34th International Symposium on
Lattice Field Theory

University of Southampton
24–30 July 2016

LATTICE
2016

Looking forward to see you in Southampton!

http://www.southampton.ac.uk/lattice2016/

Fundamental Parameters from Lattice QCD, MITP

31.08-11.09.2015

$|V_{ud}|$, $|V_{us}|$ from lattice QCD

Andreas Jüttner

University of Southampton

this talk

Hypothesis for this talk:

$$\Gamma(\text{decay}) = \text{CKM}\,(\text{WEAK})\,(\text{EM})\,(\text{STRONG})$$

Kaon, pion leptonic and semileptonic decays

V_{ud}, V_{us}

see Vittorio Lubicz's talk next week for how to go beyond this hypothesis

objective: 1st row unitarity test

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = \delta$$

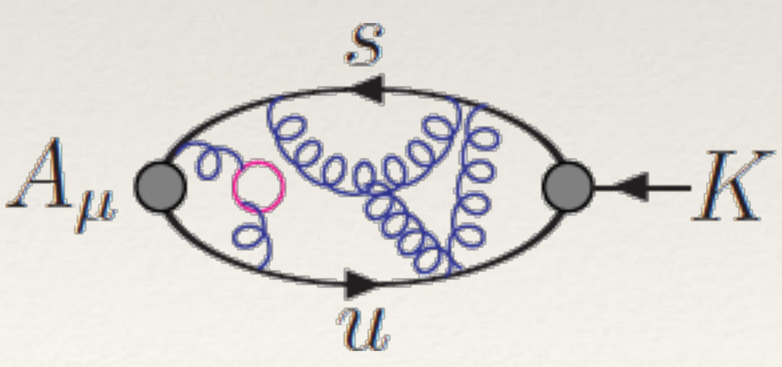
"tree" kaon/pion decays - leptonic

ratio of decay constants → ratio of CKM MEs: Marciano, PRL. 93 (2004) 231803 **hep-ph/0402299**

$$\frac{\Gamma(K^+ \to l^+ \nu_l(\gamma))}{\Gamma(\pi^+ \to l^+ \nu_l(\gamma))} = \left(\frac{|V_{us}|}{|V_{ud}|}\frac{f_{K^+}}{f_{\pi^+}}\right)^2 \frac{m_K\left(1 - m_l^2/m_K^2\right)^2}{m_\pi\left(1 - m_l^2/m_\pi^2\right)^2}\left(1 + \underbrace{\delta_{\mathrm{EM}}^{\mathrm{ChPT}}}_{0.0069(17)}\right)$$

Knecht et al. Eur. Phys. J. C 12, 469–478 (2000)
hep-ph/9909284
Cirigliano & Neufeld, PLB 700 (2011) 7-10
arXiv:1102.0563

leptonic kaon decay constants:



$$\langle 0|\bar{s}/\bar{d}\gamma_\mu\gamma_5 u|K^+/\pi^+(p)\rangle = i f_{K^+/\pi^+} p_\mu$$

“tree” kaon/pion decays - leptonic

experimental status (FLAG1,2):

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_{K^+}}{f_{\pi^+}} = 0.2758(5)$$

FLAVIA Kaon WG EPJ C 69, 399-424 (2010)
arXiv:1005.2323 KTeV, Istra, KLOE

update by Moulson arXiv:1411.5252

Parameter	Value	S
BR($\mu\nu$)	63.58(11)%	1.1
BR($\pi\pi^0$)	20.64(7)%	1.1
BR($\pi\pi^+\pi^-$)	5.56(4)%	1.0
BR($\pi^0 e\nu$)	5.088(27)%	1.2
BR($\pi^0\mu\nu$)	3.366(30)%	1.9
BR($\pi\pi^0\pi^0$)	1.764(25)%	1.0
$\tau_{K^\pm}$	12.384(15) ns	1.2

thanks to updated $K\to\mu\nu_\mu$ BRs

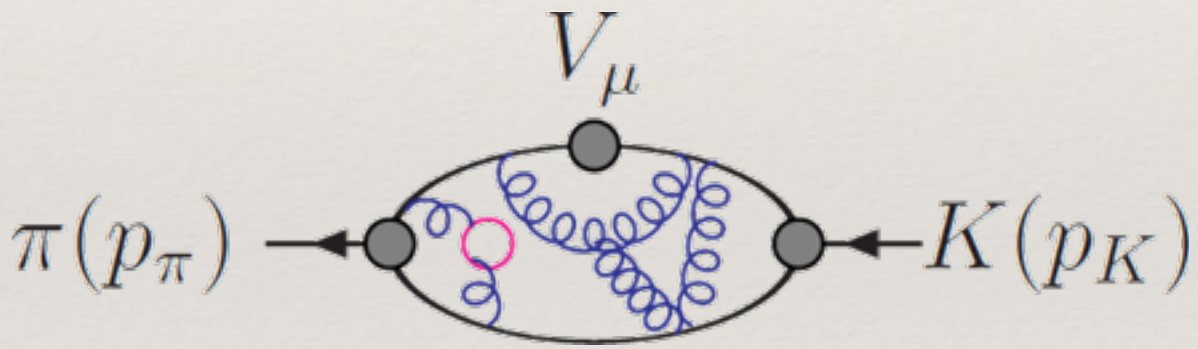
$$\left|\frac{V_{\rm us}}{V_{\rm ud}}\right|\frac{f_{K^+}}{f_{\pi^+}} = 0.2760(4)$$

δ≈0.14%

“tree” kaon/pion decays - semileptonic

semi-leptonic kaon decay:

$$\Gamma(K \to \pi l \nu) = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} S_{\rm EW} (1 + \delta_{SU(2)}^{\rm ChPT} + \delta_{\rm EM}^{\rm ChPT})^2 \, I \, \left(f_+^{K^0\pi^-}(0) |V_{us}| \right)^2$$



matrix element and form factors:

$$\langle \pi(p_\pi) | V_\mu(0) | K(p_K) \rangle = f_+^{K\pi}(q^2)(p_K + p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K - p_\pi)_\mu$$

“tree” kaon/pion decays - semileptonic

experimental status (FLAG1,2):

$$|V_{us}|f_+^{K^0\pi^-}(0) = 0.2163(5)$$

FLAVIA Kaon WG EPJ C 69, 399-424 (2010) **arXiv:1005.2323**
KTeV, Istra, KLOE

update Moulson

0.2163(6)
0.2166(6)
0.2155(13)
0.2160(11)
0.2158(14)

arXiv:1411.5252 →

			Approx contrib to % err			
Mode	$V_{us}\, f_+(0)$	% err	BR	τ	Δ	I
K_{Le3}	0.2163(6)	0.26	0.09	0.20	0.11	0.05
$K_{L\mu3}$	0.2166(6)	0.28	0.15	0.18	0.11	0.06
K_{Se3}	0.2155(13)	0.61	0.60	0.02	0.11	0.05
$K^{\pm}_{e3}$	0.2172(8)	0.36	0.27	0.06	0.23	0.05
$K^{\pm}_{\mu3}$	0.2170(11)	0.51	0.45	0.06	0.23	0.06

Improvement due to new computation of strong isospin correction $\delta^{\rm ChPT}_{\rm SU(2)}$ which depends on quark mass ratio $Q^2 = \dfrac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$ now taken from FLAG (Q=22.6(7)(6))

$$|V_{us}|f_+^{K^0\pi^-}(0) = 0.2165(4)$$

δ≈0.18%

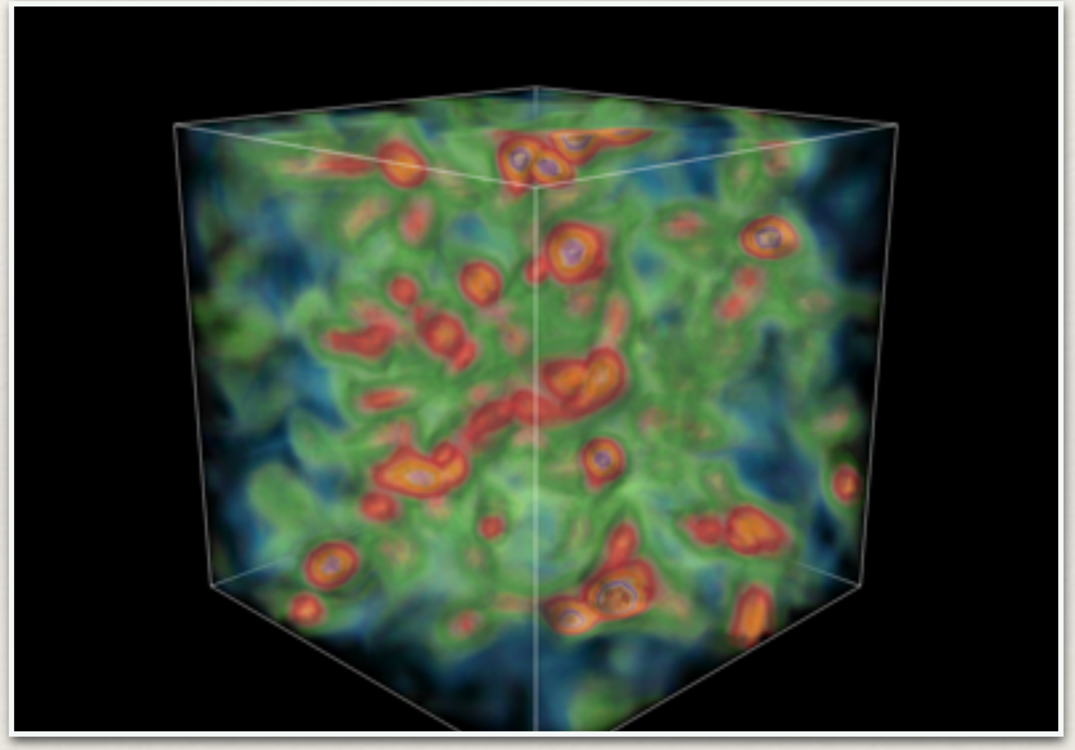
State of the art simulations

What we can do

- mass degenerate up and down quarks at their physical point
- physical strange and charm quarks ($\rightarrow N_f = 2,\ 2+1,\ 2+1+1$ QCD)
- bottom needs special treatment
- cut-off $a^{-1} \leq 4\text{GeV}$
- volume $L \leq 6\,fm$

light quarks u,d,s fit into the window:

$$L^{-1} \ll \begin{array}{c}\text{relevant}\\ \text{energy scales}\end{array} \ll a^{-1}$$



action density of RBC/UKQCD physical point DWF ensemble

What comes next

- add isospin breaking
- add electromagnetism

$K \to \pi l \nu$

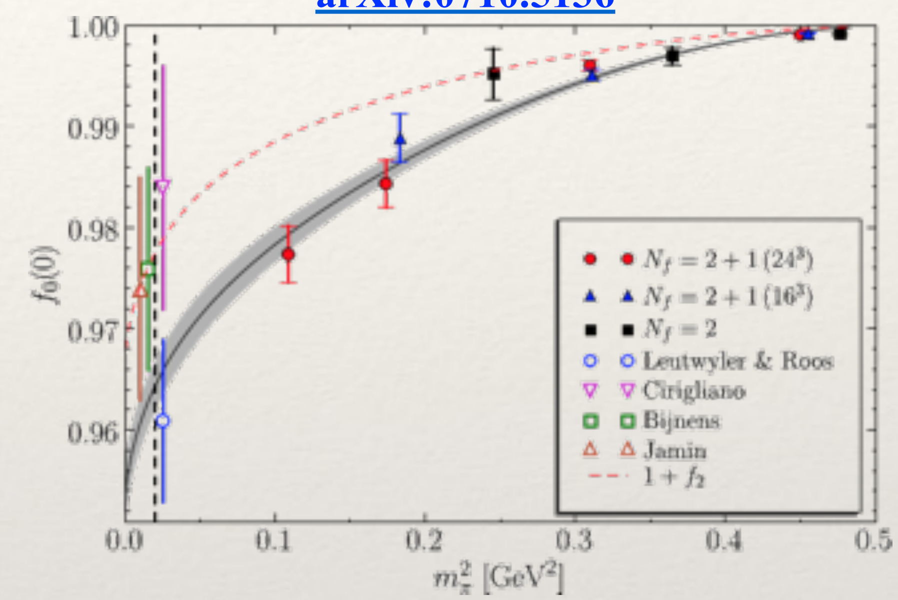
How is sub-percent precision possible in lattice QCD?

