EGB supergravity and black holes with unbroken supersymmetries

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arXiv:2106.01876 (JHEP'21)

YOUNGST@RS - Supergravity and Holography Workshop 2021

13 – 15 December 2021

Plan

Plan of the talk

- Chern-Simons AdS supergravity
- Hairy black hole solution and BPS state
- Physical properties
- Discussion and open questions

Motivation

Quadratic curvature gravity in AdS_5 space

- Higher-curvature terms in the framework of AdS/CFT correspondence
- Black holes solutions in supergravity that leave some of SUSY unbroken

Simplest case – AdS_5 space with the radius ℓ_0

- General Relativity with negative cosmological constant $\Lambda = -\frac{6}{\ell_0^2}$

$+$ First correction given by the Gauss-Bonnet term

$$\mathcal{L}_{\mathrm{EGB}} = \kappa \left[R - 2\Lambda + \alpha \left(R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta} - 4R^{\mu\nu} R_{\mu\nu} + R^2 \right) \right]$$

Equations of motion can be factorized

- $\delta \int \mathcal{L}_{\rm EGB} = 0 \quad \Rightarrow \quad \boxed{\delta^{\mu\mu_1\cdots\mu_4}_{\nu\nu_1\cdots\nu_4}\left(R^{\nu_1\nu_2}_{\mu_1\mu_2} + \frac{1}{\ell_+^2}\,\delta^{\nu_1\nu_2}_{\mu_1\mu_2}\right)\left(R^{\nu_3\nu_4}_{\mu_3\mu_4} + \frac{1}{\ell_-^2}\,\delta^{\nu_3\nu_4}_{\mu_3\mu_4}\right) = 0}$
- There are $k=2$ AdS vacua: $R^{\nu_1\nu_2}_{\mu_1\mu_2} = -\frac{1}{\ell_\pm^2}\,\delta^{\nu_1\nu_2}_{\mu_1\mu_2}$, $\quad \ell_\pm^2 = \frac{4\alpha}{1\pm\sqrt{1-\frac{8\alpha}{\ell_0^2}}}$
- When $\ell_+ \neq \ell_-$, eqs. can be linearized around the AdS background

Chern-Simons coupling point: two vacua coincide, unique AdS radius

$\ell \equiv \ell_+ = \ell_- = \ell_0/\sqrt{2}\,, \qquad \alpha^* = \ell_0^2/8\,, \qquad \alpha \to \alpha^*$ is not continuous

- Static black holes change their asymptotics compared to Schwarzschild AdS

$g_{tt} \sim 1 + \frac{r^2}{\ell_{\rm eff}^2} - \frac{\mu}{r^{(D-2k-1)/k}}$ (k-fold AdS vacuum in D dimensions)

$g_{tt} = 1 + \frac{r^2}{\ell_{\rm eff}^2} - \mu$ $\quad D = 2k+1$, Dimensionally Continued Black Hole

$g_{tt} = 1 + \frac{r^2}{\ell_{\rm eff}^2} - \mu$ $\quad D = 3$, BTZ black hole

$\Rightarrow$ **The GB term with the coupling α^* is not just a correction**

Chern-Simons AdS supergravity in D=5

Einstein-Gauss-Bonnet AdS gravity with the coupling $\alpha^* = \frac{\ell^2}{4}$

- **First order formalism** (forms in the basis $\mathrm{d}x^\mu$, without writing $\wedge$)

  (e^a, ω^{ab}) vielbein, spin-connection 1-forms (independent)

  $R^{ab} = \mathrm{d}\omega^{ab} + \omega^{ac}\omega_c{}^b$ (curvature) $T^a = De^a$ (torsion)

- **Chern-Simons AdS$_5$ gravity**

  $L_{\mathrm{G}} = \frac{1}{8\ell}\,\epsilon_{abcde}\left(R^{ab}R^{cd}e^e + \frac{2}{3\ell^2}\,R^{ab}e^c e^d e^e + \frac{1}{5\ell^4}\,e^a e^b e^c e^d e^e\right)$

  The fields e, ω are defined over the flat background

- **New features compared to EGB gravity**

- It can have $T^a \neq 0$ on-shell
- Extended local symmetries: local Lorentz $(\mathbf{J}_{ab}) \rightarrow$ local AdS $(\mathbf{P}_a, \mathbf{J}_{ab})$
- More degrees of freedom than EH (generically)
- A number of d.o.f. can vary depending on a sector of the phase space
- Topological sectors exist in any (odd) D

