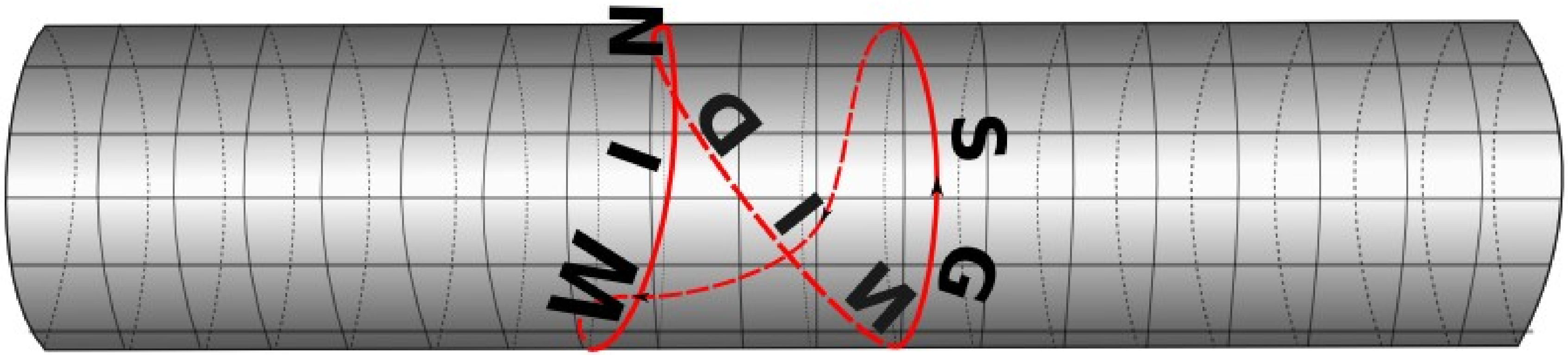


Enhancing and Double Field Theory

G. Aldazabal, CAB-IB, Bariloche

Stringy Geometry,
MITP, 2015

In collaboration with: M. Graña, S. Iguri, M. Mayo, C. Nuñez, A. Rosabal (in progress).

Motivation:

Hull, Zwiebach (2009)

Hohm, Hull, Zwiebach (2010)

- Windings are a key *stringy* ingredient of T-duality.
- DFT aims to incorporate stringy T-duality in an effective field theory.

compact momentum $p \leftrightarrow y$ compact coordinate

winding $\tilde{p} \leftrightarrow \tilde{y}$ New dual coordinate

$$D = d + 2n$$

$$T(x, \mathbb{Y}) = T(x, y, \tilde{y}) \qquad O(n, n) \text{ tensor}$$

- However DFT requires constraints:

Strong constraint $\partial_y \otimes \partial_{\tilde{y}} = 0 \rightarrow \Phi(x, y)$

Generalized Scherk-Schwarz $\Phi(x, \mathbb{Y}) = \hat{\Phi}(x) T(y, \tilde{y})$ Twist of KK zero mode

- Windings have not been clearly included in DFT, yet

Duff, Siegel, Tseytlin, (1990-1993)

Hull, Zwiebach (2009)

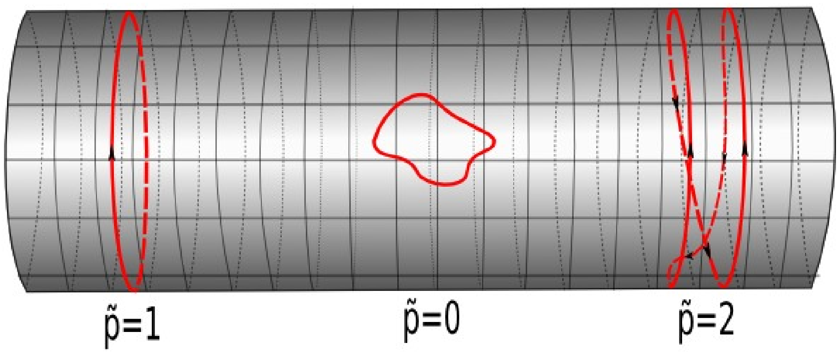
Hohm, Hull, Zwiebach (2010)

G.A, Andriot, Baron, Bedoya, Berkeley, Berman, Betz, Blair, Blumenhagen, Dall Agata, Dibitetto, Cederwall, Coimbra, Copland, Geissbuller, Fernandez-Melgarejo, Graña, Hohm, Hull, Iguri, Jeon, Kleinschmidt, Larfors, Lee, Lust, Malek, Marques, Mayo, Minasias, Nibbelink, Nuñez, Park, Patalong, Penas, Perry, Petrini, Pezzella, Pradisi, Renecke, Riccioni, Roest, Rosabal, Rudolph, Samtleben, Shahbazi, Strickland-Constable, Thomson, Waldram, West, Zweibach, …

Many others...

Circle compactification

$$z = e^{i\sigma+\tau}$$



$$Y(z,\bar{z}) = y(z) + \bar{y}(\bar{z}) \to Y(z,\bar{z}) + 2\pi\tilde{p}R$$

$$\updownarrow$$

$$\tilde{Y}(z,\bar{z}) = y(z) - \bar{y}(\bar{z}) \to \tilde{Y}(z,\bar{z}) + 2\pi p\tilde{R}$$

Left

Right

$$k = \frac{p}{R} + \frac{\tilde{p}}{\tilde{R}}\,, \qquad \bar{k} = \frac{p}{R} - \frac{\tilde{p}}{\tilde{R}}$$

Dual radius

$$\tilde{R} = \alpha'/R$$

String states

$$\sim : e^{[iky(z)+i\bar{k}\bar{y}(\bar{z})]} e^{iK\cdot[\overbrace{x(z)+\bar{x}(\bar{z})}^{X^\mu(z)}]} :\equiv e^{[ipY(z,\bar{z})+\tilde{p}\tilde{Y}(z,\bar{z})]} e^{iK\cdot X(z,\bar{z})} :$$

$$\begin{aligned} X^\mu(z) &\to x^\mu \\ Y(z) &\to Y \\ \tilde{Y}(\bar{z}) &\to \tilde{Y} \end{aligned}$$

$\tilde{p}, p \neq 0$

$$S^1 \times M_{st} \to S^1(R) \times \tilde{S}^1(\tilde{R}) \times M_{st}$$

?

String $\to$ DFT

Gauge enhancing

Gauge enhancing

$$M^2 = \frac{2}{\alpha'}(N + \tilde{N} - 2) + [(\frac{p}{R})^2 + (\frac{\tilde{p}}{\tilde{R}})^2] \qquad N = N_x + N_y$$

$$\tilde{N} - N = p.\tilde{p}$$ Level matching

Gauge enhancing

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$\tilde{N}_x = 1, N_y = 1, p = \tilde{p} = 0$ KK massless vector boson $U(1)_L$

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$$\begin{aligned} M^2 &= \frac{2}{\alpha'}(0+1-2)+(\frac{p}{R})^2+(\frac{\tilde{p}}{\tilde{R}})^2 \\ 1-0 &= 1 = p.\tilde{p} \end{aligned}$$

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Slide to $R = \tilde{R} = \sqrt{\alpha'} = R_{sd}$ Self dual radius

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2 new massless vector bosons

$$U(1)_L \rightarrow SU(2)_L$$

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2 new massless vector bosons

$$U(1)_L \to SU(2)_L$$

Same for Right sector, extra massless scalars..

