Double Field Theory and strings at the self-dual radius

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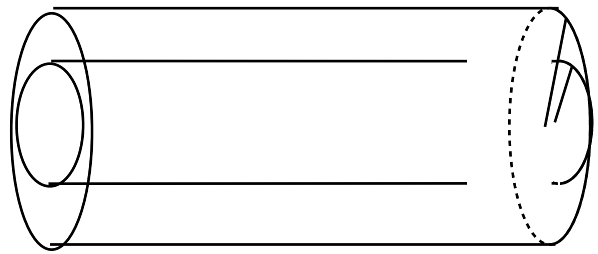
Motivation

momentum # winding #

$$M^2 = \frac{2}{\alpha'}(N + \bar{N} - 2) + \frac{p^2}{R^2} + \frac{\tilde{p}^2}{\tilde{R}^2}$$

Bosonic closed string

ℓ_s



$R \gg \ell_s = \sqrt{\alpha'}$ / $R \approx \ell_s$

$$\tilde{R} = \frac{\alpha'}{R}$$

Massless states:

$N = 0 \quad \bar{N} = 1 \quad p = \tilde{p} = \pm 1$

g_{mn}, B, dilaton → $g_{\mu\nu}$

2 vectors

$g_{\mu y}$ vector

2 scalars

g_{yy} scalar

\+ $N = 1 \quad \bar{N} = 0 \quad p = -\tilde{p} = \pm 1$

B_{mn} → $B_{\mu\nu}$

2 vectors

$B_{\mu y}$ vector

2 scalars

$N = \bar{N} = 0 \quad p = \pm 2$ 2 scalars

$\tilde{p} = \pm 2$ 2 scalars

1 scalar $SU(2) \times SU(2)$ / $U(1) \times U(1)$ 9 scalars

Can we describe the effective action using DFT ? / Can we find the physics using DFT ?

$$\bar{N} - N = p\tilde{p}$$

Some easy math...

$$\mathcal{M} = \mathcal{M}_d \times S^1$$

		dof
$g_{\mu\nu}$ $B_{\mu\nu}$	$\rightarrow$	d^2
6 vectors	$\rightarrow$	$6d$
9 scalars	$\rightarrow$	3^2
		$(d+3)^2$

$$\dim\left[\frac{O(d+3,d+3)}{O(d+3)\times O(d+3)}\right] = (d+3)^2$$

Outline

- Strings on S^1
- Effective action from string theory
- DFT description
- Effective action from DFT
- “Internal double space”

String theory on R

Momentum state for non-compact coordinate $x = x^L + x^R$

$$e^{ik(x^L(z)+x^R(\bar{z}))} \qquad k \in \mathbb{R} \qquad X(z,\bar{z}) = x^L(z) + x^R(\bar{z})$$

String theory on S^1

Momentum & winding state for compact coordinate $y = y^L + y^R \simeq y + 2\pi R$

$$e^{i(k_L y^L(z)+k_R y^R(\bar{z}))} \qquad k_{L,R} = \frac{p}{R} \pm \frac{\tilde{p}}{\tilde{R}}$$

$$Y(z,\bar{z}) = y^L(z) + y^R(\bar{z})$$

$$\tilde{Y}(z,\bar{z}) = y^L(z) - y^R(\bar{z})$$

DFT $\quad \tilde{y} = y^L - y^R \simeq y + 2\pi\tilde{R}$

$$\mathcal{M}_d \times S^1 \longrightarrow \mathcal{M}_d \times S^1 \times \tilde{S}^1$$

Massless states at $R = \tilde{R} = 1$

$$M^2 = 2(N + \bar{N} - 2) + \frac{p^2}{R^2} + \frac{\tilde{p}^2}{\tilde{R}^2}$$

Level-matching $\bar{N} - N = p\tilde{p}$

$\alpha' = 1$

- SU(2)L Vectors $\bar{N}_x = 1$
  - $N_y = 1 \quad (g_{\mu y} + B_{\mu y})$: A^3_μ $\qquad V \sim J^3(z) \cdot (\bar{\partial} X^\mu e^{ikX})$
  - $N_y = 0 \quad p = \tilde{p} = \pm 1 \quad (k_L = \pm 2)$: $A^\pm_\mu$ $\qquad V \sim J^\pm(z) \cdot (\bar{\partial} X^\mu e^{ikX})$

