Semileptonic heavy-light decays

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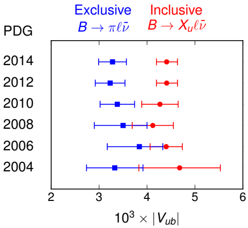
Fundamental Parameters from Lattice QCD

2015, Aug 31 – Sep 11

JG|U

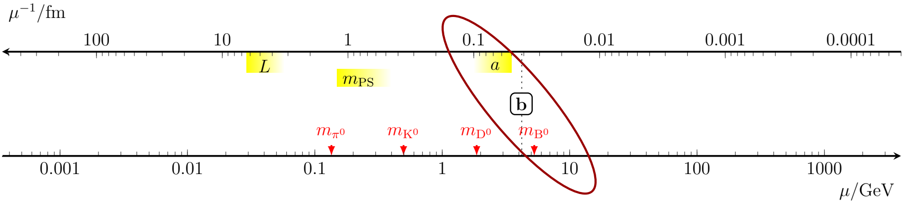
Semileptonic heavy-light decays $+$ Fundamental parameters from lattice QCD

- *determination of $V_{\rm ub}$*



- no light-cone sum rule
- present status of form factor determinations
  - $B \to \pi\ell\nu$
  - $B_{\rm s} \to K\ell\nu$
  - $\Lambda_{\rm b} \to p\ell^-\bar{\nu}_\ell$

  from lattice QCD (LATTICE 2015)
- discussion of systematics

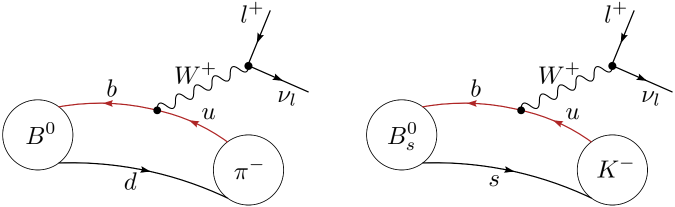


Typically: $a \gtrsim 0.05\,\mathrm{fm}$ vs. $L \lesssim 6\,\mathrm{fm}$ vs. $m_{\mathrm{PS}} \geq m_\pi$

- lattice discretisation & cont. limit — autocorrelations (top. freezing, algorithm, . . .)
- finite volume effects — to be corrected (if small)
- almost physical pion masses — χPT & beyond
- excited states, signal/noise problems, . . . — the staff of life
- renormalization, RG running & scheme dependences — RI-MOM, SF, lat, $\overline{\mathrm{MS}}$, . . .
- effective theory treatment of heavy quark — HQET, RHQ, NRQCD
- matching eff. theory and QCD — NP vs. PT

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 \sqrt{E_P^2 - m_P^2}}{q^4 m_{B_q}^2} \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) m_{B_q}^2 (E_P^2 - m_P^2) |f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2} (m_{B_q}^2 - m_P^2)^2 |f_0(q^2)|^2 \right]$$

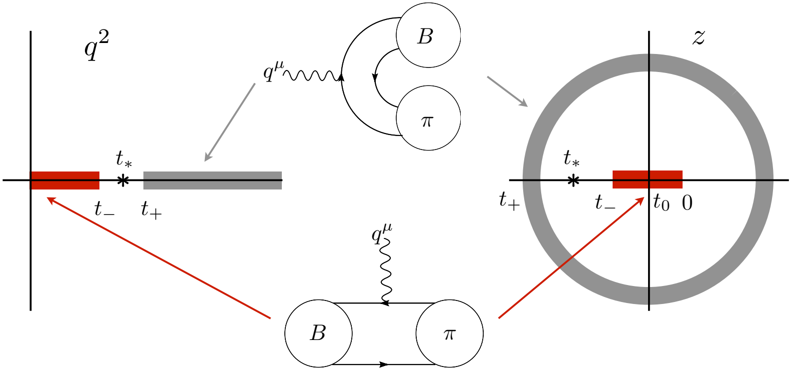
Example:



FF from lattice 3-point function:

$$\langle P(p') | \bar{b}\gamma_\mu q | B_q(p) \rangle = f_+(q^2) \left[p_\mu + p'_\mu - \frac{m_{B_q}^2 - m_P^2}{q^2} q_\mu \right] + f_0(q^2) \frac{m_{B_q}^2 - m_P^2}{q^2} q_\mu \ , \quad q = p - p'$$

Model-independent parameterisation: UNITARITY, ANALYTICITY (CROSSING-SYMMETRY) [1, 2]



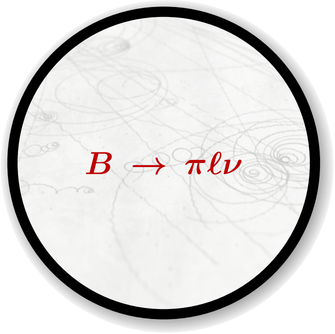
$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

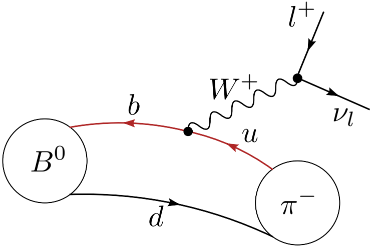
$$t_+ = (m_{\rm B} + m_P)^2 \,, \qquad t_0 < t_+$$

t_* : poles below threshold

$$f(q^2) = \frac{1}{B(q^2, t_*)\phi(q^2, t_0)} \sum_{n \geq 0} a_n z(q^2, t_0)^n$$

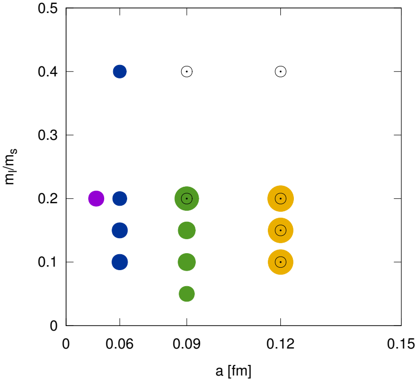
unitarity bound: $\sum_{m,n} B^{(\phi)}_{mn} a_n a_m \leq 1$


