Conformal Renormalization of AdS gravity

YOUNGST@RS Supergravity and Holography

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Dec 15th, 2021

Holography and its applications to High Energy Physics, Quantum Gravity and Condensed Matter Systems

Anillo de Investigación en Ciencia y/o Tecnología
ACT-210100

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- **Conformal Renormalization**

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$$g_{ij}(x,\rho) = g_{(0)ij}(x) + z^2 g_{(2)ij}(x) + z^4 g_{(4)ij}(x) + \cdots .$$

- $g_{(0)ij}$ **is the boundary data for the holographic reconstruction of the spacetime, i.e., solving** $g_{(k)}$ **as a covariant funtional of** $g_{(0)}$

Counterterm method

- **Renormalized AdS gravity action** (Holographic Renormalization)
  [Henningson, Skenderis JHEP 9807:023(1998)]

$$I_{ren} = \frac{1}{16\pi G}\int\limits_{M} d^{d+1}x\,\sqrt{-\hat{g}}\,(R-2\Lambda) - \frac{1}{8\pi G}\int\limits_{\partial M} d^{d}x\,\sqrt{-h}\,K + \\ + \int\limits_{\partial M} d^{d}x\,\mathcal{L}_{ct}(h,\mathcal{R},\nabla\mathcal{R})$$

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- **Holographic stress tensor** $T^{ij}[g_{(0)}] = \lim_{z\to 0}\left(\frac{1}{z^{d-2}}T^{ij}[h]\right)$.
  Contains the holographic information of the theory (e.g., Weyl anomaly)

Counterterms in AdS gravity

- $$8\pi G\,\mathcal{L}_{ct} = \frac{d-1}{\ell}\sqrt{-h} + \frac{\ell\sqrt{-h}}{2(d-2)}\mathcal{R} + \frac{\ell^3\sqrt{-h}}{2(d-2)^2(d-4)}\left(\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{d}{4(d-1)}\mathcal{R}^2\right)$$
$$+\frac{\ell^5\sqrt{-h}}{(d-2)^3(d-4)(d-6)}\left(\frac{3d-2}{4(d-1)}\mathcal{R}\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{d(d+2)}{16(d-1)^2}\mathcal{R}^3\right.$$
$$\left.-2\mathcal{R}^{ij}\mathcal{R}^{kl}\mathcal{R}_{ijkl} - \frac{d}{4(d-1)}\nabla_i\mathcal{R}\nabla^i\mathcal{R} + \nabla^k\mathcal{R}^{ij}\nabla_k\mathcal{R}_{ij}\right) + \ldots$$

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$$T^{ij} = \frac{1}{8\pi G}\left(K^{ij} - h^{ij}K\right) + \frac{1}{8\pi G}\left(\frac{d-1}{\ell}h^{ij} - \frac{\ell}{(d-2)}\left(\mathcal{R}^{ij} - \frac{1}{2}\mathcal{R}h^{ij}\right) + \ldots\right)$$

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- No general form of $\mathcal{L}_{ct}$ for arbitrary dimensions.
- Far more complicated in higher-curvature gravity theories.

Boundary conditions in AdS gravity

- Dirichlet boundary condition $\delta h_{ij} = 0$ does not make sense in AAdS spaces

  [I.Papadimitriou and K.Skenderis, hep-th/0404176]

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- Finite variation of the action and expressed in terms of $\delta g_{(0)ij}$.

Conformal Mass in AdS gravity

- Ashtekar-Magnon-Das charges

$$Q_{AMD}[\xi] = \frac{1}{8\pi G}\frac{\ell}{D-3}\int\limits_{\Sigma} dS_i\, \mathcal{E}^i_j\, \xi^j \qquad \mathcal{E}^i_j = W^{i\beta}{}_{j\nu}\, n_\beta n^\nu = W^{iz}{}_{jz}$$

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- **In** $d+1=5$, **one can prove that**

$$T^i_j = \mathcal{E}^i_j + \Delta^i_j\,, \quad \Delta^i_i = z^4 \mathcal{A}$$

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- Are counterterms the truncation of boundary terms nonlinear in K?