Chern-Simons AdS supergravity in D=5

Einstein-Gauss-Bonnet AdS gravity with $\alpha^* =$ Chern-Simons AdS theory

- Gauge theory for $SO(2,4)$ on the flat background

  $[\mathbf{J},\mathbf{J}]=\mathbf{J},\ [\mathbf{J},\mathbf{P}]=\mathbf{P},\ [\mathbf{P},\mathbf{P}]=\mathbf{J}$ ($\mathbf{P}_a, \mathbf{J}_{ab} =$ AdS generators)

  $\mathbf{A}=\frac{1}{\ell}\,e^a\mathbf{P}_a+\frac{1}{2}\,\omega^{ab}\mathbf{J}_{ab}$ (gauge field)

  $\mathbf{F}=\mathrm{d}\mathbf{A}+\mathbf{A}^2=\frac{1}{\ell}\,T^a\mathbf{P}_a+\frac{1}{2}\,(R^{ab}+\frac{1}{\ell^2}\,e^ae^b)\mathbf{J}_{ab}$

- **Chern-Simons action**

  $I[\mathbf{A}]=\frac{k}{3}\int\limits_{\mathcal{M}}L(\mathbf{A}),$ $\mathrm{d}L(\mathbf{A})=\frac{1}{3}\left\langle\mathbf{F}^3\right\rangle,$ $L(\mathbf{A})$ is a CS form

  $\epsilon_{abcde}=\langle\mathbf{J}_{ab}\mathbf{J}_{cd}\mathbf{P}_e\rangle$ (symmetric invariant tensor)

- In $D=3$, General Relativity is Chern-Simons gravity for (A)dS/Poincaré gauge group

Chern-Simons AdS supergravity in D=5

Now it is straighforward to perform a supersymmetric extension

$$\begin{aligned} SO(2,4) &\to SU(2,2|\mathcal{N}) \\ \mathbf{P}_a, \mathbf{J}_{ab} &\to \mathbf{G}_M = \{\mathbf{P}_a, \mathbf{J}_{ab}, \mathbf{T}_\Lambda, \mathbf{G}_1, \mathbf{Q}_s^\alpha, \overline{\mathbf{Q}}_\beta^u\} \\ & \qquad SO(2,4)\times SU(\mathcal{N})\times U(1)_q \quad \text{(bosonic sector)} \\ \epsilon_{abcde} &\to g_{MNK} = \langle \mathbf{G}_M \mathbf{G}_N \mathbf{G}_K \rangle \quad \text{(interaction)} \end{aligned}$$

- Dimension: $\mathfrak{D} = \mathcal{N}^2 + 8\mathcal{N} + 15, \quad \mathfrak{D}_{\text{bosonic}} = \mathcal{N}^2 + 15 \neq \mathfrak{D}_{\text{fermionic}} = 8\mathcal{N}$
- $U(1)_q$**-charged fermions** $[\mathbf{G}_1, \mathbf{Q}] = -\mathrm{i}\left(\frac{1}{4} - \frac{1}{\mathcal{N}}\right)\mathbf{Q}$
- Matter fields = gauge fields with fixed interactions, only grav. coupling
- $U(1)_q$ field couples gravity with the internal non-Abelian field

$$g_{1[ab][cd]} = \frac{1}{2}\eta_{a[c}\eta_{d]b}, \qquad g_{1ab} = -\frac{1}{4}\eta_{ab}, \qquad g_{111} = \frac{1}{4^2} - \frac{1}{\mathcal{N}^2}$$

$\mathcal{N} = 4$: Fermions become neutral, $U(1)_q$ turns to central extension, the $U(1)_q$ field without kinetic term ($g_{111} = 0$)

Chern-Simons AdS supergravity in D=5

Field content when $\mathcal{N}=4$

e^a, ω^{ab}	gravitational field;	$\mathcal{A}^\Lambda$	$SU(4)$ gauge field;
$\bar{\psi}^s, \psi_s$	fermions;	A	$U(1)_q$ gauge field;