$B \to \pi\ell\nu$


l^+
W^+
b
u
ν_l
B^0
π^-
d

	Fermilab/MILC	**RBC/UKQCD**	**HPQCD**
N_{f}	$2+1$	$2+1$	$2+1$
Ensembles	MILC	RBC/UKQCD	MILC
a [fm]	4: $[0.045, 0.12]$	2: $[0.086, 0.11]$	2: $[0.09, 0.12]$
Lm_{π}^{min}	3.8	4.0	3.8
m_{π}^{min} [MeV]	220	289	260
Light quarks	asqtad	DW	asqtad
Heavy quark	PT–RHQ	NP–RHQ	NRQCD
Refs.	[3]	[4]	[5]

$B \to \pi\ell\nu$ by Fermilab/MILC[3]



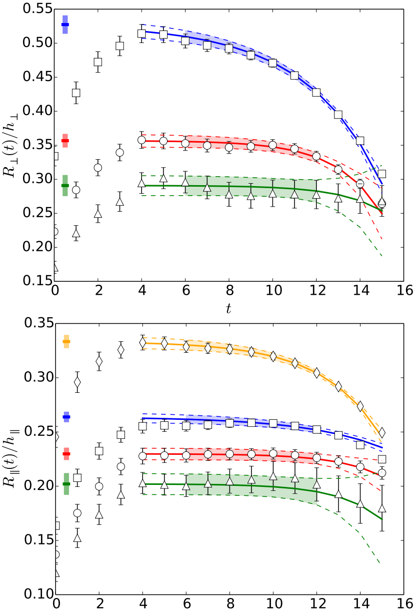
- 2,3-pt functions on each ensemble & construct appropriate ratios R_J
- determine excited state contaminations to $E_\pi(\mathbf{p})$, m_{B}
- test for autocorrelations (here: negative)

- FF parametrisation $\{f_+, f_0\} \to \{f_\parallel, f_\perp\}$
- mixed action approach to heavy-light physics [asqtad & Fermilab-RHQ]
- momenta: $|\mathbf{p}| = r \cdot \frac{2\pi}{L}$, $r = 0, 1, \sqrt{2}, \sqrt{3}\,(, 2)$
- cont. DR: $E_\pi(\mathbf{p}) \mapsto E_\pi(\mathbf{p}) = \sqrt{(\mathbf{p})^2 + (m_\pi^{(0)})^2}$
  - "$m_\pi^{(0)}$ has significantly smaller stat. error"
  - $\to$ "more stable & precise $\mathcal{M}_J^{(00)}$"
- 2 simultaneous fit ansaetze to determine ratios R_J ansatz with smaller ΔR_J chosen (p.t.o.)
- matching continuum QCD and lattice currents via *mostly non-perturbative renormalization* method

$$Z_{J_{bl}} = \rho_{J_{bl}} \sqrt{Z_{J_{bb}} Z_{J_{ll}}}\,,$$

$$\rho_J = 1 + \rho_J^{[1]} \alpha_V(q^*) + \mathrm{O}(\alpha_V^2)$$

$B \to \pi\ell\nu$ by Fermilab/MILC[3]



← coarsest ensemble ($a \approx 0.12\,\mathrm{fm}$, $m_l'/m_h' = 0.1$)

$$\frac{R_J(t)}{h_J} = f_J^{\mathrm{lat}}\left[1 + \mathcal{A}_J \mathrm{e}^{-\Delta M_B(T-t)}\right]$$

$$J \in \{\parallel, \perp\}$$

- ◊ $\mathbf{p} = \frac{2\pi}{\mathbf{L}}(0,0,0)$, p value=0.82
- □ $\mathbf{p} = \frac{2\pi}{\mathbf{L}}(1,0,0)$, p value=0.29 (combined fit)
- ○ $\mathbf{p} = \frac{2\pi}{\mathbf{L}}(1,1,,0)$, p value=0.98 (combined fit)
- △ $\mathbf{p} = \frac{2\pi}{\mathbf{L}}(1,1,1)$, p value=0.99 (combined fit)

→

- correct for mistuning in heavy-quark (kinetic) mass
- 1^{st}: extrapolation to physical point:

  SU(2) hard-pion HMrSχPT (NLO+NNLO analytic)
  + HQ discretisation effects + $g_{B^*B\pi}$
- 2^{nd}: z-expansion

- z-expansion: $t_0 = (m_B + m_\pi)(\sqrt{m_B} - \sqrt{m_\pi})^2$

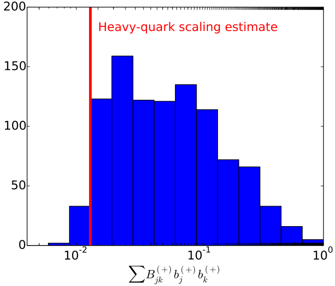
$$B_+(z)\phi_+(z)f_+(z) = \sum_{n\geq 0} a_n^+ z^n$$

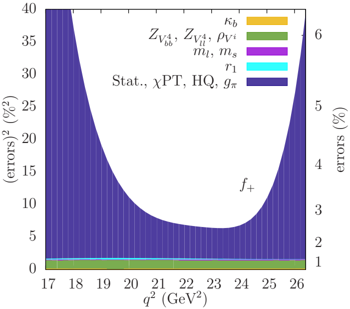
with

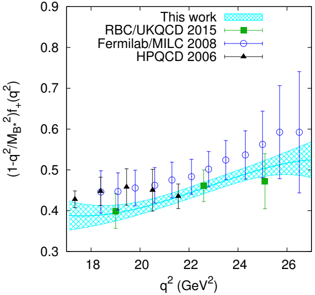
$$B_+(z) = 1 - q^2/m_{B^*}^2 \ ,$$
$$\phi_+(z) = 1$$

BCL parametrisation incl. asympt. behaviour ($q^2 \to \infty$)

- Check unitarity bound: $\sum_{m,n}^{N_z} B_{mn}^+ b_n^+ b_m^+ \leq 1$??







