Leptonic decays and mixing of the *D* and *B* mesons

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Outline

During the last decade LQCD is giving important contributions to heavy quark CKM me & UT

***D*-mesons** → 2nd row: $\delta(1 - |V_{cd}|^2 - |V_{cs}|^2 - |V_{cb}|^2) \sim$ 4% (~ 2σ discrepancy)

$[\sigma_{lat} < \sigma_{expt}]$

***B*-mesons** →

- *B*-mixing ⇨ $\Delta m_{d/s}$, $\Delta m_s / \Delta m_d$ ⇔ $|V_{td}|$, $|V_{ts}|$, $|V_{ts} / V_{td}|$ $[\sigma_{lat} >> \sigma_{expt}]$
- *B*-decays ⇨ $|V_{ub}|$, $|V_{cb}|$ $[\sigma_{lat} < \sigma_{expt}$ for leptonic decay$]$
- $\varepsilon_K \sim |V_{cb}|$ (^4)

& Neutral *D* and *B*-meson mixing ➡ New Physics scale estimates

Outline

- In Flavour Physics we witness a continuous and 'day-after-day' interplay between

  Experimental achievements (of high or even very high precision)

  Theoretical ideas & progress in studying processes dominated by
  Non Perturbative dynamics

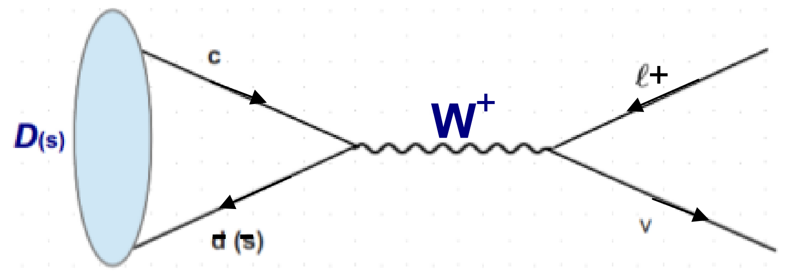
- **There is a huge experimental investment (and upgrades)** in flavour physics (**BESIII, BelleII, LHCb**) with horizon ~ 2020.

- **Full capitalisation of the experimental program** in the quark flavour sector requires continuous improvement in the precision of theoretical (lattice) calculations

D – leptonic decays

Charmed decay constants

- Leptonic Decays : $D^+_{(s)} \longrightarrow \ell^+\nu$
- $\Gamma(D^+_{(s)} \to \ell^+\nu) = \frac{G_F^2}{8\pi} m_\ell^2 M_{D(s)} \left(1 - \frac{m_\ell^2}{M^2_{D(s)}}\right)^2 f^2_{D(s)} |V_{cd(s)}|^2$ (to lowest order)
- Knowledge of $f_{D(s)} \Longrightarrow$ necessary to extract $|V_{cd(s)}|$



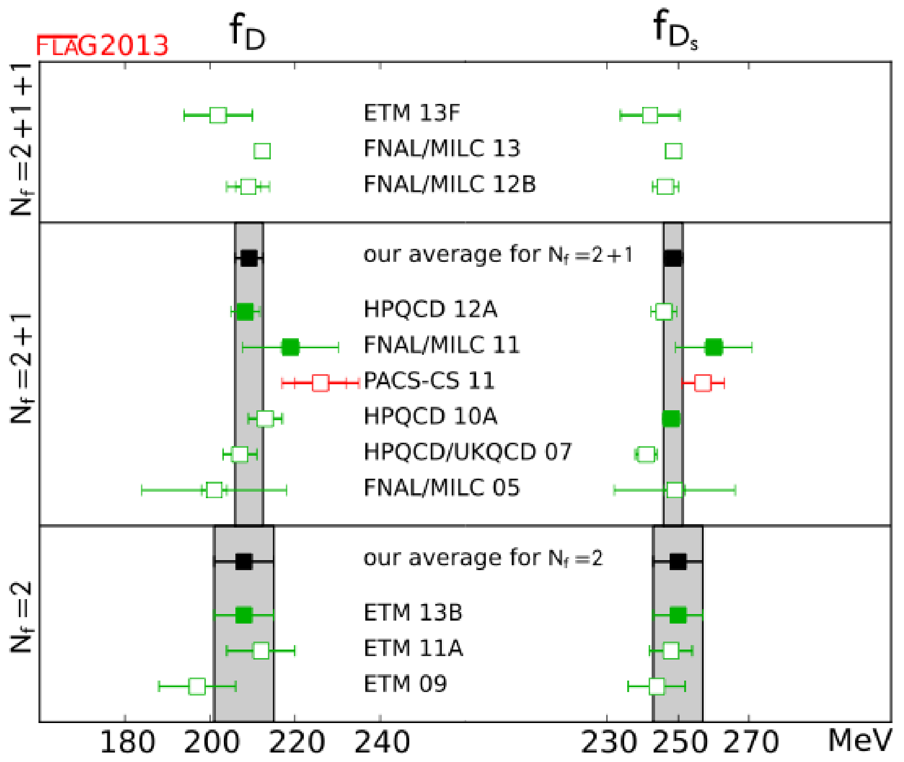
[$D_s^+ \to \mu^+\nu, \tau^+\nu$: CLEO-c, Belle, BaBar]
[$D \to \mu^+\nu, \tau^+\nu$: CLEO-c, BESIII]

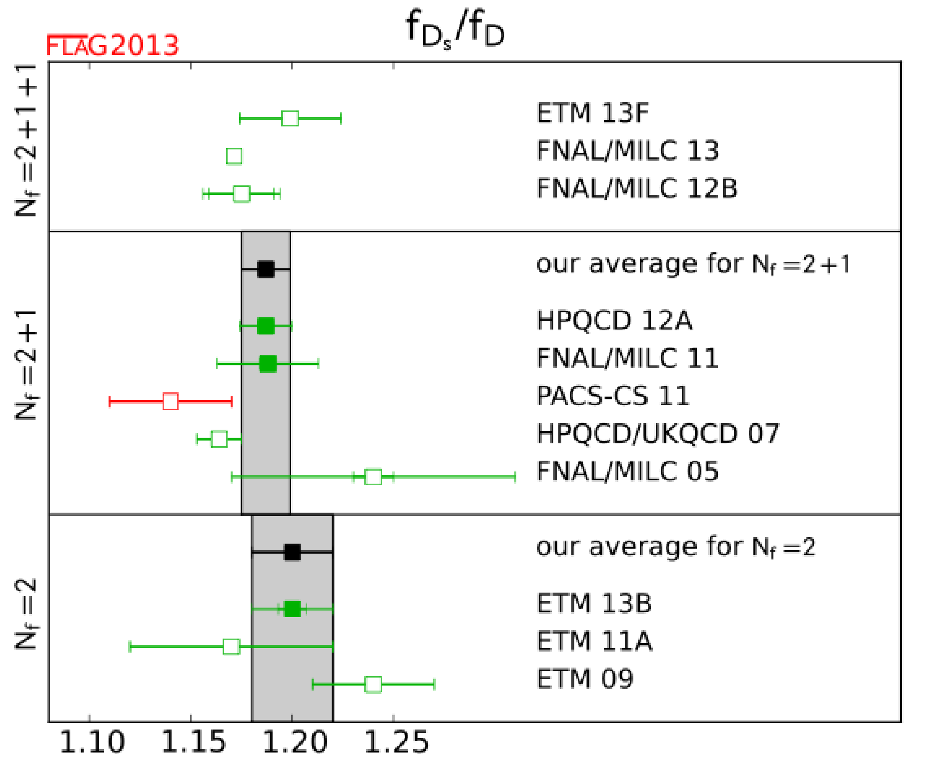
Precision $[\mathcal{B}(D^+_{(s)} \longrightarrow \ell^+\nu), \ell \equiv \mu, \tau]$: $4 - 5.5$ % → (> 2018; BelleII; BESIII) ~ 2 %

Presision for decay constants $\sim$ 1 %

- On the lattice it is straightforward to compute decay constants $\langle 0|\bar{d}(\bar{s})\gamma_\mu\gamma_5 c|D_{(s)}(p)\rangle = f_{D(s)} p_\mu$
- Cutoff effects effects driven by $(am_c) \lesssim 0.3$
- Relativistic action computations seem safe for charm-light quantities

Comparison of results from FLAG-2013 (1310.8555)





	$N_f = 2$	$N_f = 2+1$
f_D (MeV)	208(7)	209.2(3.3)
f_{Ds} (MeV)	250(7)	248.6(2.7)
f_{Ds}/f_D	1.20(2)	1.187(12)

FLAG-2103 Averages

Recent computations

$N_f = 2+1+1$

- **FNAL/MILC** (HISQ) (`1407.3772`)
  4 $a \in [0.06, 0.15]$ fm @M_π with $(M_\pi L) \in [3.2, 3.9]$
- **ETMC** (tmWilson) (`1411.7908`)
  3 $a \in [0.06, 0.09]$ fm $M_{ps}^{\min} \simeq 210$ MeV with $(M_{ps}^{\min} L) \simeq 3.2$

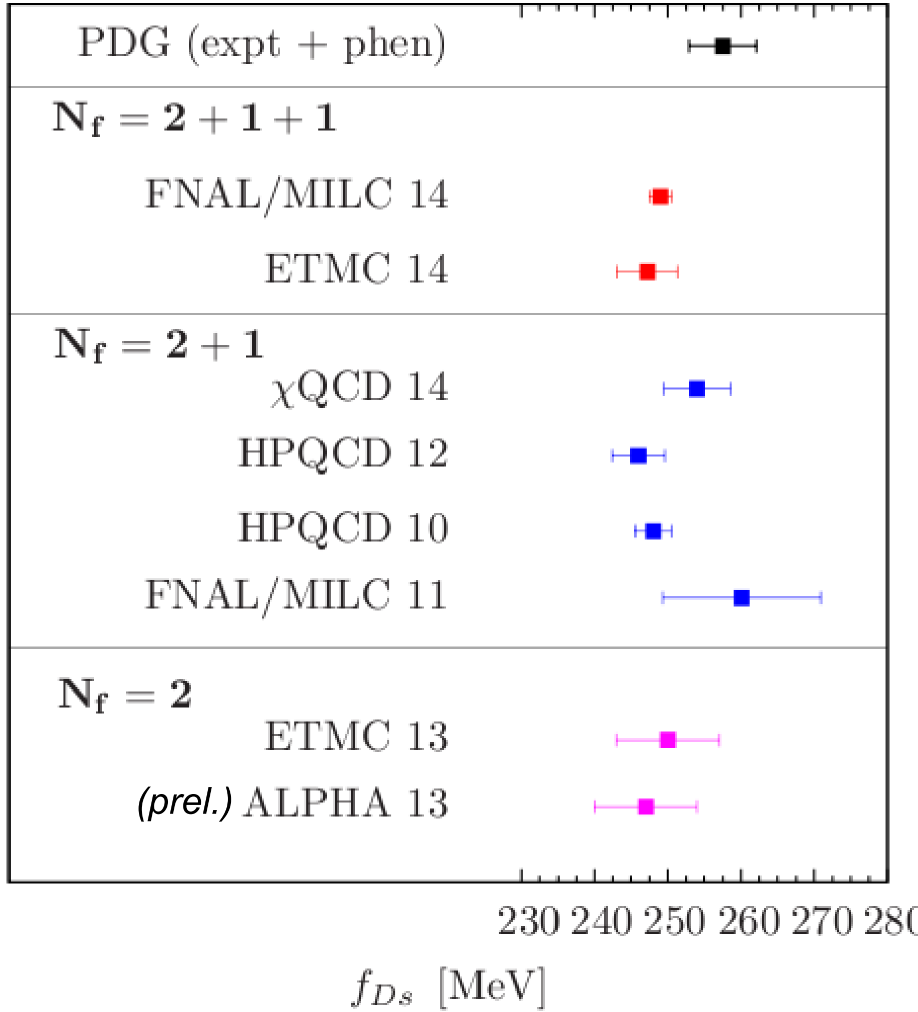
$N_f = 2+1$

- **χQCD** (DW+OV) (`1410.3343`)
  2 $a = \{0.09, 0.11\}$ fm $M_{ps}^{\min} \simeq 290$ MeV with $(M_{ps}^{\min} L) \simeq 4$
- **RBC/UKQCD** (DW)
  Computation @ phys. point, *work in progress* (`1502.0084`) & J.T.Tsang at LAT2015

$N_f = 2$

- **ALPHA** (Wilson) (`1312.7693`) (LAT13)
  2 $a \in [0.05, 0.07]$ fm $M_{ps}^{\min} \simeq 190$ MeV with $(M_{ps}^{\min} L) \simeq 4$
- **TWQCD** (DW) (`1404.3648`)
  1 $a = 0.06$ fm $M_{ps}^{\min} \simeq 259$ MeV with $(M_{ps}^{\min} L) \simeq 2$
- **ETMC** (tmWilson) (`1507.05068`)
  1 $a = 0.09$ fm @M_π with $(M_\pi L) \simeq 3.0$

Comparison of results for f_{Ds}



(refs not included in FLAG-2)

(1310.8555)

(1407.3772)

(1411.7908)

(1410.3343)

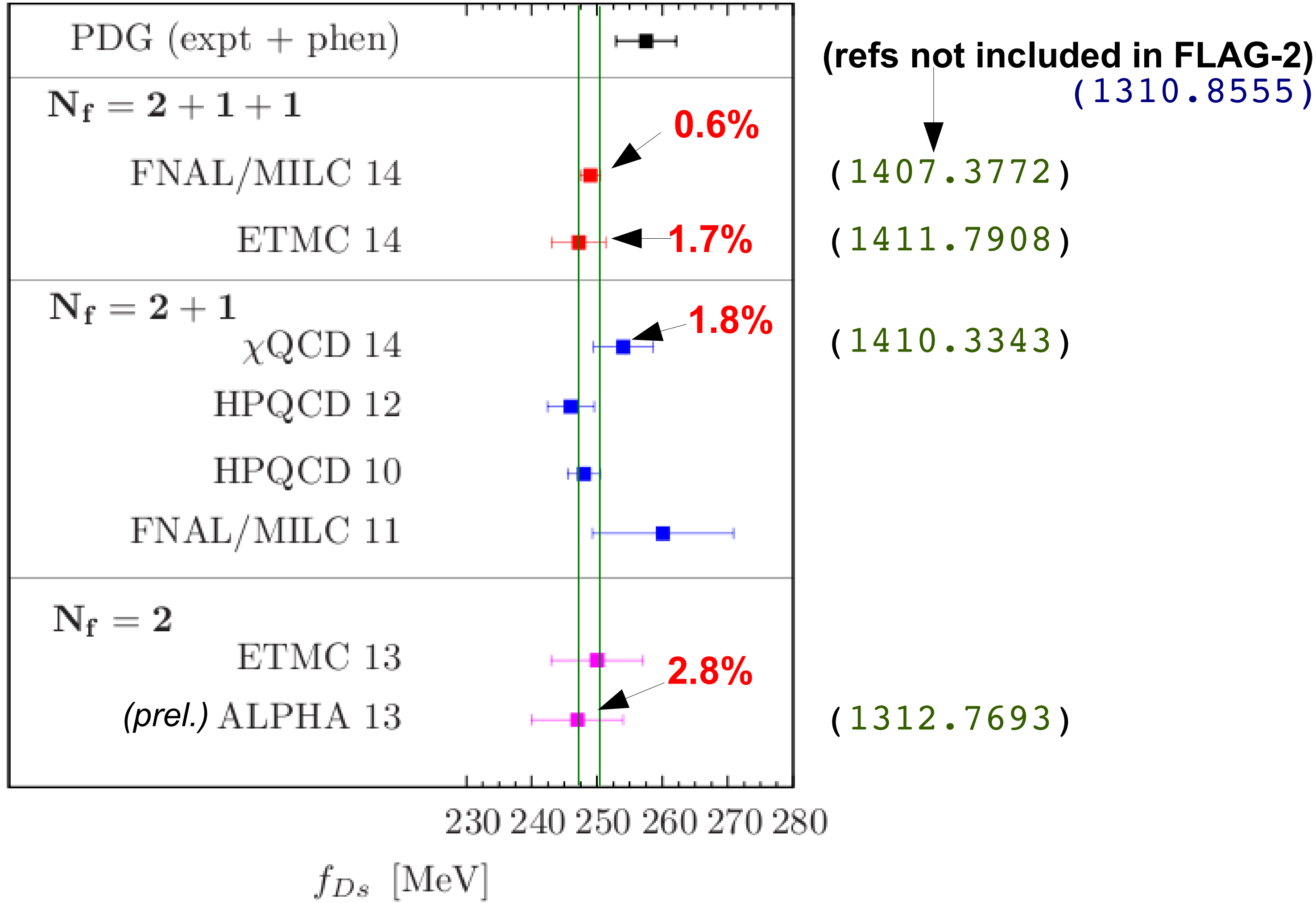
(1312.7693)

- Comparison plot with most recent results obtained at *CL* from at least two lat. spacings

+ TWQCD - Nf=2 (DW); 1a / 0.06 fm (1404.3648)

+ ETMC - Nf=2 (tmW); 1a / 0.09 fm @ phys. point (1507.05068)