Bosonic action

$$\begin{aligned} L &= L_{\mathrm{G}}(e,\omega) + L_{SU(4)}(\mathcal{A}) + L_{U(1)}(A,e,\omega,\mathcal{A}) \\ L_{SU(\mathcal{N})} &= -\tfrac{1}{3}\,\mathrm{Tr}\left(\mathcal{A}\mathcal{F}^2 - \tfrac{1}{2}\,\mathcal{A}^3\mathcal{F} + \tfrac{1}{10}\,\mathcal{A}^5\right) \\ L_{U(1)_q} &= -\tfrac{1}{4\ell^2}\left(T^a T_a - \tfrac{\ell^2}{2}\,R^{ab}R_{ab} - R^{ab}e_a e_b\right)A + \tfrac{1}{4}\,\mathcal{F}^\Lambda\mathcal{F}_\Lambda A \end{aligned}$$

Equations of motion $\mathcal{E}_M = g_{MNK}\,F^N F^K = 0$

$$\begin{aligned} \mathcal{E}_a &= \tfrac{1}{8}\,\epsilon_{abcde}\,F^{bc}F^{de} - \tfrac{1}{2\ell}\,T_a F \\ \mathcal{E}_{ab} &= \tfrac{1}{2\ell}\,\epsilon_{abcde}\,F^{cd}T^e + \tfrac{1}{2}\,F_{ab}F \\ \mathcal{E} &= \tfrac{1}{8}\left(F^{ab}F_{ab} - \tfrac{2}{\ell^2}\,T^a T_a\right) + \tfrac{1}{4}\,\mathcal{F}^\Lambda\mathcal{F}_\Lambda \\ \mathcal{E}_\Lambda &= \tfrac{1}{2}\,\mathcal{F}_\Lambda F + \gamma_{\Lambda\Lambda_1\Lambda_2}\mathcal{F}^{\Lambda_1}\mathcal{F}^{\Lambda_2} \end{aligned}$$

Static black hole solutions

Static, spherically symmetric black hole solutions

Metric field ansatz with the spherical horizon

$$\mathrm{d}s^2 = -N^2(r)f^2(r)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f^2(r)} + r^2\mathrm{d}\Omega^2$$

$$\mathbb{S}^3 : \mathrm{d}\Omega^2 = \sigma_{mn}(\theta)\,\mathrm{d}\theta^m\mathrm{d}\theta^n = \delta_{ij}\,\tilde{e}^i\tilde{e}^j$$

Torsion field ($T^a{}_{\mu\nu}$) $\rightarrow$ two scalar fields

$$T_{nrm} = \tau(r)\,r\sigma_{nm}, \qquad T_{nmk} = 2r^2\varphi(r)\sqrt{\sigma}\,\epsilon_{nmk}$$

Particularity of a three-dimensional geometry

It can have constant curvature and torsion

Our solution will be asymptotically with constant curvature and torsion

Static black hole solutions

Dimensionally continued black holes

- Special case $f^2 = \frac{r^2}{\ell^2} - M\,,\; T^a = 0$ [Banados, Teitelboim, Zanelli '94]

$M > 0$	Black holes
$M = 0$	Black hole ground state
$-1 < M < 0$	Naked singularities
$M = -1$	AdS space

- Global AdS space with $SU(4) \times U(1)_q$ fields, BPS states [Miskovic, Troncoso, Zanelli '06]
- Black holes with $\varphi(r) = C$, BPS states [Canfora, Giacomini, Troncoso '08]
- Black holes with $U(1)_q$ charges [Giribet, Merino, Miskovic, Zanelli '14]
- *Goal*: BPS black holes with $SU(4)$ fields

Static black hole solutions

Hairy black hole with four integration constants (μ, C, b_0, b)

$$\tau = \frac{b\ell^2}{2r + b\ell^2}\,, \qquad f = \sqrt{\frac{r^2}{\ell^2} + br + 1 - \mu}\,, \qquad N = 1 - \frac{b_0\ell}{2f}$$
$$\varphi^2 = \left[C^2 - \frac{b\ell^2}{2}\frac{\mu + \frac{b^2\ell^2}{4}}{r + \frac{b\ell^2}{2}} + \frac{b^2\ell^4}{4}\frac{\left(r + \frac{b\ell^2}{3}\right)\left(\mu + \frac{b^2\ell^2}{4} - \kappa\right)}{\left(r + \frac{b\ell^2}{2}\right)^3}\right]\left(1 + \frac{b\ell^2}{2r}\right)$$