- until recently the error budget was dominated by either
  - statistical noise
  - chiral extrapolation

- the latest/most competitive results are from ensembles with physical quark masses

- leading syst. errors typically
  - cutoff effects
  - finite volume effects
  - isospin breaking effects

Phys.Rev.Lett. 100 (2008) 141601
arXiv:0710.5136



How is sub-percent precision possible in lattice QCD? Example: K→π ff

$$\langle\pi(p_\pi)|V_\mu(0)|K(p_K)\rangle = f_+^{K\pi}(q^2)(p_K+p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K-p_\pi)_\mu$$

- $f_+(0)$ dimensionless

- SU(3) symmetry - exactly preserved:
  $f_+(0)\to 1$ in SU(3) symmetric limit exactly manifest in finite volume and at finite lattice spacing
  finite volume errors and cutoff effects apply only to difference of form factor from the SU(3)-limit

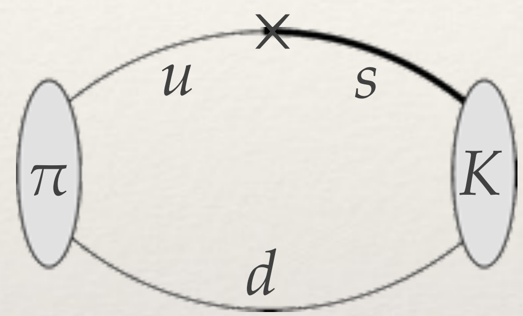
- RBC/UKQCD results
  N_f=2+1 DWF, m_π=139MeV,…,693MeV
  $a = 0.11,\ 0.08\ fm$
  $m_\pi L \geq 3.8$

RBC/UKQCD JHEP06(2015)164
arXiv:1504.01692

correlation functions

$$\langle \pi(p_\pi)|V_\mu|K(p_K)\rangle = f_+^{K\pi}(q^2)(p_K+p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K-p_\pi)_\mu$$

at q^2=0



$$C_{\pi K}(t_{\rm snk}, t, \vec{p}_{\rm snk}, \vec{p}_{\rm src}) \equiv \sum_{\vec{x},\vec{y},\vec{z}} e^{i\vec{p}_{\rm snk}(\vec{z}-\vec{y})} e^{i\vec{p}_{\rm src}(\vec{y}-\vec{x})} \langle\, O_\pi(t_{\rm snk},\vec{z})\, V_\mu(t,\vec{y})\, O_K^\dagger(0,\vec{x})\,\rangle$$

$$\rightarrow \frac{Z_K\, Z_\pi}{4E_K\, E_{\rm src}} \langle\, P_\pi(\vec{p}_\pi)\,|\,V_\mu\,|\,P_K(\vec{p}_K)\,\rangle$$

$$\times \left\{ \theta(t_\pi - t)\, e^{-E_K t - E_\pi(t_\pi - t)} + c_\mu\, \theta(t-t_\pi)\, e^{-E_K(T-t)-E_\pi(t-t_\pi)} \right\}$$

(here ground state only)

all-mode-averaging

- light quark propagators (inverse of lattice Dirac operator by CG type algorithm) very CPU-time intense
- ideally generate many propagators from different source positions
- all-mode-averaging is a powerful new trick to reduce statistical errors at moderate computational cost

Blum, Izubuchi, Shintani PRD88 (2013) 9, 094503
arXiv:1208.4349

- idea:

$$\delta O(t, t_{\rm src}) = O_{\rm exact}(t, t_{\rm src}) - O_{\rm sloppy}(t, t_{\rm src})$$

$$O_{\rm AMA}(t) = \delta O(t, t_{\rm src}) + \frac{1}{N_t} \sum_{t_{\rm src}} O_{\rm sloppy}(t, t_{\rm src})$$

	m_π	m_K	E_π	Z_V^π
Exact	0.08046(16)	0.28859(33)	0.15632(59)	0.71172(555)
AMA	0.08049(11)	0.28862(17)	0.15683(33)	0.71080(38)
$\sigma_{exact}/\sigma_{AMA}$	1.5	1.9	1.8	14.6

renormalisation of the vector current

- vector current renormalisation via $f_+^{\pi\pi}(0) = f_+^{KK}(0) = 1$ extracted from

$$\text{e.g.} \qquad Z_V^\pi = \frac{C_\pi(t_{\rm snk}, \vec{0})}{C_{\pi\pi}^{(B)}(t_{\rm snk}, t, 0, \vec{0}, \vec{0}\,)}$$

- at finite lattice spacing *a* the two definitions can differ by mass-dependent cutoff effects → two scaling trajectories

Ensemble	Z_V^π	Z_V^K
48I	0.71094(20)	0.71407(9)
64I	0.74295(18)	0.74517(5)

- either compute Z_V individually or build ratios of correlation functions where normalisation cancels

$$2\sqrt{E_K E_\pi}\sqrt{\frac{C_{K\pi}^{(\mu)}(t,\mathbf{p}_K,\mathbf{p}_\pi)\,C_{\pi K}^{(\mu)}(t,\mathbf{p}_\pi,\mathbf{p}_K)}{C_{KK}^{(0)}(t,\mathbf{p}_K,\mathbf{p}_K)\,C_{\pi\pi}^{(0)}(t,\mathbf{p}_\pi,\mathbf{p}_\pi)}} = f_+^{K\pi}(q^2)(p_K+p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K-p_\pi)_\mu$$

vector WI and relation to scalar form factor

$$\langle\pi(p_\pi)|V_\mu|K(p_K)\rangle = f_+^{K\pi}(q^2)(p_K+p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K-p_\pi)_\mu$$

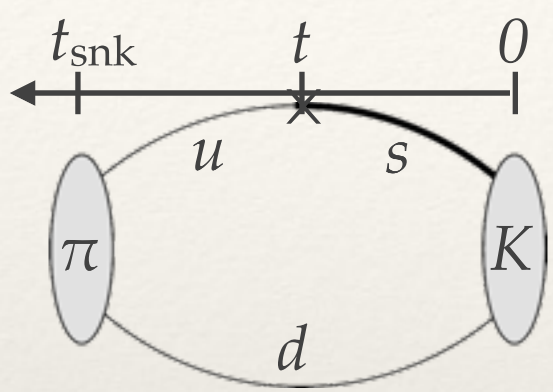
we can also make use of $f_+(0)=f_0(0)$ and the vector Ward identity:

$$\langle\pi(p_\pi)|S|K(p_K)\rangle|_{q^2=0} = f_0^{K\pi}(0)\frac{m_K^2-m_\pi^2}{m_s-m_u}$$

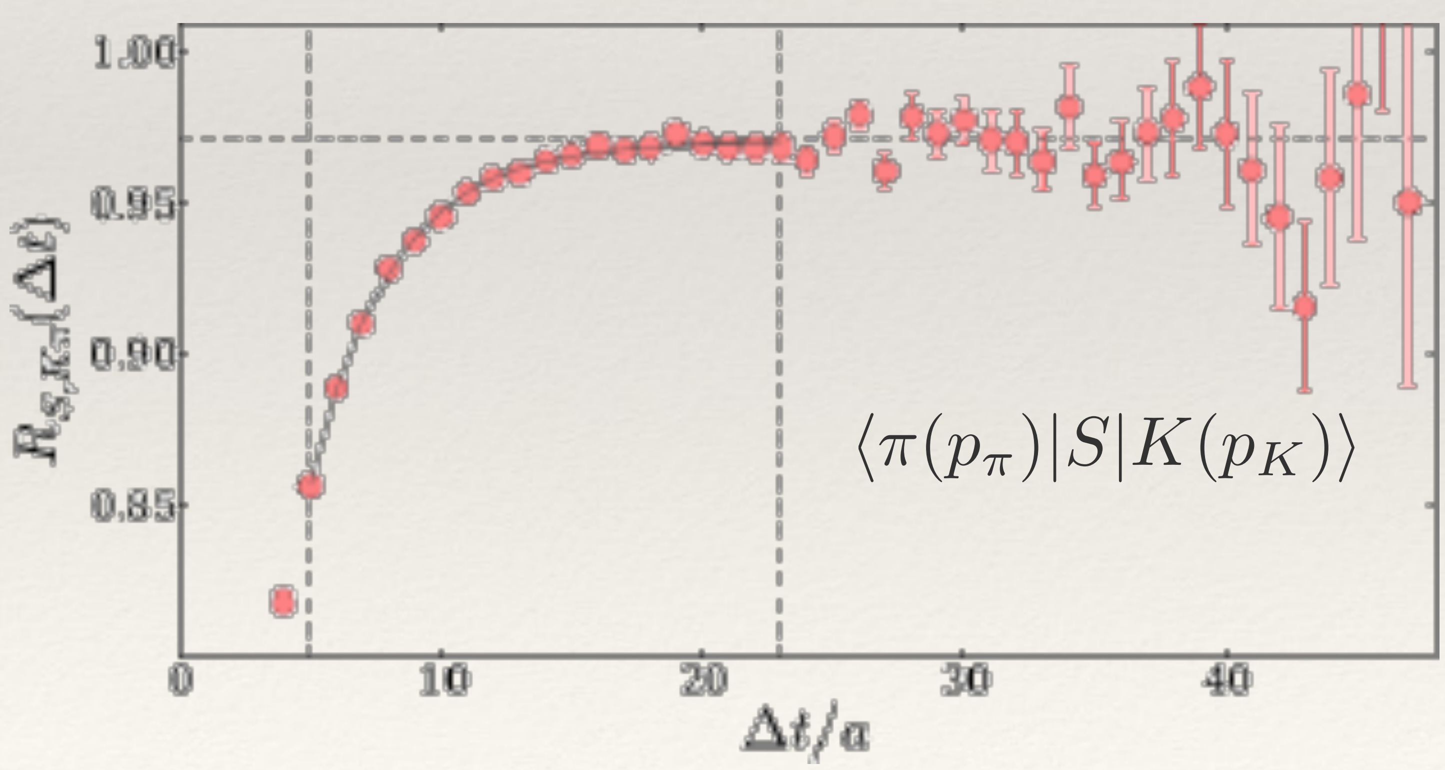
- this provides for an independent way to compute the form factor with partly different systematic errors
  Na et al., PRD 82 (2010) 114506
  arXiv:1008.4562
- the scalar current renormalisation Z_S cancels the mass renormalisation Z_m
- S-correlation function directly determines f_0 - in the vector current case one needs two constraints
  - two values of μ
  - two kinematical situations (keeping $q^2=0$)

excited state contribution

- correlation functions $\langle O_\pi(t_{\rm snk}) V_\mu(t) O_K(0)\rangle$ receive contributions from ground- and excited states