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- Kounterterms = counterterms of unusual sort (depend on K_{ij} and $\mathcal{R}_{ij}^{kl}(h)$)
- $D = 2n$ dimensions [R.O., hep-th/0504233]

$$B_{2n-1} = 2n\sqrt{-h} \int\limits_0^1 dt \, \delta_{[j_1 \cdots j_{2n-1}]}^{[i_1 \cdots i_{2n-1}]} K_{i_1}^{j_1} \left(\frac{1}{2} \mathcal{R}_{i_2 i_3}^{j_2 j_3} - t^2 K_{i_2}^{j_2} K_{i_3}^{j_3} \right) \times \cdots$$
$$\cdots \times \left(\frac{1}{2} \mathcal{R}_{i_{2n-2} i_{2n-1}}^{j_{2n-2} j_{2n-1}} - t^2 K_{i_{2n-2}}^{j_{2n-2}} K_{i_{2n-1}}^{j_{2n-1}} \right)$$

Kounterterms

- Kounterterms in $D = 2n + 1$ [R.O., hep-th/0610230]

$$B_{2n} = 2n \int_0^1 dt \int_0^t ds\, \delta^{[j_1 \cdots j_{2n}]}_{[i_1 \cdots i_{2n}]} K^{i_1}_{j_1} \delta^{i_2}_{j_2} \left(\frac{1}{2} \mathcal{R}^{i_3 i_4}_{j_3 j_4} - t^2 K^{i_3}_{j_3} K^{i_4}_{j_4} + \frac{s^2}{\ell^2} \delta^{i_3}_{j_3} \delta^{i_4}_{j_4} \right) \times \cdots$$

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- **Global issues (topology)**

$$\int_{M_{2n}} (\mathrm{Euler})_{2n} = (4\pi)^n n! \chi(M_{2n}) + \int_{\partial M_{2n}} B_{2n-1}$$

Kounterterms

- $D = 2n + 1$ dimensions

$$
\begin{aligned}
L^{TF}_{2n+1}(A, \bar{A}) &= (n+1) \int_0^1 dt \, \mathrm{tr}\left[(A - \bar{A}) \, F_t^n \right] \\
F_t &= dA_t + A_t^2 \,, \quad A_t = tA + (1-t)\bar{A}
\end{aligned}
$$

Kounterterms

- $D = 2n+1$ dimensions

$$L_{2n+1}^{TF}(A,\bar{A}) = (n+1)\int_0^1 dt\,\mathrm{tr}\left[(A-\bar{A})\,F_t^n\right]$$

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- Gauge-invariant version of Chern-Simons forms

$$L_{2n+1}^{TF}(A,\bar{A}) = L_{2n+1}^{CS}(A) - L_{2n+1}^{CS}(\bar{A}) + d\beta_{2n}(A,\bar{A})$$

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- *Contact* term

$$\begin{aligned} \beta_{2n}(A,\bar{A}) &= \int_0^1 dt \int_0^t ds\,\mathrm{tr}\left[A_t\,(A-\bar{A})\,F_{st}^{n-1}\right] \\ F_{st} &= sF_t + s(s-1)A_t^2 \end{aligned}$$

Renormalized Action = Renormalized Volume

- Black Hole Thermodynamics $G = TI^{E} = U - TS$

$$
\begin{aligned}
ds^2 &= f^2(r)d\tau^2 + \frac{dr^2}{f^2(r)} + r^2 d\Omega_{D-2}^2 \\
f^2(r) &= 1 - \frac{2\omega_D GM}{r^{D-3}} + \frac{r^2}{\ell^2}
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$$TI_{bulk}^E = \frac{(D-3)}{(D-2)}M - TS + \lim_{r\to\infty} \frac{V(S^{D-2})}{8\pi G}\frac{r^{D-1}}{\ell^2}$$

