Local SUSY: An unconventional approach

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Supergravity and Holography Workshop
December 13-15, 2021

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1109.3944, 1306.1247, 1505.03834, 1606.05239, 1910.03508, 202504178, 2104.05133, 2105.14606, 2110.06828

Supersymmetry (SUSY) is a spacetime symmetry between bosons and fermions. In SUSY, each particle from one class would have an associated particle in the other, known as its *superpartner*, the spin of which differs by a half-integer.

Wikipedia

$$\begin{bmatrix} B' \\ F' \end{bmatrix} = Q \begin{bmatrix} B \\ F \end{bmatrix}$$

F.A. Berezin and G.I. Kac, Math. Sbornik **82** (1970) 343
Yu.A. Golfand and E. P. Likhtman, JETP Lett.**13** (1971) 323
J.-L. Gervais and B. Sakita, Nucl. Phys. **B34** (1971) 632

In our youth, we all fell in love with supersymmetry SUSY:

- Makes fermions and bosons necessary
- Restricts particle multiplets
- Relates masses and coupling constants
- Provides $\boldsymbol{E} \geq \mathbf{0}$ theorems, stability
- Improves renormalizability (cancellation of infinities)
- Respects hierarchies (protects mass scales)

… after five decades of active search, no evidence of SUSY has been found.

PRL 107, 221804 (2011) — Selected for a Viewpoint in *Physics* — PHYSICAL REVIEW LETTERS — week ending 25 NOVEMBER 2011

Search for Supersymmetry at the LHC in Events with Jets and Missing Transverse Energy

S. Chatrchyan *et al.**
(CMS Collaboration)

(Received 12 September 2011; published 21 November 2011)

A search for events with jets and missing transverse energy is performed in a data sample of pp collisions collected at $\sqrt{s} = 7$ TeV by the CMS experiment at the LHC. The analyzed data sample corresponds to an integrated luminosity of 1.14 fb^{-1}. In this search, a kinematic variable α_T is used as the main discriminator between events with genuine and misreconstructed missing transverse energy. No excess of events over the standard model expectation is found. Exclusion limits in the parameter space of the constrained minimal supersymmetric extension of the standard model are set. In this model, squark masses below 1.1 TeV are excluded at 95% C.L. Gluino masses below 1.1 TeV are also ruled out at 95% C.L. for values of the universal scalar mass parameter below 500 GeV.

Is Supersymmetry Dead?

The grand scheme, a stepping-stone to string theory, is still high on physicists' wish lists. But if no solid evidence surfaces soon, it could begin to have a serious PR problem

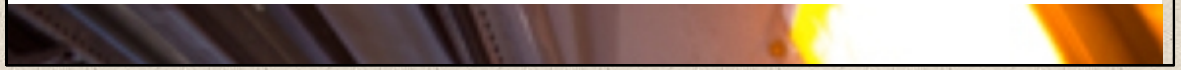
By Davide Castelvecchi | April 25, 2012 | 29

THE SCIENCES

Supersymmetry Fails Test, Forcing Physics to Seek New Ideas

With the Large Hadron Collider unable to find the particles that the theory says must exist, the field of particle physics is back to its "nightmare scenario"

By Natalie Wolchover, Quanta Magazine on November 29, 2012



I. What was SUSY expected to solve?

1960s:

Spacetime symmetries

(Lorentz, Poincaré, conformal)

$\mathcal{G}_S$: $[J, J] \sim J$

$[J, T] = 0$

Internal gauge symmetries

($U(1)$, $SU(2)$, $SU(3)$…)

$\mathcal{G}_I$: $[T, T] \sim T$

Conserved charges → Quantum numbers, classification of particles ← Conserved charges

Can these two symmetries combine into a larger one?

Coleman–Mandula Theorem (1967): Lie algebras representing spacetime and internal symmetries of the S-matrix can only be combined in a trivial manner (as a direct sum):

$$\mathcal{G} = \mathcal{G}_S \oplus \mathcal{G}_I$$

C-M theorem notwithstanding, a few years later, a nontrivial extension of the Poincaré group was found…

Volume 46B, number 1 PHYSICS LETTERS 3 September 1973

IS THE NEUTRINO A GOLDSTONE PARTICLE?

D.V. VOLKOV and V.P. AKULOV
Physico-Technical Institute, Academy of Sciences of the Ukrainian SSR, Kharkov 108, USSR

Received 5 March 1973

Using the hypotheses, that the neutrino is a goldstone particle, a phenomenological Lagrangian is constructed, which describes an interaction of the neutrino with itself and with other particles.

where a Lagrangian invariant under combined spacetime translations and shifts in the fermion field was proposed,

$$x^\mu \to x^\mu + \bar{\epsilon}\gamma^\mu\psi\,, \qquad \psi \to \psi + \epsilon$$

which seems to violate the C-M theorem…

Haag–Łopuszański–Sohnius theorem (1975)