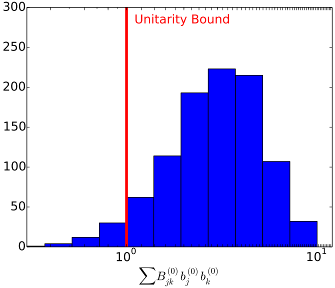
- z-expansion: $t_0 = (m_B + m_\pi)(\sqrt{m_B} - \sqrt{m_\pi})^2$

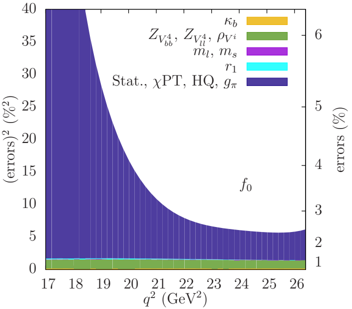
$$B_0(z)\phi_0(z)f_0(z) = \sum_{n\geq 0} a_n^0 z^n$$

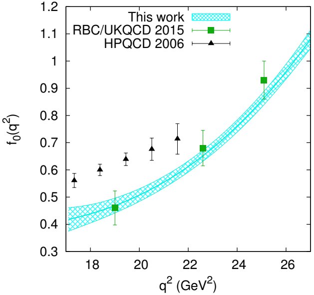
with (no poles incl.)

$$B_0(z) = 1 \,,$$
$$\phi_0(z) = 1$$

- Check unitarity bound: $\sum_{m,n}^{N_z} B_{mn}^0 b_n^0 b_m^0 \leq 1$??

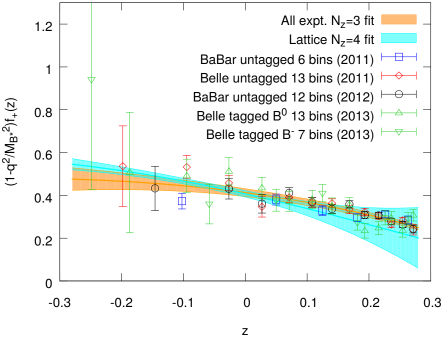


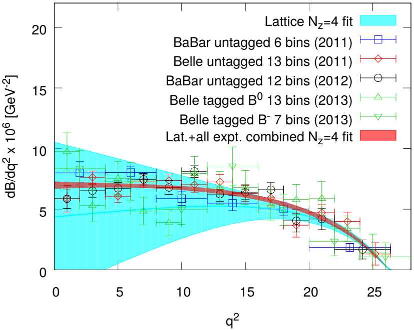




Combined analysis from form factor $f_+(q^2)$ gives

$$|V_{\rm ub}| = (3.72 \pm 0.16) \times 10^{-3}$$





- low p-value (0.02) due to tension between experimental data sets
- largest χ^2/d.o.f. contribution from BABAR11 (6-bin) data set

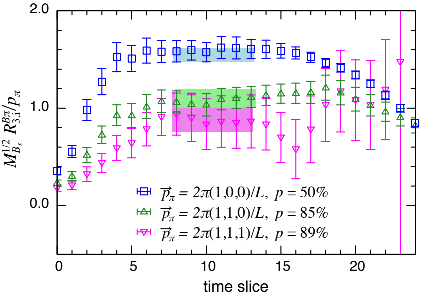
SAME LATTICE TECHNOLOGY AS BEFORE.

DIFFERENCES:

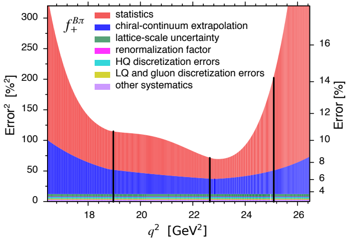
- 2+3 pion masses
  $422\,\mathrm{MeV} \geq m_\pi \geq 290\,\mathrm{MeV}$
- 2 lattice spacings $a \approx 0.11\,\mathrm{fm}, 0.86\,\mathrm{fm}$
- momenta $q^2_{\mathrm{max}} > q^2 \gtrsim 19.0\,\mathrm{GeV}^2$
- domain wall light quarks
- non-perturbatively tuned RHQ action
- simpler SU(2) (hard-π) HMχPT expressions
- scale enters via $m^{\mathrm{exp}}_{\mathrm{B_s}}$
- optimized src-snk separations to reduce excited states contaminations $\Rightarrow$

SIMILARITIES:

- computing $f_\parallel, f_\perp$
- mostly non-perturbative renorm. for $Z^{bl}_{V_\mu}$
  via 1-lp MF-impr. PT at $\alpha_{\overline{\mathrm{MS}}}(a^{-1})$
- continuum disp. relation for $E_\pi(\mathbf{p})$



$B \to \pi \ell \nu$ by RBC/UKQCD[4]



- z-expansion: $t_0 = (m_B + m_\pi)(\sqrt{m_B} - \sqrt{m_\pi})^2$

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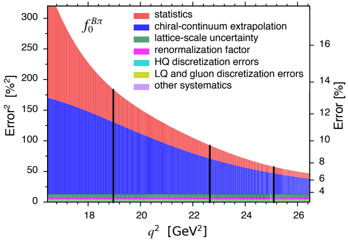
BCL parametrisation incl. asympt. behaviour ($q^2 \to \infty$)



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with (no poles incl.)

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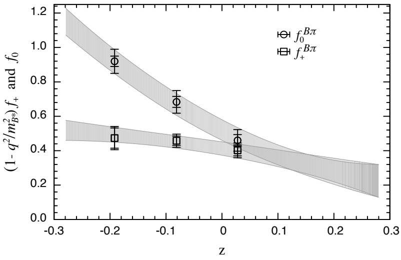
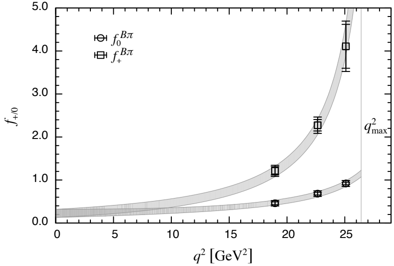
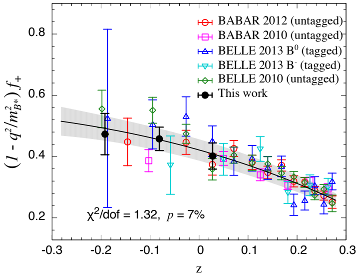
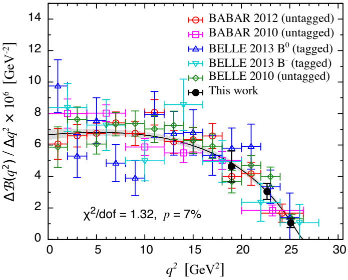
$B \to \pi\ell\nu$ by RBC/UKQCD[4]

