

The string origin of dyonic  $\mathcal{N} = 8$  supergravity  
(and its simple Chern-Simons duals)

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- Adolfo Guarino, Daniel Jafferis, OV, Phys. Rev. Lett. **115** (2015) 9 [arXiv:1504.08009]
- Adolfo Guarino, OV, arXiv:1508.04432
- Adolfo Guarino, OV, arXiv:1509.02526

# Two puzzles

- 1 Two puzzles
- 2 Dyonic ISO(7) supergravity
- 3 Consistent truncation of massive type IIA on  $S^6$
- 4 An  $\mathcal{N} = 2$  AdS<sub>4</sub> solution of massive IIA
- 5 Final comments

## Puzzle 1: AdS/CFT for simplest CS?

- A natural way to obtain a  $\text{CFT}_3$  is to start from Chern-Simons (CS) theory and add couplings. CS with a simple gauge group  $\text{SU}(N)$  and adjoint matter seemed like a good candidate to describe the M2-brane  $\text{CFT}_3$ .

However: [Schwarz '04]

- Such  $\text{CFT}_3$ s cannot preserve maximal supersymmetry.
- The CS term seems to be related to the Romans mass.

## Puzzle 1: AdS/CFT for simplest CS?

- The M2-brane  $CFT_3$  is now known to be described by an ( $\mathcal{N} = 6$ ) CS-matter theory with non-simple gauge group  $SU(N) \times SU(N)$  at levels  $k$  and  $-k$ .

[Aharony, Bergman, Jafferis, Maldacena '08]

- The question remained: do the simplest type of CS-matter theory enjoy  $AdS_4$  duals? Prospects looked bleak. For most of these theories the spectrum has light higher spin operators and exponential growth. [Minwalla, Narayan, Sharma, Umesh, Yin '11]

- Some of these simplest CS-matter theories can still have conventional  $AdS_4$  duals. But none has been found until now.

Puzzle 2: dyonic  $\mathcal{N} = 8$  supergravity

- $D = 4$   $\mathcal{N} = 8$  gauged supergravity often admits continuous or discrete symplectic deformations that respect  $\mathcal{N} = 8$  supersymmetry and the gauge group

[Dall'Agata, Inverso, Trigiante, '12]

- E.g. the covariant derivatives acquire a new coupling to the magnetic vectors proportional to a parameter  $c$ ,

$$D = d - g (\mathcal{A}^\Lambda - c \tilde{\mathcal{A}}_\Lambda) .$$

- At finite gauge coupling  $g$ , electric/magnetic duality is broken and the theory typically becomes sensitive to the symplectic frame specified by  $c$ . The physical couplings of the supergravity develop a  $c$  dependence.

Puzzle 2: dyonic  $\mathcal{N} = 8$  supergravity

- Do these  $\mathcal{N} = 8$  gaugings enjoy a string or M-theory origin? For dyonic gaugings with AdS vacua, do these have  $\text{CFT}_3$  duals?
- For example, the purely electric  $\mathcal{N} = 8$   $\text{SO}(8)$  gauging arises from consistent truncation of  $D = 11$  supergravity on  $S^7$ . [de Wit, Nicolai '87]
- All the solutions of the  $D = 4$  theory give rise to solutions in  $D = 11$ . In particular, the  $D = 4$  vacua uplift to  $\text{AdS}_4 \times S^7$  M-theory backgrounds.
- Some of these have known  $\text{CFT}_3$  duals. Eg, the central vacuum uplifts to Freund-Rubin, which is dual to ABJM.

# Our new results

- Massive type IIA supergravity admits an  $\mathcal{N} = 8$  consistent truncation on  $S^6$ .
- The resulting  $D = 4$  theory has a dyonically-gauged  $ISO(7) = SO(7) \ltimes \mathbb{R}^7$  gauge group.
- All the solutions of the  $D = 4$  theory give rise to solutions in  $D = 10$ . In particular, the  $D = 4$  vacua uplift to  $AdS_4 \times S^6$  massive type IIA backgrounds.

We found the first explicit  $\mathcal{N} = 2$  such solution.



