New AdS/CFT duals through non-Abelian T-duality

Yolanda Lozano (U. Oviedo)

Stringy Geometry Workshop (Mainz), September 2015

Universidad de Oviedo

1. Motivation and summary of results:

- Non-Abelian T-duality (NATD) very useful in generating new supergravity backgrounds with CFT duals

  (Sfetsos, Thompson'10)

- Interesting relations between AdS/CFT duals:

$AdS_5 \times S^5$	$\xrightarrow{\text{NATD}}$ Uplift to 11 dim	Gaiotto & Maldacena geometries (dual to Gaiotto's N=2 SCFTs)
$AdS_5 \times T^{1,1}$		Bah, Beem, Bobev, Wecht sols. (dual to N=1 SCFTs (Sicilian quivers))

(Sfetsos, Thompson'10; Itsios, Nuñez, Sfetsos & Thompson'13)

(This may be so because NATD has not been proved to be a String Theory symmetry, unlike its Abelian counterpart)

What happens in other dimensions?

In this talk we will use NATD to generate new AdS_6, AdS_4 and AdS_3 backgrounds and will explore properties of their field theory duals

Review talk of results in:

- Y.L., O Colgain, Rodriguez-Gomez, Sfetsos, PRL (2013)
- Y.L., O Colgain, Rodriguez-Gomez, JHEP (2014)
- Y.L., Macpherson, JHEP (2014)
- Y.L., Macpherson, Montero, JHEP (2015)
- Y.L., Macpherson, Montero, O Colgain, JHEP (2015)

AdS_6 : AdS_6 backgrounds are quite unique ("no-go" theorem) (Passias'12)

Dual to 5d fixed point theories with interesting properties, intrinsically strongly coupled, whose ST realization is only known in very specific cases

(Seiberg'96; Intriligator, Morrison, Seiberg'97; Aharony, Hanany'97)

The only known AdS_6 background (before 1212.1043) arises as a near horizon geometry (Brandhuber, Oz'99)

Search for new realizations of 5d fixed point theories by scanning over possible AdS_6 vacua in SUGRA

New AdS_6 background through NATD with SUSY fully preserved (Y.L., O Colgain, Rodriguez-Gomez, Sfetsos, PRL (2013))

Only known AdS_6 background in IIB

(Apruzzi, Fazzi, Passias, Rosa, Tomasiello'14)

NATD as a powerful tool to construct explicit backgrounds unaccessible by other means

New fixed point theory?

(Y.L., O Colgain, Rodriguez-Gomez, JHEP (2014))

AdS_4 : AdS_4 backgrounds associated to wrapped M5-branes are scarce (Gauntlett, Mac Conamhna, Mateos, Waldram'06)

(New infinite family in massive IIA (N=1) (Rota, Tomasiello'15))

For N=2 they are candidate duals to the 6d (2,0) CFT's compactified in Σ_3 that appear in 3d-3d duality

The only known explicit solution before 1507.02660 was the sol. in Pernici, Sezgin'85

New N=2 IIB AdS_4 background through NATD

(Y.L., Macpherson, JHEP (2014)) $\rightarrow$ New N=2 11d AdS_4 solution with purely magnetic flux in the general classification in Gauntlett, Mac Conamhna, Mateos, Waldram'06

(Y.L., Macpherson, Montero, 1507.02660)

Hints on its dual CFT

AdS_3 : 1/4-BPS $AdS_3 \times S^2$ 11d solutions classified in Kim, Kim, Kim'07 (see also Gauntlett, Mac Conamhna, Mateos, Waldram'06)

General class with SU(2)-structure to which there was no explicit solution

In Y.L., Macpherson, Montero, O Colgain, 1507.02659:

- The NATD of $AdS_3 \times S^3 \times T^4$ (Sfetsos, Thompson'10) (plus T-dualities and uplift) gives the only known example
- The NATD of $AdS_3 \times S^3 \times S^3 \times S^1$ (plus T-dualities and uplift) falls outside these classifications $\rightarrow$ More general class of $AdS_3 \times S^2$ solutions with SU(2)-structure