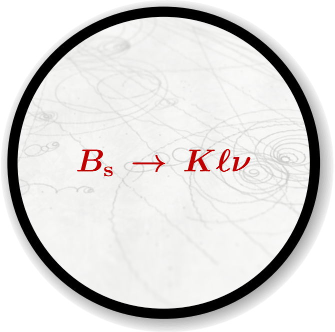


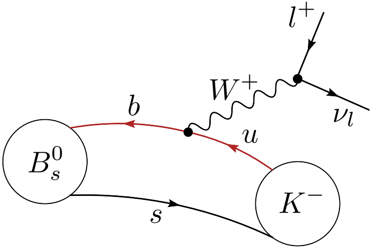


$$\Rightarrow \qquad |V_{\mathrm{ub}}| = (3.61 \pm 0.32) \times 10^{-3} \qquad\qquad (16\,\mathrm{GeV}^2 \le q^2 \le q^2_{\max})$$






$B_s \to K\ell\nu$


l^+
W^+
b
ν_l
u
B_s^0
K^-
s

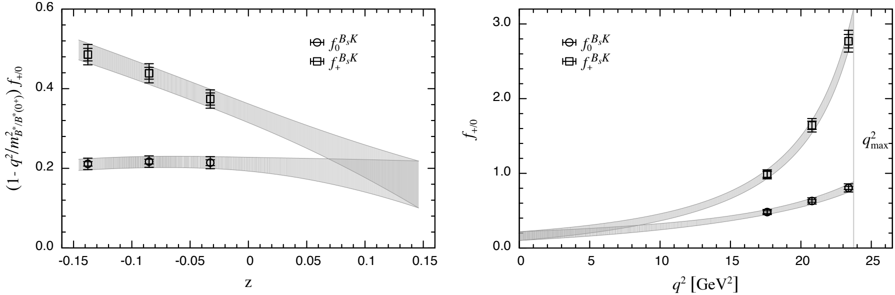
$B_s \to K\ell\nu$

	RBC/UKQCD	HPQCD	ALPHA
N_f	$2+1$	$2+1$	2
ENSEMBLES	RBC/UKQCD	MILC	CLS
a [fm]	2: $[0.086, 0.11]$	2: $[0.09, 0.12]$	3: $[0.048, 0.075]$
$Lm_\pi^{\rm min}$	4.0	3.8	4.0
$m_\pi^{\rm min}$ [MeV]	289	260	310
LIGHT QUARKS	DW	asqtad	NP O(a) improved
HEAVY QUARK	NP–RHQ	NRQCD	NP–HQET
REFS.	[4]	[5]	[6]

$B_s \to K\ell\nu$ by RBC/UKQCD[4]

SAME ENSEMBLES AS FOR $B \to \pi\ell\nu$

$q^2\ [\mathrm{GeV}^2] = 17.6, 20.8, 23.4$



- analogous analysis
- smaller uncertainties in $B_s \to K$ channel

$B_s \to K\ell\nu$ by HPQCD[5]

EXCERPT OF PROCEDURE:

- triply mixed-action approach
  sea: asqtad MILC / *valence:* HISQ light + NRQCD b quark
- 5 MILC ensembles, 2 lattice spacings, $|\mathbf{p}_K| \leq \sqrt{3} \cdot 2\pi/L$
- matching (improved) lattice NRQCD vector current using 1-loop HISQ lattice-PT
- simultaneous Bayesian fit of 2,3-pt function with priors
  + 'marginalization' : using $(N, \tilde{N}) = (8, 8)$ states
  + 'chaining' : iterative update of priors by best-fit mean & cov.
- *modified z-expansion:* hard-π 'inspired' factorization + z-expansion per ensemble

$\Rightarrow$ simultaneous extrapolation (chiral + continuum + kinematic) ???

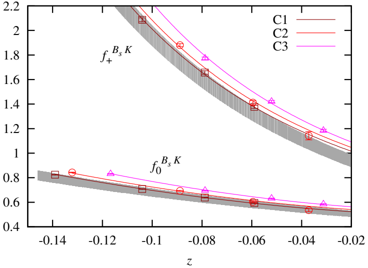
$$B(z) f(z, a) = (1 + \text{[logs]}) \sum_{k=0}^{K} D_k(a) \cdot a_k \, z(q^2, t_0)^k$$

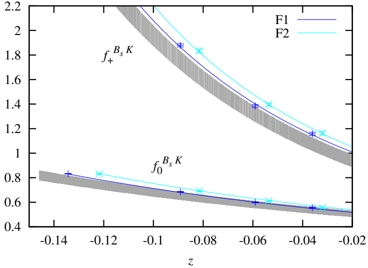
and imposing $f_0 \equiv f_+$ at $(q^2, a) = (0, 0)$

$$m^+_{\text{pole}} = 5.3252(5)\ \text{GeV}$$
$$m^0_{\text{pole}} = 5.6794(10)\ \text{GeV}$$

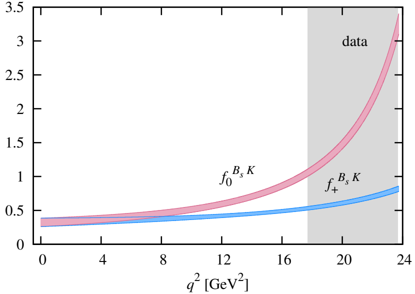
actually: $D(a) = D(a, m_l^{(v,s)}, m_s^{(v,s)})$

$B_s \to K\ell\nu$ by HPQCD[5]





$$\Rightarrow \qquad |V_{\rm ub}| = (3.47 \pm 0.22) \times 10^{-3} \qquad\qquad (17 \lesssim q^2\ [\mathrm{GeV}^2] \lesssim 22)$$

