Determination of α_s from the QCD static energy

Xavier Garcia i Tormo
Universität Bern

Based on:

A. Bazavov, N. Brambilla, XGT, P. Petreczky, J. Soto and A. Vairo, Phys. Rev. D **86**, 114031 (2012) [arXiv:1205.6155 [hep-ph]];

Phys. Rev. D **90**, 074038 (2014) [arXiv:1407.8437 [hep-ph]]

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Introduction

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Does the uncertainty in the world average really reflects our current understanding of the value of α_s?
Increasing corroboration of α_s value, by extracting it from independent quantities, is crucial; exhaustively analyze theoretical errors entering in each determination.

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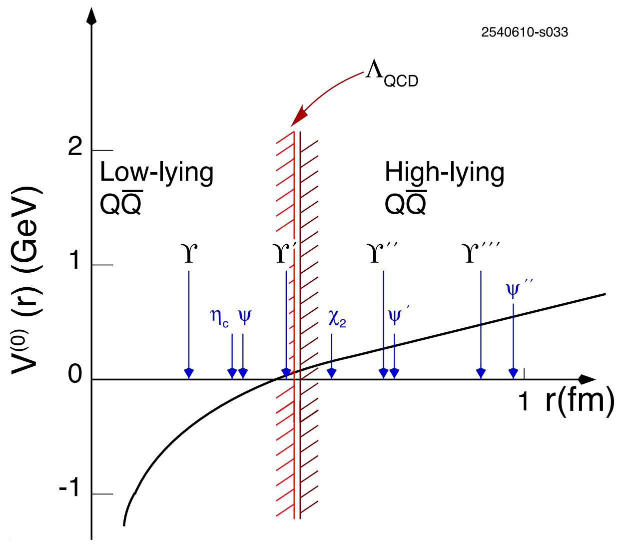

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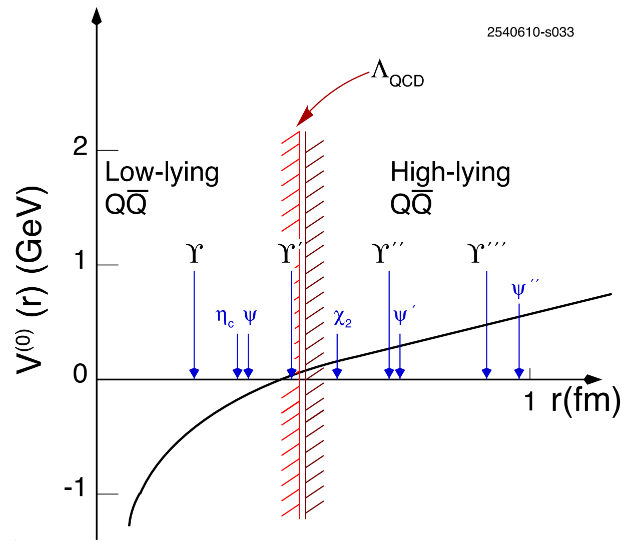

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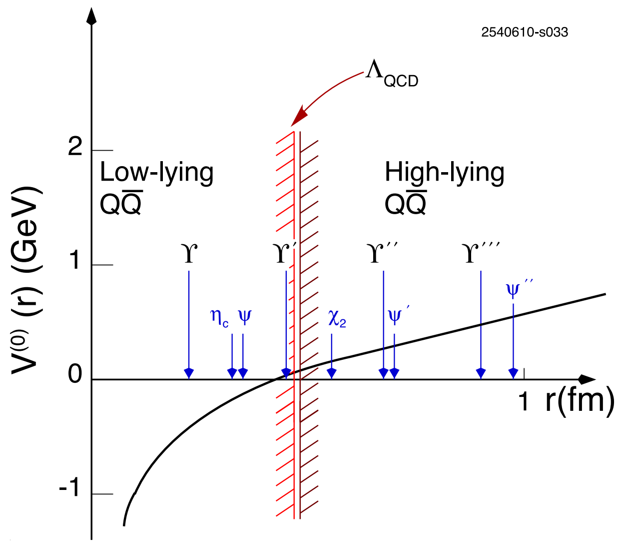

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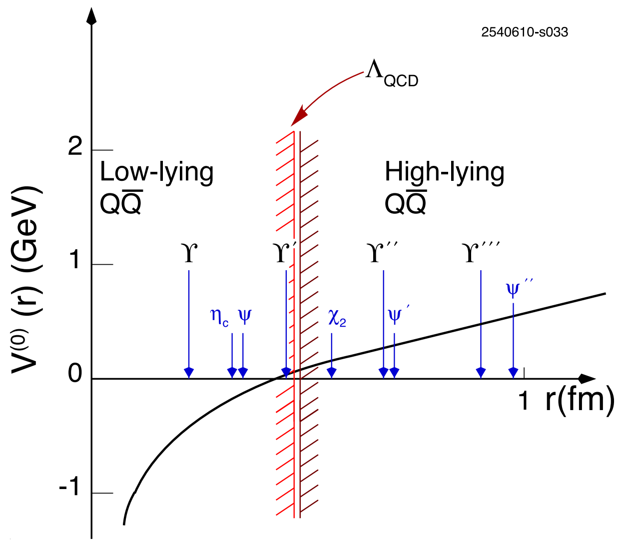

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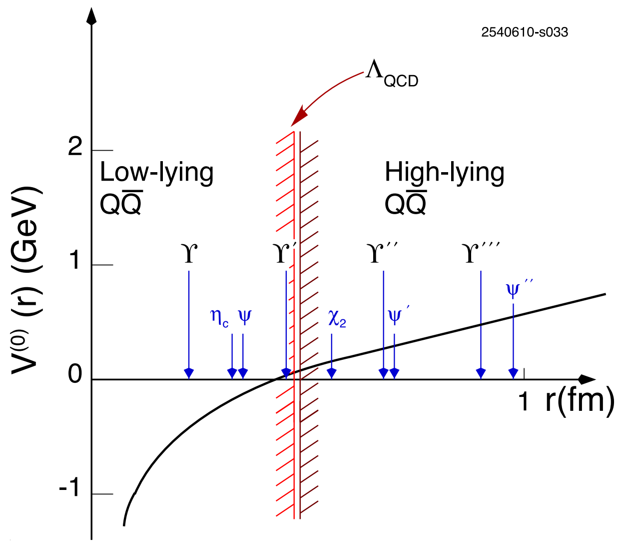

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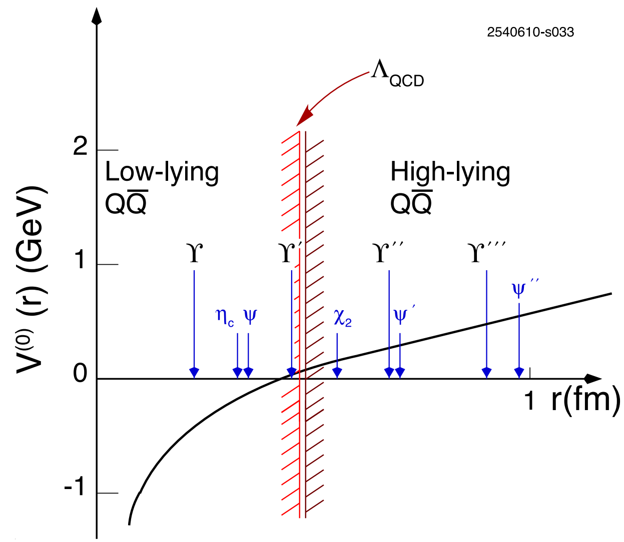

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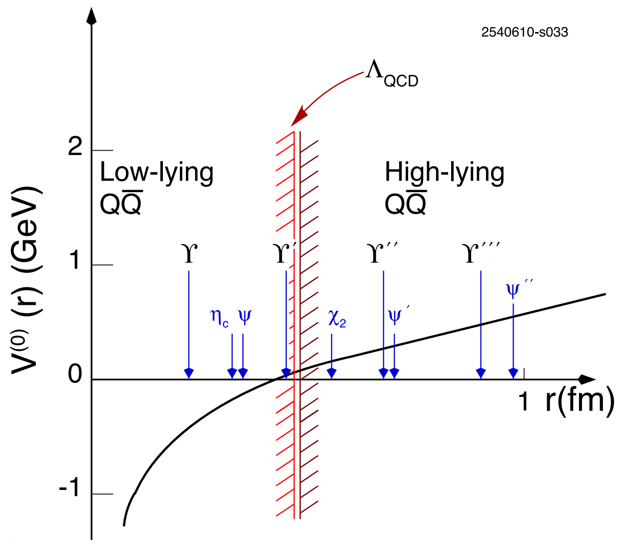

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Compare perturbative and lattice results for the static energy *at short distances* to extract α_s

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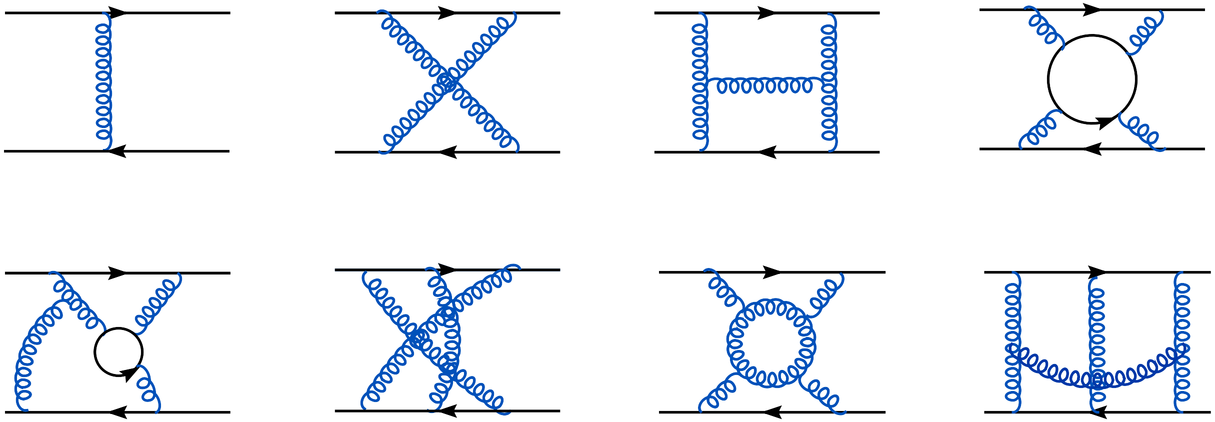