Internal gauge symmetries
$[T, T] \sim T$

$[J, T] = 0$

External space-time symmetries
$[J, J] \sim J$

SUSY

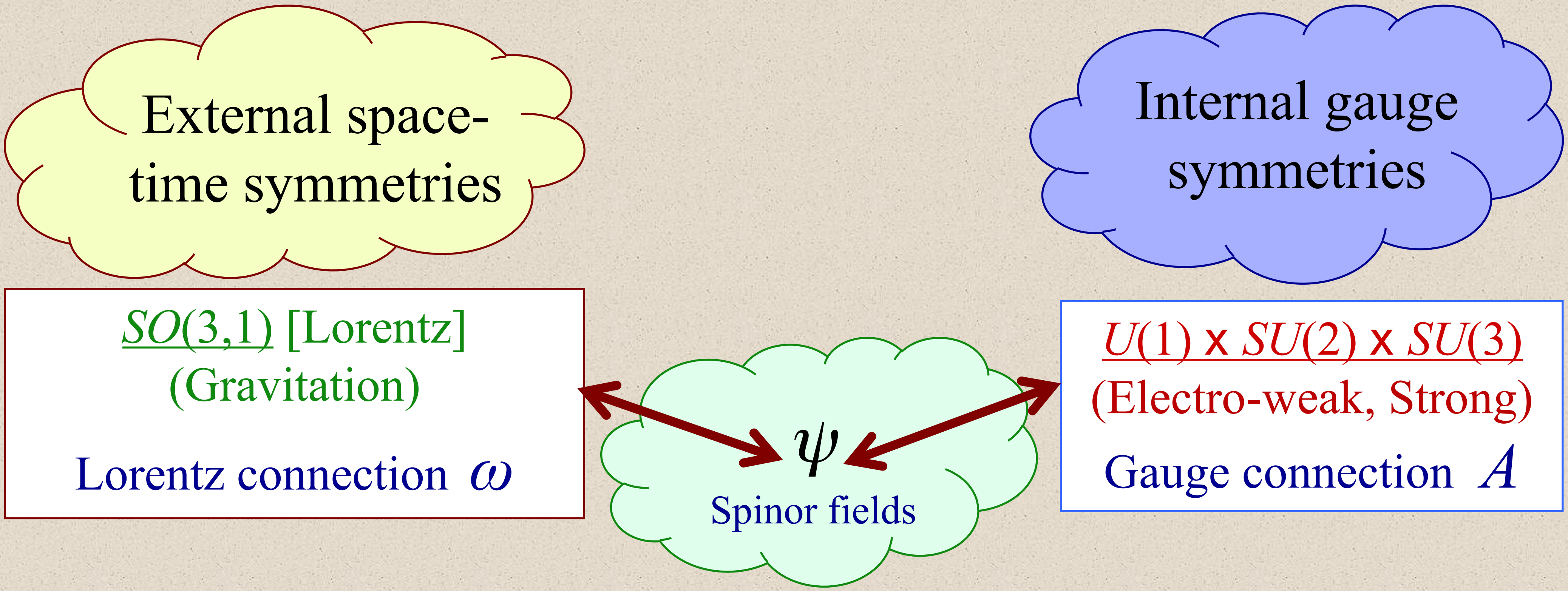
$[T, Q] \sim Q$

$[J, Q] \sim Q$

Supersymmetry
$\{Q, Q\} \sim T + J$

*Spacetime and gauge symmetries can be part of a larger **graded** Lie algebra*

Fermions link spacetime and internal symmetries



Fermions belong to nontrivial irreducible representations of Lorentz and gauge groups; they couple naturally to the connections for spacetime and internal symmetries

$$\bar{\psi}\, \not{\omega}\, \psi\,, \qquad \bar{\psi}\, \not{A}\, \psi$$

Fermions (matter) couple to internal gauge fields and to the spacetime geometry

$$L_{Int} \sim \bar{\psi}\, \not{A}\, \psi$$

$$L_{Int} \sim \bar{\psi}\, \not{\varphi}\, \psi$$

Standard (rigid/global) SUSY

$$\begin{bmatrix} B' \\ F' \end{bmatrix} = Q \begin{bmatrix} B \\ F \end{bmatrix} = \begin{bmatrix} s_{BB} & s_{BF} \\ s_{FB} & s_{FF} \end{bmatrix} \begin{bmatrix} B \\ F \end{bmatrix}$$

"Vector" representation

Global (rigid) SUSY [Wess&Zumino, Ramond, Salam&Strathdee, Iliopoulos, Shifman, …]

$$\{Q^{\alpha}, \overline{Q}_{\beta}\} = H(\Gamma_0)^{\alpha}{}_{\beta}, \quad [H, Q] = 0$$

- ➔ Energy states are degenerate: $m_B = m_F$
- ➔ Equal numbers of *B*- and *F*-states