- Auxiliary function $\quad \Xi(r) = 1 + \frac{r^2}{\ell^2} - \varphi^2 - \nu^2$
- **Asymptotic behaviour**

$$f^2 \to \frac{r^2}{\ell^2} + br, \qquad N \to 1, \qquad \varphi^2 \to C^2, \qquad \tau \to 0, \qquad F|_{\mathbb{S}^3} \to \text{const}$$
$$\Xi \to \mu - C^2 + \frac{b^2\ell^2}{4} = \text{const}$$

- μ is the mass parameter, C is the axial torsion charge,
- b, b_0 are gravitational hair

Static black hole solutions

Singularity

- Solution with $\Xi \neq 0$ has scalar curvature singularity at $r = 0$
- Extremality parameter of two BH horizons
  $p^2 = \left(\frac{r_+ - r_-}{4\ell}\right)^2 = \mu - 1 + \frac{b^2\ell^2}{4} \geq 0$
- Hawking temperature is $T = \frac{N(r_+)}{2\pi} \left(\frac{r_+}{\ell^2} + \frac{b}{2}\right)$
- When $p^2 < 0$, $p^2 \neq -1$ we get *naked singularities*
- When $p^2 = -1$, the space is maximally symmetric globally (AdS_5)

Static black hole solutions

We are interested in the solutions that leave some supersymmetries unbroken (BPS states)

- Let us choose $b = b_0 = 0$ (usual asympotically AdS space)
- The only torsion component, the axial torsion field $\varphi(r)$, can be seen as a scalar matter field on the curved (metric) background
- **As a result, we get $U(1)_q$-neutral and Ξ-charged black hole**

$$
\begin{aligned}
&\mathrm{d}s^2 = -\left(1 + \frac{r^2}{\ell^2} - \mu\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1+\frac{r^2}{\ell^2}-\mu} + r^2\mathrm{d}\Omega^2 \\
&A_t = 0\,, \qquad \Xi = \mu - C^2 \neq 0\,, \qquad \varphi = C\,, \\
&\tau = 0, \qquad N = 1\,,
\end{aligned}
$$

Internal non-Abelian field

Non-Abelian field $\mathcal{F} = \frac{1}{2}\,\mathcal{F}_{mn}\,\mathrm{d}\theta^m \wedge \mathrm{d}\theta^n$

- Transversal space is the maximally symmetric 3-sphere $\mathbb{S}^3$ or its identifications, the (locally) isometry group is $SO(4)$

$$\begin{cases} \mathbb{S}^3 \simeq \frac{SO(4)}{SO(3)} \simeq SU(2) \\ \pi_3(SU(2)) = \mathbb{Z} \end{cases} \Rightarrow \text{ Nontrivial winding of } SU(2) \text{ around the sphere}$$

- **Breaking internal symmetry**

$$SU(4) \simeq SO(6) \to SU(2)_+ \times SU(2)_- \times U(1)_c \quad \text{(maximal subgroup)}$$

$$SU(4) \to SU(2)_D \times U(1)_c \qquad \text{(suitable for static solutions)}$$

- The Killing spinor will be a $U(1)_c$-charged Dirac spinor on $\mathbb{S}^3$
- We can also consider quotient spaces of the 3-sphere, for example the topological space $\mathbb{RP}^3 \equiv \mathbb{S}^3/\mathbb{Z}_2$ (real projective 3-space) that has identified antipodal points on the sphere

Internal non-Abelian field

Explict construction of the non-Abelian field

- $SU(4) \simeq SO(6)$ generators $\mathbf{T}_{IJ}$ with $I = 0, 1, \hat{\imath}, 5$

$$\boxed{\begin{array}{ccccc} SU(4) & \rightarrow & SU(2)_+ \times SU(2)_- \times U(1)_c & \rightarrow & SU(2)_D \times U(1)_c \\ \mathbf{T}_{IJ} & \rightarrow & \{\mathbf{T}_{+\hat{\imath}}\,, \quad \mathbf{T}_{-\hat{\imath}}\,, \quad \mathbf{T}_c = \mathbf{T}_{15}\} & \rightarrow & \{\mathbf{T}_{\hat{\imath}}\,, \quad \mathbf{T}_c\} \end{array}}$$

