Unconventional supersymmetry and AdS_4 Supergravity

...from SuGra to GraPhene?

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Outline

AVZ CS model

Unconventional SuSy in 3D CS:

- CS theory of the supergroup $OSp(2/2) \supset SO(1,2) \times SO(2)$:
  Gauge connection: $\mathbb{A} = \frac{1}{2}\omega_{ij}\mathbb{J}^{ij} + A\,\mathbb{K} + \bar{\mathbb{Q}}_I\psi_I \in \mathfrak{osp}(2/2)$,

$$\mathcal{S}^{CS} = \int_{\mathcal{M}_{(3)}} \mathcal{L}^{CS}, \qquad \mathcal{L}^{CS} = -STr(\mathbb{A}d\mathbb{A} + \frac{2}{3}\mathbb{A}^3)$$

- with the condition: $\psi_{\mu\ \alpha I} = \mathrm{i}(\gamma_i)_{\alpha\beta}\chi_{\beta I}\, e^i_\mu$
  dreibein of base space $\mathcal{M}_{(3)}$ $\hookleftarrow$
  $\delta_{\bar{\mathbb{Q}}\epsilon} e^i_\mu = 0$

[AVZ = P. D. Alvarez, M. Valenzuela and J. Zanelli, “Supersymmetry of a different kind,” JHEP **1204**, 058 (2012).]

AVZ field equations

$$\frac{\delta \mathcal{L}^{CS}}{\delta \omega^{ij}} = 0 \quad : \quad R^{ij} = -\mathrm{i}\bar{\psi}_I \gamma^{ij} \psi_I|_\chi = -\mathrm{i}\bar{\chi}\chi e^j e^k \epsilon^i{}_{jk}$$

$$\frac{\delta \mathcal{L}^{CS}}{\delta A} = 0 \quad : \quad dA = \epsilon_{IJ}\bar{\psi}_I \psi_J|_\chi = \bar{\chi}\gamma^i \chi e^j e^k \epsilon_{ijk}$$

$$\frac{\delta \mathcal{L}^{CS}}{\delta \chi} = 0 \quad : \quad \boxed{-\mathrm{i}\gamma^i \nabla_i \chi = \frac{1}{2}\kappa\,\chi} \qquad \text{with } \kappa \equiv e_i \mathcal{D} e^i \text{ (contorsion)}$$

where $\chi_\alpha = \chi_{\alpha\,1} + \mathrm{i}\chi_{\alpha\,2} \in \mathbb{C}$, $\nabla\chi_I \equiv \mathcal{D}\chi_I + \frac{1}{2}A\chi_J\epsilon_{IJ}$,
$\mathcal{D}$: Lorentz covariant derivative.

Unconventional supersymmetry

(We named $\psi \leftrightarrow \chi$ with respect to original AVZ ...sorry for this)

AVZ model describes a SuSy invariant theory, with spin-$\frac{1}{2}$ field χ as the only propagating d.o.f.

χ massive for $\mathcal{D}e^i \neq 0 \;\Leftrightarrow\; AdS_3$ background.

It finds application in (2+1)D cond-matt models such as graphene-like systems.

It is a fascinating model: Could it be found from standard supergravity in higher dimensions?

AdS_4 SuGra with non trivial $\partial\mathcal{M}_4$

Consider the Lagrangian of pure 4D SUGRA, with FI term $\frac{1}{\ell}$, in $N = 2$ superspace (at first-order for the spin connection ω^{ab}):

$$\mathcal{L}_{\text{bulk}} = \frac{1}{4}\mathcal{R}^{ab}V^cV^d\epsilon_{abcd} + \bar{\Psi}^I\Gamma_a\Gamma_5\rho_I V^a + \frac{\mathrm{i}}{2}dA\,\bar{\Psi}^I\Gamma_5\Psi^J\epsilon_{IJ} + \\ -\frac{\mathrm{i}}{2\ell}\bar{\Psi}^I\Gamma_{ab}\Gamma_5\Psi_I V^aV^b - \frac{1}{8\ell^2}V^aV^bV^cV^d\epsilon_{abcd} + \\ +\frac{1}{4}\left[\tilde{F}^{cd}V^aV^b\left(dA - \frac{1}{2}\bar{\psi}^I\psi^J\epsilon_{IJ}\right) - \frac{1}{12}\tilde{F}_{lm}\tilde{F}^{lm}V^aV^bV^cV^d\right]\epsilon_{abcd}\,,$$

where (V^a, Ψ_I) is the $N = 2$ supervielbein, $\mathcal{R}^{ab} = d\omega^{ab} + \omega^a{}_c\omega^{cb}$, $\rho_I = \mathcal{D}\Psi_I + \frac{1}{\ell}A\epsilon_{IJ}\Psi_J$, $\Lambda = -\frac{1}{\ell^2}$ is the AdS$_4$ cosmological constant.