Comparison of results for f_{Ds}



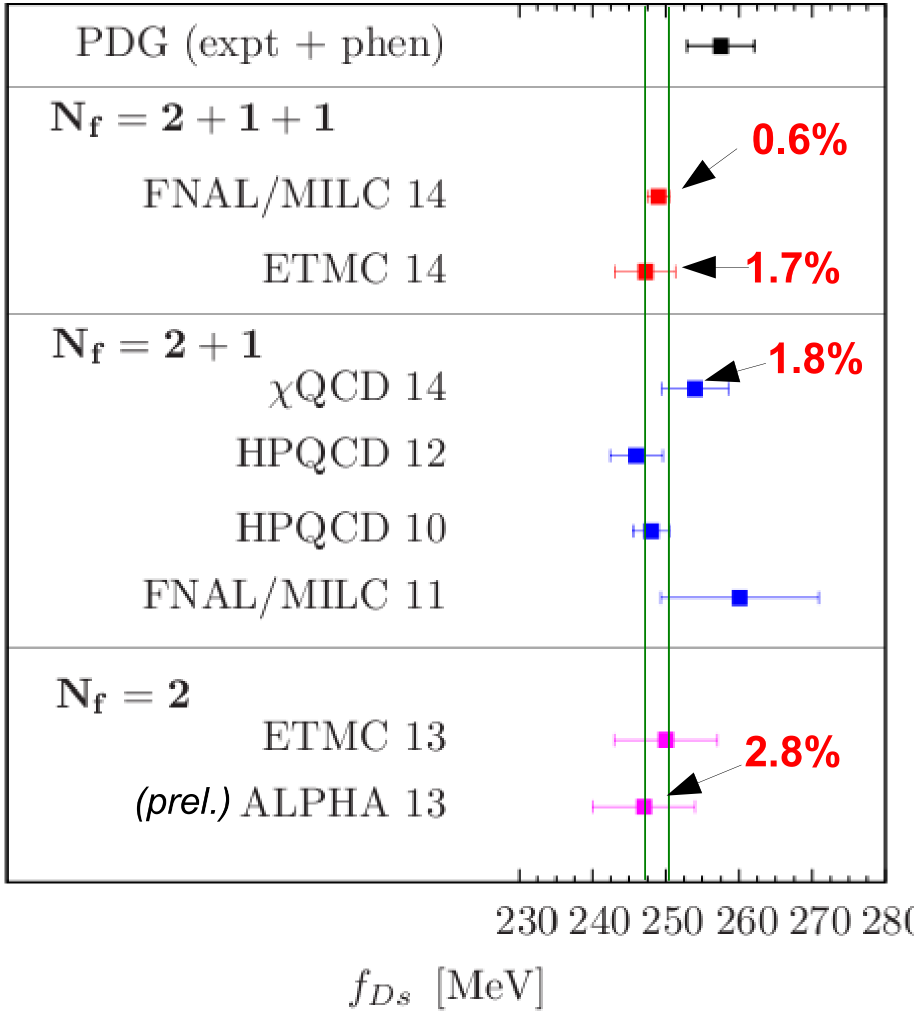
- Comparison plot with most recent results obtained at *CL* from at least two lat. spacings

➡ Vertical (green) lines follow the "FNAL/MILC 14" uncertainty

+ TWQCD - Nf=2 (DW); 1a / 0.06 fm (1404.3648)
+ ETMC - Nf=2 (tmW); 1a / 0.09 fm @ phys. point (1507.05068)

- **FNAL/MILC 14** result: remarkable precision 0.6%

Comparison of results for f_{Ds}



(refs not included in FLAG-2)
(1310.8555)

(1407.3772)

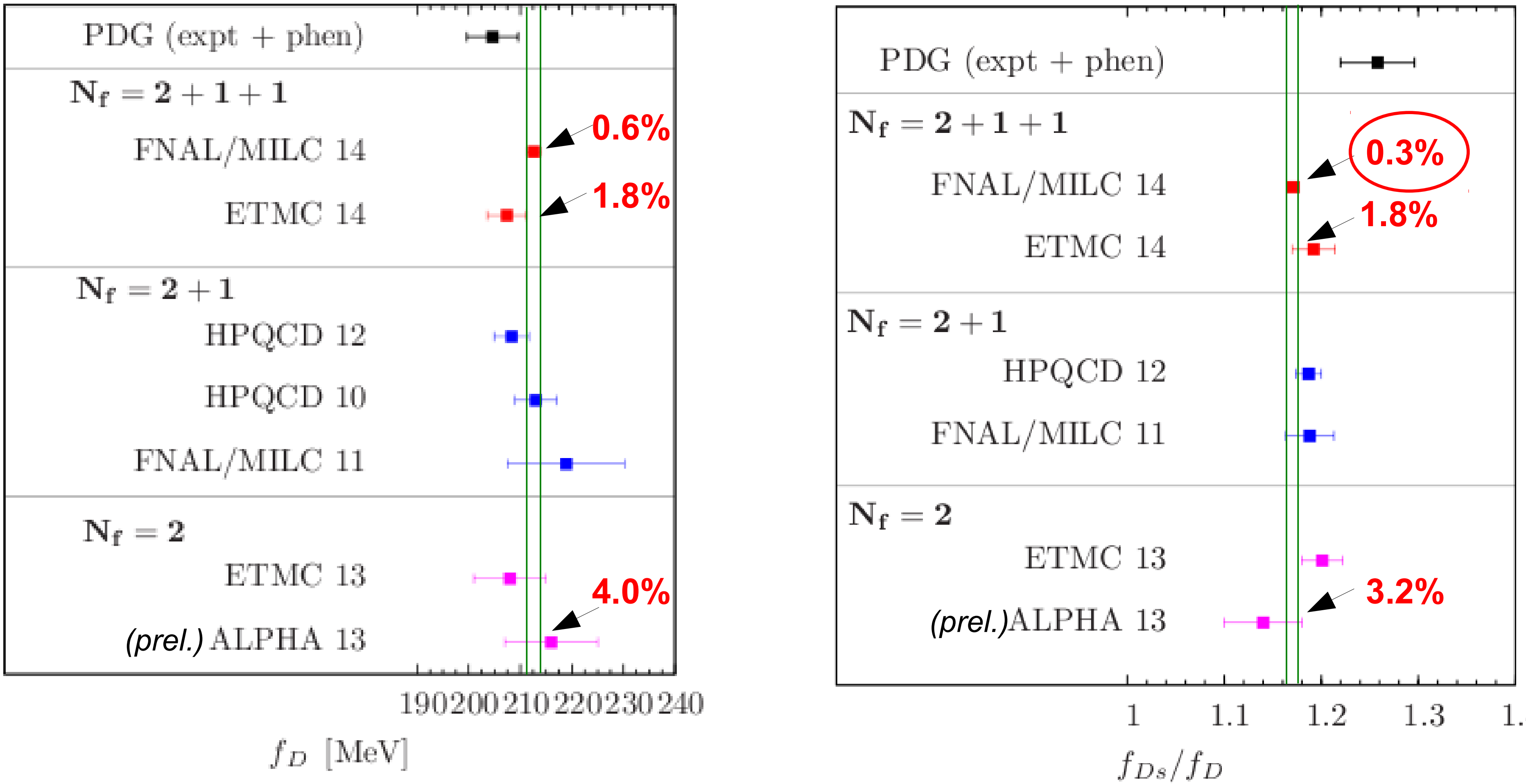
(1411.7908)

(1410.3343)

(1312.7693)

- All lattice results are in very good agreement
- Within ~2% no detectable dependence on the number of flavours
- 'Tension' of ~ 2 σ between more recent Lattice (with reduced errors) and PDG -> expt. + unit. assumptions: $|V_{cs}| \simeq |V_{ud}| - |V_{cb}|^2/2, |V_{ud}| = 0.97425(22), |V_{cb}| = 0.04$
- Lattice precision in the D_s - leptonic decay *better* than experimental one

Comparison of results for f_D and f_{Ds}/f_D

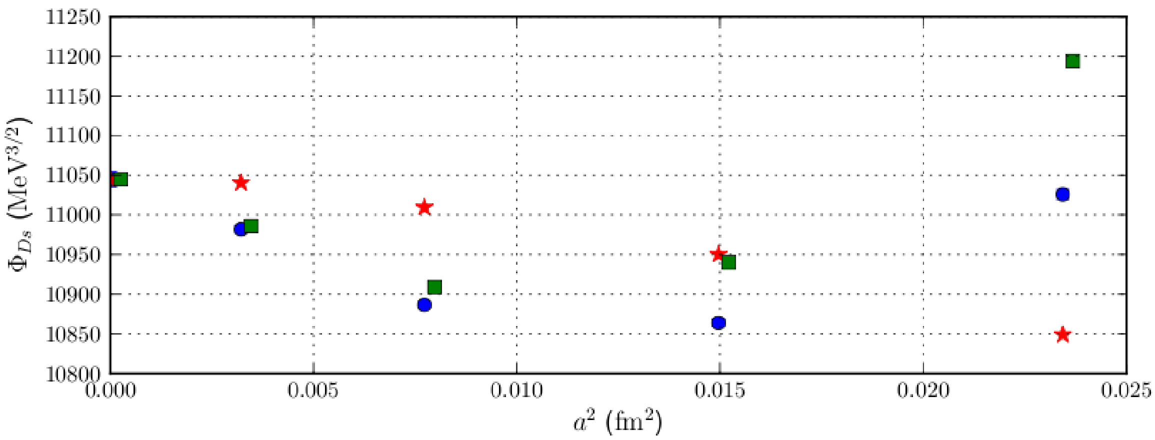


- Comparison plot with most recent results obtained at *CL* from at least two lat. spacings
  - \+ TWQCD - Nf=2 (DW); 1a / 0.06 fm (1404.3648)
  - \+ ETMC - Nf=2 (tmW); 1a / 0.09 fm @ phys. point (1507.05068)

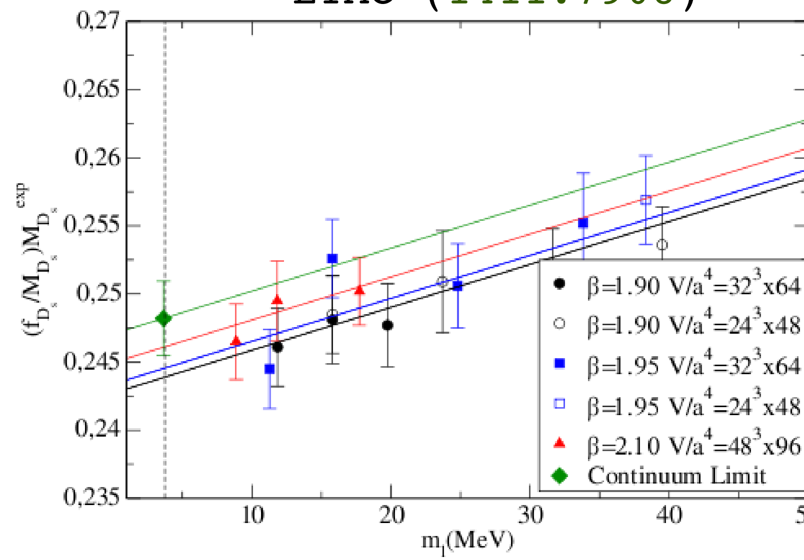
➡ Vertical (green) lines follow the “FNAL/MILC 14” uncertainty in both plots

- Are discretisation errors under control?

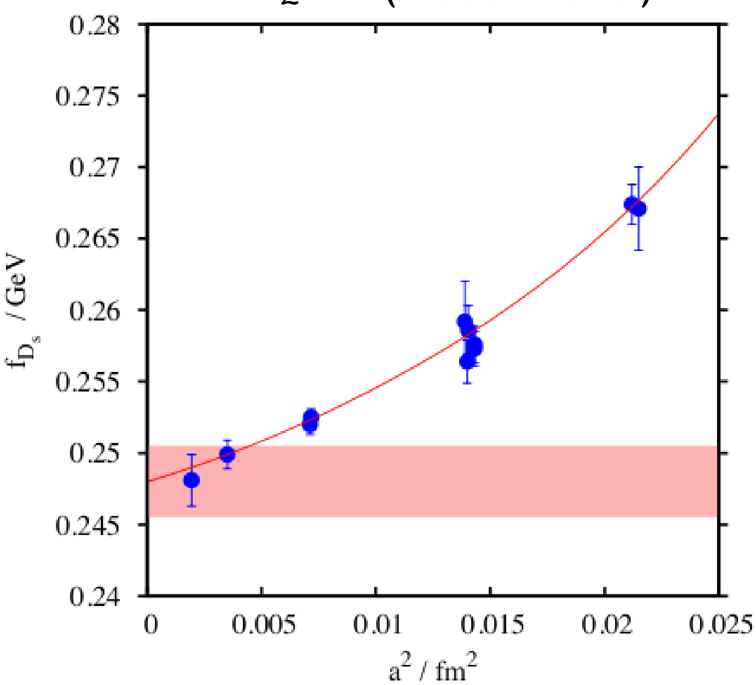
FNAL/MILC (1407.3772)



ETMC (1411.7908)



HPQCD (1008.4818)



→ At a~0.09 fm roughly estimated cutoff eff. are only *O(1-3%)*

→ Residual estimated systematic cutoff uncertainties *e.g.* :

FNAL/MILC	14 (2+1+1)	: ~ 0.4%
ETMC	14 (2+1+1)	: ~ 0.5%
HPQCD	10 (2+1)	: ~ 0.4%

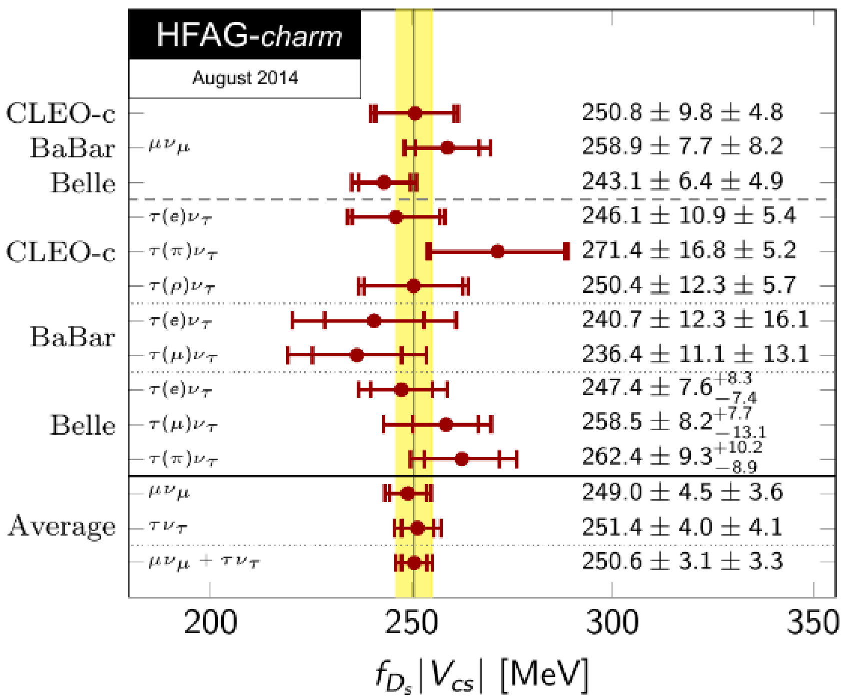
- Is a smaller lat. spacing (a ~ 0.045 fm or less) indispensable? (What about autocorrelations?)

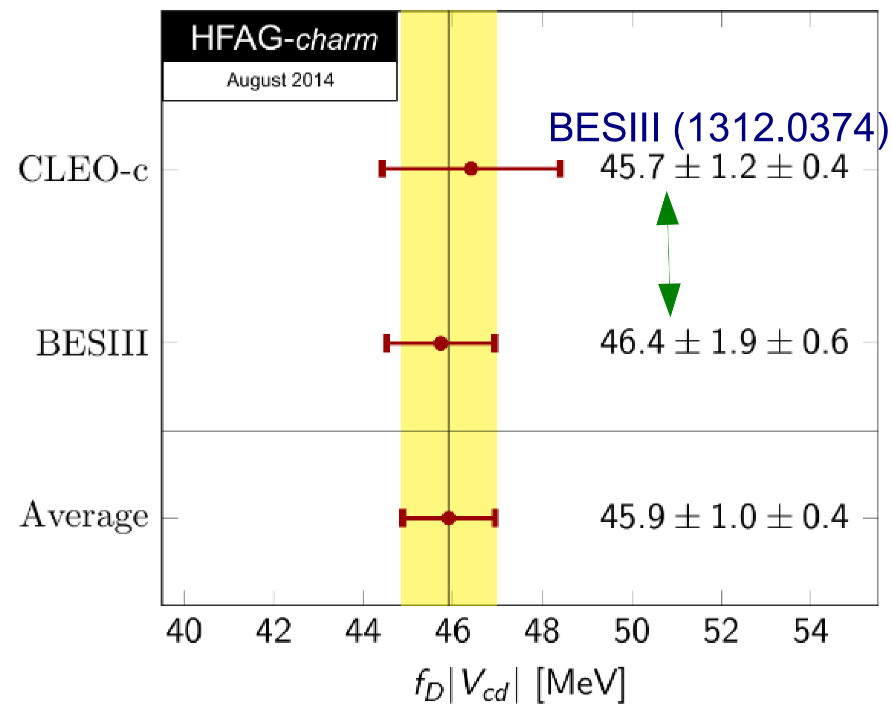
- Now (given the current precision) EM effects need to be carefully addressed!