(Picture from A. V. Smirnov, V. A. Smirnov and M. Steinhauser, Phys.Rev.Lett. **104** (2010) 112002

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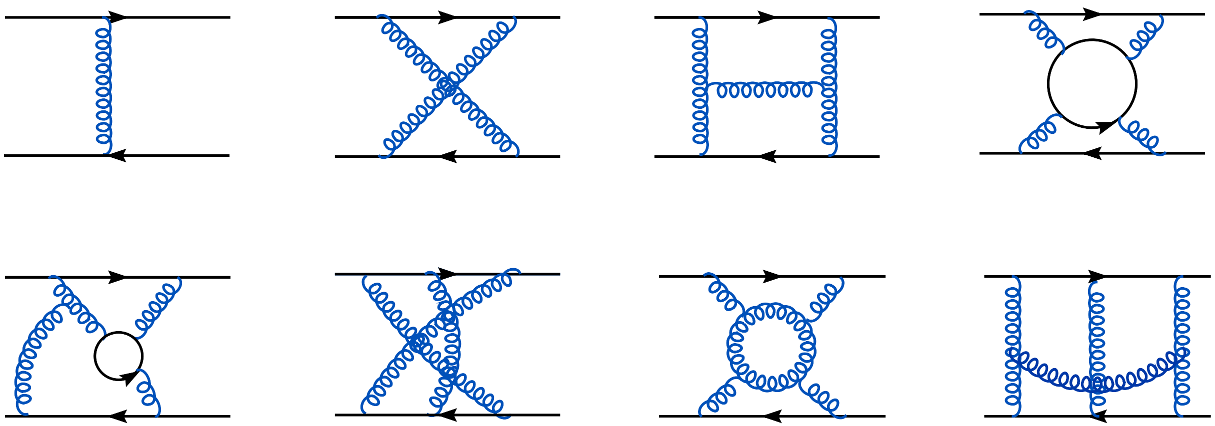

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Virtual emissions that change the color state of the pair (*Ultrasoft gluons*) Appelquist Dine Muzinich'78

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$E_0(r)$ calculated on the lattice in $n_f = 2 + 1$ flavor QCD

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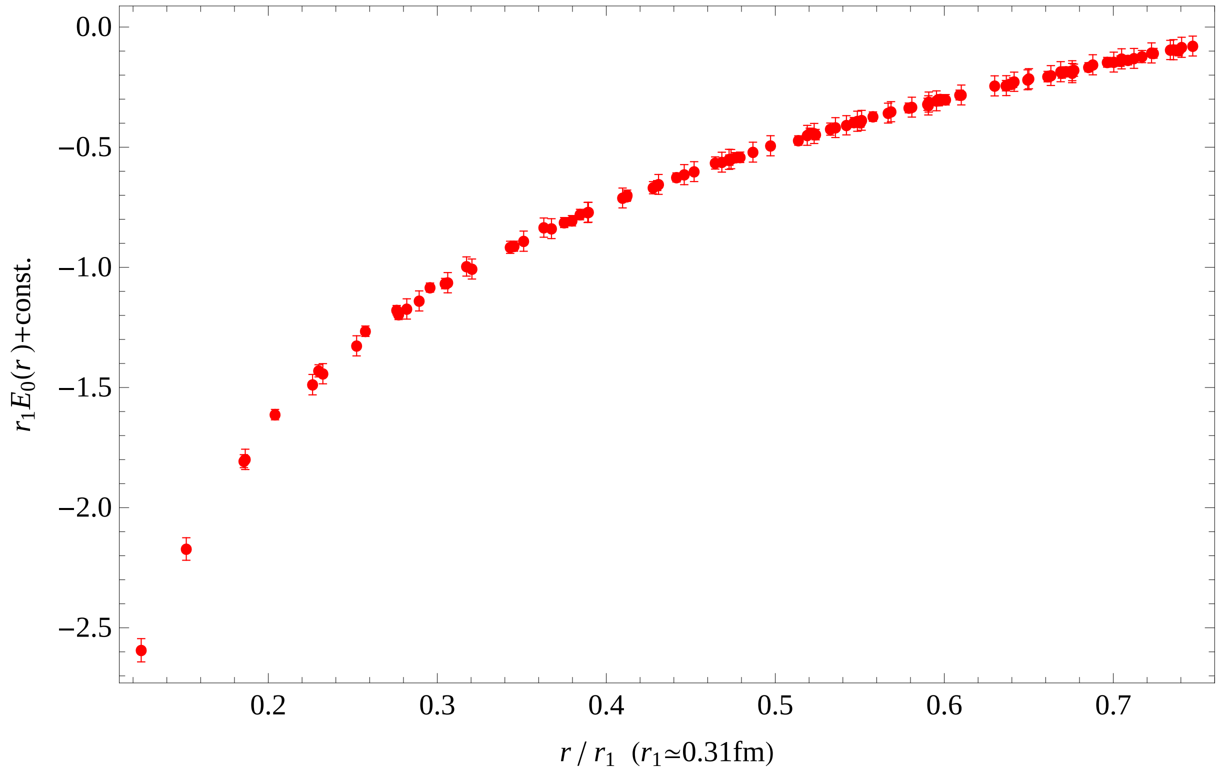

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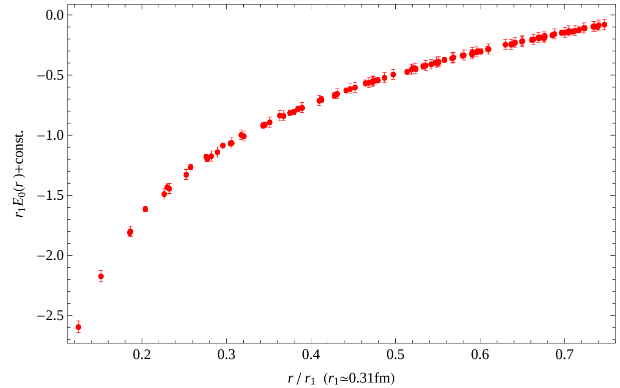

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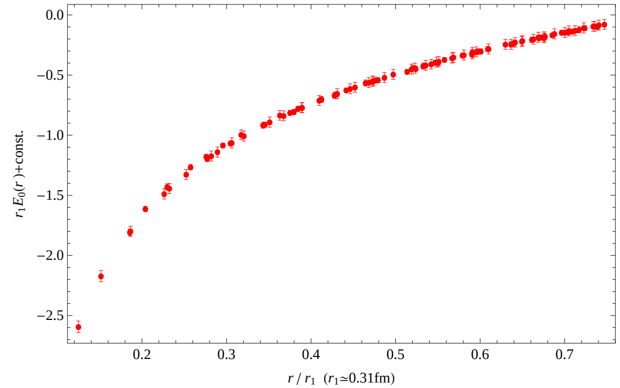

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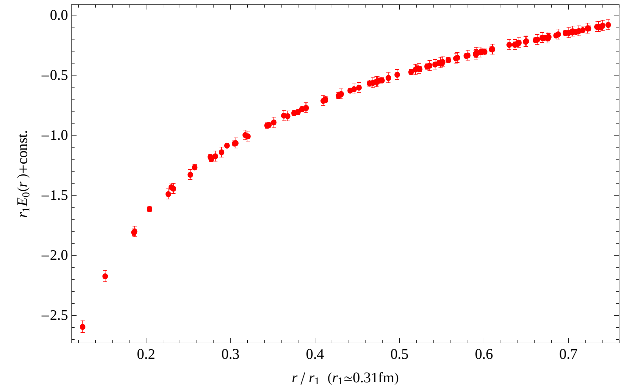

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Energy calculated in units of r_1

$$r^2 \frac{dE_0(r)}{dr}\Big|_{r=r_1} = 1$$

Lattice data for several gauge couplings
$\beta = 7.150, 7.280, 7.373, 7.596, 7.825,$
the smallest lattice spacing is $a = 0.041$ fm

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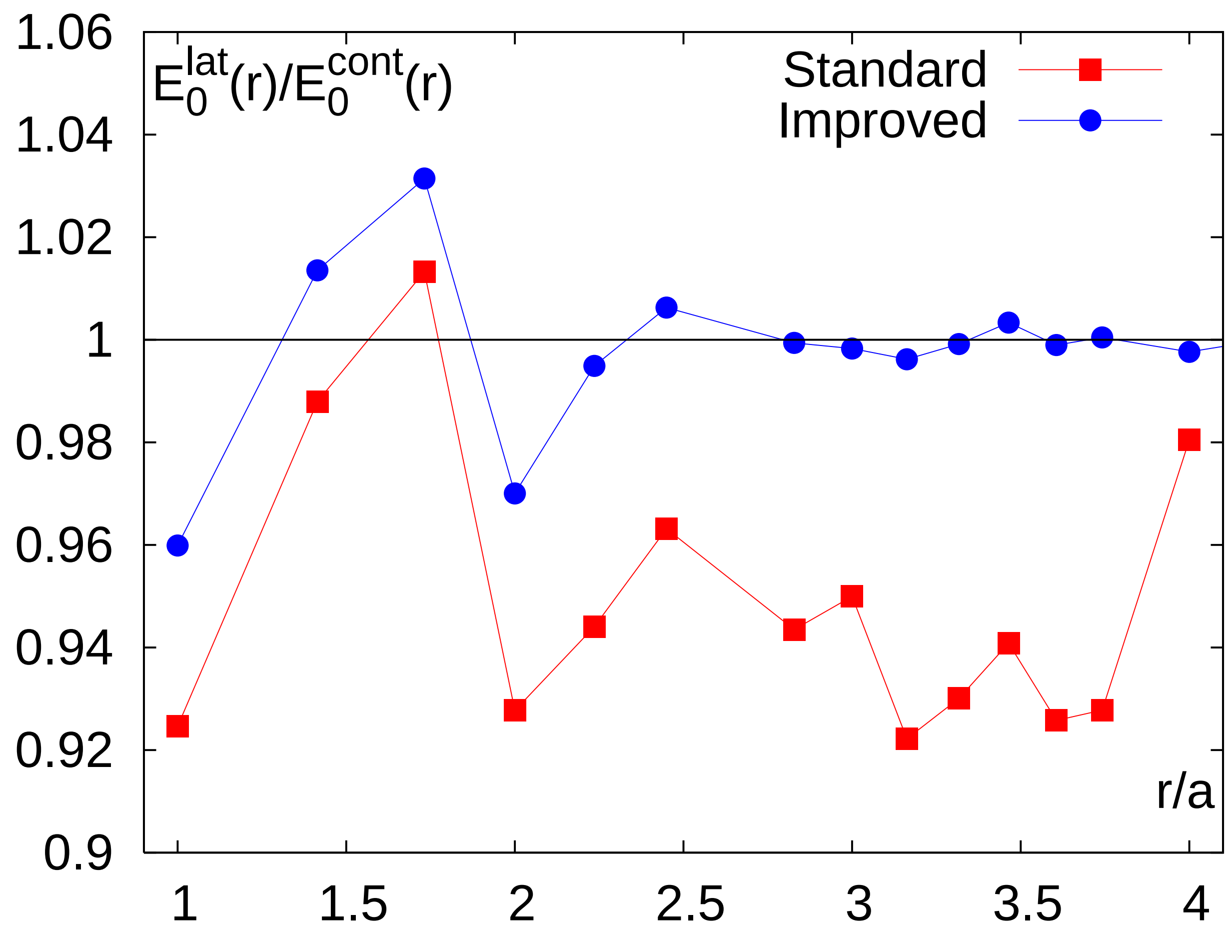
Replace r by improved distance $r_I = (4\pi C_L(r))^{-1}$