AdS_4 SuGra with non trivial $\partial\mathcal{M}_4$

For non trivial boundary conditions, $\delta\mathcal{L}_{\text{bulk}} \neq 0$.

$\Rightarrow$ Symmetries are restored by adding a boundary term $\mathcal{L}_{bdy} = d\mathcal{B}^{(3)}$ [York (1972); Gibbons-Hawking (1977]

A background independent way to do: by adding as topological terms: Gauss-Bonnet+ SuSy partners

[Aros-Contreras-Olea-Troncoso-Zanelli (1999); Olea (2005)]; [L. A.-D'Auria, (2014)]

$$\mathcal{L}_{bdy} = -\frac{\ell^2}{8}\mathcal{R}^{ab} \wedge \mathcal{R}^{cd}\epsilon_{abcd} + 2\mathrm{i}\ell\bar{\rho}^I\rho_I - \frac{1}{2}\theta dA \wedge dA + \\ + \frac{\mathrm{i}\ell}{8}\mathcal{R}^{ab}\bar{\Psi}^I\Gamma_{ab}\Psi_I + \frac{\mathrm{i}\ell}{2}dA\bar{\Psi}^I\Psi^J\epsilon_{IJ}$$

The invariant lagrangian is $\mathcal{L} = \mathcal{L}_{\text{bulk}} + \mathcal{L}_{\text{bdy}}$.

Conditions for SuSy invariance

SuSy invariance $\delta\mathcal{L} = 0$ requires the vanishing on $\partial\mathcal{M}_4$ of the $OSp(2|4)$ supercurvatures

$$\mathbb{R}^{ab} \equiv \mathcal{R}^{ab} - \left[\frac{1}{\ell^2} V^a \wedge V^b + \frac{1}{2\ell} \bar{\Psi}_I \Gamma^{ab} \wedge \Psi_I\right] ,$$

$$\mathbb{R}^{a} \equiv \mathcal{D}^{(4)} V^a - \frac{i}{2} \bar{\Psi}_I \Gamma^a \wedge \Psi_I ,$$

$$\mathbb{F} \equiv dA^{(4)} - \bar{\Psi}_I \wedge \Psi_J \epsilon_{IJ} ,$$

$$\mathbb{D}\Psi_I \equiv \mathcal{D}\Psi_I + \frac{1}{\ell} A \epsilon_{IJ} \Psi_J - \frac{\mathrm{i}}{2\ell} \Gamma_a \Psi_I \wedge V^a ,$$

$$\delta\mathcal{L} = 0 \Leftrightarrow \mathbb{R}^{ab}|_{\partial\mathcal{M}_4} = 0,\ \mathbb{R}^{a}|_{\partial\mathcal{M}_4} = 0,\ \mathbb{F}|_{\partial\mathcal{M}_4} = 0,\ \mathbb{D}\Psi|_{\partial\mathcal{M}_4} = 0$$

Maurer-Cartan eq.s of $OSp(2|4)$ superalgebra

$D = 3$ description on asymptotic $\partial\mathcal{M}_4$

The explicit description on $\partial\mathcal{M}_4$ depends on choice of $\partial\mathcal{M}_4$.

- Consider a Fefferman-Graham decomposition, with $\partial\mathcal{M}_4$ at $r \to \infty$, and $a = (i, 3)$, $\hat{\mu} = (\mu, r)$; $i, \mu = 0, 1, 2$.
- We introduce the 1-forms

$$E^i_\pm \equiv \frac{1}{2}(V^i \mp \omega^{3i}), \; \Psi_{I\pm} \equiv \frac{1}{2}(\mathbb{I} \mp \mathrm{i}\Gamma_3)\Psi_I,$$

  covariant under $SO(1,2) \times SO(1,1) \subset SO(2,3)_{AdS_4}$
- On $\partial\mathcal{M}_4$: $V^3 = dV^3 = 0 \;\Rightarrow\; \bar{\Psi}_{I+}\Psi_{I-} = 0$ (half-SuSy)

The boundary conditions ($OSp(2|4)$ M-C eq.s on $\partial\mathcal{M}$) read

$$
\begin{aligned}
\mathcal{R}^{ij} &= \frac{4}{\ell^2}\, E_+^{[i} \wedge E_-^{j]} + \frac{1}{\ell}\, \bar{\Psi}_{I+}\Gamma^{ij}\Psi_{I-}\,, \\
dV^3 &= 2\, E_+^i \wedge E_{-i} + \bar{\Psi}_{I+}\Psi_{I-}\,, \\
\mathcal{D}E_\pm^i &= \frac{\mathrm{i}}{2}\, \bar{\Psi}_{I\pm}\Gamma^i\Psi_{I\pm} \pm \frac{1}{\ell}\, E_\pm^i \wedge V^3\,, \\
\mathcal{D}\Psi_{I\pm} &= -\frac{\mathrm{i}}{\ell}\, E_{\pm i} \wedge \Gamma^i\Psi_{I\mp} + \frac{1}{2\ell}\, \epsilon_{IJ}\, A^{(4)} \wedge \Psi_{\pm J} \pm \frac{1}{2\ell}\, \Psi_{\pm I} \wedge V^3\,, \\
dA^{(4)} &= 2\, \epsilon_{IJ}\bar{\Psi}_{I+} \wedge \Psi_{J-}
\end{aligned}
$$