- SU(2)R Vectors $N_x = 1 \quad A^i \to \bar{A}^i$ $\qquad J^i(z) \to \bar{J}^i(\bar{z}) \qquad y^L \to y^R$

- Scalars (3,3) $N_x = \bar{N}_x = 0$

$N_y = 1, \bar{N}_y = 1 \ (g_{yy})$: M^{33}

$N_y = 1, p = -\tilde{p} = \pm 1 \ (\bar{k} = \pm 2)$: $M^{3\pm}$

$\bar{N}_y = 1, p = \tilde{p} = \pm 1 \ (k = \pm 2)$: $M^{\pm 3}$

$p = \pm 2, \tilde{p} = 0 \ (k = \bar{k} = \pm 2)$: $M^{\pm\pm}$

$p = 0, \tilde{p} = \pm 2 \ (k = -\bar{k} = \pm 2)$: $M^{\pm\mp}$

$$V^{ij} \sim J^i J^j e^{ikX}$$

$$J^3(z) = \partial y^L(z)$$
$$J^\pm(z) = \mathrm{e}^{\pm 2 i y^L(z)}$$

$$J^i(z) = \sum J^i_m \, z^{-(m+1)}$$

$$[J^i_m, J^j_n] = \frac{m}{2}\delta^{ij}\delta_{m,-n} + \epsilon^{ijk} J^k_{m+n}$$

SU(2)L current algebra

Effective action from string theory

Computing 3-point functions <VVV> we read off

Gerardo Aldazabal's talk

$$\mathcal{L} = R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} + \frac{1}{4} \bar{F}^i_{\mu\nu} \bar{F}^{i\mu\nu}$$

$$+ \frac{1}{4} M^{ij} F^i_{\mu\nu} \bar{F}^{j\mu\nu} + D_\mu M^{ij} D^\mu M^{ij} - \det M$$

$M^{\pm\pm}, M^{\pm\mp}$ acquire mass$^2 = \epsilon$

$A^\pm$, $\bar{A}^\pm$ acquire mass$^2 = \epsilon^2$

$$H = dB + A^i \wedge F^i - \bar{A}^i \wedge \bar{F}^i$$

$$F^i = dA^i + \epsilon^{ijk} A^j \wedge A^k$$

$$D_\mu M^{ii} = \partial_\mu M^{ii} + f^{ijk} A^j_\mu M^{ki} + f^{ijk} \bar{A}^j_\mu M^{ik}$$

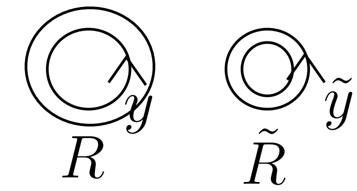
SU(2) x SU(2) $\rightarrow$ U(1) x U(1)

Higgs mechanism

$$M^{ij} \rightarrow \epsilon \, \delta^{ij}_{33} + M'^{ij}$$

Double field theory

$$\mathcal{M}_d \times S^1 \longrightarrow \mathcal{M}_d \times S^1 \times \tilde{S}^1$$



In DFT on S^1

natural pairing

$TS^1 \oplus T^*S^1$ $T\tilde{S}^1$

$$< \partial_y + dy, \partial_y + dy >= 2\iota_{\partial_y} dy = 2$$

$$\partial_y + dy \cong \partial_{\tilde{y}}$$

$$\eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$< V, V >= \eta_{MN} V^M V^N$$

$$< \partial_y, \partial_{\tilde{y}} >= 1$$

$$\eta^{LR} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Vertex operators: depend on y^L and y^R $\Rightarrow$ to reproduce string theory action we need dependence on y and $\tilde{y}$

$$y^L = y + \tilde{y} \qquad y^R = y - \tilde{y}$$

Violating weak / strong constraint ?