## Our new results

- Massive type IIA on these  $\text{AdS}_4 \times S^6$  backgrounds is dual to the simple CS theories of type discussed above. We gave the first  $\text{AdS}_4/\text{CFT}_3$  precision match.
- The  $D = 4$  magnetic coupling  $m \equiv gc$ , the  $D = 10$  Romans mass  $\hat{F}_{(0)}$  and the CS level  $k$  are related by

$$m = \hat{F}_{(0)} = k/(2\pi\ell_s),$$

where  $\ell_s = \sqrt{\alpha'}$  is the string length.

# Outline

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# Dyonic ISO(7) supergravity

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The  $D = 4 \mathcal{N} = 8$  tensor hierarchy

The bosonic fields of  $D = 4 \mathcal{N} = 8$  supergravity come in irreps of  $E_{7(7)}$ . The  $p$ -forms,  $p = 1, 2, 3, 4$ , generate a ‘tensor hierarchy’. [de Wit, Nicolai, Samtleben '08]

<b>1</b>	metric :	$ds_4^2$
<b>56</b>	coset representatives :	$\mathcal{V}_M^{ij}$ ,
<b>56</b>	vectors :	$\mathcal{A}^M$ ,
<b>133</b>	two-forms :	$\mathcal{B}_\alpha$ ,
<b>912</b>	three-forms :	$\mathcal{C}_\alpha^M$ ,
<b>133 + 8645</b>	four-forms	

The  $D = 4$   $\mathcal{N} = 8$  duality hierarchy

The higher-rank forms carry dynamical degrees of freedom, albeit not independent ones. They can be expressed in terms of the lower-rank forms, scalars and metric via the ‘duality hierarchy’ [Bergshoeff, Hartong, Hohm, Huebscher, Ortin '09]

$$\begin{aligned}\tilde{\mathcal{H}}_{(2)\Lambda} &= \mathcal{R}_{\Lambda\Sigma} \mathcal{H}_{(2)}^\Sigma - \mathcal{I}_{\Lambda\Sigma} * \mathcal{H}_{(2)}^\Sigma, \\ \mathcal{H}_{(3)\alpha} &= -\frac{1}{12} (t_\alpha)_M{}^P \mathcal{M}_{NP} * D\mathcal{M}^{MN}, \\ \mathcal{H}_{(4)\alpha}{}^M &= -\frac{1}{84} (t_\alpha)_P{}^R X_{NQ}{}^S \mathcal{M}^{MN} \left( \mathcal{M}^{PQ} \mathcal{M}_{RS} + 7 \delta_S^P \delta_R^Q \right) \text{vol}_4.\end{aligned}$$

# The embedding tensor

In  $\mathcal{N} = 8$  supergravity, all effects of the gauging are codified in the embedding tensor  $\Theta_{\mathbb{M}}^{\alpha}$ .

[de Wit, Samtleben, Trigiante '07]

This is subject to

- quadratic constraints, which ensure the consistency of the gauging, and
- linear constraints, which restrict it to the **912** of  $E_{7(7)}$ .

## The dyonic ISO(7) embedding tensor

- To formulate the ISO(7) gauging, it is natural to branch out  $E_{7(7)}$  into  $SL(7)$ , since

$$ISO(7) \equiv SO(7) \ltimes \mathbb{R}^7 \subset SL(7) \ltimes \mathbb{R}^7 \subset GL(7) \ltimes \mathbb{R}^7 \subset SL(8) \subset E_{7(7)} .$$

- The embedding tensor of dyonic ISO(7) supergravity takes values in the  $\mathbf{28} + \mathbf{1}$  of  $SL(7)$ :

[Dall'Agata, Inverso, '11]

$$\Theta_{[AB]}{}^C{}_D = 2\delta_{[A}^C\theta_{B]D} \quad , \quad \Theta^{[AB]C}{}_D = 2\delta_D^{[A}\xi^{B]C} .$$

where

$$\theta = \text{diag}(\mathbb{1}_7, 0) \quad , \quad \xi = \text{diag}(0_7, 1) \quad ,$$

