# FLAG: Lattice QCD tests of the SM and foretaste for beyond

A.Vladikas
INFN - TOR VERGATA

Mainz Institute for Theoretical Physics
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# The FLAG Collaboration

## FLAG: Flavour Lattice Averaging Group

# The FLAG Collaboration

- Lattice simulations performed by different groups involve different choices both at the level of formalism (lattice actions, number of sea flavours etc.) and at the level of resources (lattice volumes, quark masses etc.).

- Often this amounts to making different compromises which in turn introduce different systematic effects; thus not all lattice results of a given quantity are directly comparable.

- FLAG aim: answer, in a way which is readily accessible to non-experts, the question: What is currently the "best lattice value" for a particular quantity?

- 2011: end of phase 1 (FLAG-1 consisted of 12 European members): G. Colangelo et al., "Review of Lattice Results Concerning Low-Energy Particle Physics", Eur. Phys. J. C 71 (2011) 1695

- 2014: end of phase 2 (FLAG-2 consisted of 28 American/Asian/European members): S. Aoki et al., "Review of Lattice Results Concerning Low-Energy Particle Physics", Eur. Phys. J. C 74 (2014) 2890

- Lattice collaborations which participated in FLAG-2: Alpha/CLS, BMW, ETMC, FNAL, HPQCD, JLQCD, PACS-CS, RBC/UKQCD

- Here a selection of FLAG-2 results are presented (NB: Closing date for reviewing lattice papers: 30th November 2013). Currently working on FLAG-3; should be ready by spring 2016; some FLAG-3 *PRELIMINARY* results also shown.

# FLAG-2 composition

## Advisory Board:

Sinya Aoki
Claude Bernard
Chris Sachrajda

## Editorial Board:

Gilberto Colangelo
Heiri Leutwyler
Tassos Vladikas
Urs Wenger

## Working Groups:

### Quark masses (u,d,s)

Laurent Lellouch
Tom Blum
Vittorio Lubicz

### $B_K$

Hartmut Wittig
Jack Laiho
Steve Sharpe

### $f_D$, $f_B$, $B_B$

Aida El-Khadra
Michele Della Morte
Yasumichi Aoki
Junko Shigemitsu

### $f_K$, $f_K/f_\pi$, $f_+^{K\pi}(0)$

Andreas Jüttner
Silvano Simula
Takashi Kaneko

### $\alpha_S$

Rainer Sommer
Roger Horsley
Tetsuya Onogi

### *B*, *D* semi-leptonic

Ruth van de Water
Enrico Lunghi
Carlos Pena

### LEC

Stefan Dürr
Hinedori Fukaya
Silvia Necco

# FLAG-3 composition

## Advisory Board:

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Claude Bernard
Maarten Golterman
Heiri Leutwyler
Chris Sachrajda

## Editorial Board:

Gilberto Colangelo
Andreas Jüttner
Shoji Hashimoto
Steve Sharpe
Tassos Vladikas
Urs Wenger

## Working Groups:

### Quark masses (u,d,s,c,b)

Laurent Lellouch
Tom Blum
Vittorio Lubicz

### $B_K$ SM & BSM

Hartmut Wittig
Petros Dimopoulos
Bob Mawhinney

### $f_D$, $f_B$, $B_B$

Michele Della Morte
Yasumichi Aoki
David Lin

### $V_{us}$ / $V_{ud}$

Silvano Simula
Takashi Kaneko
Peter Boyle

### $\alpha_s$

Rainer Sommer
Roger Horsley
Tetsuya Onogi

### *B*, *D* semi-leptonic

Enrico Lunghi
Damir Becirevic
Steve Gottlieb
Carlos Pena

### LEC

Stefan Dürr
Hinedori Fukaya
Urs Heller

FLAG-3 preprint expected in early 2016

# FLAG topics in this seminar

Quark masses (u,d,s,c,b)

$B_K$ SM & BSM

$f_D$, $f_B$, $B_B$

*V*us / *V*ud

$\alpha_S$

*B*, *D* semi-leptonic

LEC

# FLAG Criteria

- A number of criteria have been fixed; these are subjective and time dependent
- We aim at providing compact information on the quality of a computation
- Criteria:

★ systematic error estimated in a satisfactory manner and under control

○ a reasonable attempt at estimating systematic error; can be improved

🟥 no attempt or unsatisfactory attempt at controlling a systematic error (result is dropped!)

- Example: for light-flavour masses, decay constants, LECs, $B_K$-parameters, criteria rate quality of:
  - chiral extrapolations ($M_\pi$ cutoffs at 200 MeV and 400 MeV)
  - continuum extrapolations (number of points below $a \simeq 0.1$ fm ; quantities scaling like $a$ or $a^2$)
  - finite volume effects (e.g. $[M_\pi \, L]_{\mathrm{min}} > 3$ or 4..)
  - renormalization (non-perturbative, 2-loop PT, 1-loop PT)
- For heavy flavours and $\alpha_{\mathrm{strong}}$ the criteria are different

# FLAG Criteria

- A number of criteria have been fixed; these are subjective and time dependent
- We aim at providing compact information on the quality of a computation
- Criteria:
  - ★ systematic error estimated in a satisfactory manner and under control
  - ○ a reasonable attempt at estimating systematic error; can be improved
  - 🟥 no attempt or unsatisfactory attempt at controlling a systematic error (result is dropped!)
- FLAG-3: wording will change (FLAG meeting in spring 2015 in Berne)

# FLAG Criteria

- Many more issues; e.g. how to average, how to make an estimate if and average is not possible, how to combine/correlate errors, how (not) to take conference proceedings into account, ...

- Simulations are carried out either for $N_f = 2$, or $N_f = 2+1$, or $N_f = 2+1+1$ sea quarks (two light flavours are isospin symmetric).

- Quenched results ($N_f = 0$) are omitted, except for $\alpha_{\mathrm{strong}}$, where they are reported without averages

- NB: FLAG averages/estimates reported at fixed $N_f$ and are **not averaged** for different $N_f$

- **FIGURES:** for each $N_f$ value, we use different symbols as follows:

  ■ FLAG average or estimate;

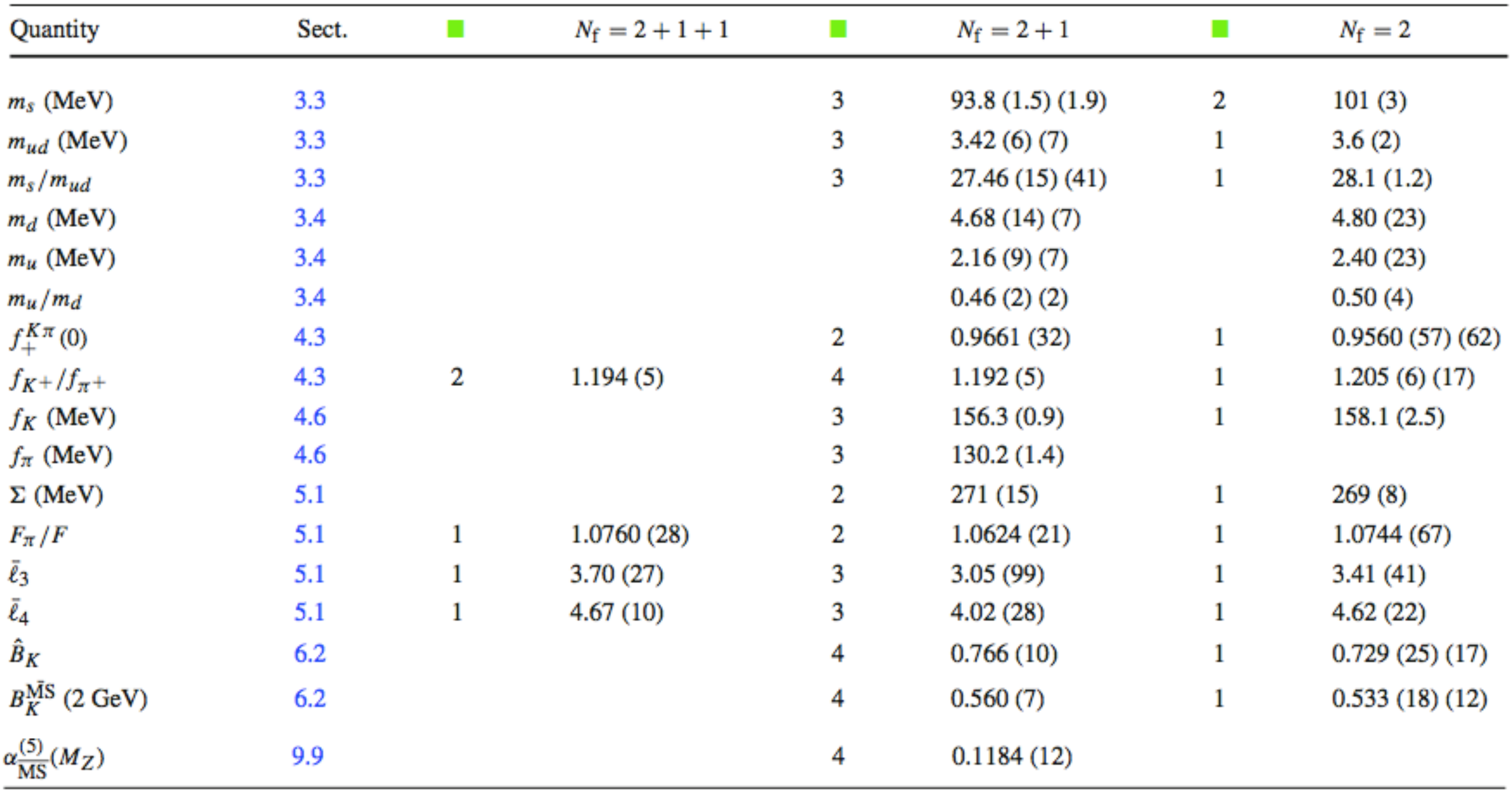
  🟩 results which from which the FLAG average/estimate is obtained;

  □ results without red tags (i.e. good control of the systematics) but not included in the average for some reason; e.g. not published in peer reviewed journals, superseded by later results of the same collaboration, some other effect has not been controlled...

  □ results are not included in the average because they do not pass the criteria;

  🔵 non-lattice results.

# FLAG-2 results -light flavours

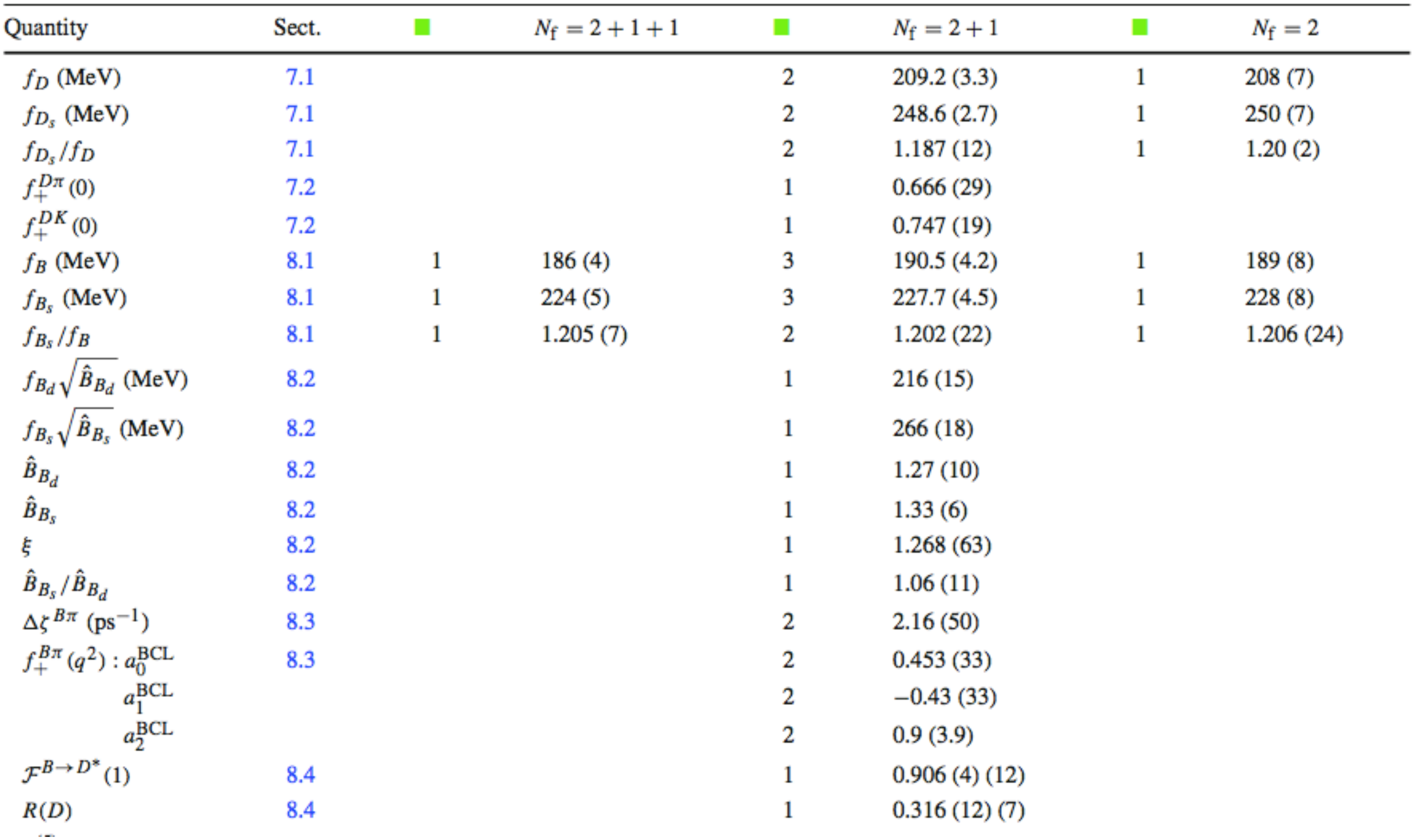


| Quantity | Sect. | 🟩 | $N_f = 2+1+1$ | 🟩 | $N_f = 2+1$ | 🟩 | $N_f = 2$ |
|---|---|---|---|---|---|---|---|
| $m_s$ (MeV) | 3.3 | | | 3 | 93.8 (1.5) (1.9) | 2 | 101 (3) |
| $m_{ud}$ (MeV) | 3.3 | | | 3 | 3.42 (6) (7) | 1 | 3.6 (2) |
| $m_s/m_{ud}$ | 3.3 | | | 3 | 27.46 (15) (41) | 1 | 28.1 (1.2) |
| $m_d$ (MeV) | 3.4 | | | | 4.68 (14) (7) | | 4.80 (23) |
| $m_u$ (MeV) | 3.4 | | | | 2.16 (9) (7) | | 2.40 (23) |
| $m_u/m_d$ | 3.4 | | | | 0.46 (2) (2) | | 0.50 (4) |
| $f_+^{K\pi}(0)$ | 4.3 | | | 2 | 0.9661 (32) | 1 | 0.9560 (57) (62) |
| $f_{K^+}/f_{\pi^+}$ | 4.3 | 2 | 1.194 (5) | 4 | 1.192 (5) | 1 | 1.205 (6) (17) |
| $f_K$ (MeV) | 4.6 | | | 3 | 156.3 (0.9) | 1 | 158.1 (2.5) |
| $f_\pi$ (MeV) | 4.6 | | | 3 | 130.2 (1.4) | | |
| $\Sigma$ (MeV) | 5.1 | | | 2 | 271 (15) | 1 | 269 (8) |
| $F_\pi/F$ | 5.1 | 1 | 1.0760 (28) | 2 | 1.0624 (21) | 1 | 1.0744 (67) |
| $\bar{\ell}_3$ | 5.1 | 1 | 3.70 (27) | 3 | 3.05 (99) | 1 | 3.41 (41) |
| $\bar{\ell}_4$ | 5.1 | 1 | 4.67 (10) | 3 | 4.02 (28) | 1 | 4.62 (22) |
| $\hat{B}_K$ | 6.2 | | | 4 | 0.766 (10) | 1 | 0.729 (25) (17) |
| $B_K^{\overline{\mathrm{MS}}}$ (2 GeV) | 6.2 | | | 4 | 0.560 (7) | 1 | 0.533 (18) (12) |
| $\alpha_{\overline{\mathrm{MS}}}^{(5)}(M_Z)$ | 9.9 | | | 4 | 0.1184 (12) | | |

🟩 indicates number of results participating in the average

NB: Quark masses & condensate are in the MS-bar scheme at μ= 2 GeV

# FLAG results -charm and bottom flavours



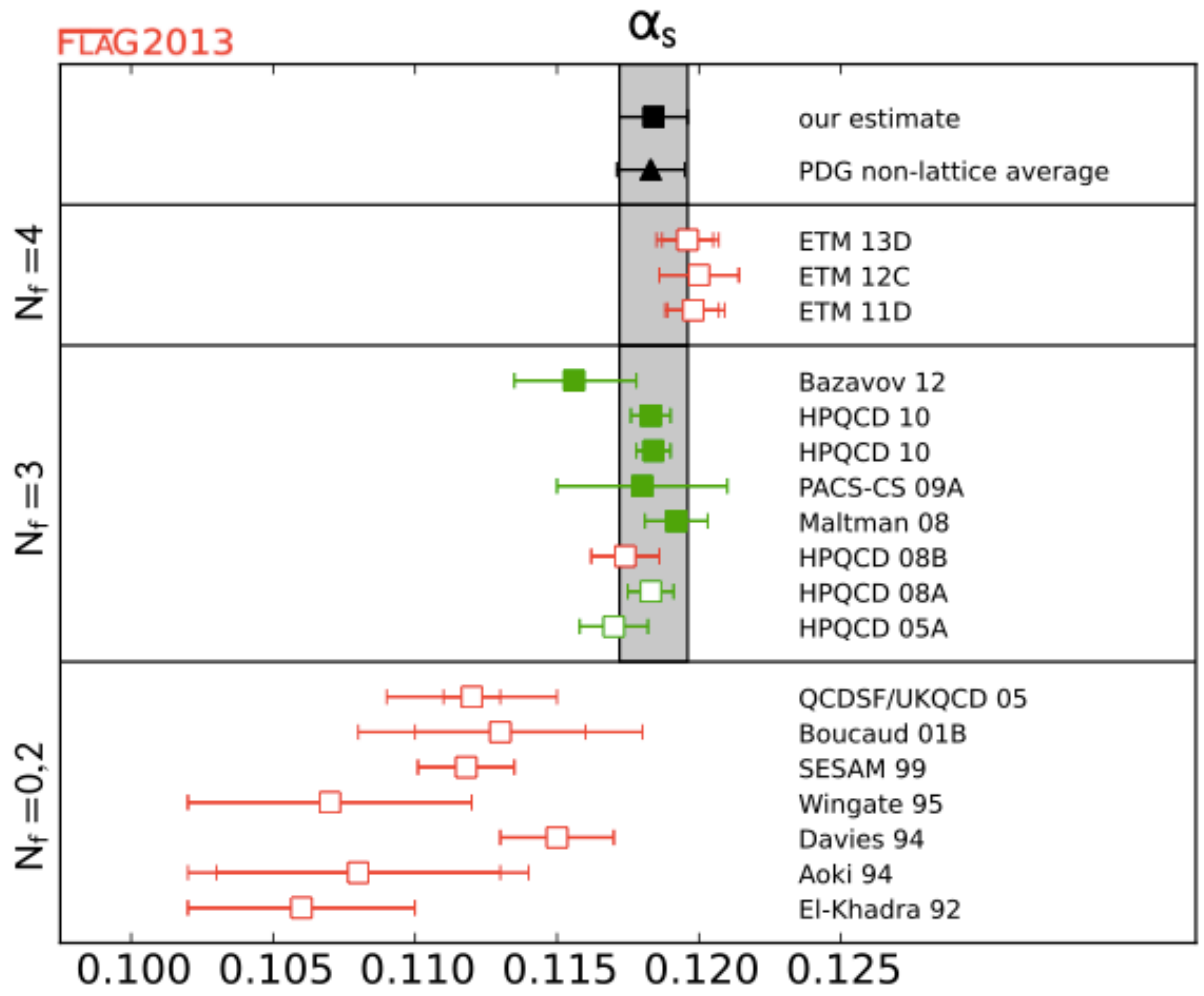
| Quantity | Sect. | 🟩 | $N_f = 2+1+1$ | 🟩 | $N_f = 2+1$ | 🟩 | $N_f = 2$ |
|---|---|---|---|---|---|---|---|
| $f_D$ (MeV) | 7.1 | | | 2 | 209.2 (3.3) | 1 | 208 (7) |
| $f_{D_s}$ (MeV) | 7.1 | | | 2 | 248.6 (2.7) | 1 | 250 (7) |
| $f_{D_s}/f_D$ | 7.1 | | | 2 | 1.187 (12) | 1 | 1.20 (2) |
| $f_+^{D\pi}(0)$ | 7.2 | | | 1 | 0.666 (29) | | |
| $f_+^{DK}(0)$ | 7.2 | | | 1 | 0.747 (19) | | |
| $f_B$ (MeV) | 8.1 | 1 | 186 (4) | 3 | 190.5 (4.2) | 1 | 189 (8) |
| $f_{B_s}$ (MeV) | 8.1 | 1 | 224 (5) | 3 | 227.7 (4.5) | 1 | 228 (8) |
| $f_{B_s}/f_B$ | 8.1 | 1 | 1.205 (7) | 2 | 1.202 (22) | 1 | 1.206 (24) |
| $f_{B_d}\sqrt{\hat{B}_{B_d}}$ (MeV) | 8.2 | | | 1 | 216 (15) | | |
| $f_{B_s}\sqrt{\hat{B}_{B_s}}$ (MeV) | 8.2 | | | 1 | 266 (18) | | |
| $\hat{B}_{B_d}$ | 8.2 | | | 1 | 1.27 (10) | | |
| $\hat{B}_{B_s}$ | 8.2 | | | 1 | 1.33 (6) | | |
| $\xi$ | 8.2 | | | 1 | 1.268 (63) | | |
| $\hat{B}_{B_s}/\hat{B}_{B_d}$ | 8.2 | | | 1 | 1.06 (11) | | |
| $\Delta\zeta^{B\pi}$ ($\mathrm{ps}^{-1}$) | 8.3 | | | 2 | 2.16 (50) | | |
| $f_+^{B\pi}(q^2)$ : $a_0^{\mathrm{BCL}}$ | 8.3 | | | 2 | 0.453 (33) | | |
| $a_1^{\mathrm{BCL}}$ | | | | 2 | −0.43 (33) | | |
| $a_2^{\mathrm{BCL}}$ | | | | 2 | 0.9 (3.9) | | |
| $\mathcal{F}^{B\to D^*}(1)$ | 8.4 | | | 1 | 0.906 (4) (12) | | |
| $R(D)$ | 8.4 | | | 1 | 0.316 (12) (7) | | |

🟩 indicates number of results participating in the average

NB: Quark masses & condensate are in the MS-bar scheme at μ= 2 GeV

# Quality Criteria

- The importance of quality criteria is seen in our estimate of $\alpha_{\text{strong}}$



- FLAG estimate has conservative error (not all FLAG agrees)
- PDG total average takes all lattice results at face value
- PDG without lattice agrees with FLAG