Gauge enhancing

$$U(1)_L \times U(1)_R \to SU(2)_L \times SU(2)_R$$

2d vectors

6d vectors

1 massless KK scalar
(+ "slightly" massive states)

$R \to \tilde{R}$

9 massless scalars $(3,3)$

+

d^2 Universal gravity sector G, B

+

Massive states+tachyons I

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~~Massive states+tachyons I~~

$$d^2 + 2d + 1 = \dim \frac{O(d+1,d+1)}{O(d+1)\times O(d+1)} \quad \to \quad d^2 + 6d + 9 = \dim \frac{O(d+3,d+3)}{O(d+3)\times O(d+3)}$$

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DFT description?

PLAN

PLAN

- String: (3-point) scattering amplitudes for $R = \tilde{R}$ and $R \neq \tilde{R}$

  Derivation of Effective gauge field theory action

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String theory action

String vertex operators

String vertex operators

$$V(z,\bar{z}) \sim : \Phi(\epsilon\partial y, \partial X) e^{[iky(z)+i\bar{k}\bar{y}(\bar{z})]} e^{iK\cdot[x(z)+\bar{x}(\bar{z})]} :\equiv \Phi(\epsilon\partial y, \partial X) e^{[ipY(z,\bar{z})+\tilde{p}\tilde{Y}(z,\bar{z})]} e^{iK\cdot X(z,\bar{z})} :$$

$$k = \frac{p}{R} + \frac{\tilde{p}}{\tilde{R}}, \qquad \bar{k} = \frac{p}{R} - \frac{\tilde{p}}{\tilde{R}}$$

mixes Left and Right

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Level matching

$$p.\tilde p = k.k - \bar k.\bar k = \bar N - N$$

$$\partial_Y \partial_{\tilde Y} V = (\partial_y\partial_y - \partial_{\bar y}\partial_{\bar y})V = p.\tilde p V = (\bar N - N)V$$

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i.e. $\quad p = \tilde{p} = 1 \qquad p.\tilde{p} = 1$

$$V^{\pm}(z,\bar{z}) = i\sqrt{2}\frac{g'_c}{\alpha'^{1/2}}\epsilon^{\pm}_{\mu} : \bar{\partial}X^{\mu} e^{iK\cdot X} exp[\pm i m_+ y(z)] exp[\pm i m_- \bar{y}(\bar{z})] :$$

$$\begin{aligned} m_- &= R^{-1} - \tilde{R}^{-1} = \frac{1}{\alpha'}(\tilde{R} - R)\,, \\ m_+ &= R^{-1} + \tilde{R}^{-1} = \frac{1}{\alpha'}(\tilde{R} + R)\,. \end{aligned}$$

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Massive vector

$$m_V = m_- \to 0$$

$$R \to \tilde{R} \to \sqrt{\alpha'}$$

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- Massless Left gauge bosons

String vertex operators

$$R = \tilde{R} = \sqrt{\alpha'}$$

- Massless Left gauge bosons $SU(2)_L$

$\bar{N}_x = 1, N_y = 1$	$p = \tilde{p} = 0 \, (k = \bar{k} = 0)$	$V^3(z,\bar{z}) = i\sqrt{2}\frac{g_c'}{\alpha'^{1/2}}\epsilon^3_\mu : J^3(z)\bar{\partial}X^\mu e^{iK\cdot X}$	$A^3_\mu dx^\mu$
$\bar{N}_x = 1$	$p = \tilde{p} = \pm 1 \; (k = \pm\frac{2}{\sqrt{\alpha'}}, \bar{k} = 0)$	$V^\pm(z,\bar{z}) = i\sqrt{2}\frac{g_c'}{\alpha'^{1/2}}\epsilon^\pm_\mu : J^\pm(z)\bar{\partial}X^\mu e^{iK\cdot X}$	$A^\pm_\mu dx^\mu$

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- Massless Right gauge bosons $SU(2)_R$

$N_x = 1, \bar{N}_y = 1$	$p = \tilde{p} = 0\,(k = \bar{k} = 0)$	$\bar{V}^3(z,\bar{z}) = i\sqrt{2}\frac{g_c'}{\alpha'^{1/2}}\epsilon^3_\mu : \bar{J}^3(z)\bar{\partial}X^\mu e^{iK\cdot X}$	$\bar{A}^3_\mu dx^\mu$
$N_x = 1$	$p = -\tilde{p} = \pm 1\ (k = 0, \bar{k} = \pm\frac{2}{\sqrt{\alpha'}})$	$\bar{V}^\pm(z,\bar{z}) = i\sqrt{2}\frac{g_c'}{\alpha'^{1/2}}\epsilon^\pm_\mu : \bar{J}^\pm(z)\partial X^\mu e^{iK\cdot X}$	$\bar{A}^\pm_\mu dx^\mu$

String vertex operators

$$R = \tilde{R} = \sqrt{\alpha'}$$

- Massless Left gauge bosons $\quad SU(2)_L$

$\bar{N}_x = 1, N_y = 1$	$p = \tilde{p} = 0 \, (k = \bar{k} = 0)$	$V^3(z,\bar{z}) = i\sqrt{2}\frac{g_c'}{\alpha'^{1/2}}\epsilon^3_\mu : J^3(z)\bar{\partial}X^\mu e^{iK\cdot X}$	$A^3_\mu dx^\mu$
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- Massless scalars $\quad (3,3) \quad SU(2)_L \times SU(2)_R$

$N_y = 1, \bar{N}_y = 1$	$p = \tilde{p} = 0$	M^{33}
$N_y = 1, \bar{N}_y = 0$	$p = -\tilde{p} = \pm 1$	$M^{3\pm}$
$N_y = 0, \bar{N}_y = 1$	$p = \tilde{p} = \pm 1$	$M^{\pm 3}$
$N_y = 0, \bar{N}_y = 0$	$p = \pm 2, \tilde{p} = 0$	$M^{\pm\pm}$
$N_y = 0, \bar{N}_y = 0$	$p = 0, \tilde{p} = \pm 2$	$M^{\pm\mp}$

$$V_S(z,\bar{z}) = g_c'\sqrt{2}M^{ab}(K) : J^a(z)\bar{J}^b(\bar{z})e^{iK\cdot X} :$$

CFT Currents

$$V(z,\bar{z}) = i\sqrt{2}\frac{g_c'}{\alpha'^{1/2}}\epsilon^a_\mu(K) : J^a(z)\bar{\partial}X^\mu e^{iK\cdot X} dz d\bar{z}$$

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$$J^3(z) \quad = \quad \frac{i}{\sqrt{\alpha'}}\partial_z y(z), \qquad J^\pm(z) =: exp(\pm 2i\alpha'^{-1/2}y(z)) :$$

$$\mathrm{J}^a(z)J^b(0) \sim \frac{\kappa^{ab}}{z^2} + \frac{f_c^{ab}}{z}J^c(0) \quad \rightarrow \quad SU(2)_L$$

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$$A^a J^a(z)dz = (A^+ e^{+2i\alpha'^{1/2}y(z)} + A^- e^{-2i\alpha'^{-1/2}y(z)})dz + A^3 dy(z)$$

CFT Currents

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Mode expansion

CFT Currents

$$V(z,\bar{z}) = i\sqrt{2}\frac{g'_c}{\alpha'^{1/2}}\epsilon^a_\mu(K) : J^a(z)\bar{\partial}X^\mu e^{iK\cdot X} dz d\bar{z}$$

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$$A(x,\mathbb{Y}) = \sum_{(\mathbb{P}=p,\tilde{p})} A^{(\mathbb{P})}(x)e^{i\mathbb{P}.\mathbb{Y}} + \text{level} \quad \text{matching} \quad \mathbb{P}.\mathbb{P} = p.\tilde{p} = 1 \equiv \partial_{\mathbb{Y}}.\partial_{\mathbb{Y}} = \mathbf{1}$$