- generate 3pts for all all possible src-sink seps. → **study excited states contamination**

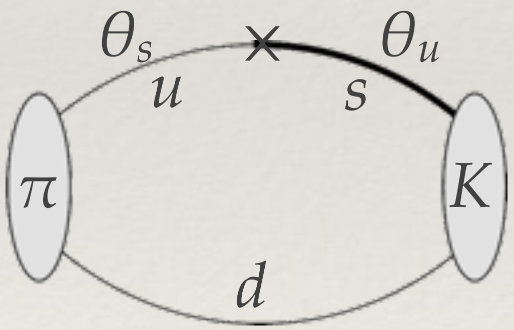


RBC/UKQCD JHEP06(2015)164
[arXiv:1504.01692]

partially twisted boundary conditions

JHEP 0705 (2007) 016
hep-lat/0703005

- conventionally the normalisation of the SM prediction is done at $q^2=0$ (the expression in ChPT reduces to a particularly simple form in this case but in principle any other point could be chosen)
- $q^2=0$ is not straight forward to achieve in lattice QCD due to $\vec{p} = \frac{2\pi}{L}\vec{n}$
- use **partially twisted boundary conditions** $\psi(x_k + L) = e^{i\theta_k}\psi(x_k)$



$$q^2 = \left\{ \left(E_K(\vec{\theta}_s) - E_\pi(\vec{\theta}_u) \right)^2 - \left(\vec{\theta}_s/L - \vec{\theta}_u/L \right) \right\}$$

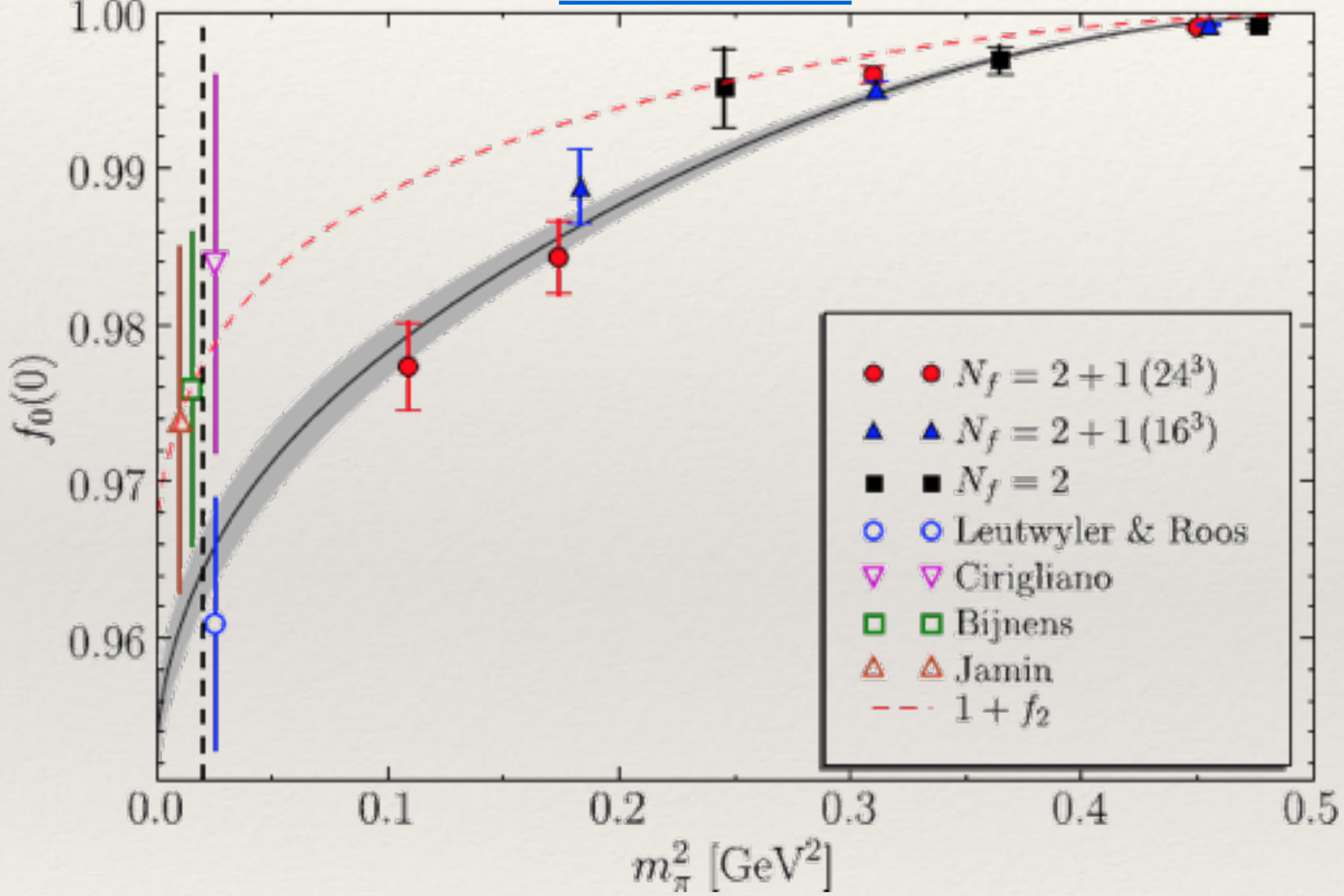
e.g. $q^2=0$ with moving pion with kaon at rest

$$|\vec{\theta}_\pi| = L\sqrt{\left(\frac{m_K^2 + m_\pi^2}{2m_K}\right)^2 - m_\pi^2}\,, \qquad \vec{\theta}_K = \vec{0}$$

