Determining the strong coupling from Lattice QCD

R. Horsley

– University of Edinburgh –

together with: T. Onogi, R. Sommer

[Mainz Workshop, September 2015, Mainz]

Introduction

Strong coupling constant:

$$\alpha_s = \frac{g^2}{4\pi}$$

- Fundamental parameter of QCD sector of the Standard Model along with quark masses, θ, ...
- Key role for (eg):
  - LHC collider physics ($H \to b\bar{b}$, $H \to gg$, ...)
  - vacuum stability
  - ...

Continuum QCD

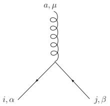
QCD – the theory of strong interactions

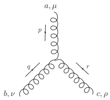
$$\mathcal{L} = -\tfrac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + \sum_{f=1}^{n_f} \overline{q}_f (i\gamma^\mu D_\mu - m_{q_f}) q_f$$

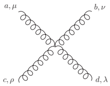
$$\left.\begin{array}{lcl} F_{\mu\nu} & = & \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu] \\ D_\mu & = & \partial_\mu + ig\lambda^a A^a_\mu \end{array}\right\} \text{'generalised' QED, } U(1) \to SU(3)$$

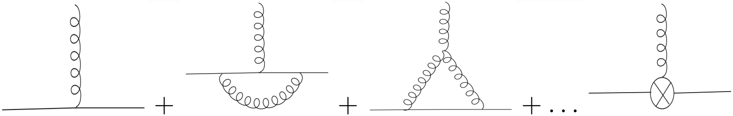
6 quarks: $\begin{pmatrix} u \\ d \end{pmatrix}$ $\begin{pmatrix} s \\ c \end{pmatrix}$ $\begin{pmatrix} t \\ b \end{pmatrix}$ 8 gluons: A^a_μ

Vertices:







$i\Gamma^{(3)}_{q\bar{q}A} \quad =$ 

Divergencies in loops need regularisation (eg dimensional) and then a renormalisation procedure, $\mathcal{S}$ (eg $\mathcal{S} = \overline{MS}$). This procedure introduces a scale μ

$g_{\mathcal{S}}(\mu)^2$ – the QCD coupling constant runs

Change from one scheme (eg $\mathcal{S}$) to another (eg $\overline{MS}$)

$$g^2_{\overline{MS}} = g^2_{\mathcal{S}} \left(1 + c_g^{(1)} g^2_{\mathcal{S}} + \dots\right)$$

The 'running' of the QCD coupling constant as the scale changes is controlled by the β function,

$$\frac{\partial g_{\mathcal{S}}(\mu)}{\partial \log \mu} = \beta^{\mathcal{S}}(g_{\mathcal{S}}(\mu))$$

with

$$\beta^{\mathcal{S}}(g_{\mathcal{S}}) = -b_0 g_{\mathcal{S}}^3 - b_1 g_{\mathcal{S}}^5 - b_2^{\mathcal{S}} g_{\mathcal{S}}^7 - b_3^{\mathcal{S}} g_{\mathcal{S}}^9 - \ldots ,$$

Integrating

$$\frac{\Lambda^{\mathcal{S}}}{\mu} = \exp\left(-\frac{1}{2 b_0 g_{\mathcal{S}}^2}\right) \left(b_0 g_{\mathcal{S}}^2\right)^{-\frac{b_1}{2 b_0^2}} \exp\left\{-\int_0^{g_{\mathcal{S}}} d\xi \left[\frac{1}{\beta^{\mathcal{S}}(\xi)} + \frac{1}{b_0 \xi^3} - \frac{b_1}{b_0^2 \xi}\right]\right\}$$

with (scheme dependent) integration constant $\Lambda^{\mathcal{S}}$

To leading order

$$\alpha_s(\mu) \sim \frac{4\pi}{b_0 \ln(\mu/\Lambda_{\mathcal{S}})^2}$$

b coefficients

The first two coefficients are scheme independent:

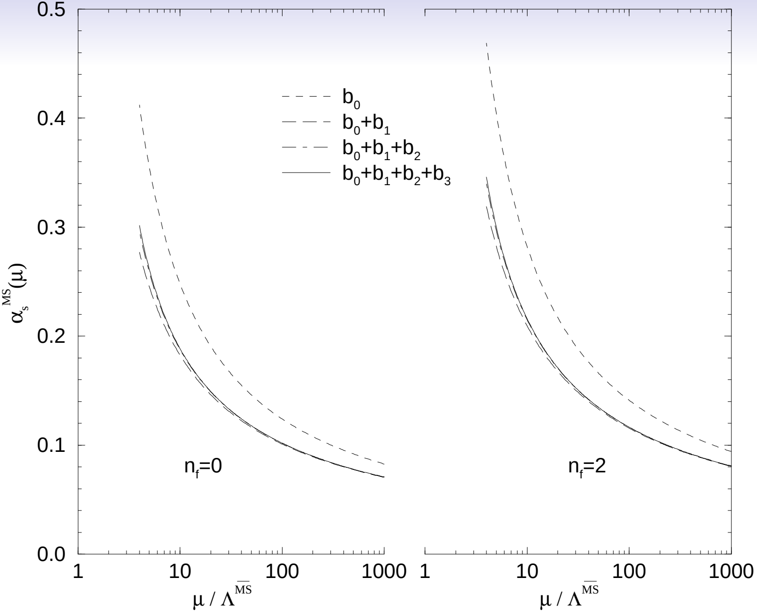
$$b_0 = \frac{1}{(4\pi)^2}\left(11 - \frac{2}{3}n_f\right), \qquad b_1 = \frac{1}{(4\pi)^4}\left(102 - \frac{38}{3}n_f\right)$$

