Heavy-light spectra and decays

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Outline

- Heavy-light spectra: B, B_s, D, D_s
  - Motivation and general considerations.
  - $1S$ hyperfine splittings.
  - Higher states: near threshold ($D^*_{s0}(2317)$) and resonances.
- Heavy-light-light, heavy-heavy-light spectra: cqq, ccq, bbq, bqq, bcq.
- Decays: D_s semi-leptonic decays to flavour singlets.
- Summary

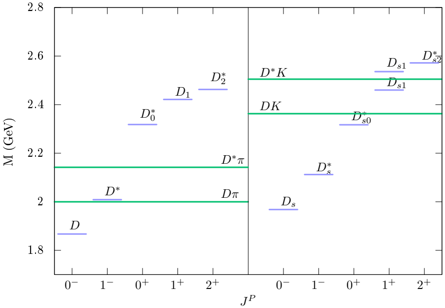
Heavy-light spectra

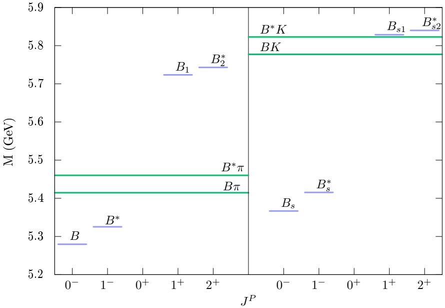
Motivation:

- Postdiction of states well established experimentally.
  - Demonstration of lattice techniques.
  - (Precision) tests show systematics are under control, supports determinations of other quantities, m_c, m_b, f_D, f_{D_s}.
- Postdiction of states less established experimentally.
  - Help with spin and parity assignments.
  - Whether a bound state/resonance exists.
- Prediction of new states (better test of lattice methods).
  - Expected from quark model.
  - Non-standard, $q\bar{q}q\bar{q}$, hybrids.
- Investigating internal structure of non-standard candidates.
- Testing theoretical descriptions: HQET.

Heavy-light spectra

Experimentally observed meson spectra:

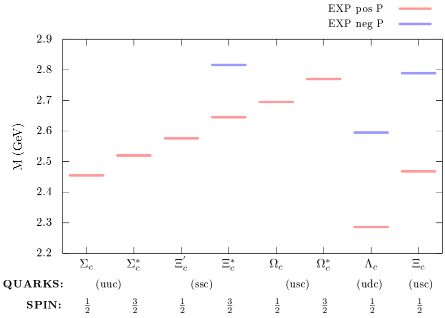


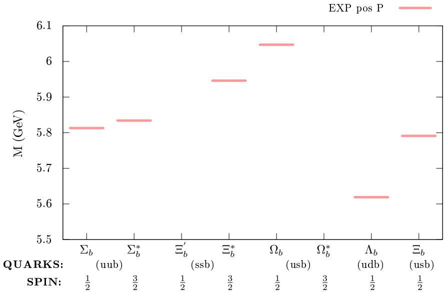


- Additional states: $D_1(2430)^0$, $D(2550)^0$, $D(2600)$, $D^*(2640)^\pm$, $D(2750)$, $D_{s1}^*(2700)^\pm$, $D_{sJ}(2860)$, $D_{sJ}(3040)^\pm$, $B_J^*(5732)$, $B_{sJ}(5850)$.
- Additional thresholds: $D\pi\pi$, $D_s\pi$, ...
- Widths: $D^* < 2.1$ MeV, $D_0^* = 267(40)$ MeV, $D_1 = 27(3)$ MeV, $D_2^* = 49(1)$, $D_{s0}^* < 3.8$ MeV, $D_{s1}(2460) < 3.5$ MeV, $D_{s1}(2536) = 0.92$ MeV, $D_{s2}^* = 17(4)$ MeV, $B_2^* = 23^{+5}_{-11}$ MeV, $B_{s2}^* = 1.6(5)$ MeV.