Necco Sommer'01

$$C_L(r) = \int \frac{d^3k}{(2\pi)^3} D_{00}(k_0 = 0, \vec{k}) e^{i\vec{k}\vec{r}}$$

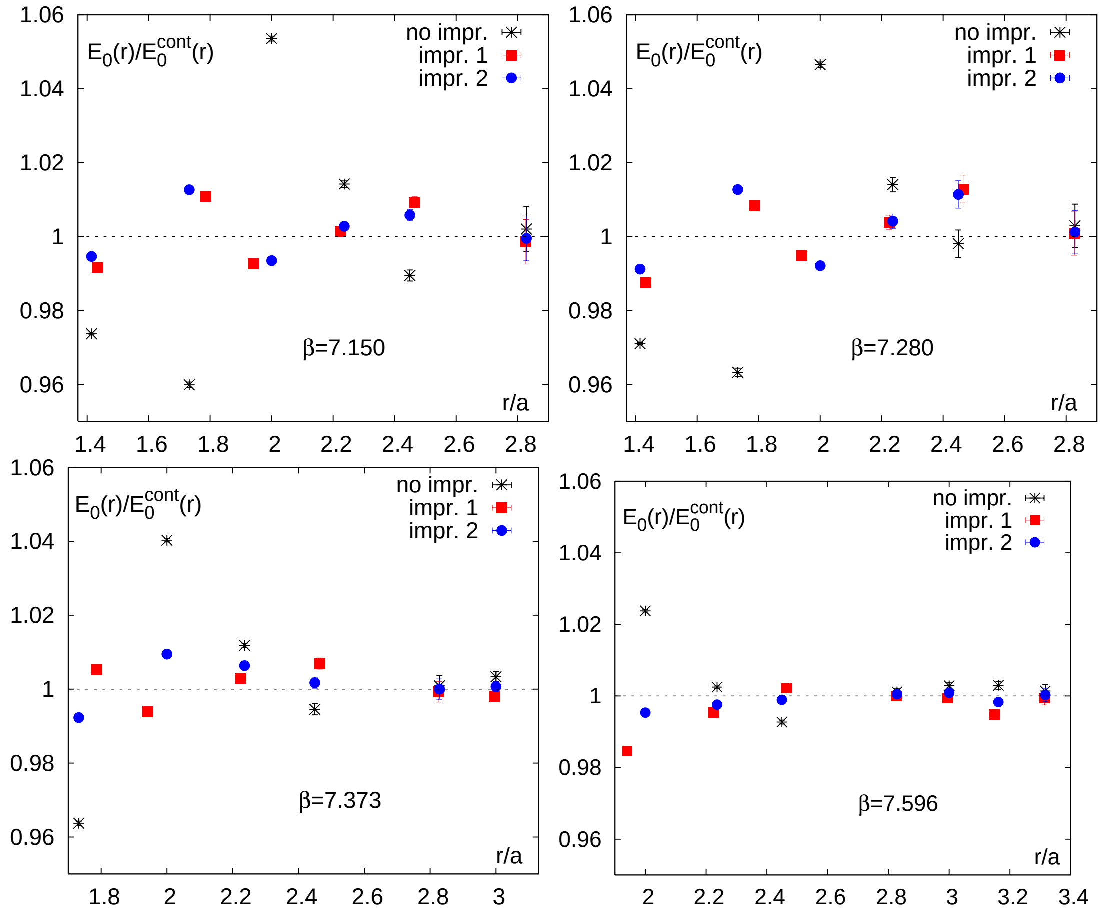
(D_{00} is the tree-level gluon propagator on the lattice)

Tree level



Discretization effects $\lesssim 1\%$ for $r/a > 2$

To estimate cutoff effects in actual calculation, need continuum estimate of E_0. Assume cutoff effects negligible for $r/a > 2$, fit $\beta = 7.825$ results to Coulomb plus linear plus constant form, to get continuum estimate.



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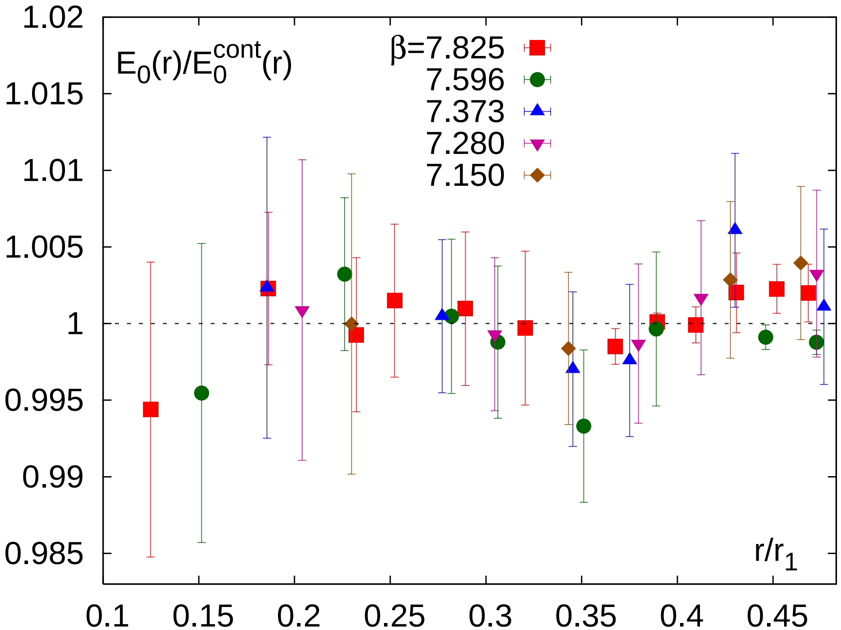
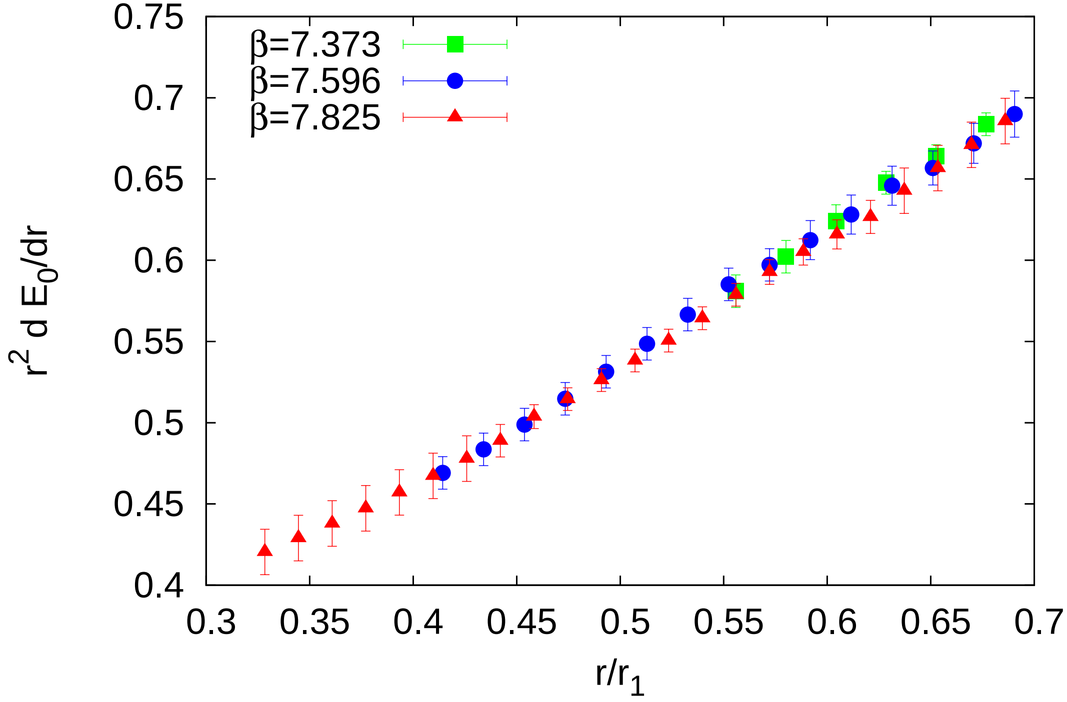


Can also calculate force from the lattice data. Use only $r/a > 2$ to avoid problems with lattice artifacts. Obtained with smoothing splines



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Perturbative expression best suited for the comparison. Use pert. expression for the force

$$E_0 \sim -\frac{C_F}{r}\alpha_E(r,\nu) + RS(\rho)$$

Beneke'98; Hoang *el al.*'99; Pineda'01

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$\alpha_E(r,\nu)$: series in $\alpha_{\rm s}(\nu)$, contain $\ln(r\nu)$ terms
$RS(\rho)$: series in $\alpha_{\rm s}(\rho)$, affected by uncertainties in computation of renormalon