- The ISO(7)-covariant derivatives that follow from this are

$$D = d - g\mathcal{A}^{IJ} t_{[I}{}^K \delta_{J]K} + (g\delta_{IJ} \mathcal{A}^I - m\tilde{\mathcal{A}}_J) t_8^J .$$

## A restricted duality hierarchy

For the ISO(7) gauging, a restricted duality hierarchy can be identified, still  $\mathcal{N} = 8$  but only SL(7)-covariant:

<b>1</b>	metric :	$ds_4^2$
<b>21' + 7' + 21 + 7</b>	coset representatives :	$\mathcal{V}^{IJij}$ , $\mathcal{V}^{I8ij}$ , $\tilde{\mathcal{V}}_{IJ}{}^{ij}$ , $\tilde{\mathcal{V}}_{I8}{}^{ij}$ ,
<b>21' + 7' + 21 + 7</b>	vectors :	$\mathcal{A}^{IJ}$ , $\mathcal{A}^I$ , $\tilde{\mathcal{A}}_{IJ}$ , $\tilde{\mathcal{A}}_I$ ,
<b>48 + 7'</b>	two-forms :	$\mathcal{B}_I{}^J$ , $\mathcal{B}^I$ ,
<b>28'</b>	three-forms :	$\mathcal{C}^{IJ}$ .



## A restricted duality hierarchy

This restricted duality hierarchy has closed supersymmetry transformations and field equations. For example, the Bianchi identities close into

$$D\mathcal{H}_{(2)}^{IJ} = 0 \quad , \quad D\mathcal{H}_{(2)}^I = m \mathcal{H}_{(3)}^I \quad , \quad D\tilde{\mathcal{H}}_{(2)IJ} = -2g \mathcal{H}_{(3)[I}{}^K \delta_{J]K} \quad , \quad D\tilde{\mathcal{H}}_{(2)I} = g \delta_{IJ} \mathcal{H}_{(3)}^J \quad ,$$

$$D\mathcal{H}_{(3)I}{}^J = \mathcal{H}_{(2)}^{JK} \wedge \tilde{\mathcal{H}}_{(2)IK} + \mathcal{H}_{(2)}^J \wedge \tilde{\mathcal{H}}_{(2)I} - 2g \delta_{IK} \mathcal{H}_{(4)}^{JK} - \frac{1}{7} \delta_I^J (\text{trace}) \quad ,$$

$$D\mathcal{H}_{(3)}^I = -\mathcal{H}_{(2)}^{IJ} \wedge \tilde{\mathcal{H}}_{(2)J} \quad , \quad D\mathcal{H}_{(4)}^{IJ} \equiv 0 \quad .$$

## A restricted duality hierarchy

The duality relations close into

$$\tilde{\mathcal{H}}_{(2)IJ} = -\frac{1}{2}\mathcal{I}_{[IJ][KL]} * \mathcal{H}_{(2)}^{KL} - \mathcal{I}_{[IJ][K8]} * \mathcal{H}_{(2)}^K + \frac{1}{2}\mathcal{R}_{[IJ][KL]} \mathcal{H}_{(2)}^{KL} + \mathcal{R}_{[IJ][K8]} \mathcal{H}_{(2)}^K ,$$

$$\tilde{\mathcal{H}}_{(2)I} = -\frac{1}{2}\mathcal{I}_{[I8][KL]} * \mathcal{H}_{(2)}^{KL} - \mathcal{I}_{[I8][K8]} * \mathcal{H}_{(2)}^K + \frac{1}{2}\mathcal{R}_{[I8][KL]} \mathcal{H}_{(2)}^{KL} + \mathcal{R}_{[I8][K8]} \mathcal{H}_{(2)}^K ,$$

$$\mathcal{H}_{(3)I}^J = -\frac{1}{12}(t_I^J)_{\mathbb{M}^{\mathbb{P}}} \mathcal{M}_{\mathbb{NP}} * D\mathcal{M}^{\mathbb{MN}} - \frac{1}{7}\delta_I^J (\text{trace}) ,$$

$$\mathcal{H}_{(3)}^I = -\frac{1}{12}(t_8^I)_{\mathbb{M}^{\mathbb{P}}} \mathcal{M}_{\mathbb{NP}} * D\mathcal{M}^{\mathbb{MN}} ,$$