AVZ from AdS_4 SUGRA

The AVZ model can be reproduced as asymptotic limit of pure AdS_4 SUGRA in the "ultraspinning limit" (inspired by AdS-Kerr BH, but here $M_{ADM} = 0$) [M.Caldarelli, R.Emparan, M.J.Rodriguez, JHEP 0811 (2008) 011]
for $\partial\mathcal{M}_4 = AdS_3$:

$$E^i_+(x,r) = \frac{r}{\ell}\left(E^i(x) + \cdots\right) , \quad E^i_-(x,r) = \frac{\ell}{r}\left(E^i(x) + \cdots\right) ,$$

$$V^3(r) = \frac{\ell}{r}(dr + \cdots), \; \omega^{ij}(x,r) = \omega^{ij}(x) + \cdots , \; A^{(4)}(x,r) = -A_\mu(x)\, dx^\mu + \cdots$$

$$\Psi_{+\,I\,\mu}(x,r) = \sqrt{\frac{r}{\ell}}\left((\psi_{I\,\mu}, \mathbf{0}) + \cdots\right), \; \Psi_{-\,I\,\mu}(x,r) = \sqrt{\frac{\ell}{r}}\left((\mathbf{0}, -\psi_{I\,\mu}) + \cdots\right).$$

the ellipses refer to $\mathcal{O}(\frac{\ell}{r})$.

[L.A., Cerchiai, D'Auria, Trigiante (2018)]

AVZ from AdS$_4$ SUGRA

Putting the "ultraspinning" ansatz in the boundary supercurvatures

$$OSp(2|4) \to SO(1,2)_{(+)} \times OSp(2/2)_{(-)} \supset SO(1,2)_{+} \times SO(1,2)_{-} \times SO(2)$$

$$OSp(2|4) \text{ M-C eq.s} \Rightarrow \begin{cases} \mathcal{R}^{ij} = \frac{1}{\ell^2} E^i E^j - \frac{1}{2\ell} \bar{\psi}_I \gamma^{ij} \psi_I \,, \\ \mathcal{D}E^i = \frac{\mathrm{i}}{2} \bar{\psi}_I \gamma^i \psi_I \,, \\ \mathcal{D}\psi_I = \frac{\mathrm{i}}{2\ell} E_i \gamma^i \psi_I - \frac{1}{2\ell} \epsilon_{IJ} A \psi_J \,, \\ dA = \epsilon_{IJ} \bar{\psi}_I \psi_J \,. \end{cases}$$

They can be obtained as field eq.s from A-T SUGRA

$$\mathcal{L}^{(3)}_{(AT)} = (\mathcal{R}^{ij} - \frac{1}{3\,\ell^2} E^i E^j + \frac{1}{2\,\ell} \bar{\psi}_I \gamma^{ij} \psi_I) E^k \epsilon_{ijk} + \frac{1}{2\,\ell} A dA + 2\bar{\psi}_I (\mathcal{D}\psi_I + \frac{1}{2\ell} \epsilon_{IJ} A \psi_J)$$

[A. Achucarro and P. K. Townsend (1986)].

AVZ from AdS_4 SUGRA

$$\mathcal{L}^{(3)}_{(AT)} + d\mathcal{B}^{(2)} = \mathcal{L}^{CS}_{\mathfrak{so}(1,2)_+ \times \mathfrak{osp}(2/2)_-} = \mathcal{L}^{CS}_{\mathfrak{so}(1,2)_+} - \mathcal{L}^{CS}_{\mathfrak{osp}(2/2)_-}$$

In $\mathcal{L}^{(3)}_{(AT)}$: $SO(1,2)_{Lorentz} = \mathrm{diag}[SO(1,2)_+ \times SO(1,2)_-]$
The full symmetry is manifest with torsionful connections:

$$\omega^{ij}_{(\pm)} = \omega^{ij} \mp \frac{1}{\ell} E_k \epsilon^{ijk} = \epsilon^{ijk}\,\omega_{k(\pm)}\,, \quad \text{giving}$$

$$\mathcal{R}^i_{(+)} = 0, \quad \mathcal{R}^i_{(-)} = -\frac{\mathrm{i}}{\ell}\,\bar{\psi}_I \gamma^i \psi_I, \quad \mathcal{D}_{(-)}\psi_I = -\frac{1}{2\ell}\,\epsilon_{IJ} A\psi_J, \quad dA = \epsilon_{IJ}\bar{\psi}_I\psi_J$$