Example with large superconformal symmetry, which is preserved by the NATD

2. Non-Abelian T-duality back in the 90's

Rocek and Verlinde's formulation of Abelian T-duality for ST in a curved background (Rocek, Verlinde'92) :

i) Identify an Abelian isometry: $\theta \to \theta + \epsilon$

ii) Gauge the isometry: $d\theta \to D\theta = d\theta + A$

iii) Add a Lagrange multiplier term: $\tilde{\theta}\, dA$

iv) Integrate the gauge field

$\to$ Dual sigma model with $\{\theta, X^\alpha\} \to \{\tilde{\theta}, X^\alpha\}$ and $(\tilde{g}, \tilde{B}_2, \tilde{\phi})$ given by Buscher's formulae

v) For non-trivial world-sheets:

Invariance under $\oint_\gamma d\epsilon = 2\pi n$ fixes $\oint_\gamma d\tilde{\theta} = 2\pi m$

Same thing in the non-Abelian case: (De la Ossa, Quevedo'93)

i) $X^m \to g^m_n X^n \, , g \in G$

ii) Add a Lagrange multiplier term: $\mathrm{Tr}(\chi F)$

$\rightarrow$ Dual sigma model with $\{X^m, X^\alpha\} \to \{\chi^m, X^\alpha\}$

However,

- Non-involutive
- Higher genus generalization? Set to zero $W_\gamma = P\, e^{\oint_\gamma A}$
- Global properties?
- Conformal invariance not proved in general

True symmetry in ST?

3. NATD as a solution generating technique

(Sfetsos, Thompson'10)

Need to know how the RR fields transform

In the Abelian case: Reduce to a unique N=2, d=9 SUGRA

(Bergshoeff, Hull, Ortín'95)

Hassan'99: Implement the relative twist between left and right movers in the bispinor formed by the RR fields:

$$\hat{P} = P\Omega^{-1} \qquad P = \frac{e^{\phi}}{2}\sum_k \frac{1}{k!} F_{\mu_1\ldots\mu_k}\Gamma^{\mu_1\ldots\mu_k}$$

with $\Omega = \sqrt{g_{00}^{-1}}\Gamma_{11}\Gamma^0$

Same thing in the non-Abelian case

Interesting AdS solutions have been found

Given that NATD is not guaranteed to be a symmetry of ST:

New CFTs may arise, that may only exist in the strong coupling regime

(This seems to be the case with the NATD of $AdS_5 \times S^5$ and $AdS_5 \times T^{1,1}$!)

Explore new examples in other dimensions

4. Non-Abelian T-duality and 5d fixed point theories

5d gauge theories are non-renormalizable:

$$[g^2] = M^{-1} \quad \rightarrow \quad g^2 E \quad \rightarrow \quad \text{UV completion}$$

5d SYM with minimal SUSY can be at fixed points for specific gauge groups and matter content

(Seiberg'96; Intriligator, Morrison, Seiberg'97)

- They are intrinsically strongly coupled $\rightarrow AdS/CFT$
- The string theory realization is however only known in very specific cases

- In particular, $Sp(N)$ (with specific matter content) can be realized in Type I' in a D4/D8/O8 system. (Seiberg'96)

The only known AdS_6 background (before arXiv:1311.4842) arises as its near horizon geometry (Brandhuber, Oz'99)

The Brandhuber-Oz background

Fibration of AdS_6 over half-S^4 with an S^3 boundary at the position of the O8-plane, preserving 16 SUSYs

$$ds^2 = \frac{W^2\,L^2}{4}\left[9\,ds^2(AdS_6) + 4\,ds^2(S^4)\right] \qquad \theta \in [0, \frac{\pi}{2}]$$

$$F_4 = 5\,L^4\,W^{-2}\,\sin^3\theta\,d\theta \wedge \mathrm{Vol}(S^3)$$

$$e^{-\phi} = \frac{3\,L}{2\,W^5}\,, \qquad W = (m\,\cos\theta)^{-\frac{1}{6}} \qquad m = \frac{8-N_f}{2\pi l_s}$$