FLAG estimate: $$\alpha^{(5)}_{\overline{MS}}(M_Z) = 0.1184(12)$$

PDG average $$\alpha^{(5)}_{\overline{MS}}(M_Z) = 0.1185(5)$$

PDG average (non lattice) $$\alpha^{(5)}_{\overline{MS}}(M_Z) = 0.1183(12)$$

# Light Flavour Physics

$f_\pi$, $f_K$, $f_+(0)$, $|V_{ud}|$, $|V_{us}|$

CKM first row unitarity

# Form factor, decay constants and unitarity

- Leptonic pion and Kaon decays associated with hadronic matrix elements, expressed in terms of decay constants $f^{\pm}_{\pi}$ and $f^{\pm}_{K}$:

$$\langle 0|\bar{d}\gamma_\mu\gamma_5 u|\pi^\pm(\vec{p})\rangle \;=\; i p_\mu f_{\pi^\pm} \qquad\qquad \langle 0|\bar{s}\gamma_\mu\gamma_5 u|K^\pm(\vec{p})\rangle \;=\; i p_\mu f_{K^\pm}$$

- Semi-leptonic Kaon decays associated with form factor $f_+(q^2)$ at momentum transfer to lepton pair $q^2$:

$$K^0 \;\to\; \pi^-\,\nu\, l^+$$

- M.Antonelli et al., Eur.Phys.J. C69(2010)399 results from high accuracy experimental data:

$$|V_{us}|\; f_+(0) \;=\; 0.2163(5) \qquad\qquad \left|\frac{V_{us}}{V_{ud}}\right| \frac{f_{K^\pm}}{f_{\pi^\pm}} \;=\; 0.2758(5)$$

form factor @ zero momentum transfer

# Form factor, decay constants and unitarity

- M.Antonelli et al., Eur.Phys.J. C69(2010)399 provide from high accuracy experimental data:

$$|V_{us}|\ f_+(0) \ = \ 0.2163(5)$$

$$\left|\frac{V_{us}}{V_{ud}}\right| \frac{f_{K^\pm}}{f_{\pi^\pm}} \ = \ 0.2758(5)$$

Experimental data corrected for strong and EM isospin-breaking effects (NLO χPT)

Lattice data obtained in isospin limit

Experimental data corrected for EM isospin-breaking effects (NLO χPT)

Lattice data mostly obtained in isospin limit, denoted by $f_\pi$ and $f_K$

For now lattice data corrected by NLO χPT

Early progress in including strong and EM corrections in simulations

# Form factor, decay constants and unitarity

NLO χPT use to get $f_{K^\pm}/f_{\pi^\pm}$ from $f_K/f_\pi$ and vice versa

$$\frac{f_K}{f_\pi} = \frac{1}{\sqrt{\delta_{\mathrm{SU}(2)}+1}}\,\frac{f_K^\pm}{f_\pi^\pm}$$

$$\delta_{\mathrm{SU}(2)} \approx \sqrt{3}\,\epsilon_{\mathrm{SU}(2)}\left[-\frac{4}{3}\left(\frac{f_K^\pm}{f_\pi^\pm}-1\right)+\frac{2}{3(4\pi)^2 f_0^2}\left(M_K^2-M_\pi^2-M_\pi^2\ln\frac{M_K^2}{M_\pi^2}\right)\right]$$

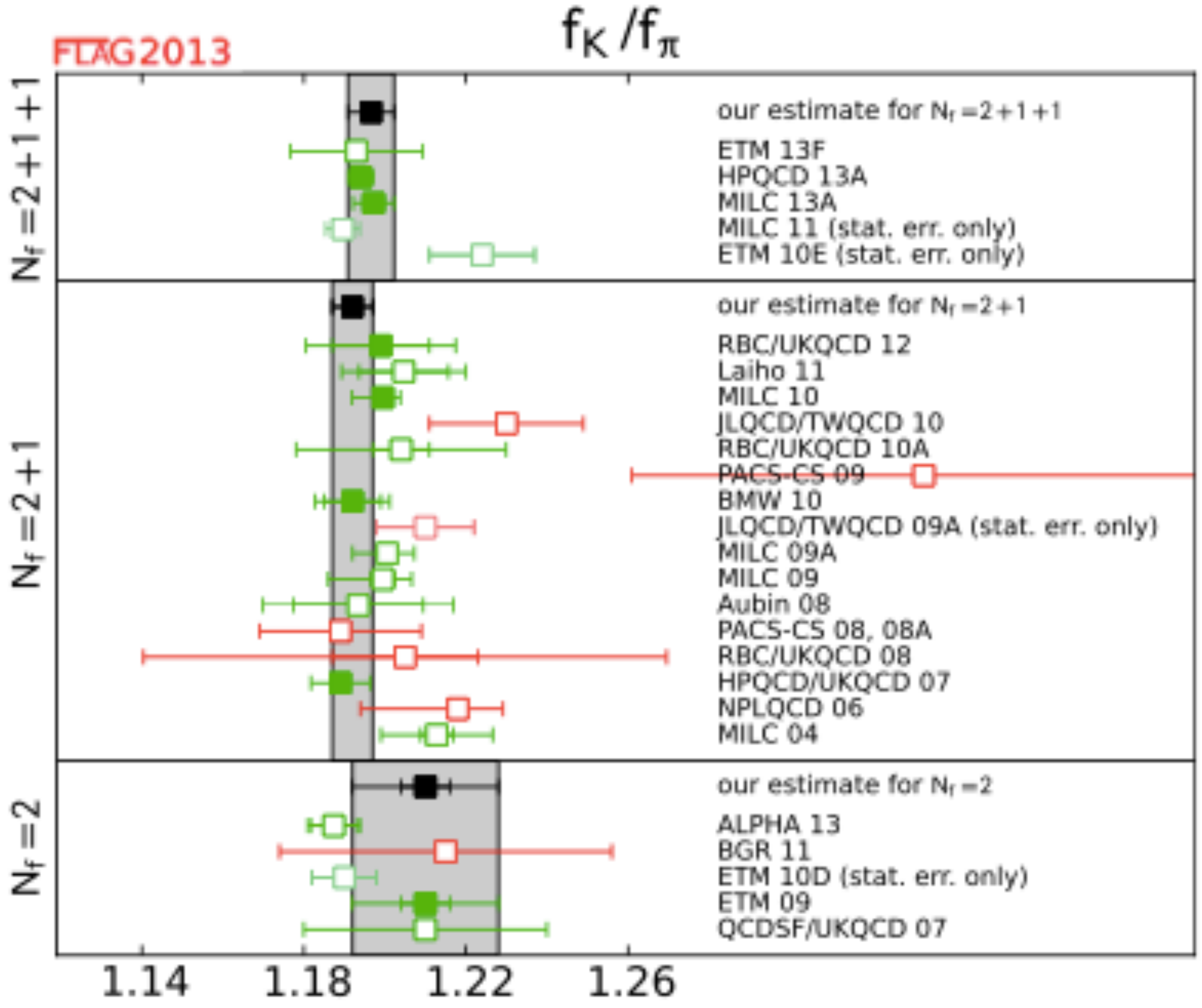
$$\epsilon_{\mathrm{SU}(2)} = \frac{\sqrt{3}}{4R} \qquad R = \frac{m_s-m_{ud}}{m_d-m_u} = 35.8(1.9)(1.8)$$ ⟵ from FLAG 2

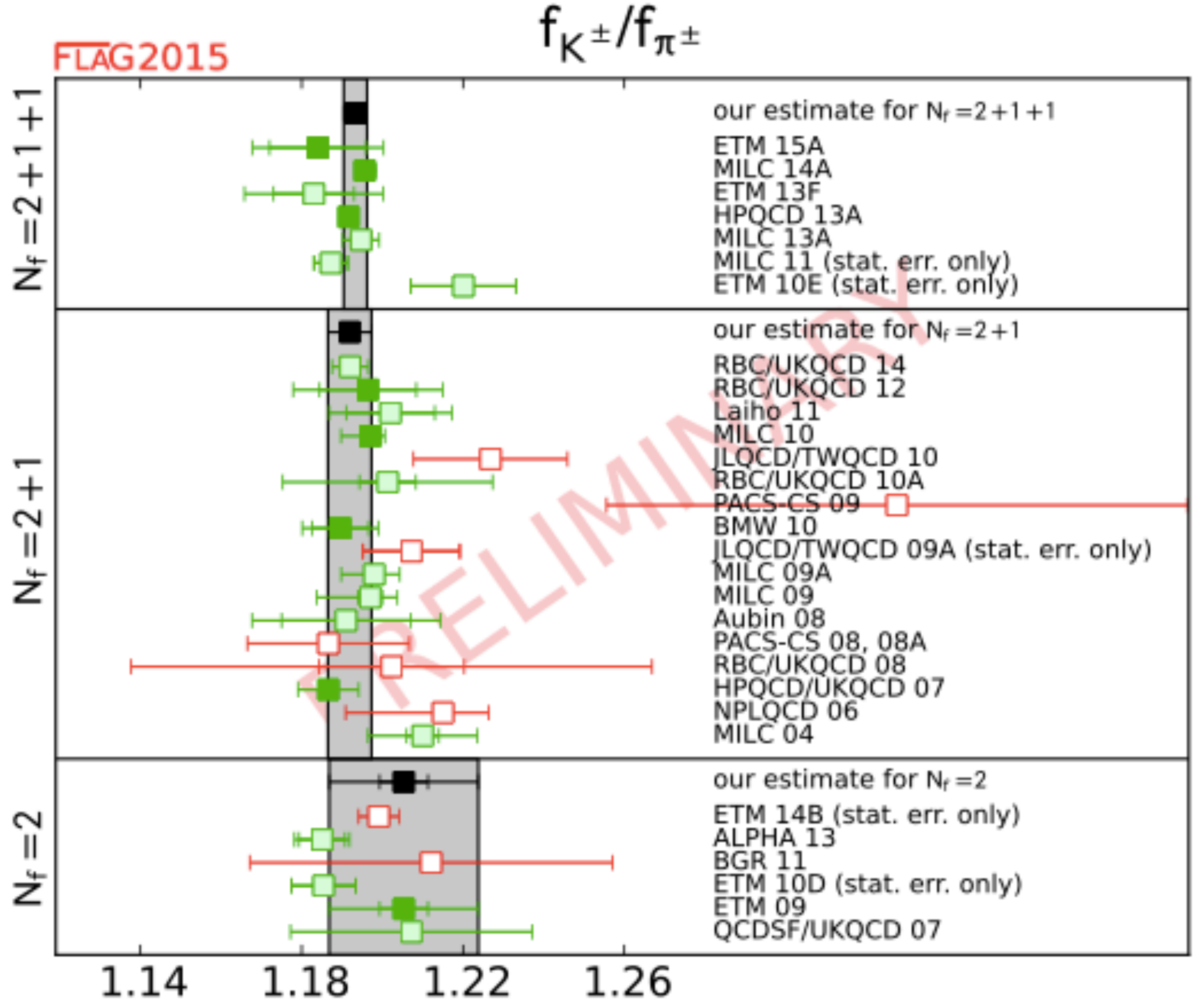
$$M_\pi = 135 \text{ MeV} \qquad M_K = 495 \text{ MeV} \qquad \frac{f_0}{\sqrt{2}} = 80(2) \text{ MeV}$$

NB: $\delta_{\mathrm{SU}(2)} \approx -0.0042(7)$ from various simulations $N_f = 2+1$ simulations, but $\delta_{\mathrm{SU}(2)} \approx -0.0078(7)$ from $N_f = 2$ simulations with isospin breaking corrections de Divitiis et al., JHEP04 (2012)124

Discrepancy: Strange loop effects (unlikely)? Higher χPT? Other effects?

# Form factor, decay constants and unitarity





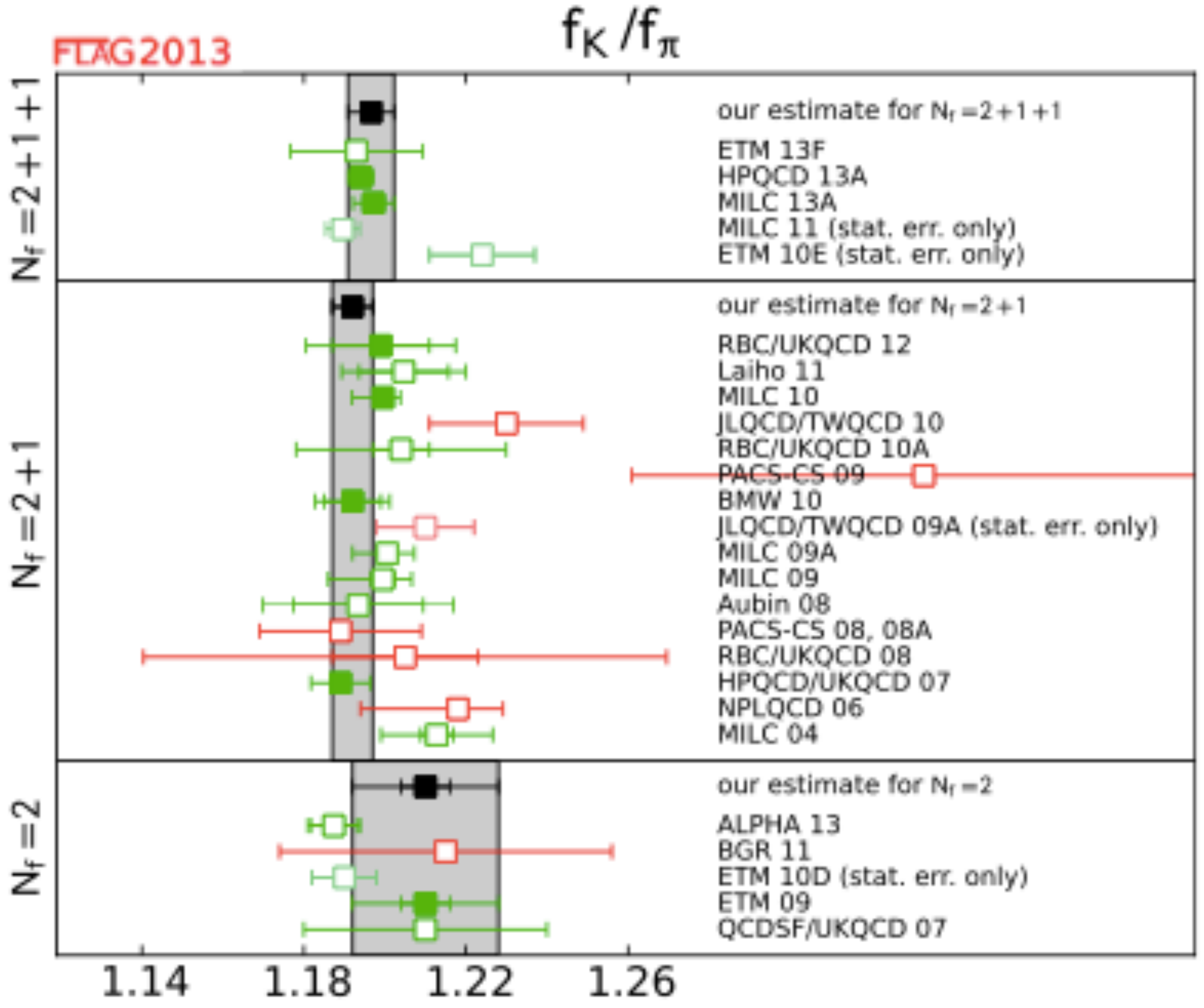
NB: the two plots are not directly comparable

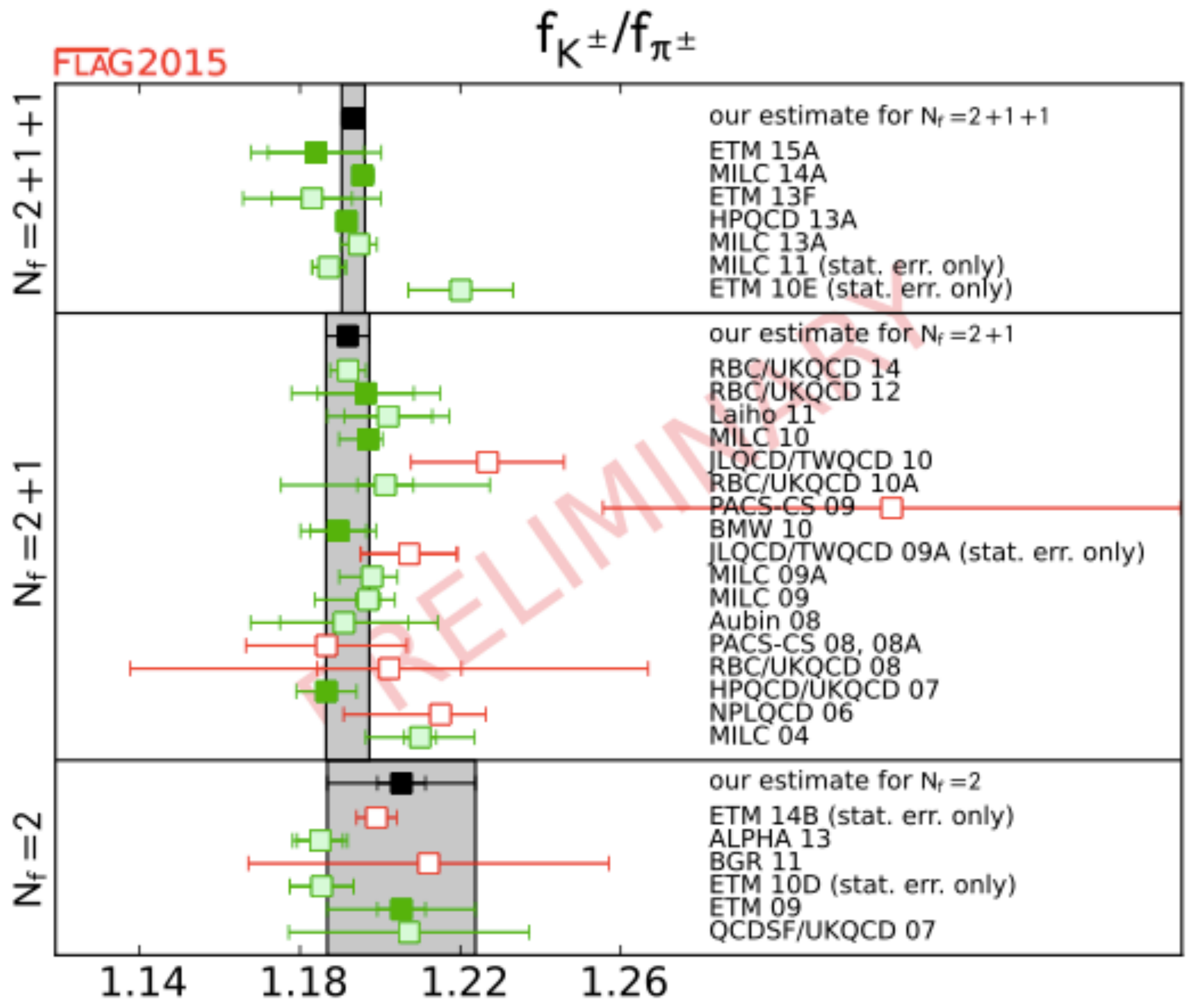
Simulations habitually compute $f_K/f_\pi$

Some groups quote only $f_{K^\pm}/f_{\pi^\pm}$ while others (the most recent and the majority of those entering FLAG averages) give both $f_K/f_\pi$ and $f_{K^\pm}/f_{\pi^\pm}$

NLO χPT used to get $f_{K^\pm}/f_{\pi^\pm}$ from $f_K/f_\pi$ and vice versa

# Form factor, decay constants and unitarity





$$\frac{f_K^\pm}{f_\pi^\pm} = 1.194(5)\ \mathrm{MeV} \qquad N_f = 2+1+1 \qquad \frac{f_K^\pm}{f_\pi^\pm} = 1.193(3)\ \mathrm{MeV}$$

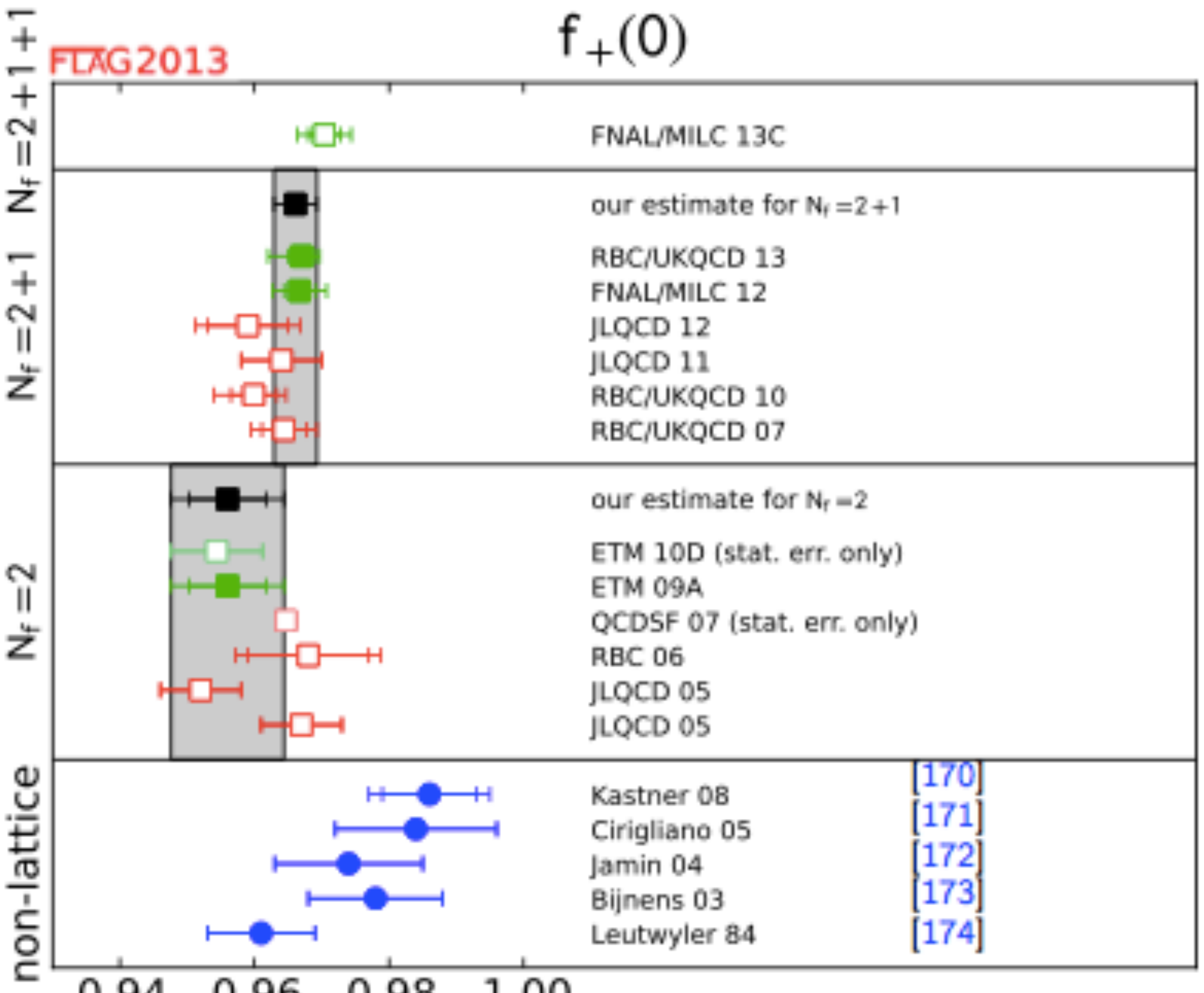
$$\frac{f_K^\pm}{f_\pi^\pm} = 1.192(5)\ \mathrm{MeV} \qquad N_f = 2+1$$