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J^a Internal base

No strong constraint

String 3-point amplitudes

$$R = \tilde{R} = \sqrt{\alpha'}$$

$\langle GGG \rangle$ gravity sector

$\langle VVG \rangle + \langle VVV \rangle$

gauge kinetic terms

$\langle \bar{V}\bar{V}\bar{G} \rangle + \quad \langle \bar{V}\bar{V}\bar{V} \rangle$

$\langle V_S V_S G \rangle + \langle V V_S V_S \rangle$ scalar kinetic terms

$\langle V_S V_S V_S \rangle$ cubic scalar potetial

$\langle V \bar{V} V_S \rangle$ mixings

$\ldots$

Effective action

$$R = \tilde{R} = \sqrt{\alpha'}$$

$$\begin{aligned}
\frac{1}{\sqrt{g}}\mathcal{L} &= R - \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} \\
&- \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \frac{1}{4} \bar{F}^a_{\mu\nu} \bar{F}^{a\mu\nu} - \frac{1}{2} D_\mu M^{a\tilde{a}} D_\nu M^{a\tilde{a}} g^{\mu\nu} \\
&- det M - \frac{1}{2} M^{a\tilde{a}} F^a_{\mu\nu} \bar{F}^{\tilde{a}\mu\nu} + \ldots
\end{aligned}$$

$$\begin{aligned}
F^a_{\mu\nu} &= 2\partial_{[\mu} A^a_{\nu]} + f^{abc} A^b_\mu A^c_\nu \,, \quad F^{\tilde{a}}_{\mu\nu} = 2\partial_{[\mu} A^{\tilde{a}}_{\nu]} + f^{\tilde{a}\tilde{b}\tilde{c}} A^{\tilde{b}}_\mu A^{\tilde{c}}_\nu \,, \\
D_\mu M^{a\tilde{a}} &= \partial_\mu M^{a\tilde{a}} + f^{abc} A^b_\mu M^{c\tilde{a}} + f^{\tilde{a}\tilde{b}\tilde{c}} A^{\tilde{b}}_\mu M^{a\tilde{c}} \\
H_{\mu\nu\rho} &= \partial_\mu B_{\nu\rho} + A^a_{[\mu} F^a_{\nu\rho]} + f^{abc} A^a_\mu A^b_\nu A^c_\mu + \ldots
\end{aligned}$$

$$R \neq \tilde{R}$$

String vertex operators

$$R \neq \tilde{R}$$

Only field that are massles at $R = \tilde{R} = \sqrt{\alpha}'$

$M^{3\pm}$,M$^{\pm 3}$?

scalars	m^2
M^{33}	0
$M^{\pm\pm}$	$\frac{4}{R} m_-$
$M^{\pm\mp}$	$\frac{4}{\tilde{R}} m_-$

vectors	m^2
V^3	0
$\bar{V}^3$	0
$V^{\pm}$	m_-^2

$$\begin{aligned} m_- &= R^{-1} - \tilde{R}^{-1} = \frac{1}{\alpha}(\tilde{R} - R) \\ m_+ &= R^{-1} + \tilde{R}^{-1} = \frac{1}{\alpha}(\tilde{R} + R) \end{aligned}$$

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$$\bar{T}(\bar{z})V(0) \sim k \cdot \epsilon^+ \frac{1}{z^3} + V(0)\frac{1}{z}$$

anomalous

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anomalous

$$V^{\pm,3}(z,\bar{z}) = \frac{g'_c}{\alpha'^{1/2}}\epsilon^{\pm,3}\bar{\partial}\bar{y}(\bar{z}) e^{\pm i m_+ y} e^{\pm i m_- \bar{y}} e^{iK\cdot X} \,.$$

Goldstone boson

$$V'^{\pm} = V^{\pm} - \xi V^{\pm,3}$$

Massive vector boson

String vertex operators

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$$\bar{T}(\bar{z})V(0) \sim \cancel{k\cdot\epsilon^+}\frac{1}{z^3} + V(0)\frac{1}{z}$$ anomalous

$$V^{\pm,3}(z, \bar{z}) = \frac{g_c'}{\alpha'^{1/2}}\epsilon^{\pm,3}\bar{\partial}\bar{y}(\bar{z})e^{\pm i m_+ y}e^{\pm i m_- \bar{y}}e^{iK\cdot X} .$$ Goldstone boson

$$V'^{\pm} = V^{\pm} - \xi V^{\pm,3}$$ Massive vector boson

$$K \cdot \epsilon^{\pm} \mp \xi m_- \epsilon^{\pm,3} = 0$$ Anomaly cancellation, longitudinal polarization

$$\downarrow$$

$$\partial_\mu A^{\pm\mu} \pm i\xi m_- M^{\pm,3} = 0$$ 't Hooft gauge fixing

Effective action

$R \neq \tilde{R}$

$$
\begin{aligned}
\frac{1}{\sqrt{g}}\mathcal{L} &= \\
& \frac{1}{2k_d^2}R - \frac{1}{4}(\partial_\mu \phi)^2 - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} \\
&- \frac{1}{4}F^3_{\mu\nu}F^{\mu\nu 3} - \frac{1}{4}\bar{F}^3_{\mu\nu}\bar{F}^{\mu\nu 3} \\
&- \frac{1}{2}F'^{+}_{\mu\nu}F'^{\mu\nu -} - m_-^2 A'_\mu A'_\nu G^{\mu\nu} - \frac{1}{2}\bar{F}'^{+}_{\mu\nu}\bar{F}'^{\mu\nu -} - m_-^2 \bar{A}'_\mu \bar{A}'_\nu G^{\mu\nu} \\
&+ \frac{1}{2}\partial_\mu M^{33}\partial^\mu M^{33} + D_\mu M^{\pm,\pm}D^\mu M^{\mp,\mp} + D_\mu M^{\pm,\mp}D^\mu M^{\mp,\pm} \\
&- i\frac{g}{\sqrt{\alpha'}}\frac{\sqrt{\alpha'}m_+}{2}A'^{+\mu}A'^{-\nu}\frac{1}{2}F^3_{\mu\nu} + i\frac{g}{\sqrt{\alpha'}}\frac{\sqrt{\alpha'}m_-}{2}A'^{+\mu}A'^{-\nu}\frac{1}{2}\bar{F}^3_{\mu\nu} \\
&- i\frac{g}{\sqrt{\alpha'}}\frac{\sqrt{\alpha'}m_+}{2}\bar{A}'^{+\nu}\bar{A}'^{-\nu}\frac{1}{2}\bar{F}^3_{\mu\nu} + i\frac{g}{\sqrt{\alpha'}}\frac{\sqrt{\alpha'}m_-}{2}\bar{A}'^{+\mu}\bar{A}'^{-\nu}\frac{1}{2}F^3_{\mu\nu} \\
&+ 2\frac{g}{\sqrt{\alpha'}}\frac{m_+\sqrt{\alpha'}}{2}A'^{\pm,\mu}A'^{\mp}_{,\mu}M^{33}m_- + 2\frac{g}{\sqrt{\alpha'}}\frac{m_+\sqrt{\alpha'}}{2}\bar{A}'^{\pm,\mu}\bar{A}'^{\mp}_{\mu}M^{33}m_- \\
&- \frac{1}{2}F'^{+}_{\mu\nu}\bar{F}'^{+\mu\nu}M^{-,-} - \frac{1}{2}F'^{+}_{\mu\nu}\bar{F}'^{-\mu\nu}M^{-,+} - \frac{1}{2}F^3_{\mu\nu}F^{3\mu\nu}M^{3,3} \\
&+ \frac{4g}{\alpha'}M^{+,-}M^{-,+}M^{33}\left(\frac{\sqrt{\alpha'}}{\tilde{R}}\right)^2 - \frac{4g}{\alpha'}M^{+,+}M^{-,-}M^{33}\left(\frac{\sqrt{\alpha'}}{R}\right)^2
\end{aligned}
$$