- $SO(5)$ symmetry broken to $SO(4) \sim SU(2) \times SU(2)$:

  $SU(2) \leftrightarrow SU(2)_R$ R-symmetry of the field theory

  $SU(2) \leftrightarrow$ global symmetry massless antisym. hyper

- $SO(2,5) \leftrightarrow$ Conformal symmetry

The non-Abelian T-dual

- Take the $AdS_6 \times S^4$ background

$$ds^2 = \frac{W^2 L^2}{4}\left[9 ds^2(AdS_6) + 4\Big(d\theta^2 + \sin^2\theta\, ds^2(S^3)\Big)\right]$$

$$F_4 = 5 L^4 W^{-2} \sin^3\theta\, d\theta \wedge \mathrm{Vol}(S^3)$$

- Dualize it w.r.t. one of the $SU(2)$ symmetries

In spherical coordinates adapted to the remaining $SU(2)$:

$$ds^2 = \frac{W^2 L^2}{4}\left[9\, ds^2(AdS_6) + 4\, d\theta^2\right] + e^{-2A}\, dr^2 + \frac{r^2 e^{2A}}{r^2 + e^{4A}}\, ds^2(S^2)$$

$$B_2 = \frac{r^3}{r^2 + e^{4A}}\, \mathrm{Vol}(S^2) \qquad e^{-\phi} = \frac{3L}{2W^5}\, e^{A}\sqrt{r^2 + e^{4A}}$$

$$F_1 = -G_1 - m\, r\, dr \qquad F_3 = \frac{r^2}{r^2 + e^{4A}}\left[-r\, G_1 + m\, e^{4A}\, dr\right] \wedge \mathrm{Vol}(S^2)$$

- It solves the IIB equations of motion
- SUSY preserved! First example of a non-Abelian T-dual geometry with supersymmetry fully preserved

  This is because the internal symmetry is really $SU(2) \times SU(2)_R$ and we dualize on the $SU(2)$ global symmetry *

- What about r ?
  - Background perfectly smooth for all $r \in \mathbb{R}^+$
  - No global properties inferred from the NATD
  - Puzzle to the dual CFT (!)
- New singularity at $\theta = 0$

* NATD and SUSY: Kelekci, Y.L., Macpherson, O Colgain, arXiv:1409.7406

5. Non-Abelian T-duality in ABJM

Take the $AdS_4 \times CP^3$ IIA dual +

parameterize the CP^3 as a foliation in $T^{1,1} = S^2 \times S^3$ +

T-dualize w.r.t. the $SU(2)$ symmetry of the S^3

$\longrightarrow$ IIB AdS_4 solution with B_2, F_1, F_3, F_5

- N=2 supersymmetric
- What about r ?
  - Background perfectly smooth for all $r \in \mathbb{R}^+$
  - No global properties inferred from the non-Abelian transf.
  - Puzzle to the dual CFT (!)

6. Some hints on the dual CFTs

After an $SU(2)$ NATD:

$$F_q \to F_{q+1}\ (M_1), \qquad F_q \to F_{q+3}\ (M_1 \times S^2)$$

$$F_q \to F_{q-1}, \qquad F_q \to F_{q-3}$$

AdS_6

N D4	→ N_5^θ D5 → N_7^θ D7	N_8 D8/O8	→ N_7^r 7-brane → N_5^r 5-brane

AdS_4

N D2	→ N_3 D3 → N_5 D5	k D6	→ k_5 D5 → k_3 D3

In AdS_6 :