Heavy(-light)-light spectra

Experimentally observed baryon spectra:





- Additional states: $\Lambda_c(2625)$, $\Lambda_c(2880)$, $\Lambda_c(2940)$, $\Sigma_c(2800)$, $\Xi_c(2880)$, $\Xi_c(3080)$, $\Lambda_b(5912)$, $\Lambda_b(5920)$.
- Thresholds: $\Lambda_c\pi$, $\Lambda_c\pi\pi$, $\Xi_c\pi$, $\Xi_c\pi\pi$, $\Sigma_c\pi$, $\Sigma_c K$, $\Lambda_c K$, ...
- Stable: Λ_c, Ξ_c, Ξ_c', Ω_c, Λ_b, Ξ_b, Ω_b.
- Widths: Λ_c $(\frac{1}{2}^-)$=2.6(6) MeV, Σ_c $(\frac{1}{2}^+)$=2.2(3) MeV, Σ_c^* $(\frac{3}{2}^+)$=14.5(1.5) MeV, Ξ_c^* $(\frac{3}{2}^+)$<5.5 MeV, Ξ_c $(\frac{1}{2}^-)$<12 MeV, Ξ_c^* $(\frac{3}{2}^-)$<6.5 MeV.

Lattice considerations

General:

- QCD, QED effects neglected (not for much longer).
- Identification of quantum numbers: construct lattice operations respecting lattice cubic symmetry. Example bosons:
  - $A_1 \to J = 0, 4, \ldots$, $T_1 \to J = 1, 3, 4, \ldots$
  - $E \to J = 2, 4$, $T_2 \to J = 2, 3, 4$, $A_2 \to J = 3, \ldots$
- Stability under strong decay (lattice simulation).

Simulation:

- Statistics
- Systematics: light quark mass, volume, discretisation effects, ...

Landscape of lattice simulations

m_u, m_s

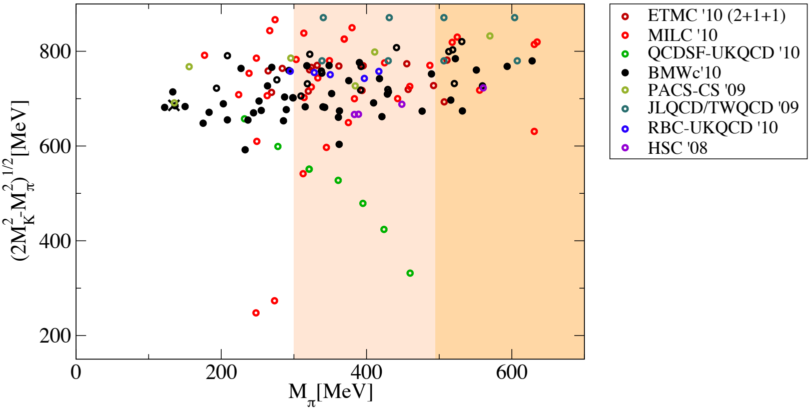


Figure taken from C Hoelbling, arXiv:1410.3403

Landscape of lattice simulations

Volume.

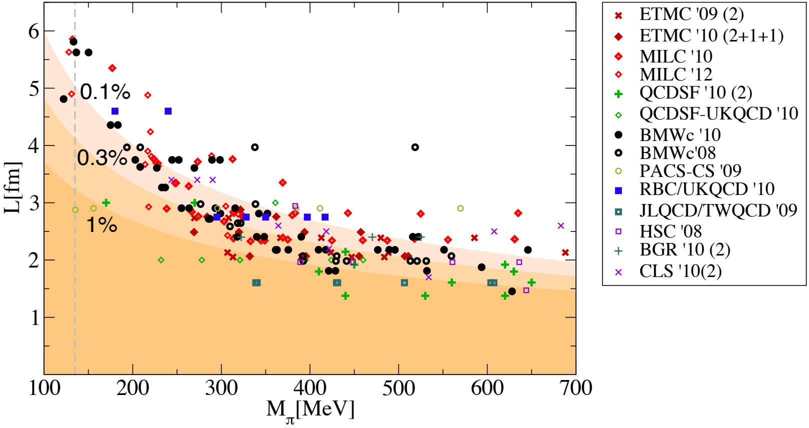


Figure taken from C Hoelbling, arXiv:1410.3403

Landscape of lattice simulations

Lattice spacing.