V_{cd} & V_{cs}

	Lattice($N_f = 2+1+1$)	HFAG (12/2014)
f_D(MeV)	212.1(1.2)	203.7(4.9)
f_{Ds}(MeV)	248.8(1.4)	257.4(4.6)
f_{Ds}/f_D	1.1717(33)	1.264(38)

*no significant variations reported from **Belle II** & **BESIII** at **CHARM15** (still relatively large stat. errors)*





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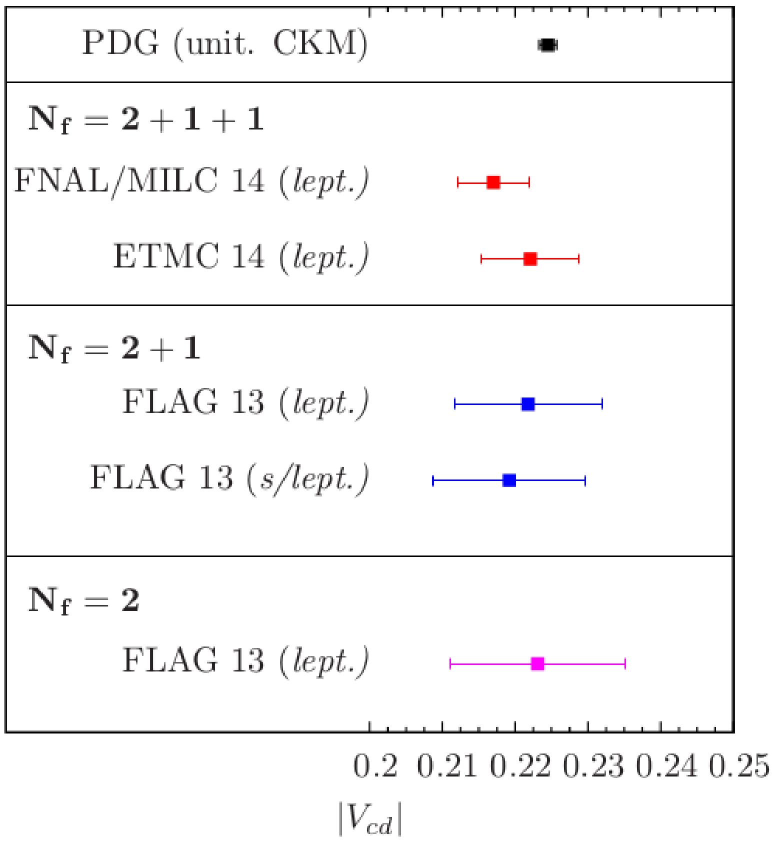
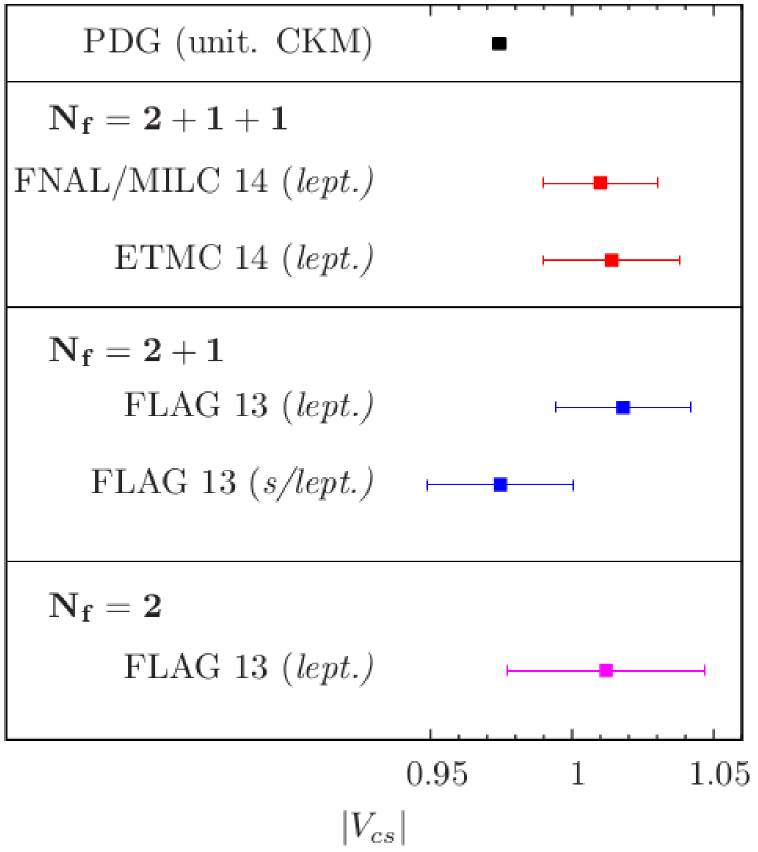


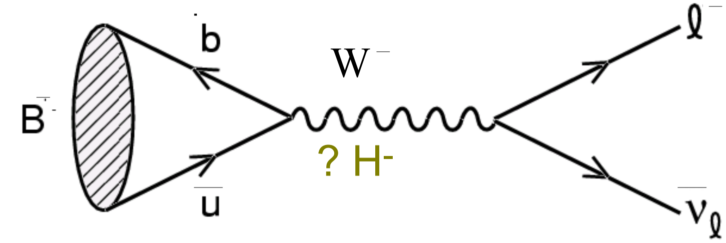


B – leptonic decays

$B \to \ell\nu$

$\mathcal{B}(B^- \to \tau^- \bar{\nu}) \sim 10^{-4}$

[Belle, BaBar]



$$r_H = \frac{\mathcal{B}(B \to \tau\nu)_{meas}}{\mathcal{B}(B \to \tau\nu)_{SM}} = 1.14 \pm 0.40$$

(P.Križan (BELLEII) @ *Flavorful ways to NP* 10/2014)

$$\Gamma(B \to \ell\nu_\ell) = \frac{m_B}{8\pi} G_F^2 f_B^2 |V_{ub}|^2 m_\ell^2 \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2$$

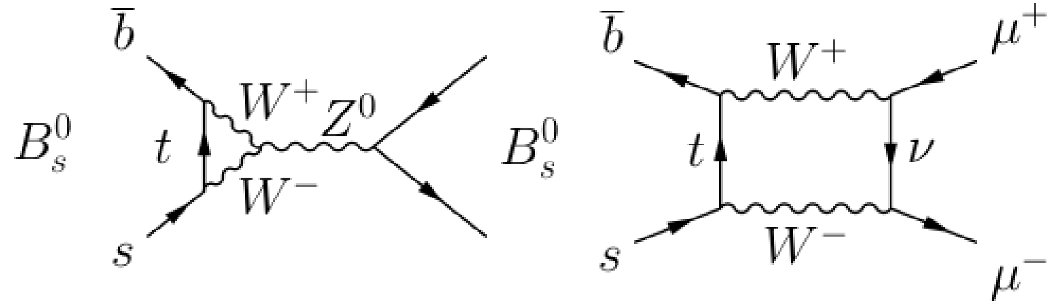
(to lowest order)

lattice

CKM prediction

current expt precision ~ 20-25 % → **Expected ~ 5 % (BelleII, 2020)**

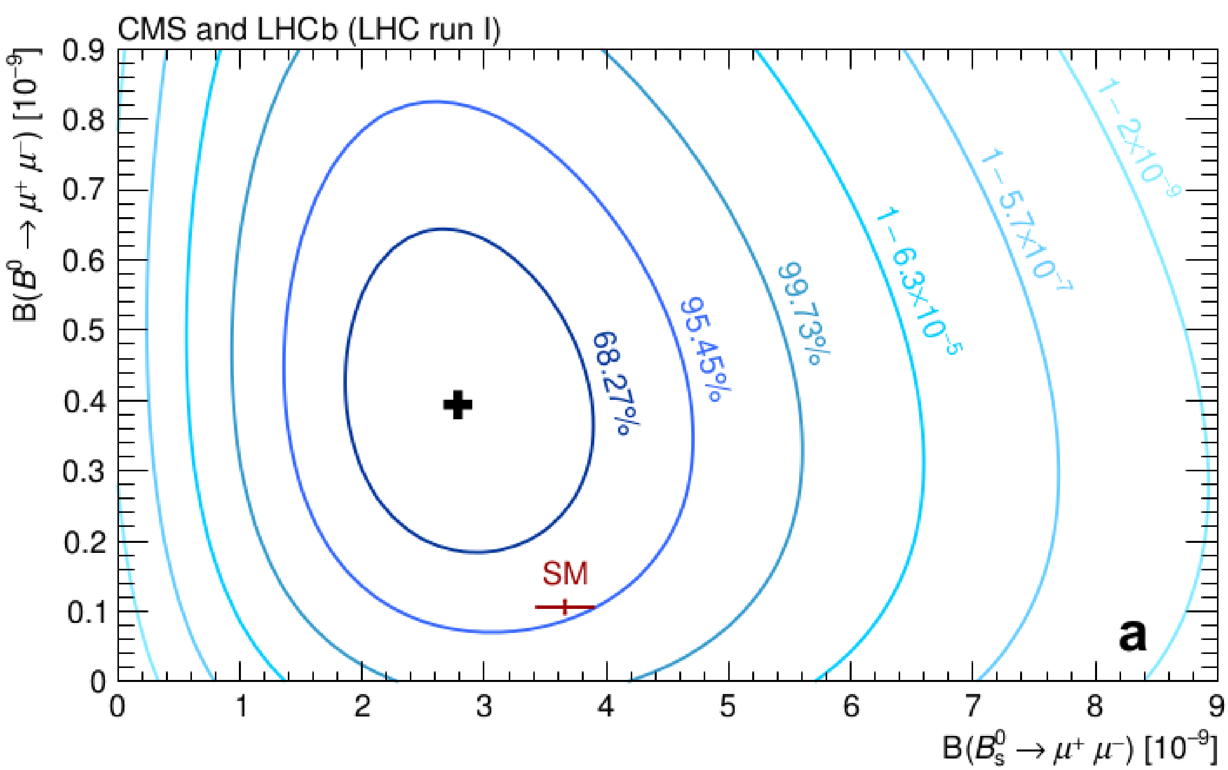
$B_{(s)} \to \ell^+\ell^-$



$$\Gamma(B_s \to \ell^+\ell^-) \propto f_{Bs}^2 |V_{tb}^* V_{ts}|^2$$

$$\mathcal{B}(B_s^0 \to \mu^+\mu^-) \sim 10^{-9}$$

$$\mathcal{B}(B_d^0 \to \mu^+\mu^-) \sim 10^{-10}$$



[CMS + LHCb (joint analysis)]

(1411.4413)

- Agreement at 1.2σ and 2.2σ with SM estimates for the BR for $\boldsymbol{B_s}$ and $\boldsymbol{B_d}$, respectively.
- Statistics for $\boldsymbol{B_d}$ decay are low (error on BR ~ 38% *vs.* 24% for $\boldsymbol{B_s}$).

b- quark on the lattice

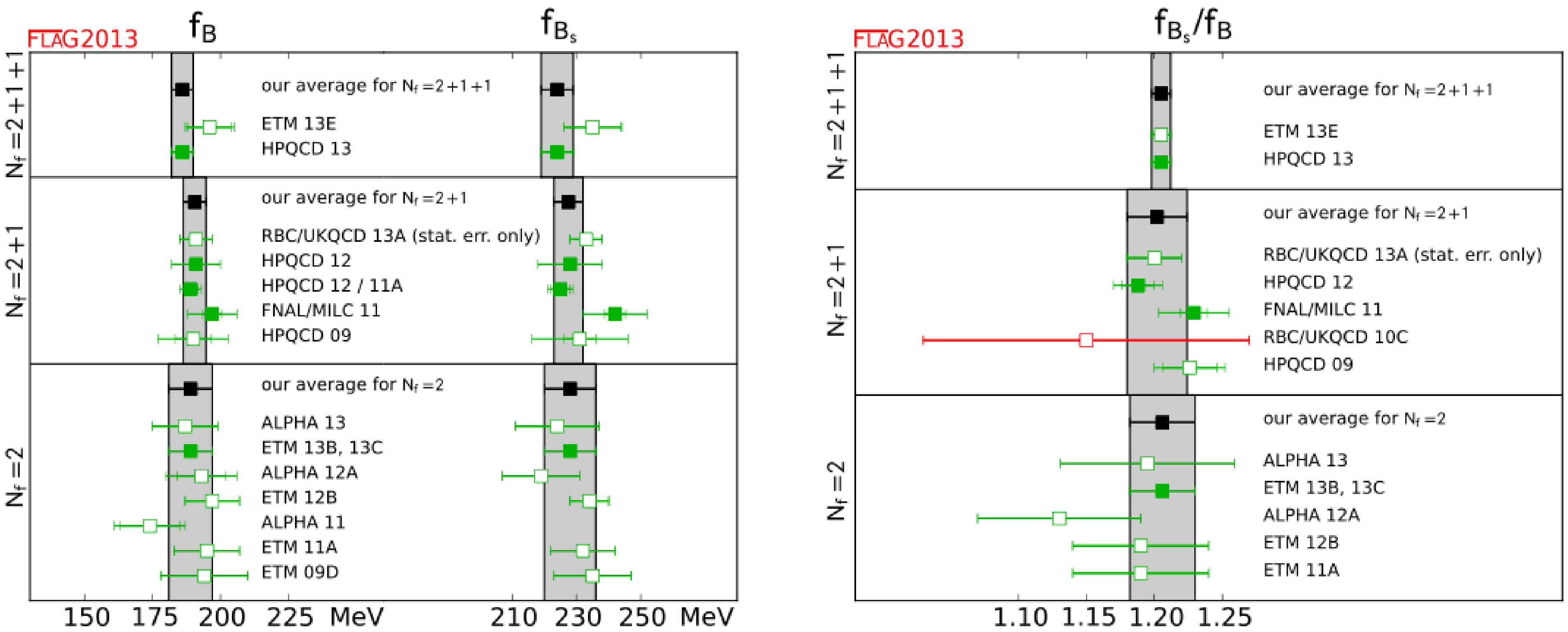
- Direct simulations ($a \leq 0.03$ fm) appropriate for b-quark are NOT YET possible (→ a two scale problem)
- Simulations at current lattices [$a \geq 0.05$ fm] + effective theories predictions

  - HQET on the lattice -O($1/m_h$)- and NP matching to QCD
  - NRQCD (+ pert. matching to QCD)
  - Appropriately (HQET) modified relativistic action (+ CT) for improved scaling behaviour
  - Interpolation between charm region and LO HQET
  - Interpolation of chain of ratios computed up to ~ $m_b/2$ (or even higher) and exactly known static limit

➡ Different lattice approaches (with pros and cons) are welcome for

- testing the methodology of lattice computations
- controlling the systematics

Comparison of results from FLAG-2013 (1310.8555)



	$N_f = 2$	$N_f = 2+1$	$N_f = 2+1+1$
f_B(MeV)	189(8)	190.5(4.2)	186(4)
f_{Bs}(MeV)	228(8)	227.7(4.5)	224(5)
f_{Bs}/f_B	1.206(24)	1.202(22)	1.205(7)

FLAG-2103 Averages

Recent computations

$N_f = 2 + 1 + 1$

- **ETMC** (tmWilson + **ratio method**) (`to appear`)

  $3\ a \in [0.06, 0.09]$ fm $\quad M_{ps}^{\min} \simeq 210$ MeV with $(M_{ps}^{\min} L) \simeq 3.2$

$N_f = 2 + 1$

- **RBC/UKQCD** (DW + **RHQ**) (`1404.4670`)

  $2\ a = \{0.09, 0.11\}$ fm $\quad M_{ps}^{\min} \simeq 290$ MeV with $(M_{ps}^{\min} L) \simeq 4$

- **RBC/UKQCD** (DW + **static b**) (`1406.6192`)

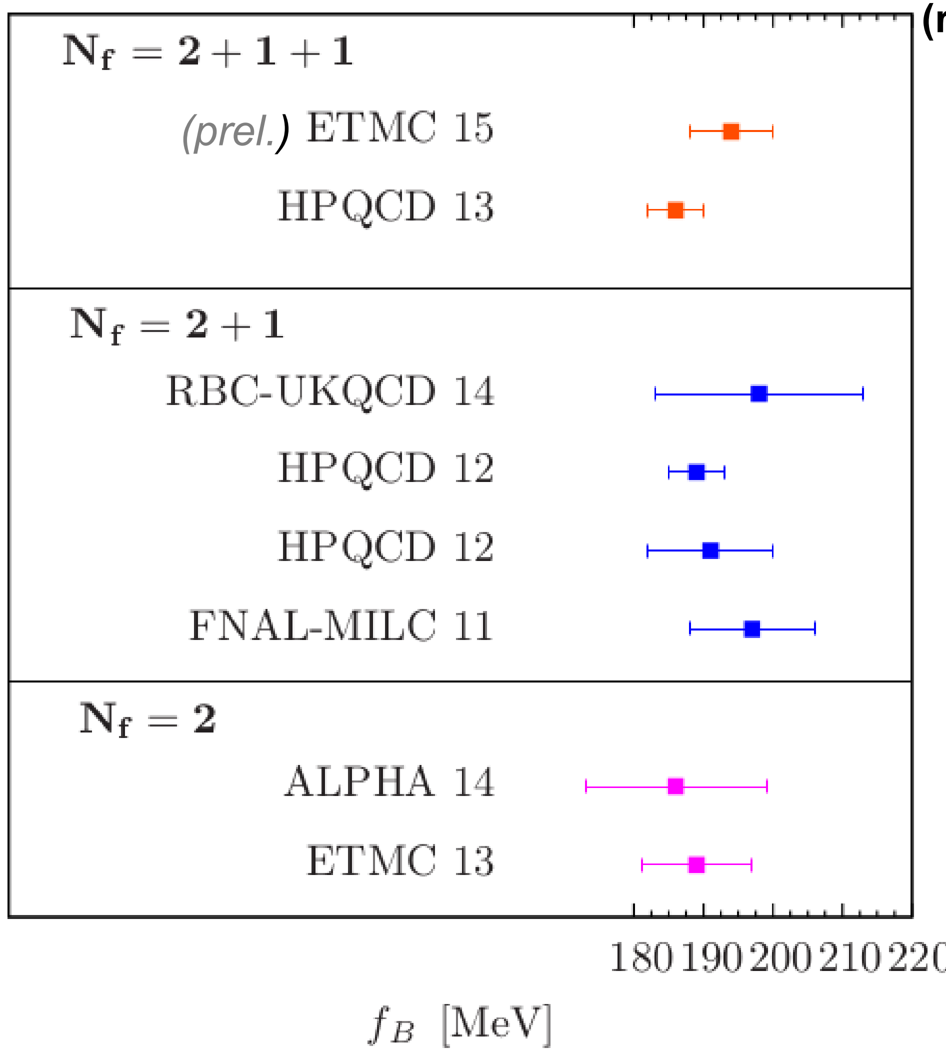
  $2\ a = \{0.09, 0.11\}$ fm $\quad M_{ps}^{\min} \simeq 290$ MeV with $(M_{ps}^{\min} L) \simeq 4$

$N_f = 2$

- **ALPHA** (Wilson + **NPHQET**) (`1404.3590`)

  $3\ a \in [0.05, 0.08]$ fm $\quad M_{ps}^{\min} \simeq 190$ MeV with $(M_{ps}^{\min} L) \simeq 4.0$