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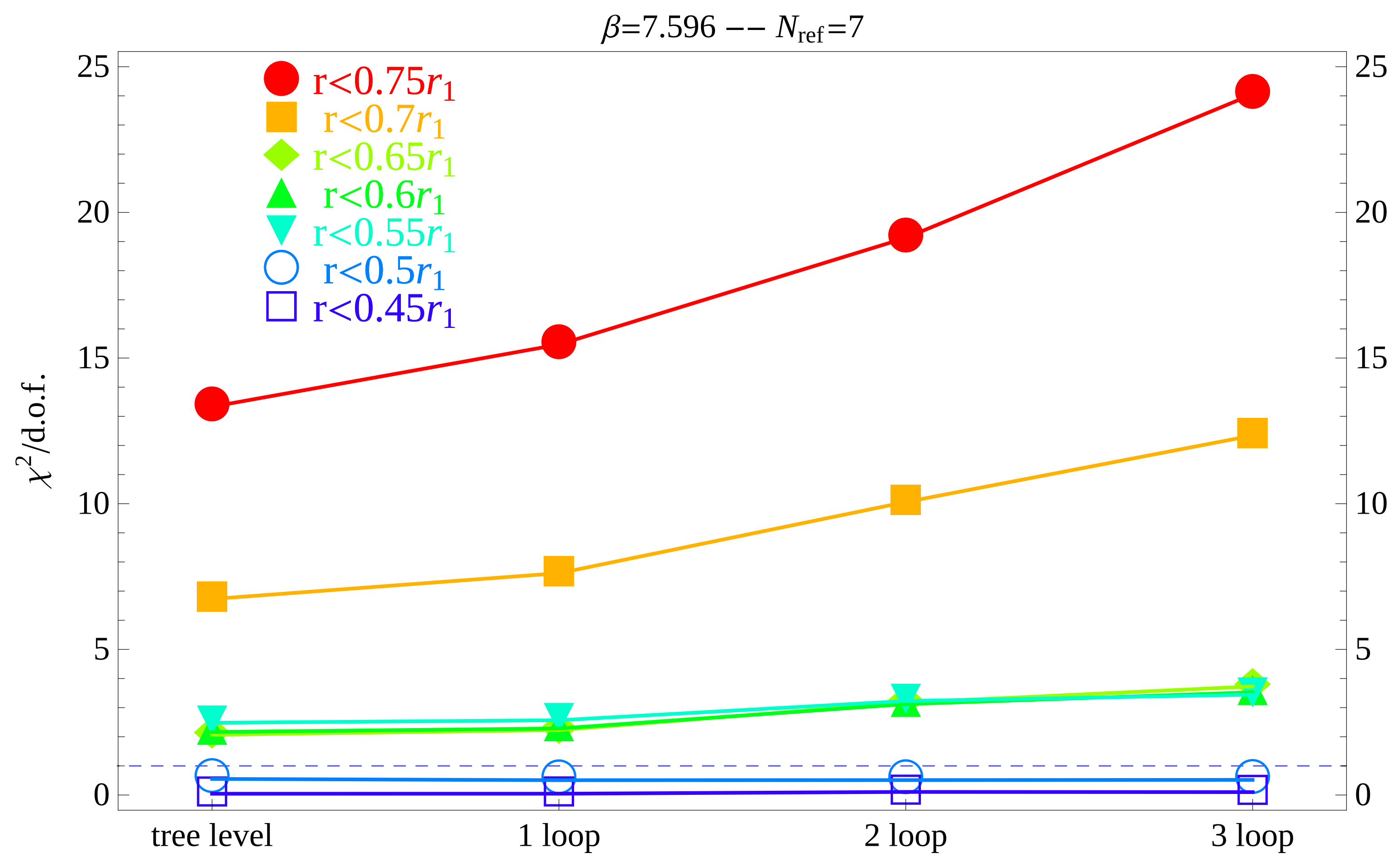
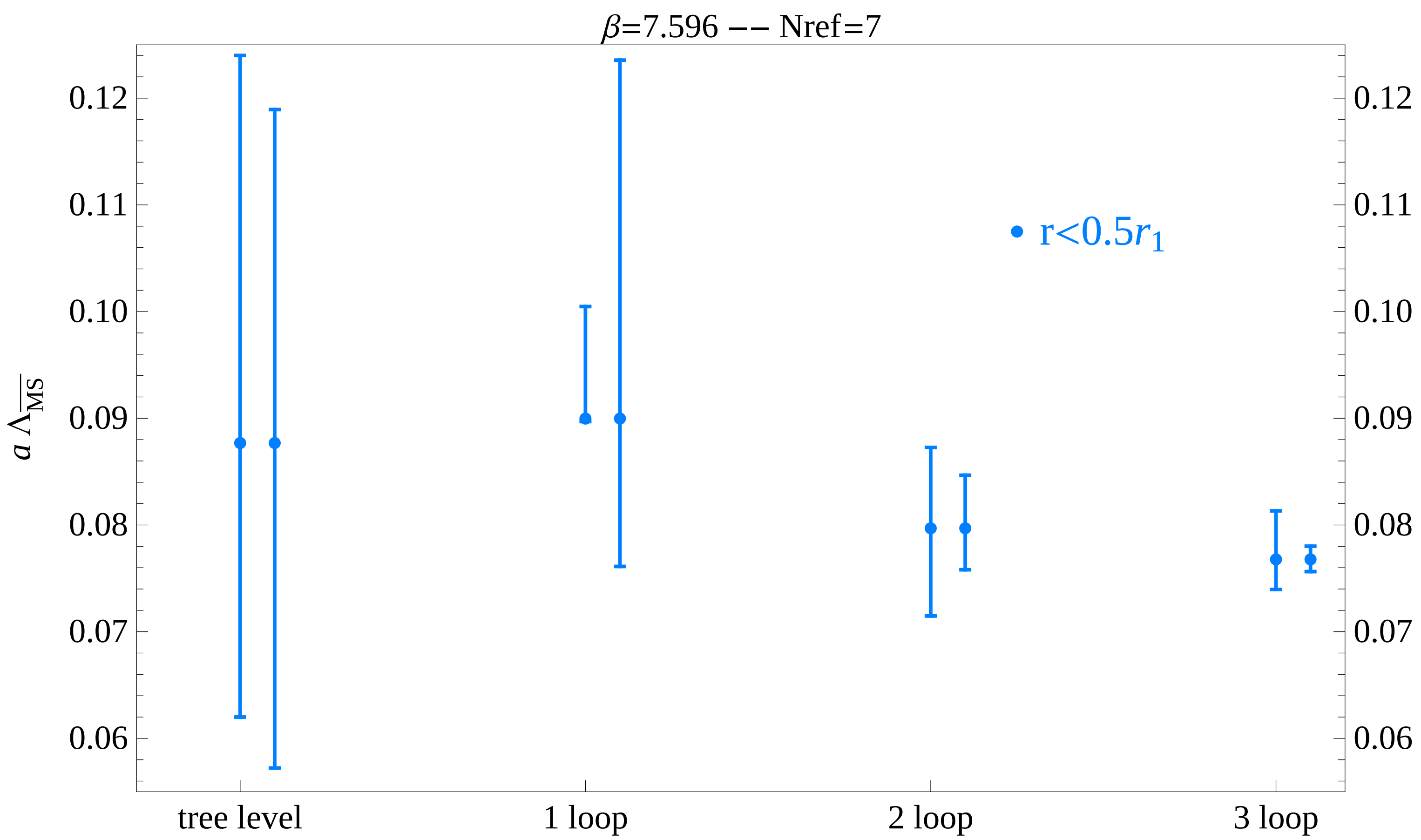
- Use ranges where χ^2 does not increase when increasing pert. order, or is $\lesssim 1$
- Estimate pert. uncertainty: Repeat fits with scale variation, and adding $\pm(C_F/r^2)\alpha_{\rm s}^{n+2}$