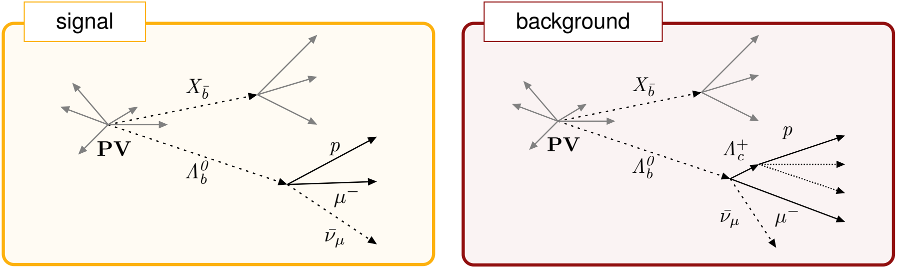


1st DETERMINATION OF V_{ub} FROM BARYONIC DECAY

$$\frac{|V_{\mathrm{ub}}|^2}{|V_{\mathrm{cb}}|^2} = \frac{\mathcal{B}(\Lambda_{\mathrm{b}}^0 \to p\mu^- \bar{\nu}_\mu)}{\mathcal{B}(\Lambda_{\mathrm{b}}^0 \to \Lambda_{\mathrm{c}}^+ \mu^- \bar{\nu}_\mu)} R_{\mathrm{FF}}$$

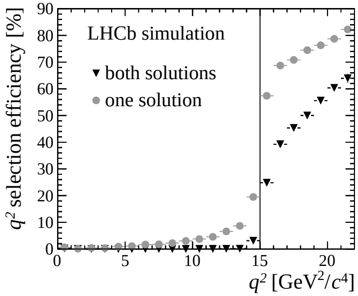
- LHCb measurement [7]
- Lattice determination of R_{FF} [8]

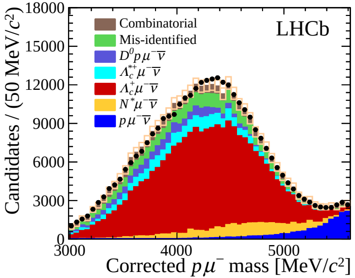
LHCb measurement [7]



- Λ_{b}^0 constitutes $\sim 20\%$ of b-hadrons produced at LHC
- huge branching fraction of $\Lambda_{\mathrm{b}} \to p\ell\nu$ compared to other decays
- $X_{\bar{b}}$ decay products reconstructed far away from signal

LHCb measurement [7]





- stringent particle identification (PID) requirements applied to proton
- proton momentum $> 15\,\mathrm{GeV}$ for PID performance (best above Cherenkov light threshold)
- 2-fold ambiguity in q^2 due to missing neutrino
  $\leftarrow$ both solutions required to exceed
  $15\,\mathrm{GeV}^2$ $(\Lambda_\mathrm{b} \to p)$ and $7\,\mathrm{GeV}^2$ $(\Lambda_\mathrm{b} \to \Lambda_\mathrm{c})$
  to avoid influence

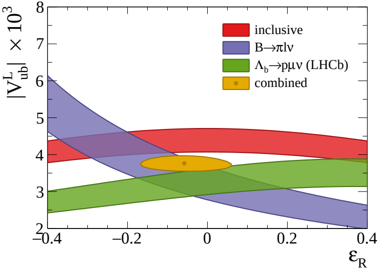
$$\frac{\mathcal{B}(\Lambda_b^0 \to p\mu^-\bar{\nu}_\mu)_{q^2>15\,\mathrm{GeV}^2}}{\mathcal{B}(\Lambda_b^0 \to \Lambda_c^+\mu^-\bar{\nu}_\mu)_{q^2>7\,\mathrm{GeV}^2}} = (1.00 \pm 0.04 \pm 0.08) \times 10^{-2}$$

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.083 \pm 0.004 \pm 0.004$$

$$R_{FF} = 0.68 \pm 0.07$$

With w.a. $|V_{cb}|_{\mathrm{excl.}} = (39.5 \pm 0.8) \times 10^{-3}$:

$$|V_{ub}| = (3.27 \pm 0.15 \pm 0.16 \pm 0.06) \times 10^{-3}$$



Source	Relative uncertainty (%)
$\mathcal{B}(\Lambda_c \to pK^+\pi^-)$	$^{+4.7}_{-5.3}$
Trigger	3.2
Tracking	3.0
Λ_c selection efficiency	3.0
$\Lambda_b \to N^*\mu^-\bar{\nu}_\mu$ shapes	2.3
Λ_b lifetime	1.5
Isolation	1.4
Form factor	1.0
Λ_b kinematics	0.5
q^2 migration	0.4
PID	0.2
Total	$^{+7.8}_{-8.2}$

Summary of systematic uncertainties. The table shows the relative systematic uncertainty on the ratio of the $\Lambda_b \to p\mu\bar{\nu}_\mu$ and $\Lambda_b \to \Lambda_c\mu\bar{\nu}_\mu$ branching fractions broken into its individual contributions. The total is obtained by adding them in quadrature. Uncertainties on the background levels are not listed here as they are incorporated into the fits.

Definition

$$R_{\text{FF}}^{-1} = \frac{\zeta_{p\mu\bar{\nu}}(15\ \text{GeV}^2)}{\zeta_{\Lambda_c\mu\bar{\nu}}(7\ \text{GeV}^2)}\,, \qquad \zeta_{p\mu\bar{\nu}}(x) \equiv \frac{1}{|V_{ub}|^2}\int_x^{q^2_{\text{max}}} \frac{\mathrm{d}\Gamma(\Lambda_b \to p\,\mu^-\bar{\nu}_\mu)}{\mathrm{d}q^2}\mathrm{d}q^2\,,$$

$$\zeta_{\Lambda_c\mu\bar{\nu}}(x) \equiv \frac{1}{|V_{cb}|^2}\int_x^{q^2_{\text{max}}} \frac{\mathrm{d}\Gamma(\Lambda_b \to \Lambda_c\,\mu^-\bar{\nu}_\mu)}{\mathrm{d}q^2}\mathrm{d}q^2$$

