F-theory at order α'^3

with T. Pugh & R. Savelli [arXiv:1506.0675]

- 4D $\mathcal{N}=2$ Kähler potential (Type II theories on CY manifold X_3)

  ◇ $$K = -2\log\left(\mathcal{V}_3 - \frac{\zeta(3)}{32\pi^3 \cdot g_s^{3/2}}\,\chi(X_3)\,\right)$$

  ▷ $\mathcal{V}_3$ - classical volume of X_3

  ▷ $\chi(X_3)$ - Euler number

  ▷ origin - higher derivative ($\sim \alpha'^3$) corrections in 10D

  ▷ This form of K survives orientifolding: open strings (O7,D7), breaking susy ...

- (Genuine) $\mathcal{N}=1$ effect (small string coupling):

  ◇ $$\chi(X_3) \longrightarrow \chi(X_3)\ +\ 2\int_{X_3} D_{\rm O7}^3$$

  ▷ $D_{\rm O7}$ - class Poincaré dual to the divisor wrapped by the O7-plane in X_3

- F-theory:

  ▷ Geometrisation of all g_s effects (only α' corrections needed)

  ▷ Backreaction - all open string effects

Plan:

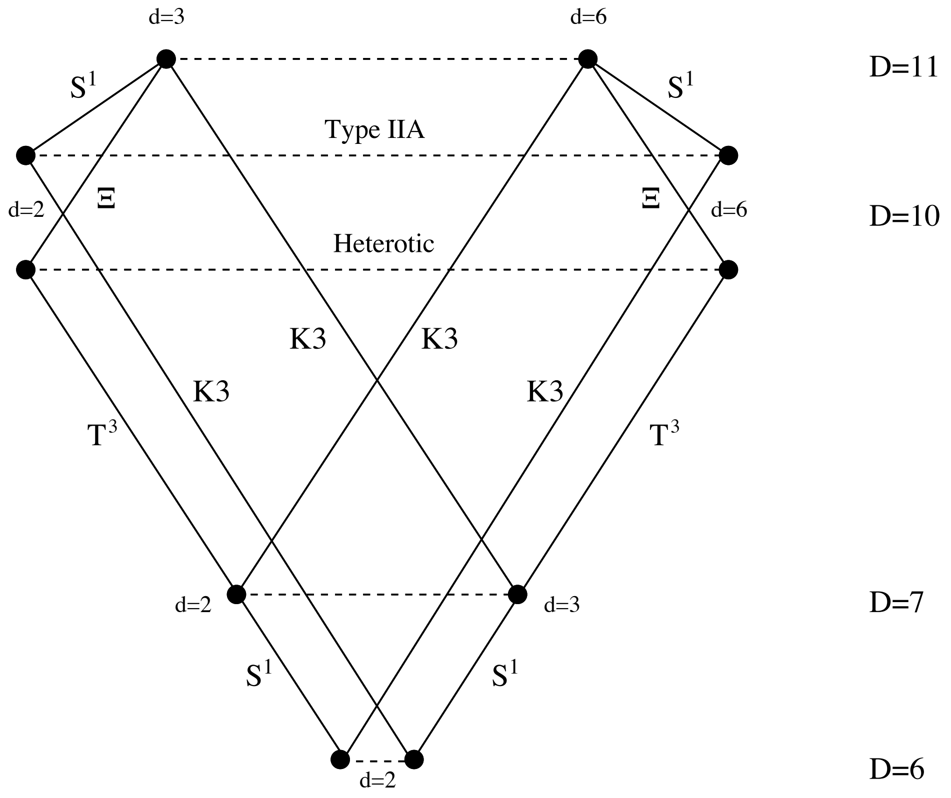
- Introduction
  - ▷ F-theory
    - ◇ dualities
  - ▷ α' corrections
    - ◇ type II theories
    - ◇ view from higher dimensions
- Higher-derivative couplings with varying dilaton-axion
- 4D $\mathcal{N} = 1$
- Some open questions