β=7.596 –– N_ref=7
r<0.75r_1
r<0.7r_1
r<0.65r_1
r<0.6r_1
r<0.55r_1
r<0.5r_1
r<0.45r_1
χ^2/d.o.f.
0
5
10
15
20
25
tree level
1 loop
2 loop
3 loop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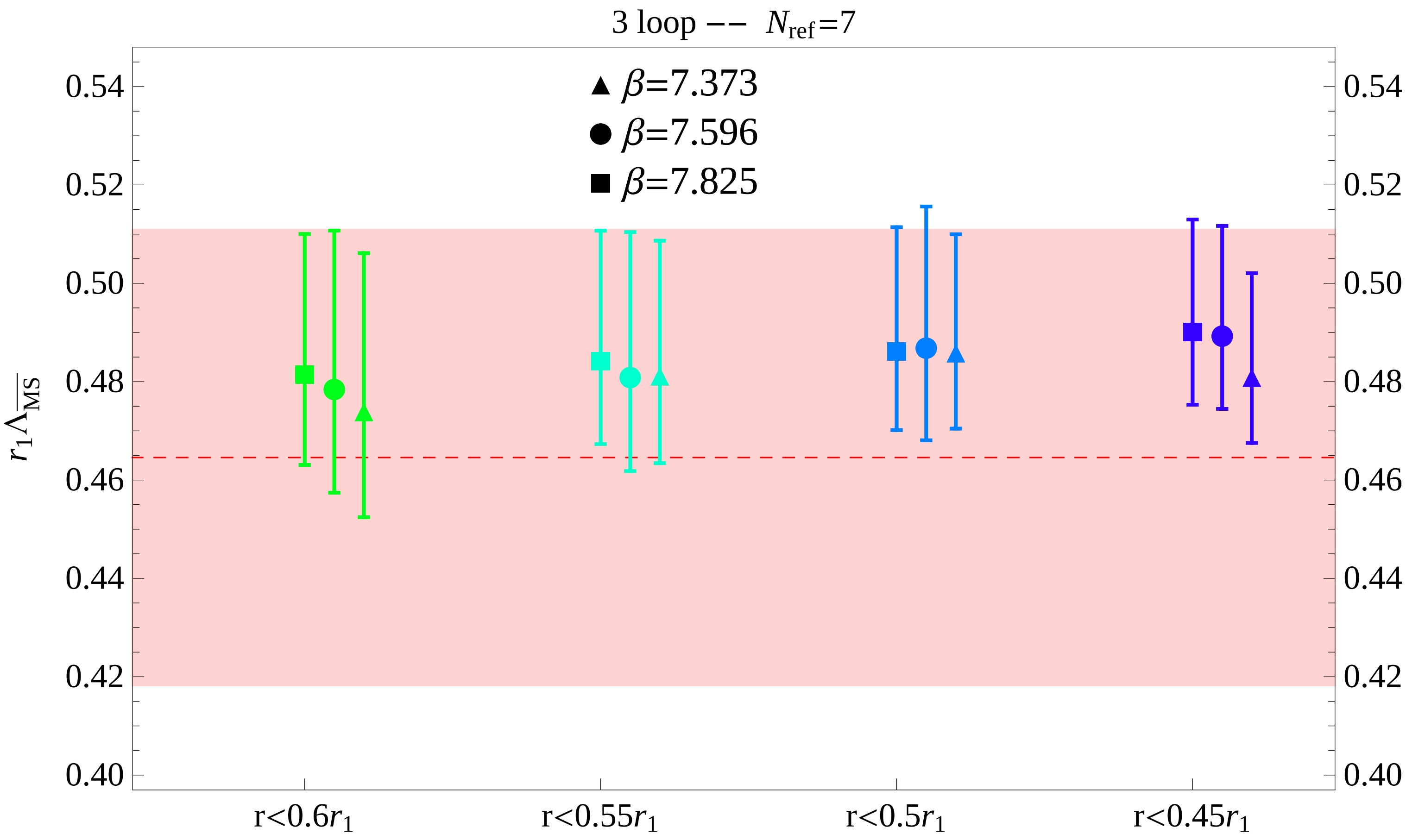
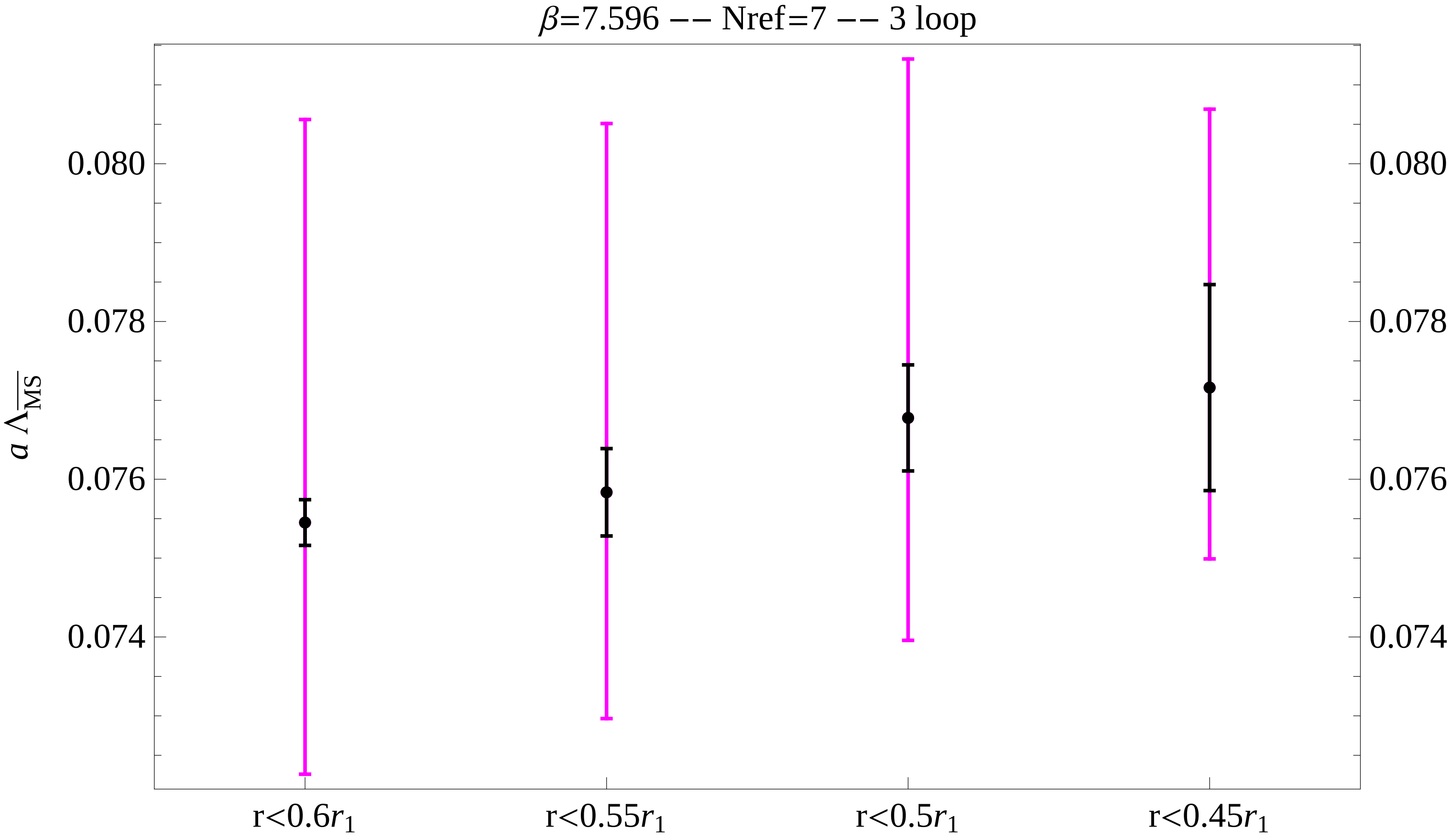
2 loop
3 loop

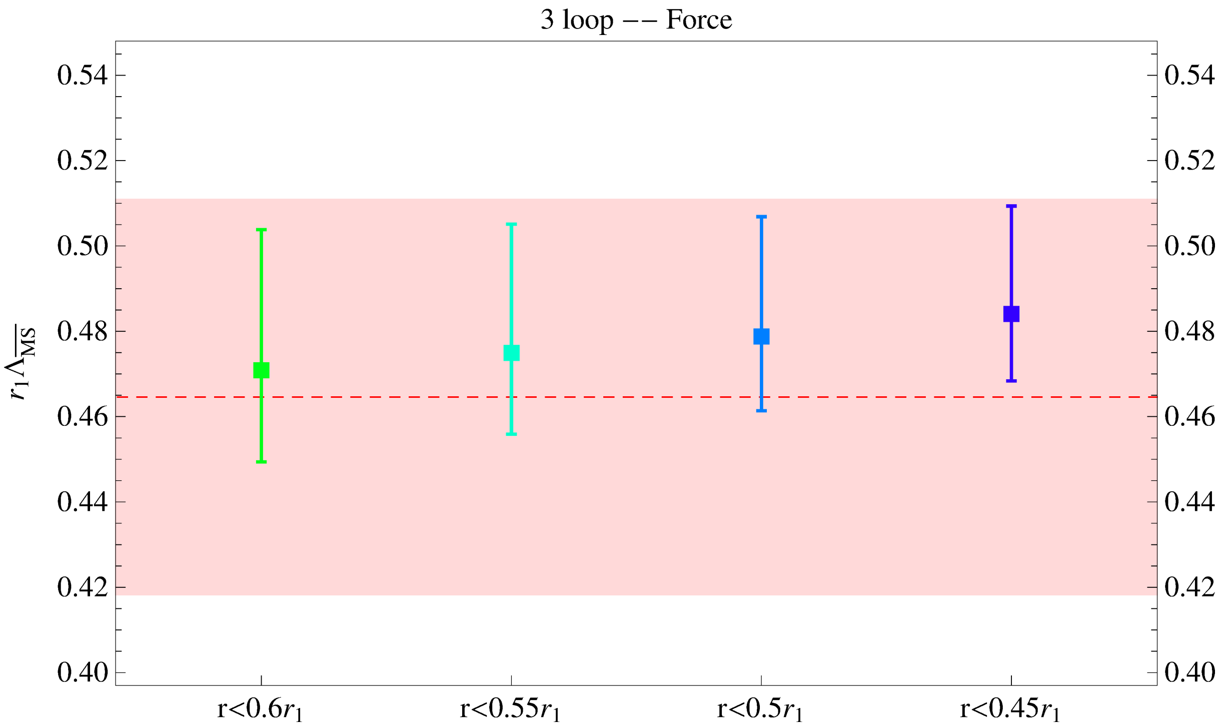

3 loop -- N_{ref}=7
▲ β=7.373
● β=7.596
■ β=7.825
$r_1\Lambda_{\overline{MS}}$
0.54
0.52
0.50
0.48
0.46
0.44
0.42
0.40
r<0.6r_1
r<0.55r_1
r<0.5r_1
r<0.45r_1


β=7.596 −− Nref=7 −− 3 loop
a Λ_MS
0.080
0.078
0.076
0.074
r<0.6r_1
r<0.55r_1
r<0.5r_1
r<0.45r_1