Yes, as expected:

Level matching condition $\bar{N} - N = p\tilde{p}$ ($p \leftarrow \partial_y$, $\tilde{p} \leftarrow \partial_{\tilde{y}}$)

$\bar{N} - N$: $\neq 0$ here; in usual massless states $= 0$. $p\tilde{p}$: $\neq 0$

$$\Rightarrow \quad \partial_y \partial_{\tilde{y}}(\quad) \neq 0$$

$$\eta^{MN} \partial_M \partial_N (\quad) \neq 0$$

~~weak constraint~~

GG/DFT

Frame on $T\mathcal{M} \oplus T^*\mathcal{M}$

frame e_a dual frame e^a

$$E_A = \begin{pmatrix} e_a - \iota_{e_a} B \\ e^a \end{pmatrix}$$

Generalized metric $\mathcal{H} = \delta^{AB} E_A \otimes E_B = \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g - Bg^{-1}B \end{pmatrix}$

Contains g, B

dof: $\frac{O(D,D)}{O(D)\times O(D)}$

Circle reduction

Frame on $T\mathcal{M} \oplus T^*\mathcal{M}$

frame e_a (on $T\mathcal{M}$), dual frame e^a (on $T^*\mathcal{M}$)

$$E_A = \begin{pmatrix} e_a - \iota_{e_a} B \\ e^a \end{pmatrix}$$

$e_a - \iota_{e_a} B \to$ #, $\phi^{-1}(\partial_y + B_1)$ (B_1 ← $B_{\mu y}$)

$e^a \to e^{\hat{a}}$, $\phi\,(dy + V_1)$ (V_1 ← $g_{\mu y}$; ϕ ← $\sqrt{g_{yy}} = R$)

$$T\mathcal{M} = T\mathcal{M}_d \oplus TS^1$$

$$y \sim y + 2\pi$$

Circle reduction

Frame on $T\mathcal{M} \oplus T^*\mathcal{M}$

frame e_a — dual frame e^a

$$E_A = \begin{pmatrix} e_a - \iota_{e_a} B \\ e^a \end{pmatrix}$$

$\#$; $\phi^{-1}(\partial_y + B_1)$; $B_{\mu y}$

$\phi\,(dy + V_1)$; $g_{\mu y}$; $e^{\hat{a}}$

$$\sqrt{g_{yy}} = R = <\exp(\tfrac{1}{2}M^{33})>$$

$$\approx 1 + \tfrac{1}{2} < \underbrace{M^{33}}_{\epsilon} >$$

$$T\mathcal{M} = T\mathcal{M}_d \oplus TS^1$$

$$y \sim y + 2\pi$$

$$\begin{pmatrix} E_d \\ E^d \end{pmatrix} = \begin{pmatrix} \phi^{-1} & 0 \\ 0 & \phi \end{pmatrix} \begin{pmatrix} \partial_y + B_1 \\ dy + V_1 \end{pmatrix} \overset{LR}{\longrightarrow} \begin{pmatrix} E^L \\ E^R \end{pmatrix} = \begin{pmatrix} U^+ & U^- \\ U^- & U^+ \end{pmatrix} \approx \begin{pmatrix} 1 & \frac{1}{2}M^{33} \\ \frac{1}{2}M^{33} & 1 \end{pmatrix} \begin{pmatrix} J + A \\ \bar{J} - \bar{A} \end{pmatrix}$$

$$E_A(x,y) = U_A{}^{A'}(x) \quad E'_{A'}(y)$$

Scherk-Schwarz reduction

$$U^+ \approx 1 \qquad U^- \approx \tfrac{1}{2}M^{33}$$

$$U^\pm = \frac{1}{2}(\phi^{-1} \pm \phi)$$

$$A = V_1 + B_1 \qquad J = \partial_y + dy$$

$$\bar{A} = V_1 - B_1 \qquad \bar{J} = \partial_y - dy$$

Effective action valid at energies $E \sim \frac{1}{\sqrt{\alpha'}}\epsilon << \frac{1}{\sqrt{\alpha'}}$

So far, no enhancement of symmetry, no double field theory

DFT & Enhancement of symmetry

$$T\mathcal{M} \oplus T^*\mathcal{M} \longrightarrow T\mathcal{M}_d \oplus TS^1 \oplus T\tilde{S}^1 \oplus T^*\mathcal{M}_d$$

$$dy \approx \partial_{\tilde{y}}$$

$$J = \partial_y + dy = \partial_y + \partial_{\tilde{y}} = \partial_{y^L}$$

$$\bar{J} = \partial_y - dy = \partial_y - \partial_{\tilde{y}} = \partial_{y^R}$$