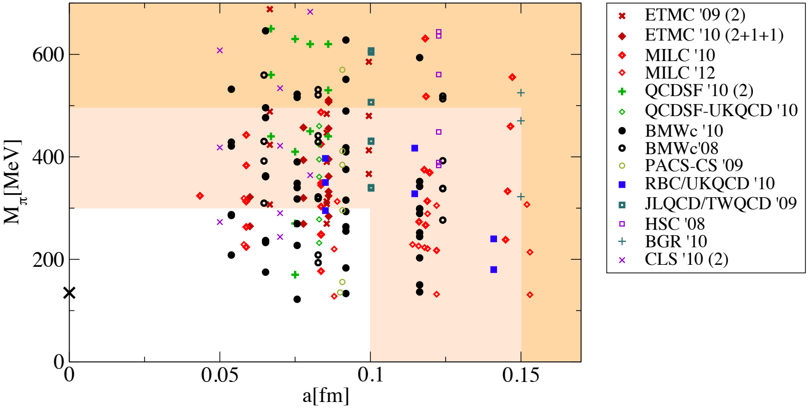


Figure taken from C Hoelbling, arXiv:1410.3403

Heavy quark approaches on the lattice

$a = 0.05$ fm, $a^{-1} \approx 4$ GeV, $am_c \sim 1/3$, $am_b > 1$.

Effective field theories:

- HQET
- NRQCD
- Relativistic heavy quark actions

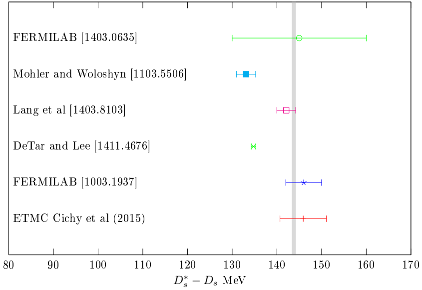
Relativistic actions:

- staggered (HISQ)
- Wilson (Clover)
- Twisted mass
- ...

Lattice action: systematically improveable.

Hyperfine splittings: D_s

Sensitive to many systematics: discretisation effects, quark mass tuning, ...

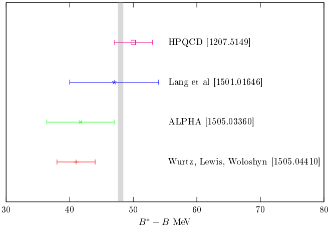


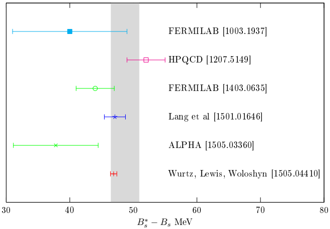
NB: ETMC $D_s^* = 2.1107(52)$ GeV, $D_s = 1.9648(36)$ GeV $\rightarrow \Delta M = 145.9(5.2)$ MeV.

Also SFB-TRR 55, T. Rae (Wuppertal), $N_f = 2+1+1$, BMW-c, 3-HEX clover, $a = 0.064 - 0.102$ fm.

Fermilab: MILC $N_f = 2+1$, $a = 0.09 - 0.15$ fm, update $a = 0.045 - 0.15$ fm.
DeTar and Lee: MILC $N_f = 2+1+1$, $a = 0.15$ fm, Fermilab charm.
ETMC: $N_f = 2+1+1$, $a = 0.062 - 0.089$ fm, twisted mass.
Lang at al: PACS-CS $N_f = 2+1$, $a = 0.091$ fm, Fermilab charm.
Mohler and Woloshyn: PACS-CS $N_f = 2+1$, $a = 0.091$ fm, Fermilab charm.