differential decay rate in terms of helicity form factors

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} = \frac{G_{\text{F}}^2|V_{qb}|^2\sqrt{s_+s_-}}{768\pi^3 m_{\Lambda_b}^3}\left(1-\frac{m_\ell^2}{q^2}\right)^2 \Bigg\{4\left(m_\ell^2+2q^2\right)\left[s_+ g_\perp(q^2)^2 + s_- f_\perp(q^2)^2\right]$$
$$+\,2\frac{m_\ell^2+2q^2}{q^2}\left(s_+\left[(m_{\Lambda_b}-m_X)\,g_+(q^2)\right]^2 + s_-\left[(m_{\Lambda_b}+m_X)\,f_+(q^2)\right]^2\right)$$
$$+\,\frac{6m_\ell^2}{q^2}\left(s_+\left[(m_{\Lambda_b}-m_X)\,f_0(q^2)\right]^2 + s_-\left[(m_{\Lambda_b}+m_X)\,g_0(q^2)\right]^2\right)\Bigg\}$$

$\Lambda_b \to X \ell^- \bar{\nu}_\ell$ FF decomposition

Helicity-based definition: [9] $s_\pm = (m_{\Lambda_b} \pm m_X)^2 - q^2\,, \quad X \in \{p, \Lambda_c\}$

vector

$$\langle X(p',s')|\bar{q}\,\gamma^\mu\, b|\Lambda_b(p,s)\rangle = \bar{u}_X(p',s')\Big[f_0(q^2)\,(m_{\Lambda_b} - m_X)\frac{q^\mu}{q^2} + f_+(q^2)\frac{m_{\Lambda_b}+m_X}{s_+}\Big(p^\mu + p'^\mu - (m_{\Lambda_b}^2 - m_X^2)\frac{q^\mu}{q^2}\Big) + f_\perp(q^2)\Big(\gamma^\mu - \frac{2m_X}{s_+}p^\mu - \frac{2m_{\Lambda_b}}{s_+}p'^\mu\Big)\Big]u_{\Lambda_b}(p,s),$$

axial vector

$$\langle X(p',s')|\bar{q}\,\gamma^\mu\gamma_5\, b|\Lambda_b(p,s)\rangle = -\bar{u}_X(p',s')\,\gamma_5\Big[g_0(q^2)\,(m_{\Lambda_b} + m_X)\frac{q^\mu}{q^2} + g_+(q^2)\frac{m_{\Lambda_b}-m_X}{s_-}\Big(p^\mu + p'^\mu - (m_{\Lambda_b}^2 - m_X^2)\frac{q^\mu}{q^2}\Big) + g_\perp(q^2)\Big(\gamma^\mu + \frac{2m_X}{s_-}p^\mu - \frac{2m_{\Lambda_b}}{s_-}p'^\mu\Big)\Big]u_{\Lambda_b}(p,s).$$

$$(f_0, f_+, f_\perp), (g_0, g_+, g_\perp) \leftrightarrow (f_1^V, f_2^V, f_3^V), (f_1^A, f_2^A, f_3^A)$$

Setup

Endpoint constraints

$$f_0(0) = f_+(0)\,, \qquad g_0(0) = g_+(0)\,, \qquad g_\perp(q^2_{\max}) = g_+(q^2_{\max})$$

- 3 RBC/UKQCD ensembles, 2 lattice spacings, $m_{u,d}^{(\mathrm{val})} \leq m_{u,d}^{(\mathrm{sea})}$ (domain wall)

Set	β	$N_s^3 \times N_t \times N_5$	am_5	$am_s^{(\mathrm{sea})}$	$am_{u,d}^{(\mathrm{sea})}$	a (fm)	$am_{u,d}^{(\mathrm{val})}$	$m_\pi^{(\mathrm{val})}$ (MeV)
C14	2.13	$24^3 \times 64 \times 16$	1.8	0.04	0.005	0.1119(17)	0.001	245(4)
C24	2.13	$24^3 \times 64 \times 16$	1.8	0.04	0.005	0.1119(17)	0.002	270(4)
C54	2.13	$24^3 \times 64 \times 16$	1.8	0.04	0.005	0.1119(17)	0.005	336(5)
F23	2.25	$32^3 \times 64 \times 16$	1.8	0.03	0.004	0.0849(12)	0.002	227(3)
F43	2.25	$32^3 \times 64 \times 16$	1.8	0.03	0.004	0.0849(12)	0.004	295(4)
F63	2.25	$32^3 \times 64 \times 16$	1.8	0.03	0.006	0.0848(17)	0.006	352(7)

- RHQ action, but different tuning of charm and bottom quark ! $\{am_Q, \nu^Q, c_E^Q, c_B^Q\}$

C-QUARK: PT & NP tuning[10]

- (c_E, c_B): tadpole-impr. tree-level PT
- (am_Q, ν): spin-averaged η_c, J/ψ mass & J/ψ dispersion relation

B-QUARK: NP'ly tuned parameters[11]

- $(am_Q, \nu, c_E = c_B)$: $m_{\mathrm{B_s}}^{\mathrm{avg}}$, relativistic dispersion relation, $m_{\mathrm{B_s^*}} - m_{\mathrm{B_s}}$

$\Rightarrow$ triply mixed-action approach with PQ light quarks

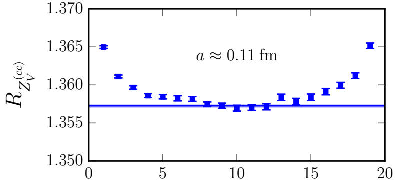
Computational details / renormalization

NEEDED FOR (IMPROVED) VECTOR AND AXIAL VECTOR 3-PT FUNCTIONS

mostly NP renormalization $\quad Y = A_0, A_i, V_0, V_i$

$$Z_Y^{(uh)} = \rho_{Y,h} \cdot \sqrt{Z_Y^{(uu)} Z_Y^{(hh)}} \,, \qquad h = c, b$$

$$\rho_{Y,h} = 1 + \rho_{Y,h}^{(1)} \alpha_{\overline{\mathrm{MS}}}(a^{-1}) + \ldots$$