D4 $\longleftrightarrow$ D8 flavor
D0 instanton

$\nearrow$ D5 $\longleftrightarrow$ D7 flavor
D1 instanton

$\searrow$ D7 $\longleftrightarrow$ D5 flavor
D3 instanton

Baryon vertex: D4 $\nearrow$ D1
$\searrow$ D3

In AdS_4 :

D2 $\longleftrightarrow$ D6 level

$\nearrow$ D3 $\longleftrightarrow$ D5 level

$\searrow$ D5 $\longleftrightarrow$ D3 level

D6 baryon vertex $\nearrow$ D5
$\searrow$ D3

D2 ‘t Hooft monopole $\nearrow$ D3
$\searrow$ D5

...

CFT interpretation?

Look at large gauge transformations:

Close to the singularity the geometry is conformal to a singular cone with S^2 boundary, where $B_2 = r\,\mathrm{Vol}(S^2)$

Large gauge transformations must be defined such that

$$\frac{1}{4\pi^2}|\int B_2| \in [0,1) \quad \text{for} \quad r > \pi \qquad \Rightarrow$$

We need $B_2 = (r - n\pi)\,\mathrm{Vol}(S^2)$ for $r \in [n\pi, (n+1)\pi)$

This modifies the Page charges, such that $N_{p-2} = n\,N_p$

$\Rightarrow$ Moving from an $r \in [n\pi, (n+1)\pi)$ interval to the previous one: $N_{p-2} \to N_{p-2} - N_p$

Geometric realization of Seiberg duality

But same CFT!

- In the 5d case:

Moving from an r interval to the previous one:

$$N_5^\theta \to N_5^\theta - N_7^\theta$$

This suggests a dual CFT with gauge group, as r varies:

$$Sp(N_5^\theta + N_7^\theta) \times Sp(N_5^\theta)$$

For $r \in [0,\pi)$ the CFT would be $Sp(N_7^\theta)$.

$\to$ Fixed point theory from 7-branes?

(DeWolfe, Hanany, Iqbal, Katz' 99)

- In 3d the same idea suggests a dual CFT:

$$U(N_3+N_5)_{k_3} \times U(N_3)_{k_5} \times U(N_3+N_5)_{-k_3} \times U(N_3)_{-k_5}$$

For $r \in [0,\pi)$: $U(N_5)_{k_3} \times U(N_5)_{-k_3}$

$\to$ N=2 CFT generated by D5-branes?

What happens as we move in r?

Invariance of ST under large gauge transformations suggests that the dual CFTs could be related by some duality

If this is true, in 5d all CFT would be equivalent to the one dual to the solution for $r \in [0, \pi)$: $Sp(N_7^\theta)$ with N_5^r flavors

$$\updownarrow$$

New fixed points associated with product gauge groups ruled out in the classification of Intriligator, Morrison, Seiberg

Same idea in 3d: D5-color branes with D3 “level” branes, N=2

But, we are moving in an internal direction, not in the holographic direction! Also, the number of degrees of freedom (the free energy) seems to change as we move in r...

A comment about the free energies

5d: In the original theory: $F \sim \frac{N^{5/2}}{m^{1/2}}$ (Jafferis, Pufu'12)

In the dual: $F \sim \frac{(N_7^\theta)^{5/2}}{(N_5^r)^{1/2}}$ $(r \in [0,\pi])$

3d: In the original theory: $F(S^3) \sim \sqrt{k}\, N^{3/2}$

(Drukker, Mariño, Putrov'10)

In the dual: $F(S^3) \sim \sqrt{k_3}\, N_5^{3/2}$

But the proportionality factors depend on n for the various $[n\pi, (n+1)\pi)$ intervals. Duality?