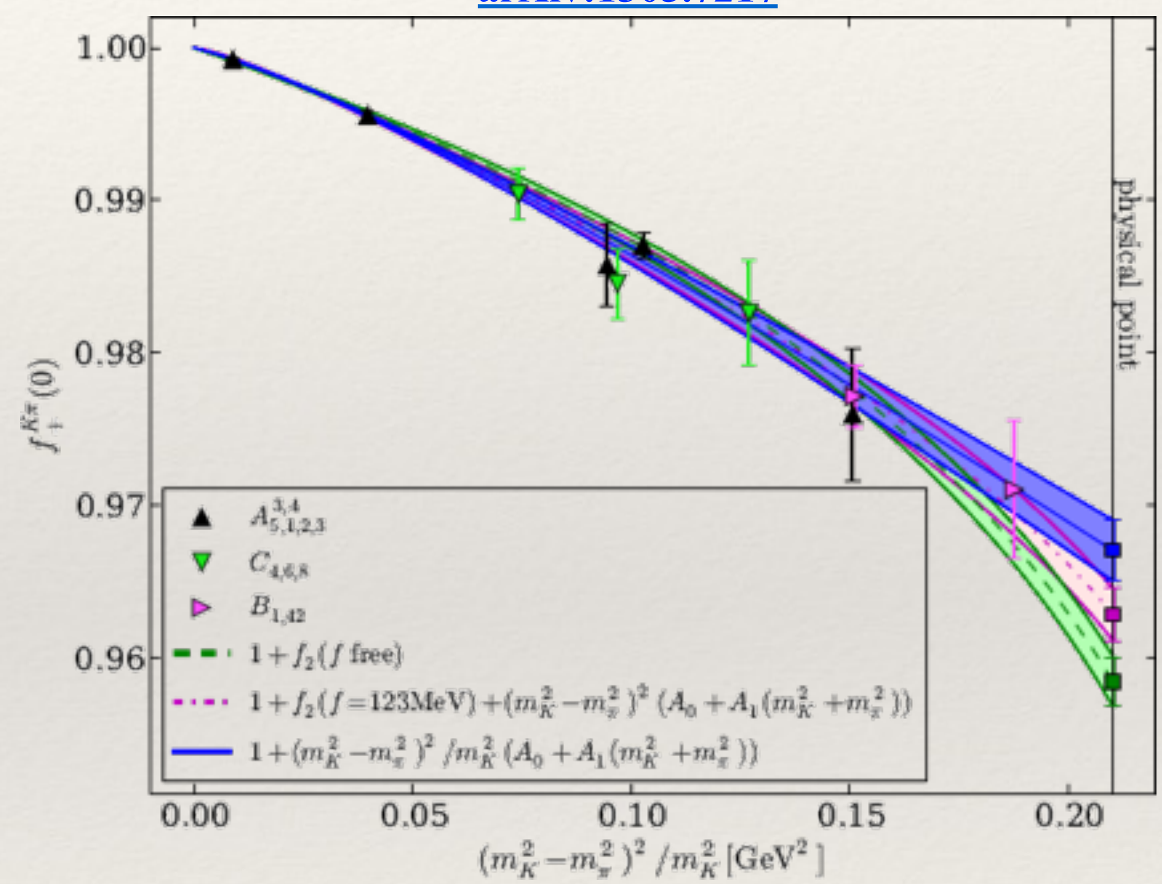
results

RBC/UKQCD results - history

Phys.Rev.Lett. 100 (2008) 141601
arXiv:0710.5136

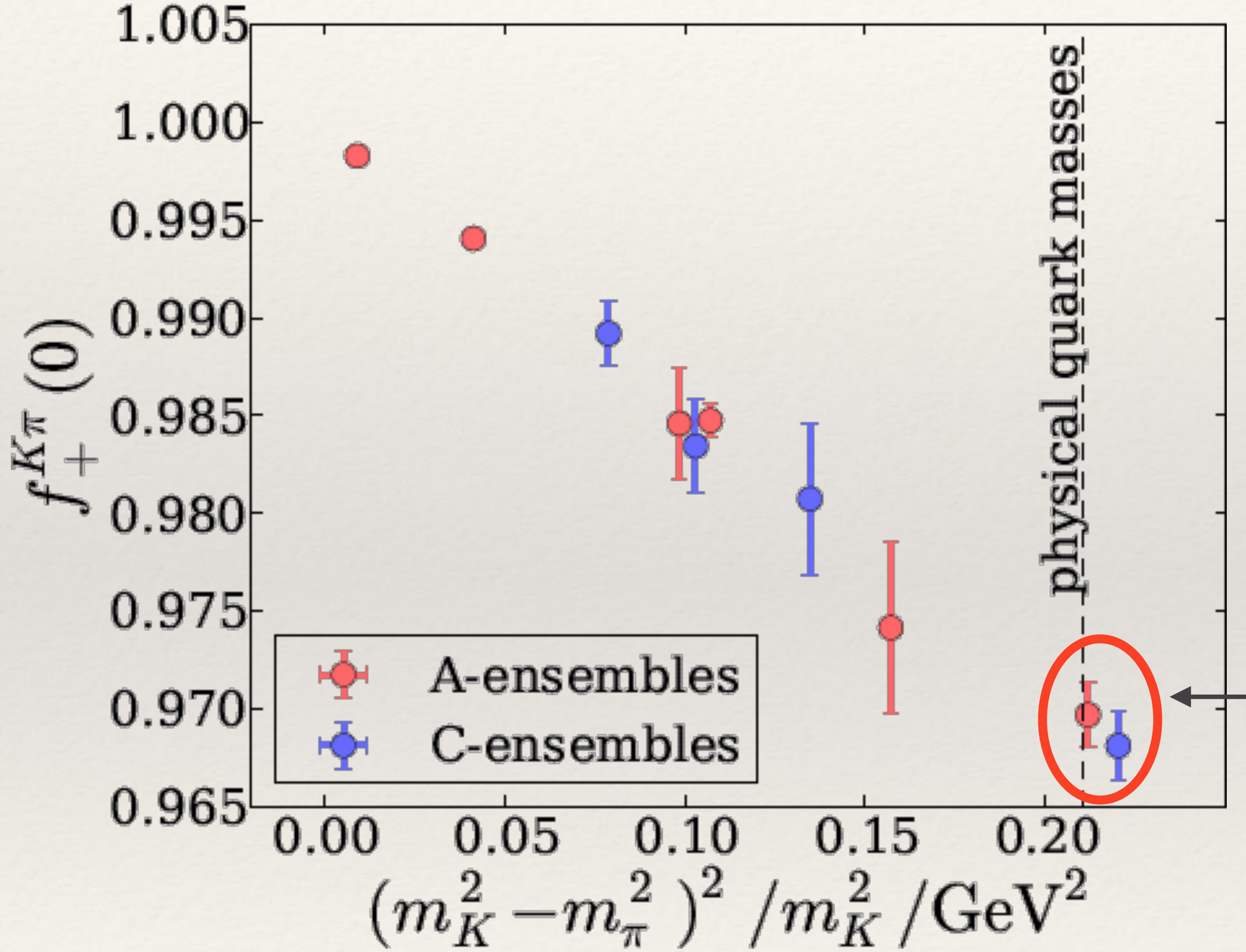


JHEP 1308 (2013) 132
arXiv:1305.7217



RBC/UKQCD results

RBC/UKQCD JHEP06(2015)164
arXiv:1504.01692



we can now interpolate to physical quark masses

mass interpolation

RBC/UKQCD JHEP06(2015)164
arXiv:1504.01692

- expectation in NLO ChPT

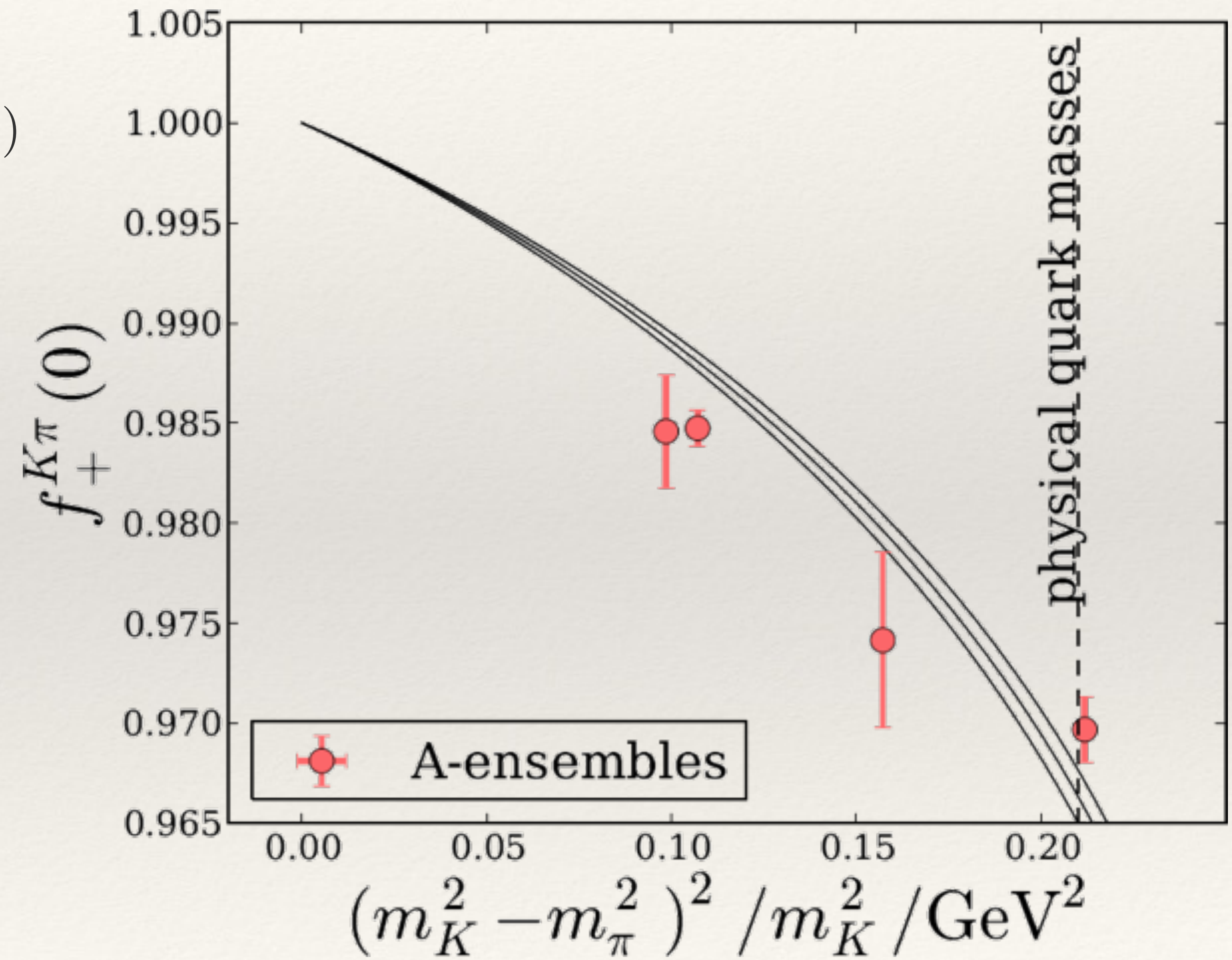
  $f_+^{K\pi}(0) = 1 + f_2(f^2, m_\pi^2, m_K^2)$

  Gasser & Leutwyler Nucl.Phys. B250 (1985) 517-538

  found to be incompatible with data

- need to add NNLO terms

- or: data suggests much simpler, linear ansatz to interpolate to the phys. value of the meson masses

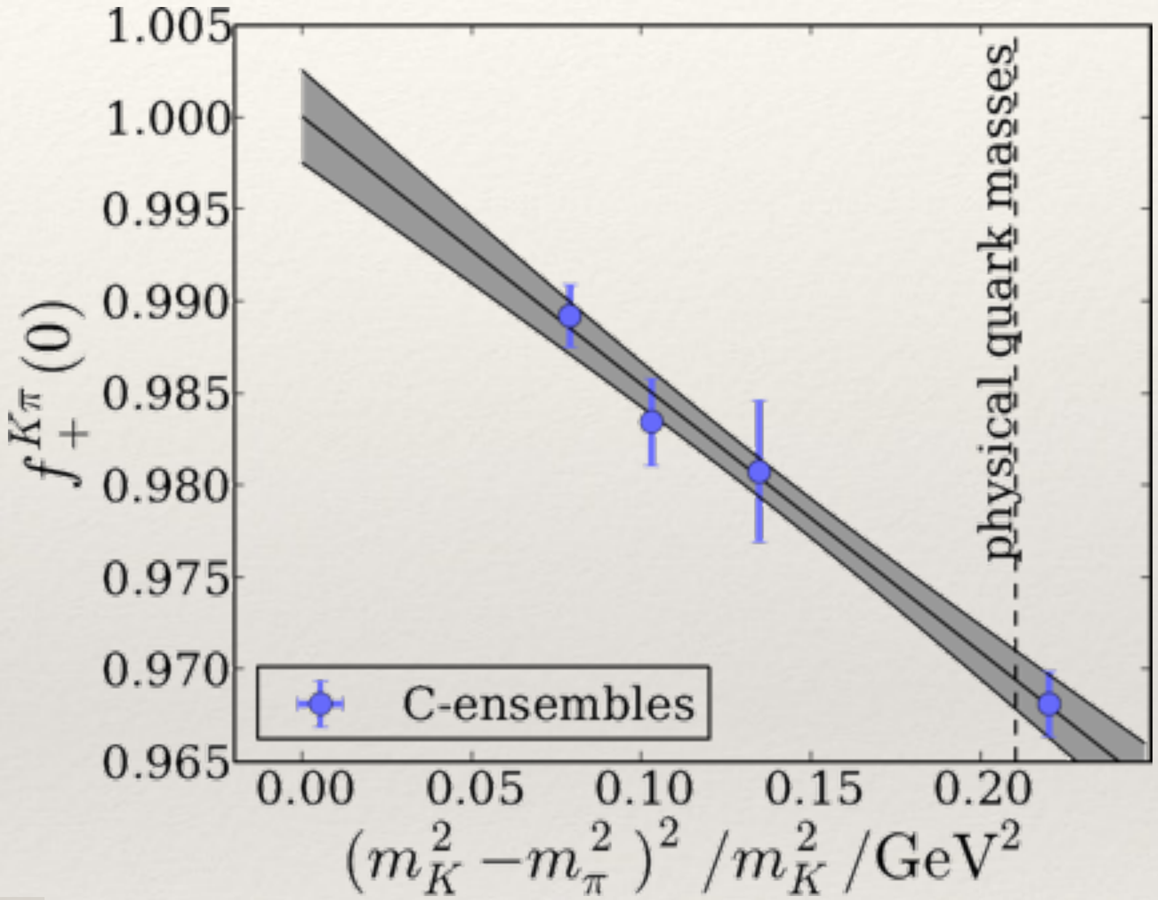


mass interpolation

RBC/UKQCD JHEP06(2015)164
arXiv:1504.01692

mass-interpolation

- just linear in SU(3)-breaking is sufficient
- only tiny correction
- induced systematic entirely subdominant



- dependence on largest included m_π:

$m_\pi{}^{min}$/MeV	355	450	600
$f_+^{K\pi}(0)$	0.9690(33)	0.9689(25)	0.9691(22)