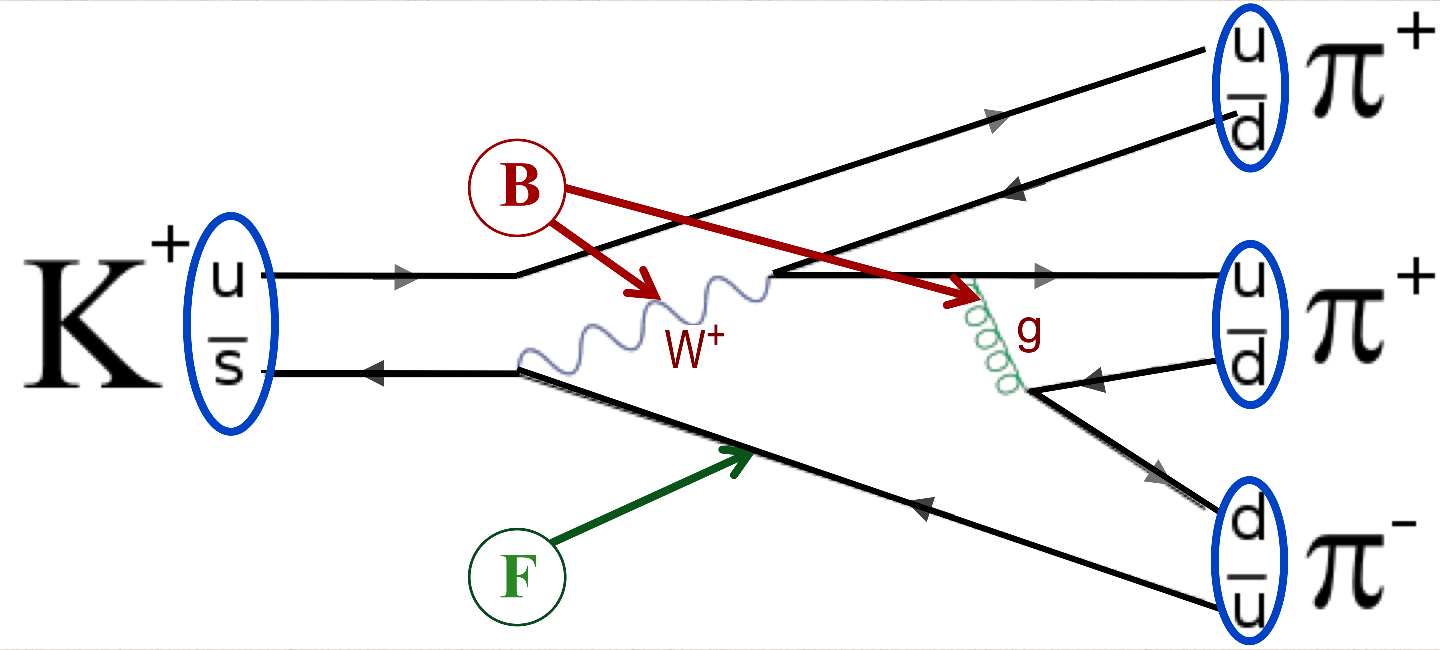
Not even approximately true: *SUSY must be severely broken*

"If this symmetry were true it would have been discovered long ago."

P. A. M. Dirac to A. Salam, 1976(?)

II. Standard Model of particle physics

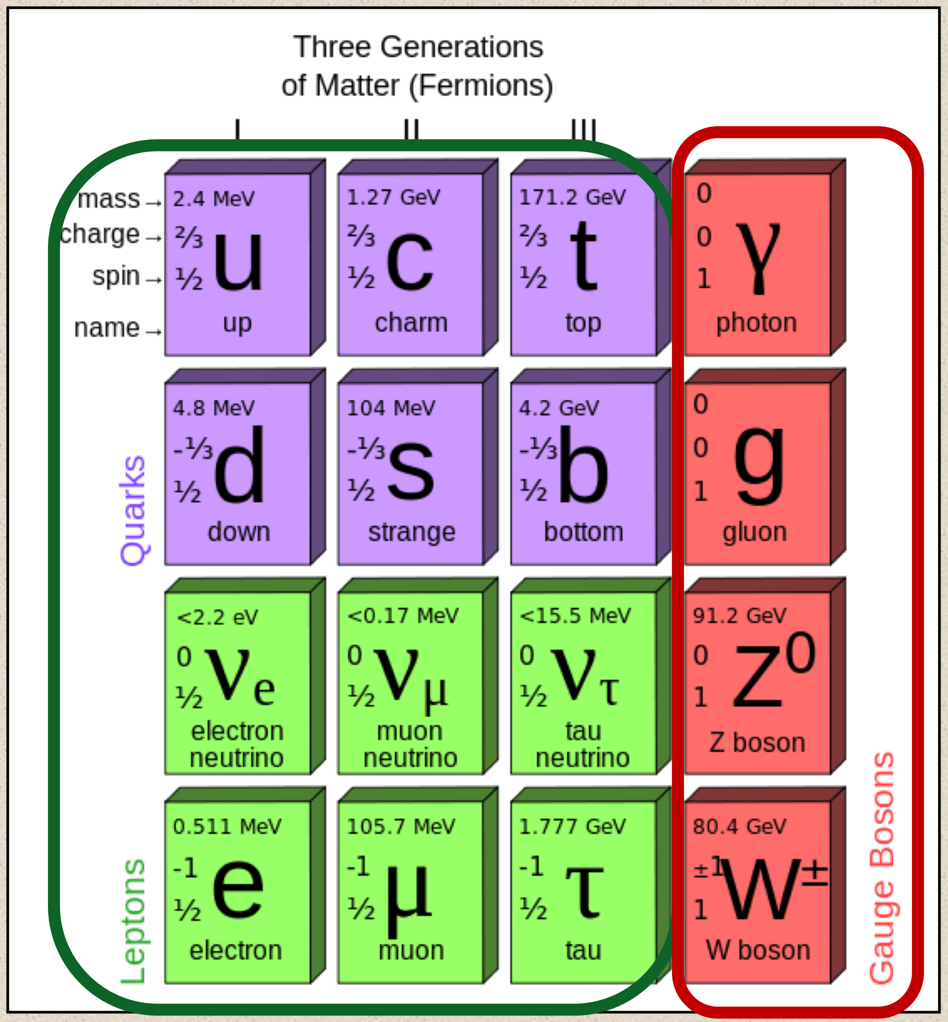
Fermions (matter)
Bosons (interaction carriers)



Standard Model

Matter Sources (ψ)

- Fermions, S=1/2
- Gauge vectors (fundamental rep.)
- Spacetime scalars (zero forms)
- Lorentz spinors
- 1st order field eqs.