$\overline{MS}$ scheme: [only defined perturbatively]

$$\begin{aligned}
b_2^{\overline{MS}} &= \frac{1}{(4\pi)^6}\left(\frac{2857}{2} - \frac{5033}{18}n_f + \frac{325}{54}n_f^2\right)\\
b_3^{\overline{MS}} &= \frac{1}{(4\pi)^8}\left[\frac{149753}{6} + 3564\,\zeta_3 - \left(\frac{1078361}{162} + \frac{6508}{27}\zeta_3\right)n_f\right.\\
&\qquad\left. + \left(\frac{50065}{162} + \frac{6472}{81}\zeta_3\right)n_f^2 + \frac{1093}{729}n_f^3\right]
\end{aligned}$$

[4-loops: T. van Ritbergen et al., hep-ph/9701390]

Mass independent, fixed n_f scheme



- $\mu/\Lambda^{\overline{MS}} = 10 \iff \mu \sim 2.5\,\text{GeV}$

$\overline{MS}$ is a mass independent, fixed n_f scheme

- 'Relatively' easy to compute b coefficients
- Cross quark thresholds, need to match $n_f \to n_f + 1$

$$\alpha_{\overline{MS}}^{(n_f)}(\mu) = \alpha_{\overline{MS}}^{(n_f+1)}(\mu) \left\{ 1 + \sum_{k=1}^{\infty} \sum_{n=0}^{k} c_{kn} \left[\frac{\alpha_{\overline{MS}}^{(n_f+1)}(\mu)}{\pi} \right]^k \ln^n \left[\frac{\mu^2}{m_{\overline{MS}}^2(\mu)} \right] \right\}$$

with

$$c_{10} = 0 \,, \quad c_{20} = \frac{11}{72} \,, \quad c_{30} = \frac{564731}{124416} - \frac{82043}{27648}\zeta_3 - \frac{2633}{31104} n_f \,, \quad \ldots c_{43}$$

- Usually choose $\mu = m_{\overline{MS}}(\mu)$ (ie no logs)
- So 'secret' scale dependence of b coefficients
- Perturbative matching, only trust (?) at charm mass and above, ie $n_f = 3 \to 4$
- In a MOM scheme (more physical), explicit mass dependence – only b_0^{MOM} , b_1^{MOM} known [Jegerlehner et al., hep-ph/9809485]
  But smoother behaviour across quark thresholds

Determining $\alpha_{\overline{MS}}$

Basic method 'measure' a short distance quantity $\mathcal{O}(\mu)$ match in a perturbative expansion

$$\mathcal{O}(\mu) = c_1\alpha_{\overline{MS}}(\mu) + c_2\alpha_{\overline{MS}}(\mu)^2 + \ldots$$

- Continuum determinations
  Cross section: need to find a suitable process over a range of high enough energies, hadronisation problems, ...
- Lattice
  'Design' (Euclidean) $\mathcal{O}$
  Need 2 scales - hadron mass (M_N, r_0, ...) and also need high energies
  Main question:
  Are we in a perturbative regime? Are there non-perturbative contributions?

Phenomenological determinations of $\alpha_{\overline{MS}}$

PDG (4 categories):

- hadronic τ decays
- hadronic final states of e^+e^- annihilation
- deep inelastic lepton nucleon scattering
- precision electroweak data

This gives [PDG]:

$$\alpha_{\overline{MS}}(M_Z) = 0.1183(12)$$

For the lattice to help needs also a precision of $\sim 1\%$

Lattice determinations of $\alpha_{\overline{MS}}$

- Vertices
- Schrödinger functional
- Potential at short distances
- Current two-point functions
- Observables at the lattice spacing scale

Main question: Are we in a low order perturbative regime? Are there non-perturbative contributions?

- Estimation

$$L \gg \text{hadron size} \sim \Lambda_{\rm QCD}^{-1} \quad \text{and} \quad 1/a \gg \mu \quad \Longrightarrow L/a \ggg \mu/\Lambda_{\rm QCD}$$

$L/a \sim 20 - 64$ so

$$\mu \ll L/a \times \Lambda_{\rm QCD} \sim 5 - 20\,\text{GeV} \Longrightarrow \mu \sim 1 - 3\text{GeV at best}$$

Is this a perturbative scale ?