Cross checks

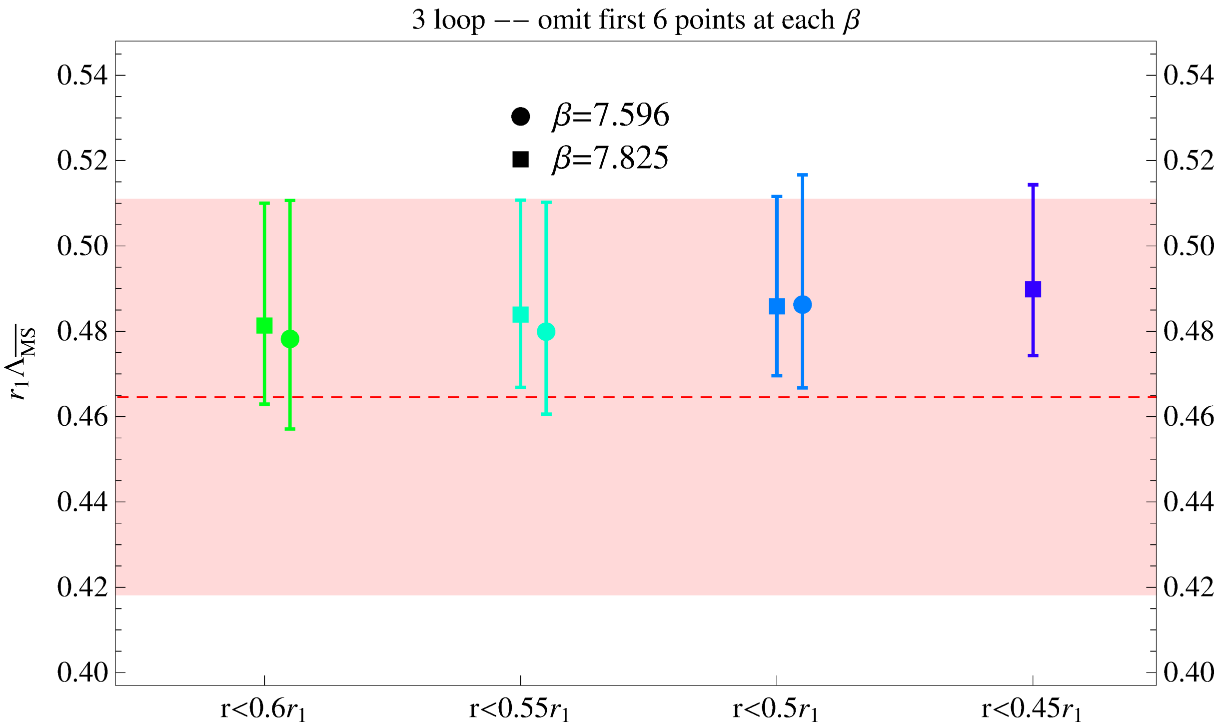
Cross checks

- Compare with lattice data for the force



Cross checks

- Compare with lattice data for the force
- Exclude lattice points with larger systematic (discretization) uncertainties, i.e. use only points were these are negligible



Cross checks

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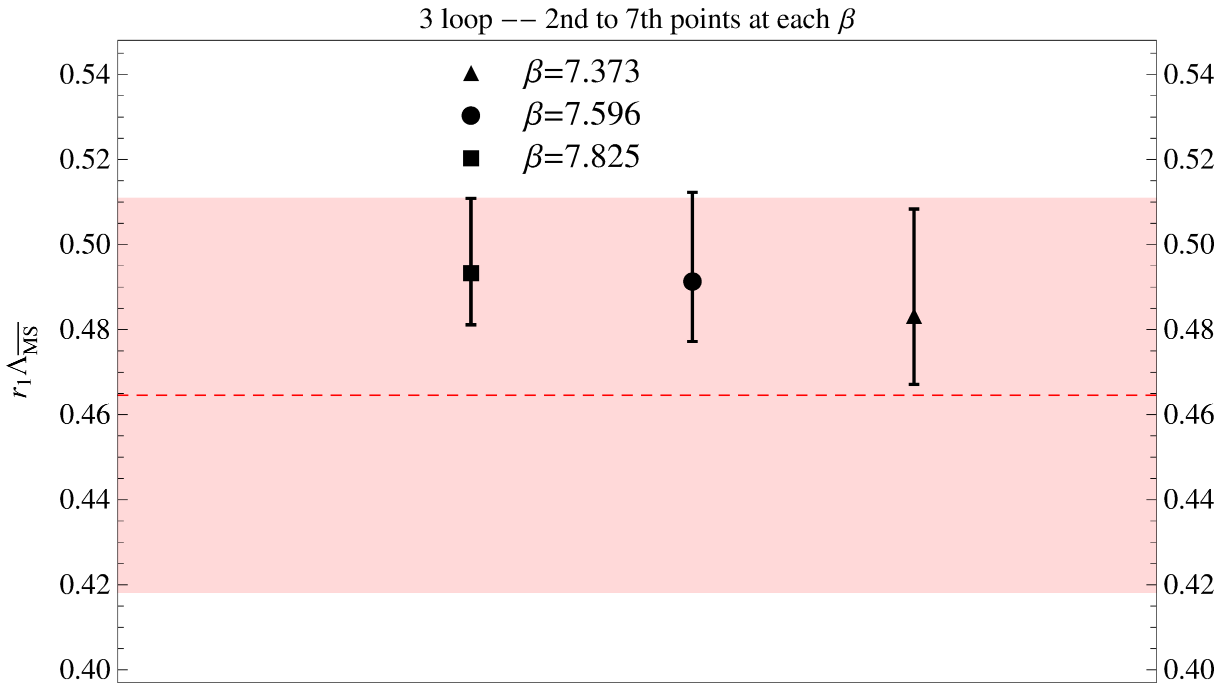
- Exclude lattice points with larger systematic (discretization) uncertainties, i.e. use only points were these are negligible
- Use only points at shorter distances



Cross checks

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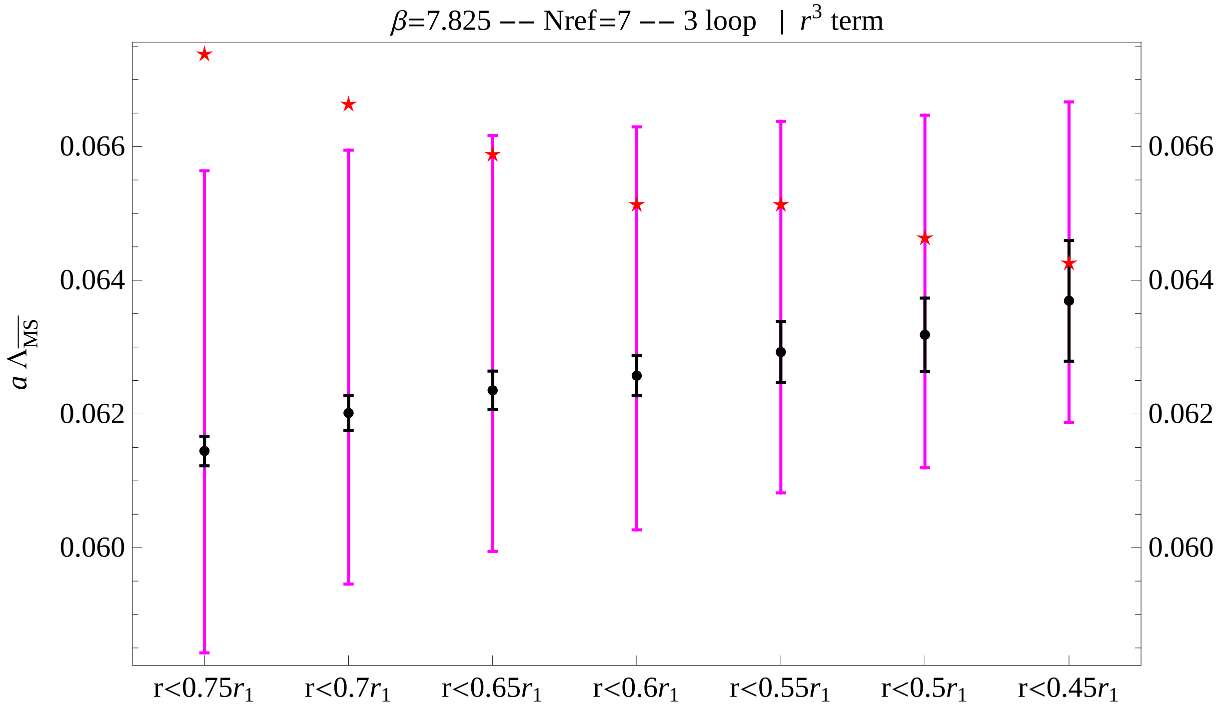
- Exclude lattice points with larger systematic (discretization) uncertainties, i.e. use only points were these are negligible
- Use only points at shorter distances
- Possible influence of non-perturbative terms



Cross checks

- Compare with lattice data for the force
- Exclude lattice points with larger systematic (discretization) uncertainties, i.e. use only points were these are negligible
- Use only points at shorter distances
- Possible influence of non-perturbative terms

All the results perfectly compatible with each other. It shows that the extraction is robust

Result for α_s

We take the 3-loop + leading ultrasoft log res. accuracy result

$$r_1\Lambda_{\overline{\rm MS}} = 0.495^{+0.028}_{-0.018} \to \Lambda_{\overline{\rm MS}} = 315^{+18}_{-12}\ \mathrm{MeV}$$

$$\alpha_s(1.5\ \mathrm{GeV}, n_f=3) = 0.336^{+0.012}_{-0.008}$$

$$\to \alpha_s(M_Z, n_f=5) = 0.1166^{+0.0012}_{-0.0008}$$

	$a\Lambda_{\overline{\rm MS}}$ $N_{\rm ref}=7$	$a\Lambda_{\overline{\rm MS}}$ $N_{\rm ref}=8$	$a\Lambda_{\overline{\rm MS}}$ $N_{\rm ref}=9$	$a\Lambda_{\overline{\rm MS}}$	$r_1\Lambda_{\overline{\rm MS}}$
$\beta=7.373$	$0.0957^{+0.0046}_{-0.0028}$ ±0.0017	$0.0957^{+0.0046}_{-0.0028}$ ±0.0017	$0.0957^{+0.0046}_{-0.0028}$ ±0.0017	$0.0957^{+0.0046}_{-0.0028}$ ±0.0017	$0.4949^{+0.0240}_{-0.0144}\pm0.0086\pm0.0025$ $=0.4949^{+0.0256}_{-0.0170}$
$\beta=7.596$	$0.0781^{+0.0046}_{-0.0029}$ ±0.0007	$0.0784^{+0.0043}_{-0.0027}$ ±0.0010	$0.0785^{+0.0046}_{-0.0029}$ ±0.0007	$0.0783^{+0.0048}_{-0.0031}$ ±0.0010	$0.4961^{+0.0303}_{-0.0197}{}^{+0.0066}_{-0.0061}\pm0.0044$ $=0.4961^{+0.0313}_{-0.0211}$
$\beta=7.825$	$0.0644^{+0.0032}_{-0.0019}$ ±0.0006	$0.0642^{+0.0033}_{-0.0020}$ ±0.0008	$0.0643^{+0.0032}_{-0.0020}$ ±0.0008	$0.0643^{+0.0033}_{-0.0021}$ ±0.0008	$0.4944^{+0.0256}_{-0.0159}\pm0.0065\pm0.0037$ $=0.4944^{+0.0267}_{-0.0175}$
Average					$\boldsymbol{r_1\Lambda_{\overline{\rm MS}}=0.495^{+0.028}_{-0.018}}$