Interaction Carriers (A)

- Bosons S=1
- Gauge connections (adjoint rep.)
- Spacetime vectors (one-forms)
- Lorentz scalars
- 2nd order field eqs.

(BEGH boson ?)

Fermions $(l, \bar{l}; q, \bar{q})$

$$L = \bar{\psi}_a (\not{\partial} - i\not{A} + m)^a{}_b \psi^b$$

$$\psi^{\alpha\, a}$$

- Spin ½ ir.-rep. of Lorentz group
- Fundamental ir.-rep. of gauge group

- Spinor under local Lorentz transformations:

$$\psi'^{\alpha}(x) = S^{\alpha}{}_{\beta}(x)\psi^{\beta}(x)$$

- Vector under internal gauge transformations:

$$\psi'^{a}(x) = [g^{-1}(x)]^a{}_b\,\psi^b(x)$$

Bosons (γ, Z, $W^{\pm}$, gluons)

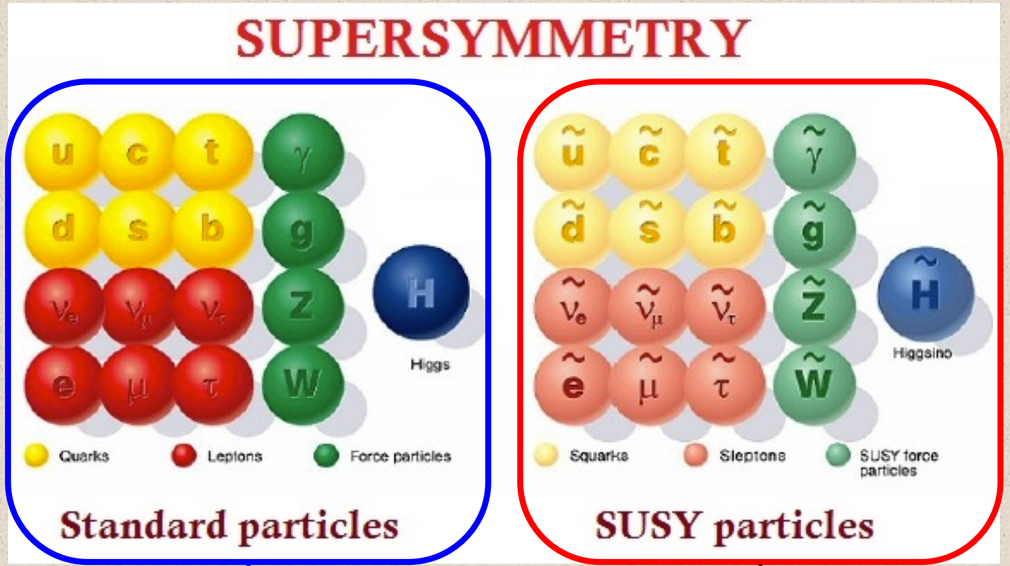
$$L = \left\langle -\tfrac{1}{4} F \wedge {}^*F - A \wedge {}^*j \right\rangle$$

- Adjoint representation of gauge group → $A^{a}{}_{b}$

Under internal gauge transformations:

$$A'(x) = g^{-1}[A(x) + d]g, \quad A = A_\mu dx^\mu$$

“SUSY-SM”: for each observed particle, include a partner with equal mass and other q-numbers, but different spin:



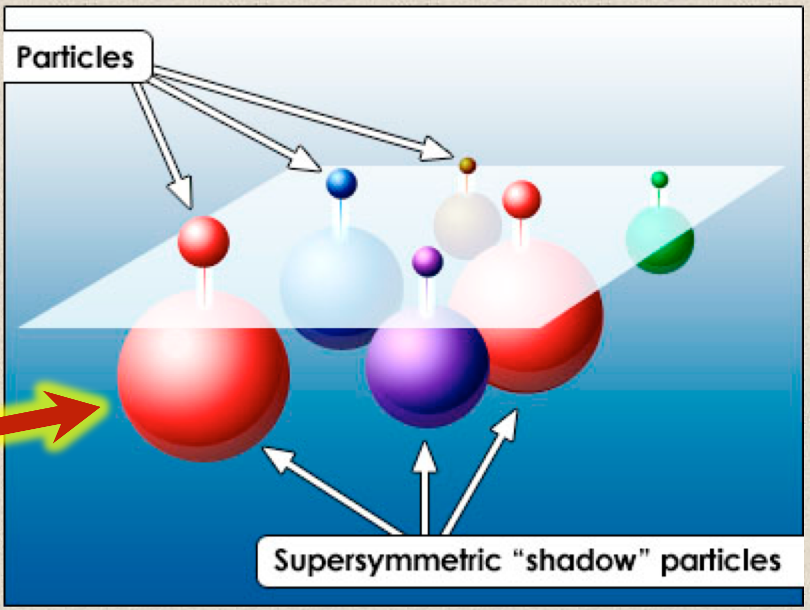
Particles	Spin	Superpartners	Spin
Graviton	2	*Gravitino*	3/2
Photon	1	*Photino*	1/2
Gluon	1	*Gluino*	1/2
$W^{\pm}$	1	*Wino*$^{\pm}$	1/2
Z^{0}	1	*Zino*	1/2
Higgs	0	*Higgsino*	1/2
Electron	1/2	*Selectron*	0
Muon	1/2	*Smuon*	0
Tau	1/2	*Stau*	0
Neutrino	1/2	*Sneutrino*	0
Quark	1/2	*Squark*	0

Observed

Not observed

(Dark matter?)