$$
\begin{aligned}
F'^{\pm}_{\mu\nu} &= \partial_{[\mu} A'^{\pm}_{\nu]} \mp i \frac{g}{\sqrt{\alpha'}} \frac{\sqrt{\alpha'} m_+}{2} A^3_{[\mu} A'^{\pm}_{\nu]} \mp i \frac{g}{\sqrt{\alpha'}} \frac{\sqrt{\alpha'} m_-}{2} \bar{A}^3_{[\mu} A'^{\pm}_{\nu]} \\
\bar{F}'^{\pm}_{\mu\nu} &= \partial_{[\mu} \bar{A}'^{\pm}_{\nu]} \mp i \frac{g}{\sqrt{\alpha'}} \frac{\sqrt{\alpha'} m_+}{2} \bar{A}^3_{[\mu} \bar{A}'^{\pm}_{\nu]} \mp ig \frac{\sqrt{\alpha'} m_-}{2} A^3_{[\mu} \bar{A}'^{\pm}_{\nu]} \\
F^3_{\mu\nu} &= \partial_{[\mu} A^3_{\nu]}
\end{aligned}
$$

$$
\begin{aligned}
D_\mu M^{\pm,\pm} &= [\partial_\mu + i(\pm) g \frac{\sqrt{\alpha'}}{R} A^3_\mu + i(\pm) g \frac{\sqrt{\alpha'}}{R} \bar{A}^3_\mu] M^{\pm,\pm} \\
D_\mu M^{\pm,\mp} &= [\partial_\mu + i(\pm) g \frac{\sqrt{\alpha'}}{\tilde{R}} A^3_\mu - i(\pm) g \frac{\sqrt{\alpha'}}{\tilde{R}} \bar{A}^3_\mu] M^{\pm,\mp}
\end{aligned}
$$

Effective theory with massless and “slightly massive” states

“Hidden” T-duality symmetry

can be understood from Higgs mechanism

$$M^{33} + \epsilon$$

Same degrees of freedom as in R_{sd}

Full dependence on R $(m_-,)$

$$R = \sqrt{\alpha'} \exp(-\tfrac{1}{2}\epsilon) = \sqrt{\alpha'}(1 - \tfrac{1}{2}\epsilon + \mathcal{O}(\epsilon^2)) \ .$$

Indicates contributions coming from higher order “non renormalizable” terms

Symmetry breaking...

$$R = \sqrt{\alpha'}\exp(-\tfrac{1}{2}\epsilon) = \sqrt{\alpha'}(1 - \tfrac{1}{2}\epsilon + \mathcal{O}(\epsilon^2)) \ .$$

$$\frac{1}{R} m_- = \frac{1}{\alpha'}(e^\epsilon - 1) = \frac{1}{\alpha'}(\epsilon + \frac{1}{2}\epsilon^2 + \dots)$$

$$\frac{1}{R} = m_- + \frac{1}{\tilde{R}}$$

$$\begin{aligned} -4gM^{+,+}M^{-,-}M^{33}\frac{1}{R^2} &= -4\frac{g}{\alpha'}M^{+,+}M^{-,-}M^{33} \\ &- g\frac{4}{R}m_- M^{+,+}M^{-,-}M^{33} \end{aligned}$$

Indicates

$$-4\tfrac{1}{\alpha'}[g(M^{33}+v) + g^2\tfrac{1}{2}(M^{33}+v)^2 + g^3\tfrac{1}{3!}(M^{33}+v)^3 + \dots]M^{++}M^{--}.$$

$$\downarrow$$

$$4M^{33}M^{++}M^{--}(v + \tfrac{1}{2}v^2 + \tfrac{1}{3!}v^3 + \dots) = m_{++}^2 M^{++}M^{--}$$

with $\quad m_{++}^2 = \tfrac{4}{R}m_-$

DFT action

PLAN

PLAN

- Brief introduction DFT frame formulation.

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  DFT generalized Scherk-Schwarz compactification.

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- $SU(2)_L \times SU(2)_R$ Effective DFT gauge field theory action

DFT

DFT

- coordinates

DFT

- coordinates

- fields

DFT

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- Symmetries

DFT

- coordinates

$p^i \quad \leftrightarrow \quad y_i \qquad\qquad \tilde{p}^i \quad \leftrightarrow \quad \tilde{y}^i$ dual coordinates

$i = 1, \dots, n$

$P_M = (p_i, \tilde{p}^i) \quad \leftrightarrow \quad \mathbb{Y} = (y^i, \tilde{y}_i)$ internal, fundamental representation of $\mathrm{O(n,n)}$

- fields

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$T(x, \mathbb{Y}) = T(x, y, \tilde{y})$ restrict to $\mathcal{H}_{MN}(X), d(X)$

Generalized metric

$$\mathcal{H}_{MN} = \begin{pmatrix} g^{ij} & -g^{ik}b_{kj} \\ b_{ik}g^{kj} & g_{ij} - b_{ik}g^{kl}b_{lj} \end{pmatrix} \in O(D,D)$$

dilaton

$$e^{-2d} = \sqrt{g}e^{-2\phi}$$

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- Symmetries

$$\mathcal{L}_{V_1} V_2^M = L_{V_1} V_2^M + Y^{MN}_{PQ} \partial_N V_{1P} V_2^Q = V_1^P \partial_P V_2^M - V_2^P \partial_P V_1^M + \partial^P V_{1P} V_2^M$$

DFT

- coordinates

$p^i \quad \leftrightarrow \quad y_i \qquad\qquad \tilde{p}^i \quad \leftrightarrow \quad \tilde{y}^i$ dual coordinates

$i = 1, \ldots, n$

$P_M = (p_i, \tilde{p}^i) \quad \leftrightarrow \quad \mathbb{Y} = (y^i, \tilde{y}_i)$ internal, fundamental representation of $\mathrm{O(n,n)}$

- fields

$T(x, \mathbb{Y}) = T(x, y, \tilde{y})$ restrict to $\mathcal{H}_{MN}(X), d(X)$

Generalized metric

dilaton

$$\mathcal{H}_{MN} = \begin{pmatrix} g^{ij} & -g^{ik} b_{kj} \\ b_{ik} g^{kj} & g_{ij} - b_{ik} g^{kl} b_{lj} \end{pmatrix} \in O(D, D)$$

$$e^{-2d} = \sqrt{g} e^{-2\phi}$$

- Symmetries

$$Y^M{}_P{}^N{}_Q = \eta^{MN} \eta_{PQ}$$

$$\mathcal{L}_{V_1} V_2^M = L_{V_1} V_2^M + Y^{MN}_{PQ} \partial_N V_{1P} V_2^Q = V_1^P \partial_P V_2^M - V_2^P \partial_P V_1^M + \partial^P V_{1P} V_2^M$$

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\+ closure $\longrightarrow$ constraints

DFT

• coordinates

$p^i \quad \leftrightarrow \quad y_i \qquad\qquad \tilde{p}^i \quad \leftrightarrow \quad \tilde{y}^i$ dual coordinates

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$T(x, \mathbb{Y}) = T(x, y, \tilde{y})$ restrict to $\mathcal{H}_{MN}(X), d(X)$