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$$

- Euclidean Kounterterms

$$
T\,c_d \int\limits_{\partial M} B_d = \frac{M}{(D-2)} + E_0 - \lim_{r\to\infty} \frac{V(S^{D-2})}{8\pi G}\frac{r^{D-1}}{\ell^2}
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- Correct Black Hole Thermo with $U = M + E_0$

From extrinsic to intrinsic renormalization in 4D

- AdS gravity action + KTs

$$I = I_{EH} + \frac{\ell^2}{16\pi G} \int\limits_{\partial M} d^3x \sqrt{-h}\, \delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K^{j_1}_{i_1} \left(\frac{1}{2} \mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3} K^{j_2}_{i_2} K^{j_3}_{i_3} \right).$$

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- Adding zero...

$$\tilde{I}_{\text{ren}} = I_{EH} - \frac{1}{8\pi G} \int\limits_{\partial M} d^3x \sqrt{-h}\, K + \int\limits_{\partial M} d^3x\, \mathcal{L}_{ct}.$$

$$\mathcal{L}_{ct} = \frac{\ell^2}{16\pi G} \sqrt{-h} \delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K^{j_1}_{i_1} \left(\frac{1}{2} \mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3} K^{j_2}_{i_2} K^{j_3}_{i_3} + \frac{1}{\ell^2} \delta^{j_2}_{i_2} \delta^{j_3}_{i_3} \right).$$

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- And expanding...

$$K^i_j = \frac{1}{\ell} \delta^i_j - \ell S^i_j(h) + \mathcal{O}(\mathcal{R}^2)$$

$$S^i_j(h) = \frac{1}{d-2} (\mathcal{R}^i_j(h) - \frac{1}{2(d-1)} \delta^i_j \mathcal{R}(h))$$

From extrinsic to intrinsic renormalization

- $$\mathcal{L}_{ct} = \frac{\ell^2}{16\pi G}\frac{\sqrt{-g}}{z^3}\delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]}\left(\frac{\delta^{j_1}_{i_1}}{\ell} - \ell S^{i_1}_{j_1}\right)\times$$
$$\times\left(\frac{1}{2}\mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3}\left(\frac{\delta^{j_2}_{i_2}}{\ell} - \ell S^{i_2}_{j_2}\right)\left(\frac{\delta^{j_3}_{i_3}}{\ell} - \ell S^{i_3}_{j_3}\right) + \frac{1}{\ell^2}\delta^{j_2}_{i_2}\delta^{j_3}_{i_3}\right) + \ldots$$

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- Kounterterms turn into *counterterms*
  [O.Miskovic and R.O., 0902.2082]

$$\mathcal{L}_{ct} = \frac{1}{8\pi G}\frac{\sqrt{-g}}{z^3}\left(\frac{2}{\ell} + \frac{\ell}{2}z^2\mathcal{R}(g)\right) + \mathcal{O}(z)$$
$$= \frac{1}{8\pi G}\sqrt{-h}\left(\frac{2}{\ell} + \frac{\ell}{2}\mathcal{R}(h)\right).$$

From extrinsic to intrinsic renormalization

- $\tilde{I}_{\text{ren}} = I_{EH} + c_{2n-1} \int\limits_{\partial M} d^{2n-1}x B_{2n-1}$

$$B_{2n-1} = 2n\sqrt{-h} \int\limits_0^1 dt\ \delta^{[i_1 \cdots i_{2n-1}]}_{[j_1 \cdots j_{2n-1}]} K^{j_1}_{i_1} \left(\frac{1}{2} \mathcal{R}^{j_2 j_3}_{i_2 i_3} - t^2 K^{j_2}_{i_2} K^{j_3}_{i_3} \right) \times \cdots$$
$$\cdots \times \left(\frac{1}{2} \mathcal{R}^{j_{2n-2} j_{2n-1}}_{i_{2n-2} i_{2n-1}} - t^2 K^{j_{2n-2}}_{i_{2n-2}} K^{j_{2n-1}}_{i_{2n-1}} \right).$$