F-theory

- IIB strings with varying dialton-axion $\tau = C_0 + ie^{-\phi}$
  - ◇ $S_{IIB} \sim \frac{1}{l_s^8} \int (R - P\bar{P}) *_{10} 1$
    - ▷ $P = \frac{i}{2\mathrm{Im}\tau} \nabla\tau$ ($U(1)_R$ covariant, charge 2)
    - ▷ *formally* obtained from $R^{(12)}$ at fixed volume ν
    - ▷ $\mathbb{T}^2$ metric: $\frac{\nu}{\mathrm{Im}\tau} \begin{pmatrix} 1 & \mathrm{Re}\tau \\ \mathrm{Re}\tau & |\tau|^2 \end{pmatrix}$
    - ▷ F-theory space - *elliptically fibered CY* ◁
    - ▷ D7/O7 - codim 2 defects (with *deficit angle*) - degenerations of el. fiber
    - ▷ 10D slice integral: $S_0^{12} \sim \frac{1}{l_s^8} \int R^{(12)} *_{10} 1$
- Decompactification limit of M-theory (on X_e)
  - ◇ $S^{11} \sim \frac{1}{l_M^9} \int R *_{11} 1 \quad \Rightarrow \quad S^9 \sim \frac{\nu}{l_M^7} \int (R - P\bar{P}) *_9 1 \quad \Rightarrow \quad S_{IIB}$
    - ▷ IIB limit $\nu \to 0$:
- $\sim \alpha'^3$ couplings:
  - ◇ Dualities (notably F-th/$K3_e \cong$ Het/$\mathbb{T}^2$)
  - ◇ Necessity of four-form flux for CY_4 (4d $\mathcal{N} = 1$)

Dualities and higher derivative couplings

Six-dimensional Heterotic/IIA duality (M-theory):



- $H^{\rm het} = e^{2\phi} * H^{\rm IIA}, \qquad g^{\rm het}_{\mu\nu} = e^{2\phi} g^{\rm IIA}_{\mu\nu}, \qquad \phi = -\varphi^{\rm IIA}$
- Het. BI : $dH_3 = \frac{\alpha'}{4}(\text{tr}\, R^2 - \text{tr}\, F^2) \quad \Leftrightarrow \quad$ Type II EOM ($\Leftarrow \; B \wedge (F^I \wedge F^J d_{IJ} - \text{tr}\, R^2)$)
  - ◇ $B \wedge F \wedge F$ descends from 11d $C_3 \wedge G_4 \wedge G_4 \qquad (d_{iJ} = \int_{K3} \omega_I \wedge \omega_J)$
  - ◇ $B \wedge \text{tr}\, R^2$ descends from 11d $C_3 \wedge X_8(\Omega^{LC})$

Summary of type II $(\alpha')^3$ couplings (10D):

	No B	With B
e-o **+** **o-e**	$\frac{1}{8}(t_8\epsilon_{10}+\epsilon_{10}t_8)BR^4$ $= B\wedge X_8(\Omega^{\rm LC})$ $= \frac{1}{192(2\pi)^4}B\wedge\left(\text{tr}\,R^4-\frac{1}{4}(\text{tr}\,R^2)^2\right)$	$\frac{1}{8}(t_8\epsilon_{10}+\epsilon_{10}t_8)BR^4(\Omega_+)$ $=\frac{1}{8}t_8\epsilon_{10}B(R^4(\Omega_+)+R^4(\Omega_-))$ $=\frac{1}{2}B\wedge[X_8(\Omega_+)+X_8(\Omega_-)]$
e-e	$t_8t_8R^4$	$t_8t_8R^4(\Omega_+)=t_8t_8R^4(\Omega_-)$
o-o	$\frac{1}{8}\epsilon_{10}\epsilon_{10}R^4$	$\frac{1}{8}\epsilon_{10}\epsilon_{10}\left(R(\Omega_+)^4+\frac{8}{3}H^2R(\Omega_+)^3+\cdots\right)$ $=\frac{1}{8}\epsilon_{10}\epsilon_{10}\left(R(\Omega_-)^4+\frac{8}{3}H^2R(\Omega_-)^3+\cdots\right)$