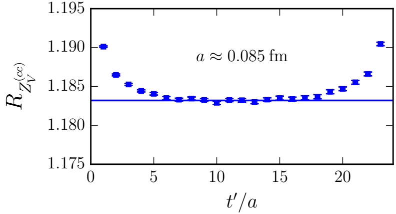


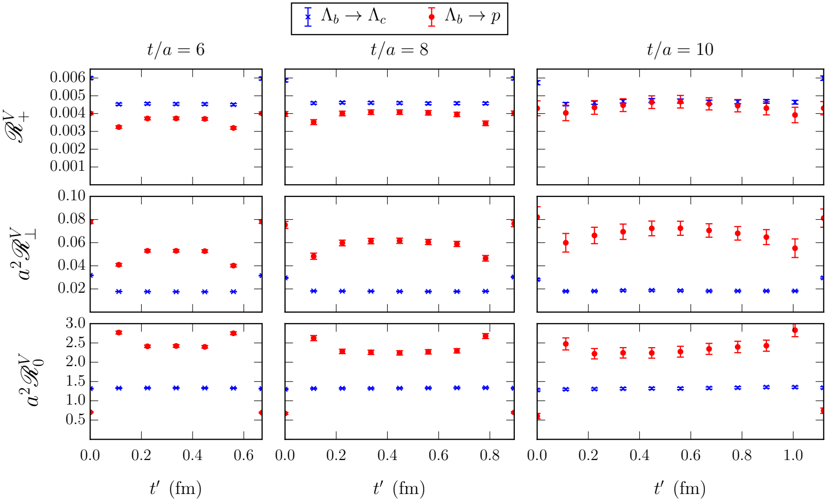
Table IV: Nonperturbative renormalization factors of the flavor-conserving temporal vector currents. For $Z_V^{(uu)}$, we use the results in the chiral limit from Ref. [12]. For $Z_V^{(bb)}$, we use the results obtained in Ref. [13] on the coarse $am_{u,d}^{(\mathrm{sea})} = 0.005$ and fine $am_{u,d}^{(\mathrm{sea})} = 0.004$ ensembles.

Parameter	coarse	fine
$Z_V^{(bb)}$	10.037(34)	5.270(13)
$Z_V^{(cc)}$	1.35725(23)	1.18321(14)
$Z_V^{(uu)}$	0.71651(46)	0.74475(12)

$\Downarrow$

$Z_V^{(ub)}$	~ 2.68	~ 1.98
$Z_V^{(uc)}$	~ 0.98	~ 0.94

Computational details / current ratios



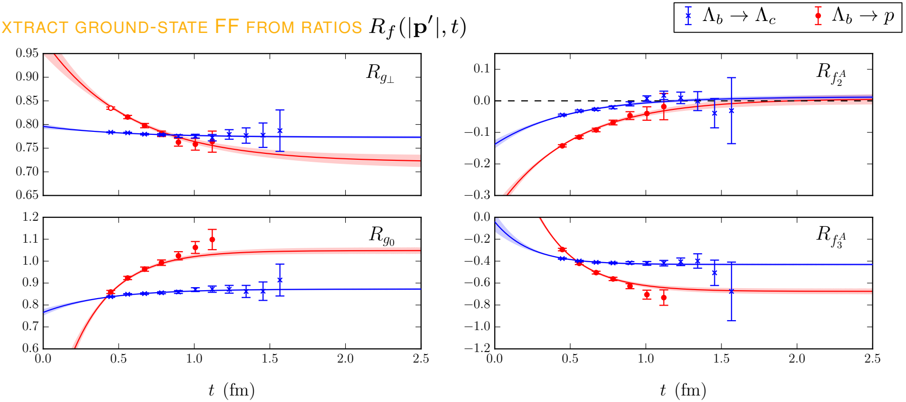
t : fixed source-sink separation ($\Lambda_\mathrm{b} \leftrightarrow X$)

t': current insertion ($\Lambda_\mathrm{b} \leftrightarrow A, V$)

ensemble C24 with $|\mathbf{p}'| = \sqrt{3} \cdot 2\pi/L$

'Infinite' source-sink separation

EXTRACT GROUND-STATE FF FROM RATIOS $R_f(|\mathbf{p}'|, t)$



$$R_{f,i,n} = f_{i,n} + A_{f,i,n} \exp(-\delta_{f,i,n} t) \ , \qquad \delta_{f,i,n} = \delta_{\min} + \exp(l_{f,i,n}) \,\mathrm{GeV}$$

f, i, n labels all form factors f (set$\times$2), ensembles i and momenta $|\mathbf{p}'|^2 = n \cdot (2\pi/L)^2$

complicated coupled fit to all vector form factors with
n fixed, $\delta_{\min} = 170\,\mathrm{MeV}$ & including Gaussian priors in $\chi^2_{V,n}$

χ/continuum/kinematic extrapolation of FFs

modified z-expansion $\quad t_0 = (m_{\Lambda_b} - m_X)^2$

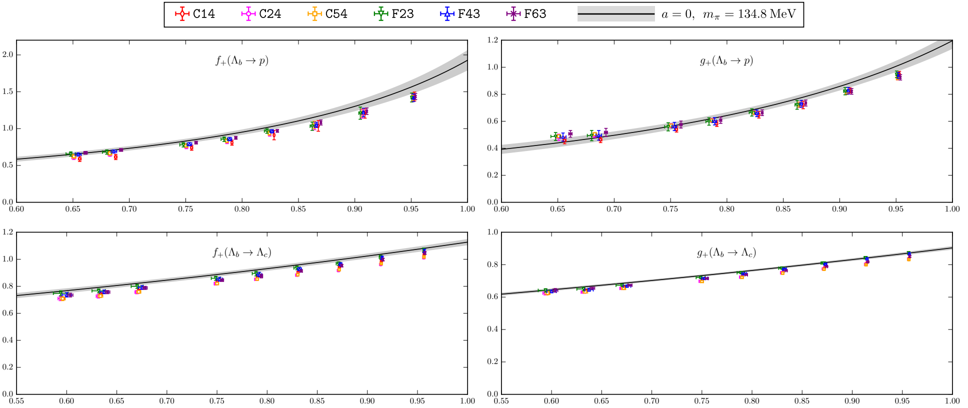
$$B(z)f(z) = \left[a_0^f\left(1 + c_0^f \frac{m_\pi^2 - m_{\pi,\text{phys}}^2}{\Lambda_\chi^2}\right) + a_1^f\, z^f\right]\left[1 + b^f \frac{|\mathbf{p}'|^2}{(\pi/a)^2} + d^f \frac{\Lambda_{\text{QCD}}^2}{(\pi/a)^2}\right],$$

$$B(z) = 1 - q^2/m_{f,\text{pole}}^2\,, \qquad am_{f,\text{pole}} = am_{\text{B}_q} + a\Delta_f\,, \qquad q = u, c$$