- SUSY breaking mechanism?
- Why are these so heavy?



The soft SUSY-breaking Lagrangian of the MSSM contains 105 new parameters not found in the Standard Model.

Graham Kribs, Supersymmetry, 2012

If the Lord had consulted my opinion, I would have suggested something considerably simpler…

Alfonso X, the Wise, commenting on Ptolemy's epicycles (~ 1280)

Wish list

Combine **B** and **F** under a local (graded)symmetry that would:

- Respect their roles as connections (**B**) and sections (**F**) in a fiber bundle
- Give the right kinetic terms ($\bar{\psi}\not{\partial}\psi, F * F$) and couplings ($\bar{\psi}\not{A}\psi$)
- No duplicate fields (no SUSY superpartners)
- Allow for massive **F** and massless **B** fields
- Contain only spins 1 and ½ (the rest can be composites)
- Allow for curved, dynamic spacetime (Gravity)

These features are generically violated by SUSY and SUGRA

III. Unconventional SUSY

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How to combine fields in different representations?

N.B.: It is often possible to combine an adjoint representation and a vector into an adjoint of a larger group:

$$\left\{ \begin{pmatrix} 0 & a^{12} & a^{13} \\ -a^{12} & 0 & a^{23} \\ -a^{13} & -a^{23} & 0 \end{pmatrix}, \begin{pmatrix} v^1 \\ v^2 \\ v^3 \end{pmatrix} \right\} \longrightarrow \begin{pmatrix} 0 & a^{12} & a^{13} & v^1 \\ -a^{12} & 0 & a^{23} & v^2 \\ -a^{13} & -a^{23} & 0 & v^3 \\ -v^1 & -v^2 & -v^3 & 0 \end{pmatrix}$$

Adjoint $SO(3)$ Fund. $SO(3)$ Adjoint $SO(4)$

$$\tfrac{1}{2}a^{ij}\, G_{ij} + v^i\, T_i \;=\; \tfrac{1}{2}A^{IJ}\,\hat{G}_{IJ}$$

This allows to combine connections and vectors of a given group into a connection for a larger group.

Generalizing the idea:

Combine an internal gauge connection $A^r{}_\mu dx^\mu$, a spinor χ^α and the Lorentz connection ω^{ab} into a single connection field:

$$\mathcal{A} \sim A^r K_r + \overline{Q}_\alpha \chi^\alpha + \overline{\chi}_\alpha Q^\alpha + \tfrac{1}{2}\omega^{ab} J_{ab} + \boxed{\ldots}$$

Internal gauge generators

Complex SUSY generators

Lorentz generators

Needed to close the algebra

This is still rather conventional

➔ McDowell-Mansouri SUGRA in 4D (1976)

➔ Chern-Simons Supergravity in odd Dimensions, any $\langle \cdots \rangle$

Matter ansatz

Technical issue:

$\mathcal{A}$ is a connection 1-form ➔ the spinor χ^{α} must also be a 1-form,

$\chi^{\alpha} = \chi^{\alpha}{}_{\mu} dx^{\mu}$ ➔ s = $^3/_2$ gravitino (not in the SM)

There is an alternative: $\chi^{\alpha}{}_{\mu} \equiv (\Gamma_{\mu})^{\alpha}{}_{\beta} \psi^{\beta}$ (*Matter Ansatz*)

where $\Gamma_{\mu} = \Gamma_a e^a_{\mu}$

Standard s = $^1/_2$ *spinor*

Dirac matrices (tangent space)

$\{\Gamma_a, \Gamma_b\} = 2\eta_{ab} I$

Vielbein/soldering form: Projects from tangent space onto the spacetime manifold.

Fermion: $\chi^{\alpha}{}_{\mu} \in 1 \otimes \frac{1}{2} = \frac{3}{2} \oplus \frac{1}{2}$

Matter ansatz: $\chi_\mu = \Gamma_\mu \psi \quad \Longrightarrow \psi = \frac{1}{4}\Gamma^\mu \chi_\mu$ (D=4)

$$\Longrightarrow \left(\delta^\nu_\mu - \tfrac{1}{4}\Gamma_\mu \Gamma^\nu\right)\chi_\nu = 0$$

Standard Supergravity: $\Gamma^\mu \chi_\mu = 0$ ➔ $\chi^{\alpha}{}_{\mu} \in 1 \otimes \frac{1}{2} = \frac{3}{2} \oplus \cancel{\frac{1}{2}}$

U-SUSY: $\left(\delta^\mu_\nu - \frac{1}{4}\Gamma_\nu \Gamma^\mu\right)\chi_\mu = 0$ ➔ $\chi^{\alpha}{}_{\mu} \in 1 \otimes \frac{1}{2} = \cancel{\frac{3}{2}} \oplus \frac{1}{2}$

Unconventional SUSY uses the discarded spin-1/2 sector of Supergravity

Example in 3 dimensions

Consider a connection for an algebra that includes internal, spacetime, and supersymmetry generators

$$\mathcal{A} = A^A \mathrm{T}_A + \tfrac{1}{2}\omega^{ab}\mathrm{J}_{ab} + \overline{\psi}^r_\alpha (\Gamma)^\alpha{}_\beta Q^\beta_r + \overline{Q}^r_\alpha (\Gamma)^\alpha{}_\beta \psi^\beta_r$$

1-form bosons (A^A, ω^{ab})