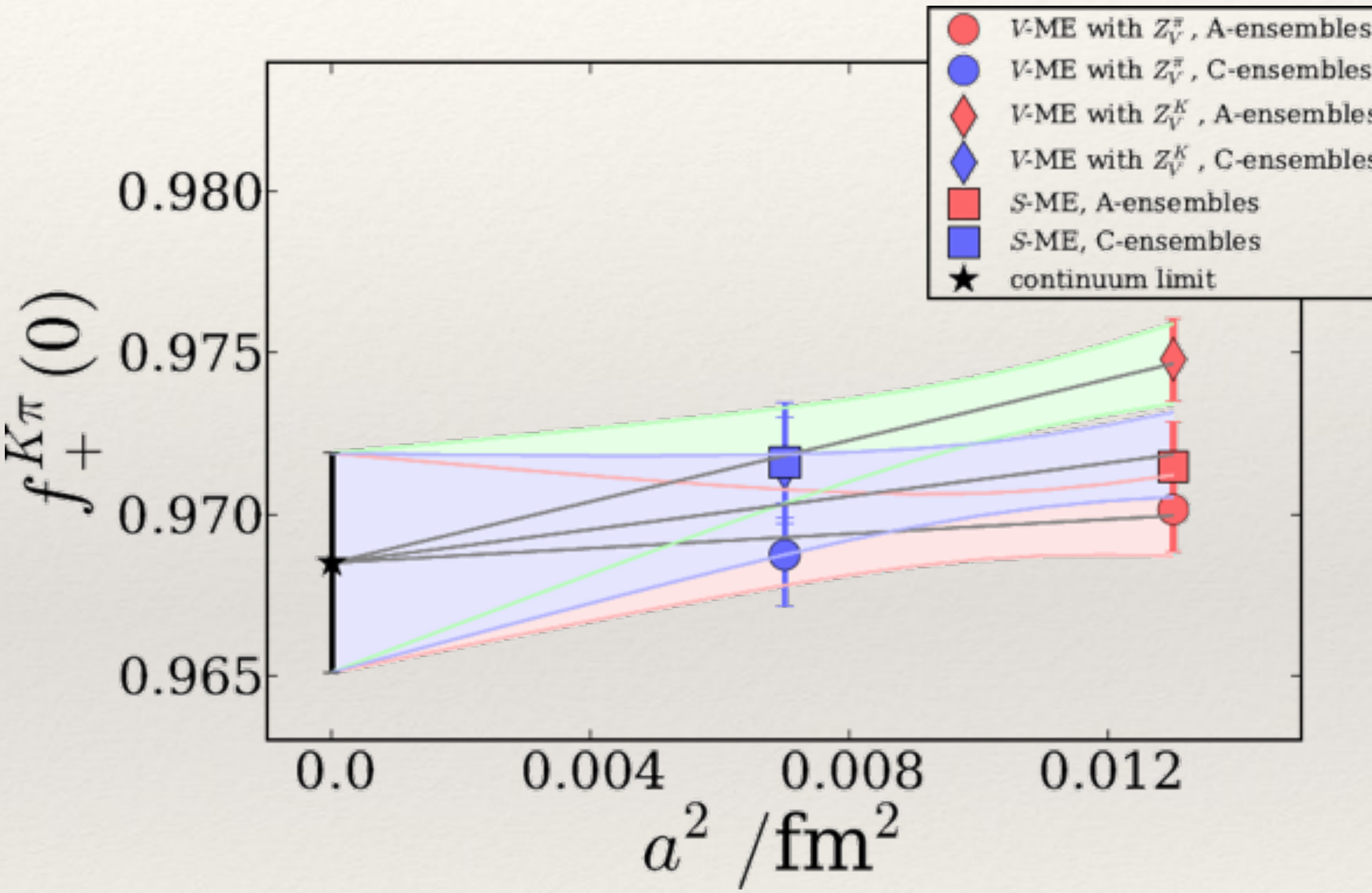
high-m_π cut: small stat., potentially syst. interpolation error
low-m_π cut : larger stat., basically no syst. interpolation error

continuum limit

RBC/UKQCD JHEP06(2015)164
arXiv:1504.01692

Continuum limit

- currently two values of the lattice spacing
- three continuum limits:
  - from scalar ME with Z_V^π
  - from vector ME with Z_V^K
  - from scalar ME
- all remaining systematics errors estimated to be smaller than stat. error
- dominant systematic error likely from finite volume effects

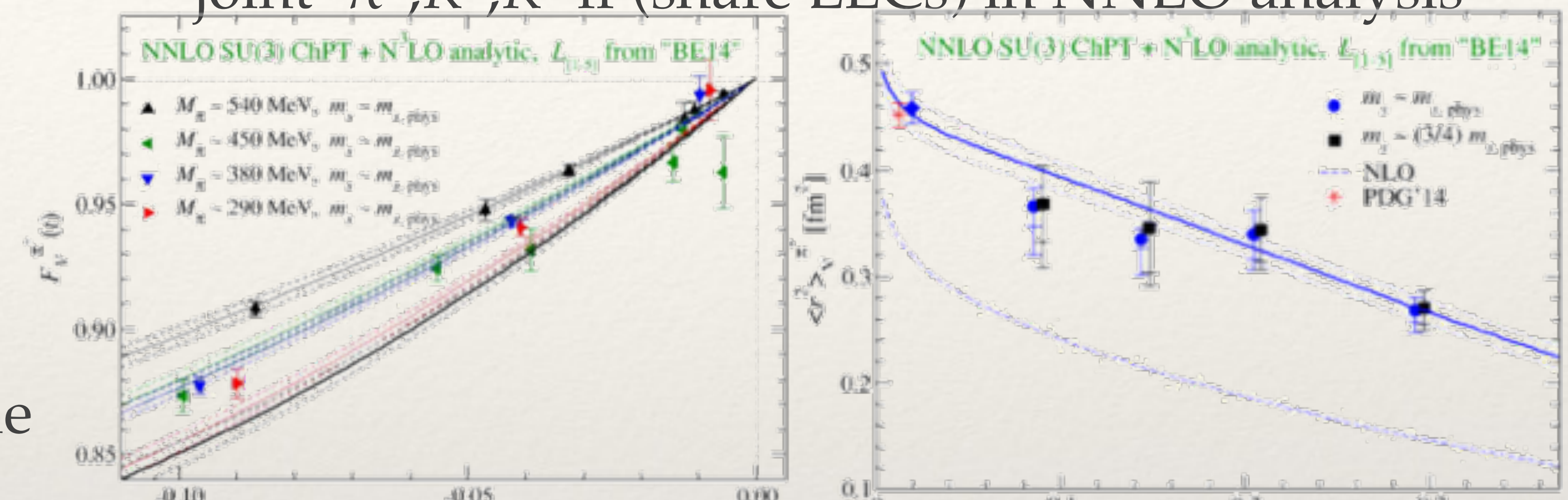


$$f_+^{K\pi}(0) = 0.9685(34)(14)$$

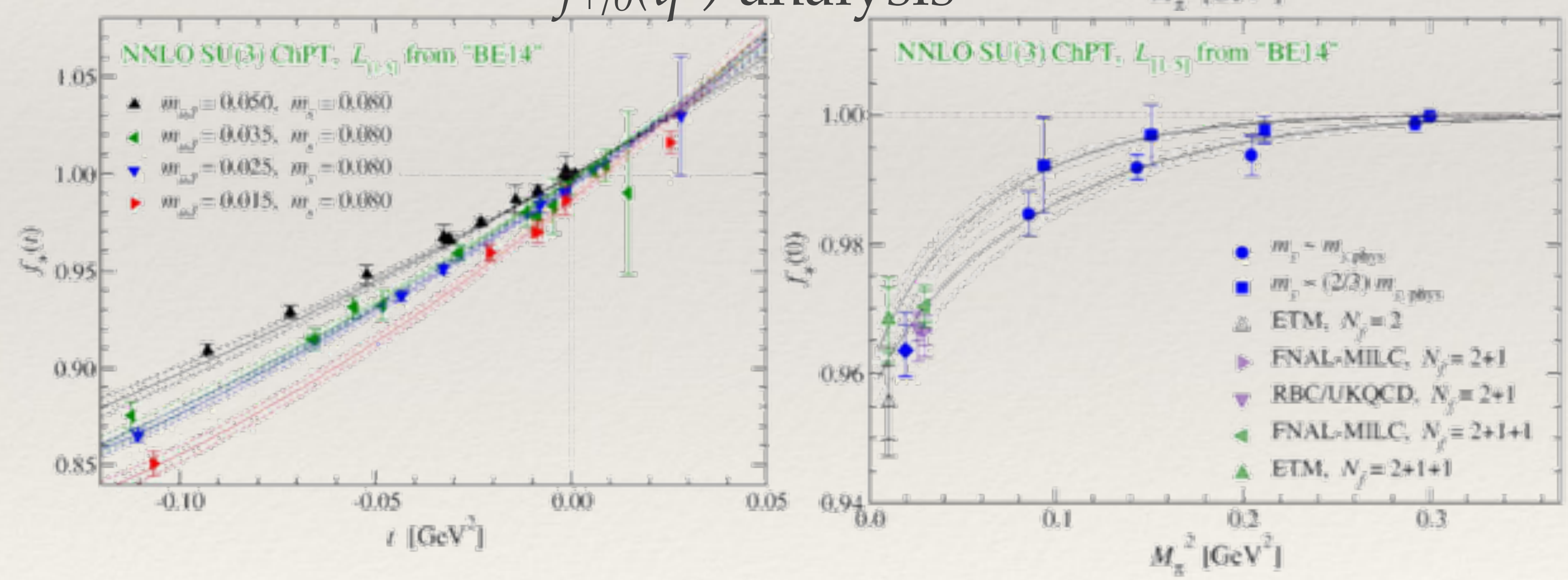
we shouldn’t forget LECs …

- JLQCD
- N_f=2+1 overlap fixed Q
- a=0.11fm
- m_π=290-540MeV
- $m_\pi L \gtrsim 4$
- except for a^2 cont. ChPT applicable here: NNLO (+N^{3}LO terms) with L_{1-8} from Bijnens, Ecker Ann.Rev.Nucl.Part.Sci. 64 (2014) 149-174 arXiv:1405.6488
- EM ff and $K\to\pi$ ff share LECs do combined fit
- besides results for charge radii and $K\to\pi$ ff also predictions for ff shape and LECs

joint π^+,K^+,K^0 ff (share LECs) in NNLO analysis



$f_{+/0}(q^2)$ analysis



$$f_+(0) = 0.9636(36)_{\rm stat}\left(^{+0}_{-45}\right)_{\rm N^3NLO}\left(^{+41}_{-3}\right)_{L_i}(29)_{a\neq 0} = 0.9636\left(^{+62}_{-65}\right)$$

What's next – K→π ff. ?

$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} S_{\rm EW}(1+\Delta_{SU(2)}+\Delta_{\rm EM})^2\, I\,|f_+^{K\pi}(0)|^2|V_{us}|^2$$

Take N_f=2+1+1 FLAG average $f_+(0)=0.970(3)$ and $|V_{us}|f_+^{K^0\pi^-}(0) = 0.2163(5)$

FLAVIA Kaon WG EPJ C 69, 399-424 (2010)
KTeV, Istra, KLOE

$\delta_{exp}\approx 0.2\%$ $\delta_{th}\approx 0.3\%$

What's to be expected from the experimental side?