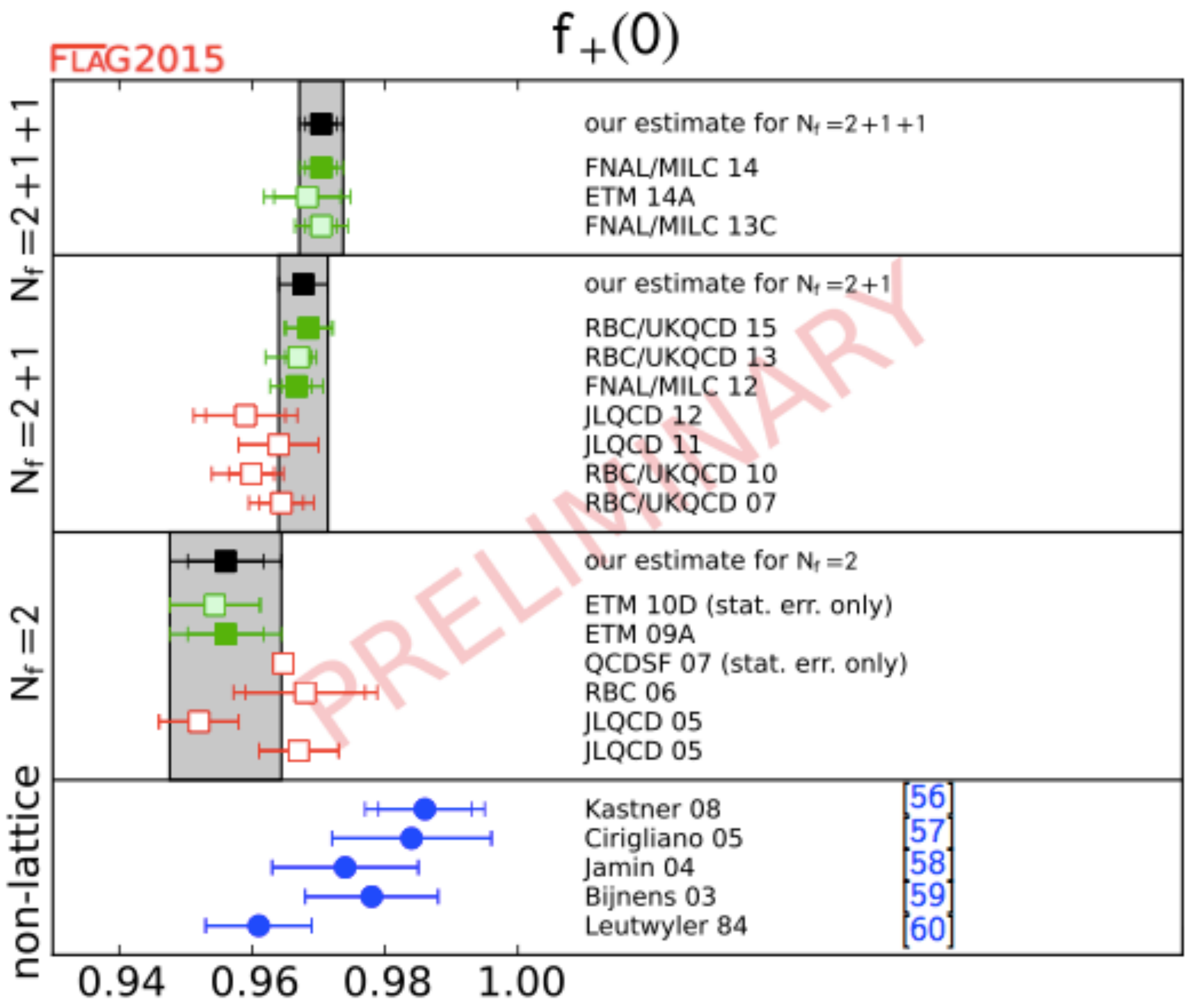
unchanged

$$\frac{f_K^\pm}{f_\pi^\pm} = 1.205(6)(17)\ \mathrm{MeV} \qquad N_f = 2$$

unchanged

# Form factor, decay constants and unitarity



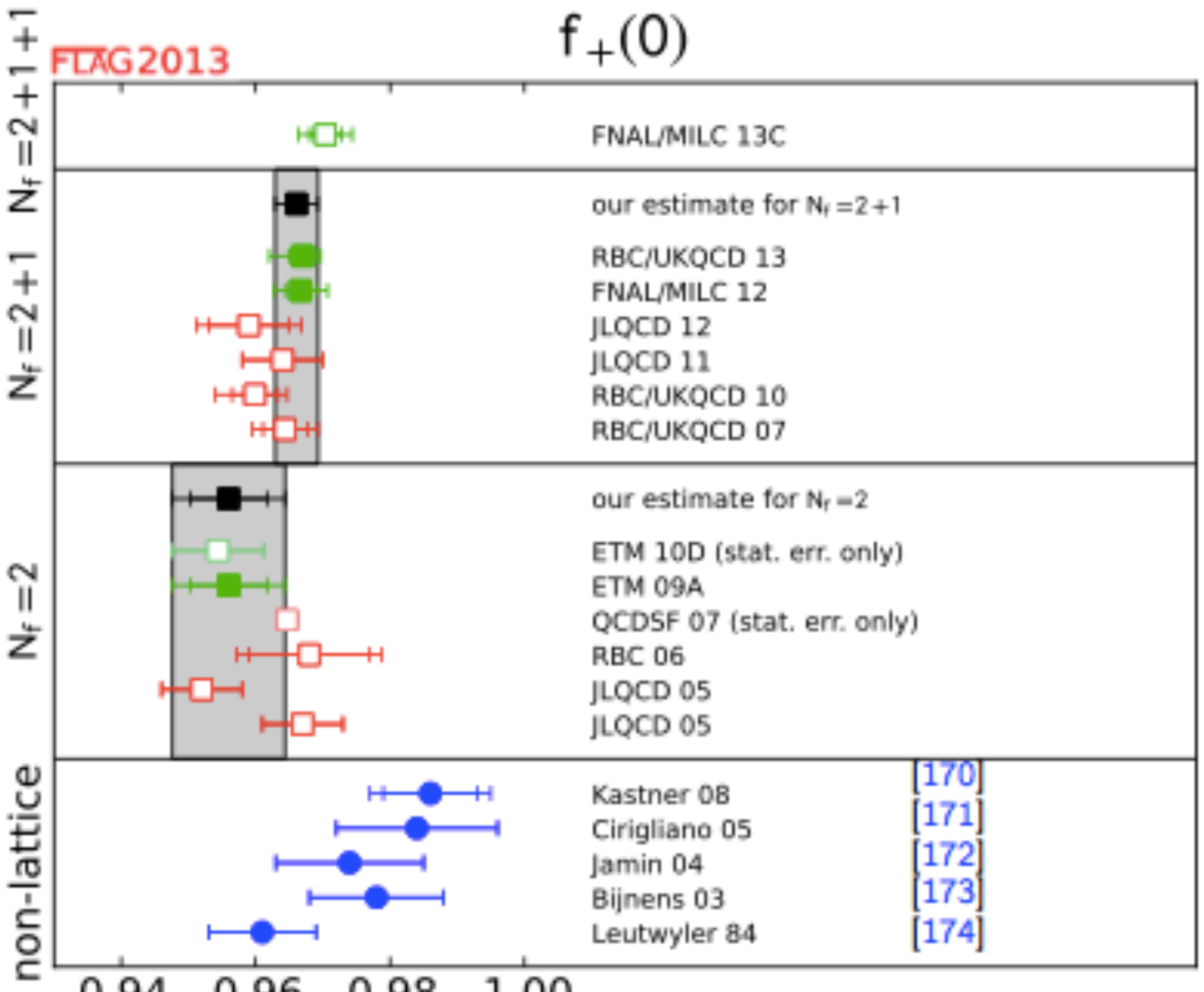


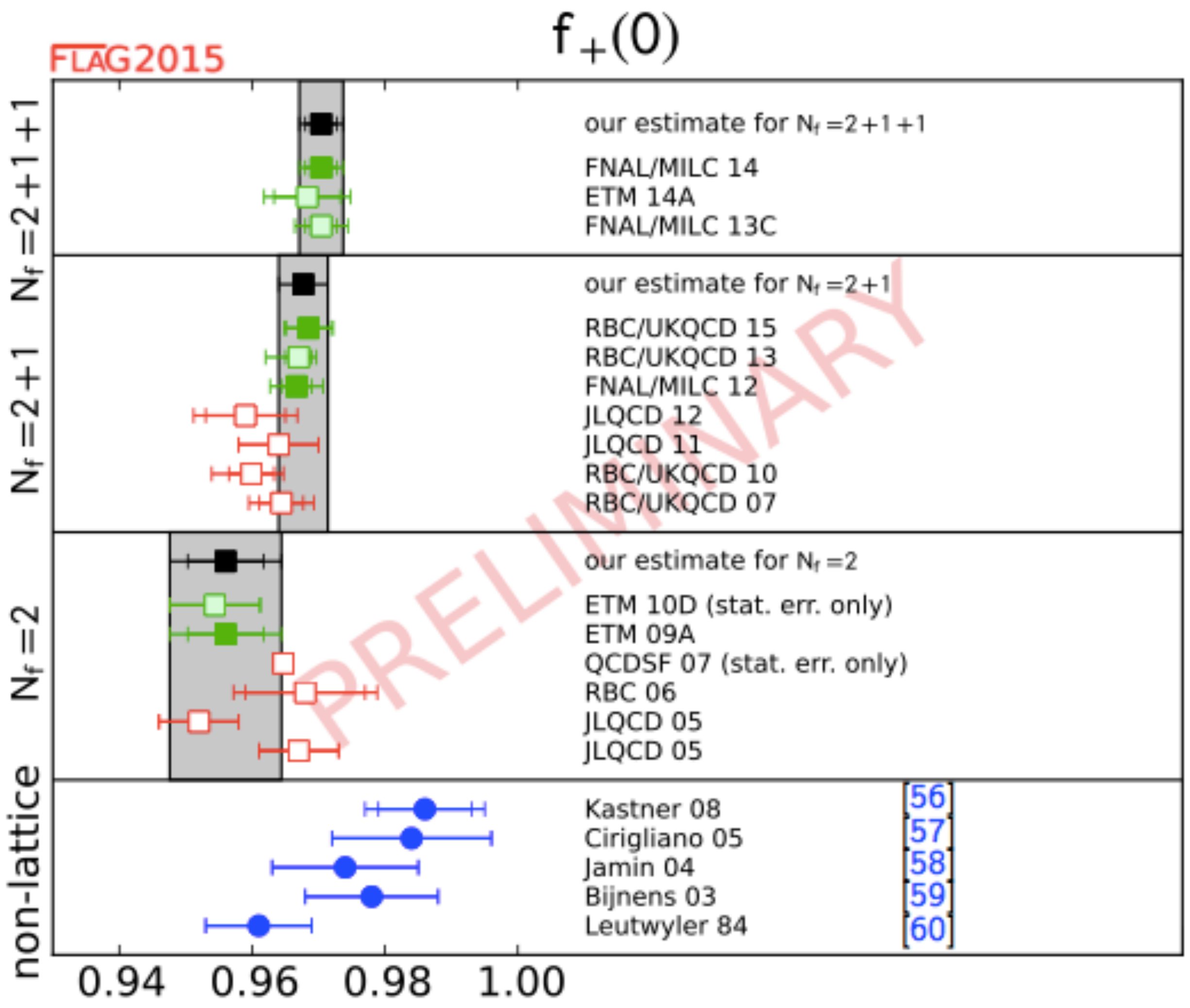
$N_f = 2+1+1$ $\quad f_+(0) = 0.9704(24)(22)$ MeV

$N_f = 2+1$ $\quad f_+(0) = 0.9661(32)$ MeV $\quad f_+(0) = 0.9677(37)$ MeV

$N_f = 2$ $\quad f_+(0) = 0.9560(57)(62)$ MeV $\quad$ unchanged

# Form factor, decay constants and unitarity





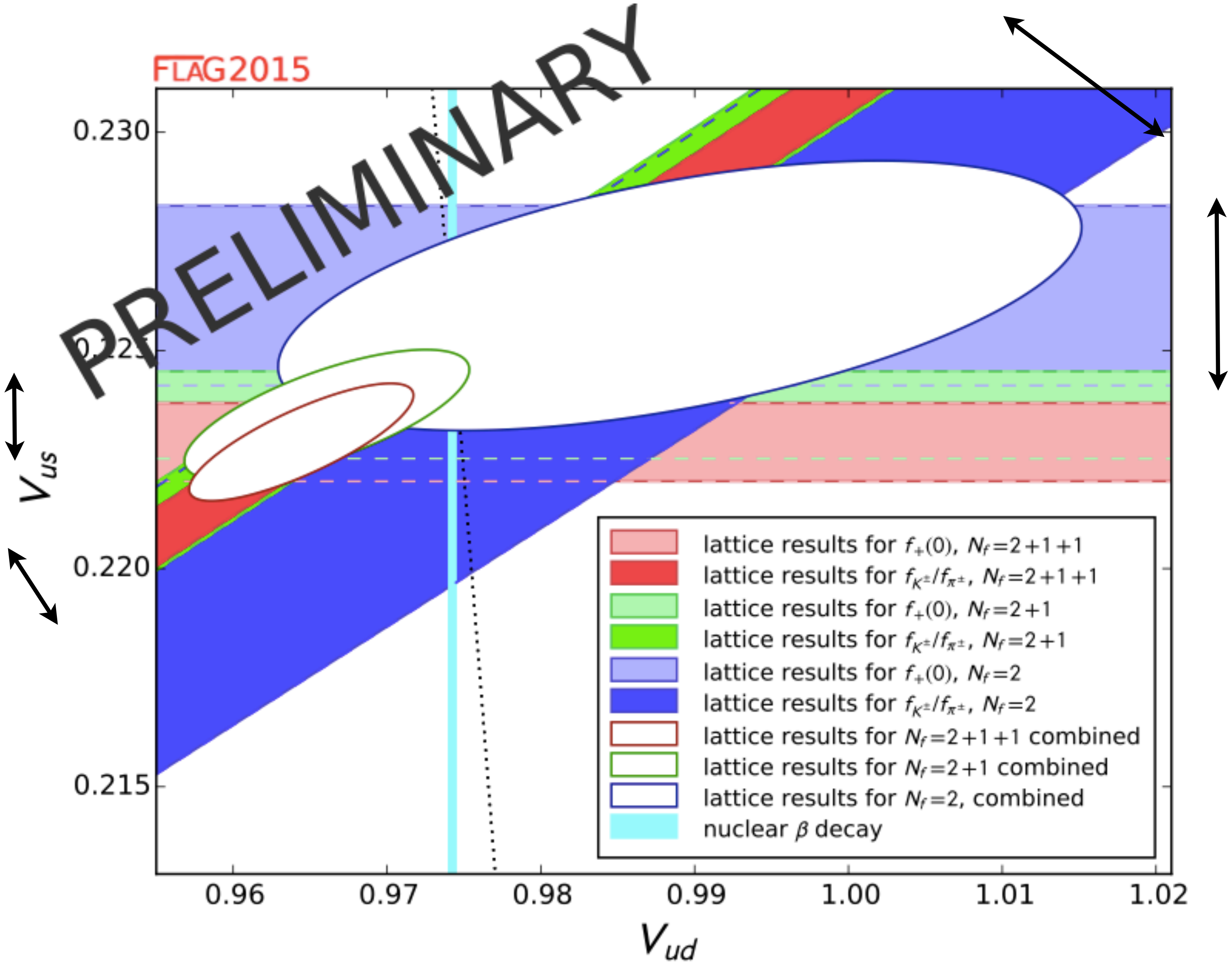
- χPT expansion: $f_+(0) \;=\; 1 \,+\, f_2 \,+\, f_4 \,+\, \cdots$

- from LO χPT: $\Delta f \;\equiv\; f_+(0) \,-\, 1 \,-\, f_2 \;=\; f_+(0) \,-\, 0.977$

- from most recent χPT estimates of $f_4$ , we have $\Delta f > 0$
- lattice suggests $\Delta f < 0$

- NB: simulation of $f+(q^2)$ at $q^2=0$ requires twisted boundary conditions in space

# Form factor, decay constants and unitarity



$N_f$=2+1

$|V_{us}|$ from $f_+(0)$

$|V_{us}|/|V_{ud}|$ from $f_{K^\pm}/f_{\pi^\pm}$

$|V_{us}|/|V_{ud}|$ from $f_{K^\pm}/f_{\pi^\pm}$

$|V_{us}|$ from $f_+(0)$

$N_f$=2

$$|V_{us}|\ f_+(0)\ =\ 0.2163(5)$$

$$\left|\frac{V_{us}}{V_{ud}}\right|\frac{f_{K^\pm}}{f_{\pi^\pm}}\ =\ 0.2758(5)$$

M.Antonelli et al., Eur.Phys.J. C69(2010)399

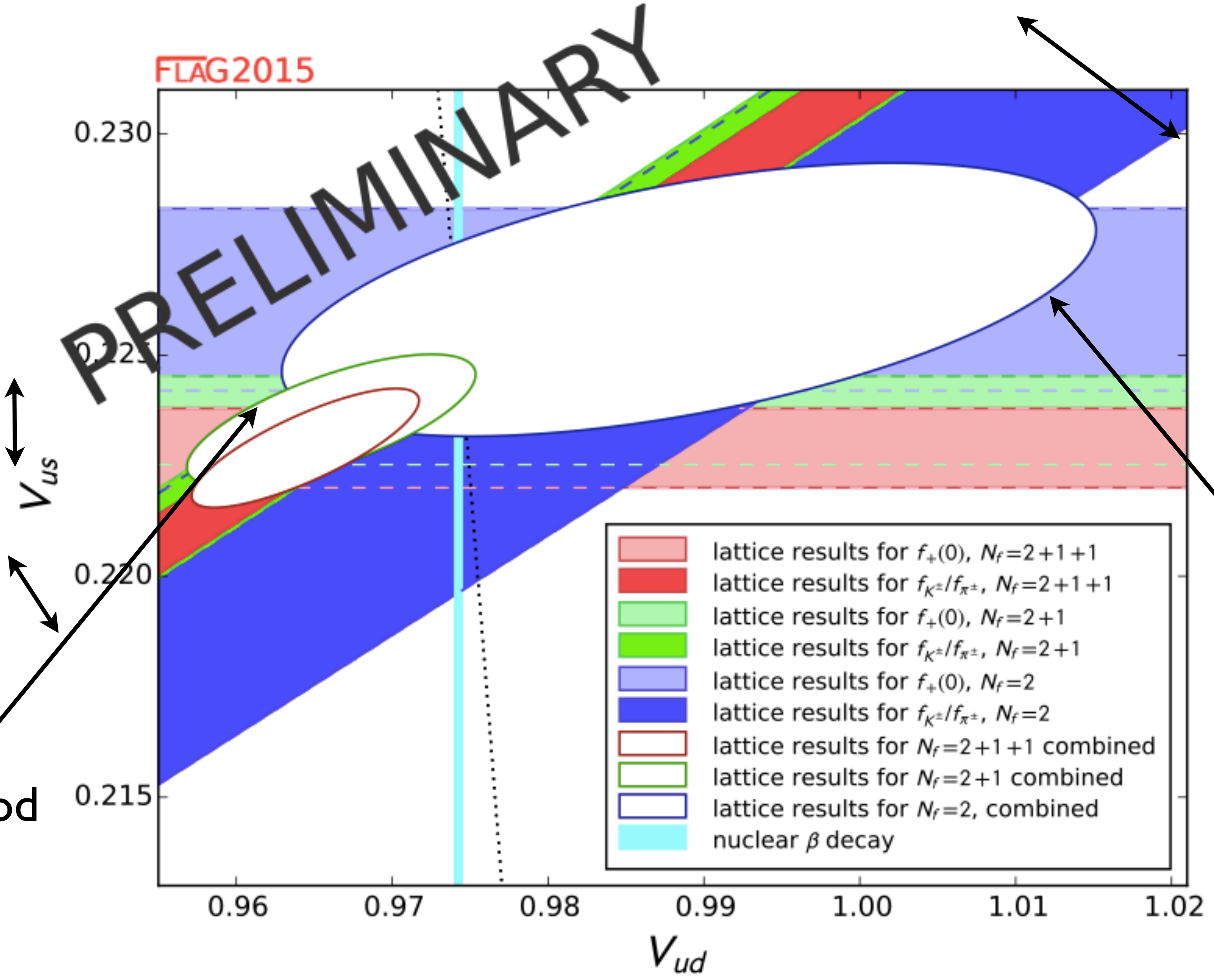
# Form factor, decay constants and unitarity

$N_f$=2+1

$|V_{us}|$ from $f_+(0)$

$|V_{us}|/|V_{ud}|$ from $f_{K^\pm}/f_{\pi^\pm}$

68% likelihood



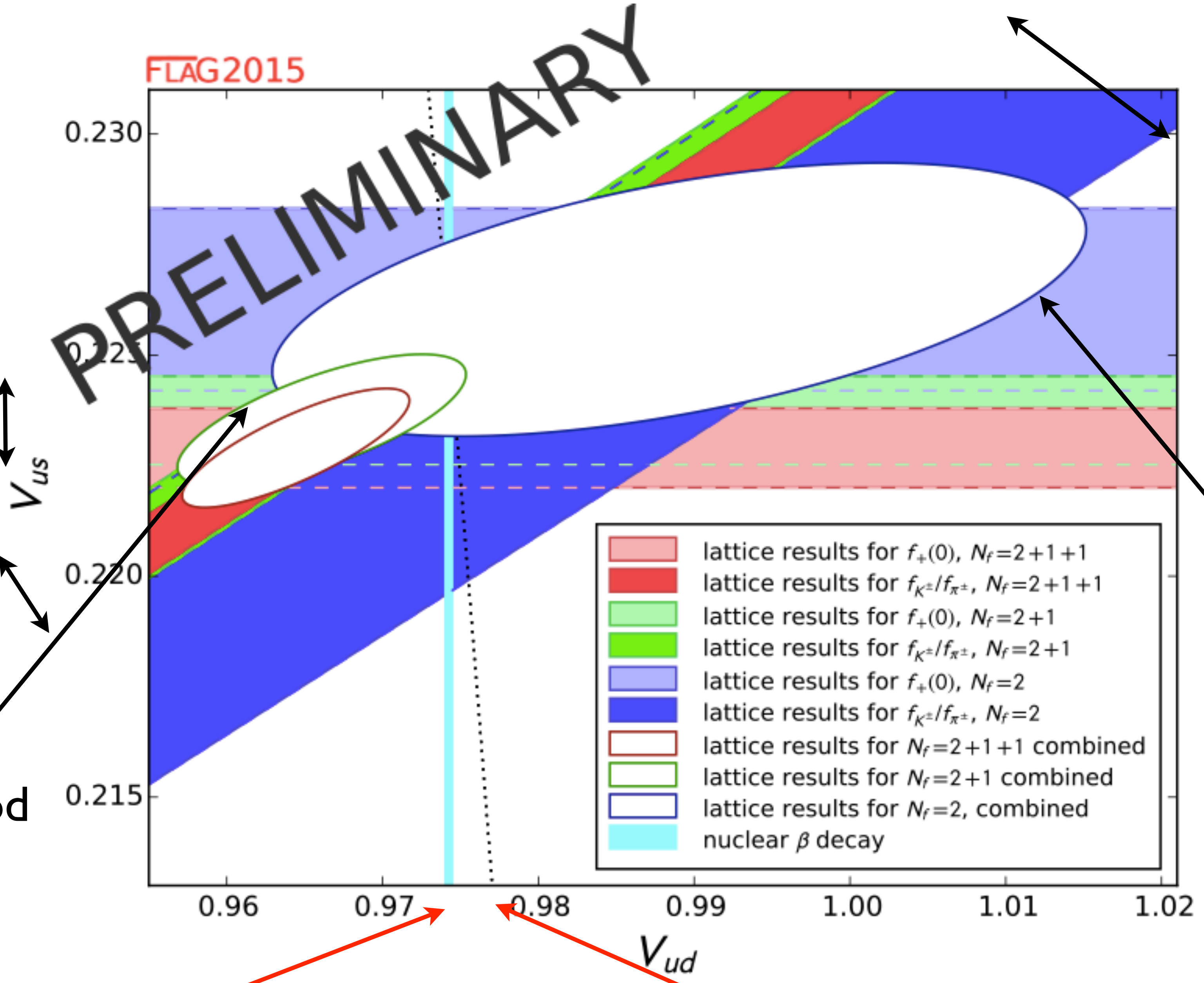
$|V_{us}|/|V_{ud}|$ from $f_{K^\pm}/f_{\pi^\pm}$

$|V_{us}|$ from $f_+(0)$

$N_f$=2

68% likelihood

# Form factor, decay constants and unitarity



$N_f=2+1$

$|V_{us}|$ from $f+(0)$

$|V_{us}|/|V_{ud}|$ from $f_{K^\pm}/f_{\pi^\pm}$

68% likelihood

$|V_{us}|/|V_{ud}|$ from $f_{K^\pm}/f_{\pi^\pm}$

$|V_{us}|$ from $f+(0)$

$N_f=2$

68% likelihood

$|V_{ud}|$ from nuclear β-decay; tension with $N_f=2+1+1$

$|V_{us}|$ vs. $|V_{ud}|$ from CKM unitarity --- small tension

# Form factor, decay constants and unitarity

- 1st row unitarity: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$
- PDG experiment: $|V_{ub}| = 4.15(49) \cdot 10^{-3}$
- From lattice data for $N_f$=2+1+1 and kaon decay branching rations we see a slight tension of previous plot (small ellipse vs dotted curve); UNITARITY OK within 2σ!