Imposing the AVZ constraint $\psi_I = \mathrm{i}\, e_i \gamma^i \chi_I$

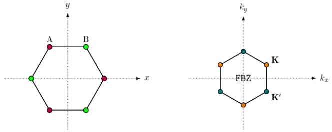
Massive AVZ model with spinor mass $\kappa = e^i \mathcal{D} e_i = -\frac{6}{\ell} \neq 0$

Several open directions of investigation:

1. as a corner of AdS_4/CMT duality, since AVZ describes graphene-like systems
2. directly in 3 dimensions, in the spirit of Rozansky-Witten/Kapustin-Saulina/Gaiotto-Witten (quantum BRST description)
3. ...

Graphene lattice and Dirac equation

Graphene is a 2D layer of carbon atoms, that form a bipartite lattice composed by two sublattices (sites A and sites B).



- The electronic wave function has spinorial nature

$$\varphi = (\sqrt{n_A} e^{i\alpha}, \sqrt{n_B} e^{i\beta}) \quad \text{Pseudospin}$$

- Close to the Fermi level: $E_k = \pm\hbar v_F |\vec{k}|, \quad v_F = c/300$

Shrödinger eq. $\Rightarrow$ Dirac eq. for Dirac spinor φ

In the AVZ model: $\varphi \propto \chi \equiv \chi_1 + i\chi_2$

Graphene from pure Sugra

In our description, the wave function φ of electrons in graphene-like systems near the Dirac points is identified as the radial component $\Psi_r \equiv \Psi_{-r\,1} + i\Psi_{-r\,2}$ of the N=2, D=4 gravitino at the boundary of AdS_4 SUGRA:

$$\Psi_r(r \to \infty) \propto \varphi \propto \chi = \left(\sqrt{n_A}e^{i\alpha}, \sqrt{n_B}e^{i\beta}\right)$$

Indeed, from $\Gamma^{\hat{\mu}}\Psi_{\hat{\mu}I} = \Gamma^r\Psi_{rI} + \Gamma^\mu\Psi_{\mu I} = 0$:

$$\chi = -\frac{\mathrm{i}}{3}\gamma^\mu\psi_\mu \propto \Psi_r$$

In our case $\Lambda \neq 0 \Rightarrow m_\chi = \kappa \neq 0$

- in graphene framework “analog speed of light”: $v_F = c/300$
  $\hookrightarrow$ Analog supergravity?

[L.A., Cerchiai, D’Auria, Trigiante (2018); L.A., Cerchiai, D’Auria, Gallerati, Noris, Trigiante, Zanelli (2019)]

Graphene from pure Sugra

General features, results and open directions

[L.A., Cerchiai, D'Auria, Trigiante (2018); L.A., Cerchiai, D'Auria, Gallerati, Noris, Trigiante, Zanelli (2019)]

- We investigated the general case in A-T family of CS SUGRAS $OSp(p/2)_{(+)} \times OSp(N-p)_{(-)}$, finding extra symmetries for $N = 2p$
- In the absence of torsional anomalies, $\bar{\chi}\chi = n_A - n_B$ is a constant index related to NYW invariance of AVZ ansatz:

$$e^i \to e^{\phi} e^i, \quad \chi \to e^{-\phi}\chi$$

  ... to further explore
- We developed a holographic framework for $N = 2$ AdS_4 SUGRA, including all fermionic contributions

  [L.A., Cerchiai, Matrecano, Miskovic, Noris, Olea, Ravera, Trigiante]

  It is ready to be applied to check if the corresponce AVZ/AdS_4 SUGRA fits in a holographic renormalization scheme ...in progress

Let us come back to AVZ CS model

Unconventional SuSy in 3D CS:

- CS theory of the supergroup $OSp(2/2)$ (topological): Gauge connection of $\mathfrak{osp}(2/2) \supset \mathfrak{so}(1,2) \times \mathfrak{so}(2)$:

$$\mathbb{A} = \frac{1}{2}\omega_{ij}\,\mathbb{J}^{ij} + A\,\mathbb{K} + \bar{\mathbb{Q}}_{\alpha I}\psi_{\alpha I},$$

$$\mathcal{S}^{CS} = \int_{\mathcal{M}_{(3)}} \mathcal{L}^{CS}, \qquad \mathcal{L}^{CS} = -STr(\mathbb{A}d\mathbb{A} + \frac{2}{3}\mathbb{A}^3)$$

- with the condition: $\psi_{\alpha I} = \mathrm{i}(\gamma_i)_{\alpha\beta}\chi_{\beta I}\, e^i_\mu dx^\mu = \psi_{\alpha I\,\mu} dx^\mu$

  Here α, i, I are gauge indices, μ are $\mathcal{M}_{(3)}$ base space indices; e^i_μ dreibein of $\mathcal{M}_{(3)}$, $\chi_{\beta I}$ spinor field on $\mathcal{M}_{(3)}$.