Hyperfine splittings: B, B_s



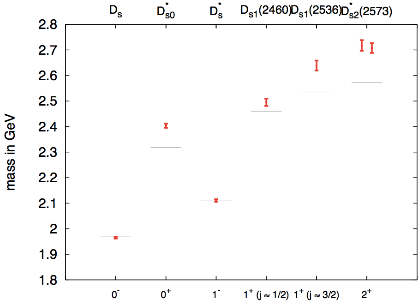


Wurtz et al: PACS-CS $N_f = 2 + 1$, $a = 0.091$ fm, NRQCD bottom.
ALPHA: $N_f = 2$, $a = 0.048 - 0.075$ fm, HQET bottom.
Lang et al: PACS-CS $N_f = 2 + 1$, $a = 0.091$ fm, Fermilab bottom.
HPQCD: MILC $N_f = 2 + 1 + 1$, $a = 0.09 - 0.15$ fm, NRQCD bottom.
Fermilab: MILC $N_f = 2 + 1$, $a = 0.09 - 0.15$ fm, update $a = 0.045 - 0.15$ fm.

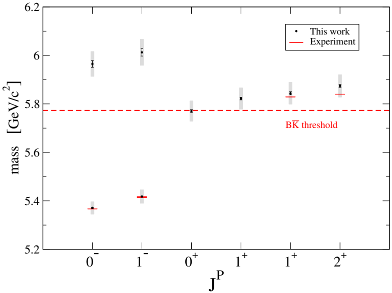
Higher states with $q\bar{q}$

First step

D_s: ETMC (Lattice 2015)



B_s: Wurtz et al. [1505.04410]

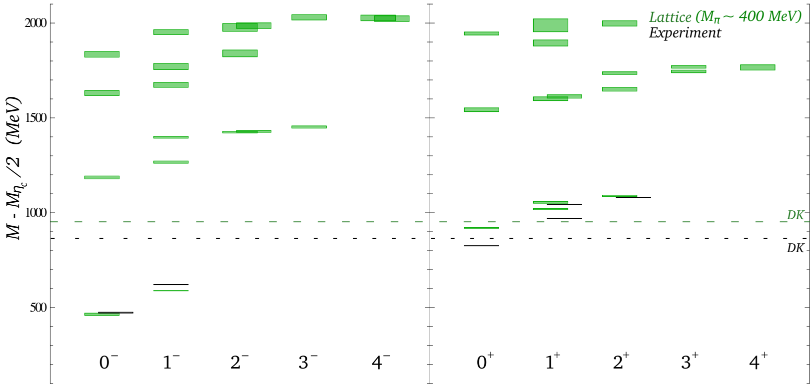


D/D_s: Mohler and Woloshyn [1103.5506], De Tar and Lee [1411.4676], ETMC (Lattice 2015), Hadron Spectrum Collaboration [1301.7670],...

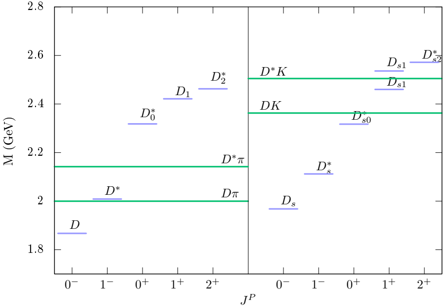
B/B_s: ALPHA [1505.03360], ...

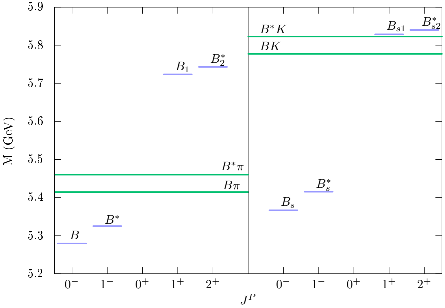
Higher states with $q\bar{q}$

Hadron Spectrum Collaboration [1301.7670]: $N_f = 2 + 1$, anisotropic lattices, $a_t^{-1} = 0.035$ fm, $a_s = 0.12$ fm, tree-level clover quark action, $L = 1.9$ fm and 2.9 fm (shown).



