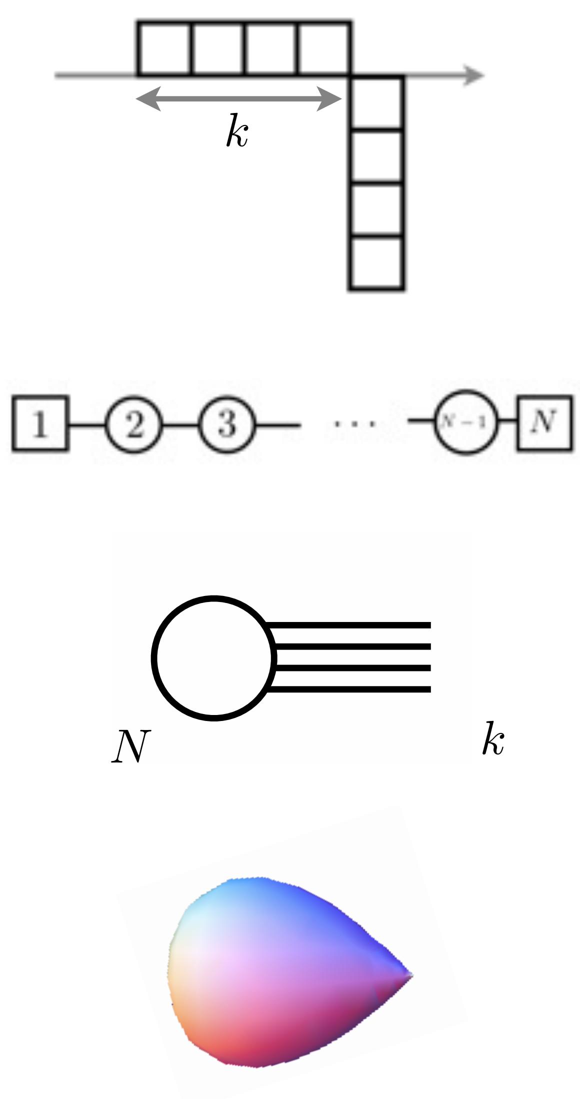


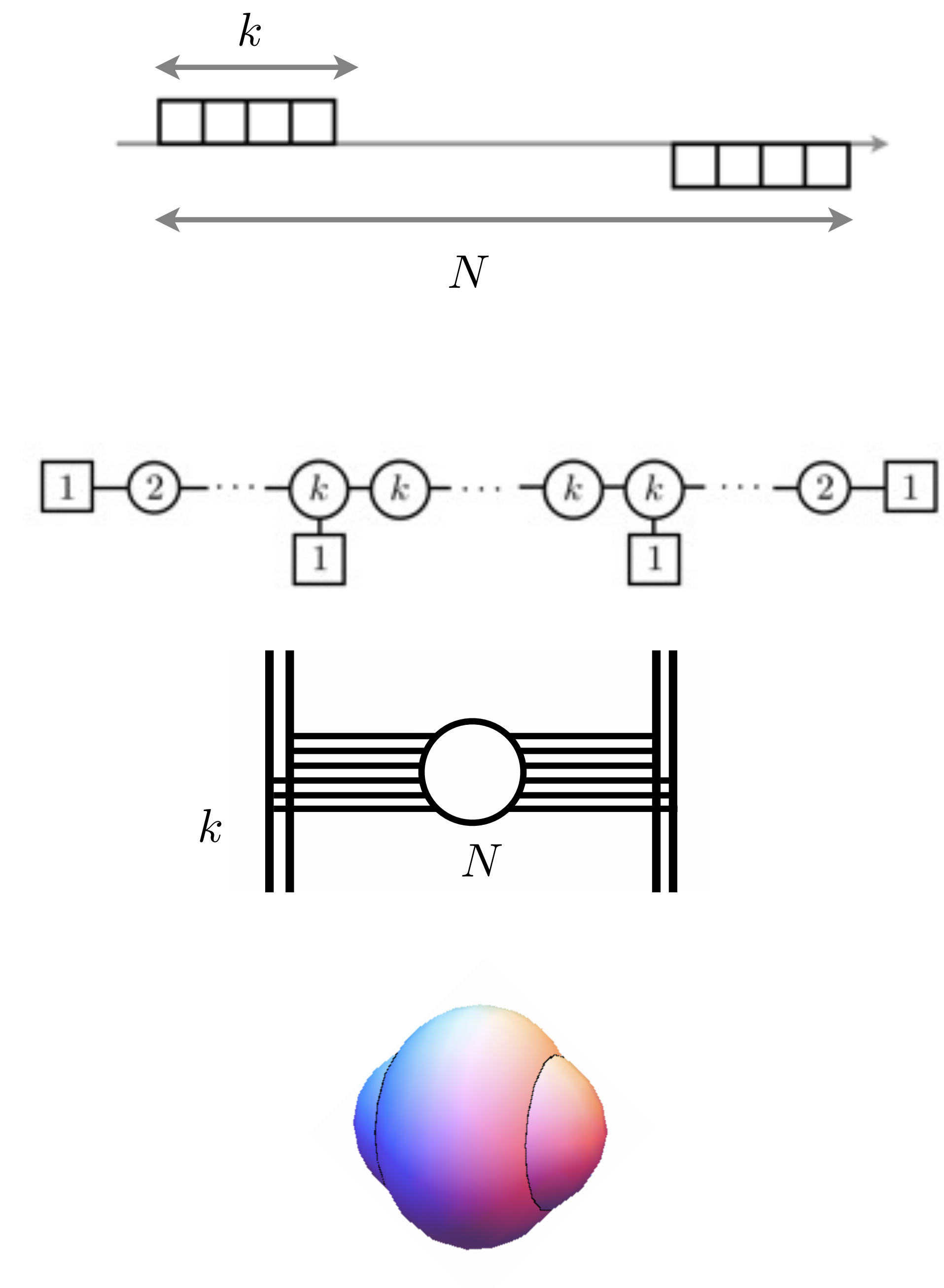
Some notable examples:



reduction of
$\mathrm{AdS}_7 \times S^4/\mathbb{Z}_k$

an orbifold of
the $(2,0)$ theory

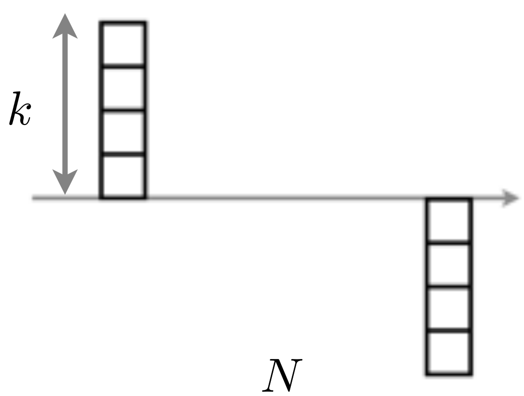


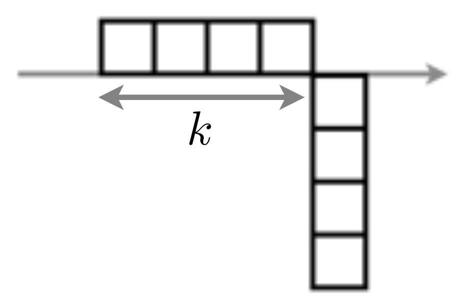


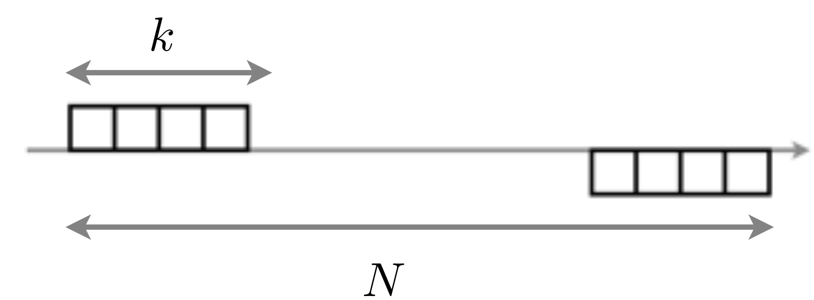
A check of this conjectured correspondence