- We define $\mathbf{T}_{\pm\hat{\imath}} = \frac{1}{2}\left(\pm\mathbf{T}_{0\hat{\imath}} - \mathbf{T}_{\hat{\imath}}\right)$ and $\mathbf{T}_{\hat{\imath}} = \frac{1}{2}\,\epsilon_{\hat{\imath}}{}^{\hat{\jmath}\hat{k}}\,\mathbf{T}_{\hat{\jmath}\hat{k}}$ $(\hat{\imath} = 2, 3, 4)$
- Constant $SU(2)_D$-curvature solution of the strength $\tilde{\Xi}$

$$\begin{aligned} \mathcal{A}^i &= \tilde{\omega}^i - B\,\tilde{e}^i\,, \quad \tilde{\omega}_i \equiv \tfrac{1}{2}\,\epsilon_i{}^{jk}\,\tilde{\omega}_{jk} \\ \mathcal{F}^i &= \tfrac{1}{2}\,\tilde{\Xi}\,\epsilon^{ijk}\,\tilde{e}_i\tilde{e}_j\mathbf{T}_{\hat{k}}\,, \quad \tilde{\Xi} = 1 - B^2 \\ \mathcal{A}^{15} &= \mathrm{d}\Omega(\theta) \qquad \text{(pure gauge)} \end{aligned}$$

- $B =$new integration constant, the amplitude of the internal $SU(2)_D$ soliton

BPS states

- **Parameters of the solution** $(B, C, p^2 = \mu - 1)$ $\boxed{\Xi = 1 - C^2,\ \tilde{\Xi} = 1 - B^2}$

$$
\begin{aligned}
\mathbf{A} &= \left(\tfrac{r}{\ell}\,\mathbf{J}_{01} + f\,\mathbf{P}_0\right)\tfrac{\mathrm{d}t}{\ell} + \tfrac{\mathrm{d}r}{\ell f}\,\mathbf{P}_1 + \tilde{e}^i\left(\tfrac{r}{\ell}\,\mathbf{P}_i - f\,\mathbf{J}_{1i}\right) \\
&\quad + (\tilde{\omega}^i - C\,\tilde{e}^i)\,\mathbf{J}_i + (\tilde{\omega}^i - B\,\tilde{e}^i)\mathbf{T}_i + \mathrm{d}\Omega\,\mathbf{T}_c \\
\mathbf{F} &= C\,\epsilon_{ij}{}^k\tilde{e}^i\tilde{e}^j\left(\tfrac{r}{\ell}\,\mathbf{P}_k - f\,\mathbf{J}_{1k}\right) + \tfrac{1}{2}\,\epsilon_{ij}{}^k\,\tilde{e}^i\tilde{e}^j\left(\Xi\,\mathbf{J}_k + \tilde{\Xi}\,\mathbf{T}_k\right)
\end{aligned}
$$

- The field does *not* vanish at the boundary, but *it has finite energy*
- **Supersymmetry transformations** $\psi_s = 0 \Rightarrow \delta_\epsilon\psi_s = \nabla\epsilon_s =$ $\left(\mathrm{d} + \frac{1}{4}\,\omega^{ab}\Gamma_{ab} + \frac{1}{2\ell}\,e^a\Gamma_a\right)\epsilon_s - \frac{1}{4}\,\mathcal{A}^{IJ}(\tilde{\Gamma}_{IJ})_s^u\epsilon_u = 0$
- **BPS states** $\boxed{p^2 = 0,\ B = C,\ \ \Xi = \tilde{\Xi}}$ (extremal black holes)

BPS states

Killing spinors

- **Solutions for the parameters** $\epsilon(r,\theta) = \sqrt{\frac{r}{\ell}}\, \mathrm{e}^{\Omega(\theta)\mathbf{T}_c}\eta_0$, η_0-constant spinor
- Ω gives a phase to the spinor on the sphere

$$\eta_s(\theta) = \mathrm{e}^{-\frac{\mathrm{i}}{2}\,\Omega(\theta)}\eta_{0(+)s} + \mathrm{e}^{\frac{\mathrm{i}}{2}\,\Omega(\theta)}\eta_{0(-)s}\,, \qquad (\tilde{\Gamma}_1)_s^u\,\eta_{0(\pm)u} = \pm\eta_{0(\pm)s}$$

- *First projection condition:* $\boxed{\Gamma_1\eta_0 = -\eta_0}$
- *Three more projection conditions*: The tangent space $\widehat{SU}(2)_D$ symmetry on $\mathbb{S}^3$ (and the corresponding indices $i, j, \ldots$) has to match with the internal $SU(2)_D$ symmetry (labeled with indices $\hat{\imath}, \hat{\jmath}, \ldots$)