Near threshold states and resonances





$D^*_{s0}(2317)$, $J^P = 0^+$, $D_{s1}(2460)$, $J^P = 1^+$, narrow states just below (S-wave) DK and D^*K thresholds.

B_s analogues not yet discovered.

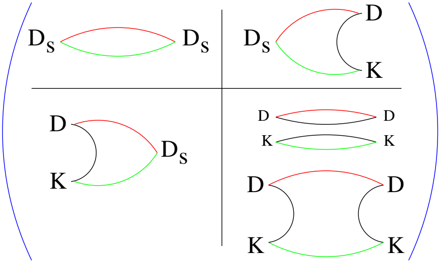
What are the natures of the states?

$D_{s0}^*(2317)$, $J^P = 0^+$

Lattice calculation of "bound" states close to threshold

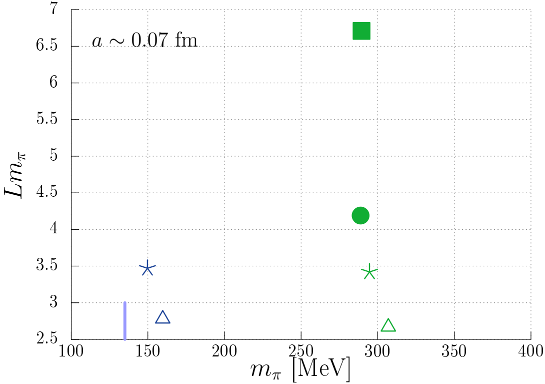
- Physical DK threshold: close to physical light quark mass, study the volume dependence.
- DK in S-wave, consider $D(0)K(0)$ ($D(p)K(-p)$ omitted).

Diagonalise



Lattice details

RQCD+QCDSF: $N_f = 2$ non-perturbatively improved clover.

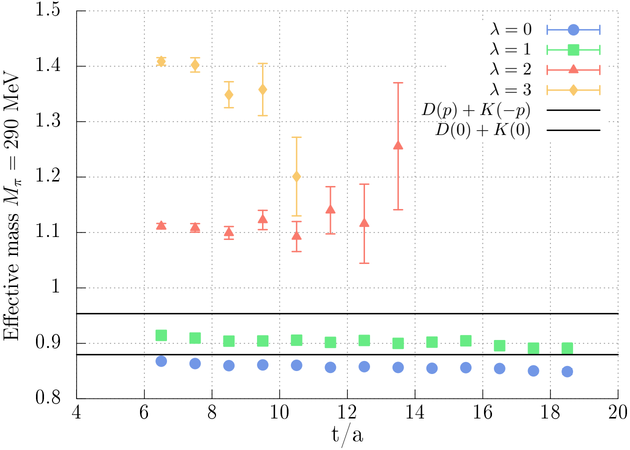


Operators: $c\bar{s}$, $c\gamma_4\bar{s}$, 3 smearings, $c\gamma_5\bar{\ell}(0)\ell\gamma_5\bar{s}(0)$, 1 smearing.

Use stochastic estimation: one-end trick + sequential propagators following CP-PACS [0708.3705] ($\rho \to \pi\pi$). Statistics: 800-2000 configurations.

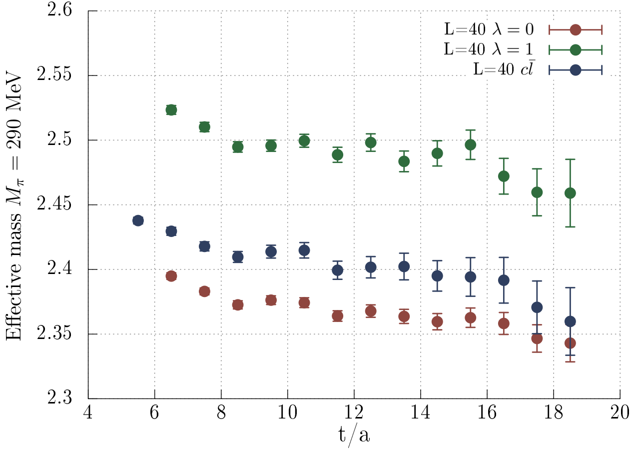
Eigenvalues, $M_\pi = 290$ MeV, $L = 40$