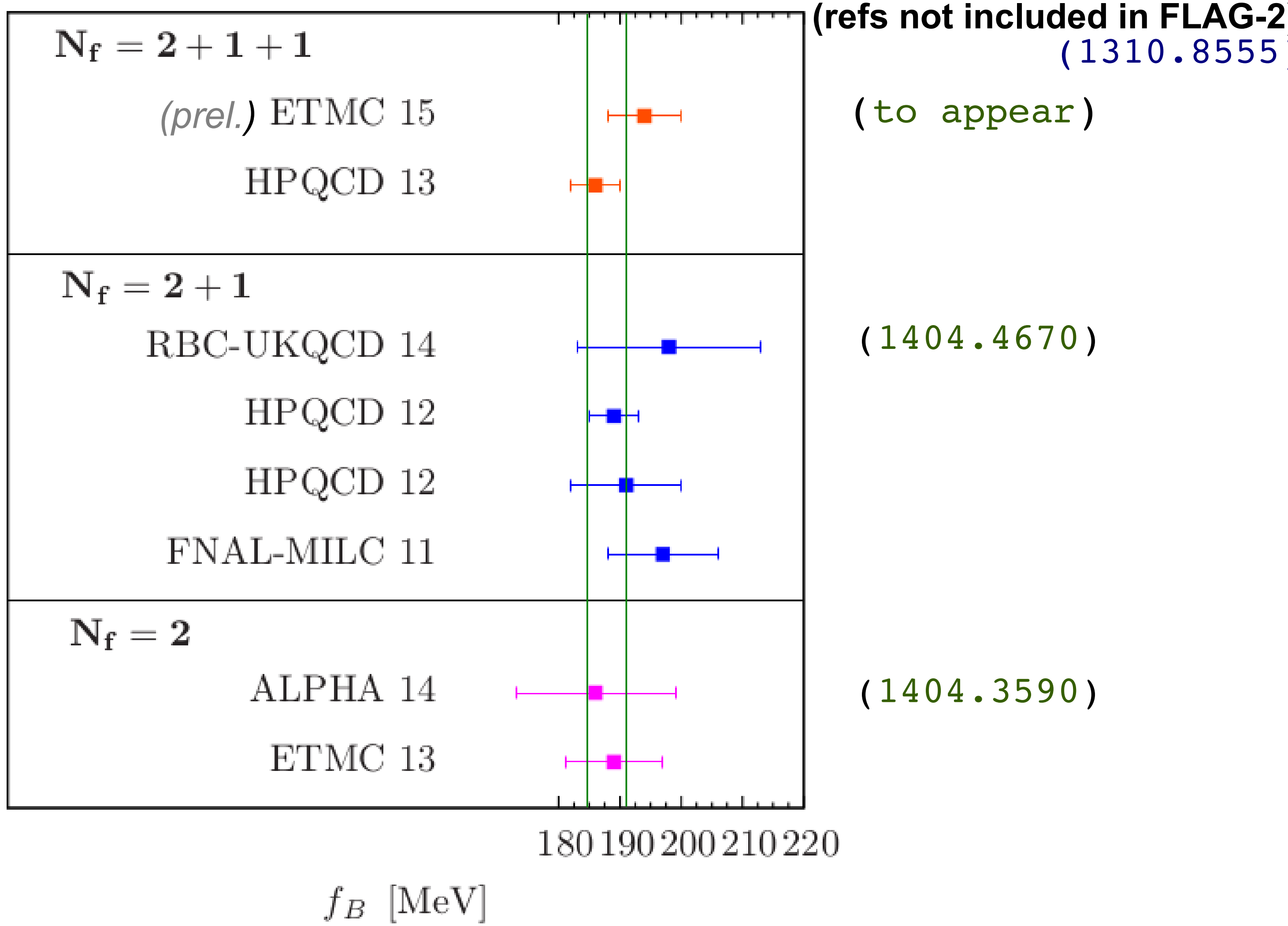
Comparison of results for f_B



(refs not included in FLAG-2)
(1310.8555)
(to appear)
(1404.4670)
(1404.3590)

- Comparison plot with most recent results obtained at *CL* from at least two lat. spacings
  - + C. DeTar for FNAL-MILC (LAT2015) - Nf=2+1+1 (HISQ); over 5 a ≥ 0.045 fm; much reduced total uncertainty *(work in progress)*
  - + T. Kawanai for RBC-UKQCD (LAT2015) - Nf=2+1(DW) @ phys. point, 1a (*work in progress)*
  - + FNAL-MILC (LAT2014) – Nf=2+1 over 5 a (1501.01991 and *work in progress)*
  - + RBC/UKQCD – Nf=2+1 (static *b*) over 2 a (1406.6192*)* [not included due to relatively large syst. uncertainty]

Comparison of results for f_B

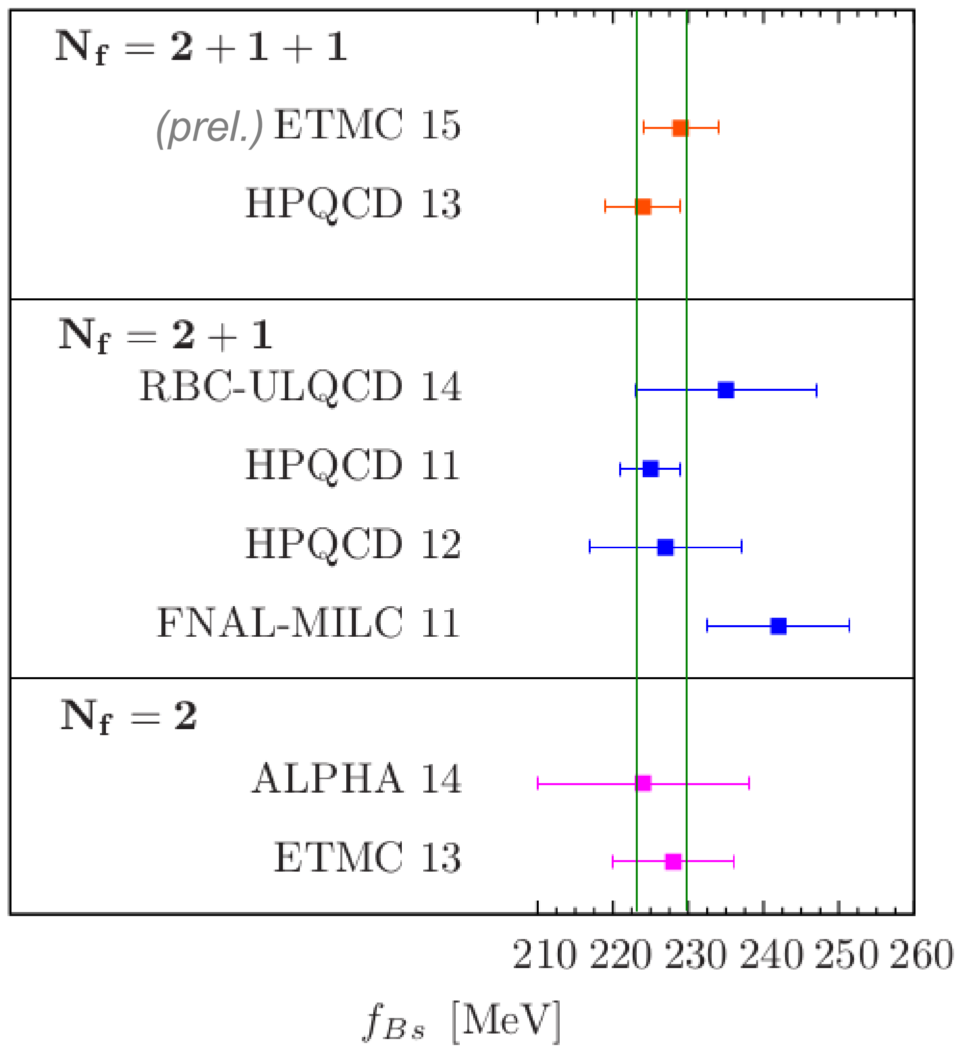


- Comparison plot with most recent results obtained at *CL* from at least two lat. spacings

➡ Vertical (green) lines show -1σ:1σ of the average over the two 2+1+1 results

- **current uncertainty ~ 2%; all results compatible – no visible Nf dependence**

Comparison of results for f_{Bs}



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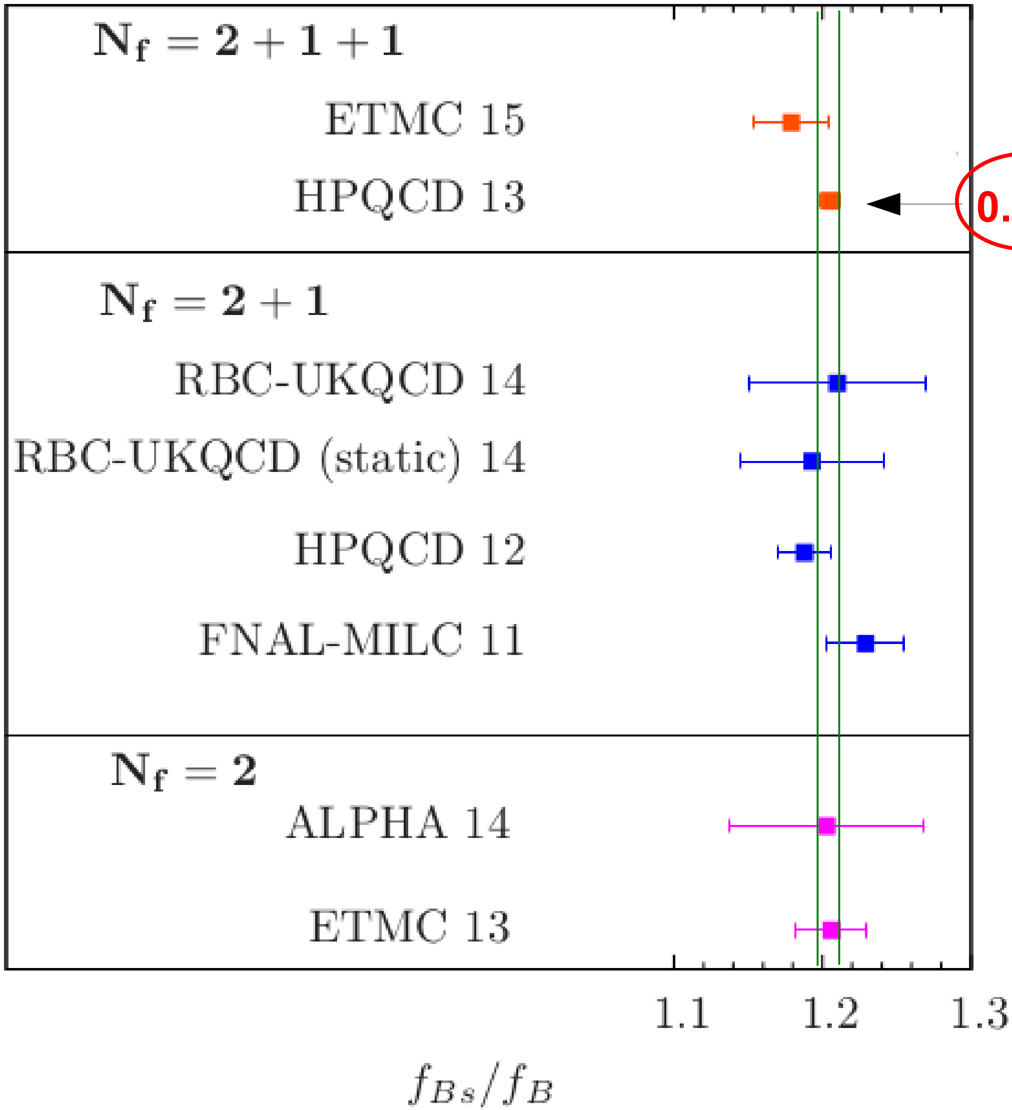
- **current uncertainty ~ 2%; all results compatible – no visible Nf dependence**

Comparison of results for f_{Bs}/f_B

Most of systematics cancel out:

- cutoff effects
- b-quark tuning



(refs not included in FLAG-2)

(1310.8555)

(to appear)

(1404.4670)

(1406.6192)

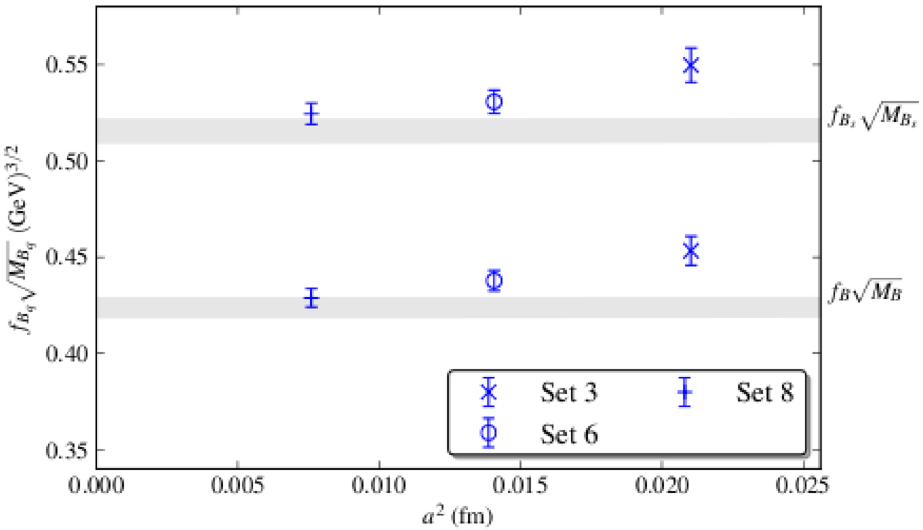
(1404.3590)

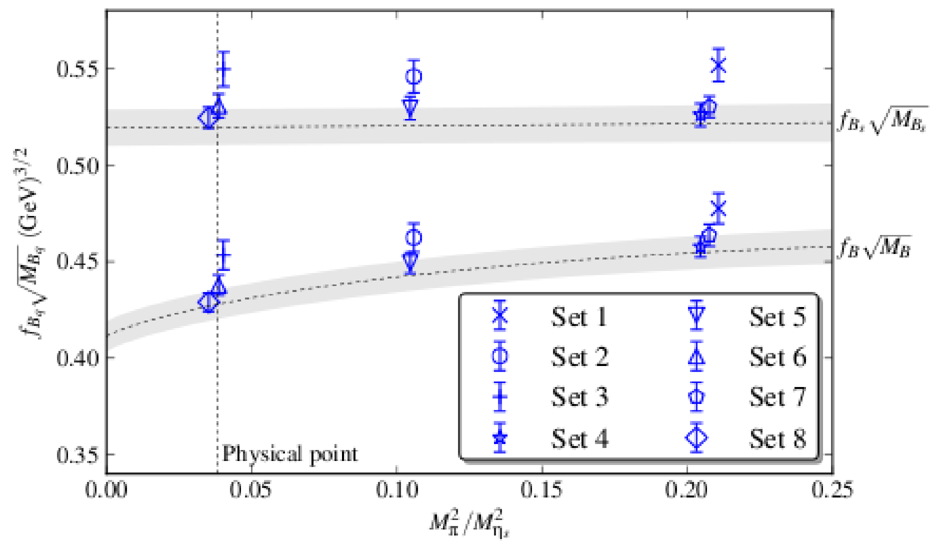
+ FNAL-MILC (LAT2014) – Nf=2+1 over 5 a's - projected error ~ 0.9% (1501.01991 and *work in progress)*

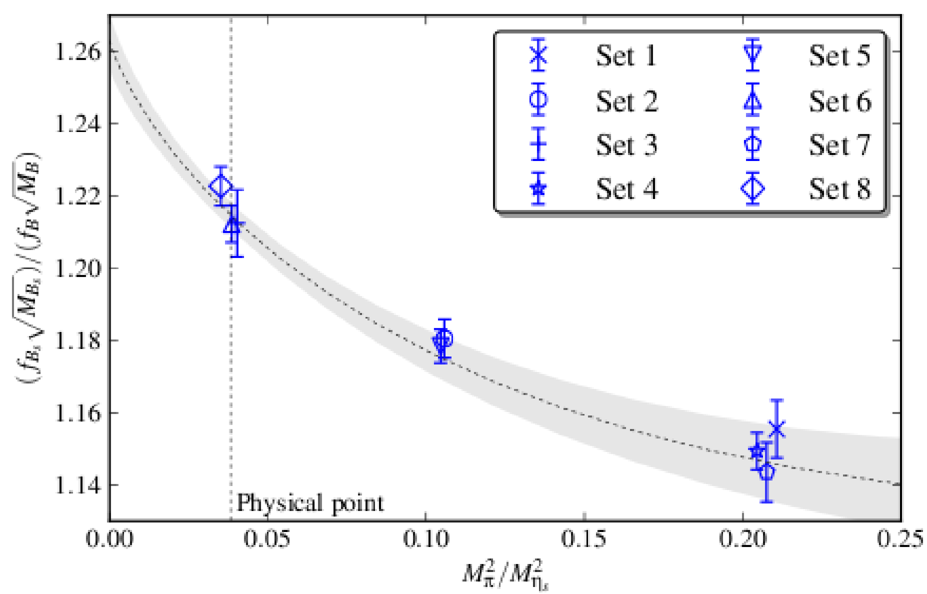
➡ Vertical (green) lines follow the "HPQCD 13" uncertainty

Cutoff effects & chiral extrapolations

HPQCD (1302.2644)