- ◇ $\Omega^{\rm LC} \quad \longrightarrow \Omega_\pm=\Omega^{\rm LC}\pm\frac{1}{2}\mathcal{H}$
- ◇ Curvature: $R(\Omega_\pm)_{\mu\nu}{}^{\alpha\beta}=R_{\mu\nu}{}^{\alpha\beta}\pm\nabla_{[\mu}H_{\nu]}{}^{\alpha\beta}+\frac{1}{2}H_{[\mu}{}^{\alpha\gamma}H_{\nu]\gamma}{}^{\beta}$
- ◇ $t_8M^4=24\left(\text{tr}\,M^4-\frac{1}{4}(\text{tr}\,M^2)^2\right)$
- ◇ $\epsilon_{10}\epsilon_{10}R^4=\epsilon_{\alpha\beta\mu_1\cdots\mu_8}\epsilon^{\alpha\beta\nu_1\cdots\nu_8}R^{\mu_1\mu_2}{}_{\nu_1\nu_2}R^{\mu_3\mu_4}{}_{\nu_3\nu_4}R^{\mu_5\mu_6}{}_{\nu_5\nu_6}R^{\mu_7\mu_8}{}_{\nu_7\nu_8}$
- ◇ lifting to 11d: $\quad H \;\mapsto\; G_4$

$\mathcal{N}=2$: M-theory/IIA on Calabi-Yau threefolds

- 4D quantum corrected effective action:

$$S=\frac{1}{2\kappa_4^2}\quad\int d^4x\sqrt{g^\sigma}\left[\left((1+\tfrac{\chi_T}{v_6})\,e^{-2\phi_4}-\chi_1\right)\mathcal{R}_{(4)}+\left((1-\tfrac{\chi_T}{v_6})\,e^{-2\phi_4}-\chi_1\right)G_{vv}(\partial v)^2 + \left((1+\tfrac{\chi_T}{v_6})\,e^{-2\phi_4}+\chi_1\right)G_{hh}(\partial h)^2\right]$$

  - ▷ $v_6=\mathcal{V}_3(2\pi l_s)^{-6}$
  - ▷ G_{vv} - the metric of the $h_{(1,1)}-1$ vector-multiplets
  - ▷ G_{hh} - the metric of the $h_{(1,2)}$ non-universal hypermultiplets
  - ▷ $\chi_T=2\zeta(3)\chi/(2\pi)^3$
  - ▷ $\chi_1=4\zeta(2)\chi/(2\pi)^3$

- Weyl rescaling:
  - ▷ Quantum corrections to vector and hyper moduli space metrics
  - ▷ Corrections to the Kähler potential

Lifting the duality to 8D: F-th/$K3_e \cong$ Het/$\mathbb{T}^2$

- Effective theory 8D minimal sugra (16 supercharges) coupled to VM, rk G=18
  - ▷ 2-form field; 3-form flux with non-trivial BI

Het/$\mathbb{T}^2$ $\Rightarrow$
- Het. Bianchi identity
$$\mathrm{d}H_3 = \frac{\alpha'}{4}(\mathrm{tr}\, R^2 - \mathrm{tr}\, F^2)$$
- Green-Schwarz term ($SO(8)^4$ points) with
$$\mathrm{tr}\, R^4 + \frac{1}{4}(\mathrm{tr}\, R^2)^2 - \mathrm{tr}\, F^2 \mathrm{tr}\, R^2 + 8\mathrm{tr}\, F^4$$

  - ▷ 4-form with non-trivial CS couplings (Sen limit: restricted c.s of $K3$, such that $g_s \to 0$ is well-defined)

$$C \wedge \Big[\underbrace{\mu e^{-iF}\sqrt{\hat{A}(R)} + \mu'\sqrt{L(R/4)}}_{\sum_{i=1}^{4}(\mathrm{O7}+4\mathrm{D7})_i}\Big] \Rightarrow \begin{cases} \sim C_4 \wedge \left(\frac{1}{4}\mathrm{tr}\, R^2 - \mathrm{tr}\, F_i^2\right) \\ \sim C_0 \wedge \left(\frac{1}{4}\mathrm{tr}\, R^4 + \frac{1}{16}(\mathrm{tr}\, R^2)^2 - \mathrm{tr}\, R^2 \mathrm{tr}\, F_i^2 + 8\mathrm{tr}\, F_i^4\right) \end{cases}$$

  - ▷ Can recover CP-even part by susy
  - ▷ Generic points of moduli space?!