Antonio Cox (Regensburg)



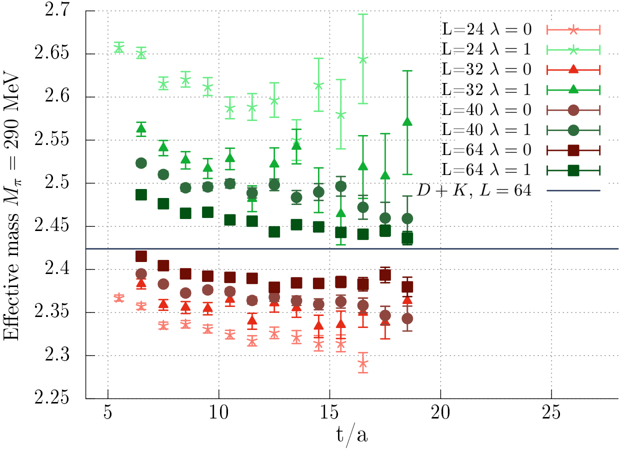
Comparison with $c\bar{s}$

Antonio Cox (Regensburg)



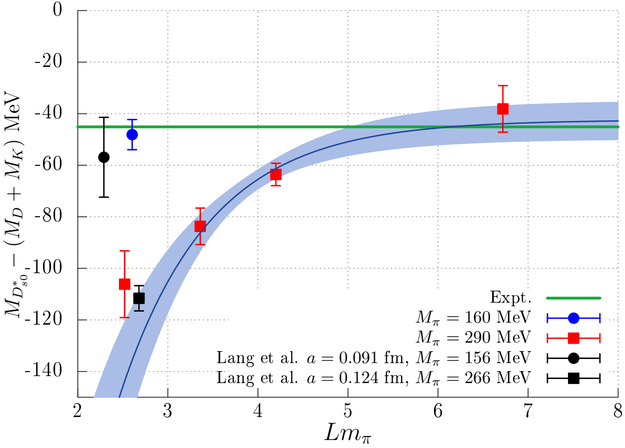
Volume dependence

A. Cox (Regensburg)



Splitting with threshold

A. Cox (Regensburg): (VERY) PRELIMINARY



Comparison with Lang et al. [1403.8103].
Preliminary fit function $a + be^{-Lm_\pi}$.

Other studies

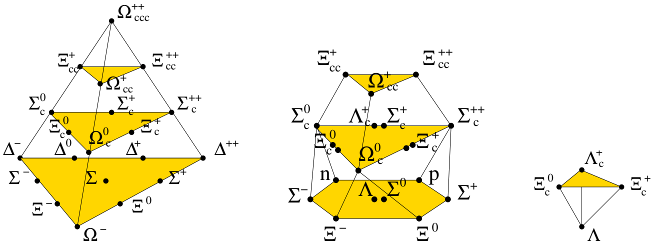
Lang et al. [1403.8103] use the effective range approximation below threshold to estimate infinite volume masses:

Ens (M_π)	$m_K + m_D - m_{D_{s0}^*}$	$m_{D_{s0}^*} - \frac{1}{4}(m_{D_s} + 3m_{D_s})$
266 MeV	79.9(5.4)(0.8)	287(5)(3)
156 MeV	36.6(16.6)(0.5)	266(17)(4)
Expt	45.1	241.5

Also

- For $M_\pi = 156$ MeV ens. $D_{s1}(2460)$, $D_{s1}(2536)$ and $D_{s2}^*(2573)$ (only $q\bar{q}$) consistent with expt..
- For $M_\pi = 266$ MeV ens. $D\pi$ scattering study for $D_0^*(2400)$ and $D_1(2430)$ resonances, Mohler et al. [1208.4059].
- For $M_\pi = 156$ MeV ens. $B_{s1}(5830)$, $B_{s2}^*(5840)$ (both $q\bar{q}$) and find B_{s0}^* and B_{s1} are bound states below the $B^{(*)}K$ thresholds, Lang et al. [1501.01646].
- Hadron Spectrum Collaboration: D_s/DK and $D/D\pi$ spectra Ryan et al. (Lattice 2014).