0-form fermions ($\overline{\psi}^r_\alpha$, ψ^β_r)

- $\mathrm{T}_A \rightarrow SU(2)$
- $\mathrm{J}_{ab} \rightarrow SO(1,2)$
- $Q^\beta_r,\ \overline{Q}^r_\alpha \rightarrow SUSY$

Superalgebra $su(1,2|2)$

The one-form $\mathcal{A} = A^A \mathrm{T}_A + \frac{1}{2}\omega^{ab}\mathrm{J}_{ab} + \bar{\psi}^r_\alpha (\Gamma)^\alpha_\beta Q^\beta_r + \bar{Q}^r_\alpha (\Gamma)^\alpha_\beta \psi^\beta_r$ transforms as a *connection* under the *su*(1,2|2) superalgebra: $\delta\mathcal{A} = d\Lambda + [\mathcal{A}, \Lambda] = D_{\mathcal{A}}\Lambda$

SO(1,2): $$\left[\mathrm{J}^{ab}, \mathrm{J}^{cd}\right] = \eta^{bc}\mathrm{J}^{ad} - \eta^{ac}\mathrm{J}^{bd} + \eta^{ad}\mathrm{J}^{bc} - \eta^{bd}\mathrm{J}^{ac},$$

SU(2): $$\left[\mathrm{T}_A, \mathrm{T}_B\right] = i\varepsilon_{ABC}\mathrm{T}_C, \qquad \left[\mathrm{T}_A, \mathrm{J}^{ab}\right] = 0$$

SUSY: $$\left\{Q^\alpha_r, \bar{Q}^s_\beta\right\} = i\delta^\alpha_\beta \mathrm{T}_A \left(\sigma^A\right)^s_r + \frac{1}{2}\delta^s_r \mathrm{J}^{ab}(\Gamma_{ab})^\alpha_\beta$$

$$\left[\mathrm{J}^{ab}, Q\right] = \frac{1}{2}\Gamma^{ab}Q; \quad \left[\mathrm{J}^{ab}, \bar{Q}\right] = -\frac{1}{2}\Gamma^{ab}\bar{Q},$$

$$[\mathrm{T}, Q] \sim Q; \quad [\mathrm{T}, \bar{Q}] \sim -\bar{Q},$$

All fields are scalars under general coordinate transformations (diff. forms).

→ *General covariance is automatically built in*

The system has a metric structure and local Lorentz symmetry: Gravity!

IV. Unconventional Actions

The action is the integral of a gauge-invariant *D*-form.

$$I = \int L(\mathcal{A}) = \int L(A, \psi, \ldots)$$

There are two standard options:

- Chern-Simons (odd *D* only)

$$L_{2n+1} = \left\langle \mathcal{A}(d\mathcal{A})^n + c_1 \mathcal{A}^3 (d\mathcal{A})^{n-1} + \cdots + c_n \mathcal{A}^{2n+1} \right\rangle$$

and

- Yang-Mills (any *D*)

$$L_{\mathrm{YM}} = \langle \mathcal{F} \wedge \circledast \mathcal{F} \rangle$$

where $\mathcal{F} = d\mathcal{A} + \mathcal{A}\mathcal{A}$, $\langle\ \rangle$ = invariant trace, and $\circledast$ = Hodge dual

$3D$: The Chern-Simons form gives a gauge (quasi-) invariant action for $\mathcal{A}$:

$$L = \frac{1}{2}\left\langle \mathcal{A} d\mathcal{A} + \frac{2}{3}\mathcal{A}\mathcal{A}\mathcal{A} \right\rangle$$

where the bracket is the invariant symmetric trace in the algebra.

$SU(2)$-CS | $SO(1,2)$-CS (gravity)

$$L = \underbrace{\frac{1}{2}\mathrm{Tr}[AdA + \frac{2}{3}A^3]}_{SU(2)\text{-}CS} + \underbrace{\frac{1}{2}[\omega^a{}_b d\omega^b{}_a + \frac{2}{3}\omega^a{}_b\omega^b{}_c\omega^c{}_a]}_{SO(1,2)\text{-}CS}$$

$$-2\bar{\psi}(\not{\partial} + i\not{A} - \frac{1}{4}\Gamma_a \not{\omega}^{ab}\Gamma_b)\psi|e|d^3x \leftarrow \text{Dirac}$$

$$-\bar{\psi}\psi\, e^a T_a \leftarrow \textit{Torsional coupling (mass)}$$

Long wavelength limit of graphene, including curvature, torsion, SU(2). The only propagating degree of freedom is the spin-1/2 Dirac fermion.

Field equations:

$$\delta A: \qquad F^A = \tfrac{i}{2}\varepsilon_{abc}\overline{\psi}\mathrm{T}^{A}\Gamma^{a}\psi e^{b}e^{c} \qquad (1)$$

$$(F^{A}_{\mu\nu} = \varepsilon_{\mu\nu\lambda}j^{\lambda A})$$

$$\delta\omega^{ab}: \qquad R^{ab} = -2\overline{\psi}\psi e^{a}e^{b} \qquad (2)$$

Fermion mass

$$\delta\overline{\psi}: \qquad [\partial\!\!\!/ + iA\!\!\!/ - \tfrac{1}{4}\Gamma^{a}\omega_{ab}\Gamma^{b} + \mu]\psi = 0 \qquad (3)$$

$$\delta e^{a}: \qquad \overline{\psi}\varepsilon_{abc}\Gamma^{c}[\overleftarrow{d}e^{b} - e^{b}\overrightarrow{d} + 2iAe^{b}]\psi = 2\overline{\psi}\psi T_{a} \qquad (4)$$