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$$\mathcal{H}_{MN} = \begin{pmatrix} g^{ij} & -g^{ik} b_{kj} \\ b_{ik} g^{kj} & g_{ij} - b_{ik} g^{kl} b_{lj} \end{pmatrix} \in O(D, D)$$

dilaton

$$e^{-2d} = \sqrt{g} e^{-2\phi}$$

• Symmetries

$$Y^M{}_P{}^N{}_Q = \eta^{MN} \eta_{PQ}$$

$$\mathcal{L}_{V_1} V_2^M = L_{V_1} V_2^M + Y^{MN}_{PQ} \partial_N V_{1P} V_2^Q = V_1^P \partial_P V_2^M - V_2^P \partial_P V_1^M + \partial^P V_{1P} V_2^M$$

+ closure $\longrightarrow$ constraints i.e. $\partial_M \partial^M \cdots = 0\,, \qquad \partial_M \cdots \partial^M \cdots = 0\,,$

DFT

Frame formulation:

Geissbuhler, (2011)
Marques, Nuñez, Penas,
G.A, Marques, Nuñez (2014)

$V = v + \xi$ vectors+forms

$$V_1 \cdot V_2 = \tfrac{1}{2}(\iota_{v_1}\xi_2 + \iota_{v_2}\xi_1) = \eta(V_1, V_2) = V_1^M \eta_{MN} V_2^N$$

$E_A \equiv E^a \oplus E_a$ generalized frame $\in O(D,D)/H$

$$A \in H = O(1, D-1) \times O(D-1, 1)$$

$$\eta_{MN} = E^A{}_M \; \eta_{AB} \; E^B{}_N$$

$$\downarrow$$

$\mathcal{H}_{MN} = E^A{}_M \; S_{A\bar{B}} \; E^B{}_N$ generalized metric

can be parametrized as

$$E^A{}_M = \begin{pmatrix} e_a{}^i & e_a{}^j b_{ji} \\ 0 & e^a{}_i \end{pmatrix} \,, \qquad S_{AB} = \begin{pmatrix} s^{ab} & 0 \\ 0 & s_{ab} \end{pmatrix}$$

with $g_{ij} = e^a{}_i s_{ab} e^b{}_j$ and $s_{ab} = \mathrm{diag}(- + \cdots +)$

DFT

Generalized (dynamical) fluxes $\mathcal{F}_{\bar{A}\bar{B}\bar{C}}(X)$

$$\mathcal{L}_\xi E_A{}^M = \xi^P \partial_P E_A{}^M + (\partial^M \xi_P - \partial_P \xi^M)\; E_A{}^P$$

transforms as a vector

in particular

$$\mathcal{L}_{E_A} E_B{}^M \;=\; \mathcal{F}_{AB}{}^C E_C{}^M$$

Fluxes (dynamical)

$$\delta_\xi \mathcal{F}_{ABC} \;=\; \xi^D \partial_D \mathcal{F}_{ABC} + \Delta_\xi \mathcal{F}_{ABC}$$

$\downarrow$

scalar

if closure is satisfied

$$\Delta_\xi \mathcal{F}_{ABC} \;=\; E_{CM} \Delta_\xi (\mathcal{L}_{E_A} E_B{}^M) = 0$$

DFT action

$$S_{DFT} = \int dX e^{-2D} \mathcal{R}$$

$$\mathcal{R} = \mathcal{F}_{ABC}\, \mathcal{F}_{DEF} \left[\frac{1}{4}S^{AD}\eta^{BE}\eta^{CF} - \frac{1}{12}S^{AD}S^{BE}S^{CF} - \frac{1}{6}\eta^{AD}\eta^{BE}\eta^{CF}\right]$$

Scherk-Schwarz dimensional reductions

$D = d + n$

G. A, Baron, Marques, Nuñez, (2011)
Geissbuller

$$\boxed{E_A(x,y) = U_A{}^{A'}(x)E'_{A'}(y)}$$

frame twist

gauged

$$\hat{\mathcal{F}}_{ABC}(x,y) = \mathcal{F}_{ABC}(x) - f_{IJK}U_A{}^I U_B{}^J U_C{}^K$$

$$\mathcal{L}_{E_A} E_I{}^M = f_{IJ}{}^K E_K{}^M$$

$$\mathcal{F}_{ABC}(x) = 3\Omega_{[ABC]}$$

$$\tilde{\Omega}_{ABC}(U) = U_A{}^I \partial_I U_B{}^J U_{CJ}$$

$$\tilde{\Omega}_{IJK}(E) = E_I{}^M \partial_M E_J{}^N E_{KN}$$

$$f_{IJK} = 3\tilde{\Omega}_{[IJK]}$$

constant

$$f_{[MN}{}^P f_{Q]P}{}^R = 0\,,$$

Quadratic constraints

$$\hat{\mathcal{F}}_{ABC}(x) \xrightarrow{A \to (\mu, I)}$$

$$\begin{aligned}
\mathcal{G}_{\mu\rho\lambda} &= 3\partial_{[\mu}b_{\rho\lambda]} - f_{IJK}A^I{}_\mu A^J{}_\rho A^K{}_\lambda + 3\partial_{[\mu}A^I{}_\rho A_{\lambda]J} \\
\mathcal{F}^I{}_{\mu\nu} &= \partial_\mu A^I{}_\nu - \partial_\nu A^I{}_\mu - f_{JK}{}^I A^J{}_\mu\ A^K{}_\nu \\
(D_\mu \mathcal{H})_{IJ} &= (\partial_\mu \mathcal{H})_{IJ} + f^K{}_{LI}A^L_\mu \mathcal{H}_{KJ} + f^K{}_{LJ}A^L_\mu \mathcal{H}_{IK}\,.
\end{aligned}$$

DFT Effective action

$$
\begin{aligned}
S_{eff} \quad = \quad & \int d^dx\sqrt{g}e^{-2\varphi}\Big(\Lambda + \mathcal{R} + 4\partial^\mu\varphi\partial_\mu\varphi - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} \\
& \qquad -\frac{1}{4}\mathcal{H}_{IJ}F^{I\mu\nu}F^J_{\mu\nu} + \frac{1}{8}(D_\mu\mathcal{H})_{IJ}(D^\mu\mathcal{H})^{IJ} \\
& -\frac{1}{12}f_{IJK}f_{LMN}\left(\mathcal{H}^{IL}\mathcal{H}^{JM}\mathcal{H}^{KN} - 3\,\mathcal{H}^{IL}\eta^{JM}\eta^{KN} + 2\,\eta^{IL}\eta^{JM}\eta^{KN}\right)\Big)
\end{aligned}
$$

$$\mathcal{H}_{IJ}(x) = \mathcal{S}_{I'J'}U_I{}^{I'}(x)U_J{}^{J'}(x)$$

scalars

Is there a DFT frame

$$E_A(x,y) = U_A{}^{A'}(x)E'_{A'}(y)$$

?