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$$\mathcal{L}_{ct} = c_{2n-1} B_{2n-1} + \frac{1}{8\pi G}\sqrt{-h} K$$
$$= \frac{(-\ell^2)^n}{8\pi G (2n-2)!}\sqrt{-h}\, \delta^{[i_1\cdots i_{2n-1}]}_{[j_1\cdots j_{2n-1}]} K^{j_1}_{i_1} \int\limits_0^1 dt \left[\left(\frac{1}{2}\mathcal{R}^{j_2 j_3}_{i_2 i_3} - t^2 K^{j_2}_{i_2} K^{j_3}_{i_3} \right) \times \cdots \right.$$
$$\left. \cdots \times \left(\frac{1}{2}\mathcal{R}^{j_{2n-2} j_{2n-1}}_{i_{2n-2} i_{2n-1}} - t^2 K^{j_{2n-2}}_{i_{2n-2}} K^{j_{2n-1}}_{i_{2n-1}} \right) + \frac{(-1)^n}{\ell^{2n-2}} \delta^{j_2}_{i_2} \cdots \delta^{j_{2n-1}}_{i_{2n-1}} \right].$$

From extrinsic to intrinsic renormalization

- And expanding...

$$\mathcal{L}_{ct} = \frac{\sqrt{-h}}{8\pi G}\left[\frac{(2n-2)}{\ell} + \frac{\ell}{2(2n-3)}\mathcal{R} + \right.$$
$$\left. + \frac{\ell^3}{2(2n-3)^2(2n-5)}\left(2\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{(2n+1)}{4(2n-2)}\mathcal{R}^2 - \frac{(2n-3)}{4}\mathcal{R}^{ijkl}\mathcal{R}_{ijkl}\right) + \cdots\right]$$

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- Boundary Weyl tensor $\mathcal{W}^{ijkl}\mathcal{W}_{ijkl}$ implies

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- And, finally

$$\mathcal{L}_{ct} = \frac{\sqrt{-h}}{8\pi G}\left[\frac{(2n-2)}{\ell} + \frac{\ell}{2(2n-3)}\mathcal{R} + \right.$$
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Holographic Renormalization = Kounterterms?

- Well...almost. [G.Anastasiou, O.Miskovic, R.O. and I.Papadimitriou, 2003.06425]

$$\tilde{I}_{\text{ren}} = I_{HR} - \frac{\ell^3}{64\pi G(2n-3)(2n-5)} \int\limits_{\partial M} \sqrt{-h}\mathcal{W}^{ijkl}\mathcal{W}_{ijkl} + ...$$

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- Patching up the theory

$$I_{HR} = \tilde{I}_{\text{ren}} + \frac{\ell^3}{64\pi G(2n-3)(2n-5)} \int_{\partial M} \sqrt{-h}\mathcal{W}^{ijkl}\mathcal{W}_{ijkl} + \ldots$$

Renormalized AdS Action

- **Euler-Gauss-Bonnet Theorem in 4D**

$$\int_{\partial M} d^3x \, B_3(K, \mathcal{R}) = \int_{M} d^4x \, GB - 32\pi^2 \chi(M)$$

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[R. Aros et al, gr-qc/9909015]

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$$I_{\text{ren}} = \frac{\ell^2}{256\pi G} \int_M d^4x \sqrt{-\hat{g}}\, \delta^{[\nu_1 \cdots \nu_4]}_{[\mu_1 \cdots \mu_4]} \left[R^{\mu_1\mu_2}_{\nu_1\nu_2} + \frac{\delta^{\mu_1\mu_2}_{\nu_1\nu_2}}{\ell^2} \right] \left[R^{\mu_3\mu_4}_{\nu_3\nu_4} + \frac{\delta^{\mu_3\mu_4}_{\nu_3\nu_4}}{\ell^2} \right],$$

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Renormalized AdS Action

- **Weyl tensor**

$$W^{\alpha\beta}_{\mu\nu} = R^{\alpha\beta}_{\mu\nu} - S^{[a}_{[\mu}\delta^{\beta]}_{\nu]},$$