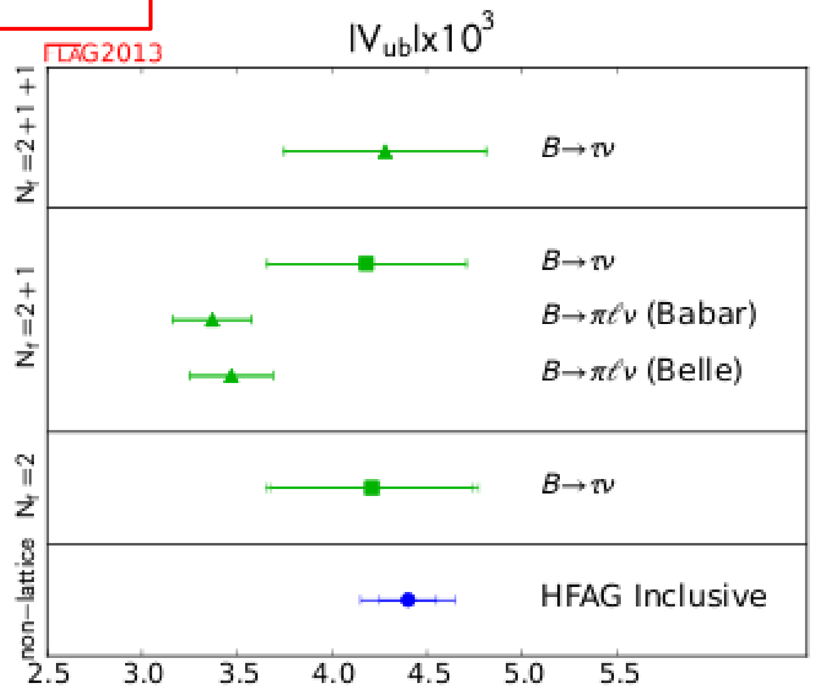


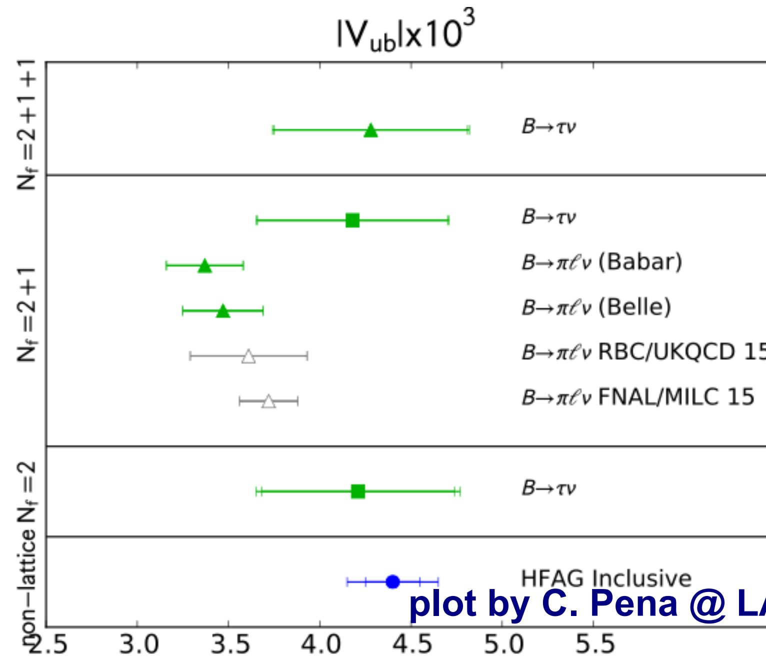




$N_f = 2+1+1 \quad a \geq 0.09$ fm, @ (phys. u/d); $\quad b -$ NRQCD

V_{ub}

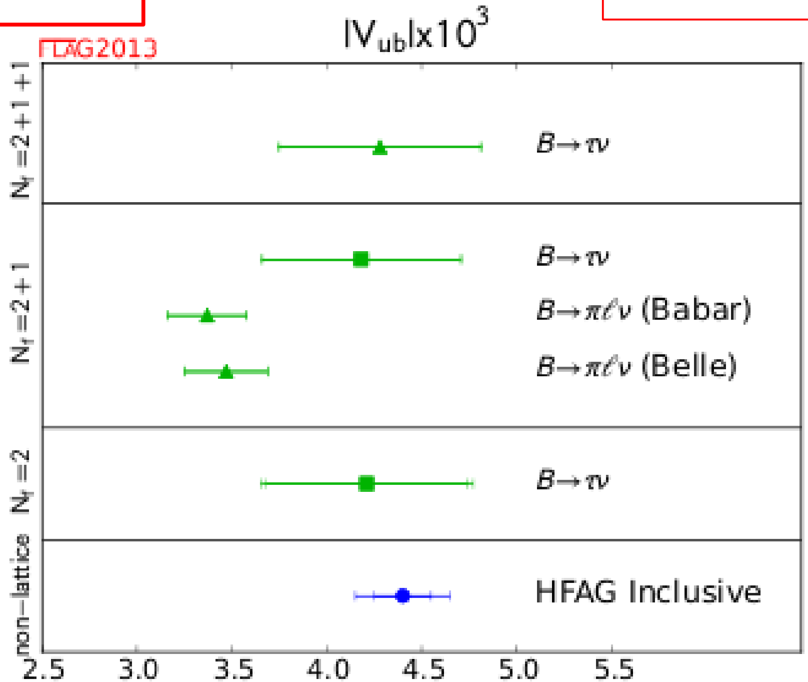


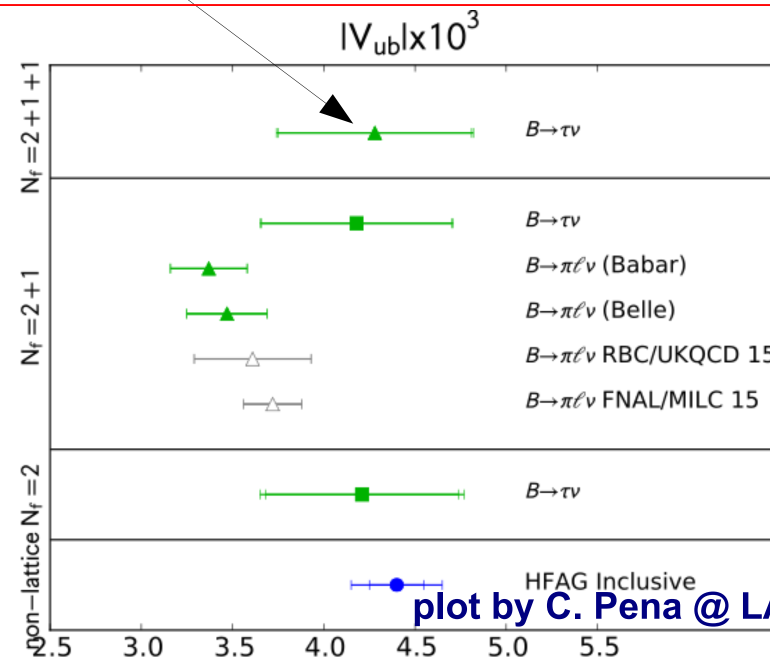


plot by C. Pena @ LAT2015

V_{ub}

- new f_B(ETMC) result would slightly reduce central value of $|V_{ub}|$ (lept.)



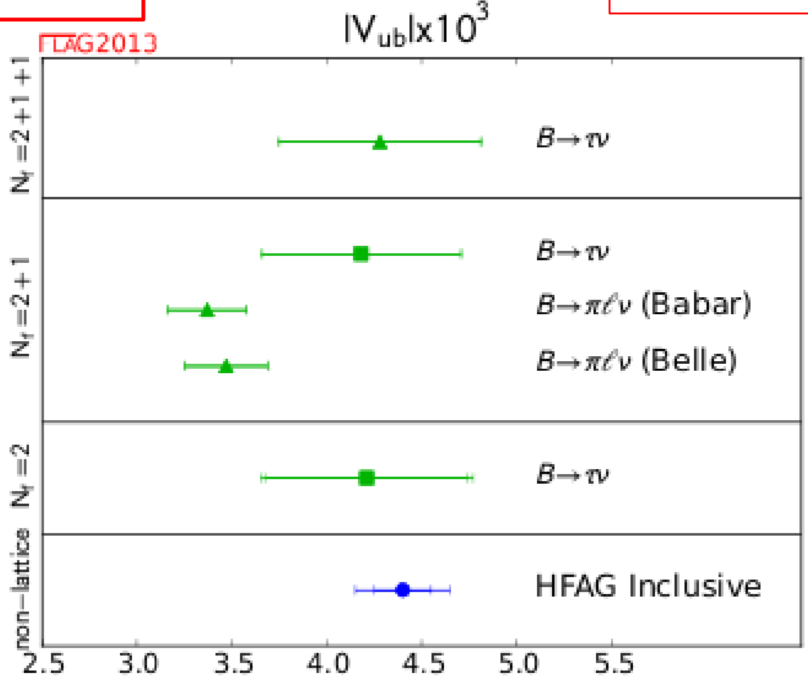


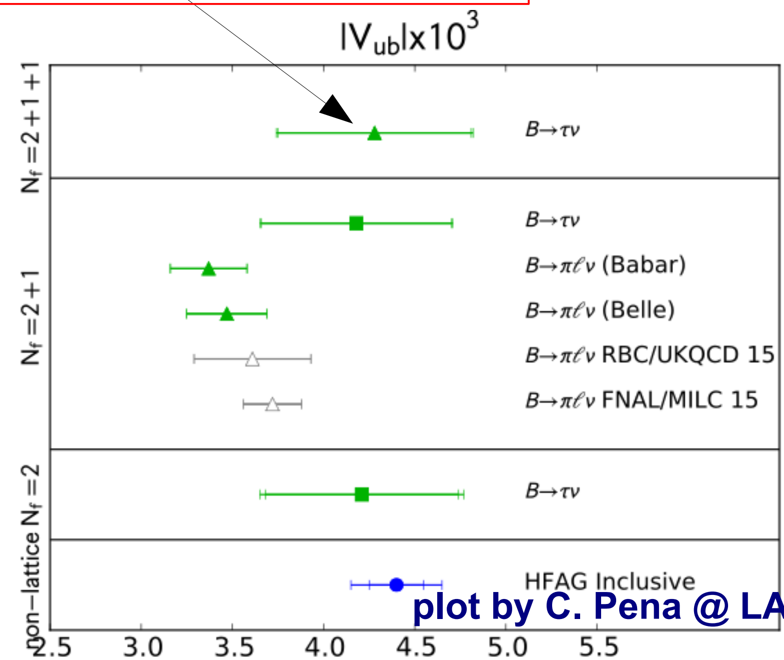
plot by C. Pena @ LAT2015

N.B. Belle and Babar discrepancy BR($B \to \tau\, \nu$)

V_{ub}

- new f_B(ETMC) result will slightly reduce $|V_{ub}|$ (lept.)





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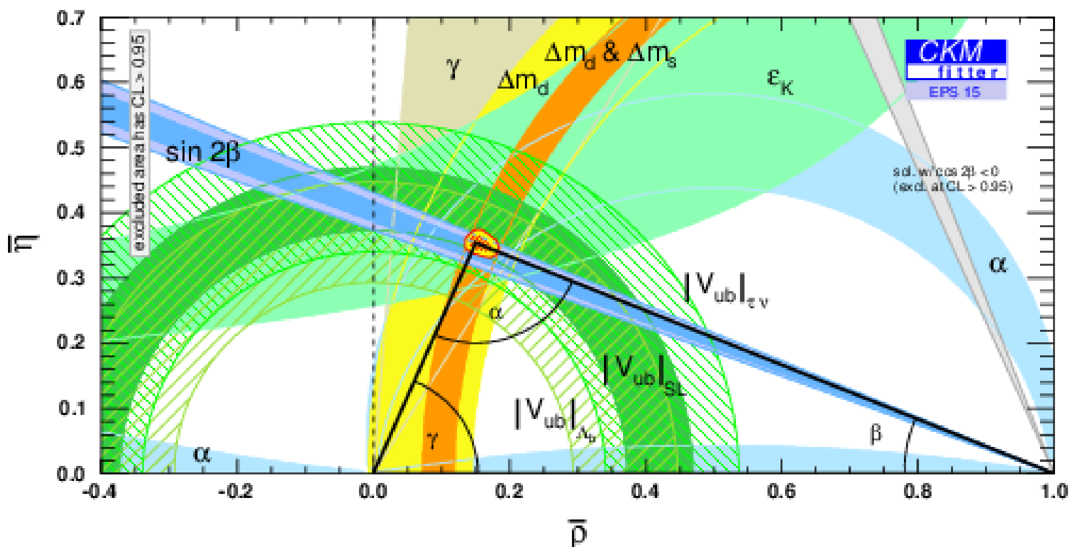

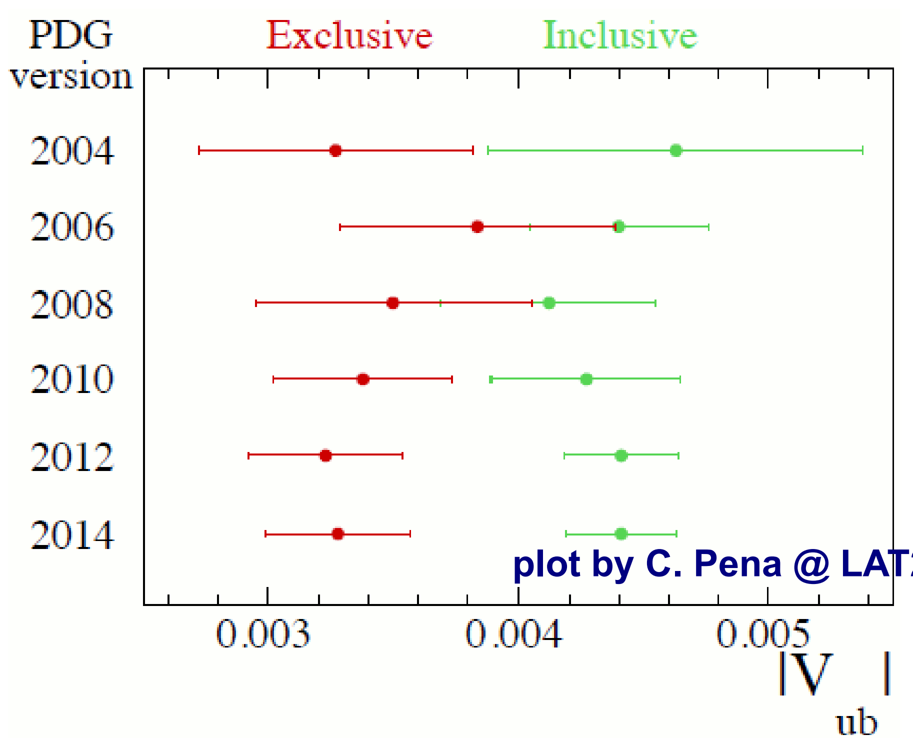

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D & *B* – mixing

$D^0 - \overline{D^0}$ mixing

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle$$

Oscillation (FCNC process) of up-type quarks!

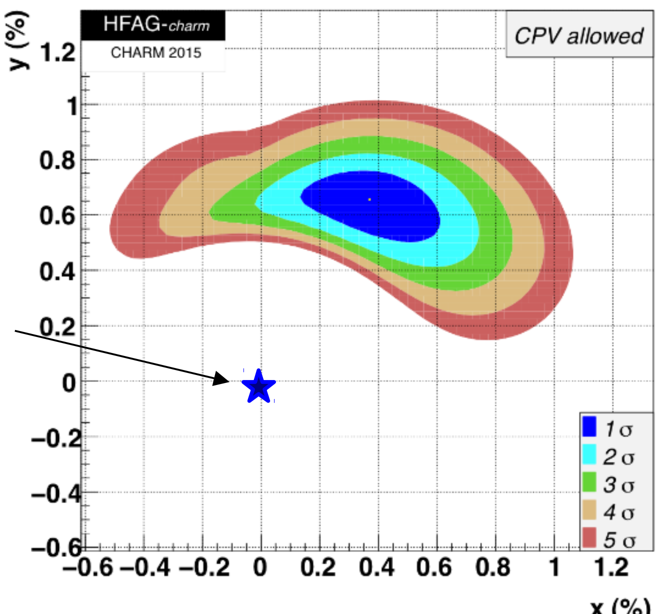
$$x \equiv (m_1 - m_2)/\Gamma = \Delta m/\Gamma$$
$$y \equiv (\Gamma_1 - \Gamma_2)/2\Gamma = \Delta\Gamma/2\Gamma$$

Doubly Cabibbo & GIM supressed

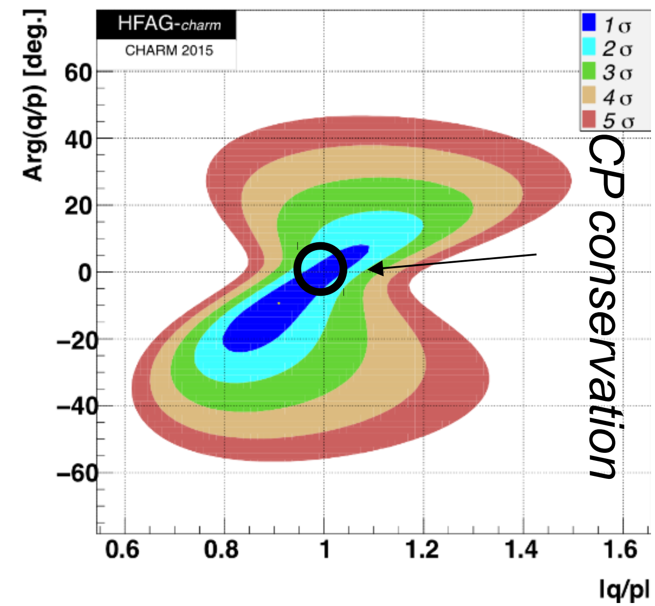
- small contribution from box diagram

Mixing observed in 2007 by **Belle** & **BaBar**

Also observed and confirmed by **CDF** & **LHCb**



Mixing at *11.5σ* significance



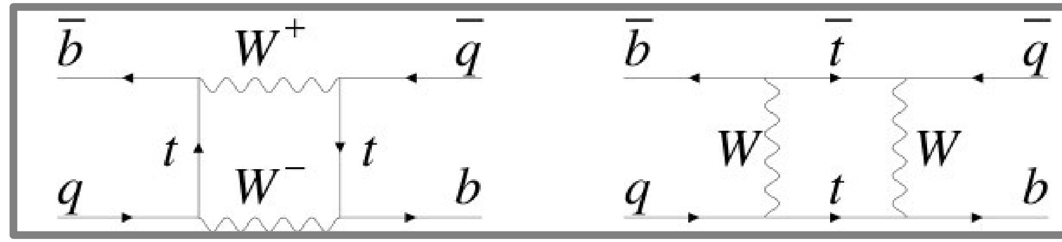
CPV at 30% confidence

- large distance behavior due to ***d*** and ***s*** dominates → CPV expected negligible (in SM).
- CP violating signals associated to short distance interactions described by 4-quark operators

→ *CPV in D-neutral meson system will be a clear sign of New Physics*

- Neutral D-meson mixing may be useful as a probe for NP scale.