Peculiarities of AVZ ansatz $\psi_\mu = i\gamma_\mu\chi$:

1. it introduces background dependence (e^i_μ) and local dynamics ($\chi_{I\beta}$) in topological CS theory
   $\hookrightarrow$ typical of gauge fixing (e.g. $\Phi_{g.f.} = \partial^\mu A_\mu = g^{\mu\nu}\partial_\mu A_\nu$):

$$\Phi_{g.f}(\psi) = \partial^\mu\psi_\mu - \frac{i}{3}m\gamma^\mu\psi_\mu = 0 \Rightarrow \quad \mathrm{i}\partial\!\!\!/\chi + m\chi = 0$$

2. it implies identification of $SO(1,2)$ gauge indices (α, i) with Lorentz indices of base space (through e^i_μ):

$$SO(1,2)_L \simeq SO(1,2)_{gauge} \subset [OSp(2|2) \times SO(1,2)]$$

   $\hookrightarrow$ typical of solitons

[L.A., Cerchiai, Grassi, Trigiante (2019)];[L.A., Cerchiai, Matrecano, Noris, Ravera, Trigiante (2021)]

AVZ as a covariant gauge-fixing? BRST procedure

$\mathcal{S}$ cohomological Grassman op. of ghost number (g.n.) 1, $\mathcal{S}^2 = 0$

- Given a bosonic gauge symmetry $\delta A_\mu = D_\mu \Lambda$:

$$\mathcal{S}A_\mu = D_\mu c, \quad \mathcal{S}c = [c, c], \quad \mathcal{S}\bar{c} = t, \quad \mathcal{S}t = 0$$

$c, \bar{c}$: Grassman scalars (opposite spin-stat. than Λ) of g.n. 1, -1
t: ordinary scalar (same spin-stat. as Λ) of g.n. 0

- For fermionic gauge symmetry $\delta\psi_{\mu\alpha} = D_\mu \epsilon_\alpha$ (α gauge index):

$$\mathcal{S}\psi_{\mu\alpha} = D_\mu \phi_\alpha, \quad \mathcal{S}\phi = [\phi, \phi], \quad \mathcal{S}\bar{\phi}_\alpha = \eta_\alpha, \quad \mathcal{S}\eta = 0$$

$\phi_\alpha, \bar{\phi}_\alpha$: ordinary scalars (opposite spin-stat. than ϵ_α) of g.n. 1, -1
η_α: Grassman field (same spin-stat. as ϵ_α) of g.n. 0

[Rozansky-Witten (1996)], then applied in [Kapustin-Saulina (2009)] to a D3-NS5 model [Gaiotto-Witten (2008)]

AVZ as a covariant gauge-fixing? BRST procedure

$(\bar{c}, t)$, $(\bar{\phi}, \eta)$ implement the gauge-fixing $\Phi_{g.f}$ in the lagrangian:

$$\mathcal{L}_0 \to \mathcal{L}_{g.f.} = \mathcal{L}_0 + \mathcal{L}_S$$

For $\mathcal{L}_0$ invariant under bosonic gauge symmetry:

$$\mathcal{L}_{\mathcal{S}} = \mathcal{S}\left(\bar{c}\Phi_{g.f}(A)\right) = t\partial^\mu A_\mu + \partial^\mu \bar{c}\partial_\mu c + \partial^\mu(\ldots)$$

For $\mathcal{L}_0$ invariant under fermionic gauge symmetry:

$$\mathcal{L}_{\mathcal{S}} = \mathcal{S}\left(\bar{\phi}_\alpha \Phi^\alpha_{g.f}(\psi)\right) = \eta_\alpha \partial_\mu \psi^{\alpha\mu} + \boxed{\partial^\mu \bar{\phi}_\alpha \partial_\mu \psi^\alpha} + \partial^\mu(\ldots)$$

standard kin. term for holomorphic scalars $\phi_\alpha, \bar{\phi}_\alpha \hookleftarrow$

$\mathcal{L}_{g.f.}$ is S-invariant, and also invariant under $\bar{\mathcal{S}}$ of g.n. -1, $\bar{\mathcal{S}}^2 = 0$:

$$\bar{\mathcal{S}}\psi_{\mu\alpha} = D_\mu \bar{\phi}_\alpha, \quad \bar{\mathcal{S}}\bar{\phi} = [\bar{\phi}, \bar{\phi}], \quad \bar{\mathcal{S}}\phi = \eta, \quad \bar{\mathcal{S}}\eta = 0 \quad \text{(same for } A_\mu, c, \bar{c}, t\text{)}$$

$$\mathcal{S}\bar{\mathcal{S}} + \bar{\mathcal{S}}\mathcal{S} = 0 \quad \Rightarrow \quad \mathcal{S}^A = (\mathcal{S}, \bar{\mathcal{S}}) \in SU(2)_{ghost}, \quad \phi^A_\alpha = (\phi_\alpha, \bar{\phi}_\alpha)$$