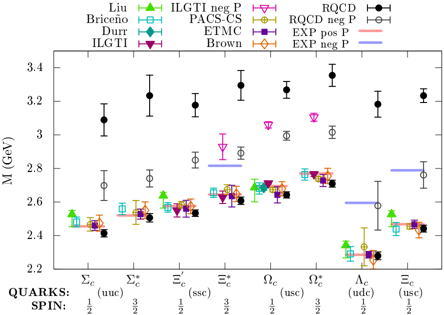
Charmed/bottomed baryons



$SU(4)$ representations : $\quad 4 \otimes 4 \otimes 4 \quad = \quad 20_S \ \oplus 20_M \oplus 20_M \ \oplus \overline{4}_A$

Ground states: 20_S has $J = \frac{3}{2}^+$, 20_M has $J = \frac{1}{2}^+$ and $\overline{4}_A$ has $J = \frac{1}{2}^-$ (non-rel. limit).

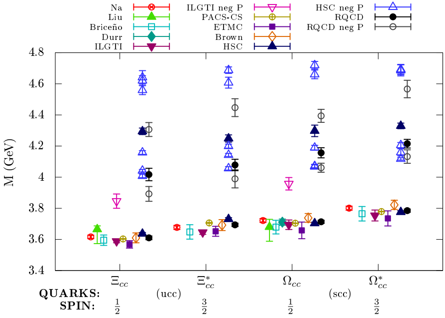
Spectrum singly charmed baryons



$N_f = 2 + 1$: Liu et al. clover/DW [0909.3294], PACS-CS NP-clover/NP-clover [1301.4743], Brown et al. FNAL-clover/Domain Wall [1409.0497].
$N_f = 2 + 1 + 1$: Briceno et al. clover/HISQ [1207.3536], ILGTI overlap/HISQ [1312.3050], ETMC Twisted Mass/Twisted Mass [1406.4310].

Also HSC [1410.8791], QCDSF [1311.5010], Na et al. [0812.1235]

Spectrum doubly charmed baryons



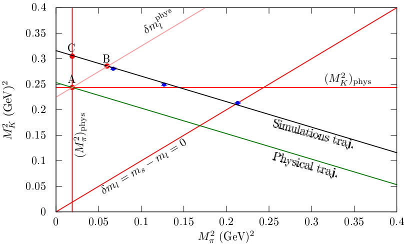
Including also $N_f = 2$: Dürr et al. Brilloin/NP-clover [1208.6270]

Lattice results are consistent and approx. 80 MeV above SELEX result for $\Xi_{cc} = 3518.7(1.7)$ [hep-ex/0406033].

Borsanyi et al. [1406.4088] QCD+QED: $\Xi_{cc}^{++} - \Xi_{cc}^{+} = 2.16(11)(17)$ MeV.

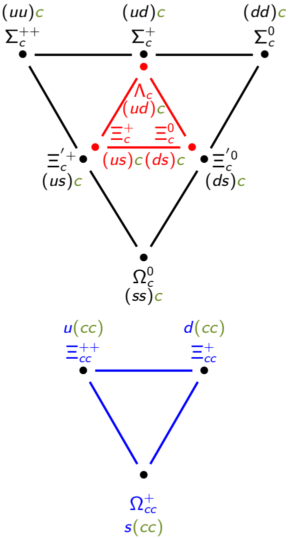
RQCD results

- 3 operator (smearings) basis for variational method.
- Found for $J^P = \frac{1}{2}^-$, Ω_c ground state degenerate with $\Xi_c + K$ and Ξ_c' with $\Lambda_c + K$. Identify as scattering states.
- Use QCDSF $N_f = 2+1$ configurations, simulate along $\bar{m} = \frac{1}{3}(m_s + 2m_{u/d}) = \text{const.} \propto (X_\pi^{phys})^2 = \frac{1}{3}(2M_K^2 + M_\pi^2)$.
- In practice, X_π is 60 MeV heavier due to change in $a \sim 0.083$ fm from average octet baryon mass QCDSF [1003.1114] to $a \sim 0.075$ fm.