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.980(10)$$

- From lattice result for $f_+(0)$ and nuclear β-decay for $|V_{ud}|$ the test sharpens:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9989(8)$$

- From lattice result for $f_{K^\pm}/f_{\pi^\pm}$ and nuclear β-decay for $|V_{ud}|$ the test sharpens:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9999(7)$$

- Unitarity confirmed at the *per-mille* level for $N_f$=2+1+1; almost identical situation with $N_f$=2+1data; full agreement with unitarity for $N_f$=2 (bigger errors)

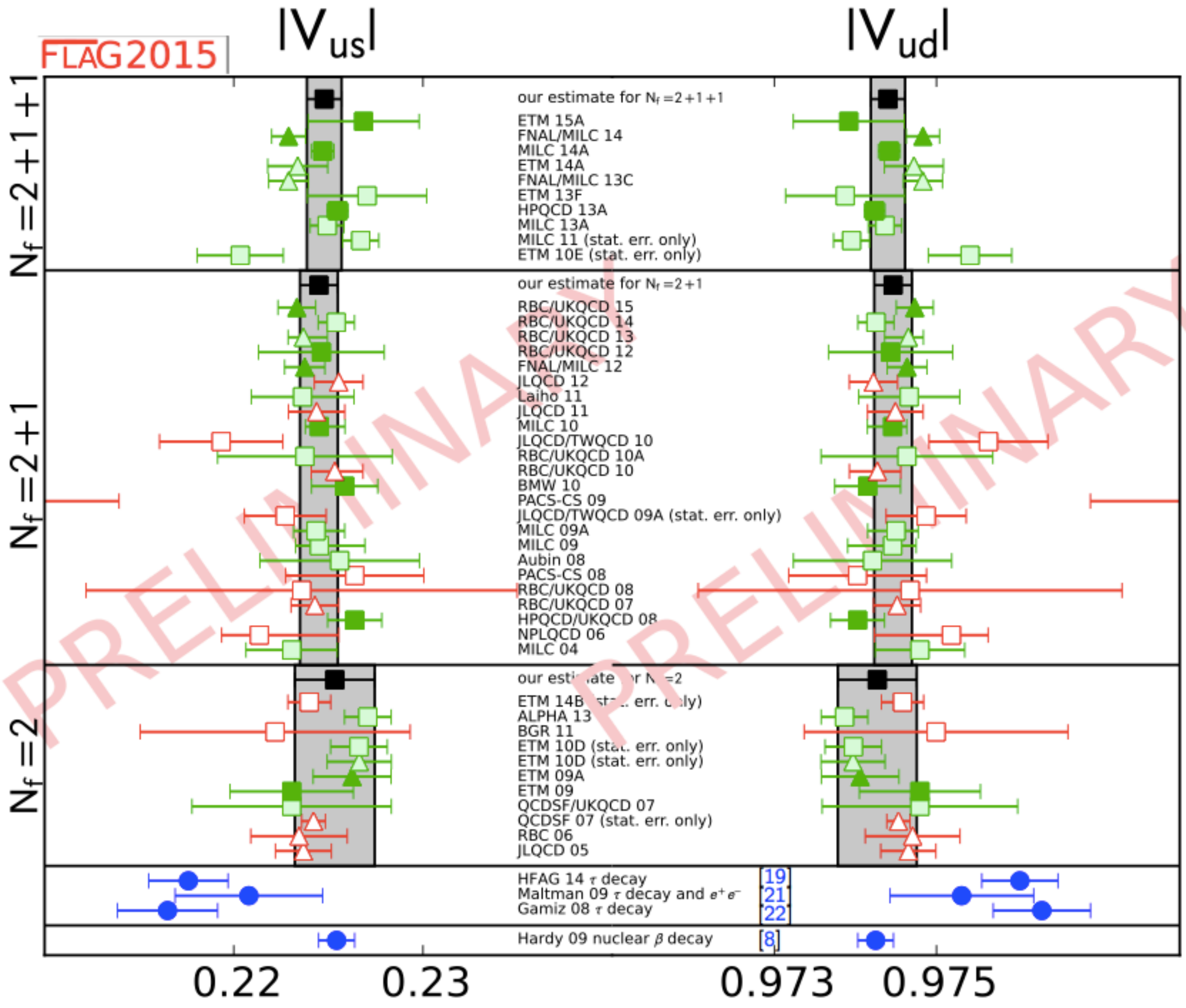
# Form factor, decay constants and unitarity

- CKM first row unitarity: $$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$
- PDG experiment: $$|V_{ub}| = 4.15(49) \cdot 10^{-3}$$
- K and π leptonic decays: $$\left|\frac{V_{us}}{V_{ud}}\right| \frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.2758(5)$$
- K → π semileptonic decays: $$|V_{us}|\ f_+(0) = 0.2163(5)$$

M.Antonelli et al., Eur.Phys.J. C69(2010)399

- 3 expressions, 4 unknowns: $f_{K^\pm}/f_{\pi^\pm}$ ; $f_+(0)$ ; $|V_{ud}|$ ; $|V_{us}|$
- need one input from lattice
- either $f_{K^\pm}/f_{\pi^\pm}$ or $f_+(0)$ to obtain $|V_{ud}|$ and $|V_{us}|$

# Form factor, decay constants and unitarity



- input $f_{K^\pm}/f_{\pi^\pm}$ 🟩
- input $f_+(0)$ ▲

some tension between lattice and τ-decay

agreement between lattice and β-decay

agreement for different $N_f$

# Light Flavour Physics

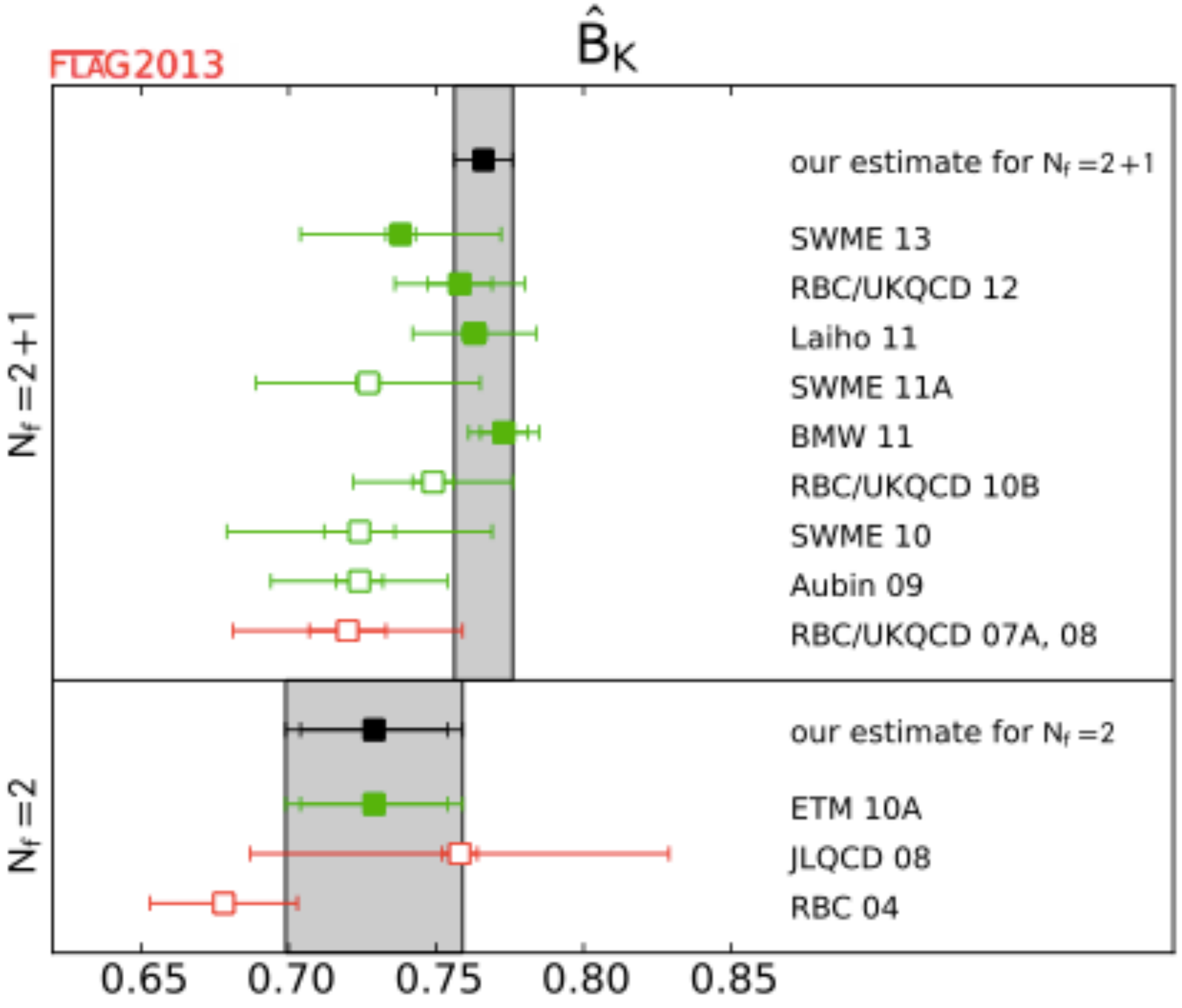
## $B_K$-in the SM and beyond

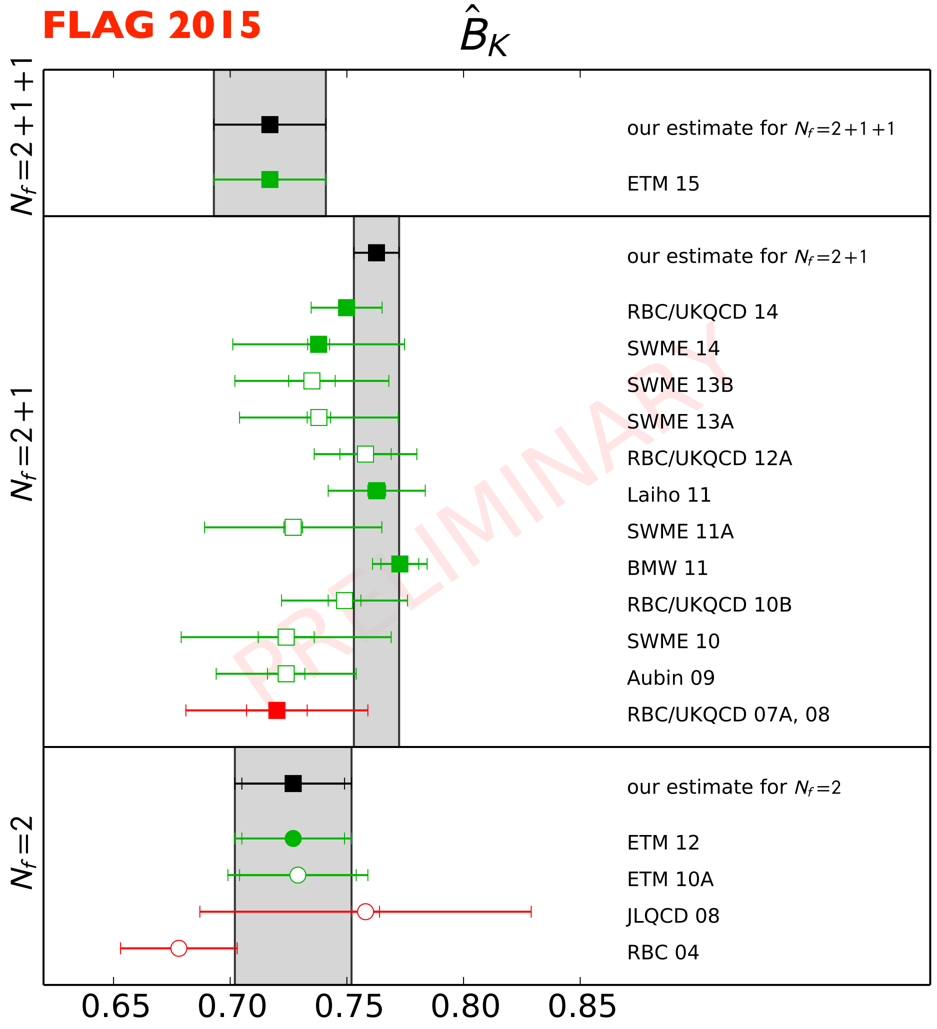
NB: some non-FLAG analysis

SWME: J.A. Bailey et al., arXiv:1503.06613

ETM: V.Bertone et al., JHEP03(2013)089

# $B_K$ in the SM





$N_f = 2+1+1$ $\quad$ $\hat{B}_{\rm K} = 0.717(24)$

$\hat{B}_{\rm K} = 0.7661(99)$ $\quad$ $N_f = 2+1$ $\quad$ $\hat{B}_{\rm K} = 0.7627(97)$

$\hat{B}_{\rm K} = 0.729(25)(17)$ $\quad$ $N_f = 2$ $\quad$ $\hat{B}_{\rm K} = 0.727(25)$

# $B_K$ in the SM

- Self consistency of $\varepsilon_K$, the role of $B_K$- and |Vcb| NB: not FLAG!

SWME: J.A. Bailey et al., arXiv:1503.06613

$$\epsilon_K = e^{i\theta}\sqrt{2}\sin\theta\left(C_\epsilon \hat{B}_K X_{\rm SD} + \xi_0 + \xi_{\rm LD}\right) \; + \; \cdots$$

known factor: $$C_\epsilon = \frac{G_F^2 F_K^2 m_{K^0} M_W^2}{6\sqrt{2}\pi^2 \Delta M_K}$$ $$x_{c,t} \equiv m_{c,t}^2/M_W^2$$

short distance:

$$X_{\rm SD} = \bar{\eta}\lambda^2|V_{cb}|^2\left[|V_{cb}|^2(1-\bar{\rho})\eta_{tt}S_0(x_t)(1+r) + \left(1-\frac{\lambda^4}{8}\right)\{\eta_{ct}S_0(x_c,x_t) - \eta_{cc}S_0(x_c)\}\right]$$

Inami-Lim functions: $S_0(x_{c,t})$ $\quad S_0(x_c, x_t)$

Coefficients known to NLO, NNLO, NNLO: $\eta_{tt}$ $\quad \eta_{ct}$ $\quad \eta_{cc}$

$$r = \{\eta_{cc}S_0(x_c) - 2\eta_{ct}S_0(x_c,x_t)\}/\{\eta_{tt}S_0(x_t)\}$$

# $B_K$ in the SM

- Self consistency of $\varepsilon_K$, the role of $B_K$- and |Vcb| NB: not FLAG!

SWME: J.A. Bailey et al., arXiv:1503.06613

$$\epsilon_K = e^{i\theta}\sqrt{2}\sin\theta\left(C_\epsilon \hat{B}_K X_{\rm SD} + \xi_0 + \xi_{\rm LD}\right) \; + \; \cdots$$

short distance:

$$X_{\rm SD} = \bar{\eta}\lambda^2|V_{cb}|^2\left[|V_{cb}|^2(1-\bar{\rho})\eta_{tt}S_0(x_t)(1+r) + \left(1-\frac{\lambda^4}{8}\right)\{\eta_{ct}S_0(x_c,x_t) - \eta_{cc}S_0(x_c)\}\right]$$

long distance effect from absorptive part (-7% effect): $\xi_0 = \mathrm{Im}(A_0)/\mathrm{Re}(A_0)$

RBC/UKQCD T. Blum et al., Phys.Rev.Lett.108 (2012)141601

long distance effect from dispersive part (2% effect - neglected): $\xi_{\rm LD}$

RBC/UKQCD N. Christ et al., Phys.Rev.D88 (2013)014508

# $B_K$ in the SM

- Self consistency of $\varepsilon_K$, the role of $B_K$- and |Vcb| NB: not FLAG!

SWME: J.A. Bailey et al., arXiv:1503.06613

$$\epsilon_K = e^{i\theta}\sqrt{2}\sin\theta\Big(C_\epsilon \hat{B}_K X_{\mathrm{SD}} + \xi_0 + \xi_{\mathrm{LD}}\Big) \; + \; \cdots$$

short distance:

$$X_{\mathrm{SD}} = \bar{\eta}\lambda^2|V_{cb}|^2\left[|V_{cb}|^2(1-\bar{\rho})\eta_{tt}S_0(x_t)(1+r) + \left(1-\frac{\lambda^4}{8}\right)\{\eta_{ct}S_0(x_c,x_t) - \eta_{cc}S_0(x_c)\}\right]$$

Wolfenstein parameters NOT from UTfit / CKMfitter (they contain unwanted dependence on $B_K$, |Vcb| and $\varepsilon_K$)

Prefer Angle-Only-Fit (AOF) of A. Bevan, M. Bona et al., Nucl.Phys.Proc.Suppl.241-242 (2013) 89 for $\rho$ and $\eta$

|Vus| ≈ λ from $K_{\mu 2}$ and $K_{l3}$

|Vcb| ≈ $A\lambda^2$

NB: 4th POWER!

# $B_K$ in the SM

- Self consistency of $\varepsilon_K$, the role of $B_K$- and |Vcb| NB: not FLAG!

SWME: J.A. Bailey et al., arXiv:1503.06613

$$\epsilon_K = e^{i\theta}\sqrt{2}\sin\theta\Big(C_\epsilon \hat{B}_K X_{\rm SD} + \xi_0 + \xi_{\rm LD}\Big) \ + \ \cdots$$

short distance:

$$X_{\rm SD} = \bar{\eta}\lambda^2|V_{cb}|^2\left[|V_{cb}|^2(1-\bar{\rho})\eta_{tt}S_0(x_t)(1+r) + \left(1-\frac{\lambda^4}{8}\right)\{\eta_{ct}S_0(x_c,x_t) - \eta_{cc}S_0(x_c)\}\right]$$

1.Use $N_f = 2+1$ FLAG-2 result for $B_K$

2. Use inclusive channel ($B \to X_c \, l \, \nu$ and $B \to X_s \gamma$ decays) for $|Vcb| = 42.21(78) \times 10^{-3}$

A. Alberti, et al., Phys.Rev.Lett.114 (2015) 061802

Use exclusive channel ($B \to D^* \, l \, \nu$ decays) for $|Vcb| = 39.04(49)(53)(19) \times 10^{-3}$

FNAL/MILC: J.A. Bailey Phys.Rev.D89 (2014)014504

3. Calculate $|\epsilon_K^{\rm SM}|$ and compare it to $|\epsilon_K^{\rm exp}| = (2.228 \pm 0.011) \times 10^{-3}$

# $B_K$ in the SM

- Self consistency of $\varepsilon_K$, the role of $B_K$- and |Vcb| NB: not FLAG!

SWME: J.A. Bailey et al., arXiv:1503.06613

$$\epsilon_K = e^{i\theta}\sqrt{2}\sin\theta\Big(C_\epsilon \hat{B}_K X_{\rm SD} + \xi_0 + \xi_{\rm LD}\Big) \ + \ \cdots$$

short distance:

$$X_{\rm SD} = \bar{\eta}\lambda^2|V_{cb}|^2\left[|V_{cb}|^2(1-\bar{\rho})\eta_{tt}S_0(x_t)(1+r) + \left(1-\frac{\lambda^4}{8}\right)\{\eta_{ct}S_0(x_c,x_t) - \eta_{cc}S_0(x_c)\}\right]$$

- $|\,\epsilon_K^{\rm exp}| = (2.228 \pm 0.011) \times 10^{-3}$
- $|\,\epsilon_K^{\rm SM}| = (1.58 \pm 0.18) \times 10^{-3}$ exclusive |Vcb|
- $|\,\epsilon_K^{\rm SM}| = (2.13 \pm 0.23) \times 10^{-3}$ inclusive |Vcb|

$$\Delta\epsilon_{\rm K} \equiv |\epsilon_{\rm K}^{\rm SM}| - |\epsilon_{\rm K}^{\rm exp}|$$

$$\Delta\epsilon_{\rm K} = 3.6(2)\sigma \qquad \text{exclusive } |V_{cb}|$$

$$\Delta\epsilon_{\rm K} = 0.44(24)\sigma \qquad \text{inclusive } |V_{cb}|$$

neglected $\xi_{\rm LD}$ contribution (2%) cannot explain this 30% gap in $\Delta\varepsilon_K$

# $B_K$ in the SM

- Self consistency of $\varepsilon_K$, the role of $B_K$- and |Vcb| NB: not FLAG!