$B^0_{(d/s)} - \overline{B^0_{(d/s)}}$ oscillations



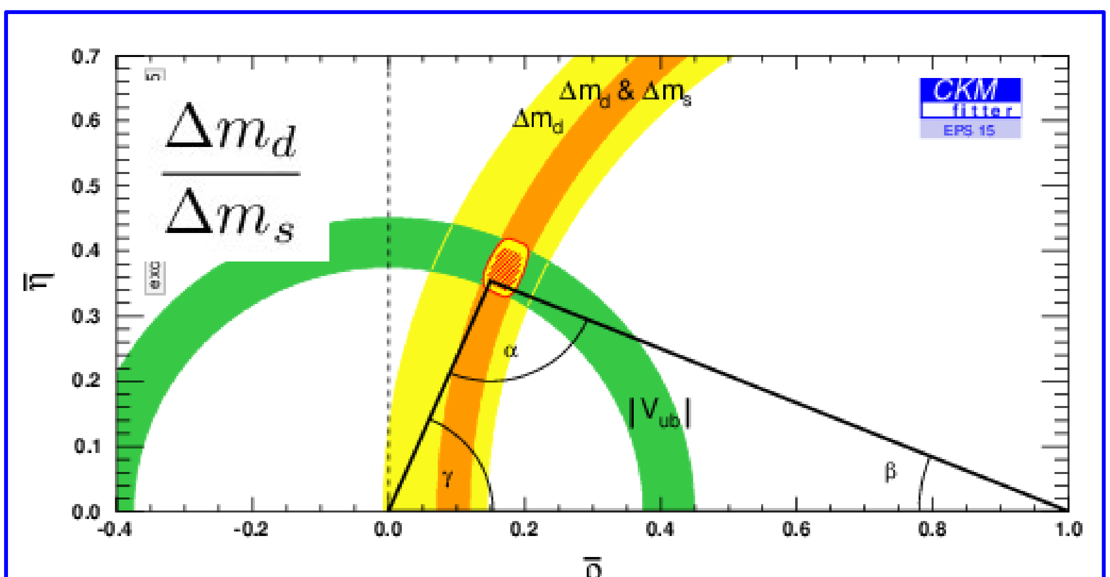
- $\Delta m_q \propto m_{B_q} B_{B_q} f^2_{B_q} |V^*_{tq} V_{tb}|^2, \quad q = d,\ s$

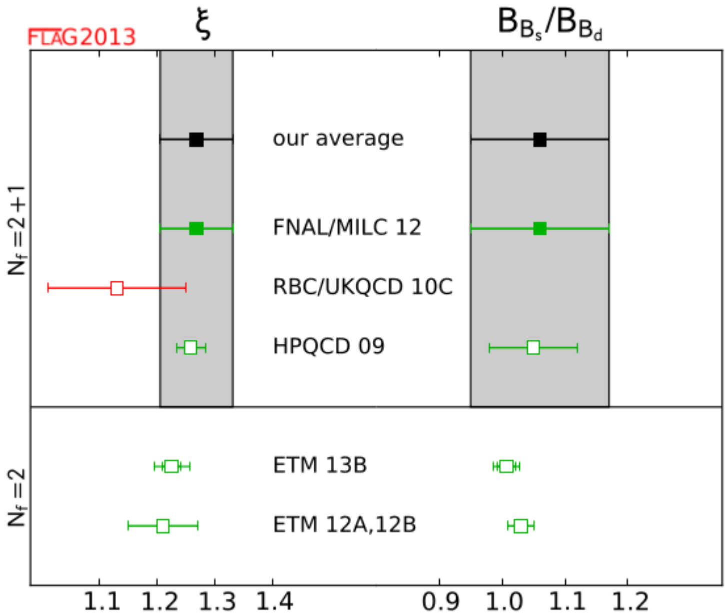
subpercent expt.precision

- $\xi = (f_{B_s}\sqrt{B_{B_s}})/(f_{B_d}\sqrt{B_{B_d}})$

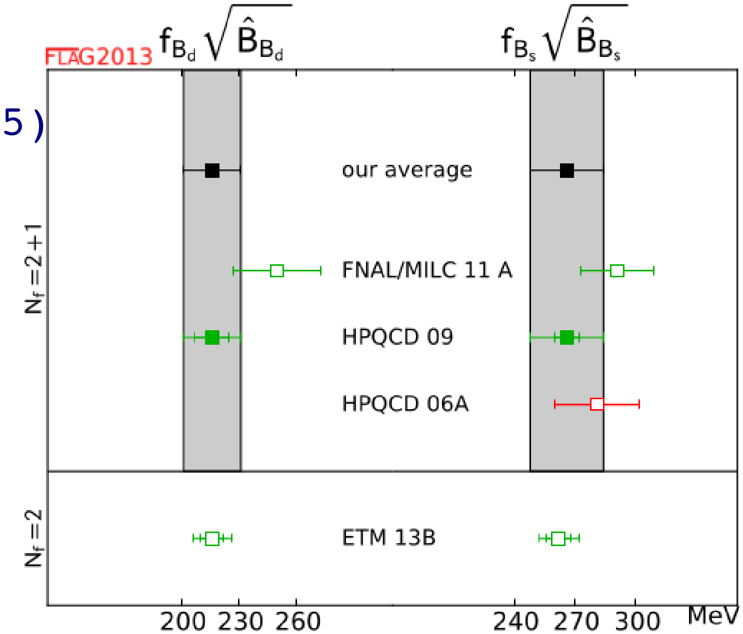
- $\dfrac{\Delta m_s}{\Delta m_d} = \dfrac{m_{B_s}}{m_{B_d}} \xi^2 \left|\dfrac{V_{ts}}{V_{td}}\right|^2$

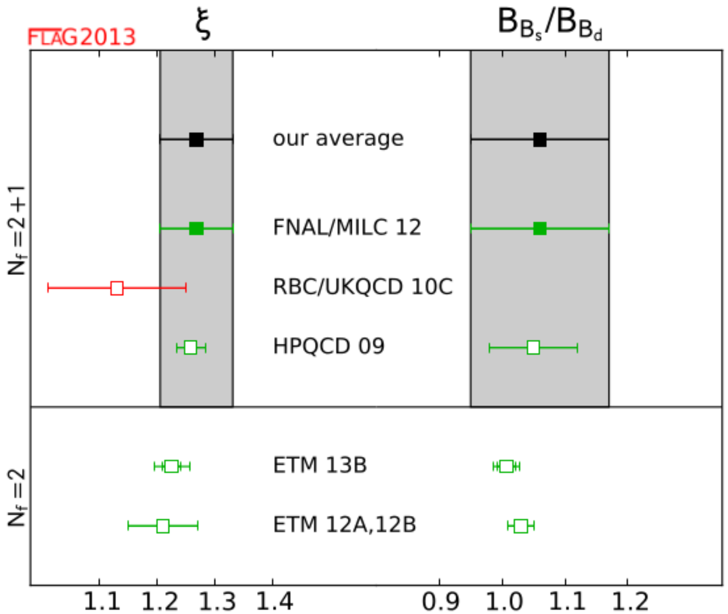
- double ratio → most of the systematics cancel out
(& no RC uncertainty from B's)



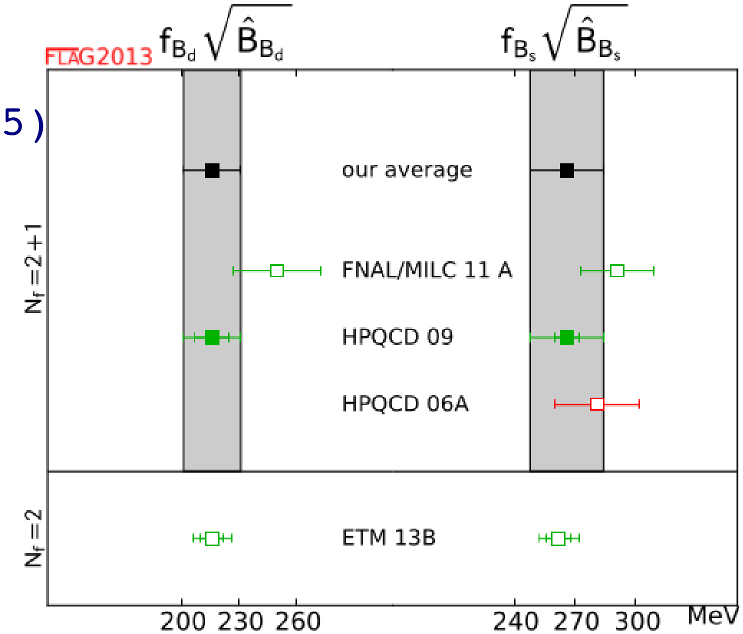

FLAG2013
ξ
B_{B_s}/B_{B_d}
our average
FNAL/MILC 12
RBC/UKQCD 10C
HPQCD 09
ETM 13B
ETM 12A,12B
$N_f=2+1$
$N_f=2$
1.1 1.2 1.3 1.4
0.9 1.0 1.1 1.2

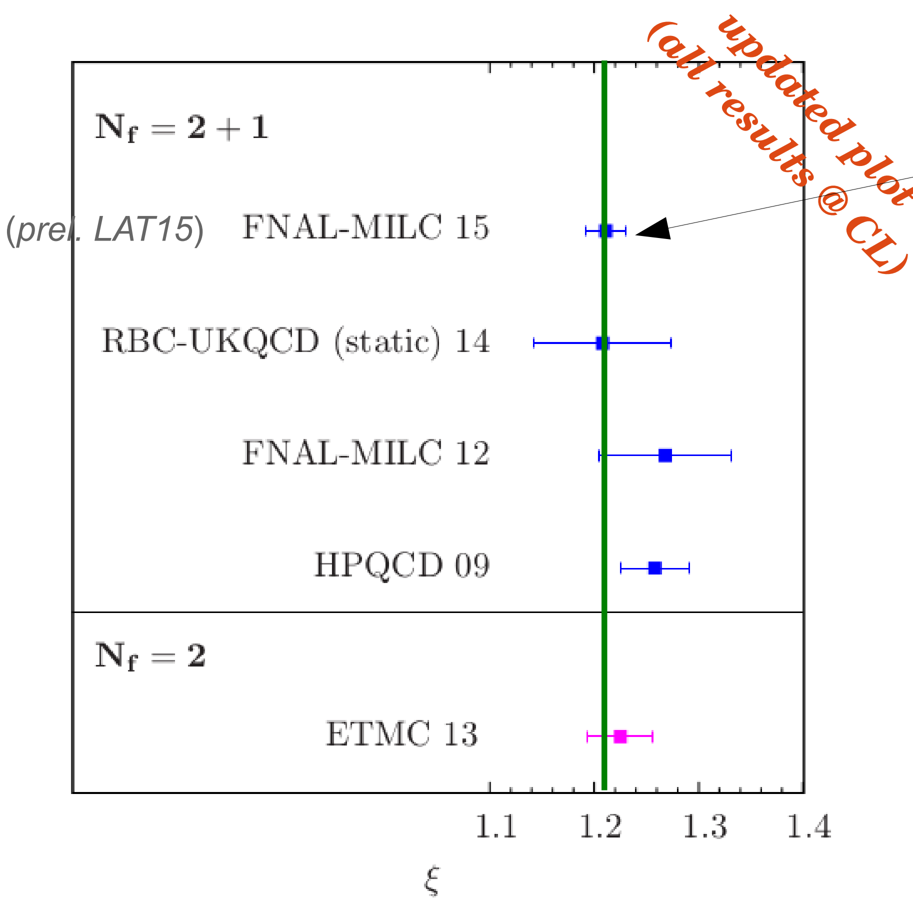
(1310.8555)


FLAG2013
$f_{B_d}\sqrt{\hat{B}_{B_d}}$
$f_{B_s}\sqrt{\hat{B}_{B_s}}$
our average
FNAL/MILC 11 A
HPQCD 09
HPQCD 06A
ETM 13B
$N_f=2+1$
$N_f=2$
200 230 260
240 270 300
MeV



(1310.8555)





- Vertical (green) line (to guide the eye) (passes through the most precise result)
- HPQCD (Nf=2+1+1 1411.6989 (LAT14 *work in progress*)
- RBC-UKQCD (Nf=2+1) O. Witzel @ LAT15 (*work in progress*)
- FNAL-MILC (Nf=2+1) J. Simone @ LAT15 (prel. results and *work in progress*, 1412.5047)
- No published results yet from Nf=2+1+1

ξ : $\sigma_{lat} >> \sigma_{expt}$

Complete 4-fermion operator basis

$$\mathcal{O}_1 = \left[\bar{h}^a \gamma_\mu (1-\gamma_5) \ell^a\right] \left[\bar{h}^b \gamma_\mu (1-\gamma_5) \ell^b\right]$$

$$\mathcal{O}_2 = \left[\bar{h}^a (1-\gamma_5) \ell^a\right] \left[\bar{h}^b (1-\gamma_5) \ell^b\right], \quad \mathcal{O}_3 = \left[\bar{h}^a (1-\gamma_5) \ell^b\right] \left[\bar{h}^b (1-\gamma_5) \ell^a\right]$$

$$\mathcal{O}_4 = \left[\bar{h}^a (1-\gamma_5) \ell^a\right] \left[\bar{h}^b (1+\gamma_5) \ell^b\right], \quad \mathcal{O}_5 = \left[\bar{h}^a (1-\gamma_5) \ell^b\right] \left[\bar{h}^b (1+\gamma_5) \ell^a\right]$$

$(h, \ell) \equiv (c, u)$ for D-mixing $\qquad (h, \ell) \equiv (b, d/s)$ for $B_{d/s}$-mixing

+ 3 more 4-fermion operators whose whose *P*-even contribution is the same as for $\mathcal{O}_{1,2,3}$

Bag parameters:

$$\langle \overline{P}^0 | \mathcal{O}_1(\mu) | P^0 \rangle = \xi_1 B_1(\mu)\, m_{P^0}^2 f_{P^0}^2$$

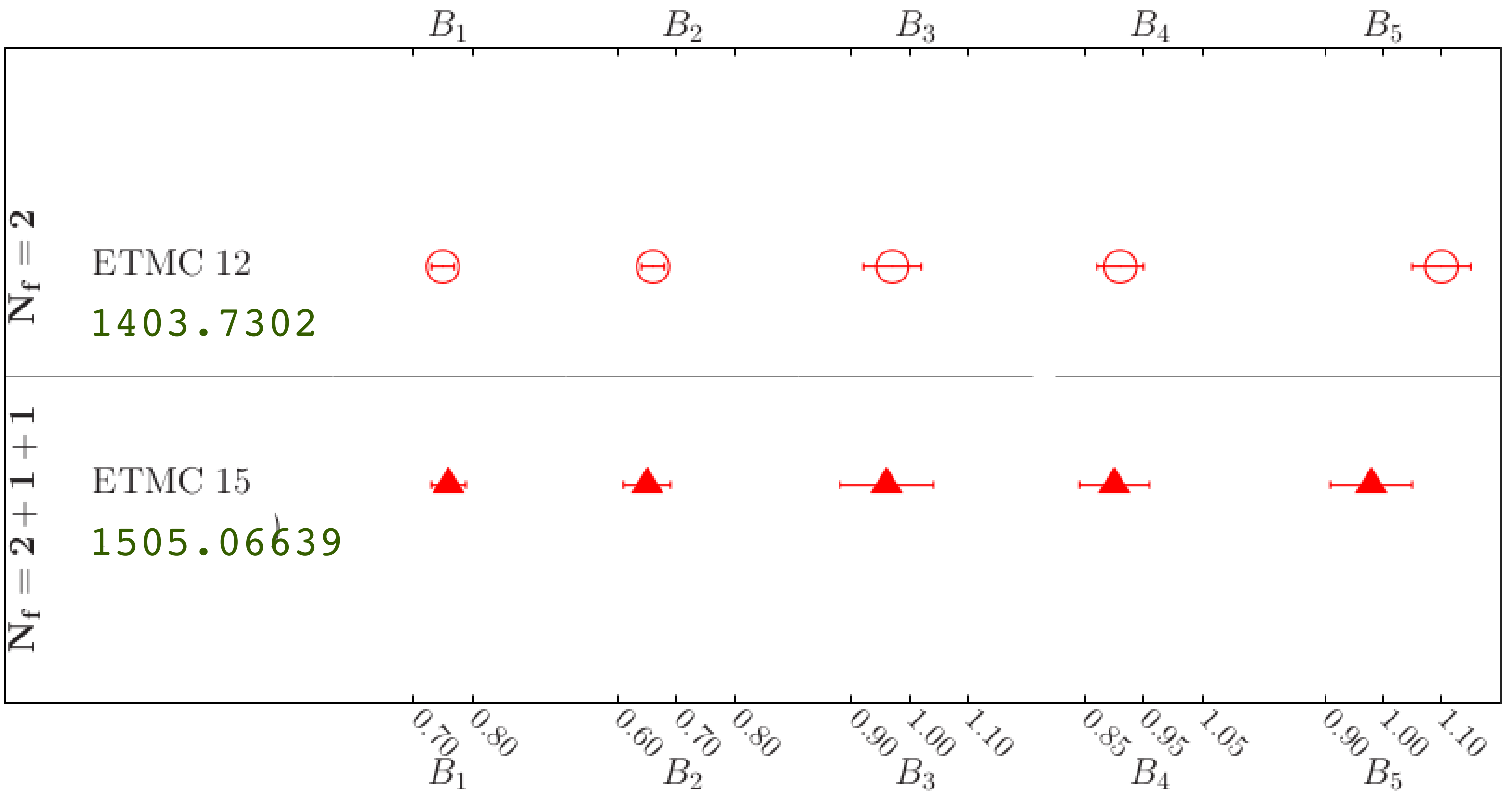
$$\langle \overline{P}^0 | \mathcal{O}_i(\mu) | P^0 \rangle = \xi_i B_i(\mu)\, \frac{m_{P^0}^4 f_{P^0}^2}{(m_\ell(\mu) + m_h(\mu))^2} \quad \text{for } i = 2, \ldots, 5$$

$$\xi_i = \{8/3, -5/3, 1/3, 2, 2/3\}$$

$$\langle \bar{P}^0 | \mathcal{H}^{\Delta F=2} | P^0 \rangle = \sum_{i=1}^{5} C_i \langle \bar{P}^0 | \mathcal{O}_i | P^0 \rangle$$

D-mixing

- **ETMC (N_f=2 & N_f=2+1+1)**



- **FNAL/MILC (N_f =2+1) –** `1411.6086(LAT14) + LAT15` (*work in progress*)

B-mixing (full operator basis)

- **ETMC (N_f=2)** (1308.1851)

($\overline{\text{MS}}$–BMU, m_b)				
$B_1^{(d)}$	$B_2^{(d)}$	$B_3^{(d)}$	$B_4^{(d)}$	$B_5^{(d)}$
0.85(4)	0.72(3)	0.88(13)	0.95(5)	1.47(12)
$B_1^{(s)}$	$B_2^{(s)}$	$B_3^{(s)}$	$B_4^{(s)}$	$B_5^{(s)}$
0.86(3)	0.73(3)	0.89(12)	0.93(4)	1.57(11)