- Moulson@CKM 2014 arXiv:1411.5252
  Cirigliano et al. Rev.Mod.Phys. 84 (2012) 399 arXiv:1107.6001
  update: $|V_{us}|f_+^{K^0\pi^-}(0) = 0.2165(4)$
- BR, τ and δ_{EM} and $\delta_{SU(2)}$ are dominant src. of uncertainty

arXiv:1411.5252

			Approx contrib to % err			
Mode	$V_{us}\, f_+(0)$	% err	BR	τ	Δ	I
K_{Le3}	0.2163(6)	0.26	0.09	0.20	0.11	0.05
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$K_{\mu3}^{\pm}$	0.2170(11)	0.51	0.45	0.06	0.23	0.06

- there is old data (e.g KLOE) and there are new experiments (NA62, KOTO, OKA) who could potentially improve the experimental number but seems that it is not on the top of todos/limited manpower

Analysis paradigm change?

Results with physical mass pions are quickly becoming standard

This is having an impact on the analysis strategies in use:

A. do a global analysis over physical AND unphysical m_l QCD simulations and make predictions in terms of global fit

B. take physical m_l result as what it is and use unphysical point results merely to correct (i.e. interpolate) for mistuning in quark masses

A. seems preferable if model/EFT is 100% trustworthy → reduce stat. error

B. preferable in cases where reliability of fit models/EFT is questionable
→ exclude unknown systematics?

$K \rightarrow l\nu$

Example (I) - f_K/f_π

RBC/UKQCD	arXiv:1411.7017	2+1	dwf	**"overweighting"**
HPQCD	PRD88, 074504 (2013) **arXiv:1303.1670**	2+1+1	stagg.	**"global analysis"**
FNAL/MILC	PRD90 (2014) 7, 074509 **arXiv:1407.3772**	2+1+1	stagg.	**"physical-mass"**

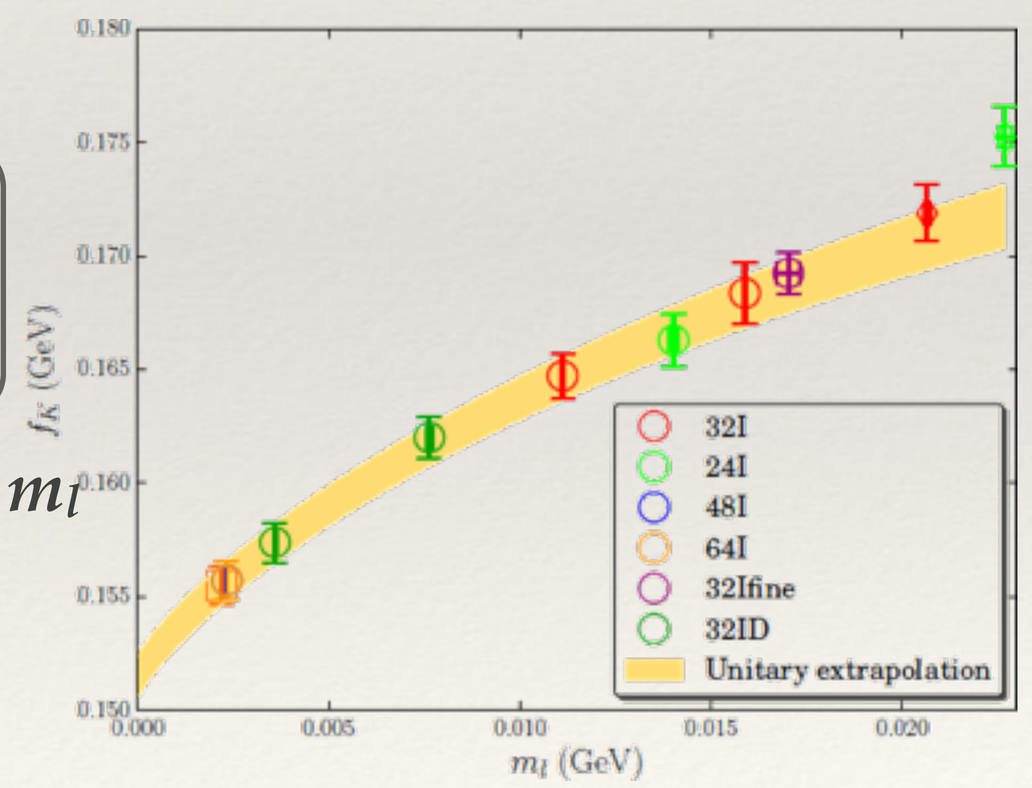
RBC/UKQCD "overweighting"

- heavy meson ChPT or polynomials
- global analysis but *overweight* physical quark mass ensembles

$$\chi^2 = \sum_i \frac{\omega_i\,[y_i - f(x_i,\vec{a})]^2}{\sigma_i^2}$$

$\omega_i =$ large for physical m_l ensembles

$\omega_i = 1$ otherwise



→ fit forced to go exactly through near-physical m_l ensemble and resampling ensures that stat. error after extrapolation not artificially deflated

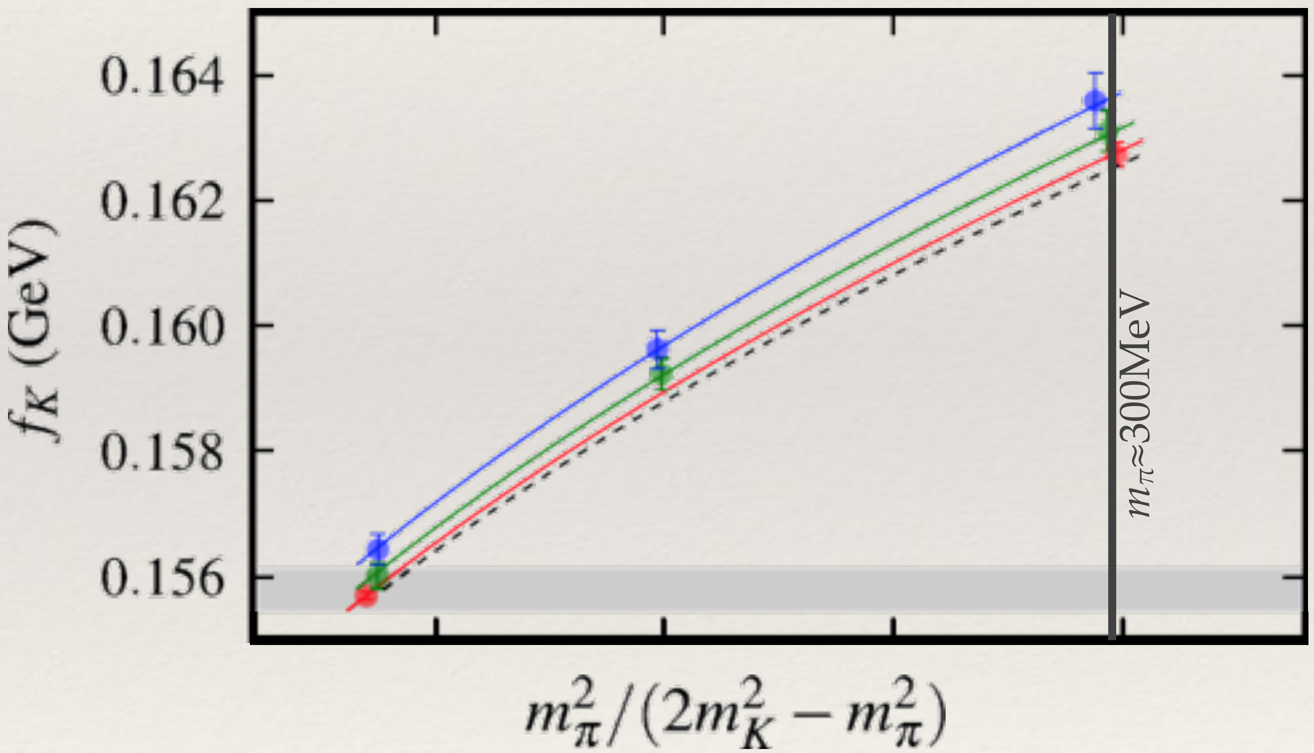
→ final error budget completely dominated by stat. error

Example (II) - f_K/f_π

RBC/UKQCD	arXiv:1411.7017	2+1	dwf	**"overweighting"**
HPQCD	PRD88, 074504 (2013) **arXiv:1303.1670**	2+1+1	stagg.	**"global analysis"**
FNAL/MILC	PRD90 (2014) 7, 074509 **arXiv:1407.3772**	2+1+1	stagg.	**"physical-mass"**

HPQCD "global analysis":

- global fit including large dataset and many observables
- NLO PQChPT + higher order models
- large set of model parameters

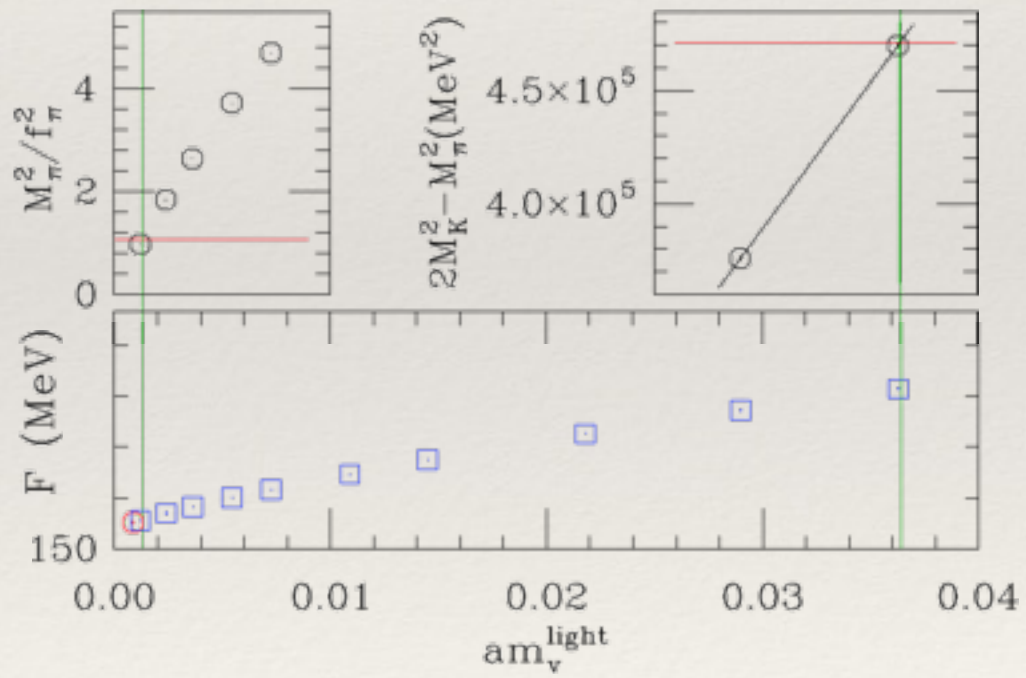


Example (III) - f_K/f_π

RBC/UKQCD	**arXiv:1411.7017**	2+1	dwf	**"overweighting"**
HPQCD	PRD88, 074504 (2013) **arXiv:1303.1670**	2+1+1	stagg.	**"global analysis"**
FNAL/MILC	PRD90 (2014) 7, 074509 **arXiv:1407.3772**	2+1+1	stagg.	**"physical-mass"**

FNAL/MILC "physical-mass" analysis:

- PQ analysis on physical sea m_l
- no global fit:
  m_l, m_s and a^{-1} obtained for individual ensembles based on continuum NLO PQChPT
  subsequent continuum limit
- **reduced impact of *heavy* m_l data**
  reduced model dependence



systematic error treatment

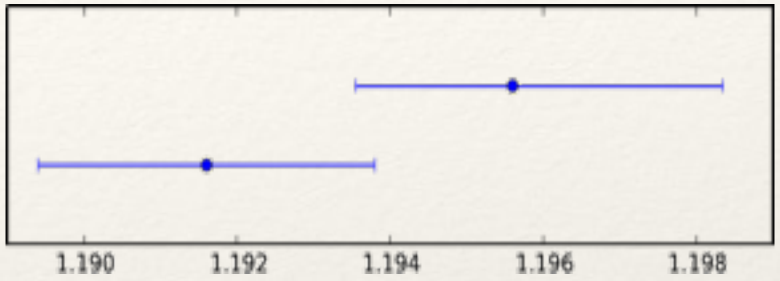
- effective theories guide the way extrapolations are done
  - finite volume effects → ChPT, non-relativisitc effective field theory, …
  - cutoff effects → Symanzik effective theory
  - heavy quarks → HQET
- but also *phenomenological* models or polynomials
- validity and range of applicability needs to be studied

- use different models for extrapolation and compare → devise systematic uncertainty
  - use spread to devise range of possible results (which distribution?)
  - histograms
  - check that other models predict similar corrections/effects and be happy (too agressive?)