Result for α_s

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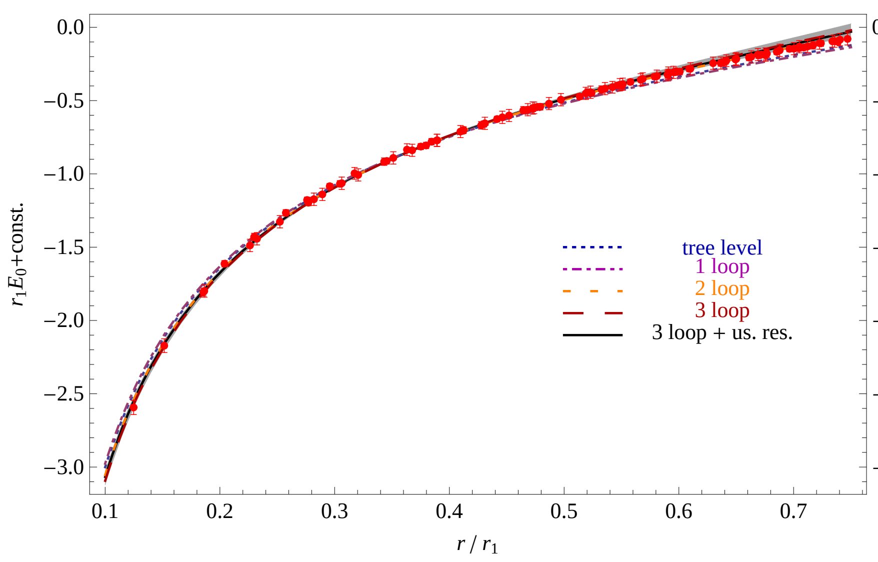
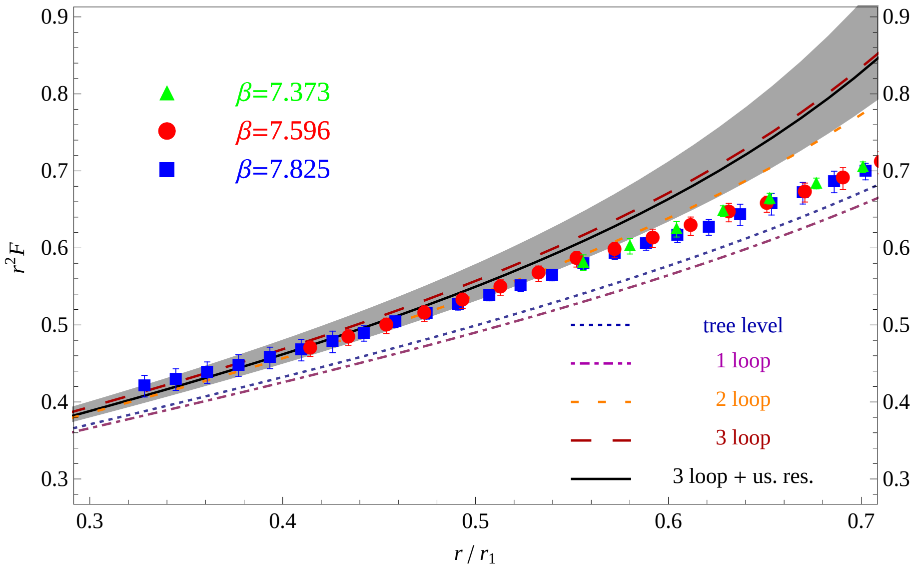
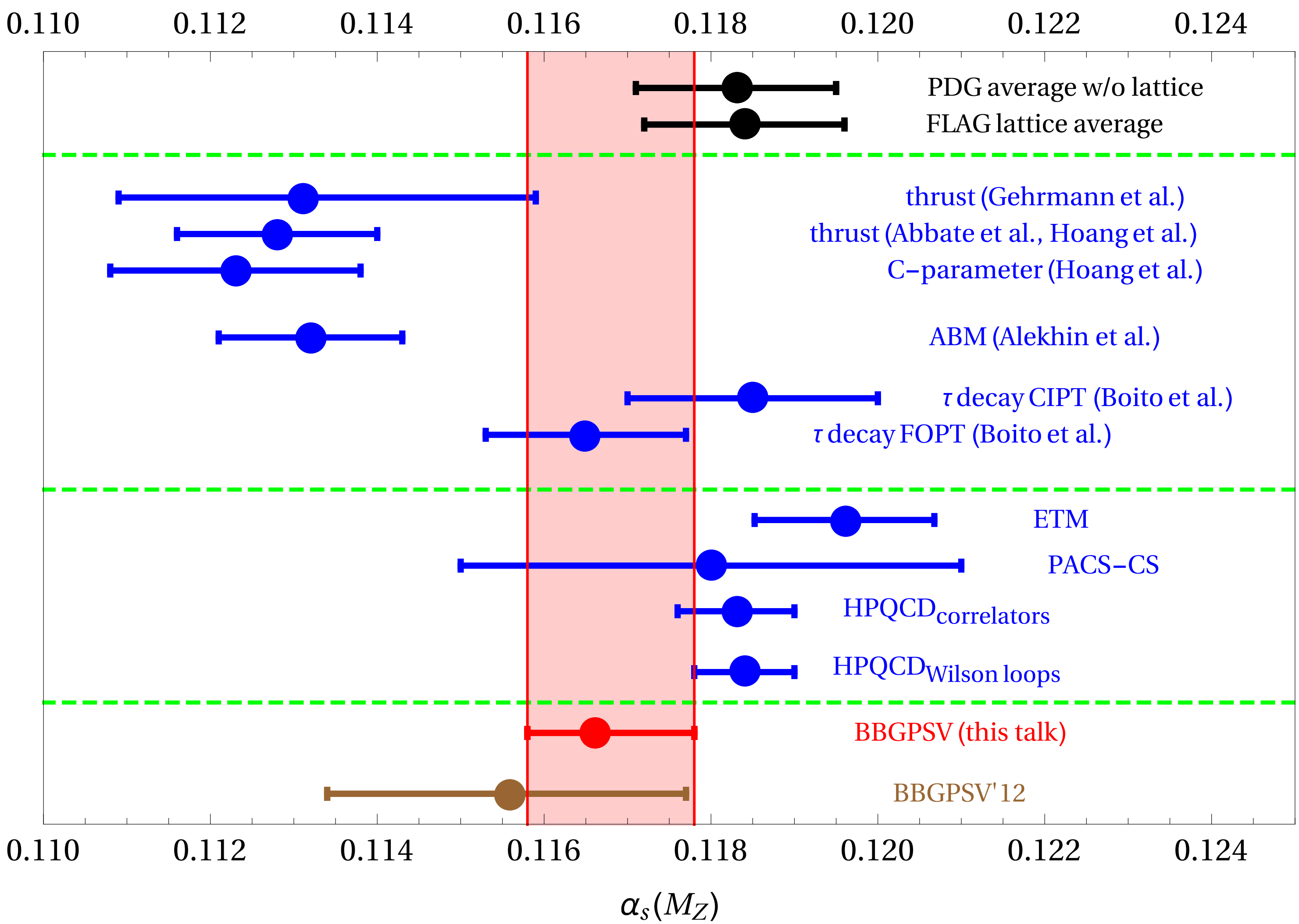




Comparison with other results



Conclusions

Determination of α_s by comparing lattice data for the short-distance part of the QCD static energy with perturbation theory (3 loop + resummation of ultrasoft logs accuracy)

$$\alpha_s\left(M_Z\right) = 0.1166^{+0.0012}_{-0.0008}$$

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Further data at shorter distance would reduce errors on $\alpha_{\rm s}$ extraction