$\boxed{(\mathbf{J}_i)^\alpha_\beta\ \eta^\beta_{0s} = (\mathbf{T}_{\hat{\imath}})^u_s\ \eta_{0u}\,,}$ $i \equiv \hat{\imath} \in \{2,3,4\} \Rightarrow$ **1/16-BPS states**

BPS states

The spinors are globally well-defined

- General solution has an additional phase factor

$$\eta = \mathcal{P}\, \mathrm{e}^{\int_{\mathcal{C}(\theta)} \left[-\frac{\mathrm{i}}{2}(\tilde{\omega}^i - C\,\tilde{\mathrm{e}}^i)\Gamma_i + \mathcal{A}\right]} \eta_0 \qquad (\mathcal{P}\text{ path ordering})$$

- Global behavior is studied by looking at the closed paths γ

$$\mathcal{P}\, \mathrm{e}^{\oint_\gamma \left[-\frac{\mathrm{i}}{2}(\tilde{\omega}^i - C\,\tilde{\mathrm{e}}^i)\Gamma_i + \mathcal{A}\right]} \eta_0 = \lambda(\gamma)\eta_0 \quad \text{(holonomy condition)}$$

- The holonomy condition depends on the manifold Σ, holonomy of the fields and the spin structure, and the (anti)periodicity $\lambda(\gamma) = \pm 1$.

Topological properties

Pontryagin numbers

- Topological conserved current: $\left\langle \mathbf{F}^2 \right\rangle|_\Gamma = \mathrm{d}(^*J_{\mathrm{top}})$ *locally*
- Globally Pontryagin topological invariant on $\partial\Gamma = \mathbb{S}^3$
- Possible Pontryagin numbers

$SU(2,2,\|4)$	$P = \frac{1}{4\pi^2}\int\limits_\Gamma \left\langle \mathbf{F}^2 \right\rangle \in \mathbb{R}$
$SU(2)_D \subset SU(4)$	$n_1 = \frac{1}{4\pi^2}\int\limits_\Gamma \left\langle \mathcal{F}^2 \right\rangle \in \mathbb{Z}$
$\widehat{SU}(2)_D \subset SO(2,4)$	$n_2 = \frac{1}{4\pi^2}\int\limits_\Gamma \left\langle \hat{\mathcal{F}}^2 \right\rangle \in \mathbb{Z}$
$\widehat{SU}(2)_D \times SU(2)_D$	$n = n_1 + n_2 \in \mathbb{Z}$

Topological properties

- Hopf's coordinates $\mathrm{d}\Omega^2 = \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi_1^2 + \cos^2\theta\,\mathrm{d}\varphi_2^2$

$$\begin{cases} \mathcal{A}^2 &= -B\,\mathrm{d}\theta\,, \\ \mathcal{A}^3 &= -\sin\theta\,(B\,\mathrm{d}\varphi_1 + \mathrm{d}\varphi_2)\,, \\ \mathcal{A}^4 &= -\cos\theta\,(B\,\mathrm{d}\varphi_2 + \mathrm{d}\varphi_1)\,. \end{cases}$$

- $B =$ the magnitude of the internal soliton, $C =$ magnitude of the axial soliton
- **Topological numbers**

$$\boxed{\begin{aligned} n_1 &= B(2-B^2) \\ n_2 &= -C\,(2-C^2) \\ n &= -C(2-C^2) + B(2-B^2) \\ P &= n - \tfrac{3}{2}\,Cp^2 \end{aligned}}$$

- The magnitudes $B(n_1)$ and $C(n_2)$ are quantized by topology

Topological properties

- **BPS condition** $B = C$, $p^2 = 0 \quad \Rightarrow \quad n_{\text{BPS}}, P_{\text{BPS}} = 0$

  (soliton-antisoliton system unwinds)

- **Balance of forces** between gravity and internal symmetry is usual for BPS states in standard supergravity.

Comparison between CS and standard supergravity

Chern-Simons: $SU(2)$ charges quantized; μ is not quantized $(p^2 = \mu - 1)$

Standard: $U(1)$ charges quantized; μ is not quantized

Hamiltonian conserved charges

Hamiltonian charge

- Charge formula linearized around an AdS background **X**
- Noether charge in CS AdS gravity without T^a