AVZ as a covariant gauge-fixing? BRST procedure

Relevant in CS case, where ψ is part of the gauge connection

3D CS case is special:

- $\mathcal{L}_0 = \mathcal{L}^{CS}_{(3)}$ is topological, while $\mathcal{L}_{g.f.}$ has local dynamics for sector of fields in $Im(\mathcal{S})$
- $\mathcal{L}_{g.f.}$ has extra $\mathcal{S}^i_A$ global inv., $i = 0, 1, 2 \in SO(1, 2)_L \sim SU(2)_L$ of $\mathcal{M}_{(3)}$: "vector SUSY"[Delduc,Lucchesi,Piguet,Sorella (90); Delcima,Landsteiner,Schweda(98)]
- $\mathcal{S}^A, \mathcal{S}^A_i$: 8 generators of global invariance of $\mathcal{L}_{g.f.}$
  $\Rightarrow$ same as number of generators of $N = 4$ rigid SUSY in D=3: $\mathcal{Q}_{\alpha'\Lambda}$, with $\alpha' \in SO(1, 2)_L$; $\Lambda = 1, ..., 4 = (\dot{\alpha}'A) \in SO(4)_R$
  N.B.: $SO(4)_R \simeq SU(2)_1 \times SU(2)_{ghost}$, with $\dot{\alpha}' = 1, 2 \in SU(2)_1$
- 3 $SU(2)$ factors, top. twist: $SU'(2)_L = \mathrm{diag}[SU(2)_L \times SU(2)_1]$

AVZ as a covariant gauge-fixing? BRST procedure

- map between $\mathcal{S}^A, \mathcal{S}^A_i$ and $\mathcal{Q}_{\alpha'\dot{\alpha}'A}$:

$$\mathcal{Q}_{\alpha'\dot{\alpha}'A} = \epsilon_{\alpha'\dot{\alpha}'}\mathcal{S}_A + \gamma^i_{\alpha'\dot{\alpha}'}\mathcal{S}_{iA}$$

$$\mathcal{Q}_{\alpha'\dot{\alpha}'A} \in (2,2,2) \text{ of } SU(2)_L \times SU(2)_1 \times SU(2)_{ghost}$$

$$\mathcal{S}_A, \mathcal{S}^i_A \in (2, 1+3) \text{ of } SU'(2)_L \times SU(2)_{ghost}$$

- BRST inv. of $\mathcal{L}_{g.f.}$ is twisted description of rigid SUSY inv. of $\mathcal{L}_{g.f.}$
- different choice of Lorentz group on $\mathcal{M}_{(3)}$
- in "untwisted" description, the BRST-extended fields $\xi^I_\alpha \equiv (\psi^I_{i\alpha}, \eta^I_\alpha, \phi^I_\alpha \bar{\phi}^I_\alpha)$ build up a "standard" hypermultiplet on $\mathcal{M}_{(3)}$:

$$\Lambda^I_{\alpha\alpha'\dot{\alpha}'} = \epsilon_{\alpha'\dot{\alpha}'}\eta^I_\alpha + \gamma^i_{\alpha'\dot{\alpha}'}\psi^I_{i\alpha} \quad (F), \quad (\phi^I_\alpha \bar{\phi}^I_\alpha) \in \mathcal{M}_{HK} \quad (B)$$

Making contact with AVZ and its AdS_4 description

- $\mathcal{L}_0 = \mathcal{L}^{CS}_{(3)}(\mathbb{A})$, $\quad \mathbb{A} \in \mathrm{Adj}(SG)$ with $SG = SO(1,2)_+ \times OSp(2|2)_-$, $G = SO(1,2)_+ \times SO(1,2)_- \times SO(2) \simeq SO(2,2) \times SO(2)$.
- Gauge-fixing $\Phi_{g.f.} = \partial^i \psi_i + \ldots$: $SG \to G$; Also AVZ ansatz $\chi \propto \gamma^i \psi_i$: $\mathbb{A} \in \mathrm{Adj}(SG) \to \mathrm{Adj}(G)$
- AVZ as gauge-fixing would imply that a $\mathcal{S}$-invariant lagrangian $\mathcal{L}_{g.f.} = \mathcal{L}_0(\mathbb{A}) + \mathcal{L}_{\mathcal{S}}(\xi)$ should exist on $\tilde{\mathcal{M}}_{(3)}$
- or, equivalently, that a SUSY-invariant lagrangian of G-charged hypers ξ^I_α coupled to CS-gauge multiplets of G (no local d.o.f) exists on a rigid N=4 SUSY extension of $\mathcal{M}_{(3)}$ spanned by $\mathcal{Q}_{\alpha'\dot{\alpha}'A}$
- $\mathcal{M}_{(3)}$, $\tilde{\mathcal{M}}_{(3)}$ same base space, they differ in the choice of $SU(2)_L$ spin connection.