- Standard equations for CS $SU(2)$, gravity and spin ½ in 2+1 dimensions.
- ψ gets "mass" from torsion: $\mu = \eta_{ab}e^{a}_{\mu}T^{b}_{\nu\lambda}\varepsilon^{\mu\nu\lambda}$

$$DT^{a} = 0 \Rightarrow T_{a} = \tfrac{1}{6}\mu\varepsilon_{abc}e^{b}e^{c}, \quad \mu = \text{const}$$

✔

Example in 4 dimensions

- Minimal susy extension of $SU(2)$, $SO(3,1)$ leads to $osp(4|2)$:

$$\mathcal{A} = A_r \mathbf{K}^r + A\mathbf{Z} + \bar{Q}_{\mathrm{i}} \Gamma \psi^{\mathrm{i}} + \overline{\psi}_i \Gamma Q^{\mathrm{i}} + f^a \mathbf{J}_a + \frac{1}{2} \omega^{ab} \mathbf{J}_{ab}$$

$SU(2)\mathrm{x}U(1)$
Internal symmetry

Complex Dirac spinor

$SO(3,2)$
anti - de Sitter

$$\left\{ \bar{Q}^{\mathrm{i}}_{\alpha}, Q^{\beta}_{\mathrm{j}} \right\} = -i\delta^{\beta}_{\alpha} \left(\sigma^r\right)^{\mathrm{i}}{}_{\mathrm{j}} \mathbf{K}_r + \left(\frac{1}{2}\left(\Gamma^a\right)^{\beta}{}_{\alpha} \mathbf{J}_a - \frac{1}{2}\left(\Sigma^{ab}\right)^{\beta}{}_{\alpha} \mathbf{J}_{ab} \right) \delta^{\mathrm{i}}_{\mathrm{j}}$$

- Curvature:

$$\mathcal{F} = d\mathcal{A} + \mathcal{A}\,\mathcal{A}$$
$$= F_r \mathbf{K}^r + F\mathbf{Z} + \bar{Q}_{\mathrm{i}} \mathcal{F}^{\mathrm{i}} + \overline{\mathcal{F}}_{\mathrm{i}} Q^{\mathrm{i}} + F^a \mathbf{J}_a + \frac{1}{2} F^{ab} \mathbf{J}_{ab}$$

Type equation here.

Where:

$$F = dA - \frac{i}{4}\bar{\psi}_i \not{e} \not{e}\, \psi^i \qquad U(1)$$

$$F_r = dA_r + \frac{1}{2}\epsilon_r{}^{st} A_s A_t - \frac{i}{2}\bar{\psi}\not{e}\sigma_r\not{e}\psi \qquad SU(2)$$

$$\mathcal{F}^i = d(\not{e}\psi^i) + iA_r(\sigma^r)^i{}_j\psi^j + \frac{1}{4}\Omega^{AB}\Gamma_{AB}\psi^i \qquad SUSY$$

$$F^a = df^a + \frac{1}{2}\omega^a{}_b f^b + \frac{1}{2}\bar{\psi}_i \not{e}\, \Gamma^a \not{e}\, \psi^i$$

$$F^{ab} = R^{ab} + f^a f^b - \bar{\psi}_i \not{e}\, \Gamma^{ab} \not{e}\, \psi^i$$

$SO(3,2)$

$SU(2)\times SO(3,1)$ - invariant trace

Lagrangian: $L_4 = \langle \mathcal{F} \wedge^{\circledast} \mathcal{F} \rangle$

$$= -\frac{1}{4} F_r \wedge * F^r + F \wedge {}^*F + \overline{\mathcal{F}}_i \, \Gamma_5 \, \mathcal{F}^i$$
$$+ \frac{1}{2} \epsilon_{abcd} F^{ab} F^{cd}$$

- *Hodge dual* $\circledast = \begin{cases} * & \text{Internal} \\ \Gamma_5 & \text{Fermions} \\ \epsilon_{abcd} & \text{Spacetime} \end{cases}$

- $\nexists$ $Osp(4|2)$ or $SO(3,2)$-*invariant traces in* $4D$. *The largest symmetry group that has an invariant trace is* $SU(2)\times U(1)\times SO(3,1)$ ➔ *Largest gauge symmetry of the action.*

- f^a *is no longer a gauge field*

4D Lagrangian (identifying $f_\mu^a = \lambda^{1/2} e_\mu^a$):

$$L = \Big\{ -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \tfrac{i}{2}[\bar\psi \overleftarrow{\nabla} \psi - \bar\psi \overrightarrow{\nabla} \psi] + \bar\psi \Gamma_5 \Gamma_a T^a \psi + \mu^{-2}[(\bar\psi\psi)^2 - (\bar\psi \Gamma_5 \psi)^2] \Big\} \sqrt{-g}\, d^4x - \tfrac{1}{16} \varepsilon_{abcd} [R^{ab} - \lambda e^a e^b][R^{cd} - \lambda e^c e^d]$$

Maxwell / YM

Dirac

Nambu-Jona Lasinio

Einstein +cc

- Standard couplings: $\nabla_\nu = \partial_\nu - iA_\nu + \frac{1}{4}\Gamma_{ab}\omega_\nu^{ab} - \frac{i\mu}{2}\Gamma_\nu$
- No $\partial_\mu \partial_\nu \psi$ - terms: fermions behave as standard matter
- Cosmological constant $\Lambda \sim -\lambda^2$
- Newton's constant $G \sim \lambda^{-1}$

Phenomenological, low energy, 4D theory.