$$E_A(x,y) = U_A{}^{A'}(x)E'_{A'}(y) \longrightarrow SU(2)_L \times SU(2)_R$$

$$\mathcal{L}_{E_A}(y)E_I{}^M(y) = f_{IJ}{}^K E_K{}^M(y) \quad \rightarrow \quad f_{IJK} \equiv \epsilon_{ijk} \oplus \bar{\epsilon}_{ijk}$$

$$\mathcal{H}_{IJ}(x) = \mathcal{S}_{I'J'}U_I{}^{I'}U_J{}^{J'} \quad \rightarrow \quad M^{a,\bar{a}}$$

$$A^I{}_\mu \quad \rightarrow \quad A^a{}_\mu \oplus \bar{A}^a{}_\mu$$

$$D = d + 3 = d + 1 + 2$$

Warming up

$$D = d + 1 \qquad M = 0, \ldots, d-1; d$$

KK reduction on a circle

$$G_{MN} \to G_{\mu\nu},\, G_{\mu d},\, G_{dd}$$

$$B_{MN} \to B_{\mu d}$$

Circle KK reduction $\quad D = d + 1$

Generalized frame

$$E_A \equiv \begin{pmatrix} E_{\hat a} \\ E^{\hat a} \end{pmatrix} = \begin{pmatrix} e_{\hat a} - \iota_{e_a} B \\ e^{\hat a} \end{pmatrix} \quad . \qquad A \equiv (\hat a, {}^{\hat a}\,) \qquad \hat a = (0, \ldots, d-1, d)$$

$$\hat e^{\hat a} = \begin{pmatrix} e^a \\ \phi(dy + V_1) \end{pmatrix} \qquad \mathrm{V}_1 = V_\mu dx^\mu$$

$$\hat e_{\hat a} = \begin{pmatrix} e_a - \iota_{e^a} V_1 \partial_y \\ \phi^{-1}\partial_y \end{pmatrix}$$

$$\hat B_2 = B_2 + B_1 \wedge (dy + V_1)$$

$$\begin{aligned} E_a &= e_a - (\iota_{e_a} V_1)\, \partial_y - (\iota_{e_a} B_1) dy - \iota'_{e_a} C^+ \\ E_d &= \phi^{-1}(\partial_y + B_1) \\ E^d &= \phi(dy + V_1) \\ E^a &= e^a \end{aligned}$$

Circle reduction

Internal frame

Scherk-Schwarz

$$\begin{pmatrix} E_d \\ E^d \end{pmatrix} = \begin{pmatrix} \phi^{-1} & 0 \\ 0 & \phi \end{pmatrix} \begin{pmatrix} \partial_y + B_1 \\ dy + V_1 \end{pmatrix} .$$

$$\phi = \exp(-\tfrac{1}{2} M^{33}).$$

M^{33} Metric fluctuations

$$TS^1 \oplus T^*S^1$$

$$\big\downarrow$$

$$\mathrm{C}_+ \oplus \mathcal{C}_-$$

$$R_A{}^B = \tfrac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$\begin{aligned} A &= V_1 + B_1 \\ \bar{A} &= V_1 - B_1 \ , \end{aligned}$$

$$U(1)_L \times U(1)_R$$

Circle reduction

$$D = d + 1$$

Circle reduction

$$D = d + 1$$

$$E(x, y) = U E'$$

Circle reduction

$$D = d + 1$$

$$E(x, y) = U E'$$

$$\begin{array}{cccc} & \text{scalar} & \text{matrix} & \\ \end{array}$$

$$\begin{pmatrix} 1_d & 0 & 0 & 0 \\ 0 & U^+ & -U^- & 0 \\ 0 & -U^- & U^+ & 0 \\ 0 & 0 & 0 & 1_d \end{pmatrix} \in \frac{O(d+1,d+1)}{O(d+1)\times O(d+1)}$$

Circle reduction

$$D = d + 1$$

$$E(x,y) = UE'$$

$$\begin{array}{cccc} & \text{scalar} & \text{matrix} & \\ \end{array}$$

$$\begin{pmatrix} 1_d & 0 & 0 & 0 \\ 0 & U^+ & -U^- & 0 \\ 0 & -U^- & U^+ & 0 \\ 0 & 0 & 0 & 1_d \end{pmatrix} \in \frac{O(d+1,d+1)}{O(d+1)\times O(d+1)}$$

$$\begin{pmatrix} E'_a \\ E'^- \\ E'^+ \\ E'^a \end{pmatrix} = \begin{pmatrix} e_a - \iota_{e_a}\bar{A}\bar{\mathcal{J}} - \iota_{e_a}A\mathcal{J} & -\iota_{e'_a}C^+ \\ \bar{\mathcal{J}} - \bar{A} & \\ \mathcal{J} + A & \\ e^a & \end{pmatrix} .$$

$$U^\pm = \tfrac{1}{2}(\phi \pm \phi^{-1})$$

$$\mathcal{J} = \partial_y + dy$$
$$\bar{\mathcal{J}} = \partial_y - dy$$

Circle reduction

$$D = d + 1$$

$$E(x,y) = UE'$$

$$\begin{array}{ccccc} & \text{scalar} & \text{matrix} & & \\ \end{array}$$

$$\begin{pmatrix} 1_d & 0 & 0 & 0 \\ 0 & U^+ & -U^- & 0 \\ 0 & -U^- & U^+ & 0 \\ 0 & 0 & 0 & 1_d \end{pmatrix} \in \frac{O(d+1,d+1)}{O(d+1)\times O(d+1)}$$

$$\begin{pmatrix} E'_a \\ E'^- \\ E'^+ \\ E'^a \end{pmatrix} = \begin{pmatrix} e_a - \iota_{e_a}\bar{A}\bar{\mathcal{J}} - \iota_{e_a}A\mathcal{J} & -\iota_{e'_a}C^+ \\ \bar{\mathcal{J}} - \bar{A} & \\ \mathcal{J} + A & \\ e^a & \end{pmatrix}.$$

$$U^\pm = \tfrac{1}{2}(\phi \pm \phi^{-1})$$

$$\mathcal{J} = \partial_y + dy$$
$$\bar{\mathcal{J}} = \partial_y - dy$$

$$\mathcal{H}_{IJ}(x) = \mathcal{S}_{I'J'}U_I{}^{I'}U_J{}^{J'}$$

Circle reduction

$$D = d + 1$$

$$E(x, y) = U E'$$

scalar matrix

$$\begin{pmatrix} 1_d & 0 & 0 & 0 \\ 0 & U^+ & -U^- & 0 \\ 0 & -U^- & U^+ & 0 \\ 0 & 0 & 0 & 1_d \end{pmatrix} \in$$

$$\frac{O(d+1,d+1)}{O(d+1)\times O(d+1)}$$

$$\begin{pmatrix} E'_a \\ E'^- \\ E'^+ \\ E'^a \end{pmatrix} = \begin{pmatrix} e_a - \iota_{e_a}\bar{A}\bar{\mathcal{J}} - \iota_{e_a}A\mathcal{J} & -\iota_{e'_a}C^+ \\ \bar{\mathcal{J}} - \bar{A} & \\ \mathcal{J} + A & \\ e^a & \end{pmatrix} .$$

$$U^\pm = \tfrac{1}{2}(\phi \pm \phi^{-1})$$

$$\mathcal{J} = \partial_y + dy$$
$$\bar{\mathcal{J}} = \partial_y - dy$$

$$\mathcal{H}_{IJ}(x) = \mathcal{S}_{I'J'} U_I{}^{I'} U_J{}^{J'}$$

$$\phi = \exp(-\tfrac{1}{2} M^{33}).$$

$$\mathcal{H}_{\mathcal{C}} = \begin{pmatrix} \cosh(M^{33}) & -\sinh(M^{33}) \\ -\sinh(M^{33}) & \cosh(M^{33}) \end{pmatrix} \approx \begin{pmatrix} 1 & -M^{33} \\ -M^{33} & 1 \end{pmatrix} + \mathcal{O}(M^{33})^2 \ .$$