- 'Perturbative' series

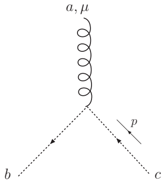
$$\mathcal{O} = c_1\alpha_s + c_2\alpha_s^2 + \ldots + c_n\alpha_s^n + \mathrm{O}(\alpha_s^{n+1}) + \mathrm{O}(\exp(-\gamma/\alpha_s))$$

  - 'NP' piece $\sim \exp(-\gamma/\alpha_s)$ (instantons, renormalons, ...) or equivalently power corrections: $\sim (\Lambda/\mu)^\gamma$
  - So ideally want small α_s, when both the $\exp(-\gamma/\alpha_s)$ term is negligible and low order PT sufficient

Determination of α_s from QCD vertices

- 'Natural' definition
- Zero incoming ghost momentum in ghost-ghost-gluon vertex
- Simplification: vertex not renormalised (Taylor)
  'T' or 'MM' (minimal mom) scheme

$$\alpha_{\mathrm{T}}(\mu) = D^{\mathrm{ghost}}_{\mathrm{lat}}(\mu, a)^2 D^{\mathrm{gluon}}_{\mathrm{lat}}(\mu, a)\, \frac{g_0^2(a)}{4\pi}$$



- $D^{\mathrm{ghost}}_{\mathrm{lat}}$, $D^{\mathrm{gluon}}_{\mathrm{lat}}$ (bare lattice) dressed ghost/gluon 'form factors' propagator functions in the Landau gauge

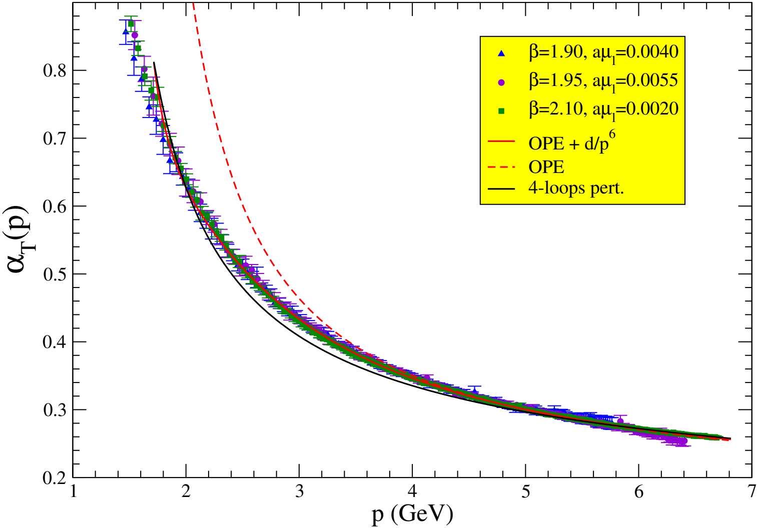
$$D^{ab}(p) = -\delta^{ab}\, \frac{D^{\mathrm{ghost}}(p)}{p^2}\,, \quad D^{ab}_{\mu\nu}(p) = \delta^{ab}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{D^{\mathrm{gluon}}(p)}{p^2}$$

[$D^{\mathrm{ghost/gluon}}(p) = D^{\mathrm{ghost/gluon}}_{\mathrm{lat}}(p, 0)$ (continuum)]

- Thus there is now no need to compute the ghost-ghost-gluon vertex, just the ghost and gluon propagators

α_T: ETM: $n_f = 4$

[arXiv:1201.5770]



- condensate necessary (to increase fit region) $\alpha_T(p) \rightarrow \alpha_T(p) + \frac{d}{p^6}$

Determination of α_s from the Schrödinger functional

- Developed by ALPHA collaboration, presently for $n_f = 0,\ 2,\ 4$
- Split determination of α_s at large μ and hadronic scale into two lattice calculations – connected by 'step scaling'
  - SF (finite volume) scheme, choose a large scale

$$\mu = \frac{1}{L} \sim 10\,,\ldots\,,100\,\mathrm{GeV} \qquad a/L << 1$$

  - Choose

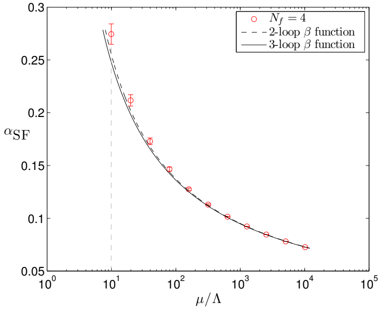
    $$g_{\max}^{\mathrm{SF}\,2} = g^{\mathrm{SF}}(1/L_{\max})^2 \quad \text{so that} \quad L_{\max} \sim 0.5\,\mathrm{fm} \sim 1/(400\,\mathrm{MeV})$$