New suggestion in Bea, Edelstein, Itsios, Kooner, Núñez, Schofield, Sierra-García'15 (see conclusions)

7. A new AdS_4 solution with purely magnetic flux

T-duality on the NATD of ABJM + uplift to 11d

$\rightarrow$ $\mathcal{N}=2$ solution to M-theory with purely magnetic flux

- Second known example in this class besides the uplift of the Pernici-Sezgin solution to 7d gauged sugra

- Pernici-Sezgin:
  - $AdS_4 \times \Sigma_3 \times S^4$ solution to M-theory
  - Candidate holographic dual to a twisted compactification on a 3d manifold of the (2,0) CFT_6 living in M5-branes (3d-3d duality)

- Our solution:
  - Not of the form $AdS_4 \times \Sigma_3 \times S^4$
  - Extra Taub-NUT charge coming from D6

- Extra U(1) isometry (this seems to appear in the analogue in 3d to Gaiotto's 4d theories with punctures) (Bah, Gabella, Halmagyi'14)

  Similar interpretation as NAT duals of AdS_5 ?

- Free energy goes as $\sqrt{k}\, N^{3/2}$ as in ABJM:

  However no M2 (Gabella, Martelli, Passias, Sparks'12)

  Is it associated to wrapped M5-branes?

8. Some AdS_3 results

AdS_3 classifications: $AdS_3 \times S^2$ 1/4-BPS geometries as solutions to the 3d analogue of the Toda equation of LLM (Kim, Kim, Kim'07)

However, no known solution besides $AdS_3 \times S^2 \times CY_3$.

In Y.L., Macpherson, Montero, O Colgain'15:

- NATD of $AdS_3 \times S^3 \times T^4$ (Sfetsos, Thompson'10) : First example with SU(2)-structure in this class.
- $AdS_3 \times S^3 \times S^3 \times S^1$:
  - $\mathcal{N} = (4,4)$ SUSY background with $SU(2) \times SU(2)$ R-symmetry (large superconformal symmetry in 2d)

- Near horizon geometry of a D1-D5-D5' system
- Recently (Tong'14): Explicit $\mathcal{N}=(4,4)$ CFT dual

Central charge: $c = 2\,N_1\,\dfrac{N_5^+ N_5^-}{N_5^+ + N_5^-} = 2\,\dfrac{k^+ k^-}{k^+ + k^-}$

with $k^\pm = N_1 N_5^\pm$, like the gravity dual

NATD:

$AdS_3 \times S^3 \times S^2$ geometry with $\mathcal{N}=(0,4)$ SUSY in 2d

- Analysis of the dual solution suggests it arises as the near horizon of a D4-D8-D2 system
- Central charge: $c = 2\,N_4\,\dfrac{N_2 N_8}{3N_2 + N_8}$ $\leftrightarrow$ Large $\mathcal{N}=(0,4)$ CFT with $k^+ = 3N_4 N_2\,,\; k^- = N_4 N_8$
- Not in the classification in (Kim, Kim, Kim'07)

9. Conclusions

- NATD useful as a solution generating technique:

  Powerful tool to construct explicit backgrounds unaccessible by other methods (AdS_6, AdS_4 and AdS_3 examples)

  It reveals striking connections between backgrounds with seemingly very different dual CFTs

  But there is an issue with the free energies:

  - Free energy invariant under NATD $\Rightarrow$ Even if the NATDs of $AdS_5 \times S^5$ and $AdS_5 \times T^{1,1}$ belong to the general class of GM and BBBW solutions, the free energies don't agree

  - Similarly, the 11d $\mathcal{N}=2$ AdS_4 sol. with purely magnetic flux does not seem to be associated to M5-branes

- First steps towards a systematic study of the dual CFTs

Most urgent is an explanation for the realization in the CFT of the running of the non-compact direction

Can the CFTs dual to the solution as r varies be equivalent?

The internal volume changes $\Rightarrow$ Different free energies

Proposal by Santiago-Swansea group: Motion in r induces D(p-2) charge $\Rightarrow$ New gauge groups through "un-Higgsing"

But, where is the energy scale?

Thanks!