Still, this is formal. No dependence on y or $\tilde{y}$

Of course, we have not included momentum/winding modes $\sim e^{2iy}/e^{2i\tilde{y}}$

To include winding modes we need DFT: $S^1, \tilde{S}^1$

To account for the enhancement of symmetry, we need to enlarge the generalized tangent space

$$T\mathcal{M}_d \oplus TS^1 \oplus T\tilde{S}^1 \oplus T^*\mathcal{M}_d$$

$$\downarrow$$

$$T\mathcal{M}_d \oplus \underbrace{V_2 \oplus TS^1 \oplus T\tilde{S}^1 \oplus V_2^*}_{\text{O(3,3)}} \oplus T^*\mathcal{M}_d$$

Enhancement of symmetry

$$T\mathcal{M}_d \oplus TS^1 \oplus T\tilde{S}^1 \oplus T^*\mathcal{M}_d$$

$$\downarrow$$

$$T\mathcal{M}_d \oplus V_2 \oplus TS^1 \oplus T\tilde{S}^1 \oplus V_2^* \oplus T^*\mathcal{M}_d$$

$$\begin{pmatrix} E^3 \\ E^{\bar{3}} \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}M^{3\bar{3}} \\ \frac{1}{2}M^{\bar{3}3} & 1 \end{pmatrix} \begin{pmatrix} J^3 + A^3 \\ \bar{J}^{\bar{3}} - \bar{A}^{\bar{3}} \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} E^i \\ E^{\bar{\imath}} \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}M^{i\bar{\jmath}} \\ \frac{1}{2}M^{\bar{\imath}j} & 1 \end{pmatrix} \begin{pmatrix} J^j + A^j \\ \bar{J}^{\bar{\jmath}} - \bar{A}^{\bar{\jmath}} \end{pmatrix}$$

$M^{i\bar{\jmath}}(x)$

9 scalar fields

$A^i(x)$

$\bar{A}^{\bar{\imath}}(x)$

6 vector fields

Should satisfy SU(2)$_L$ algebra $J^i(y,\tilde{y})$

Should satisfy SU(2)$_R$ algebra $\bar{J}^{\bar{\imath}}(y,\tilde{y})$

under some bracket

Effective action

$$\begin{pmatrix} E_a \\ E^{L} \\ E^{R} \\ E^a \end{pmatrix} = \begin{pmatrix} e_a & \iota_{e_a}A & \iota_{e_a}\bar{A} & \iota_{e_a}B \\ 0 & 1 & \frac{1}{2}M & M\bar{A} \\ 0 & \frac{1}{2}M^t & 1 & M^t A \\ 0 & 0 & 0 & e^a \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & J & 0 & 0 \\ 0 & 0 & \bar{J} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$E_A(x,y) = \qquad U_A{}^{A'}(x) \qquad E'_{A'}(y)$$

Generalized Scherk-Schwarz reduction of DFT action

Aldazabal, Baron, Marques, Nuñez 11
Geissbuhler 11

$$\square^2 \approx \begin{pmatrix} 1 & M \\ M^t & 1 \end{pmatrix}$$

$$I = i, \bar{i}$$

$$\mathcal{L} = R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} \mathcal{H}_{IJ} F^{I}_{\mu\nu} F^{J\mu\nu} + \frac{1}{4} (D_\mu \mathcal{H})_{IJ} (D^\mu \mathcal{H})^{IJ}$$
$$= \frac{1}{12} f_{IJK} f_{LMN} \left(\mathcal{H}^{IL}\mathcal{H}^{JM}\mathcal{H}^{KN} - 3\mathcal{H}^{IL}\eta^{JM}\eta^{KN} + 2\eta^{IL}\eta^{JM}\eta^{KN}\right)$$

$$\mathcal{L} = R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + F^{i}_{\mu\nu} F^{i\mu\nu} + \bar{F}^{\bar{i}}_{\mu\nu} \bar{F}^{\bar{i}\mu\nu} - F^{i}_{\mu\nu} M^{ij} \bar{F}^{\bar{j}\mu\nu} + D_\mu M^{ij} D^\mu M^{ij} - \det M$$