    Determine $L_{\max}$ in terms of hadronic scale r_0
  - Connect by changing scale $L \to L/2$ in steps to give

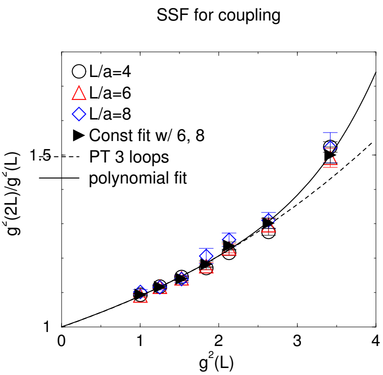
$$g^{\mathrm{SF}}(\mu)^2 \quad \text{at} \quad \mu = 2^k \times \left(\frac{r_0}{L_{\max}}\right) \times r_0^{-1}$$

  - At high scale convert $g^{\mathrm{SF}}(\mu)^2$ to $\overline{MS}$ scheme

(Can check) Non perturbative running of α_s



ALPHA: $n_f = 4$ [arXiv:1011.2332]



PACS-CS: $n_f = 3$ [arXiv:0906.3906]

$3/(4\pi) \sim 0.25$

Determination of α_s from the potential at short distances

Force/potential between (infinitely) massive quark–anti-quark pair

$$F(r) = \frac{dV(r)}{dr} = C_F \frac{\alpha_{qq}(r)}{r}$$

alternatively

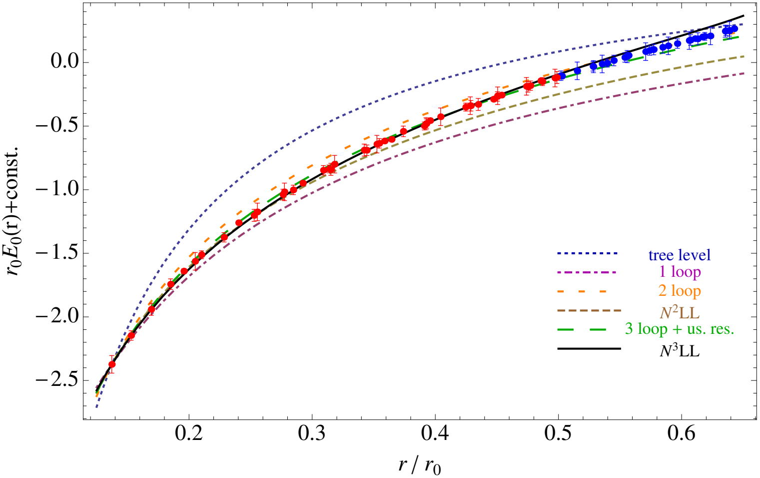
$$V(r) = -C_F \frac{\alpha_V(r)}{r}\,, \qquad \tilde{V}(r) = -C_F \frac{\alpha_V(Q)}{Q^2}\,,$$

- Defines different schemes
- Determine $V(r)$ from Wilson loops

$$\langle W(r,t)\rangle = |c_0|^2 e^{-V(r)t} + \sum_{n\neq 0} |c_n|^2 e^{-V_n(r)t}$$

- Need to fix $V(r)$ at some $r = r_{\rm ref}$ - introduces new renormalisation scale (renormalon) [Force better]
- At N^2LO, $\alpha^4_{\overline{MS}} \ln \alpha_{\overline{MS}}$ terms

Potential results: Bazarov et al arXiv:1205.6155, $n_f = 3$, NNNLO



Determination of α_s from the vacuum polarisation function at short distances

$$\langle J^a_\mu J^b_\nu \rangle = \delta^{ab}[(\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu)\Pi^{(1)}(Q) - Q_\mu Q_\nu \Pi^{(0)}(Q)]$$

- Q_μ is a space like momentum
- $J_\mu \equiv V_\mu, A_\mu$ for (non-singlet) vector/axial-vector currents

Set $\Pi_J(Q) \equiv \Pi^{(0)}_J(Q) + \Pi^{(1)}_J(Q)$, OPE of the vacuum polarisation function $\Pi_{V+A}(Q) = \Pi_V(Q) + \Pi_A(Q)$:

$$\begin{aligned} &\Pi_{V+A}|_{\mathrm{OPE}}(Q^2, \alpha_s) \\ &= c + C_0(Q^2) + C_m^{V+A}(Q^2)\frac{\bar{m}^2(Q)}{Q^2} + \sum_{q=u,d,s} C_{\bar{q}q}^{V+A}(Q^2)\frac{\langle m_Q \bar{q} q\rangle}{Q^4} \\ &\quad + C_{GG}(Q^2)\frac{\langle \alpha_s GG\rangle}{Q^4} + \mathrm{O}(Q^{-6}) \end{aligned}$$