SWME: J.A. Bailey et al., arXiv:1503.06613

$$\epsilon_K = e^{i\theta}\sqrt{2}\sin\theta\Big(C_\epsilon \hat{B}_K X_{\mathrm{SD}} + \xi_0 + \xi_{\mathrm{LD}}\Big) \ + \ \cdots$$

short distance:

$$X_{\mathrm{SD}} = \bar{\eta}\lambda^2|V_{cb}|^2\left[|V_{cb}|^2(1-\bar{\rho})\eta_{tt}S_0(x_t)(1+r) + \left(1-\frac{\lambda^4}{8}\right)\{\eta_{ct}S_0(x_c,x_t) - \eta_{cc}S_0(x_c)\}\right]$$

Error budget tells us that $B_K$ is not the dominant uncertainty

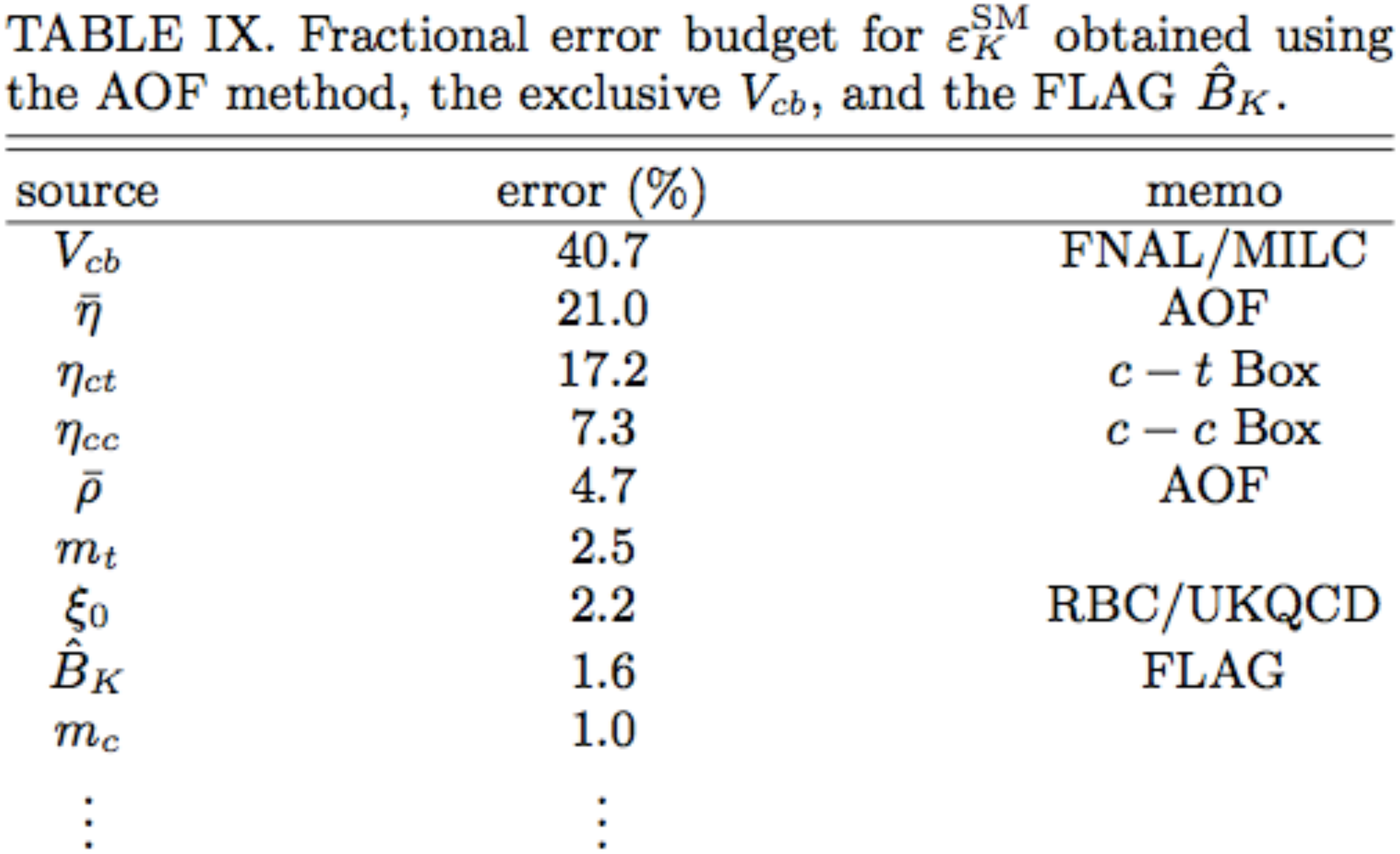


TABLE IX. Fractional error budget for $\varepsilon_K^{\mathrm{SM}}$ obtained using the AOF method, the exclusive $V_{cb}$, and the FLAG $\hat{B}_K$.

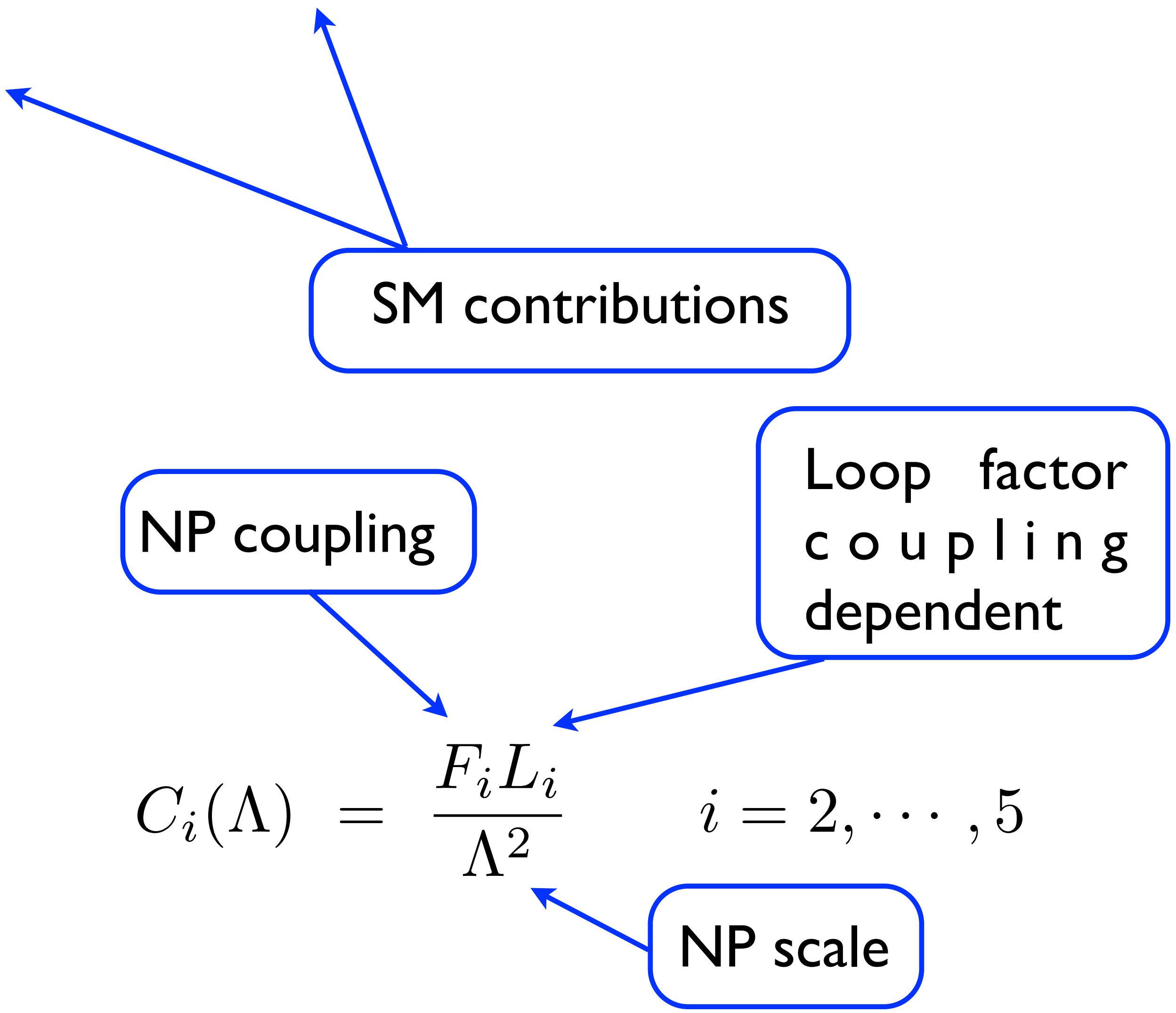
| source | error (%) | memo |
|---|---|---|
| $V_{cb}$ | 40.7 | FNAL/MILC |
| $\bar{\eta}$ | 21.0 | AOF |
| $\eta_{ct}$ | 17.2 | $c-t$ Box |
| $\eta_{cc}$ | 7.3 | $c-c$ Box |
| $\bar{\rho}$ | 4.7 | AOF |
| $m_t$ | 2.5 | |
| $\xi_0$ | 2.2 | RBC/UKQCD |
| $\hat{B}_K$ | 1.6 | FLAG |
| $m_c$ | 1.0 | |
| $\vdots$ | $\vdots$ | |

# $B_K$ beyond the SM

- Analyze New Physics (NP) effects in a model-independent way: assume a generalization of the effective $\Delta S = 2$ Hamiltonian which contains operators beyond the SM one; the amplitude is:

$$< \bar{K}^0 | \mathcal{H}_{\text{eff}}^{\Delta S=2} | K^0 > = C_1 < \bar{K}^0 | O_1 | K^0 > + \sum_{i=2}^{5} C_i < \bar{K}^0 | O_i | K^0 >$$

$$\begin{aligned}
O_1 &= [\bar{s}^\alpha \gamma_\mu (1-\gamma_5) d^\alpha][\bar{s}^\beta \gamma_\mu (1-\gamma_5) d^\beta] \\
O_2 &= [\bar{s}^\alpha (1-\gamma_5) d^\alpha][\bar{s}^\beta (1-\gamma_5) d^\beta] \\
O_3 &= [\bar{s}^\alpha (1-\gamma_5) d^\beta][\bar{s}^\beta (1-\gamma_5) d^\alpha] \\
O_4 &= [\bar{s}^\alpha (1-\gamma_5) d^\alpha][\bar{s}^\beta (1+\gamma_5) d^\beta] \\
O_5 &= [\bar{s}^\alpha (1-\gamma_5) d^\beta][\bar{s}^\beta (1+\gamma_5) d^\alpha]
\end{aligned}$$



SM contributions

NP coupling

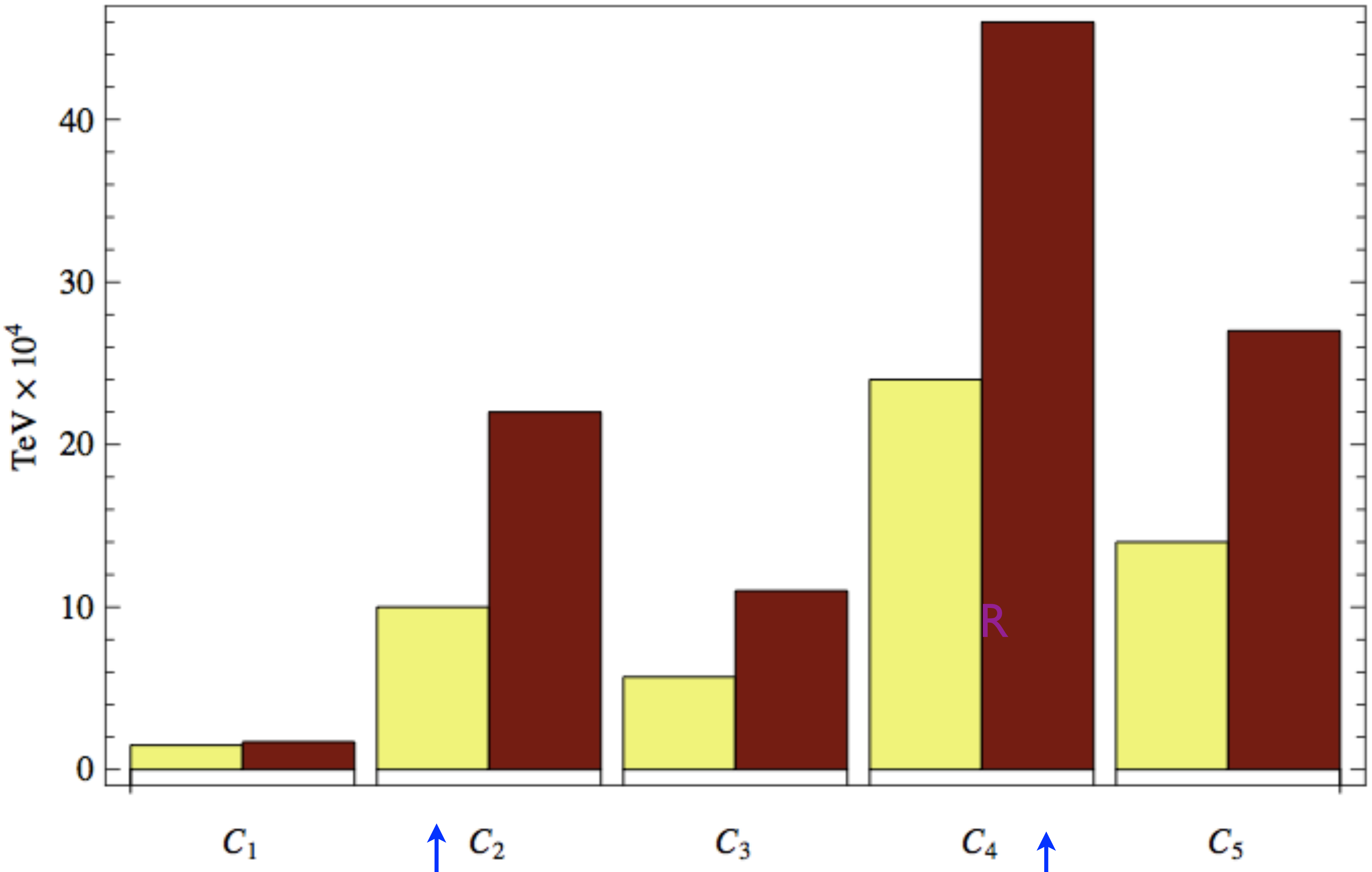
Loop factor coupling dependent

$$C_i(\Lambda) = \frac{F_i L_i}{\Lambda^2} \qquad i = 2, \cdots, 5$$

NP scale

Assuming $F_i \sim L_i \sim 1$, generalized UT-fit analysis produces lower bounds for $\Lambda$; these depend very strongly (several orders of magnitude) on this assumption.

# $B_K$ beyond the SM



$$R_i = \frac{< \bar{K}^0|O_i|K^0 >}{< \bar{K}^0|O_1|K^0 >} \qquad i = 2, \cdots, 5$$

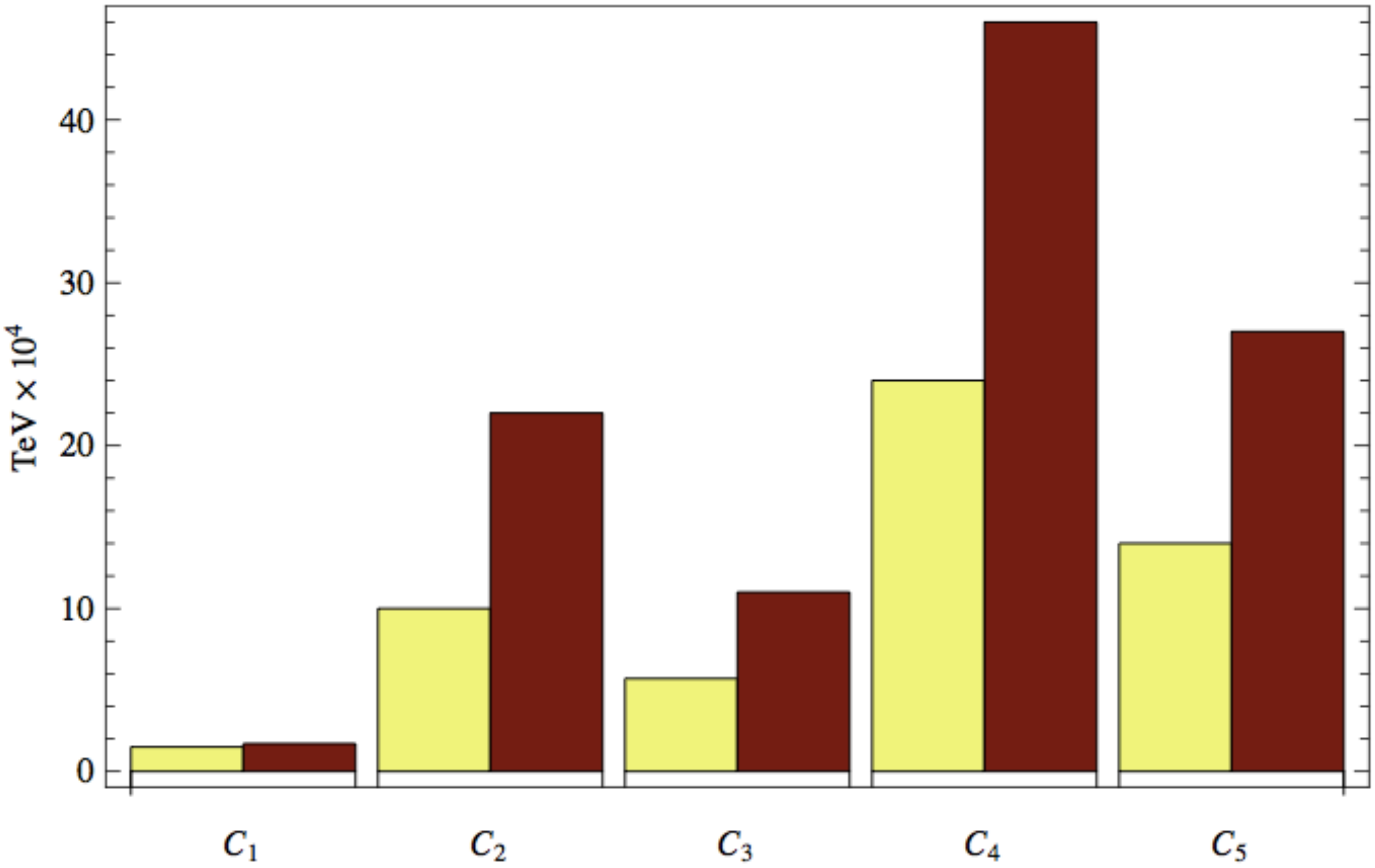
$N_f$=0 data; accuracy of ratios $R_i \sim$ 20%-23%

UTfit: M.Bona et al., JHEP03(2008)049

$N_f$=2 data; accuracy of ratios $R_i \sim$ 3%-6%

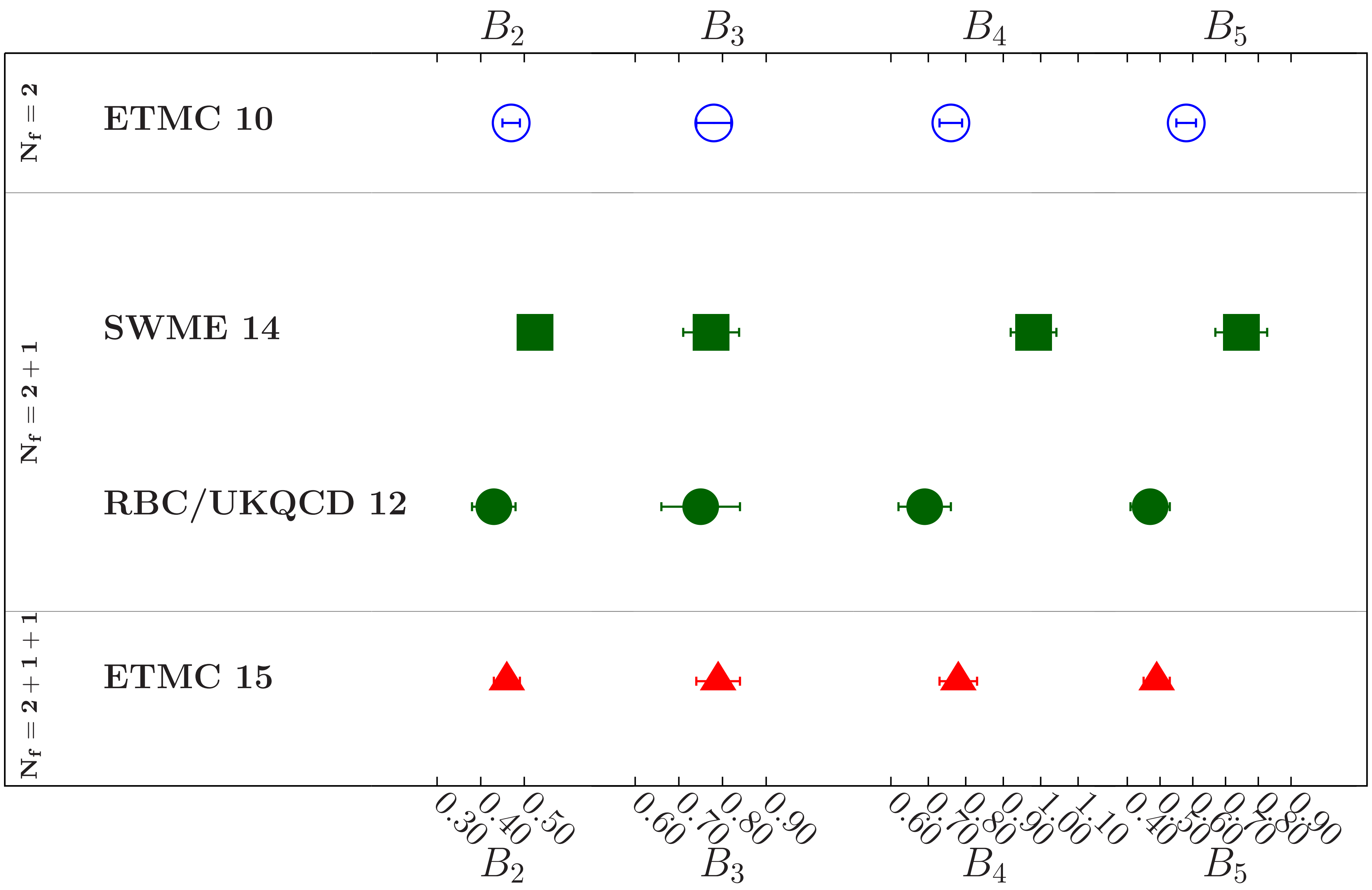
ETM: V.Bertone et al., JHEP03(2013)089

# $B_K$ beyond the SM



- NB: each contribution analyzed separately (avoids accidental cancellations).
- NB: SM bound is several orders of magnitude weaker thank those arising form BSM operators.