Enhancing

$$D = d + 3 \qquad E(x,y) = U E'$$

scalars matrix

$$\begin{pmatrix} 1_d & 0 & 0 & 0 \\ 0 & U_1^{ij} & -U_2^{ij} & 0 \\ 0 & -U_3^{ij} & U_4^{ij} & \\ 0 & 0 & 0 & 1_d \end{pmatrix} \in \frac{O(d+3,d+3)}{O(d+3)\times O(d+3)}$$

$$\begin{pmatrix} E'_a \\ E'^{Rj} \\ E'^{Lj} \\ E'^a \end{pmatrix} = \begin{pmatrix} e_a - (\iota_{e_a}\bar{A}^j)\bar{\mathcal{J}}^j - (\iota_{e_a}A^j)\mathcal{J}^j \quad -\iota_{e'_a}C^+ \\ \bar{\mathcal{J}}^j - \bar{A}^j \\ \mathcal{J}^j + A^j \\ e^a \end{pmatrix}.$$

Sufficient information

$SU(2)_L$

$$A = A^j \mathcal{J}^j .$$

$$[\mathcal{J}^i, \mathcal{J}^j] = \epsilon^{ijk}\mathcal{J}^k \ ,$$

$$[\mathcal{J}^i, \bar{\mathcal{J}}^j] = 0$$

$SU(2)_R$

$$\bar{A} = \bar{A}^j \bar{\mathcal{J}}^j$$

$$[\bar{\mathcal{J}}^i, \bar{\mathcal{J}}^j] = \epsilon^{ijk}\bar{\mathcal{J}}^k$$

Recap

$$V = \hat{v} + \hat{\xi} \qquad \in TM_D \oplus T^*_M D$$

$$O(d+1, d+1)$$

$$V = v + (v^d + \chi^d) + \xi \qquad \in TM_d \oplus (TS^1 + T^*S^1) \oplus T^*_M d$$

$$V = v + (A^i J^i + \bar{A}^i \bar{J}^i) + \xi \qquad \in TM_d \oplus (su(2)_L \oplus su(2)_R) \oplus T^*_M d$$

$$O(d+1+2, d+1+2)$$

$$\mathcal{L}_{V_1} = L_{v_1} v_2 + (L_{v_1} A_2 - L_{v_2} A_1 + [A_1, A_2]) + L_{v_1} \xi_2 - \iota_{v_2} d\xi_1$$

$$\mathcal{L}_{V_1} V_2^M \quad = \quad L_{V_1} V_2^M + Y^{MN}_{PQ} \partial_N V_{1P} V_2^Q - f^M_{PQ} V_1^P V_2^Q$$

Scalars

$$\mathcal{H}_{IJ}(x) = \mathcal{S}_{I'J'} U_I{}^{I'} U_J{}^{J'} \quad \in \quad \frac{O(d+3, d+3)}{O(d+3) \times O(d+3)}$$

$$\mathcal{H}^{IJ} = \begin{pmatrix} h^{-1} & -h^{-1}b \\ bh^{-1} & h - bh^{-1}b \end{pmatrix} \longrightarrow \mathcal{H}_{\mathcal{C}}^{IJ} = (R\mathcal{H}R^T)^{AB} = \begin{pmatrix} \mathcal{H}^{\mathcal{C}_+\mathcal{C}_+} & \mathcal{H}^{\mathcal{C}_+\mathcal{C}_-} \\ \mathcal{H}^{\mathcal{C}_-\mathcal{C}_+} & \mathcal{H}^{\mathcal{C}_-\mathcal{C}_-} \end{pmatrix} ,$$

$$\begin{aligned} \mathcal{H}^{\mathcal{C}_+\mathcal{C}_+} &= (\mathcal{H}^{\mathcal{C}_-\mathcal{C}_-})^T = \frac{1}{2}\big[(h+h^{-1}) + (h^{-1}b - bh^{-1}) - bh^{-1}b\big], \\ \mathcal{H}^{\mathcal{C}_+\mathcal{C}_-} &= (\mathcal{H}^{\mathcal{C}_-\mathcal{C}_+})^T = -\frac{1}{2}\big[(h-h^{-1}) + (h^{-1}b + bh^{-1}) - bh^{-1}b\big] . \end{aligned}$$

$h \approx 1_3 + h_0, \quad b \approx 0_3 + b_0$ as in $U(1)_L \times U(1)_R$ case

$$\mathcal{H}_{\mathcal{C}} = \begin{pmatrix} 1_3 & -M \\ -M^T & 1_3 \end{pmatrix}$$

M^{ij} 9 scalars

Needed ingredients

Needed ingredients

- ✔ frame $E(x, y)$

Needed ingredients

- ✔ frame $E(x,y)$
- ✔ SS splitting $E(x,y) = UE'$

U

E'

Needed ingredients

✔ frame $\quad E(x,y)$

✔ SS splitting $\quad E(x,y) = UE'$

U

E'

✔ fluxes

$$
\begin{aligned}
\mathcal{L}_{E_A} E_B{}^M &= \mathcal{F}_{AB}{}^C E_C{}^M \\
\hat{\mathcal{F}}_{ABC}(x,y) &= \mathcal{F}_{ABC}(x) - f_{IJK} U_A{}^I U_B{}^J U_C{}^K
\end{aligned}
$$

Needed ingredients

✔ frame

$$E(x,y)$$

✔ SS splitting

$$E(x,y) = UE' \quad \begin{matrix} \nearrow U \\ \searrow E' \end{matrix}$$

✔ fluxes

$$\begin{aligned} \mathcal{L}_{E_A} E_B{}^M &= \mathcal{F}_{AB}{}^C E_C{}^M \\ \hat{\mathcal{F}}_{ABC}(x,y) &= \mathcal{F}_{ABC}(x) - f_{IJK} U_A{}^I U_B{}^J U_C{}^K \end{aligned}$$

✔ scalars

$$\mathcal{H}_{IJ}(x) = \mathcal{S}_{I'J'} U_I{}^{I'} U_J{}^{J'} \qquad \mathcal{H}_{\mathcal{C}}^{IJ} = \begin{pmatrix} \delta^{ij} & -M^{ij} \\ -M^{ji} & \delta^{ij} \end{pmatrix}$$

DFT Effective action

$$
\begin{aligned}
S_{eff} = \int d^d x \quad \sqrt{g} \quad & e^{-2\varphi} \Big(\Lambda + \mathcal{R} + 4\partial^\mu \varphi \partial_\mu \varphi - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} \\
- \quad & \frac{1}{4} \mathcal{H}_{IJ} F^{I\mu\nu} F^J_{\mu\nu} \\
+ \quad & \frac{1}{8} (D_\mu \mathcal{H})_{IJ} (D^\mu \mathcal{H})^{IJ} \\
- \quad & \frac{1}{12} f_{IJK} f_{LMN} \left(\mathcal{H}^{IL} \mathcal{H}^{JM} \mathcal{H}^{KN} - 3\, \mathcal{H}^{IL} \eta^{JM} \eta^{KN} + 2\, \eta^{IL} \eta^{JM} \eta^{KN} \right) \Big)
\end{aligned}
$$

Gauge kinetic terms

$$-\frac{1}{4}\mathcal{H}_{IJ}F^{I\mu\nu}F^{J}_{\mu\nu}$$

Gauge kinetic terms

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$$\downarrow$$

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Gauge kinetic terms

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$$\downarrow$$

$$\mathcal{H}_{IJ} = \begin{pmatrix} \delta_{ij} & M_{ij} \\ M_{ji} & \delta_{ij} \end{pmatrix}$$

$$-\frac{1}{4}\delta_{ij}F^{i\mu\nu}F^{j}_{\mu\nu}$$
$$-\frac{1}{4}\delta_{lm}\bar{F}^{l\mu\nu}\bar{F}^{m}_{\mu\nu}$$
$$-\frac{1}{2}M_{il}F^{i\mu\nu}\bar{F}^{l}_{\mu\nu}$$

Gauge kinetic terms

$$-\tfrac{1}{4}\mathcal{H}_{IJ}F^{I\mu\nu}F^{J}_{\mu\nu}$$

$$\downarrow \qquad \mathcal{H}_{IJ} = \begin{pmatrix} \delta_{ij} & M_{ij} \\ M_{ji} & \delta_{ij} \end{pmatrix}$$

$$-\frac{1}{4}\delta_{ij}F^{i\mu\nu}F^{j}_{\mu\nu}$$
$$-\frac{1}{4}\delta_{lm}\bar{F}^{l\mu\nu}\bar{F}^{m}_{\mu\nu}$$
$$-\frac{1}{2}M_{il}F^{i\mu\nu}\bar{F}^{l}_{\mu\nu}$$

✔.