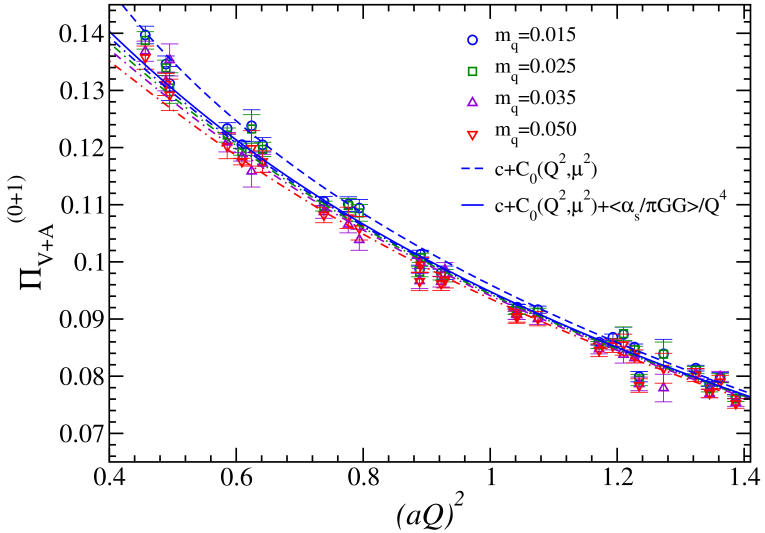
C_X^{V+A} known up to 4-loops (in $\overline{MS}$ scheme)

- c is Q–independent and divergent ultraviolet cutoff $\to \infty$
- NP condensates eg $\langle \alpha_s GG \rangle$
- terms in C_X which do not have a series expansion in α_s

[Use of Adler function, $D(Q^2) \equiv -Q^2 d\Pi(Q^2)/dQ^2$ is a scheme independent finite quantity, and so avoids some of these problems]

Π_{V+A}: JLQCD/TWQCD: $n_f = 2$

[arXiv:0807.0556]



- condensate necessary

Determination of α_s from current two-point functions

[moment method]

$$
\begin{aligned}
G(t) &= a^6 \sum_{\vec{x}} \langle J^\dagger(x) J(0) \rangle \qquad J = m_{0h} \overline{q}_h \gamma_5 q_{h'} \\
&\sim t^{-3} \text{ singularity as } t \to 0
\end{aligned}
$$

q_h, $q_{h'}$ mass degenerate,
m_{0h}, heavy valence quarks

- Consider (finite) moments ($n \geq 4$)

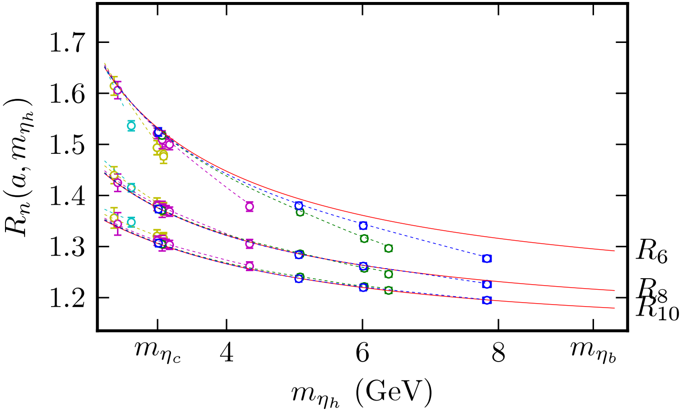
$$G_n = \sum_{t=-(T/2-a)}^{t=T/2-a} t^n G(t)$$

- moments dominated by $t \sim 1/m_{0h}$, ie short distances
- moments become increasingly perturbative for decreasing n
- $$R_n \sim \frac{G_n}{G_n^{(0)}} \sim 1 + r_{n1}\alpha_{\overline{MS}} + r_{n2}\alpha_{\overline{MS}} + r_{n3}\alpha_{\overline{MS}}^3 + \ldots$$

- $r_{ni} = r_{ni}(\mu/m_h^{\overline{MS}}(\mu))$ known (continuum PT)
- leads to a determination of both $\alpha_{\overline{MS}}$ and $m_h^{\overline{MS}}$ (heavy quark mass)

$R_n(a, m_{\eta_h})$: HPQCD: $n_f = 3$

[arXiv:1004.4285]



- $\mu = 3m_h \sim m_{\eta_h}/0.75$
- lattice spacings (dashed lines), together with continuum limit (lines)
- large masses $\Rightarrow$ large lattice artifacts:
  global fit to $(\alpha_{\overline{MS}})^n$, $(\Lambda/(m_{\eta_h}/2))^j$ [heavy quark mass], $(am_{\eta_c})^{2i}$ [lattice spacing]. $i \geq 10$ together with Bayesian fits

Determination of α_s from observables at the lattice spacing scale

General method:

- evaluate a short distance quantity $\mathcal{O}$ at the scale of the lattice spacing $\sim 1/a$
- then determine its relationship to $\alpha_{\overline{MS}}$ via a power series expansion