# $B_K$ beyond the SM



Courtesy of P. Dimopoulos

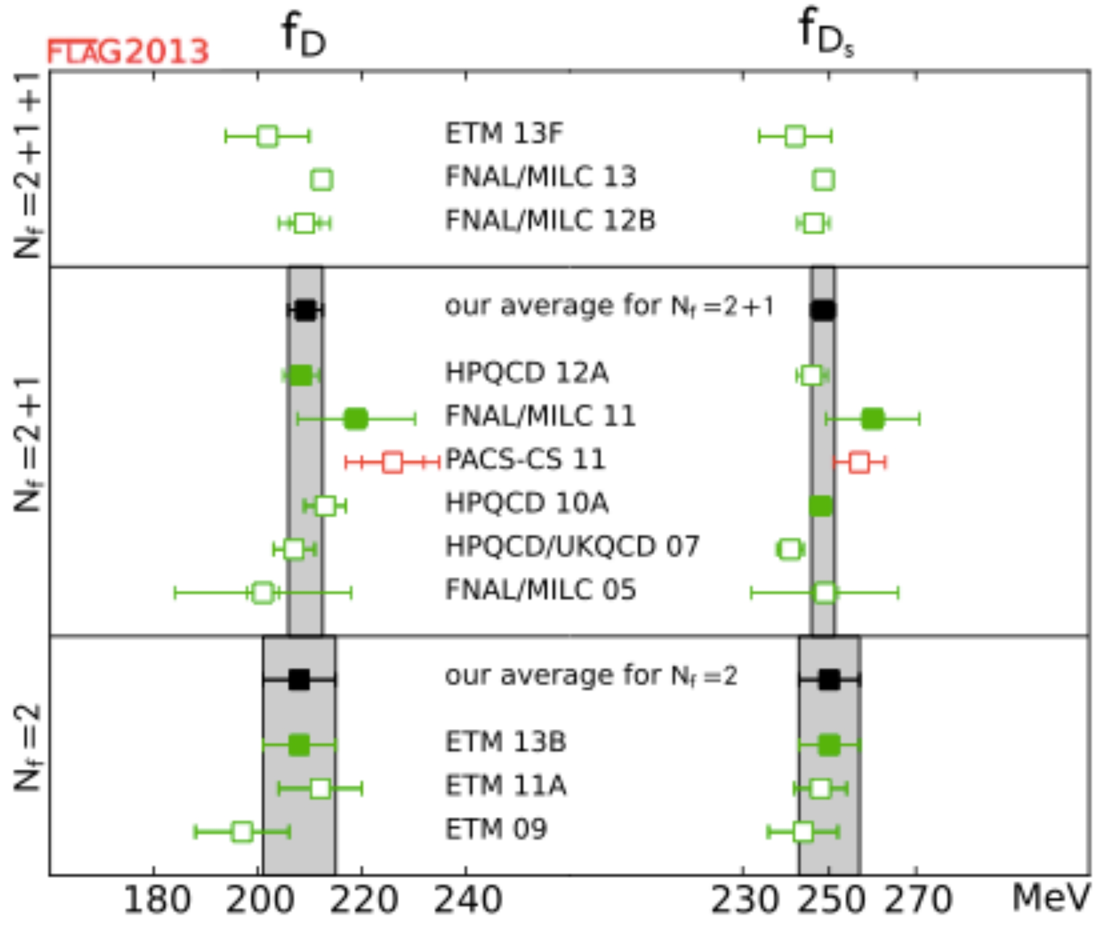
$B_k$ ($k$=2,..,5) analogous to $B_l$ (VSA)

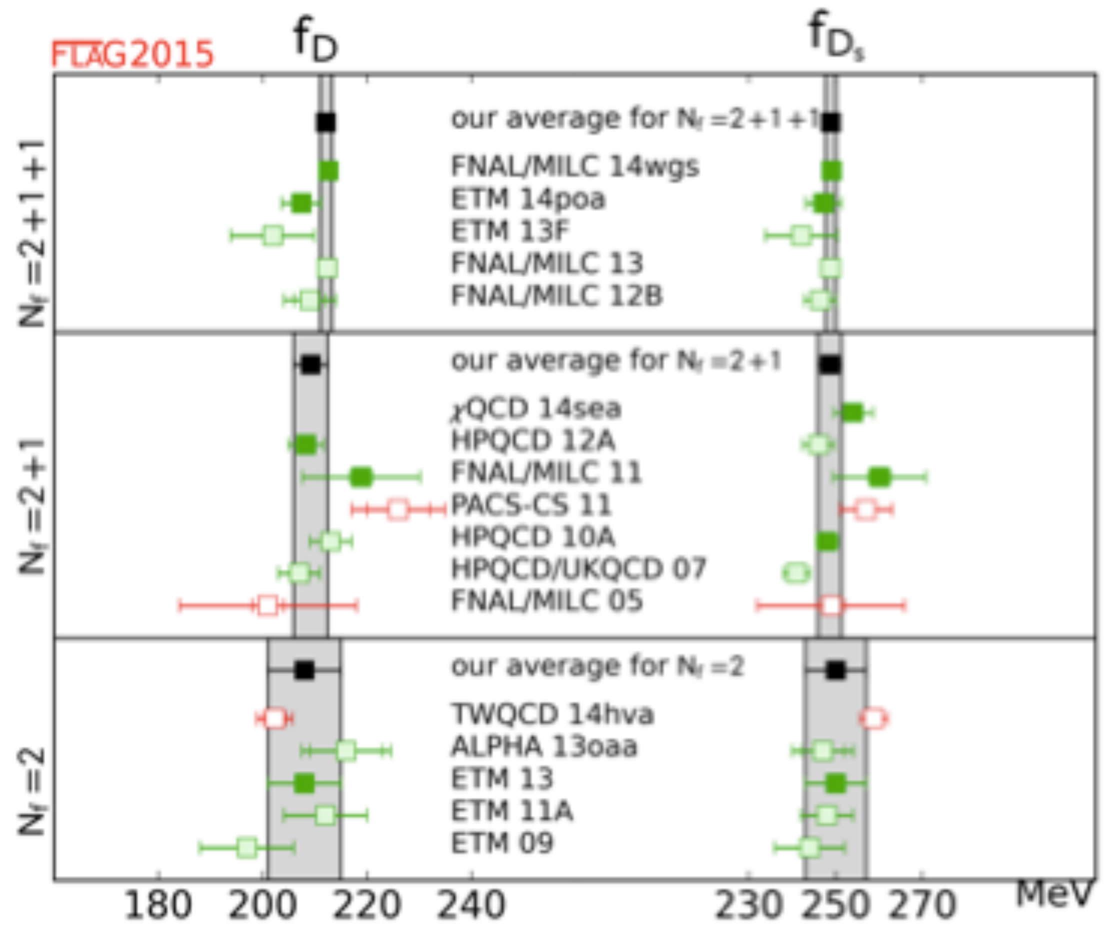
# Charm Physics

$f_D$, $f_{Ds}$, $f_+(0)$, $|V_{cd}|$, $|V_{cs}|$

CKM second row unitarity

# Leptonic decay constants $f_D$ and $f_{Ds}$

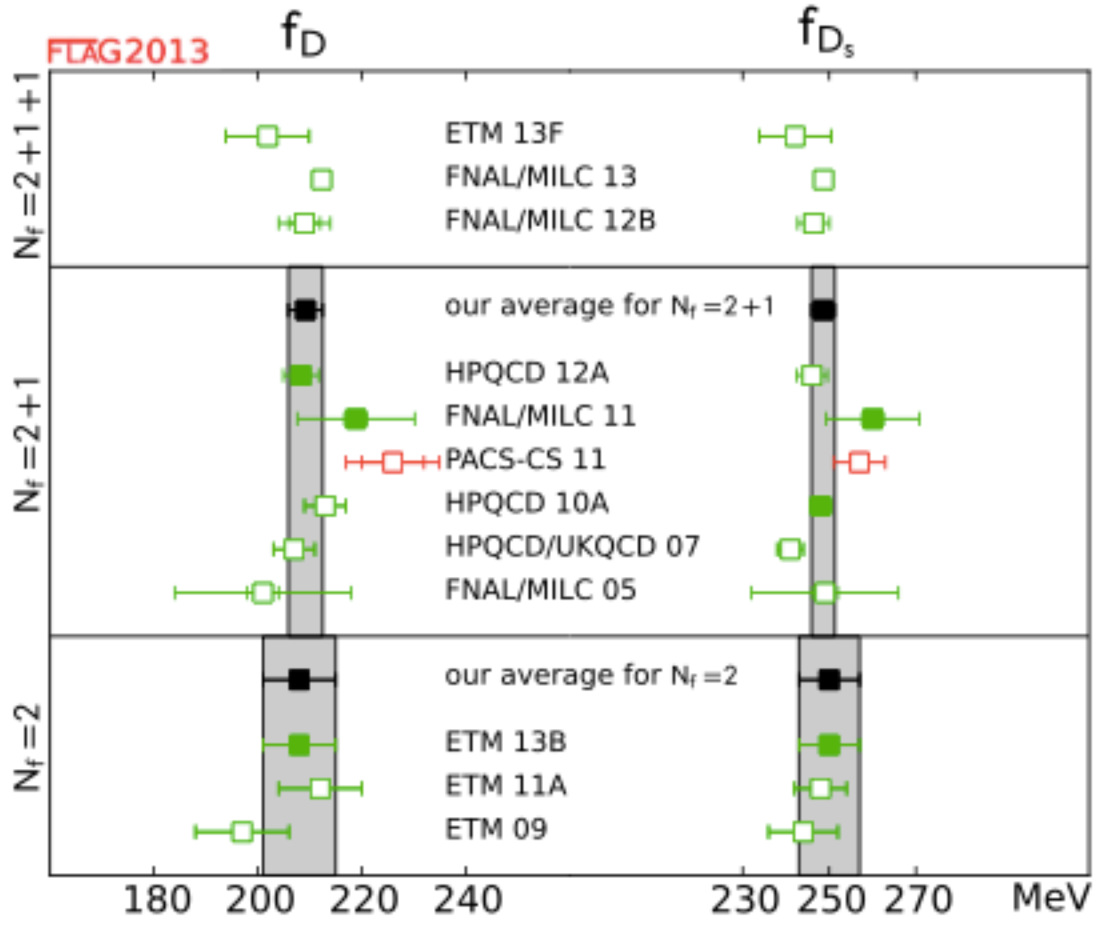


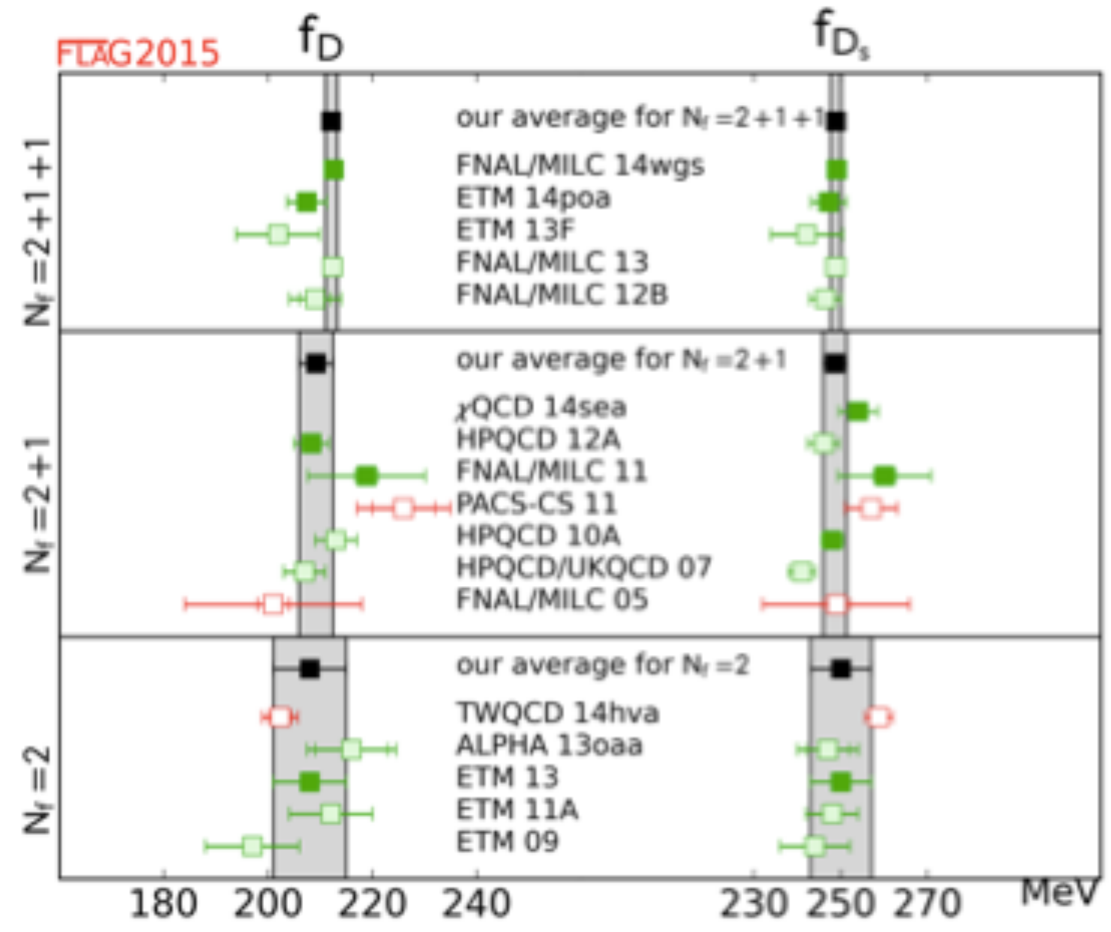


| | | |
|---|---|---|
| | $N_f = 2+1+1$ | $f_D = 212.15(1.12)$ MeV |
| $f_D = 209.2(3.3)$ MeV | $N_f = 2+1$ | unchanged |
| $f_D = 208(7)$ MeV | $N_f = 2$ | unchanged |

- NB: as the quality of the simulations improves in the near future, we should distinguish between $f_{D+}$ (FNAL/MILK) and the average between $f_{D+}$ and $f_{D0}$ (HPQCD, PACS-CS, ETM).

# Leptonic decay constants $f_D$ and $f_{D_s}$

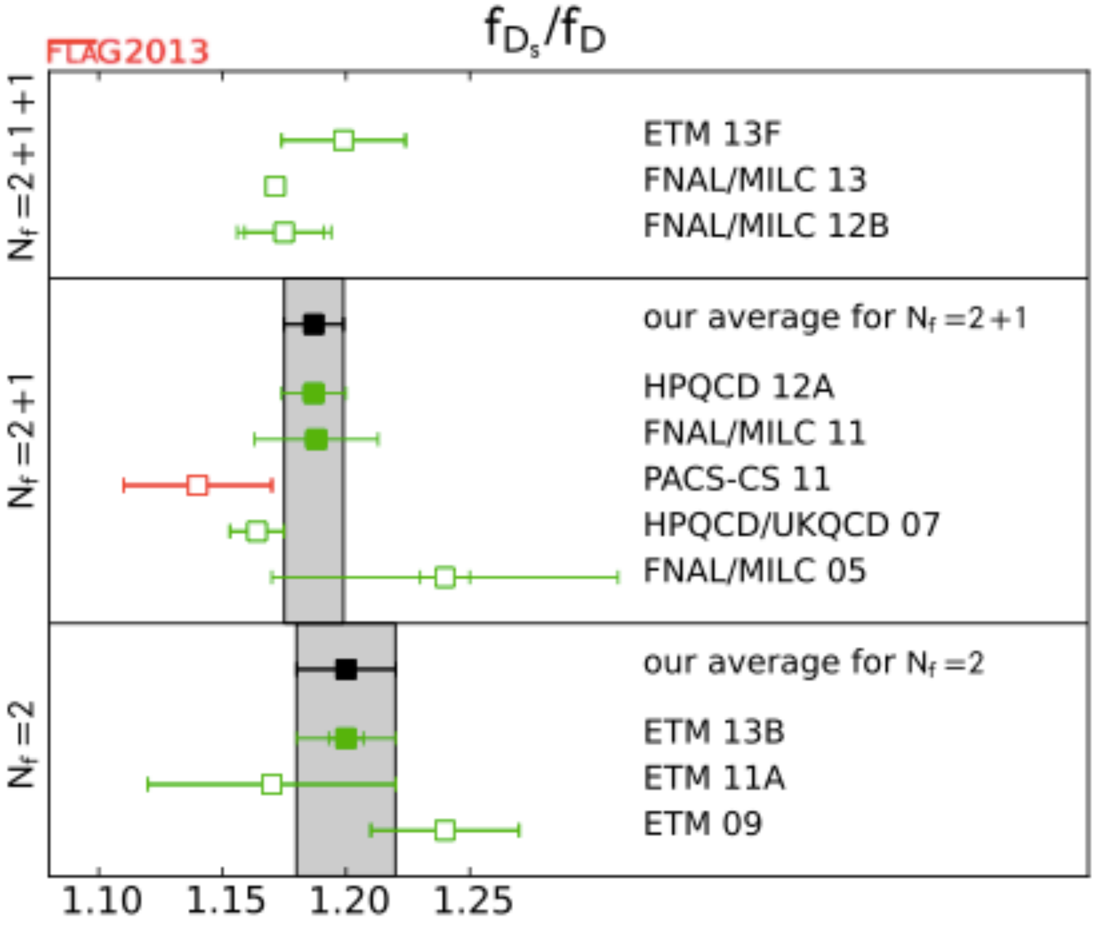


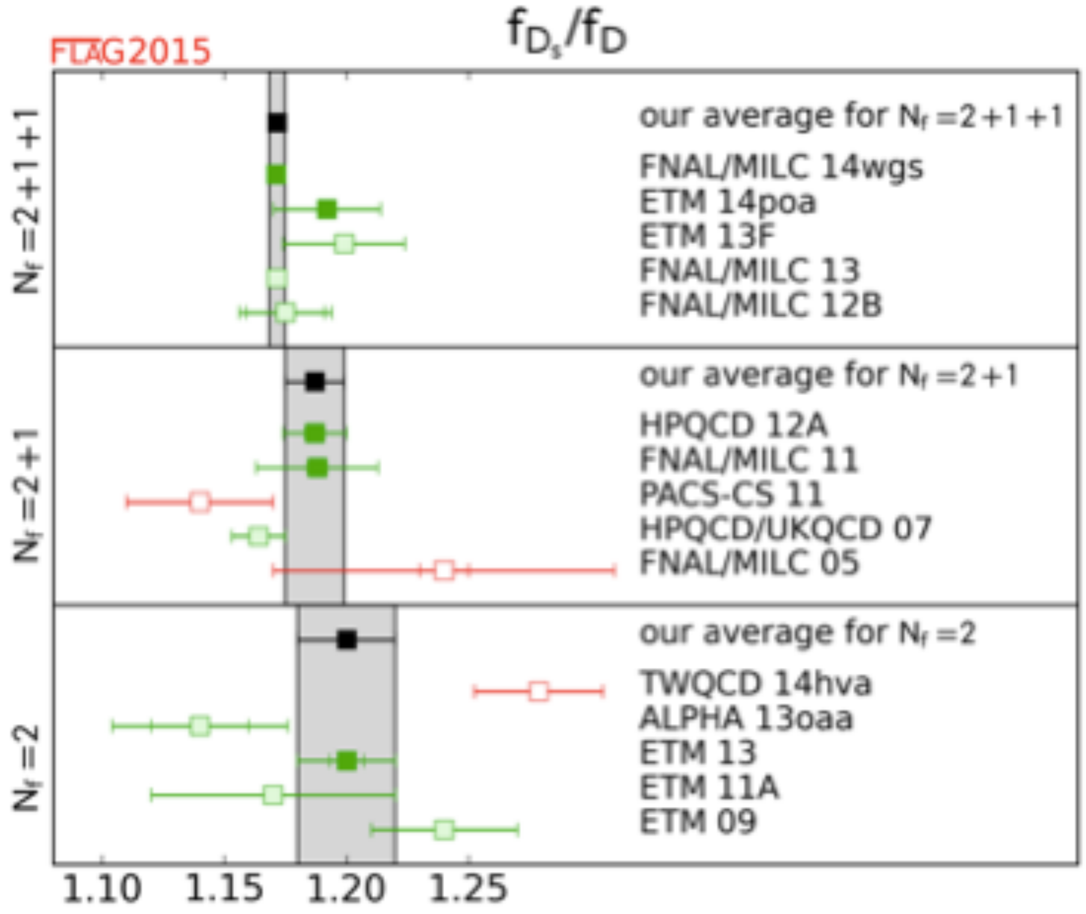


| | | |
|---|---|---|
| | $N_f = 2+1+1$ | $f_{D_s} = 248.83(1.27)$ MeV |
| $f_{D_s} = 248.6(2.7)$ MeV | $N_f = 2+1$ | $f_{D_s} = 249.8(2.3)$ MeV |
| $f_{D_s} = 250(7)$ MeV | $N_f = 2$ | unchanged |

# Leptonic decay constants $f_D$ and $f_{D_s}$

- the ratios are better determined



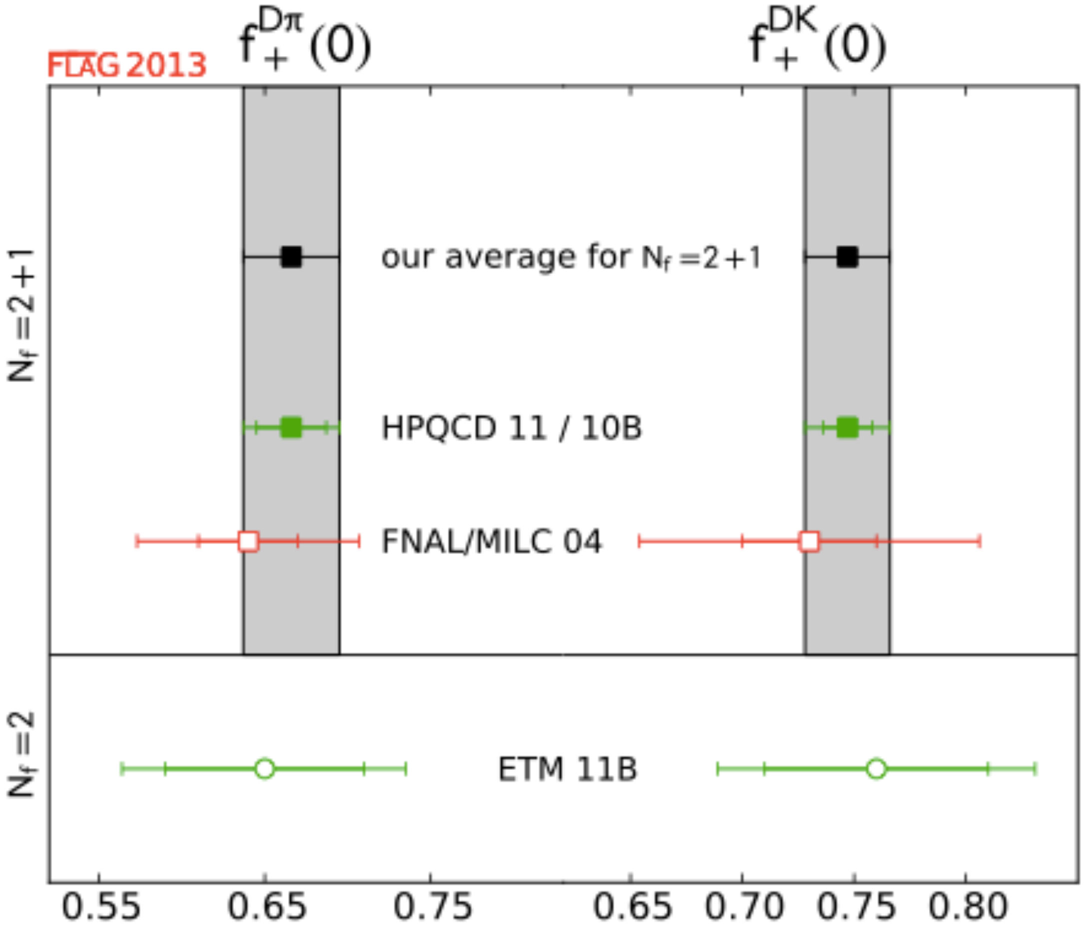


$N_f = 2+1+1$ $$\frac{f_{D_s}}{f_D} = 1.1716(32)$$

$$\frac{f_{D_s}}{f_D} = 1.187(12)$$ $N_f = 2+1$ unchanged

$$\frac{f_{D_s}}{f_D} = 1.20(2)$$ $N_f = 2$ unchanged

# Semileptonic decay form factor $f_+(0)$



- $N_f$=2
  - ETM (proceedings)
- $N_f$=2+1
  - FNAL/MILK (single lattice spacing) predicted shape of $f^{DK}_+(q^2)$ by FOCUS & Belle
  - HPQCD (more accurate)
- $N_f$=2+1+1
  - in the works (ETM)

- only HPQCD datum; no FLAG average

$$\begin{aligned} f_+^{D\pi}(0) &= 0.666 \pm 0.029 \text{ MeV} & N_f &= 2+1 \\ f_+^{DK}(0) &= 0.747 \pm 0.019 \text{ MeV} & N_f &= 2+1 \end{aligned}$$