($\overline{\text{MS}}$, m_b) [MeV]					
i	1	2	3	4	5
$f_{Bd}\sqrt{B_i^{(d)}}$	174(8)	160(8)	177(17)	185(9)	229(14)
$f_{Bs}\sqrt{B_i^{(s)}}$	211(8)	195(7)	215(17)	220(9)	285(14)

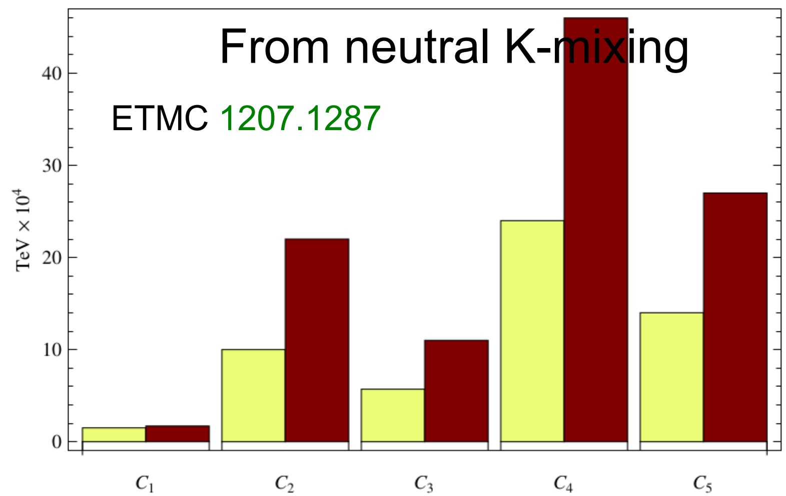
- **FNAL/MILC (N_f=2+1)** 1412.5097(LAT14) + LAT15 (*work in progress*)

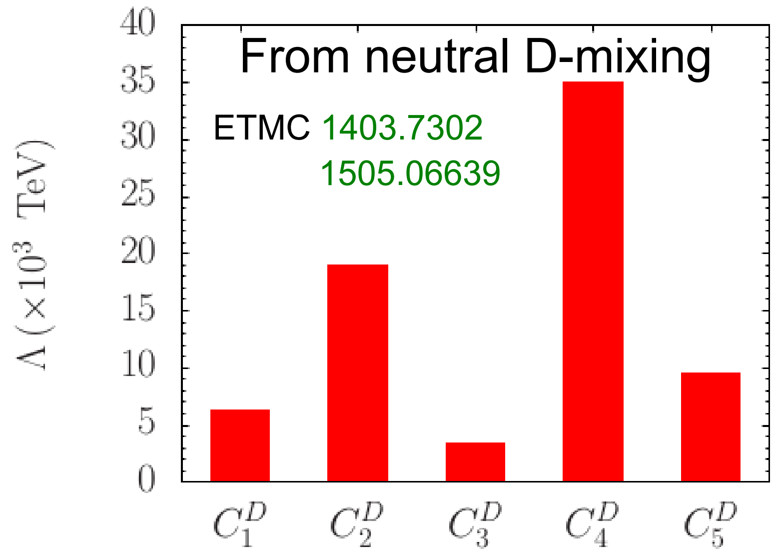
- **HPQCD (N_f=2+1)** 1412.5097(LAT14) (*work in progress*)

General Wilson coefficients parametrisation $C_i(\Lambda) = \frac{F_i L_i}{\Lambda^2}$

general NP flavour coupling   loop factor

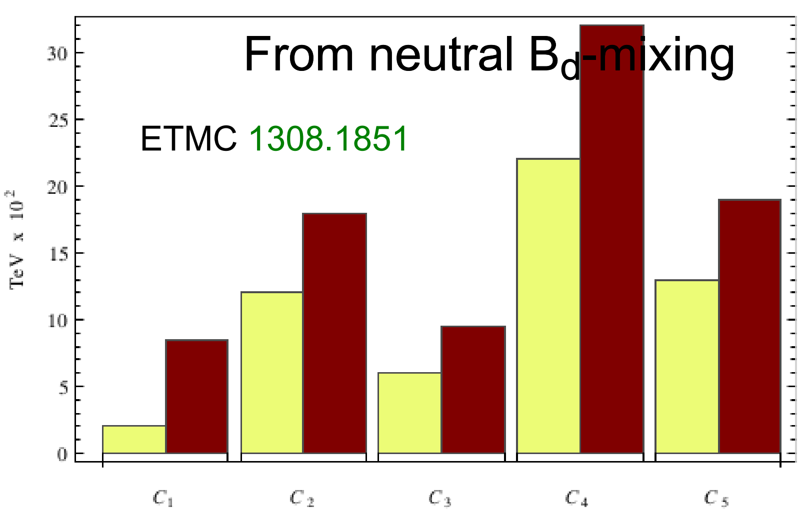
Assuming generic coupling $F_i \sim L_i \sim 1$ Wilson coeff. are translated into lower Λ bounds

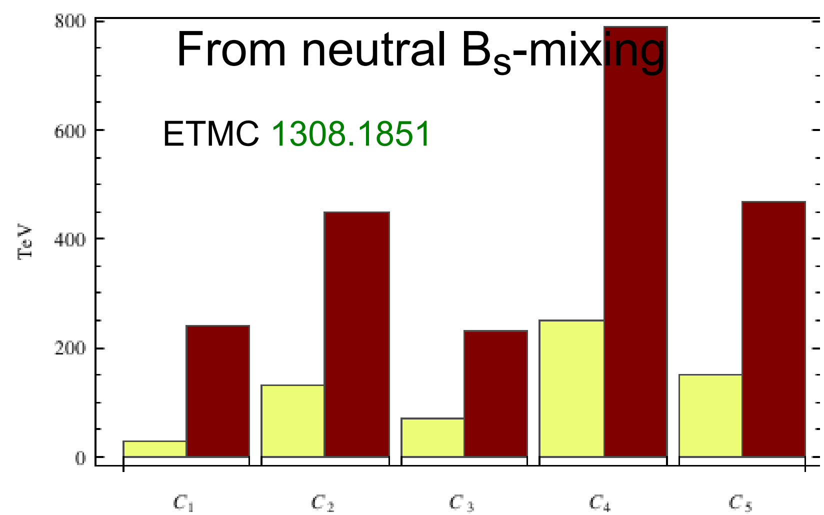




yellow bars: UTfit analysis (2007) using quenched results

brown bars: new analysis using results from Nf=2 simulations





A recent *B*-computation from ETMC
on N_f =2+1+1

[A. Bussone, N. Carrasco, P.D. R. Frezzotti, V. Lubicz,
E. Picca, G.C. Rossi, S. Simula, C. Tarantino]

RATIO METHOD [ETMC 0909.3187, 1107.1441, 1308.1851]

– Consider a *chain* of ratios for a quantity **Q(m_h)** formed at sequential heavy quark masses: $m_h^{(n)} = \lambda \, m_h^{(n-1)}$

$$Q(m_h^{(1)}) \times \underbrace{\frac{Q(m_h^{(2)})}{Q(m_h^{(1)})}} \times \underbrace{\frac{Q(m_h^{(3)})}{Q(m_h^{(2)})}} \times \ldots \times \underbrace{\frac{Q(m_h^{(K)})}{Q(m_h^{(K-1)})}}$$

$m_h^{(1)} \sim \mathbf{m_c}$

well-controlled **CL** computation in the c-region

ratios lead to large cancellation of systematics (e.g. discr. errors ~ few %) → safe **CL** determination up to ...

$m_h^{(K)} \sim 2.5 - 3 \times \mathbf{m_c}$

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Given HQET scaling law for Q:

$$\frac{Q(m_h)}{Q(m_h/\lambda)} \xrightarrow{m_h \to \infty} \mathbf{1}$$

→ static limit for ratios is exactly known.

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Interpolation
(HQET inspired)
of ratios in the b-region using *safely* computed relativistic data and the exact static limit.

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– Consider a *chain* of ratios of a quantity **Q(m_h)** formed for a sequence of heavy *q*-masses: $m_h^{(n)} = \lambda\, m_h^{(n-1)}$

$$\mathbf{Q(m_b)} \equiv \mathbf{Q(m_h^{(N+1)})} = Q(m_h^{(1)}) \times \frac{Q(m_h^{(2)})}{Q(m_h^{(1)})} \times \frac{Q(m_h^{(3)})}{Q(m_h^{(2)})} \times \ldots \times \frac{Q(m_h^{(K)})}{Q(m_h^{(K-1)})} \times \frac{Q(m_h^{(K+1)})}{Q(m_h^{(K)})} \times \ldots \times \frac{Q(m_h^{(N+1)})}{Q(m_h^{(N)})}$$

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Interpolation
(HQET inspired)
of ratios in the b-region using *safely* computed relativistic data and the exact static limit.

➡ Tune $\boldsymbol{\lambda}$ in a way to

- set $Q(m_h^{(N+1)}) = Q(m_b)|_{(\mathrm{expt.})} \longrightarrow m_b = \lambda^N m_h^{(1)}$
  then
- make predictions for any other h-quark quantity through a similar chain procedure that ends up to $m_b = m_h^{(N+1)}$

m_b computation

$$Q_m = \frac{M_{hs}}{(M_{h\ell})^{\gamma}(M_{cs})^{(1-\gamma)}}$$

[γ : free parameter (*no need for tuning*)
used for gaining extra control
of syst. uncertainties]

$$\lim_{m_h^{\rm pole}\to\infty}\left(\frac{Q_m}{(m_h^{\rm pole})^{(1-\gamma)}}\right) = {\rm const.}$$

(HQET asymptotic condition)

ratio

$$y_Q(m_h^{(n)}, \lambda; m_\ell, m_s, a) = \lambda^{(\gamma-1)} \frac{Q_m(m_h^{(n)}; m_\ell, m_s, a)}{Q_m(m_h^{(n)}/\lambda; m_\ell, m_s, a)} \left(\frac{\rho(m_h^{(n)}, \mu^*)}{\rho(m_h^{(n)}/\lambda, \mu^*)}\right)^{(\gamma-1)}$$

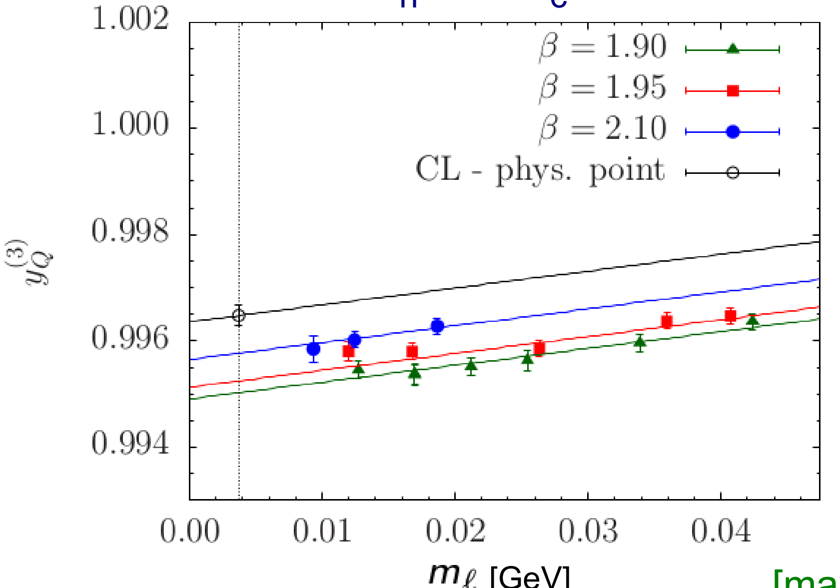
$$m_h^{(n)} = \lambda\ m_h^{(n-1)}$$

$$[m_h^{\rm pole} = \rho(m_h, \mu^*)\ \ m_h(\mu^*)]$$

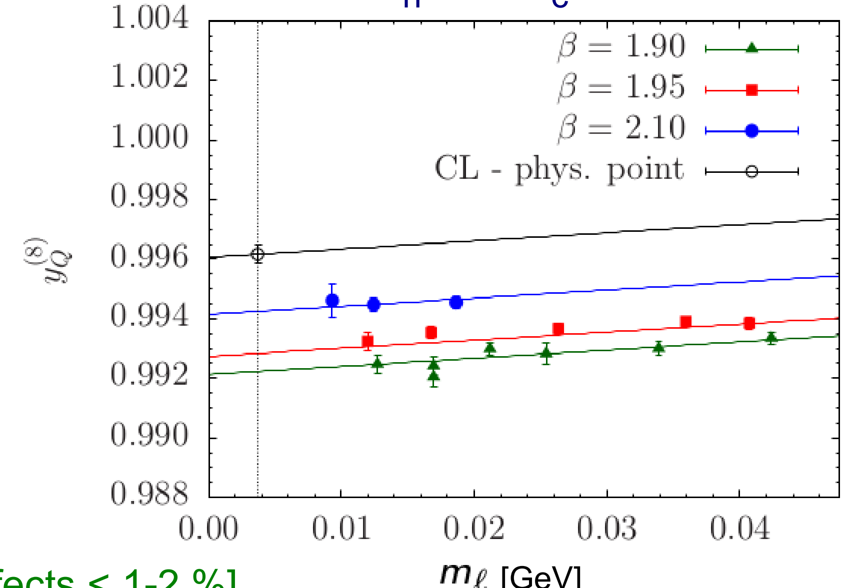
(extrapolate in CL + phys. q-light)

(known to N^3LO;
strong cancellations in ratios →
sub % effect to final results)

m_h~1.4 m_c



m_h ~ 3 m_c



[max. cutoff effects < 1-2 %]

m_b computation

$$Q_m = \frac{M_{hs}}{(M_{h\ell})^{\gamma}(M_{cs})^{(1-\gamma)}}$$

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$$m_h^{(n)} = \lambda\; m_h^{(n-1)}$$

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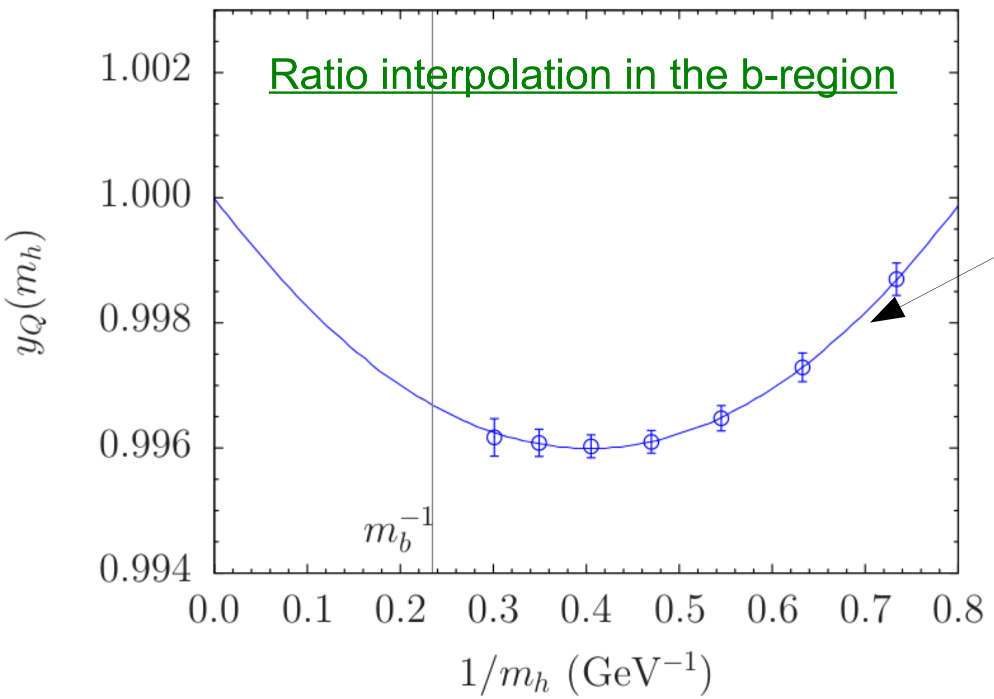
(known to N3LO;
strong cancellations in ratios →
sub % effect to final results)



HQET inspired ansatz

$$y_Q = 1 + \frac{\eta_1}{m_h} + \frac{\eta_2}{m_h^2}$$

Satisfy chain equation by tuning *step λ*, such that

$Q(m_h^{(N)})$ matches $(M_{Bs}/(M_B)^{\gamma})(M_{Ds})^{(\gamma-1)}$

$$m_b = \lambda^N m_h^{(1)}$$

Ratio method offers a simple way to determine m_b/m_c

f_{Bs} & f_{Bs}/f_B

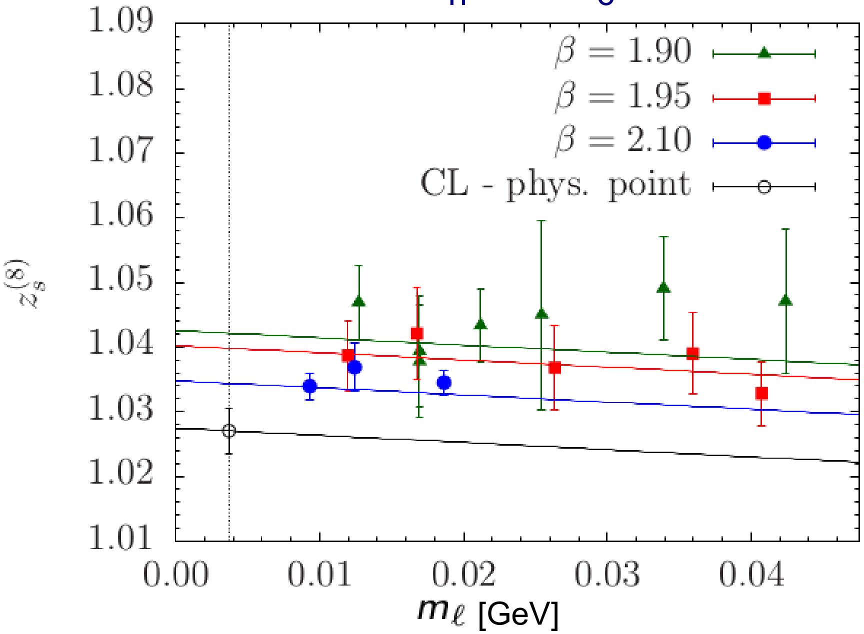
$\mathcal{F}_{hq} = f_{hq}/M_{hq}, \quad q = \ell, s$

$$\lim_{m_h^{\text{pole}}\to\infty} \mathcal{F}_{hq}\,(m_h^{\text{pole}})^{3/2} = \text{const.} \qquad \lim_{m_h^{\text{pole}}\to\infty} \left(\mathcal{F}_{hs}/\mathcal{F}_{h\ell}\right) = \text{const.}$$