Scalars kinetic terms

$$(D_\mu \mathcal{H})_{IJ}(D^\mu \mathcal{H})^{IJ}$$

$$(D_\mu \mathcal{H})_{IJ} \quad = \quad (\partial_\mu \mathcal{H})_{IJ} + f^K{}_{LI} A^L_\mu \mathcal{H}_{KJ} + f^K{}_{LJ} A^L_\mu \mathcal{H}_{IK} \, .$$

Scalars kinetic terms

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$$\Big\downarrow$$

$$\mathcal{H}_{IJ} = \begin{pmatrix} \delta_{ij} & M_{ij} \\ M_{ji} & \delta_{ij} \end{pmatrix}$$

Scalars kinetic terms

$$(D_\mu \mathcal{H})_{IJ}(D^\mu \mathcal{H})^{IJ}$$

$$(D_\mu \mathcal{H})_{IJ} \quad = \quad (\partial_\mu \mathcal{H})_{IJ} + f^K{}_{LI} A^L_\mu \mathcal{H}_{KJ} + f^K{}_{LJ} A^L_\mu \mathcal{H}_{IK}\,.$$

$$\mathcal{H}_{IJ} = \begin{pmatrix} \delta_{ij} & M_{ij} \\ M_{ji} & \delta_{ij} \end{pmatrix}$$

$$(D_\mu \mathcal{H})_{ij} \quad = \quad (\partial_\mu M)_{ij} + f^{\color{red}l}{}_{ik} A^k_\mu M_{{\color{red}l}j} + \bar{f}^{\color{red}l}{}_{jk} A^k_\mu M_{i{\color{red}l}}$$

Scalars kinetic terms

$$(D_\mu \mathcal{H})_{IJ}(D^\mu \mathcal{H})^{IJ}$$

$$(D_\mu \mathcal{H})_{IJ} \quad = \quad (\partial_\mu \mathcal{H})_{IJ} + f^K{}_{LI} A^L_\mu \mathcal{H}_{KJ} + f^K{}_{LJ} A^L_\mu \mathcal{H}_{IK} \, .$$

$$\Big\downarrow$$

$$\mathcal{H}_{IJ} = \begin{pmatrix} \delta_{ij} & M_{ij} \\ M_{ji} & \delta_{ij} \end{pmatrix}$$

$$(D_\mu \mathcal{H})_{ij} \quad = \quad (\partial_\mu M)_{ij} + f^l{}_{ik} A^k_\mu M_{lj} + \bar{f}^l{}_{jk} A^k_\mu M_{il}$$

✔.

Scalar potential

$$-\frac{1}{12} f_{IJK} f_{LMN} \left(\mathcal{H}^{IL}\mathcal{H}^{JM}\mathcal{H}^{KN} - 3\,\mathcal{H}^{IL}\eta^{JM}\eta^{KN} + 2\,\eta^{IL}\eta^{JM}\eta^{KN}\right)$$

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$$\mathcal{H}_{IJ} = \begin{pmatrix} \delta_{ij} & M_{ij} \\ M_{ji} & \delta_{ij} \end{pmatrix}$$

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$$\mathrm{det} M + \mathrm{const.}$$

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DFT effective action

$\downarrow$

String effective action $(R = \tilde{R} = \sqrt{\alpha'})$ ✔.

$O(d+1,d+1) \quad A_3; \bar{A}_3 \quad TM_d \oplus (TS^1 + T^*S^1) \oplus T^*_M d$

$O(d+1+2, d+1+2) \quad A_3, A^{\pm}; \bar{A}_3, \bar{A}^{\pm} \quad E \simeq TM_d \oplus (\mathbb{R} \oplus \mathbb{R} \oplus \mathbb{R}^4) \oplus T^*M_d = TM_d \oplus \mathbb{R}^6 \oplus T^*M_d$

Generalized (non geometric) frame

$$
\begin{aligned}
E'_{\bar{1}} &= cos\left(2y^L/R_{sd}\right) t^1 + \sin\left(2y^L/R_{sd}\right) t^2 \\
E'_{\bar{2}} &= -sin\left(2y^L/R_{sd}\right) t^1 + \cos\left(2y^L/R_{sd}\right) t^2 \\
E'_{\bar{3}} &= dy^L \\
E'_1 &= \cos\left(2y^R/R_{sd}\right) t^3 + \sin\left(2y^R/R_{sd}\right) t^4 \\
E'_2 &= -\sin\left(2y^R/R_{sd}\right) t^3 + \cos\left(2y^R/R_{sd}\right) t^4 \\
E'_3 &= dy^R
\end{aligned}
$$

Depends on y_L, y_R

$$\mathcal{L}_{E'_A} E'_B = \tfrac{1}{2}\big[{E'_A}^P \partial_P {E'_B}^M - {E'_B}^P \partial_P {E'_A}^M + \eta^{MN} \eta_{PQ} \partial_N {E'_A}^P {E'_B}^Q \big] D_M$$

$$D_M = (t^1,\ t^2,\ dy^L,\ t^3,\ t^4,\ dy^R)^T$$

$$\partial_P = (0,\ 0,\ \partial_{y^L},\ 0,\ 0,\ \partial_{y^R})$$

$$\begin{aligned} \big[E_i, E_j\big] &= \mathcal{L}_{E_i} E_j = \frac{1}{\sqrt{\alpha'}} \epsilon_{ijk} E_k \\ \big[\bar{E}_i, \bar{E}_j\big] &= \mathcal{L}_{\bar{E}_i} \bar{E}_j = \frac{1}{\sqrt{\alpha'}} \epsilon_{ijk} \bar{E}_k \\ \big[E_i, \bar{E}_j\big] &= \big[\bar{E}_i, E_j\big] = 0 \end{aligned}$$

$$\mathcal{J}'_i = \sqrt{\alpha'} E'_i \ , \quad \bar{\mathcal{J}}'_i = \sqrt{\alpha'} \bar{E}'_i \ .$$

Reproduces the needed $su(2)_L \times \mathrm{su}(2)_R$ algebra

Summary and Outlook

- Analysis of string amplitudes in D=d+1, to identify key ingredients for a DFT description.

- Built up a consistent DFT that captures winding information and reproduces string effective action at self dual point.

- Level matching is satisfied but not the strong constraint. An explicit dependence in y and $\tilde{y}$ is needed to achieve enchancing.

- $\frac{O(d+3,d+3)}{O(d+3)\times O(d+3)}$

- Hints for an internal geometry (M.G. talk)

- Higher dimensional compactifications ?

- Symmetry breaking at DFT level. Can “slightly massive” states be incorporated?

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Thank you