Much work using this method [eg HPQCD arXiv:0807.1687, Maltman et al arXiv:0807.2020]

$$Y = \sum_{n=1}^{n_{\max}} c_n \alpha_{V'}^n(q^*)$$

- Y: (logarithm) of small Wilson loops, W_{mn}, Creutz ratios, 'tadpole improved' Wilson loops, 'boosted' bare coupling,
- scale $q^* = d/a$, $d \approx 3$ (average gluon momentum at one loop)
-

$$\alpha_{\overline{MS}}(q_0) = \alpha_{V'}(q_0) + d_1 \alpha_{V'}(q_0)^2 + d_2 \alpha_{V'}(q_0)^3 + \cdots \qquad q_0 = 7.5\,\text{GeV}$$

d_1, d_2 are (known) one, two loop coefficients

- As $q \sim 1/a$ cannot separate out discretisation effects from PT, so possible a^2, a^4 (power law or condensate) corrections
- Smaller loops are at higher scale, less effected

Another example: Boosted coupling constant

- $$\alpha^{\Box}(1/a) = \frac{1}{4\pi}\frac{g_0^2}{u_0^4}\,, \qquad u_0 = W_{11}$$
- Perturbative relation

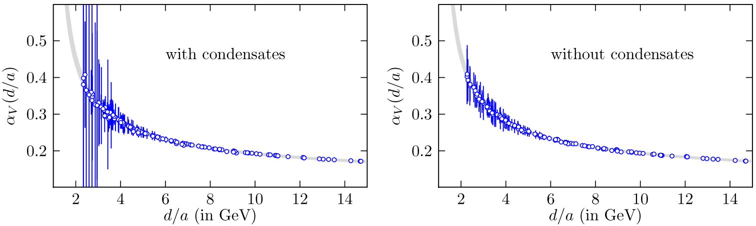
$$\frac{1}{\alpha_{\overline{MS}}(\mu)} = \frac{1}{\alpha^{\Box}(1/a)} + 4\pi(2b_0 \ln a\mu - t_1^P) + (4\pi)^2(2b_1 \ln a\mu - t_2^P)\alpha^{\Box}(1/a)$$

  $t_1^{\Box}$ and $t_2^{\Box}$ are known
- choose μ so constant term vanishes: similar $\mu \sim 3/a$ found

$\alpha_{V'}$: HPQCD: $n_f = 3$

[arXiv:0807.1687]

Running of $\alpha_{V'}$ at various scales



- various $\alpha_{V'}$ from each Y at various lattice sizes ie $q^* = d/a$ scales
- $n_{\max} = 10$, first three c_n known, then Bayesian analysis
- effect of condensates small: but more pronounced for smaller scales
- convert at $q^* = d/a = 7.5\,\text{GeV}$ to $\overline{MS}$ scheme
- independent analysis by Maltman et al [arXiv:0807.1687]

FLAG

[arXiv:1310.8555]

Development of 'goodness' criteria:

'primary' coupling: $\alpha_{\text{eff}} = \alpha_{SF}\,, \alpha_V\,, \alpha^{\Box}\,, \alpha_T\,, \mathcal{O}/c_1\,, \ldots$

- Renormalisation scale:
  - ★ all points relevant in the analysis have $\alpha_{\text{eff}} < 0.2$
  - ○ all points have $\alpha_{\text{eff}} < 0.4$ and at least one $\alpha_{\text{eff}} \leq 0.25$
  - ■ otherwise
- Perturbative behaviour:
  - ★ verified over a range of a factor 2 in α_{eff} (with no power corrections)
  - ○ agreement with perturbation theory over a range of a factor 1.5 in α_{eff} (possibly with power corrections)
  - ■ otherwise
- Continuum extrapolation[1] (at reference point of $\alpha_{\text{eff}} = 0.3$):
  - ★ three lattice spacings with $\mu a < 0.5$
  - ○ three lattice spacings with $1 < \mu a < 1.5$ (reach down to 1)
  - ■ otherwise

---

[1] for Wilson loops replace by Lattice spacings [as $q^* = d/a$]