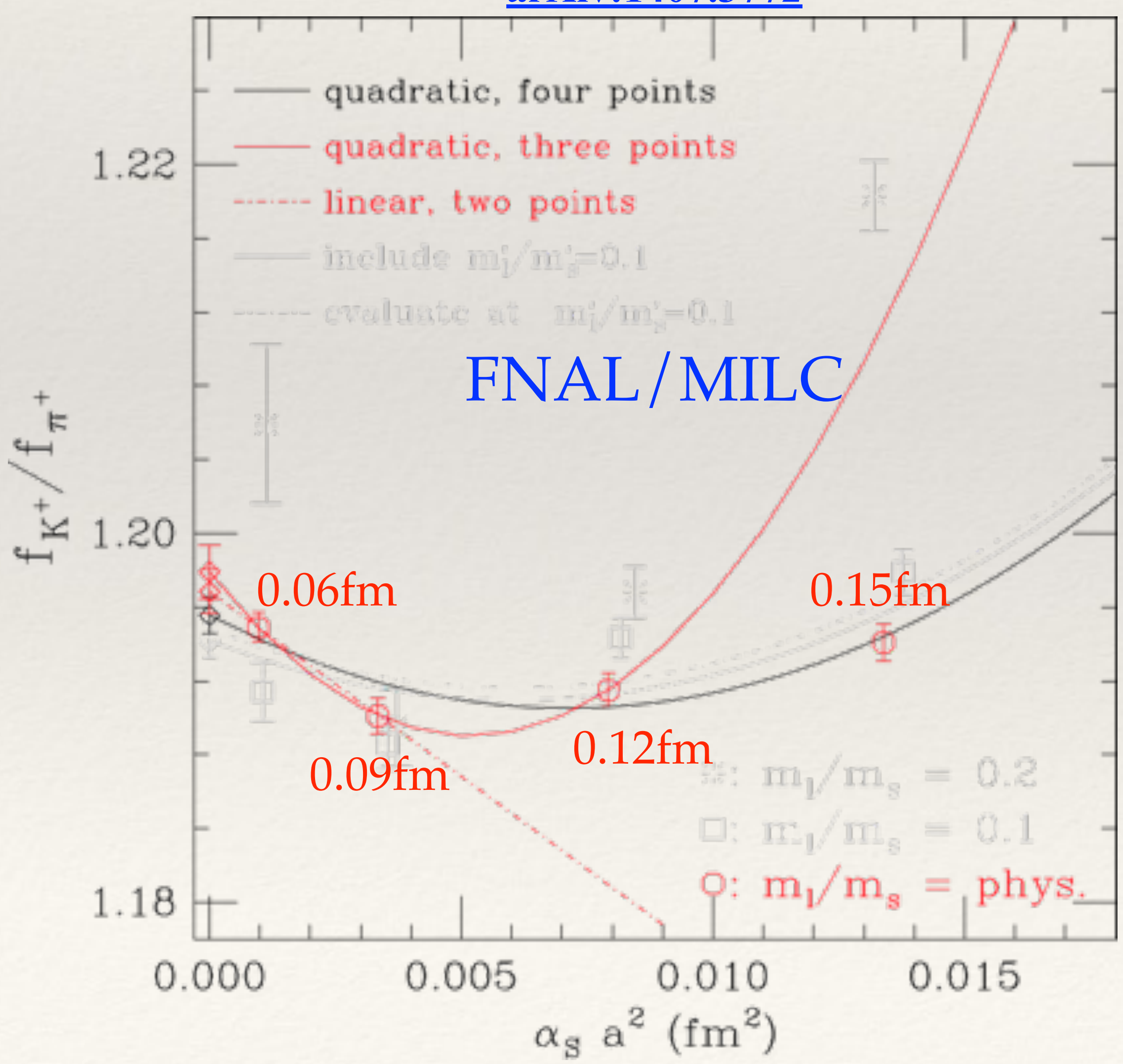
Example (II and III) - f_K/f_π

FNAL/MILC $f_{K^+}/f_{\pi^+} = 1.1956(10)_{\rm stat}\ (^{+23}_{-14})_{a^2-\rm extrapol}\ (10)_{\rm FV}\ \ (5)_{\rm EM}$

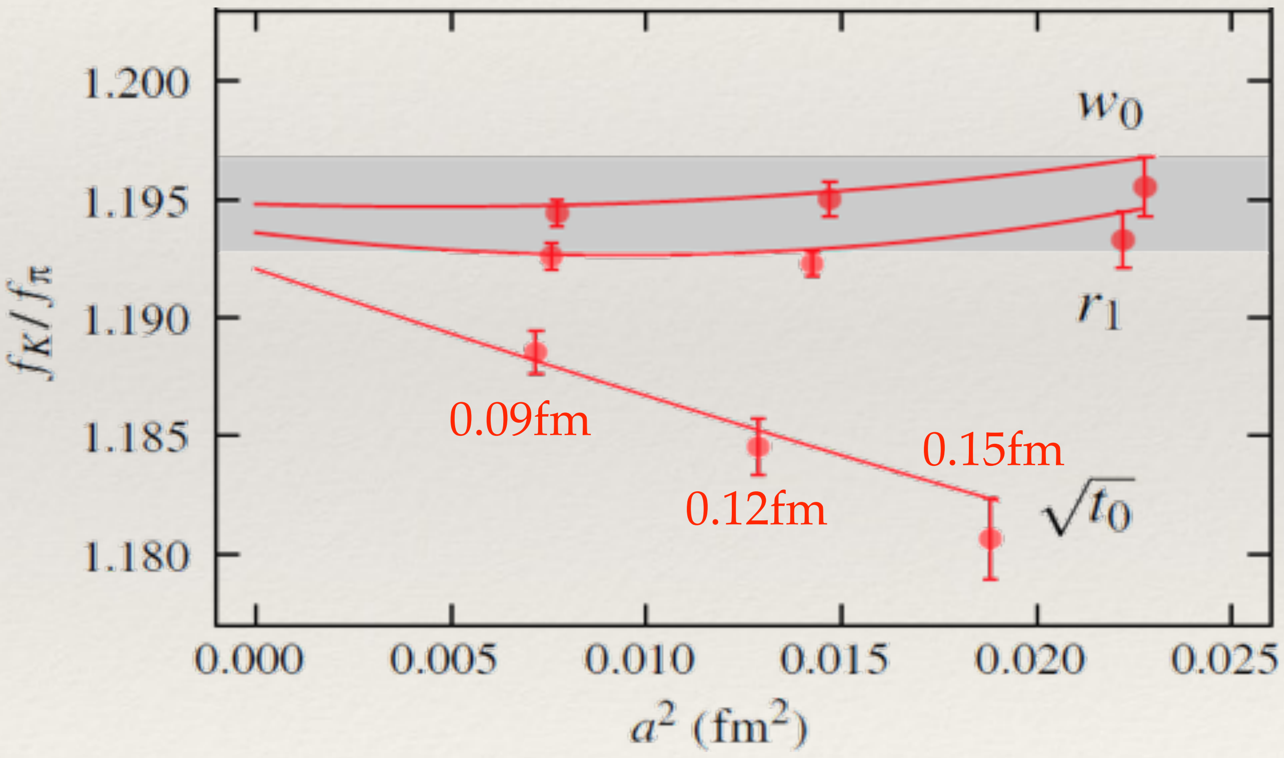
HPQCD $f_{K^+}/f_{\pi^+} = 1.1916(15)_{\rm stat}\ (12)_{a^2-\rm extrapol}\ (1)_{\rm FV}\ (10)_{\rm other}$



PRD90 (2014) 7, 074509
arXiv:1407.3772



PRD88, 074504 (2013)
arXiv:1303.1670



tension due to cutoff effect?

SU(2) correction

tension in treatment of $\delta_{SU(2)}$ for relating $f_K/f_\pi \to f_{K^\pm}/f_{\pi^\pm}$?

- SU(3) ChPT Phys.Lett. B700 (2011) 7-10 arXiv:1102.0563

$$\delta_{SU(2)} \approx \frac{3}{4} R \left[-\frac{4}{3}(f_K/f_\pi - 1) + \frac{2}{3(4\pi)^2 f_0^2}\left(M_K^2 - M_\pi^2 - M_\pi^2 \ln \frac{M_K^2}{M_\pi^2} \right) \right] = -0.004(1)$$

- ETM PRD87 (2013) 11, 114505 arXiv:1303.4896 expand action in m_u-m_d and compute 2pt function with insertions — see Vittorio's talk $\quad \delta_{SU(2)} = -0.0080(4)$
- ETM PRD91 (2015) 5, 054507 arXiv:1411.7908 correct with slope of f_K in m_l determined in two ways $\quad \delta_{SU(2)} = -0.0080(38)$
- HPQCD PRD 88, 074504 (2013) arXiv:1303.1670 correct with slope of f_K in m_l determined $\quad \delta_{SU(2)} = -0.0054(14)$

tension?