The real form matters

- symmetry on $\mathcal{M}_{(3)}$ (in $\mathbb{C}$): $SU(2)_L \times SU(2)_1 \times SU(2)_{ghost}$
- To get AVZ from AdS$_4$: $G \supset SO(2,2) = Isom(AdS_3)$ (A-T)
- AVZ ansatz $\psi = i\gamma_i e^i_\mu dx^\mu$ implies identification of $SO(1,2)_{gauge} \subset SO(2,2)_{gauge}$ with $SO(1,2)_L$ on $\mathcal{M}_{(3)} \Rightarrow \mathcal{M}_{(3)} = AdS_3$
- also on $\mathcal{M}_{(3)}$, symmetry should include $Isom(AdS_3)$:

$$\text{On } \mathcal{M}_{(3)}: \quad [SO(1,2)_L \times SO(1,2)_1] \times SU(2)_{ghost}$$

- Twist: $SO'(1,2)_L = \mathrm{diag}[SO(1,2)_L \times SO(1,2)_1]$
- It is choice of torsionless vs torsionful spin connection on $\mathcal{M}_{(3)}$ of AdS$_3$ type:

$$\text{AdS}_3 \text{ radius} \quad \Leftrightarrow \quad \text{contorsion}$$

$D^2(2,1;\alpha)$ superspace

- $\mathcal{Q}_{\alpha'\dot{\alpha}'A}$, $\Lambda^{I\alpha}_{\alpha'\dot{\alpha}'}$ are SUSY-generators and hyperinos on a rigid N=4 Super-AdS_3 background where $(\alpha'\dot{\alpha}')$ give a (redundant) 4-component description of spinor indices
- Among the AdS_3 supergroups, this hints naturally to $D^2(2,1;\alpha)$, $\alpha \in \mathbb{R}/\{-1,0,\infty)\}$:

$$D^2(2,1;\alpha) \supset SO(1,2) \times SO(1,2) \times SU(2), \quad \text{with } \mathcal{Q} \in (2,2,2)$$

possibly including its singular values for α

$D^2(2,1;\alpha)$-superspace Lagrangian

- The rigid $D^2(2,1;\alpha)$-superspace has 1-form background fields: $(e^i, \Psi^{\alpha'\dot{\alpha}'A}, A^x)$, $x = 1,2,3 \in \mathrm{Adj}(SU(2)_{ghost})$
- We have built the SUSY-invariant lagrangian for a set of hypermultiplets $\xi^{I\alpha} = (\phi^{I\alpha A}, \Lambda^{I\alpha}_{\alpha'\dot{\alpha}'})$ on $D^2(2,1;\alpha)$-superspace (not yet coupled to gauge multiplets, so far). Its space-time projection reads ($(\alpha) \equiv \alpha'\dot{\alpha}'$):

$$\mathcal{L}_{\text{sp.t.}} = \frac{1}{2}\nabla_\mu \phi_I^{\alpha A}\nabla^\mu \phi_{\alpha IA} - \frac{8}{\alpha^2-1}\Lambda^{\alpha(\alpha)I}\left(\mathbb{M}_+^\mu\right)_{(\alpha)(\beta)}\nabla_\mu \Lambda_I^{\beta(\beta)}\epsilon_{\alpha\beta} + \frac{\mathrm{i}}{(\alpha+1)}\mathcal{M}_{(\alpha)(\beta)}\Lambda^{\alpha(\alpha)I}\Lambda_I^{\beta(\beta)}\epsilon_{\alpha\beta} - \mathcal{V}(\phi)$$

with $\mathcal{M}_{(\alpha)(\beta)} = \frac{1}{\ell}[\ldots]_{(\alpha)(\beta)}$, $\mathcal{V}(\phi) = \frac{1}{2\,\ell^2}\phi_I^{\alpha A}\phi_{I\alpha A}$

Twisting $D^2(2,1;\alpha)$-superspace

- We have performed the twist (on $\mathcal{M}_{(3)}$):
  $SO'(1,2)_L = \mathrm{diag}[SO(1,2)_L \times SO(1,2)_1]$:

$$\mathcal{Q}_{\alpha'\dot{\alpha}'A} = \epsilon_{\alpha'\dot{\alpha}'}\mathcal{S}_A + \gamma^i_{\alpha'\dot{\alpha}'}\mathcal{S}_{iA}$$