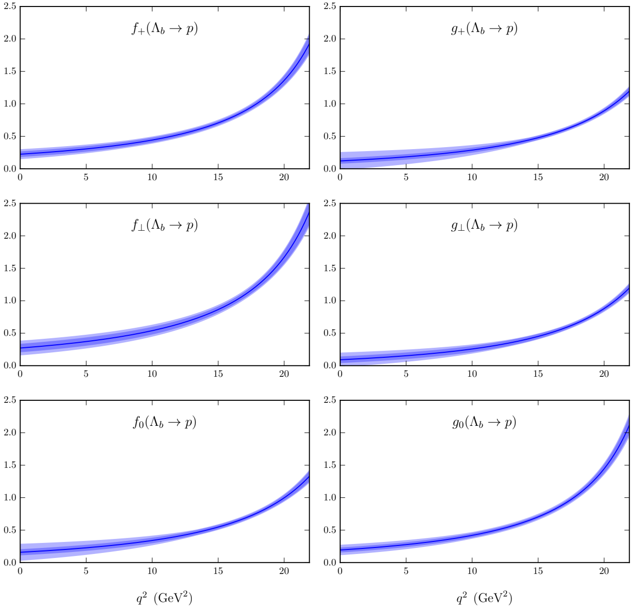
Δ_f from PDG

additional terms in fit ansatz to determine systematics

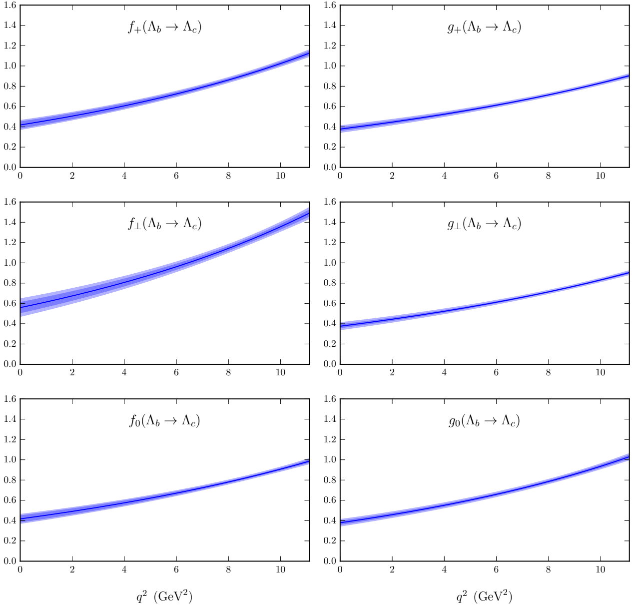
Other poles ?



FF results for $\Lambda_\mathrm{b} \to p$

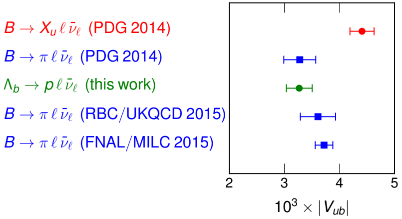


FF results for $\Lambda_b \to \Lambda_c$



Summary

- increasing impact of lattice heavy-light FF determinations on determining $V_{\rm ub}$
- standard/benchmark SL heavy-light decay $B \to \pi\ell\nu$
- 1st results on baryonic heavy-light SL decay



[14]

- Quality criteria: How good is our control of systematic errors? $\leftarrow$ FLAG
- theoretically cleaner to apply z-expansion AFTER continuum+chiral extrapolation
- preferrably at fixed q^2
- overall normalization of $f(q^2)$ controlable, but not its shape yet
- applicability domain of hard-pion χ-PT ?
- to my knowledge: many PT determinations of ρ in mostly-NPR not published yet

- poles for z-expansion NOT always known OR too many / off-axis poles, widths, more cuts, . . . ?
- w/o new techniques lattice results stucked in high-q^2 region

THANK YOU FOR YOUR ATTENTION!

I would also like to thank

Gregorio Herdoiza

Carlos Pena

for various discussions and figures for this talk.

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BACKUP SLIDES

Heavy Quark Lattice Actions

- **FermilabRHQ** *Perturbatively tuned/matched Relativistic Heavy Quark action*
- **NP–HQET** *Non-perturbatively renormalized & matched Heavy Quark Effective Theory action*
- **NPtuned–RHQ** *Non-perturbatively tuned Relativistic Heavy Quark action*
- **NRQCD** *Non-Relativistic QCD*

Relativistic Heavy Quark action (RHQ)

[15, 16]

$$\mathcal{S}_{\mathrm{RHQ}} = a^4 \sum_x \bar{Q} \Big[m_Q + \gamma_0 \nabla_0 - \tfrac{a}{2} \nabla_0^{(2)} + \nu \sum_{i=1}^{3} \Big(\gamma_i \nabla_i - \tfrac{a}{2} \nabla_i^{(2)} \Big) - c_E \tfrac{a}{2} \sum_{i=1}^{3} \sigma_{0i} F_{0i} - c_B \tfrac{a}{4} \sum_{i,\,j=1}^{3} \sigma_{ij} F_{ij} \Big] Q$$

- relativistiv heavy quark & anti-quark field $Q, \bar{Q}$
- anisotropic action with $\nu, c_E, c_B = f(am, g_0^2)$ and $\gamma D \to \{\gamma_0 D_0, \nu \cdot \gamma_i D_i\}$, $c_{\mathrm{sw}} \to \{c_E, c_B\}$
- classically improved to $\mathrm{O}(a)$, $\{\nabla_0^{(2)}, \nabla_i^{(2)}\}$