Schouten tensor $S^{\alpha}_{\mu} = \frac{1}{D-2}(R^{\alpha}_{\mu} - \frac{1}{2(D-1)}\delta^{\alpha}_{\mu}R)$

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Renormalized AdS Action

- **Weyl tensor**

$$W^{\alpha\beta}_{\mu\nu} = R^{\alpha\beta}_{\mu\nu} - S^{[a}_{[\mu}\delta^{\beta]}_{\nu]},$$

Schouten tensor $S^{\alpha}_{\mu} = \frac{1}{D-2}(R^{\alpha}_{\mu} - \frac{1}{2(D-1)}\delta^{\alpha}_{\mu}R)$

- **Weyl tensor for Einstein spaces** $R_{\mu\nu} = -\frac{D-1}{\ell^2}g_{\mu\nu}$

$$W^{\alpha\beta}_{(E)\mu\nu} = R^{\alpha\beta}_{\mu\nu} + \frac{1}{\ell^2}\delta^{[\alpha\beta]}_{[\mu\nu]}$$

- **Renormalized action for Einstein spaces**

$$I_{\text{ren}} = \frac{\ell^2}{64\pi G}\int_M d^4x\,\sqrt{-\hat{g}}\,W_{(E)\mu\nu\alpha\beta}W^{\mu\nu\alpha\beta}_{(E)}$$

- **Thermodynamic of global AdS**

$$I^{E}_{\text{ren}} = 0\,, \qquad Q^{\alpha}_{\text{Wald}} = 0\,.$$

Conformal Renormalization

- **Embedding Einstein theory in Conformal Gravity**
  (invariant under rescalings of the metric)

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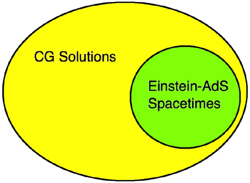

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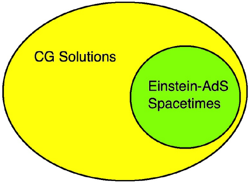

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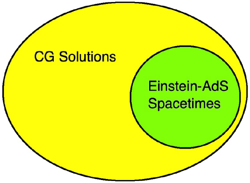

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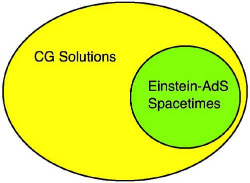

- **Why?: Conformal Gravity is finite for generic AAdS conditions.**
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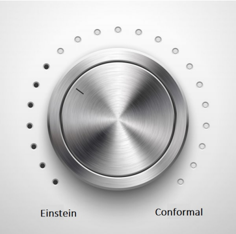

- **Einstein gravity from CG with Neumann bc's**
  [J. Maldacena, arXiv:1105.5632]

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- **Fefferman-Graham expansion for AAdS spaces in CG**

$$ds^2 = \frac{\ell^2}{z^2}\, dz^{2S} + \frac{1}{z^2}\, g_{ij}(x,z)\, dx^i dx^j$$

$$\begin{aligned} g_{ij}(x,\rho) \quad &= \quad g_{(0)ij}(x) + z^2 g_{(2)ij}(x) + \cdots \\ &\quad + z g_{(1)ij}(x) + \cdots \end{aligned}$$

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$$\begin{aligned} B_{\mu\nu} &= \nabla^{\lambda} C_{\mu\nu\lambda} + S^{\lambda\sigma} W_{\lambda\mu\sigma\nu} = 0\,, \\ C^{\mu}_{\ \nu\lambda} &= \nabla_{\nu} S^{\mu}_{\lambda} - \nabla_{\lambda} S^{\mu}_{\nu} \end{aligned}$$

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- $g_{(2)ij}$ is free data in CG $\Longrightarrow$chosen as in Einstein $g_{(2)ij} = -\ell^2 S_{(0)ij}$ [Imbimbo, Schwimmer, Theisen and Yankielowicz, hep-th/9910267]