V. SUSY breaking

For D=4, the only invariant 4-forms are *characteristic classes* (Chern-Weil theorem).

This rules out locally $SO(3,2)$ -invariant actions.

- ➔ $SO(3,2)$ is broken down to $SO(3,1)$
- ➔ Local SUSY must also be broken
- ➔ The surviving local symmetry is $U(1) \times SU(2) \times SO(3,1)$

There might exist SUSY-invariant vacua, but this is not a invariance of the action.

Local SUSY could be an approximate symmetry for some configurations or in asymptotic regions, like Poincaré or AdS invariance. (Contingent symmetries)

In odd D SUSY transformations such as

$$\left.\begin{aligned} \delta A_\mu &= -\tfrac{i}{2}(\bar{\varepsilon}\Gamma_\mu\psi + \bar{\psi}\Gamma_\mu\varepsilon) \\ \delta\omega^a_\mu &= \bar{\varepsilon}\Gamma^a\Gamma_\mu\psi + \bar{\psi}\Gamma^a\Gamma_\mu\varepsilon \\ \delta\psi &= \tfrac{1}{D}\nabla\!\!\!\!/\,\varepsilon\ , \qquad \delta e^a_\mu = 0 \end{aligned}\right\} \quad \delta\mathcal{A} = \nabla\!\!\!\!/\,\varepsilon$$

change the action by a boundary term. However, the condition

$$\left(\delta^\mu_\nu - \frac{1}{4}\Gamma_\nu\Gamma^\mu\right)\nabla_\mu\epsilon = 0$$

restricts this possibility to certain backgrounds (BPS states).

A bosonic vacuum ($\psi = 0$) is invariant provided $\boxed{\nabla\!\!\!\!/\,\varepsilon = 0}$ [Killing spinor]

$\mathcal{N}$ globally defined Killing spinors implies $\mathcal{N}$ unbroken global (rigid) SUSYs.
Hence, SUSY is again a conditional symmetry that depends on the vacuum.

VI. Overview and summary

Ingredients:

- $\mathrm{Ad}_G + \mathrm{Fund}_G \hookrightarrow \mathrm{Ad}_{G'}$, $G \subseteq G'$

- Superconnection:

$$\mathcal{A} = [\,\tfrac{1}{2}\omega^{ab}\mathbf{J}_{ab} + \cdots\,] + [\bar{Q}^i\chi_i + \bar{\chi}^i Q_i] + [\,A^r\mathbf{T}_r\,]$$

$$\begin{bmatrix}\text{spacetime}\\ \text{symmetry}\end{bmatrix} \qquad \begin{bmatrix}\text{charged}\\ \text{fermion}\end{bmatrix} \qquad \begin{bmatrix}\text{internal}\\ \text{symmetry}\end{bmatrix}$$

- Matter ansatz: $\chi_\mu^\alpha = (\Gamma_\mu)^\alpha{}_\beta\ \psi^\beta, \ \ \left(\delta_\nu^\mu - \frac{1}{D}\Gamma_\nu\Gamma^\mu\right)\chi_\mu \equiv 0$

- Invariant trace for the largest subgroup: $\langle\ \rangle$

- Hodge dual $\circledast$

Consequences of u-SUSY (What is new?)

- All fields are part of the same superconnection $\mathcal{A}$

  $\mathbb{F}$ (matter) - sections

  $\mathbb{B}$ (interactions) - connections

  } *Packaged into a single gauge connection*

- Not all internal and spacetime symmetries are allowed
- Only s = ½,1 fundamental fields (s =0, 3/2, 2 are composite)
- Only standard kinetic terms (Yang-Mills, Dirac, Chern-Simons)
- Only standard gauge couplings ($\sim\bar{\psi}\, A\, \psi$ ✓, $\bar{\psi}\, A_1 A_2 A_3 \psi$ ✗)
- No SUSY pairs, no matching d.o.f., no hidden sectors

(What is new?)

- (Bare) coupling constants and masses are fixed
- Odd D: action is invariant under gauge supergroup
- Even D: Action is not SUSY invariant
- $F = 0$ vacua have full SUSY
- Nambu – Jona-Lasinio couplings
- Gravity is necessarily included:
  - Spinors in tangent space ➔Vielbein
  - Local Lorentz symmetry ➔ spin connection

} Gravity!

In the end, what is the role of SUSY?

Guiding principle

- Relates spacetime and internal groups that can be combined
- Superalgebra, gauge couplings, coupling constants, masses
- Necessary presence of gravity
- Invariance under supergroup broken down to

  (Internal gauge group) x (Lorentz group)
- Invariance under entire supergroup for some vacua: BPS states

Open questions

- Observable effects (e.g., in graphene)
- Vacua
- Renormalizability? Hierarchy? …
- $U(1)$ x $SU(2)$ x $SU(3)$?
- Unconventional SUGRA:

$$\bar{\chi}_\mu \Gamma^{\mu\nu\lambda} \nabla_\nu \chi_\lambda \rightarrow \frac{(D-1)(D-2)}{2} \bar{\psi} \not{\partial} \psi$$

- B-E-G-H-boson?

Perhaps we have been living with SUSY all along but looking for the wrong signals.

The reports of my death have been greatly exaggerated...

(Mark Twain)

Thanks!