$\sim \alpha'^3$ in IIB

- kinematics:
  - ▷ $(t_8 t_8 + \frac{1}{8}\epsilon_{10}\epsilon_{10})R^4$
  - ▷ no CP-odd (GS-like) couplings
  - ▷ tree-level, 1 loop and non-perturbative contributions
- M-theory (11D) perspective
  - ▷ For M-th/$\mathbb{T}^2$, $\quad \nu \to 0 \quad \Rightarrow \quad S_3^9 \to 0$
  - ▷ need to account for KK modes on $\mathbb{T}^2$
- For constant τ: $\Delta S^{11} \sim \frac{1}{l_M^3} \int t_8 t_8 R^4 *_{11} 1 \quad \Rightarrow \quad \Delta S_{IIB} \sim \frac{1}{l_s^2} \int f_0(\tau,\bar\tau)\, t_8 t_8 R^4 *_{10} 1$
  - ▷ $$f_0(\tau,\bar\tau) = \sum_{(m,n)\neq(0,0)} \frac{\tau_2^{3/2}}{|m+n\tau|^3} \to \frac{2\zeta(3)}{g_3^{3/2}} + \frac{2\pi}{3} g_s^{1/2} + \mathcal{O}(e^{-1/g_s})$$
- Varying dilaton-axion:
  - ▷ $$\Delta S_{IIB}|_{4-\mathrm{pt}} \sim \frac{1}{l_s^2} \int \left\{ t_8 t_8 [R^4 + 24 R^2 |DP|^2] + \hat{\mathcal{O}}_1 \left[(|DP|^2)^2 \right] \right\} *_{10} 1$$
  - ▷ Complete agreement with 0-mode reduction of

$$S_3^{12}(t_8) \quad \sim \quad \frac{1}{l_s^2} \int \hat{t}_8 \hat{t}_8 {R^{12}}^4 *_{10} 1$$

$\sim \alpha'^3$ in F-theory...

- Odd-odd sector:
  - ▷ 0-mode reduction of

$$S_3^{12}(\epsilon_8) \quad \sim \quad \frac{1}{l_s^2}\int \hat{\epsilon}_8\hat{\epsilon}_8 R^{12^4} *_{10} 1$$

  - ▷ $S_3^{12}(\epsilon_8)$ restricted to 4pt *vanishes*
- Missing τ dynamics - all g_s corrections
- (Conjectured) complete coupling:

$$S_3^{12} \quad = \quad \frac{1}{(4\pi)^9 \cdot 3 \cdot l_s^4}\int f_0(\tau,\bar{\tau})\left[\hat{t}_8\hat{t}_8 + \frac{1}{96}\hat{\epsilon}_{12}\hat{\epsilon}_{12}\right](R^{(12)})^4 *_{10} 1$$

  - ▷ $SL(2,\mathbb{Z})$ and SUSY compatibility
  - ▷ perturbatively *tree* + 1-loop terms
  - ▷ No "cusp forms" - non-perturbative part captured by $f_0(\tau,\bar{\tau})$
  - ∗ g_s-exact $\sim \mathcal{O}(\alpha'^3)$ Type IIB action without flux
  - ∗ R and τ couplings *beyond* 4pt

4D $\mathcal{N}=1$ compactifications of F-theory

- Smooth four-fold $B_3 \to CY_4$, fibered over B_3 with zero-section
- 2 + 8 derivative reductions ($+\mathcal{O}(\alpha'^4)$):

$$S^4_{0+3} = \frac{1}{2\pi\alpha'}\int \left(\mathcal{V}_b - \frac{1}{64\pi^3}\int_{B_3} f_0(\tau,\bar\tau)\, c_3(X_4)|_{B_3}\right) R_{(4\mathrm{d})} \ast_4 1$$

  - ◇ Correction

$$\sim \int R_{(4\mathrm{d})} \ast_4 1 \int_{B_3} f_0 \ast_8 (J\wedge c_3(X_4)) \ast_6 1 + \dots ,$$

  - ◇ Use $\ast_8(J\wedge \ast_6 1) = 1 + \mathcal{O}(\alpha')$

▷ Verify that the correction is *finite*

▷ *constant* τ ($B_3 = CY_3$) known $\mathcal{N}=2$ results

▷ τ varies over B_3 - the correction is *non-topological*

▷ $\Rightarrow$ Kähler potential via Weyl rescaling (plenty of ifs and buts!)