Exactly string theory action!

$$[E'_J, E'_K]_C = f^I{}_{JK} E'_K$$

$J, \bar{J}$ $\epsilon^{ijk}, \epsilon^{\overline{ijk}}$

$$H = dB + F^I \wedge A_I$$
$$F^I = dA^I + f^I{}_{JK} A^J \wedge A^K$$

Algebra

$$V_2 + TS^1 + T\tilde{S}^1 + V_2^*$$

v_1^L, v_2^L ... v_1^R, v_2^R

C-bracket

$$[V_1, V_2]_C = \frac{1}{2}(\mathcal{L}_{V_1} V_2 - \mathcal{L}_{V_2} V_1)$$

$$(\mathcal{L}_{V_1} V_2)^I = V_1^J \partial_J V_2^I + (\partial^I V_{1J} - \partial_J V_1^I) V_2^J$$

generalized Lie derivative

The following J and $\bar{J}$ do the job

$$J = \begin{pmatrix} \cos 2y^L & \sin 2y^L & 0 \\ -\sin 2y^L & \cos 2y^L & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_1^L \\ v_2^L \\ dy^L \end{pmatrix}$$

$$\bar{J} = \begin{pmatrix} \cos 2y^R & \sin 2y^R & 0 \\ -\sin 2y^R & \cos 2y^R & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_1^R \\ v_2^R \\ dy^R \end{pmatrix}$$

$$[E'_J, E'_K]_C = f^I{}_{JK} E'_K$$

$J, \bar{J}$ ... $\epsilon^{ijk}, \epsilon^{\overline{ijk}}$

Geometry of the "internal space"

$TW_2 \oplus TS^1 \oplus TS^1 \oplus W_2$

$TM^2 \oplus TS^1 \oplus TS^1 \oplus TM^2$

$T_2 \quad T_2$

$$E'_L = J = \begin{pmatrix} \cos 2y^L & \sin 2y^L & 0 \\ -\sin 2y^L & \cos 2y^L & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v^{1L} \\ v^{2L} \\ dy^L \end{pmatrix} \qquad E'_R = \bar{J} = \begin{pmatrix} \cos 2y^R & \sin 2y^R & 0 \\ -\sin 2y^R & \cos 2y^R & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v^{1R} \\ v^{2R} \\ dy^R \end{pmatrix}$$

The space is $S^1 \times \tilde{S}^1$

$$E_A(x,y) = U_A{}^{A'}(x)\, E'_{A'}(y)$$

But if we want to "geometrize" the O(3,3)

$T^2 \rightarrow M_3 \quad (T^3) \rightarrow S^1$

$M_3 \quad (T^3) \leftarrow T^2 \rightarrow S^1$

HOWEVER $\mathcal{H}(y,\tilde{y}) = E^t\, E$

BUT $\mathcal{H}(x,y,\tilde{y}) = (U\, E')^t\, U E'$

$$\begin{pmatrix} R^t & 0 \\ 0 & R^t \end{pmatrix} \begin{pmatrix} 1 & M \\ M^t & 1 \end{pmatrix} \begin{pmatrix} R & 0 \\ 0 & R \end{pmatrix} = \begin{pmatrix} 1 & R^t M R \\ R^t M^t R & 1 \end{pmatrix}$$

$$\begin{pmatrix} R^t & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} R & 0 \\ 0 & R^t \end{pmatrix} \begin{pmatrix} R & 0 \\ 0 & R \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

All dependence on $y \quad \tilde{y}$ dissapears !

Dependence on $y \quad \tilde{y}$ when considering fluctuations M

Conclusions

- DFT description of strings very close to self-dual radius
- Enhancement of symmetry $\rightarrow$ extend the generalized tangent space O(3,3)
- Winding modes $\rightarrow$ explicit dependence on dual coordinate

  violate weak constraint

  satisfy level-matching
- When M=0, "6d double space" is a torus, no dependence on y or $\tilde{y}$
- Moduli (M≠0) bring in dependence on y and $\tilde{y}$
- By appropriate generalized Scherk-Schwarz reduction of DFT action we fully recover string theory action