$$\mathcal{H}_{(4)}^{IJ} = \frac{1}{84}X_{\mathbb{NQ}}^{\mathbb{S}}((t_K^{(I)})_{\mathbb{P}^{\mathbb{R}}} \mathcal{M}^{(J)KN} + (t_8^{(I)})_{\mathbb{P}^{\mathbb{R}}} \mathcal{M}^{(J)8N})(\mathcal{M}^{\mathbb{PQ}}\mathcal{M}_{\mathbb{RS}} + 7\delta_{\mathbb{S}}^{\mathbb{P}}\delta_{\mathbb{R}}^{\mathbb{Q}})\text{vol}_4 .$$

## Supersymmetric critical points

The dyonic ISO(7) gauging displays a rich structure of critical points, both supersymmetric and non-supersymmetric, all of them AdS. In contrast, the purely electric gauging has no known vacua.

SUSY	bos. sym.	ref.
$\mathcal{N} = 3$	SO(4)	[Gallerati, Samtleben, Trigiante '14]
$\mathcal{N} = 2$	SU(3) $\times$ U(1)	[Guarino, Jafferis, OV '15]
$\mathcal{N} = 1$	G <sub>2</sub>	[Borghese, Guarino, Roest '12]
$\mathcal{N} = 1$	SU(3)	[Guarino, OV '15]

Consistent truncation of massive type IIA on  $S^6$ 

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# The strategy

In order to find the full embedding of dyonic ISO(7) supergravity in massive IIA, we follow two steps:

- We adapt the de Wit-Nicolai  $D = 11$  approach to IIA. The IIA bosonic and fermionic field content and supersymmetry transformations are rewritten with  $\text{SO}(1, 4) \times \text{SL}(7)$  and  $\text{SO}(1, 4) \times \text{SU}(8)$  covariance.
- We develop a new technique: exploit the  $D = 4$  restricted duality hierarchy.

Type IIA with only  $SO(1,3)$  manifest

Under

$$SO(1,9) \rightarrow SO(1,3) \times SO(6),$$

the IIA fields split as

$$d\hat{s}_{10}^2 = \Delta^{-1} ds_4^2 + g_{mn} (dy^m + B^m)(dy^n + B^n),$$

$$\begin{aligned} \hat{A}_{(3)} &= \frac{1}{6} A_{\mu\nu\rho} dx^\mu \wedge dx^\nu \wedge dx^\rho + \frac{1}{2} A_{\mu\nu m} dx^\mu \wedge dx^\nu \wedge (dy^m + B^m) \\ &\quad + \frac{1}{2} A_{\mu mn} dx^\mu \wedge (dy^m + B^m) \wedge (dy^n + B^n) \\ &\quad + \frac{1}{6} A_{mnp} (dy^m + B^m) \wedge (dy^n + B^n) \wedge (dy^p + B^p), \end{aligned}$$

$$\hat{B}_{(2)} = \frac{1}{2} B_{\mu\nu} dx^\mu \wedge dx^\nu + B_{\mu m} dx^\mu \wedge (dy^m + B^m) + \frac{1}{2} B_{mn} (dy^m + B^m) \wedge (dy^n + B^n),$$

$$\hat{A}_{(1)} = A_\mu dx^\mu + A_m (dy^m + B^m),$$

Type IIA with only  $SO(1,3)$  manifest

- $SO(6)$  can be straightforwardly promoted to  $SL(6)$ . Then we have the  $SO(1,3)$ -covariant field content in  $SL(6)$  representations:

$$\begin{array}{ll}
 \mathbf{1} & \text{metric : } ds_4^2, \\
 \mathbf{21} + \mathbf{6} + \mathbf{1} + \mathbf{20} + \mathbf{15} & \text{scalars : } g_{mn}, A_m, \hat{\phi}, A_{mnp}, B_{mn}, \\
 \mathbf{6}' + \mathbf{1} + \mathbf{15} + \mathbf{6} & \text{vectors : } B_\mu{}^m, A_\mu, A_{\mu mn}, B_{\mu m}, \\
 \mathbf{6} + \mathbf{1} & \text{two-forms : } A_{\mu\nu m}, B_{\mu\nu}, \\
 \mathbf{1} & \text{three-form : } A_{\mu\nu\rho}.
 \end{array}$$

- These can be grouped up into  $SL(7)$  irreps, too.