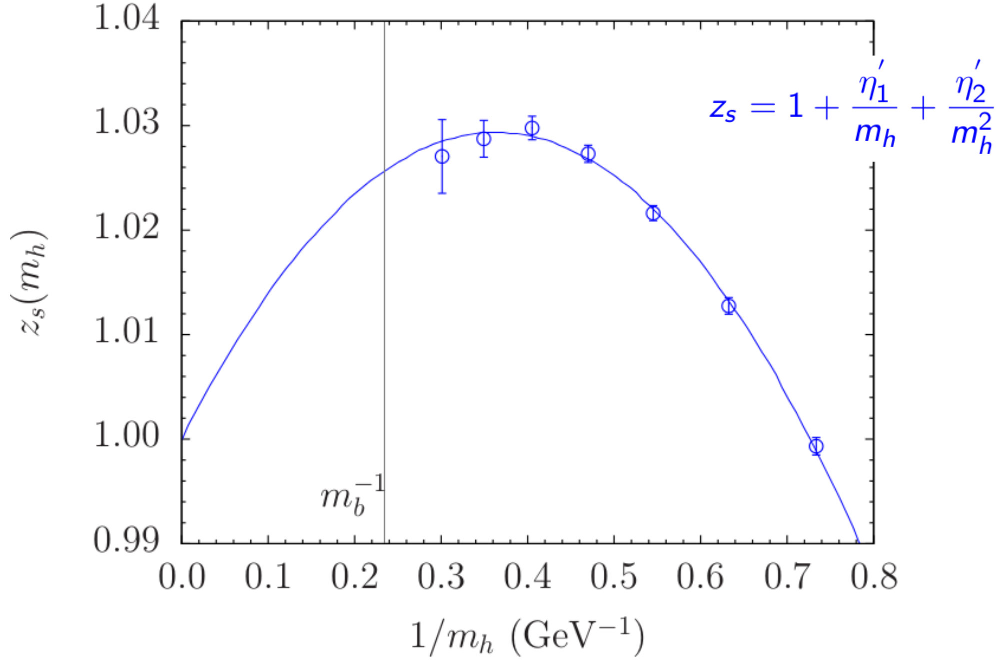
(HQET asymptotic conditions)

f_{Bs}

$m_h \sim 3\ m_c$



Ratio interpolation in the b-region



Use the chain equation up to $m_b = \lambda^N m_h^{(1)}$ to obtain f_{Bs}/M_{Bs}

and finally use $\mathbf{M_{Bs(expt)}}$ to determine f_{Bs}

f_{Bs} & f_{Bs}/f_B

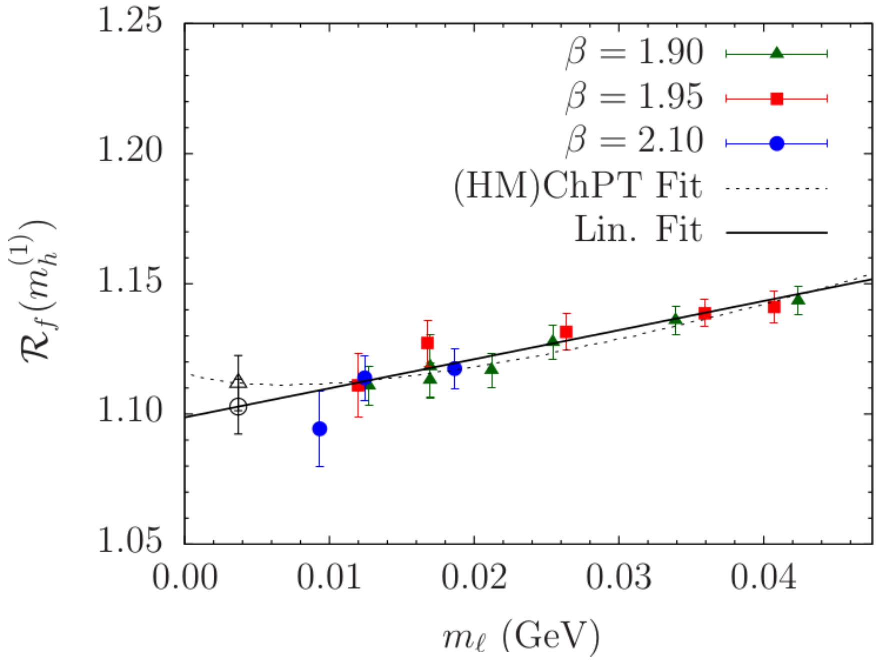
$\mathcal{F}_{hq} = f_{hq}/M_{hq}, \quad q = \ell, s$

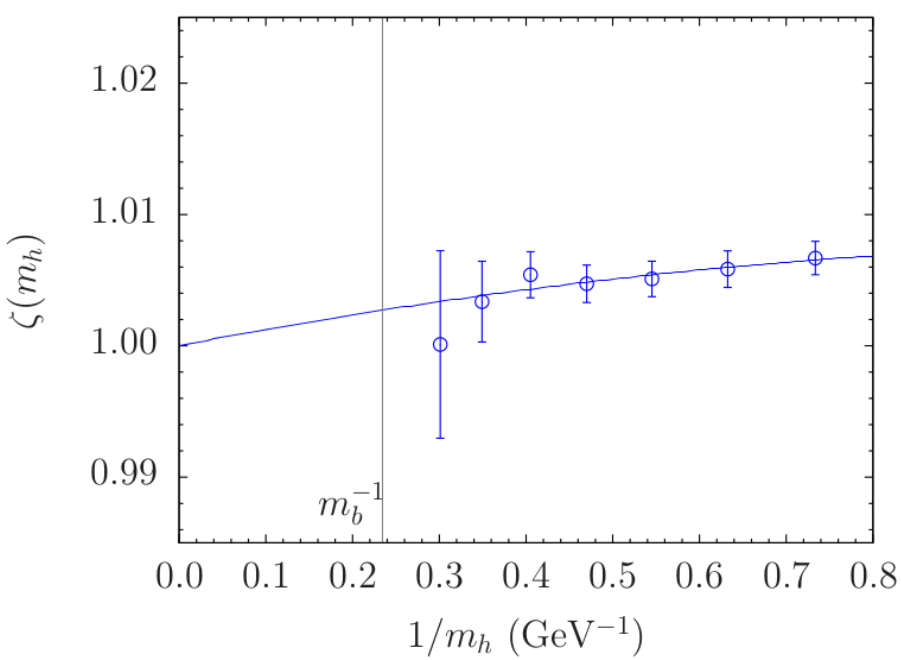
$$\lim_{m_h^{\rm pole}\to\infty} \mathcal{F}_{hq}\,(m_h^{\rm pole})^{3/2} = \text{const.} \qquad \lim_{m_h^{\rm pole}\to\infty} \left(\mathcal{F}_{hs}/\mathcal{F}_{h\ell}\right) = \text{const.}$$

(HQET asymptotic conditions)

f_{Bs}/f_B

(Double) ratio interpolation in the b-region





$\mathcal{R}_f = [(\mathcal{F}_{hs}/\mathcal{F}_{h\ell})/(f_{s\ell}/f_{\ell\ell})]$ ◄—— Control better the systematics from the fit of chiral log

Use the chain equation up to $m_b = \lambda^N m_h^{(1)}$ to obtain f_{Bs}/f_B using as input $(M_{Bs}/M_B)_{expt}$

Results & Error Budget

$$m_b(\overline{\mathrm{MS}}, m_b) = 4.26(9)\ \mathrm{GeV}$$
$$m_b/m_c = 4.42(8)$$
$$f_{Bs} = 229(5)\ \mathrm{MeV}$$
$$f_{Bs}/f_B = 1.179(26)$$
$$f_B = f_{Bs}/(f_{Bs}/f_B) = 194(6)\ \mathrm{MeV}$$

Preliminary!

ETMC

$N_f = 2+1+1$

$3\ a \in [0.06, 0.09]$ fm

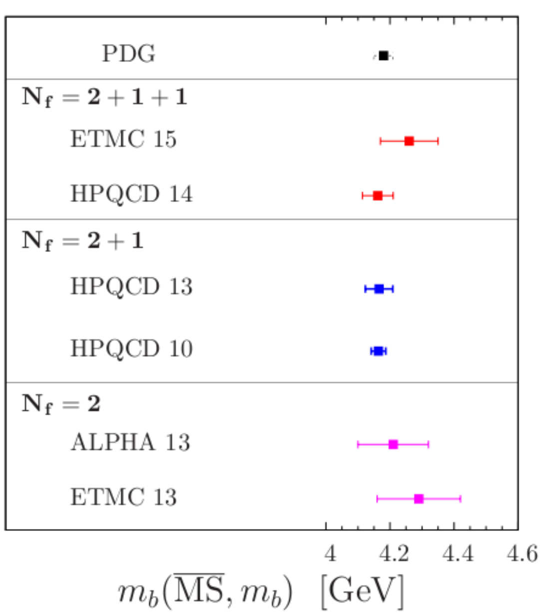
$m_\pi \in [210, 440]$ MeV

$m_\pi L \in [3.1, 5.4]$

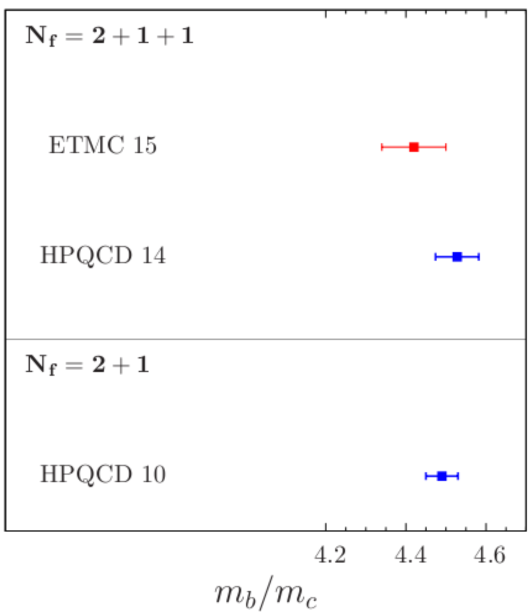
$L \in [2, 3]$ fm

uncertainty (in %)	m_b	m_b/m_c
stat+fit	1.0	1.4
syst. from discr. effects	1.8	0.9
syst. from ratios	0.9	0.8
syst. from chiral extrap.	0.4	0.3
input from experiment	< 0.01	< 0.01
Total	2.3	1.9

uncertainty (in %)	f_{Bs}	f_{Bs}/f_B	f_B
stat+fit	1.7	1.5	2.5
syst. from discr. effects	1.3	0.6	0.7
syst. from ratios	0.5	0.3	0.6
syst. from chiral extrap.	0.3	0.2	0.4
syst. from fit at the trig. point	-	1.4	1.4
input from experiment	< 0.01	< 0.01	< 0.01
Total	2.2	2.2	3.0

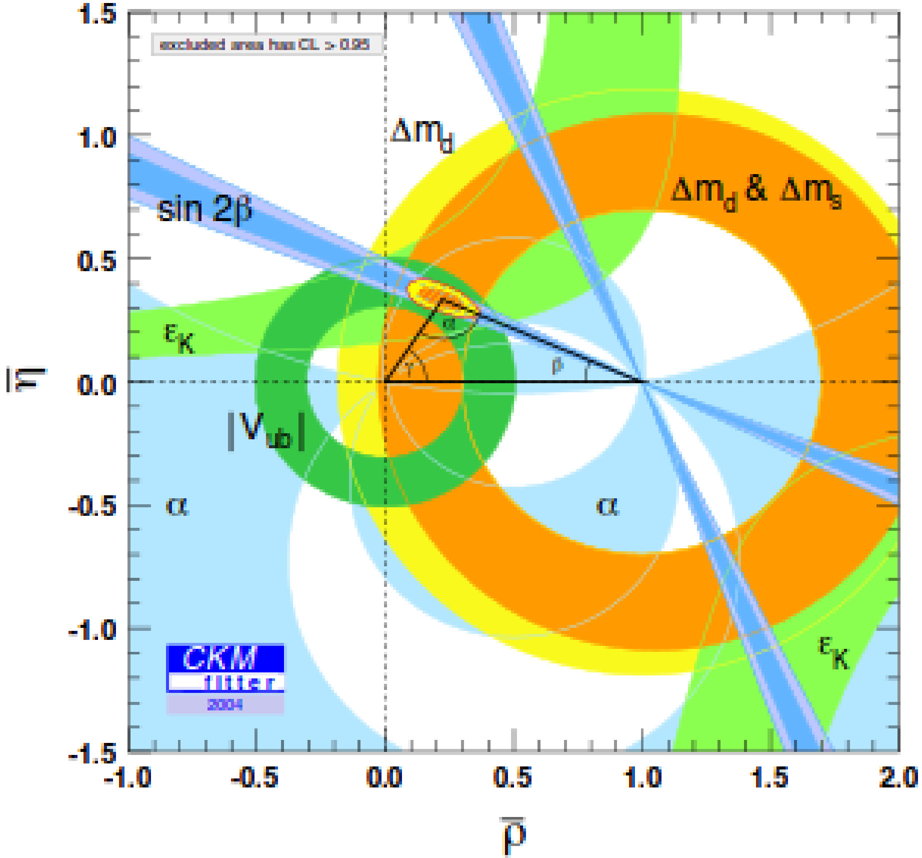


(to appear)
(1408.4169)

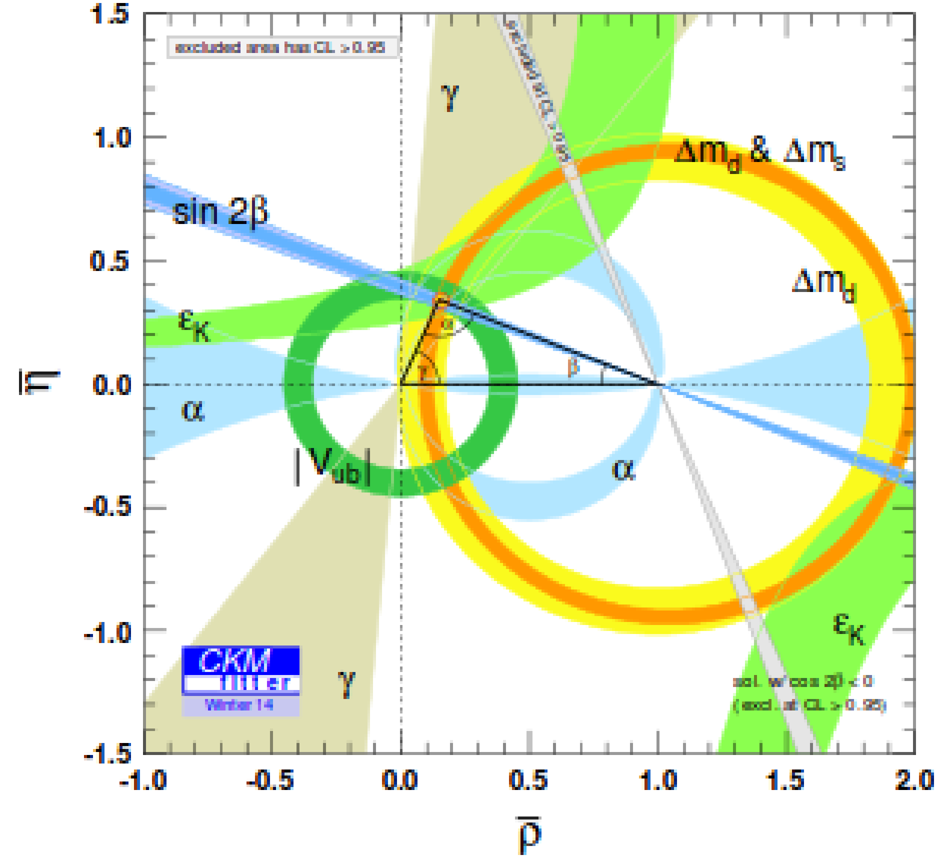


(to appear)
(1408.4169)

2004 ⟶ today



2004



2014

➡ Thanks to Experimental achievements and development (mainly B-factories) & Theoretical progress and improvements (→ important role of LQCD)

2004 → today

from LQCD	2004		2015	
f_D(MeV)	225(25)	(11%)	212.1(1.2)	(0.6%)
f_{Ds}(MeV)	263(25)	(10%)	248.8(1.4)	(0.6%)
f_B(MeV)	189(27)	(14%)	188.5(3.3)	(1.8%)
f_{Bs}(MeV)	230(30)	(13%)	226.5(3.3)	(1.5%)
f_{Bs}/f_B	1.22(6)	(5%)	1.203(7)	(0.6%)
$f_{Bd}\hat{B}_d^{1/2}$(MeV)	214(38)	(18%)	216(10)	(4.6%)
$f_{Bs}\hat{B}_s^{1/2}$(MeV)	262(35)	(13%)	262(10)	(3.8%)

2004: Hashimoto ICHEP 2004 (*B*-results)
FNAL : hep-lat/0410030 (*D*-results)

2015: averages over N_f=2+1+1 results
& FLAG2013

- Impressive improvement for phenomenologically useful observables
  (past lattice forecasts have been proved to be conservative)

FLAG	itpwiki.unibe.ch/flag
HFAG	http://www.slac.stanford.edu/xorg/hfag/
UT*fit*	www.utfit.org/UTfit
CKMfitter	ckmfitter.in2p3.fr/
LATTICE2015	www.aics.riken.jp/sympo/lattice2015/

Summary

- There is still increasing interest for lattice determinations of hadronic weak ME.

- **Precise LQCD** computations are being carried out by various collaborations. Systematic and statistical errors are being progressively reduced.

- For some interesting processes (*e.g. D* and *B* leptonic decays) in the phenomenology of SM theoretical (Lattice) precision is *competitive* with (or *better* than) the experimental one.

- Experimental precision ambitions in the quark flavour sector are high.
  A *vast* program on loop/Cabibbo suppressed processes with horizon of 2020 (BESIII, BelleII, LHCb) aim at providing footprints of NP effects.

- The goal of '1% precision' in hadronic matrix elements' determination is being achieved for many interesting cases. Depending on the relative (process by process) experimental precision, EM effects should be included.