★ 3 or more lattice spacings, at least 2 points below $a = 0.1$ fm

○ 2 lattice spacings, at least 1 point below $a = 0.1$ fm

■ otherwise

Generate tables: Observables at the lattice spacing scale

- $O(50)$ publications, for $N_f = 0, 2, 3, 4$

Collaboration	Ref.	N_f	publication status	renormalization scale	perturbative behaviour	lattice spacings	scale	$\Lambda_{\overline{\rm MS}}$[MeV]	$r_0\Lambda_{\overline{\rm MS}}$
HPQCD 10[a] [§]	[73]	2+1	A	○	★	★	$r_1 = 0.3133(23)$ fm	340(9)	0.812(22)
HPQCD 08A[a]	[514]	2+1	A	○	★	★	$r_1 = 0.321(5)$ fm[††]	338(12)[⋆]	0.809(29)
Maltman 08[a]	[517]	2+1	A	○	○	○	$r_1 = 0.318$ fm	352(17)[†]	0.841(40)
HPQCD 05A[a]	[513]	2+1	A	○	○	○	r_1[††]	319(17)[⋆⋆]	0.763(42)
QCDSF/UKQCD 05	[518]	2	A	★	■	★	$r_0 = 0.467(33)$ fm	261(17)(26)	0.617(40)(21)[b]
SESAM 99[c]	[519]	2	A	○	■	■	$c\bar{c}$(1S-1P)		
Wingate 95[d]	[520]	2	A	★	■	■	$c\bar{c}$(1S-1P)		
Davies 94[e]	[521]	2	A	★	■	■	Υ		
Aoki 94[f]	[522]	2	A	★	■	■	$c\bar{c}$(1S-1P)		
QCDSF/UKQCD 05	[518]	0	A	★	○	★	$r_0 = 0.467(33)$ fm	259(1)(20)	0.614(2)(5)[b]
SESAM 99[c]	[519]	0	A	★	■	■	$c\bar{c}$(1S-1P)		
Wingate 95[d]	[520]	0	A	★	■	■	$c\bar{c}$(1S-1P)		
Davies 94[e]	[521]	0	A	★	■	■	Υ		
El-Khadra 92[g]	[523]	0	A	★	○	○	$c\bar{c}$(1S-1P)	234(10)	0.593(25)[h]

[a] The numbers for Λ have been converted from the values for $\alpha_s^{(5)}(M_Z)$.
[§] $\alpha_{\overline{\rm MS}}^{(3)}(5\ {\rm GeV}) = 0.2034(21)$, $\alpha_{\overline{\rm MS}}^{(5)}(M_Z) = 0.1184(6)$, only update of intermediate scale and c, b quark masses,

Generate tables: Vertices

- $O(50)$ publications, for $N_f = 0, 2, 3, 4$

Collaboration	Ref.	N_f	publication status	renormalization scale	perturbative behaviour	continuum extrapolation	scale	$\Lambda_{\overline{\rm MS}}$[MeV]	$r_0\Lambda_{\overline{\rm MS}}$
ETM 13D	[544]	2+1+1	A	○	○	■	f_π	$314(7)(14)(10)^{\S}$	$0.752(18)(34)(81)^{\dagger}$
ETM 12C	[545]	2+1+1	A	○	○	■	f_π	$324(17)^{\S}$	$0.775(41)^{\dagger}$
ETM 11D	[546]	2+1+1	A	○	○	■	f_π	$316(13)(8)(^{+0}_{-9})^{\star}$	$0.756(31)(19)(^{+0}_{-22})^{\dagger}$
Sternbeck 12	[547]	2+1	C				only running of α_s in Fig. 4		
Sternbeck 12	[547]	2	C				Agreement with $r_0\Lambda_{\overline{\rm MS}}$ value of [59]		
Sternbeck 10	[548]	2	C	○	★	■			$0.60(3)(2)^{\#}$
ETM 10F	[549]	2	A	○	○	○	f_π	$330(23)(22)(^{+0}_{-33})$	$0.72(5)^{+}$
Boucaud 01B	[539]	2	A	○	○	■	$K^* - K$	$264(27)^{\star\star}$	$0.669(69)$
Sternbeck 12	[547]	0	C				Agreement with $r_0\Lambda_{\overline{\rm MS}}$ value of [505]		
Sternbeck 10	[548]	0	C	★	★	■			$0.62(1)^{\#}$
Ilgenfritz 10	[550]	0	A	★	★	■	only running of α_s in Fig. 13		
Boucaud 08	[543]	0	A	○	○	■	$\sqrt{\sigma} = 445\,$MeV	$224(3)(^{+8}_{-5})$	$0.59(1)(^{+2}_{-1})$
Boucaud 05	[540]	0	A	■	○	■	$\sqrt{\sigma} = 445\,$MeV	$320(32)$	$0.85(9)$
Soto 01	[551]	0	A	○	○	○	$\sqrt{\sigma} = 445\,$MeV	$260(18)$	$0.69(5)$
Boucaud 01A	[552]	0	A	○	○	○	$\sqrt{\sigma} = 445\,$MeV	$233(28)$ MeV	$0.62(7)$
Boucaud 00B	[553]	0	A	○	○	○		only running of α_s	