AdS gravity in 6D

- **EH Action+Euler term**

$$\tilde{I}_{\text{ren}} = \frac{1}{16\pi G}\int\limits_M d^6x\sqrt{-\hat{g}}\left(R + \frac{20}{\ell^2} - \frac{\ell^4}{72}(Euler)_6\right) \,,$$

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$$\begin{aligned}\tilde{I}_{\text{ren}} \quad = \quad & \frac{1}{16\pi G \times 192} \int\limits_M d^6x \sqrt{-\hat{g}} \delta^{[\nu_1 \cdots \nu_6]}_{[\mu_1 \cdots \mu_6]} \left[R^{\mu_1 \mu_2}_{\nu_1 \nu_2} \delta^{[\mu_3 \mu_4]}_{[\nu_3 \nu_4]} \delta^{[\mu_5 \mu_6]}_{[\nu_5 \nu_6]} \right. \\ & \left. + \frac{2}{3\ell^2} \delta^{[\mu_1 \mu_2]}_{[\nu_1 \nu_2]} \delta^{[\mu_3 \mu_4]}_{[\nu_3 \nu_4]} \delta^{[\mu_5 \mu_6]}_{[\nu_5 \nu_6]} - \frac{\ell^4}{3} R^{\mu_1 \mu_2}_{\nu_1 \nu_2} R^{\mu_3 \mu_4}_{\nu_3 \nu_4} R^{\mu_5 \mu_6}_{\nu_5 \nu_6} \right] ,\end{aligned}$$

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- Polynomial of $W_{(E)}$

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Conformal Gravity in 6D

- **There are three Conformal Invariants in 6D**

$$
\begin{aligned}
I_1 &= W_{\alpha\beta\mu\nu} W^{\alpha\sigma\lambda\nu} W_{\sigma}{}^{\beta\mu}{}_{\lambda} \,, \\
I_2 &= W_{\mu\nu\alpha\beta} W^{\alpha\beta\sigma\lambda} W_{\sigma\lambda}{}^{\mu\nu} \,, \\
I_3 &= W_{\mu\rho\sigma\lambda} \left(\delta^{\mu}_{\nu} \Box + 4 R^{\mu}_{\nu} - \frac{6}{5} R \delta^{\mu}_{\nu} \right) W^{\nu\rho\sigma\lambda} + \nabla_{\mu} J^{\mu} \,,
\end{aligned}
$$

with

$$
\begin{aligned}
J_{\mu} \quad = \quad & 4 R_{\mu}{}^{\lambda\rho\sigma} \nabla^{\nu} R_{\nu\lambda\rho\sigma} + 3 R^{\nu\lambda\rho\sigma} \nabla_{\mu} R_{\nu\lambda\rho\sigma} - 5 R^{\nu\lambda} \nabla_{\mu} R_{\nu\lambda} \\
& + \frac{1}{2} R \nabla_{\mu} R - R^{\nu}_{\mu} \nabla_{\nu} R + 2 R^{\nu\lambda} \nabla_{\nu} R_{\lambda\mu} .
\end{aligned}
$$

Lu-Pang-Pope CG in 6D

- **6D CG with an Einstein sector [Lu, Pang and Pope, 2013]**

$$I_{CG} = \alpha_{CG} \int\limits_M d^6x \sqrt{-\hat{g}} \left(\frac{1}{4!} \delta^{[\nu_1 \cdots \nu_6]}_{[\mu_1 \cdots \mu_6]} W^{\mu_1 \mu_2}_{\nu_1 \nu_2} W^{\mu_3 \mu_4}_{\nu_3 \nu_4} W^{\mu_5 \mu_6}_{\nu_5 \nu_6} + \frac{1}{2} \delta^{[\nu_1 \cdots \nu_5]}_{[\mu_1 \cdots \mu_5]} W^{\mu_1 \mu_2}_{\nu_1 \nu_2} W^{\mu_3 \mu_4}_{\nu_3 \nu_4} S^{\mu_5}_{\nu_5} \right.$$

$$\left. + 8 C^{\mu\nu\lambda} C_{\mu\nu\lambda} \right) + \alpha_{CG} \int\limits_{\partial M} d^5x \sqrt{-h} n^\mu \left(8 W^{\kappa\lambda\nu}_{\mu} C_{\kappa\lambda\nu} - W^{\kappa\lambda}_{\nu\sigma} \nabla_\mu W^{\nu\sigma}_{\kappa\lambda} \right).$$