# Non-linear redefinitions

Further redefinitions are needed so that the  $p$ -forms comply with the  $D = 4$  transformations dictated by the tensor hierarchy:

- Vectors:

$$C_\mu{}^{m8} \equiv B_\mu{}^m, \quad C_\mu{}^{78} \equiv A_\mu, \quad \tilde{C}_{\mu mn} \equiv A_{\mu mn} - A_\mu B_{mn}, \quad \tilde{C}_{\mu m7} \equiv B_{\mu m},$$

- Two-forms:

$$C_{\mu\nu m} \equiv -A_{\mu\nu m} + C_{[\mu}{}^{n8} \tilde{C}_{\nu]nm} + C_{[\mu}{}^{78} \tilde{C}_{\nu]m7}, \quad C_{\mu\nu 7} \equiv -B_{\mu\nu} + C_{[\mu}{}^{m8} \tilde{C}_{\nu]m7},$$

- Three-form:

$$C_{\mu\nu\rho} \equiv A_{\mu\nu\rho} - C_{[\mu}{}^{m8} C_{\nu}{}^{n8} \tilde{C}_{\rho]mn} + C_{[\mu}{}^{m8} C_{\nu}{}^{78} \tilde{C}_{\rho]m7} + 3 C_{[\mu}{}^{78} C_{\nu\rho]7}.$$

Similar redefinitions were first considered in type IIB. [Ciceri, de Wit, OV '14; Samtleben, Hohm '15]



## KK ansatz and consistency of the truncation

The KK ansatz naturally relates the  $SL(6)$ -covariant IIA field content to the restricted tensor hierarchy for the  $ISO(7)$  gauging and quantities on  $S^6$ :

- Vectors:

$$C_{\mu}{}^{m8}(x, y) = \frac{1}{2} g K_{IJ}^m(y) \mathcal{A}_{\mu}{}^{IJ}(x) \quad , \quad C_{\mu}{}^{78}(x, y) = -\mu_I(y) \mathcal{A}_{\mu}{}^I(x) \quad ,$$

$$\tilde{C}_{\mu mn}(x, y) = \frac{1}{4} K_{mn}^{IJ}(y) \tilde{\mathcal{A}}_{\mu IJ}(x) \quad , \quad \tilde{C}_{\mu m7}(x, y) = -g^{-1} (\partial_m \mu^I)(y) \tilde{\mathcal{A}}_{\mu I}(x) \quad ,$$

- Two-forms:

$$C_{\mu\nu m}(x, y) = -g^{-1} (\mu_I \partial_m \mu^J)(y) \mathcal{B}_{\mu\nu J}{}^I(x) \quad , \quad C_{\mu\nu 7}(x, y) = \mu_I(y) \mathcal{B}_{\mu\nu}{}^I(x) \quad .$$

- Three-form:

$$C_{\mu\nu\rho}(x, y) = (\mu_I \mu_J)(y) \mathcal{C}_{\mu\nu\rho}{}^{IJ}(x) \quad ,$$

- Similarly with the metric, scalars and fermions.

When these ansatz are introduced into the  $SL(7)$ -covariant IIA susy transformations, the  $S^6$  dependence drops out and the susy transformations of the restricted  $D=4$  hierarchy arise,