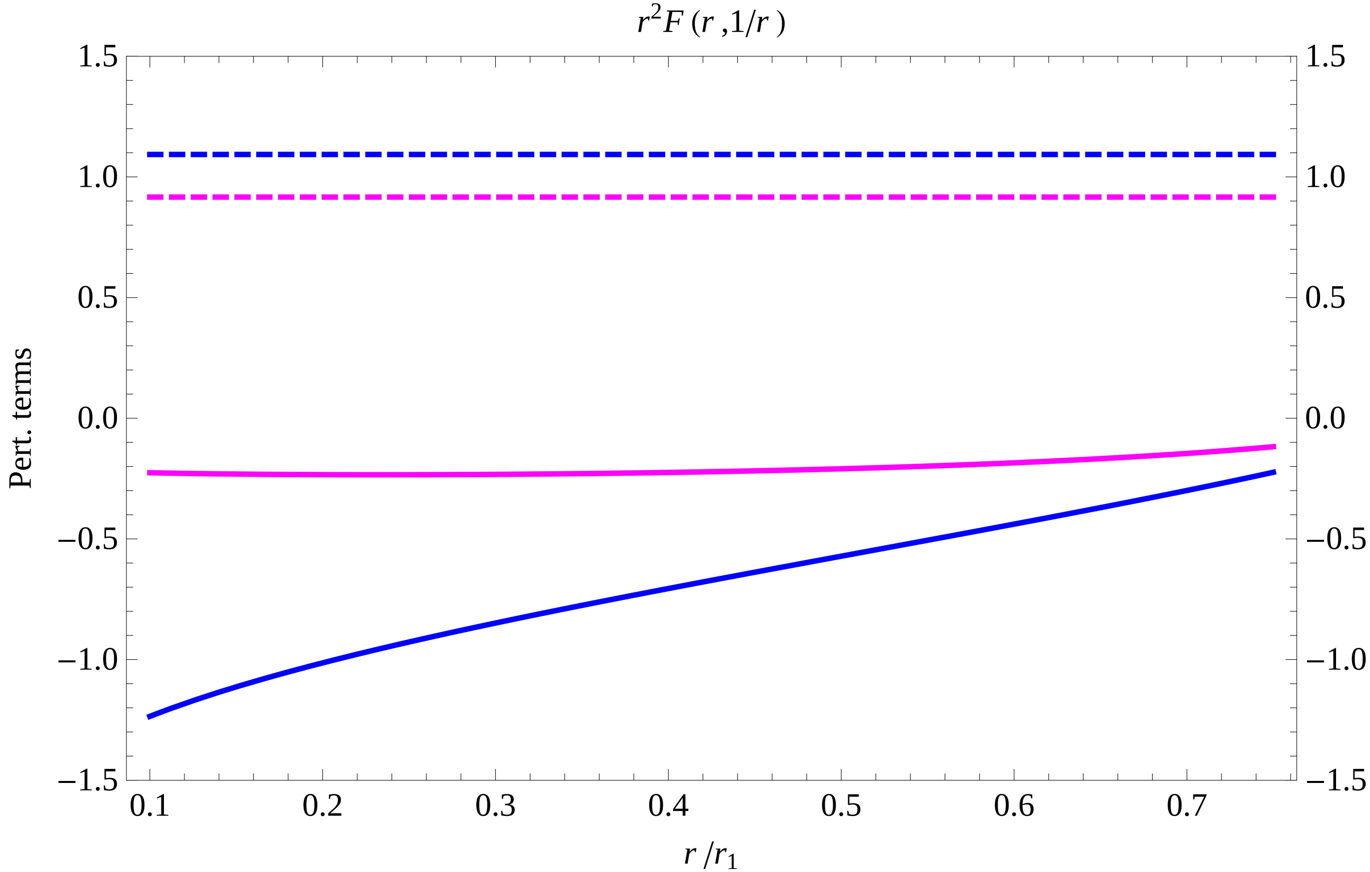
Thank you

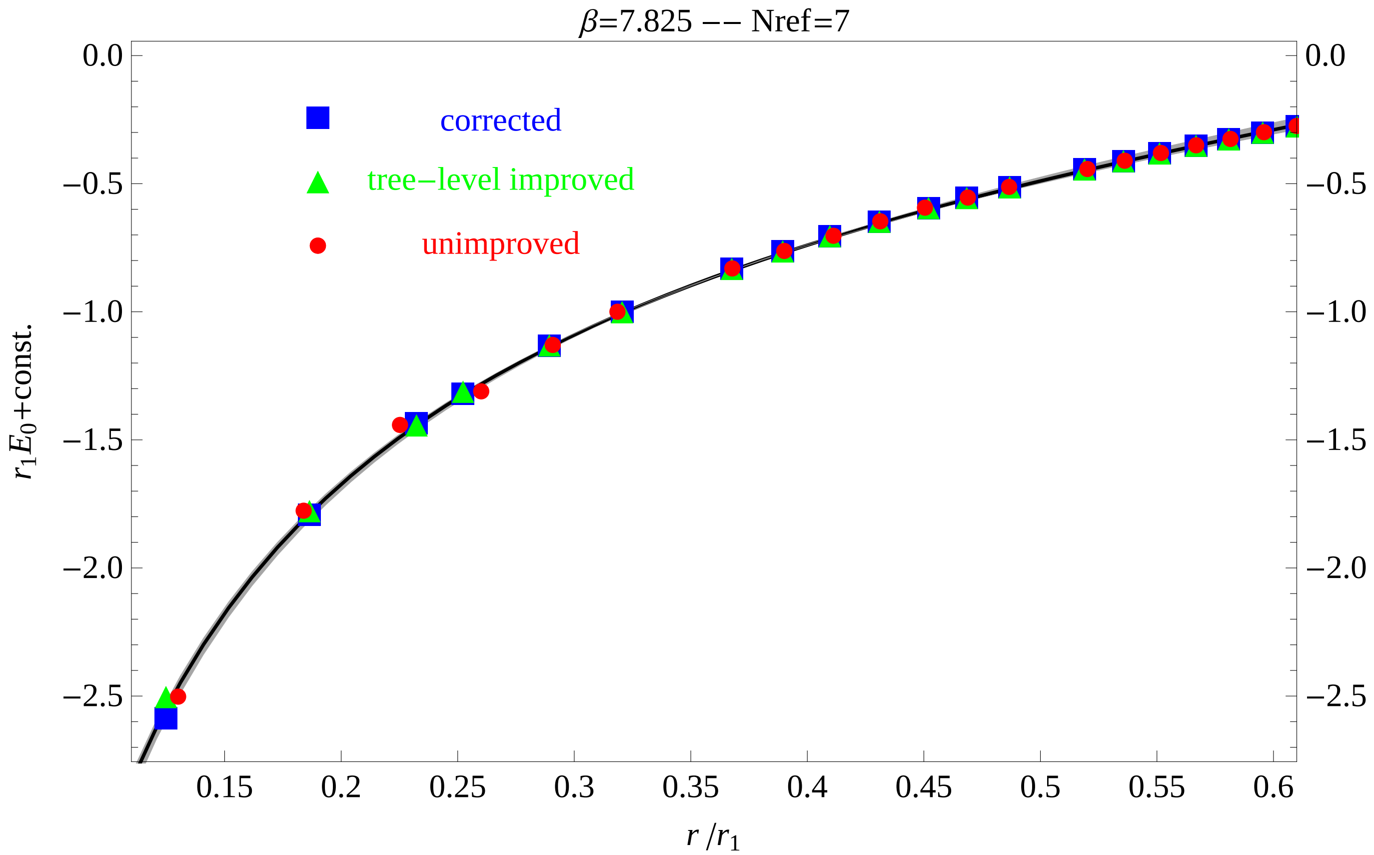
Backup slides

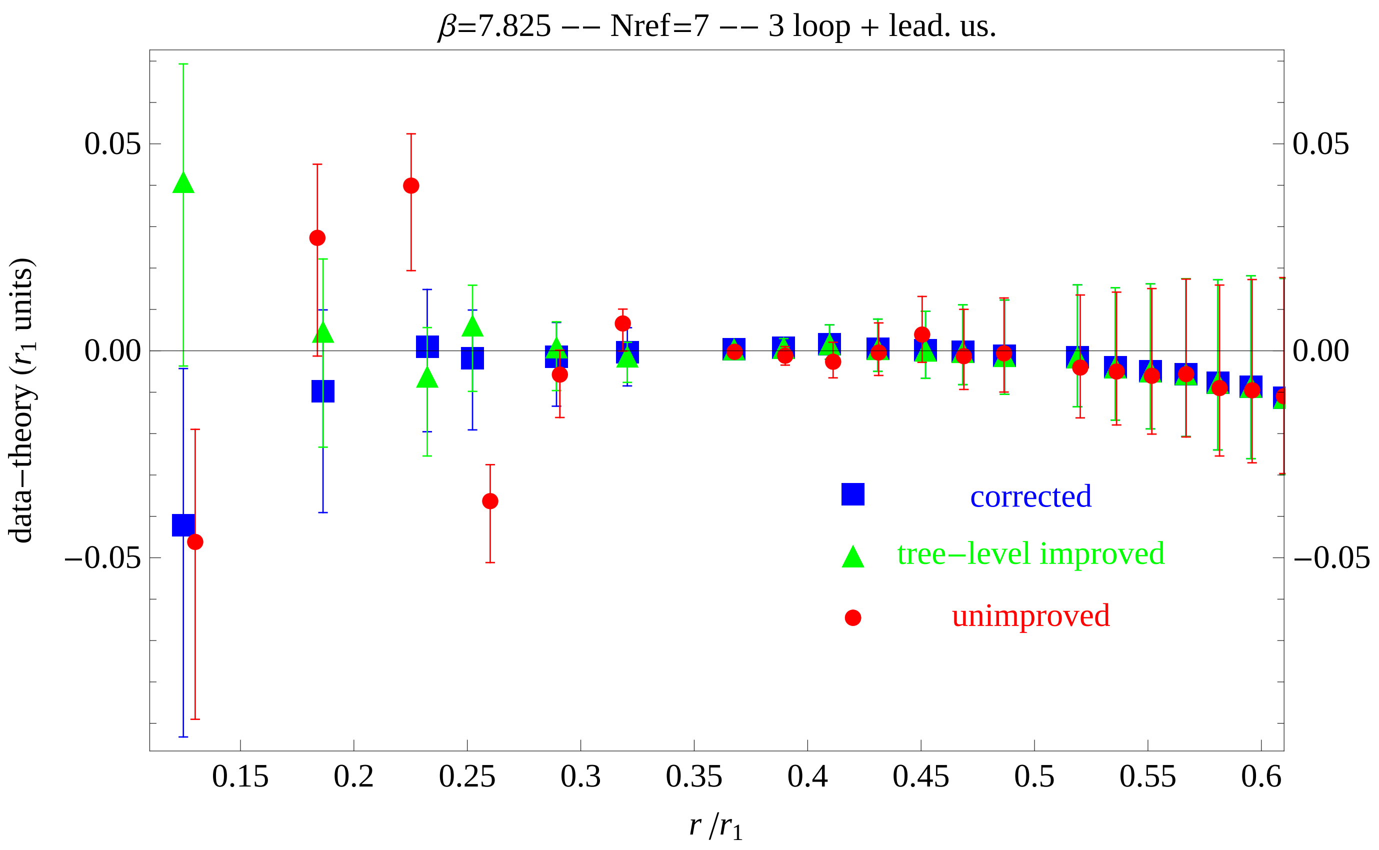
Correction factors for the static energy

β	$r/a = 1$	$r/a = \sqrt{2}$	$r/a = \sqrt{3}$	$r/a = 2$	$r/a = \sqrt{5}$	$r/a = \sqrt{6}$
7.150	0.980	0.995	1.007	0.988	1.000	1.010
7.280	0.980	0.997	1.008	0.992	1.000	1.013
7.373	0.980	0.998	1.009	0.994	0.995	1.005
7.596	0.980	0.995	1.005	0.994	1.000	1.001
7.825	0.968	0.992	1.005	0.994	0.998	1.001

Size of ultrasoft effects




β=7.825 −− Nref=7
corrected
tree−level improved
unimproved
r1E0+const.
r /r1
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6


β=7.825 –– Nref=7 –– 3 loop + lead. us.
data−theory (r_1 units)
0.05
0.00
−0.05
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6
r/r_1
corrected
tree−level improved
unimproved