1st row unitarity

1st row unitarity test - FLAG2

Numerical results from FLAG2, illustrations (preliminary) from FLAG3

Experimental results:

$$|V_{us}|f_+(0) = 0.2163(5) \qquad \frac{f_K}{f_\pi}\frac{|V_{us}|}{|V_{ud}|} = 0.2758(5)$$

FLAVIA Kaon WG EPJ C 69, 399-424 (2010)
KTeV, Istra, KLOE

First row unitarity:

- $f_+^{K\pi}(0)$ and $|V_{ud}|$ from experiment
- f_K/f_π and $|V_{ud}|$ from experiment
- $f_+^{K\pi}(0)$ and f_K/f_π from lattice

	$f_+(0)$, $\vert V_{ud}\vert$	f_K/f_K, $\vert V_{ud}\vert$	combined
N_f=2+1	0.9993(5)	1.0000(6)	0.987(10)
N_f=2	1.0004(10)	0.9989(16)	1.029(35)

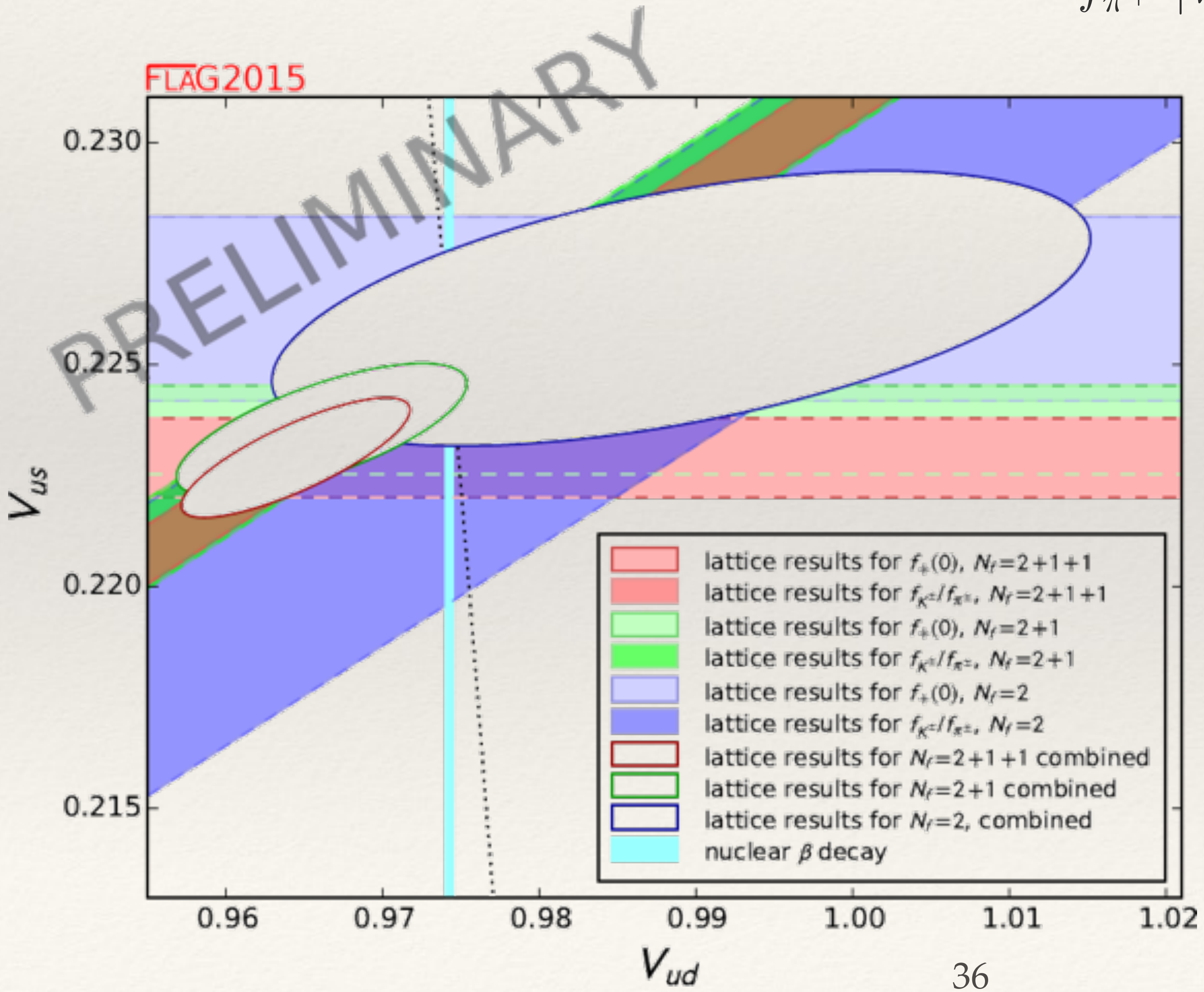
Eur.Phys.J. C74 (2014) 2890
arXiv:1310.8555

FLAG3 preliminary

FLAG V_{us} Working Group (Boyle, Kaneko, Simula)

$$|V_{us}|f_+^{K^0\pi^-}(0) = 0.2163(5) \qquad \frac{f_{K^+}}{f_{\pi^+}}\frac{|V_{us}|}{|V_{ud}|} = 0.2758(5)$$

FLAVIANet Kaon WG
EPJ C 69, 399-424 (2010)
arXiv:1005.2323



high precision test of SM unitarity - no worrisome tension at sub-percent-level precision

Analysis assuming 1st row unitarity - FLAG2

$$|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} \stackrel{?}{=} 1$$

$$|V_{us}| f_+(0) = 0.2163(5) \qquad \frac{f_K}{f_\pi} \frac{|V_{us}|}{|V_{ud}|} = 0.2758(5)$$

FLAVIA Kaon WG EPJ C 69 (2010)

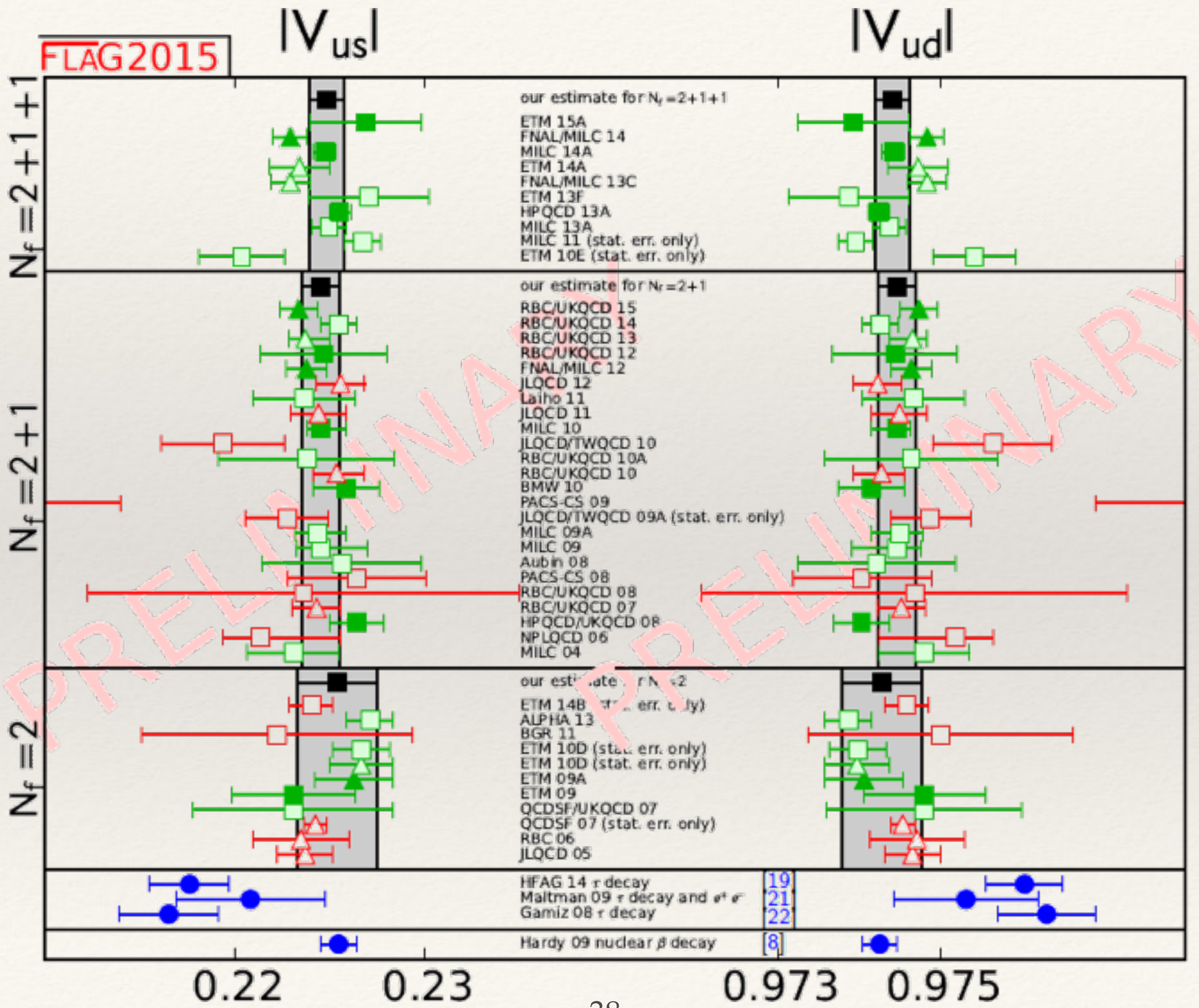
- $|V_{ub}| = 4 \times 10^{-3}$ too small to play a role, then 3 equations, 4 unknowns
- provide either $f_+(0) \rightarrow$ predict $|V_{ud}|$, $|V_{us}|$, f_K/f_π
  or $f_K/f_\pi \rightarrow$ predict $|V_{ud}|$, $|V_{us}|$, $f_+(0)$

	$\lvert V_{us}\rvert$	$\lvert V_{ud}\rvert$	$f_+(0)$	f_K/f_π
N_f=2+1+1	0.2251(10)	0.97434(22)	0.9611(47)	1.194(5)
N_f=2+1	0.2247(7)	0.97447(18)	0.9634(32)	1.197(4)
N_f=2	0.2253(21)	0.97427(49)	0.9595(90)	1.192(12)
β-decay	0.22544(95)	0.97425(22)	0.9595(46)	1.1919(57)

Eur.Phys.J. C74 (2014) 2890
arXiv:1310.8555

all nice and consistent!

Results for $|V_{ud}|$ and $|V_{us}|$ assuming 1st row unitarity - FLAG3



CKM fitter input

discussion yesterday with Andrey, Heiko, Jean, Jérôme, Sébastien

following the discussion during the talks on Monday here a list of CKMfitter inputs:
http://ckmfitter.in2p3.fr/www/results/plots_eps15/num/ckmEval_results_eps15.pdf

f_K	f_{D_s}	f_{B_s}
f_K/f_π	f_{D_s}/f_D	f_{B_s}/f_B
$f_+^{K\pi}(0)$	$f_+^{D\pi}(0)$ $f_+^{DK}(0)$	$f_+^{B\pi}$[HFAG] $f_+^{BD^{(*)}}$[HFAG]
B_K		B_{B_s} B_{B_s}/B_{B_d}
	m_c	

α_S

- chore for lattice community/FLAG:
  - make sure to predict the quantities to the left (in addition to e.g. ξ)
  - wherever possible - provide correlations
- interesting for CKM fitters to find out where a potential correlation affects their results

Summary/Conclusions

- lattice data with physical light quark masses allow for a new quality of data analysis - sub-percent precision feasible
- in particular analysis strategies which reduce the impact of data from un-physically heavy light quarks seem very attractive
- for simulations with physical quark masses statistical / finite size / continuum extrapolation errors are now the dominant source of uncertainty in pure QCD
- QCD simulations so precise that it's worth studying isospin breaking effects in matrix elements in more detail in a conceptually clean way