## The full non-linear embedding

$$d\hat{s}_{10}^2 = \Delta^{-1} ds_4^2 + g_{mn} Dy^m Dy^n ,$$

$$\begin{aligned} \hat{A}_{(3)} = & \mu_I \mu_J (C^{IJ} + \mathcal{A}^I \wedge \mathcal{B}^J + \frac{1}{6} \mathcal{A}^{IK} \wedge \mathcal{A}^{JL} \wedge \tilde{\mathcal{A}}_{KL} + \frac{1}{6} \mathcal{A}^I \wedge \mathcal{A}^{JK} \wedge \tilde{\mathcal{A}}_K) \\ & + g^{-1} (\mathcal{B}_J^I + \frac{1}{2} \mathcal{A}^{IK} \wedge \tilde{\mathcal{A}}_{KJ} + \frac{1}{2} \mathcal{A}^I \wedge \tilde{\mathcal{A}}_J) \wedge \mu_I D\mu^J + \frac{1}{2} g^{-2} \tilde{\mathcal{A}}_{IJ} \wedge D\mu^I \wedge D\mu^J \\ & - \frac{1}{2} \mu_I B_{mn} \mathcal{A}^I \wedge Dy^m \wedge Dy^n + \frac{1}{6} A_{mnp} Dy^m \wedge Dy^n \wedge Dy^p , \end{aligned}$$

$$\hat{B}_{(2)} = -\mu_I (\mathcal{B}^I + \frac{1}{2} \mathcal{A}^{IJ} \wedge \tilde{\mathcal{A}}_J) - g^{-1} \tilde{\mathcal{A}}_I \wedge D\mu^I + \frac{1}{2} B_{mn} Dy^m \wedge Dy^n ,$$

$$\hat{A}_{(1)} = -\mu_I \mathcal{A}^I + A_m Dy^m .$$

Here, the covariant derivatives are

$$Dy^m \equiv dy^m + \frac{1}{2} g K_{IJ}^m \mathcal{A}^{IJ} \quad , \quad D\mu^I \equiv d\mu^I - g \mathcal{A}^{IJ} \mu_J .$$

## The full non-linear embedding

The internal (inverse) metric and forms are given in terms of  $SL(7)$ -covariant blocks of the  $D = 4$  scalar matrix  $\mathcal{M}_{MN}$  and  $S^6$  quantities as

$$\begin{aligned}
 g^{mn} &= \frac{1}{4} g^2 \Delta K_{IJ}^m K_{KL}^n \mathcal{M}^{IJKL} , \\
 A_m &= \frac{1}{2} g \Delta g_{mn} K_{IJ}^n \mu_K \mathcal{M}^{IJK8} , \\
 B_{mn} &= -\frac{1}{2} \Delta g_{mp} K_{IJ}^p \partial_n \mu^K \mathcal{M}^{IJ}_{K8} , \\
 A_{mnp} &= A_m B_{np} + \frac{1}{8} g \Delta g_{mq} K_{IJ}^q K_{np}^{KL} \mathcal{M}^{IJ}_{KL} .
 \end{aligned}$$

## Field strengths

The embedding can be given in terms of independent  $D = 4$  degrees of freedom.

- Compute the field strengths using their type IIA definitions:

$$\hat{F}_{(4)} = \mu_I \mu_J \mathcal{H}_{(4)}^{IJ} + g^{-1} \mathcal{H}_{(3)} J^I \wedge \mu_I D\mu^J + \frac{1}{2} g^{-2} \tilde{\mathcal{H}}_{(2)IJ} \wedge D\mu^I \wedge D\mu^J + \dots ,$$

$$\hat{H}_{(3)} = -\mu_I \mathcal{H}_{(3)}^I - g^{-1} \tilde{\mathcal{H}}_{(2)I} \wedge D\mu^I + \dots ,$$

$$\hat{F}_{(2)} = -\mu_I \mathcal{H}_{(2)}^I + g^{-1} (g \delta_{IJ} \mathcal{A}^J - m \tilde{\mathcal{A}}_I) \wedge D\mu^I + \dots ,$$

Here,  $\mathcal{H}_{(4)}^{IJ}$ , etc., turn out to be the field strengths of the restricted  $D = 4$  duality hierarchy.

- These field strengths are now regarded as short-hand for the corresponding  $D = 4$  dualised expressions.