  [Mora, Olea, Troncoso, Zanelli '06] **X**
- Wald's formula without T^a **X**
- **Hamiltonian charge** $\surd$ $\delta Q\left[\Lambda\right] = 2k \int_{\mathbb{S}^3} \langle \Lambda \mathbf{F} \delta \mathbf{A} \rangle$
- Gauge parameters have to satisfy $D\Lambda = 0\,,\ \delta\Lambda \to 0$
- The $U(1)_q \times SU(2)_D \times U(1)_c$ parameters: $\Lambda^1 = 1\,,\quad \Lambda^i = 0\,,\quad \Lambda^c = 1$
- The $SO(2,4)$ parameter: $\Lambda_0 = \frac{r}{\ell^2}\,\mathbf{J}_{01} + \left(\frac{r}{\ell^2} - \frac{p^2}{2r} + \cdots\right)\mathbf{P}_0$
- Λ_0 corresponds to the time-like Killing vector $\xi = \partial_t$
- $\delta E = \delta Q\left[\Lambda_0\right]$ is integrable when $\delta C = 0$

Hamiltonian conserved charges

Total gravitational energy $E = Q[\Lambda_0] = H[\partial_t]$

$$E = M_{\mathrm{BH}} + E_{\mathrm{int}} + E_{\mathrm{AdS}} + E_{\mathrm{s}}$$

$M = \frac{3k\pi^2}{\ell}\,\mu^2$	BH mass	$E_{\mathrm{AdS}} = -\frac{3k\pi^2}{2\ell}$	AdS vacuum
$E_{\mathrm{int}} = -\frac{3k\pi^2}{2\ell}\,\mu C^2$	interaction	$E_{\mathrm{s}} = \frac{3k\pi^2}{\ell}\,C^2$	soliton vacuum

- **Torsionless limit** $E_{C\to 0} = \frac{3k\pi^2}{2\ell}\,\left(\mu^2 - 1\right)$ gives the correct result obtained from BH thermodynamics [Crisostomo, Troncoso, Zanelli '00]
- **BPS limit** $E_{\mathrm{BPS}} = 0$ (the smallest black hole, similar to the BTZ black hole)
- **Abelian charges**

$$Q_c = 0\,, \quad Q_q = -\pi^2\left(3Cp^2 + n_1 + n_2 + B - C\right), \quad Q_q^{\mathrm{BPS}} = 0$$

- $U(1)_q$ field is locally vanishing, but because it interacts with both the geometry and the internal symmetry, it leads to non-trivial global effects.

Conclusions

Similarities with standard supergravities

- BPS states are extremal states
- The space of extremal solutions is larger than the space of BPS solutions – namely, the state $B = -C$ is extremal, but it is not a BPS state.
- The energy spectrum is continuous, except in the BPS limit where it is discrete. (Superstring theory: the black p-brane charges are associated with the number of coincident D-branes.)
- The Ξ-charged solution resembles the case of the Reissner-Nordström black holes in $\mathcal{N} = 2$ supergravity where the $U(1)_q$ charge is a central extension in the superalgebra.

Conclusions

Additional results

- We compure Ξ-carged $\frac{1}{16}$-BPS states and Ξ-neutral $\frac{1}{2}$-BPS states
- The Ξ-neutral solutions with $\mu = C^2$ have fully discrete energy spectrum
- We compare $\mathbb{S}^3$ and $\mathbb{RP}^3$ topology and the results are similar, only the charges on $\mathbb{S}^3$ have the reflection symmetry under $B \leftrightarrow -B$, $C \leftrightarrow -C$
- We find inequivalent solution $(Q_c \neq 0)$ where the $SU(2)_D$ internal soliton is replaced by the right $SU(2)_+$ soliton, where the left one identically vanishes.
- **The CS theory discussed here is topological (with zero d.o.f.)**

Conclusions

Open questions

- Non-static $SU(2)_+ \times SU(2)_-$ solution, possibly (anti) self-dual
- Role of the topological invariant $\int_{\mathcal{M}} \langle (g^{-1}\mathrm{d}g)^5 \rangle$.
- Computing the charge algebra and the central charge
- Finding the Bogomol'nyi bound using $\{\mathbf{Q}_s^{\alpha}, (\mathbf{Q}^{\dagger})_{\beta}^{u}\} \geq 0$
- Black hole entropy and black hole thermodynamics

$\cdots$ etc.

THANK YOU!

Acknowledgments

Acknowledgments:

FONDECYT grant N°1190533 & ANID-SCIA- ANILLO ACT210100

Holography and its applications to High Energy Physics, Quantum Gravity and Condensed Matter Systems

Anillo de Investigación en Ciencia y/o Tecnología
ACT-210100