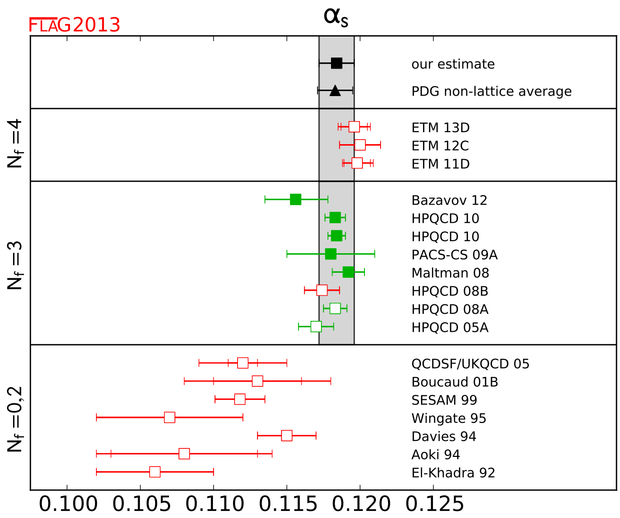
Generate tables

- $O(20)$ extrapolate to $n_f = 5$ at $\mu = M_Z$

Collaboration	Ref.	N_f	publication status	renormalisation scale	perturbative behaviour	continuum extrapolation	$\alpha_{\overline{\rm MS}}(M_{\rm Z})$	Method	Table
ETM 13D	[544]	2+1+1	A	○	○	■	0.1196(4)(8)(16)	gluon-ghost vertex	37
ETM 12C	[545]	2+1+1	A	○	○	■	0.1200(14)	gluon-ghost vertex	37
ETM 11D	[546]	2+1+1	A	○	○	■	$0.1198(9)(5)(^{+0}_{-5})$	gluon-ghost vertex	37
Bazavov 12	[503]	2+1	A	○	○	○	$0.1156(^{+21}_{-22})$	Q-$\bar{Q}$ potential	33
HPQCD 10	[73]	2+1	A	○	○	○	0.1183(7)	current two points	36
HPQCD 10	[73]	2+1	A	○	★	★	0.1184(6)	Wilson loops	35
PACS-CS 09A	[486]	2+1	A	★	★	○	$0.118(3)^{\#}$	Schrödinger functional	32
Maltman 08	[517]	2+1	A	○	○	○	0.1192(11)	Wilson loops	35
HPQCD 08B	[85]	2+1	A	■	■	■	0.1174(12)	current two points	36
HPQCD 08A	[514]	2+1	A	○	★	★	0.1183(8)	Wilson loops	35
HPQCD 05A	[513]	2+1	A	○	○	○	0.1170(12)	Wilson loops	35
QCDSF/UKQCD 05	[518]	$0, 2 \to 3$	A	★	■	★	0.112(1)(2)	Wilson loops	35
Boucaud 01B	[539]	$2 \to 3$	A	○	○	■	0.113(3)(4)	gluon-ghost vertex	37
SESAM 99	[519]	$0, 2 \to 3$	A	★	■	■	0.1118(17)	Wilson loops	35
Wingate 95	[520]	$0, 2 \to 3$	A	★	■	■	0.107(5)	Wilson loops	35
Davies 94	[521]	$0, 2 \to 3$	A	★	■	■	0.115(2)	Wilson loops	35
Aoki 94	[522]	$2 \to 3$	A	★	■	■	0.108(5)(4)	Wilson loops	35
El-Khadra 92	[523]	$0 \to 3$	A	★	○	○	0.106(4)	Wilson loops	35

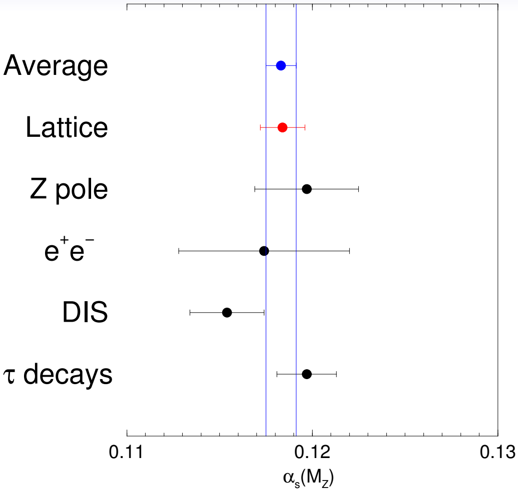
Result with a linear continuum extrapolation in a.

- In final estimation of $\alpha_{\overline{MS}}(M_Z)$ use results with
  - no red squares
  - as matching perturbative, use only $n_f = 3$ results and run to charm when match to $n_f = 4$ (and then match at bottom for $n_f = 5$)
- weighted average for final central value
  conservatively estimate error (estimate perturbative uncertainty)
  - perturbative truncation errors potentially largest source of errors
  - HPQCD 08A – Maltman 08 central difference for $\alpha_{\overline{MS}}$ (for $g^{\square}$) ~ 0.0009 (on overlapping data sets)
  - estimate taking (estimated size) of c_4 as uncertainty



- $\alpha_{\overline{MS}}(M_Z) = 0.1184(12)$

Conclusions



- Tremendous progress
- Weighted average with PDG results:

$$\alpha_{\overline{MS}}(M_Z) = 0.1183(8)$$