## The Freund-Rubin term

- An elegant expression can be found for the Freund Rubin term using the duality condition for  $\mathcal{H}_{(4)}^{IJ}$ . It can be written as  $\hat{F}_{(4)} = U \text{vol}_4$ , where

$$U = -\frac{g}{84} X'_{\text{MP}}{}^{\text{R}} X_{\text{NQ}}{}^{\text{S}} \mathcal{M}^{\text{MN}} \left( \mathcal{M}^{\text{PQ}} \mathcal{M}_{\text{RS}} + 7 \delta_{\text{S}}^{\text{P}} \delta_{\text{R}}^{\text{Q}} \right)$$

closely parallels the scalar potential of  $D = 4 \mathcal{N} = 8$  gauged supergravity

$$V = \frac{g^2}{168} X_{\text{MP}}{}^{\text{R}} X_{\text{NQ}}{}^{\text{S}} \mathcal{M}^{\text{MN}} \left( \mathcal{M}^{\text{PQ}} \mathcal{M}_{\text{RS}} + 7 \delta_{\text{S}}^{\text{P}} \delta_{\text{R}}^{\text{Q}} \right).$$

- $X'_{\text{MN}}{}^{\text{P}} = \Theta'_{\text{M}}{}^{\alpha} (t_{\alpha})_{\text{N}}{}^{\text{P}}$  is defined in terms of an  $S^6$ -dependent  $\Theta'_{\text{M}}{}^{\alpha}$ , with

$$\Theta'_{[\text{AB}]}{}^{\text{C}}{}_{\text{D}} = 2 \delta_{[\text{A}}^{\text{C}} \theta'_{\text{B}]\text{D}} \quad , \quad \Theta'^{[\text{AB}]\text{C}}{}_{\text{D}} = 2 \delta_{\text{D}}^{[\text{A}} \xi'^{\text{B}]\text{C}} \quad ,$$

and

$$\theta'_{IJ} = \mu_I \mu_J \quad , \quad \theta'_{I8} = 0 \quad , \quad \theta'_{88} = 0 \quad ; \quad \xi'^{AB} = 0 \quad .$$

An  $\mathcal{N} = 2$  AdS<sub>4</sub> solution of massive IIA

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An  $\mathcal{N} = 2$  AdS<sub>4</sub> solution of massive IIA

$$d\hat{s}_{10}^2 = L^2 (3 + \cos 2\alpha)^{1/2} (5 + \cos 2\alpha)^{1/8} \left[ ds^2(\text{AdS}_4) + \frac{3}{2} d\alpha^2 + \frac{6 \sin^2 \alpha}{3 + \cos 2\alpha} ds^2(\mathbb{CP}^2) + \frac{9 \sin^2 \alpha}{5 + \cos 2\alpha} \eta^2 \right]$$

$$e^{\hat{\phi}} = e^{\phi_0} \frac{(5 + \cos 2\alpha)^{3/4}}{3 + \cos 2\alpha} ,$$

$$L^{-2} e^{-\frac{1}{2}\phi_0} \hat{H}_{(3)} = 24\sqrt{2} \frac{\sin^3 \alpha}{(3 + \cos 2\alpha)^2} \mathbf{J} \wedge d\alpha ,$$

$$L^{-3} e^{\frac{1}{4}\phi_0} \hat{F}_{(4)} = 6 \text{vol}(\text{AdS}_4) + 12\sqrt{3} \frac{7 + 3 \cos 2\alpha}{(3 + \cos 2\alpha)^2} \sin^4 \alpha \text{vol}(\mathbb{CP}^2) \\ + 18\sqrt{3} \frac{(9 + \cos 2\alpha) \sin^3 \alpha \cos \alpha}{(3 + \cos 2\alpha)(5 + \cos 2\alpha)} \mathbf{J} \wedge d\alpha \wedge \eta ,$$

$$L^{-1} e^{\frac{3}{4}\phi_0} \hat{F}_{(2)} = -4\sqrt{6} \frac{\sin^2 \alpha \cos \alpha}{(3 + \cos 2\alpha)(5 + \cos 2\alpha)} \mathbf{J} - 3\sqrt{6} \frac{(3 - \cos 2\alpha)}{(5 + \cos 2\alpha)^2} \sin \alpha d\alpha \wedge \eta ,$$