# CKM angles $|V_{cd}|$ and $|V_{cs}|$

- Branching ratios of leptonic decays

$$\mathcal{B}(D \to l\nu_l) = \frac{G_F^2 \tau_D}{8\pi} |V_{cd}|^2 f_D^2 m_l^2 m_D \left(1 - \frac{m_l^2}{m_D^2}\right)^2$$

$$\mathcal{B}(D_s \to l\nu_l) = \frac{G_F^2 \tau_{D_s}}{8\pi} |V_{cs}|^2 f_{D_s}^2 m_l^2 m_{D_s} \left(1 - \frac{m_l^2}{m_{D_s}^2}\right)^2$$

CLEO, Belle, BaBar: 5%-6% precision

- Semileptonic decay widths

$$\frac{d\Gamma(D \to \pi l\nu)}{dq^2} = \frac{G_F^2}{24\pi^3} |\vec{p}_\pi|^3 |V_{cd}|^2 |f_+^{D\pi}(q^2)|^2 + \mathcal{O}(m_{e,\mu}^2 \, |f_0^{D\pi}(q^2)|^2)$$

$$\frac{d\Gamma(D \to K l\nu)}{dq^2} = \frac{G_F^2}{24\pi^3} |\vec{p}_K|^3 |V_{cs}|^2 |f_+^{DK}(q^2)|^2 + \mathcal{O}(m_{e,\mu}^2 \, |f_0^{DK}(q^2)|^2)$$

# CKM angles $|V_{cd}|$ and $|V_{cs}|$

- Use FLAG-2 estimates/averages

- J.L.Rosner & S.Stone, arXiv:1201.2401

$$f_D|V_{cd}| = 46.40 \pm 1.98\mathrm{MeV} \qquad f_{D_s}|V_{cs}| = 253.1 \pm 5.3\mathrm{MeV}$$

- $N_f$=2 $\quad |V_{cd}| = 0.2231(95)(75) \qquad |V_{cs}| = 1.012(21)(28)$
- $N_f$=2+1 $\quad |V_{cd}| = 0.2218\underbrace{(35)}_{\text{lattice}}\underbrace{(95)}_{\text{non-lattice th. \& exp.}} \qquad |V_{cs}| = 1.018(11)(21)$

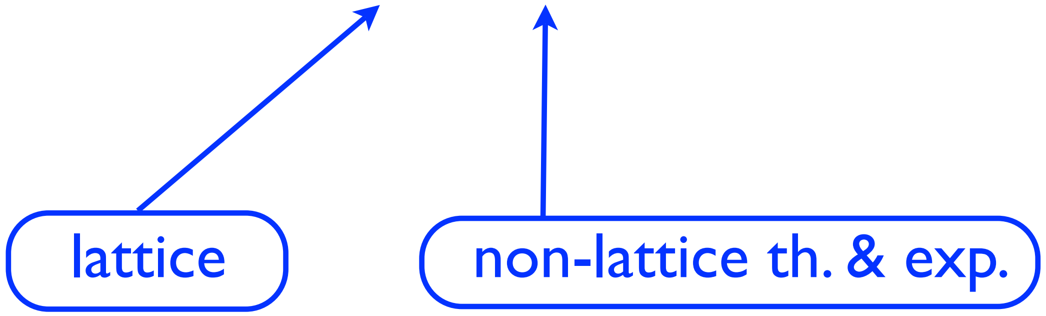
- $N_f$=2+1: lattice errors (HPQCD dominated) much smaller than other errors

# CKM angles $|V_{cd}|$ and $|V_{cs}|$

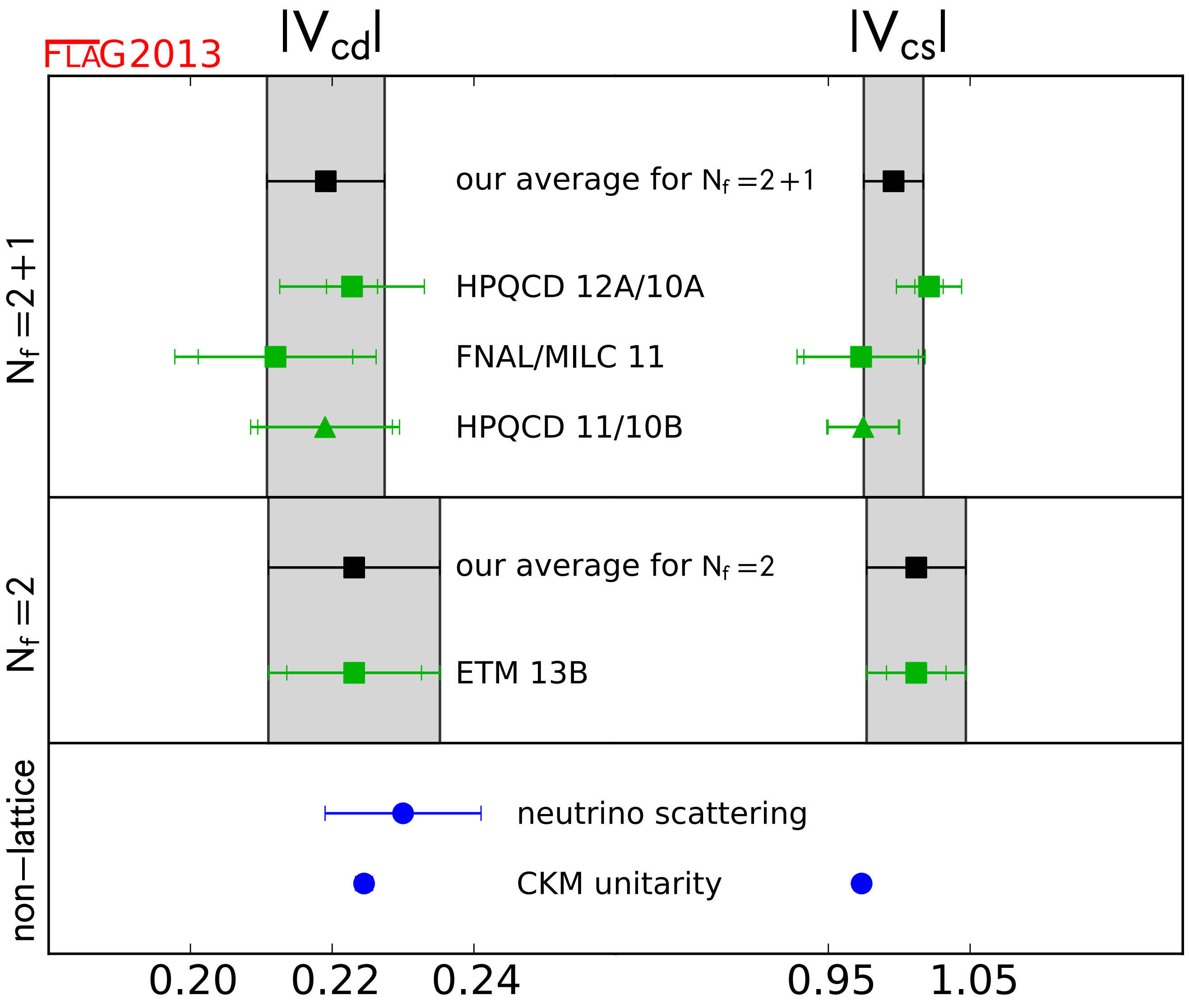
- HFAG: Y. Amhis et al., arXiv:1207.1158

$$f_+^{D\pi}(0)|V_{cd}| = 0.146 \pm 0.003 \qquad f_+^{DK}(0)|V_{cs}| = 0.728 \pm 0.005$$

- $N_f$=2+1 $\quad |V_{cd}| = 0.2192(95)(45) \qquad |V_{cs}| = 0.9746(248)(67)$



# CKM angles $|V_{cd}|$ and $|V_{cs}|$



leptonic

semileptonic

leptonic 🟩

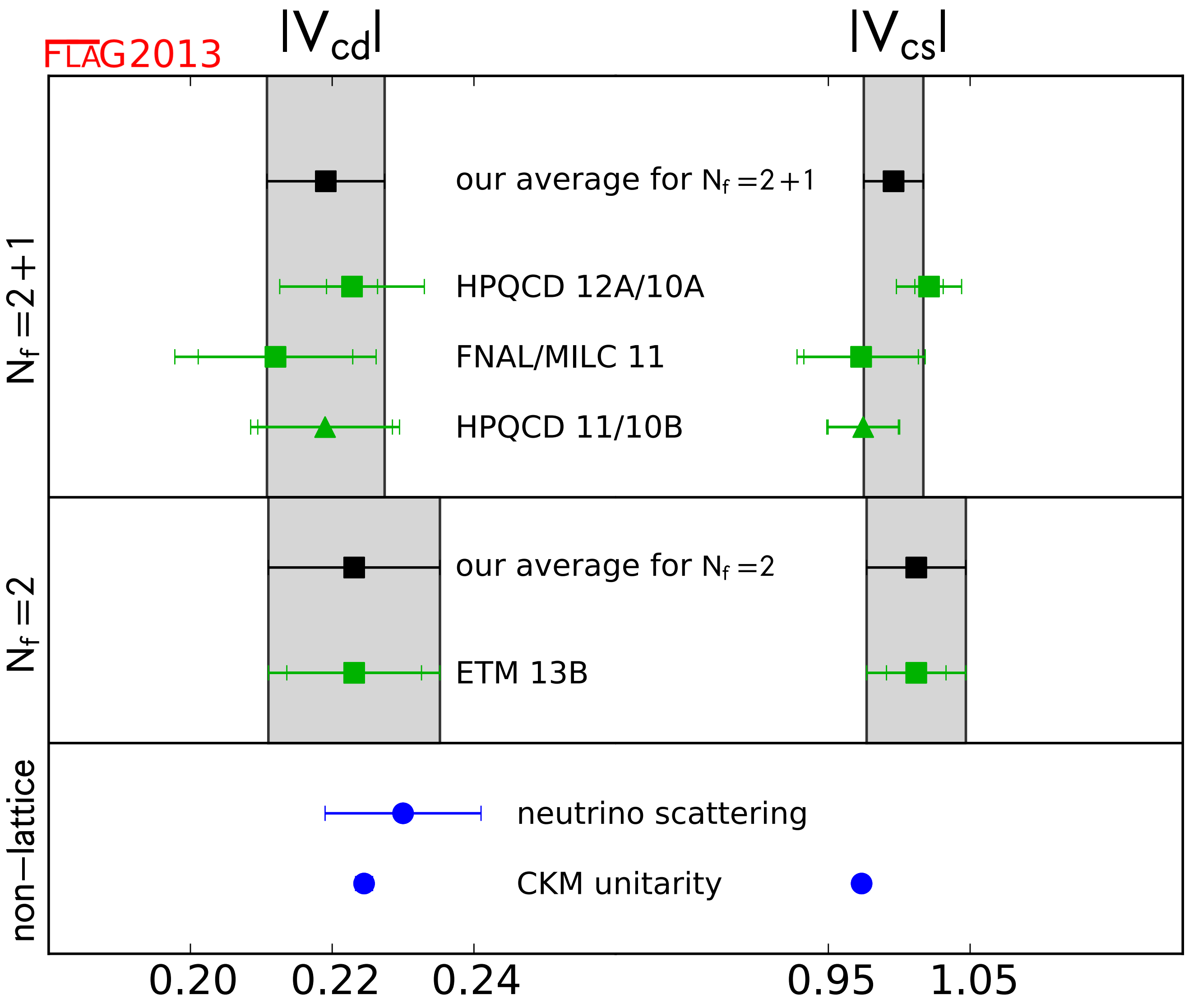
semileptonic ▲

PDG J.Beringer et al., Phys.Rev.D86(2012) 01000

J.L.Rosner & S.Stone arXiv:1201.2401

- *Vcd:* agreement
- *Vcs:* 1.2σ between leptonic/semileptonic; 1.9σ between leptonic and CKM-unit. (driven by HPQCD result; but note that the lattice estimate at $N_f$=2+1 supported by that at $N_f$=2)

# CKM angles $|V_{cd}|$ and $|V_{cs}|$



leptonic 🟩

semileptonic ▲

$$|V_{cd}| \;=\; 0.2191(83) \qquad\qquad |V_{cs}| \;=\; 0.996(21) \qquad\qquad N_f = 2+1$$

- 2nd row unitarity agrees with SM (independently of $|Vcb| = O(10^{-2})$):

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 \;=\; 0.04(6) \qquad\qquad N_f = 2+1$$

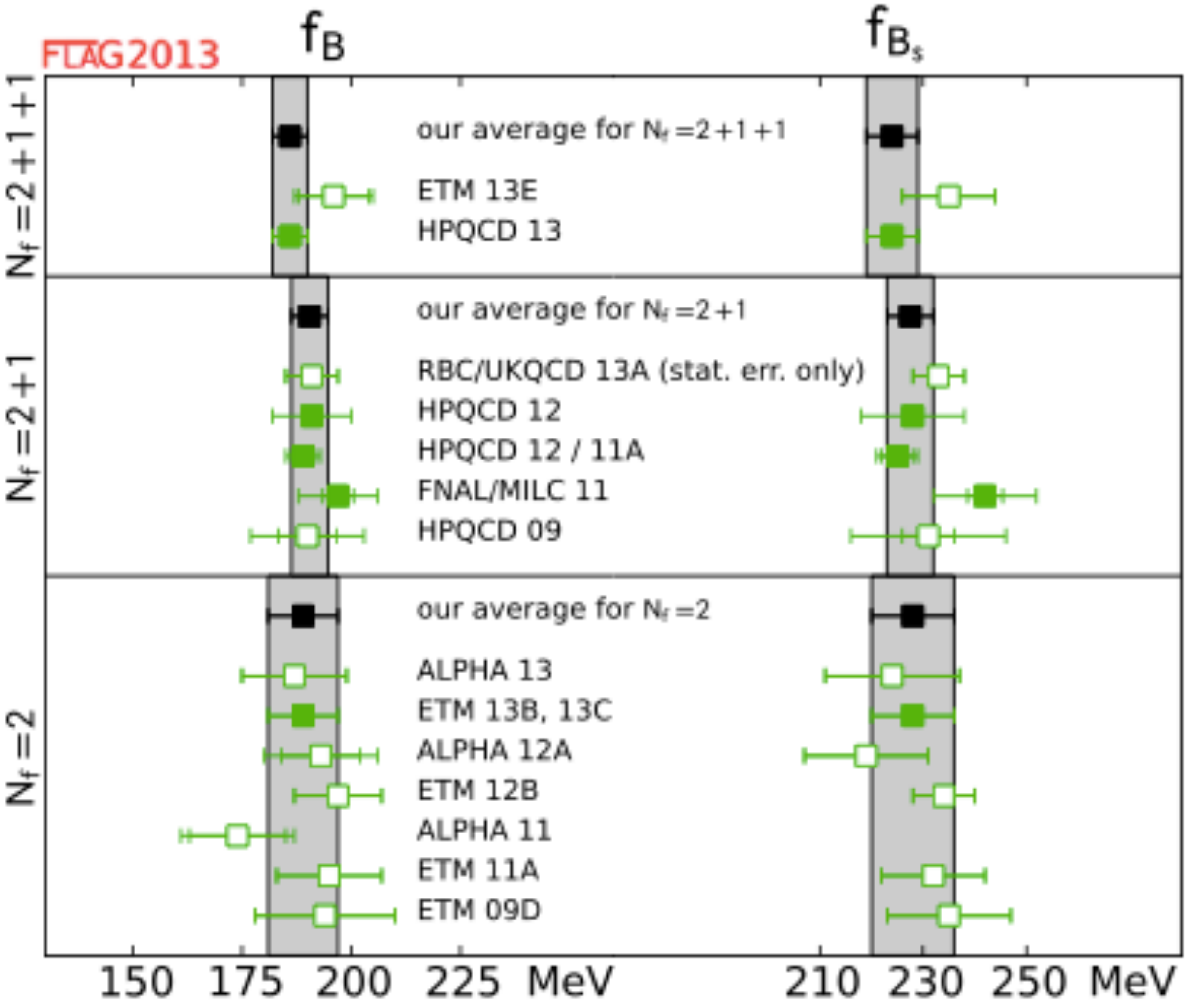
# Bottom Physics

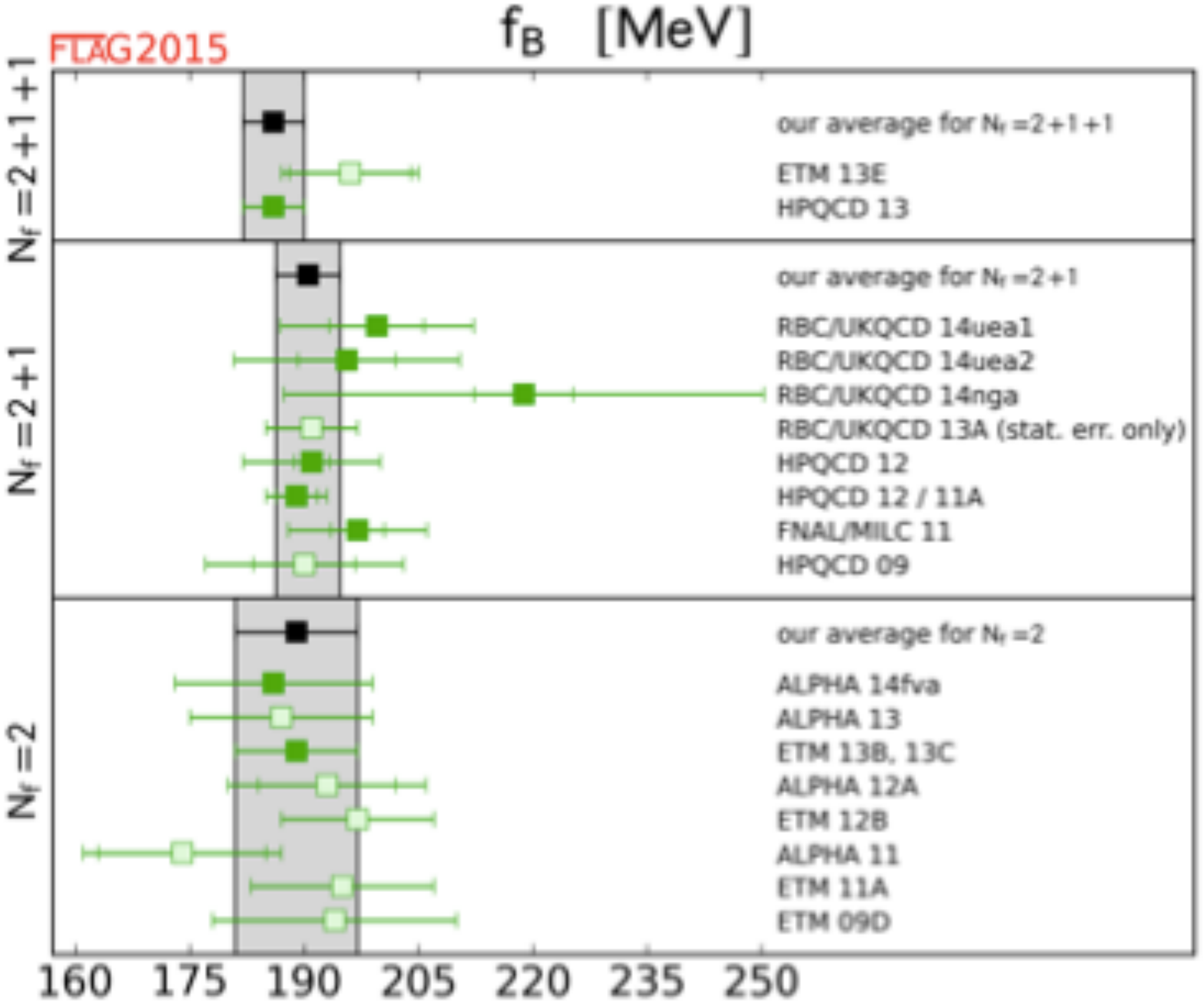
$f_B$, $f_{Bs}$, $f_+(0)$, $|V_{ub}|$

# Generalities

- In present day computations $m_b \sim \Lambda_{UV} \sim 1/a$ so this mass cannot be simulated directly. Therefore:
  - introduce effective theories (HQET, NRQCD) and a systematic expansion in $1/m_b$ (non-relativistic treatment);
  - simulate with lighter than physical bottom masses of $O(1/m_c)$ and extrapolate to physical point $m_b$, or interpolate to HQET point.
- This results to new problems (matching of HQET to QCD, renormalization, control of discretization effects).
- There are less results than in light-quark Physics; situation is rapidly improving.