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- **LPP action appears as type-B anomaly and one-loop divergences in 7D**
- **Variation of I_{CG} gives EOM in terms of Weyl, Cotton and Schouten tensors.**
- **Any Einstein-AdS spacetime is solution of LPP CG.** [Anastasiou, Araya and RO, 2010.15146]

Einstein gravity from Conformal Gravity in 6D

- **LPP CG action decomposed into Einstein and non-Einstein parts:**

$$
\begin{aligned}
I_{CG} \quad = \quad & -4!\alpha_{CG} \int\limits_{M} d^6x \sqrt{-\hat{g}} \left[P_6 \left(W_{(E)} \right) + Q \left(W_{(E)}, H \right) \right] \\
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- Hint: powers of Rie must enter into topological term.
- Einstein condition, and $\alpha_{\mathrm{E}} = -\frac{\ell^4}{384\pi G}$:

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I_{CG}[E] &= \frac{1}{16\pi G} \int\limits_{M} d^6x \sqrt{-\hat{g}} \left(R + \frac{20}{\ell^2} - \frac{\ell^4}{72} (Euler)_6 \right) \\
&\quad + \frac{\ell^4}{384\pi G} \int\limits_{\partial M} d^5x \sqrt{-h} n^\mu \left(W^{\kappa\lambda}_{(E)\nu\sigma} \nabla_\mu W^{\nu\sigma}_{(E)\kappa\lambda} \right),
\end{aligned}
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Einstein sector of LPP Conformal Gravity in 6D

- **Extra boundary term**

$$\Delta I = \frac{\ell^4}{384\pi G}\int_{\partial M} d^5x\sqrt{-h}n^{\mu}\left(W^{\kappa\lambda}_{(E)\nu\sigma}\nabla_{\mu}W^{\nu\sigma}_{(E)\kappa\lambda}\right)$$

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- In terms of the electric part of Weyl tensor $\mathcal{E}^i_j = W^{iz}_{jz}$ and the boundary Weyl

$$W^{\kappa\lambda}_{(E)\nu\sigma}W^{\nu\sigma}_{(E)\kappa\lambda} = 4\mathcal{E}^i_j\mathcal{E}^j_i + z^4\mathcal{W}^{ijkl}(g)\mathcal{W}_{ijkl}(g) + \mathcal{O}(z^6)$$

Einstein sector of LPP Conformal Gravity in 6D

- $$\begin{aligned} n^\mu &= \frac{z}{\ell}\delta^\mu_z, \quad \text{normal vector} \\ \sqrt{-h} &= \sqrt{-g_{(0)}}/z^5 + \ldots \\ \mathcal{E}^i_j &\sim \mathcal{O}(z^5)\,. \end{aligned}$$

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\end{aligned}
$$

- Extra boundary term

$$
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\Delta I &= \frac{4\ell^{3}}{2\times 384\pi G}\int\limits_{\partial M} d^{5}x\frac{\sqrt{-g}}{z^{5}}z^{4}\mathcal{W}^{ijkl}(g)\mathcal{W}_{ijkl}(g) \, , \\
&= \frac{\ell^{3}}{192\pi G}\int\limits_{\partial M} d^{5}x\sqrt{-h}\mathcal{W}^{ijkl}(h)\mathcal{W}_{ijkl}(h) \, .
\end{aligned}
$$

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- $$I_{CG}\left[E\right] = I_{HR}$$

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- Computation of HEE (codim-2 CI's) (with M.Taylor)