## Flux quantisation

- On our topologically  $S^6$  solution, one can only impose quantisation conditions on  $\hat{F}_{(0)}$  and  $\hat{F}_{(6)}$ :

$$k = 2\pi\ell_s \hat{F}_{(0)} \equiv 2\pi\ell_s m ,$$

$$N = -\frac{1}{(2\pi\ell_s)^5} \int_{S^6} e^{\frac{1}{2}\hat{\phi}} \hat{*}\hat{F}_{(4)} + \hat{B}_{(2)} \wedge d\hat{A}_{(3)} + \frac{1}{6}m \hat{B}_{(2)} \wedge \hat{B}_{(2)} \wedge \hat{B}_{(2)}$$

- The classical parameters  $g$  and  $m$  (or  $L$ ,  $e^{\phi_0}$ ) become fixed in terms of the quantum numbers  $k$ ,  $N$ .



## AdS/CFT match of the free energy

We have independently computed the free energy of the above  $\mathcal{N} = 2$  solution and in the candidate dual field theory.

- On the gravity side, the free energy is given by

$$F = \frac{16\pi^3}{(2\pi\ell_s)^8} \int_{S^6} e^{8A} \text{vol}_6 = \frac{\pi}{5} 2^{1/3} 3^{1/6} N^{5/3} k^{1/3},$$

where  $e^{2A}$  is the warp factor.

- On the field theory side, the free energy is computed by localisation to a solvable matrix model. The result matches the supergravity result.

This provides the first AdS<sub>4</sub>/CFT<sub>3</sub> test in the context of massive type IIA string theory.

# Final comments

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- 5 Final comments

# Our new results in a wider context

- The Romans mass has long been known to induce magnetic gaugings in  $D = 4$ .  
[Polchinski, Strominger '95; Louis, Micu '02] Our result is a certain  $\mathcal{N} = 8$  extension.
- Different arguments single out the usual  $D = 11$  and IIB truncations on  $S^7$ ,  $S^4$  and  $S^5$ .  
[Nastase, Vaman '00; Cvetič, Lu, Pope '00; Lee, Strickland-Constable, Waldram '14; Samtleben, Hohm '14]  
These arguments seem to miss massive IIA on  $S^6$ .
- The latter truncation is arguably different, in that dyonic ISO(7) supergravity doesn't have an  $\mathcal{N} = 8$  AdS vacuum that uplifts to a maximally supersymmetric  $\text{AdS}_4 \times S^6$  massive IIA background.

Yet, an  $\mathcal{N} = 8$  truncation does exist at the level of the supergravities.

# Our new results in a wider context

- Massless type IIA was conjectured in [Hull, Warner '88] to truncate consistently on  $S^6$  to the purely electric ( $m = 0$ ) ISO(7) gauging of [Hull, Warner '84]  
Our consistency proof is independent of the Romans mass. We thus prove that conjecture, too.
- We gave the *full* embedding of the  $D = 4$  theory in  $D = 10$  by exploiting the restricted tensor and duality hierarchies. In comparison, for  $D = 11$  on  $S^7$ , only the non-linear embedding of the scalars is known –the embedding of the vectors is only known at the linear level.

## Our new results in a wider context

- Massive IIA on  $S^6 = G_2/SU(3)$  admits a different,  $\mathcal{N} = 2$  truncation when  $S^6$  is equipped with its  $G_2$ -invariant nearly-Kähler structure. [Kashani-Poor '07] This is analogous to M-theory on Sasaki-Einstein. [Gauntlett-Varela '07]
  - The  $\mathcal{N} = 8$  truncation retains modes at the bottom of the KK towers about the AdS vacua.
  - The  $\mathcal{N} = 2$  truncation retains modes up the KK towers about the AdS vacua.
  - Both truncations have a common sector: the  $\mathcal{N} = 1$ ,  $G_2$ -invariant sector of the  $\mathcal{N} = 8$  theory.
- A certain  $\mathcal{N} = 1$  subsector of dyonic ISO(7) supergravity is described by a superpotential

$$W = -g (2 + 6T^2 U^2 + 6STU^2) + 2mU^3 ,$$

which contains non-geometric terms. This model, however, enjoys a perfectly geometric origin in massive IIA on  $S^6$ .

Thank you!