AdS/CFT: (# deg. freedom) $\cong \mathrm{vol}(M_3)$

we can compare this with the R-symmetry anomaly in field theory







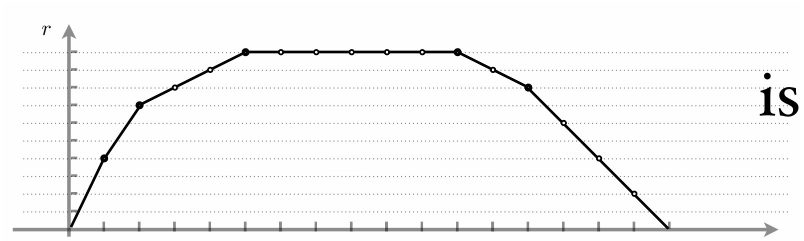
$a \sim k^2 N^3$ ✓ $\quad a \sim \frac{4}{15} k^5$ ✓ $\quad a \sim k^2 \left(N^3 - 4kN + \frac{16}{5} k^3 \right)$ ✓

...the comparison always works

[S. Cremonesi, AT, in prep.]

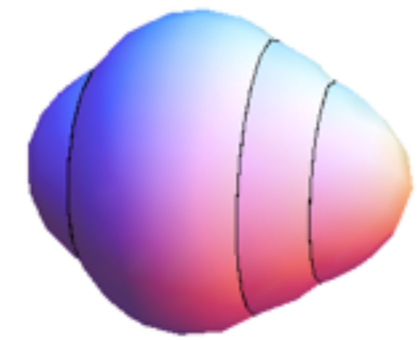
General reason:

this graph



is a discretization of the internal geometry!

Conclusions



- Classification of type II AdS7 solutions

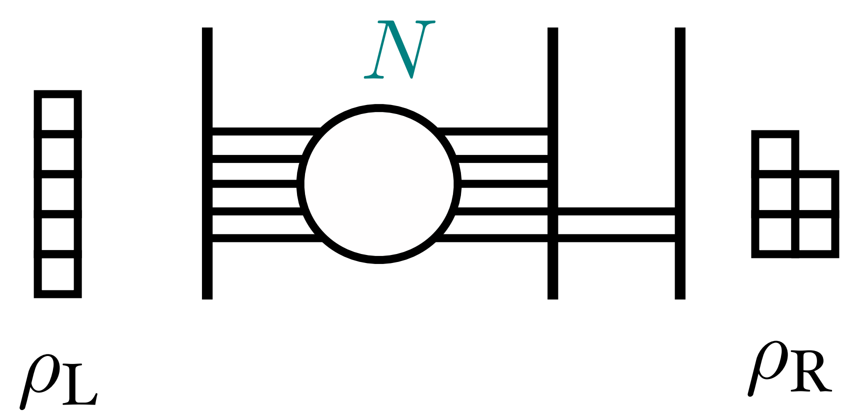
- Infinitely many analytic AdS7, AdS5, AdS4 solutions

- Dual field theories: strong coupling points in linear $\mathrm{U}(k)$ quivers

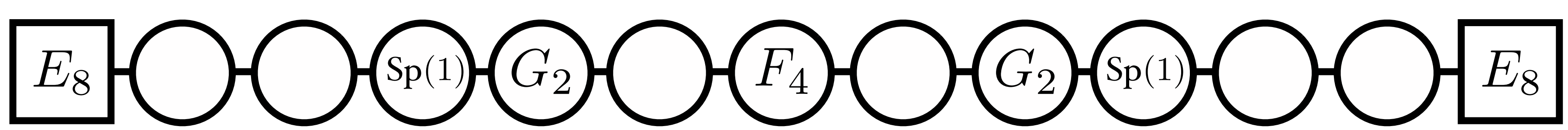
NS5-branes

$$T^{N}{}_{\rho_{\mathrm{L}},\rho_{\mathrm{R}}}$$

pattern of D6's
ending on D8's



- There are also extensions involving exceptional gauge groups



['fractional M5-branes']

[del Zotto, Heckman, AT, Vafa '14]