finding

$$\mathcal{L}_{\text{spacetime}} \to \frac{4\mathrm{i}}{1-\alpha}\epsilon^{ijk}\Psi_i^{\alpha I}\nabla_j\Psi_{kI}^{\beta}\epsilon_{\alpha\beta} + \frac{1}{2}\nabla_\mu\phi_I^{\alpha A}\nabla^\mu\phi_I^{\beta B}\epsilon_{\alpha\beta}\epsilon_{AB} - \mathcal{V}(\phi) +$$
$$+\frac{8\mathrm{i}}{(1+\alpha)}\left(-\Psi^{i\alpha I}\nabla_i\bar{\eta}_I^{\beta} + \frac{1}{\ell}\bar{\eta}^{\alpha I}\bar{\eta}_I^{\beta}\right)\epsilon_{\alpha\beta},$$
$$= \mathcal{L}_0 + \frac{4\mathrm{i}}{1+\alpha}\mathcal{S}^A\cdot\left(\nabla^i\phi_A^{\alpha I}\Psi_{iI}^{\beta} + \frac{1}{\ell}\bar{\eta}^{\alpha I}\phi_{\alpha IA}\right)$$

However: $\{\mathcal{S}^A, \mathcal{S}^B\}\cdot\phi_I^{C\alpha} = \frac{\mathrm{i}(1+\alpha)}{4\ell}\epsilon^{C(A}\phi_I^{B)\alpha} \neq 0$

For $\alpha + 1 \neq 0$, anomalous BRST!

Twisting $D^2(2,1;\alpha)$-superspace: Second twist

- Twist to AVZ $\psi_\alpha = i\gamma^i_{\alpha\alpha'}\chi_{\alpha'} e_i$ would require identification $SO(1,2)_{gauge} \sim SO'(1,2)_L$ ($\to$ second twist):

$$SO''(1,2)_L = \mathrm{diag}[SO(1,2)_{gauge} \times SO'(1,2)_L] \quad \Rightarrow \alpha \sim \alpha' \sim \dot{\alpha}'$$

$$\Lambda_I^{\alpha\alpha'\dot{\alpha}'} = \mathrm{i}\,(\gamma^k)^{\alpha\alpha'}\,\hat{\chi}^{\dot{\alpha}'}_{Ik} + \epsilon^{\alpha\alpha'}\,\zeta_I^{\dot{\alpha}'}\,, \quad \hat{\chi}_{Ii} = \mathring{\chi}_{Ii} + \frac{1}{3}\gamma_i\,\hat{\not\chi}_I$$

- We found: $\begin{cases} \psi_{Ii} & = \hat{\chi}_{Ii} - \frac{i}{2}\gamma_i\left(\zeta - i\hat{\not\chi}_I\right) = \mathring{\chi}_{Ii} - \frac{1}{3}\gamma_i\chi \\ \eta_I & = -\frac{1}{2}\left(\zeta + i\hat{\not\chi}_I\right) \end{cases}$

Twisting $D^2(2,1;\alpha)$-superspace: Second twist

- Field equations: $\begin{cases} \delta\mathring{\chi}_{I\mu} & = \dots \\ \delta\chi : & \frac{3}{2}\nabla^\mu\mathring{\chi}_{\mu I} + \mathrm{i}\not\nabla\chi_I - \mathrm{i}\frac{\alpha-1}{\alpha+1}\not\nabla\eta_I = 0\,, \\ \delta\eta : & -3\nabla^\mu\mathring{\chi}_{\mu I} + \mathrm{i}\not\nabla\chi_I - \frac{3}{2\ell}\eta_I = 0 \end{cases}$
- To make contact with AVZ: $\nabla^\mu\mathring{\chi}_{\mu I} = 0 \Rightarrow$

$$\mathrm{i}\not\nabla\sigma_I \equiv \mathrm{i}\not\nabla\left(\chi_I - \frac{\alpha-1}{\alpha+1}\eta_I\right) = 0, \quad \mathrm{i}\not\nabla\eta = m\,\eta, \quad \text{with } m \equiv \frac{3(\alpha+1)}{2\ell(\alpha-1)}$$

- Up to a free massless spinor σ_I, $\chi_I \sim \eta_I$ satisfies a massive field eq., with $m \propto \alpha + 1$

AVZ is a gauge fixing?

We found a lagrangian describing the propagation of massive $\chi = \frac{i}{3}\gamma^\mu\psi_\mu$, with $m_\chi \propto \frac{1}{\ell}$, in N=4 $\mathcal{M}_{(3)} = AdS_3$ ✓
But:

- the mass is $m \propto \frac{\alpha+1}{\ell}$, and $\{\mathcal{S}_A, \mathcal{S}_B\} \propto (\alpha+1)\epsilon_{AB}$ ✗

either we loose relation with AdS_4 ($m_\chi \neq 0$), or with BRST

Does AVZ defines an anomalous covariant gauge-fixing? ...or maybe $D^2(2,1;\alpha)$-superspace is too much?...

- Next step: to repeat the calculations for the singular value $\alpha + 1 = 0$, and to add coupling with CS gauge multiplets of $G = [SO(2,2) \times SO(2)]_{gauge}$