# Leptonic decay constants $f_B$ and $f_{Bs}$



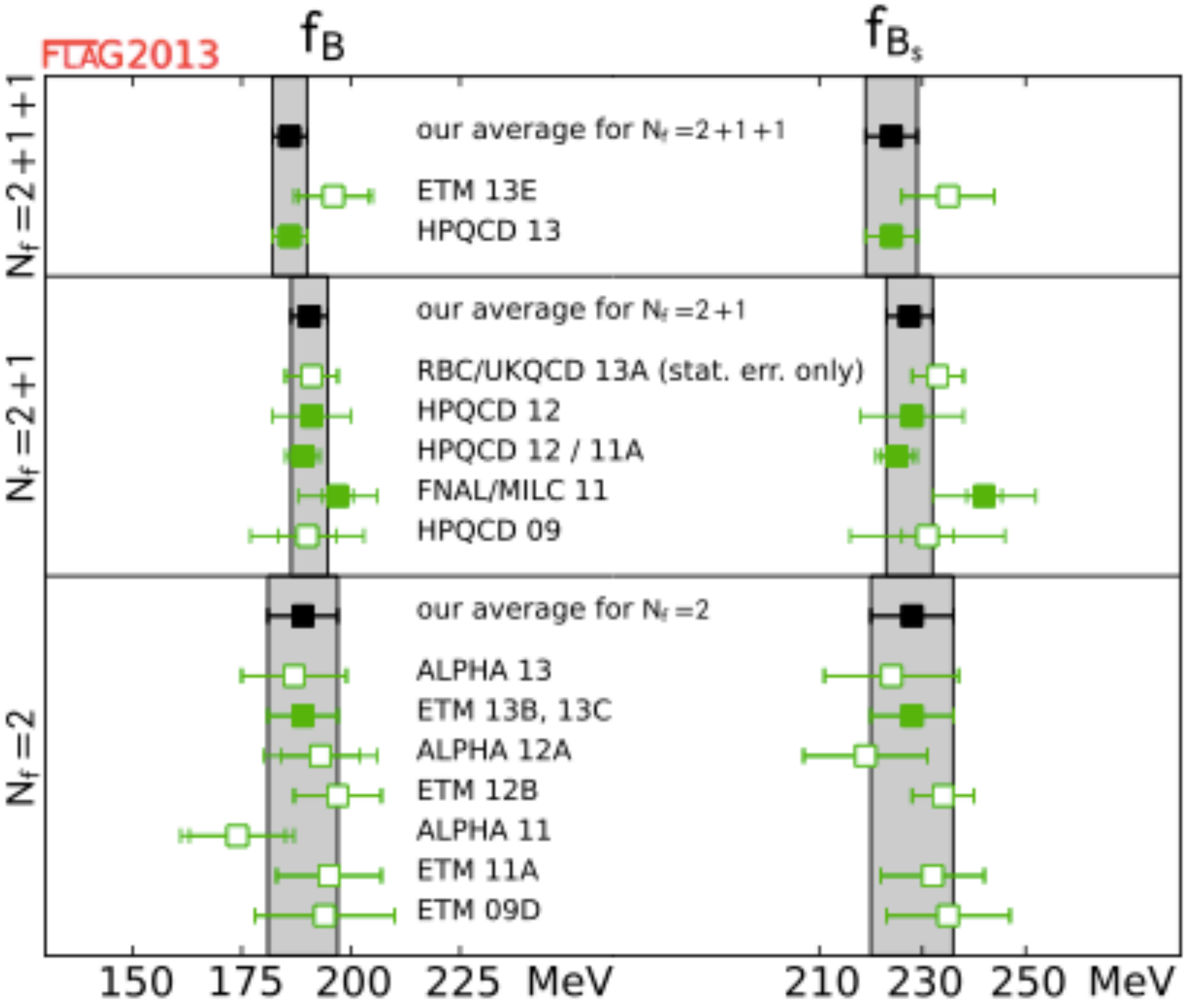


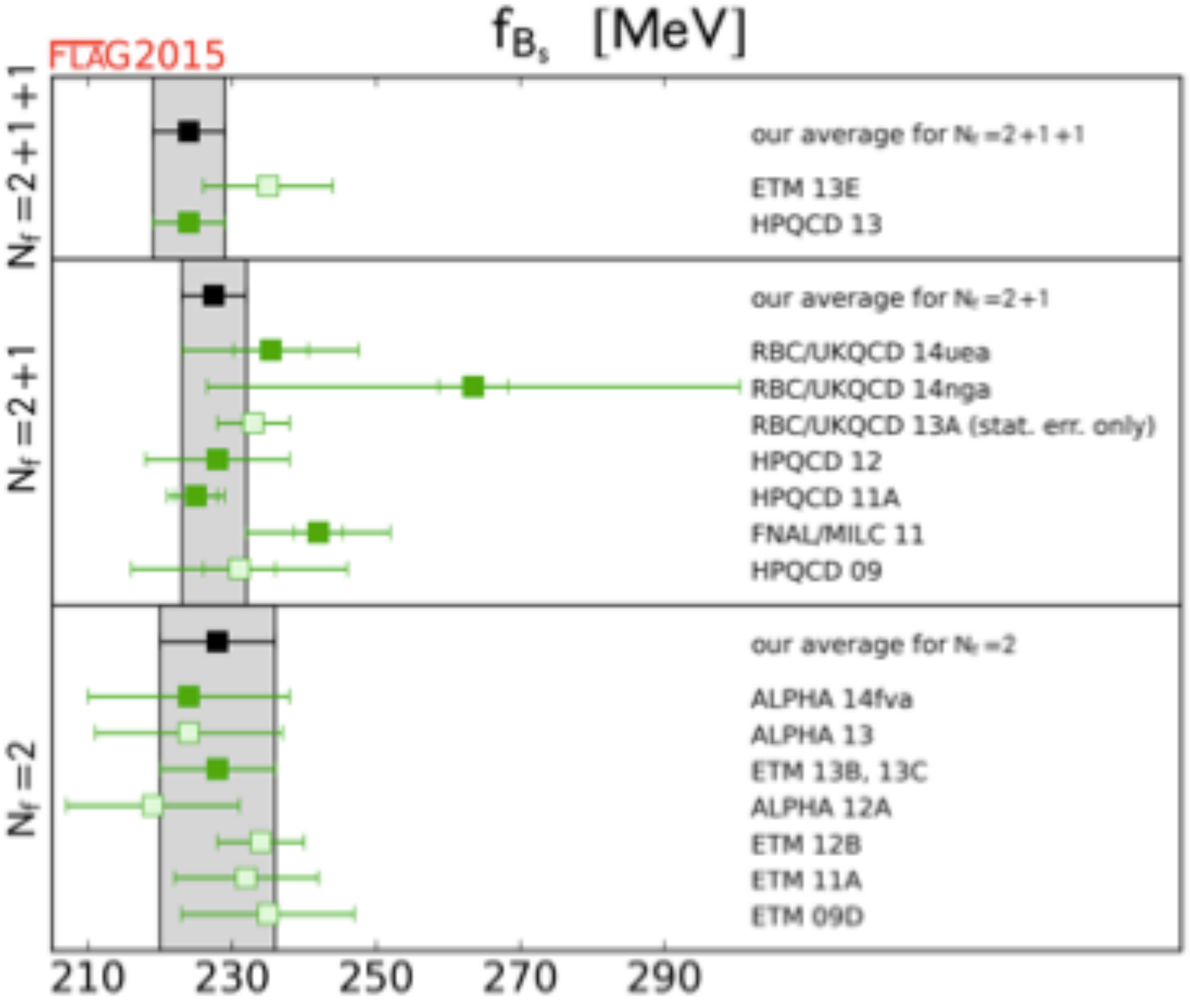
$$f_B = 186(4)\ \text{MeV} \qquad N_f = 2+1+1 \qquad \text{unchanged}$$

$$f_B = 190.5(4.2)\ \text{MeV} \qquad N_f = 2+1 \qquad f_B = 191.8(4.6)\ \text{MeV}$$

$$f_B = 189(8)\ \text{MeV} \qquad N_f = 2 \qquad f_B = 188(7)\ \text{MeV}$$

# Leptonic decay constants $f_B$ and $f_{Bs}$



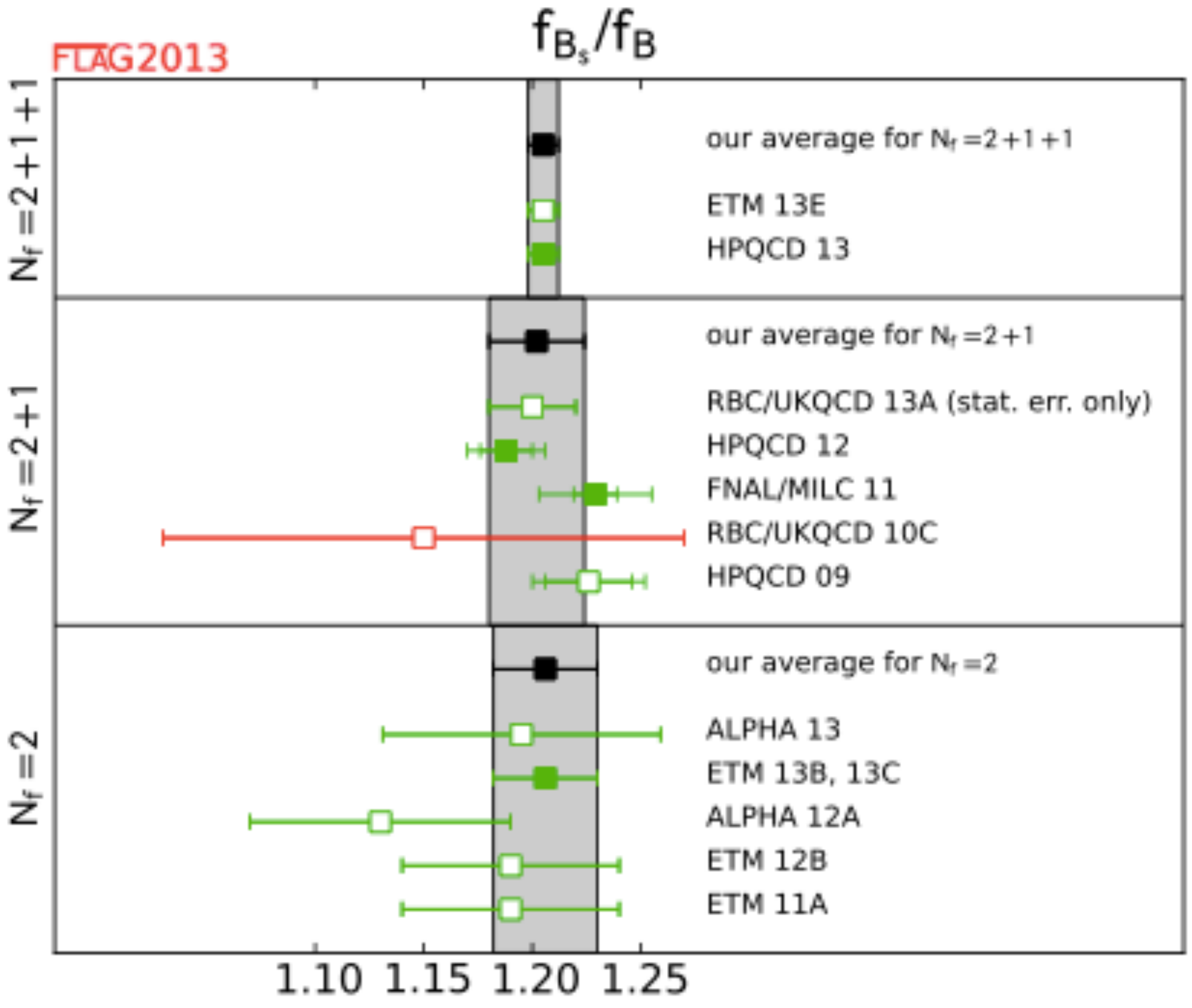


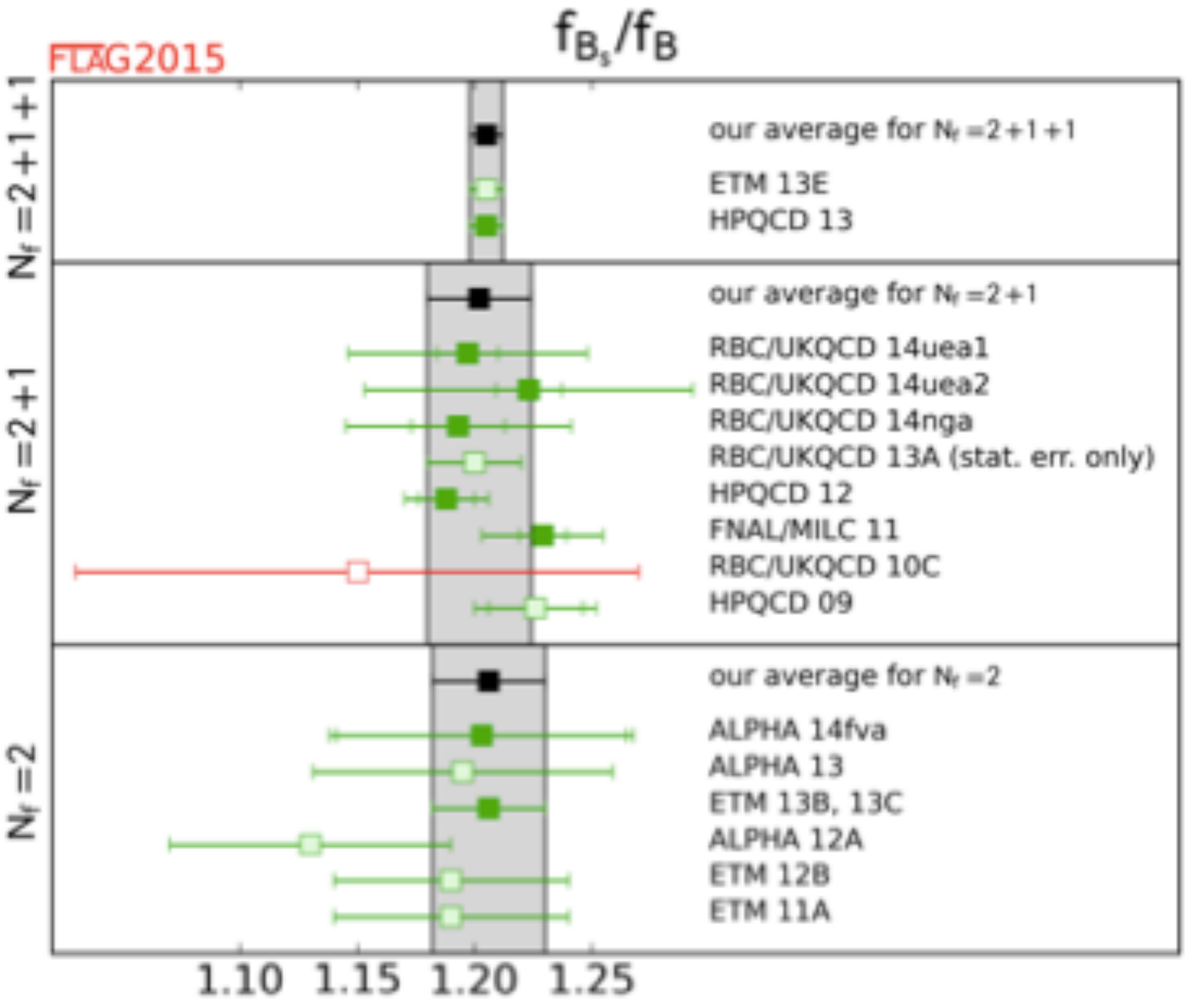
$$f_{Bs} = 224(5)\ \text{MeV} \qquad N_f = 2+1+1 \qquad \text{unchanged}$$

$$f_{B_s} = 227.7(4.5)\ \text{MeV} \qquad N_f = 2+1 \qquad f_{B_s} = 228.4(3.7)\ \text{MeV}$$

$$f_{B_s} = 228(8)\ \text{MeV} \qquad N_f = 2 \qquad f_{B_s} = 227(7)\ \text{MeV}$$

# Leptonic decay constants $f_B$ and $f_{Bs}$





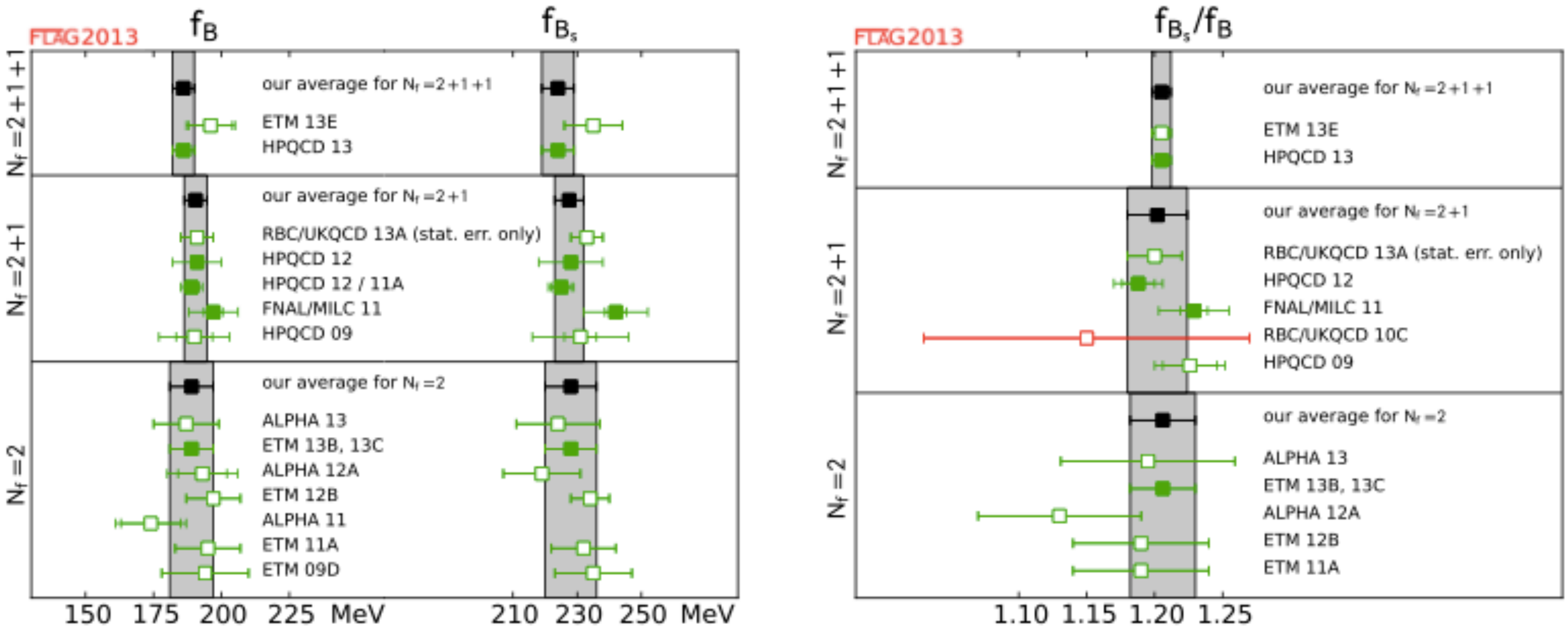
$$\frac{f_{B_s}}{f_B} = 1.205(7)\ \mathrm{MeV} \qquad N_f = 2+1+1$$

unchanged

$$\frac{f_{B_s}}{f_B} = 1.202(22)\ \mathrm{MeV} \qquad N_f = 2+1 \qquad \frac{f_{B_s}}{f_B} = 1.201(16)\ \mathrm{MeV}$$

$$\frac{f_{B_s}}{f_B} = 1.206(24)\ \mathrm{MeV} \qquad N_f = 2 \qquad \frac{f_{B_s}}{f_B} = 1.206(23)\ \mathrm{MeV}$$

# Leptonic decay constants $f_B$ and $f_{Bs}$



- *NB:*
  - Most results, obtained with degenerate light quarks, refer to average decay constants for $B^+$ and $B^0$. Some collaborations (FNAL/MILC, HPQCD) have started giving distinct results (they differ by about 2%). As errors decrease with time, collaborations should start giving $B^+$ and $B^0$ results separately.

# CKM angle $|V_{ub}|$

- Branching ratio for $B^+ \to \tau^+\nu_\tau$ measured by Belle and BaBar

BaBar: B. Aubert et al., Phys.Rev.D81 (2010) 051101; J. Lees et al., Phys.Rev.D88 (2013) 031102

Belle: K.Hara et al., Phys.Rev.D82(2010) 071101; I. Adachi et al., Phys.Rev.Lett.110 (2013)131801

$$
\begin{array}{lcll}
|V_{ub}| & = & 4.21(53)(18)\cdot 10^{-3} & N_f = 2 \\
|V_{ub}| & = & 4.18(52)(9)\cdot 10^{-3} & N_f = 2+1 \\
|V_{ub}| & = & 4.28(53)(9)\cdot 10^{-3} & N_f = 2+1+1
\end{array}
$$

- 1st error: experiment
- 2nd error: lattice

- Branching ratio for $B^0 \to \pi^- l^+\nu$ ratio measured by Belle and BaBar

BaBar: J. Lees et al., Phys.Rev.D86 (2012) 092004; J. Lees et al., Phys.Rev.D88 (2013) 031102

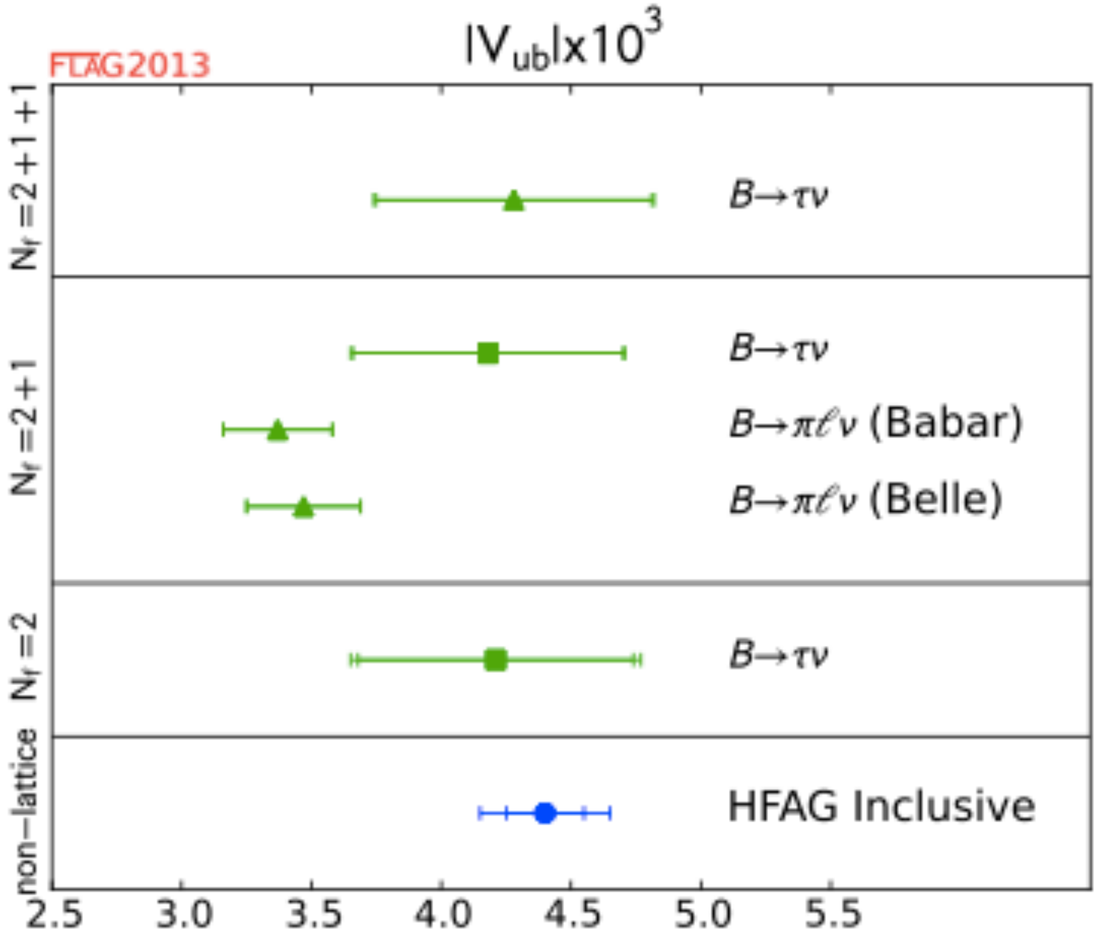
Belle: H.Ha et al., Phys.Rev.D83(2011) 071101; I. Adachi et al., Phys.Rev.Lett.110 (2013)131801

- Lattice form factor estimates are from FNAL/MILC (2008) and HPQCD (2006) for $N_f$=2+1

$$
\begin{array}{lclll}
|V_{ub}| & = & 3.37(21)\cdot 10^{-3} & N_f = 2+1 & \text{BaBar} \\
|V_{ub}| & = & 3.47(22)\cdot 10^{-3} & N_f = 2+1 & \text{Belle}
\end{array}
$$

Results reported separately, as experimental correlations cannot be properly taken into account

# CKM angle $|V_{ub}|$



- Lattice central value from $B^+ \longrightarrow \tau^+\nu_\tau$ lies between HFAG (inclusive) and lattice from $B^0 \longrightarrow \pi^- l^+\nu$ (inclusive); due to big error it agrees within ~ 1.5σ with other determinations

- Tension ~ 3σ between HFAG (inclusive) and lattice from $B^0 \longrightarrow \pi^- l^+\nu$ (inclusive)

- Situation still unclear; too much spread

- lattice improvements expected soon for the semi-leptonic $B^0 \longrightarrow \pi^- l^+\nu$ determination of $|V_{ub}|$

- Belle II data (as from 2016 ) will improve leptonic $B^+ \longrightarrow \tau^+\nu_\tau$ determination of $|V_{ub}|$

# Conclusions

- Lattice is now credible and competes with the accuracy of experiments (in recent years we moved from 5% to 1%-2%).

- It is the responsibility of the lattice community to provide experimentalists and non-lattice theorists with a review of phenomenologically relevant lattice results with conservative error estimates.

- FLAG rates lattice output according to some quality criteria, performs averages or proposes estimates and is sometimes trying to push the analysis beyond that (e.g. CKM unitarity), stopping short of a UT analysis.

- FLAG has entered its third phase with a larger group and a slightly amplified Physics scope (charm and bottom quark masses, $B_K$ beyond SM).

- The initiative is gaining momentum and the support of the lattice community as well as recognition in the wider high energy community.