Weak string coupling limit

Sen limit: a region of the complex structure moduli space of the CY fourfold X_4, where none of the monodromies acting on τ involves the string coupling τ_2^{-1}:

- ▷ τ_2^{-1} kept small in a globally well-defined way
- ▷ Type IIB on orientifolded CY threefold X_3 - branched double cover of B_3
- ▷ O7-plane - branching locus: in cohomolgy $D_{\rm O7} \equiv c_1(B_3)$
- ∗ Correction (*topological*) to the classical volume $\mathcal{V}_3$ of the CY threefold:

$$\tilde{\mathcal{V}}_3 = \mathcal{V}_3 \; - \; \frac{\zeta(3)}{32\pi^3\, g_s^{3/2}} \left(\chi(X_3) + 2 \int_{X_3} D_{\rm O7}^3 \right) \quad + \quad \mathcal{O}(g_s^{-1/2})$$

  - ◇ Note integration over X_3 (not B_3)
  - ◇ Only Weyl rescaling contribution to Kähler potential is computed here

- ▷ New terms from tree-level closed string scattering in this CY orientifold background
- ▷ *Absent* in toroidal models!

Fluxes?

- IIB limit for fluxes: $-\frac{1}{48 \cdot l_s^{10}} \int G_4 \cdot G_4 *_{11} 1 \quad \Rightarrow \quad -\frac{1}{12 \cdot l_s^8} \int G_3 \cdot \overline{G}_3 *_{10} 1$
  - ▷ only nonvanishing components $H_{mnp} = G_{mnpA}$ and $F_{mnp} = G_{mnpB}$
  - ▷ $G_3 = e^{\phi}(F_3 - \tau H_3)$
- Problems with mixed terms: $\Delta^G S_{IIB}|_{4-\mathrm{pt}} \sim$
  $\frac{1}{l_s^2} \int \left\{ t_8 t_8 \left[12 |DG|^2 (R^2 + 4|DP|^2 + 2R(DP + \overline{DP}) \right] + \hat{\mathcal{O}}_2 \left[(|DG|^2)^2 \right] \right\} *_{10} 1$
  - ▷ Invariant part: $\Leftarrow \sim \frac{1}{l_s^4} \int \left[s_6 (\nabla G_4)^2 (R^{(12)})^2 + s_{20} (\nabla G_4)^4 \right] *_{12} 1$
    - ▷ Match with 4-pt tree-level result - 26+1 free parameters
  - ▷ "Non-invariant" pieces $\Leftarrow \; f_k(\tau, \bar{\tau})\, S^{(2k)}$ with f_k of charge $-2k$ $(\bar{f_k} = f_{-k})$
    - ▷ Note (small τ_2^{-1} expansion of f_k)

$$\left(k + 2i\tau_2 \frac{\partial}{\partial \tau} \right) f_k = \left(\frac{3}{2} + k \right) f_{k+1}$$

    - ▷ $\frac{1}{l_s^2} \int t_8 t_8 \left\{ RDPDG(f_1 \overline{DG} + 2 f_2 DG) + \beta (DP)^2 [f_1 (\overline{DG})^2 - f_3 (DG)^2] \right\} *_{10} 1$
      do not come from direct reduction
- The origin of "non-invariant" couplings?

Some (more) open questions:

- F-theory at order α'^3:
  - $\triangleright$ F_5 couplings?
  - $\triangleright$ Going beyond 4pt checks?
- $\mathcal{N} = 1$ compactifications of F-theory
  - $\triangleright$ Warping? Moduli kinetic terms? Other corrections to Kähler potential?
  - $\triangleright$ Moduli kinetic terms? Other corrections to Kähler potential?
- Gen. geometry and $SL(2, \mathbb{Z})$ invariance ?
  - $\triangleright$ F-theory space (CY_e (with fluxes)) - a generalised tangent bundle? $\triangleleft$
- Towards completions of higher-derivative couplings:
  - $\triangleright$ Susy completion of GS terms for heterotic strings
  - $\triangleright$ Susy completions of R^4 couplings in type II theories and M-theory
  - $\star$ Can generalised geometry capture the systematics of the string (perturbation) theory?