Results on same configurations from QCDSF-UKQCD, R.Horsley et al. [1311.5010].

$SU(3)_F$, Gell-Mann Okubo formulae, charm spectator



Sextet

$$m_{\Sigma_c^{(*)}} = m_0 - \tfrac{2}{3} A\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$

$$m_{\Xi_c'^{(*)}} = m_0 + \tfrac{1}{3} A\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$

$$m_{\Omega_c^{(*)}} = m_0 + \tfrac{4}{3} A\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$

Anti-triplet

$$m_{\Lambda_c} = m_0 - \tfrac{2}{3} B\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$

$$m_{\Xi_c} = m_0 + \tfrac{1}{3} B\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$

Triplet

$$m_{\Xi_{cc}^{(*)}} = m_0 - \tfrac{1}{3} C\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$

$$m_{\Omega_{cc}^{(*)}} = m_0 + \tfrac{2}{3} C\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$

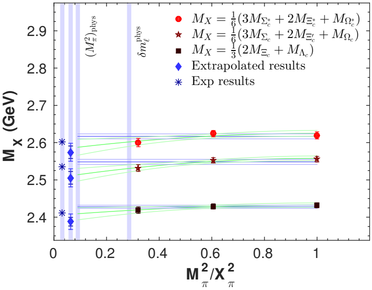
$$\delta m_\ell = m_s - m_{u/d} \propto 1 - M_\pi^2/X_\pi^2 + O((\delta m_\ell)^2)$$

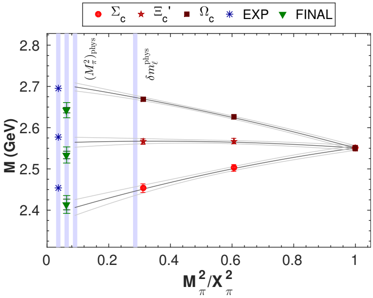
Flavour singlet combinations

$(c\ell\ell)$	$\frac{1}{6}(3m_{\Sigma_c} + 2m_{\Xi_c'} + m_{\Omega_c})$
	$\frac{1}{3}(2m_{\Xi_c} + m_{\Lambda_c})$
$(cc\ell)$	$\frac{1}{3}(m_{\Omega_{cc}} + 2m_{\Xi_{cc}})$

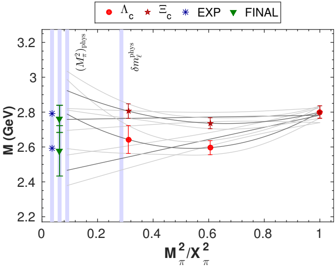
$SU(3)_F$ flavour breaking, P.Perez-Rubio

Positive parity:



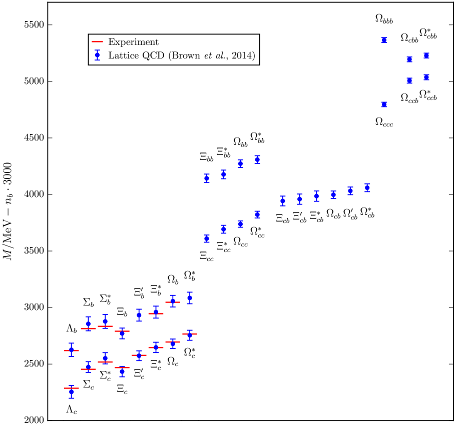


Negative parity



Bottomed baryons

Brown et al. [1409.0497], RBC/UKQCD $N_f = 2 + 1$ domain wall sea + valence.
Relativistic heavy quark action for charm, NRQCD for bottom.

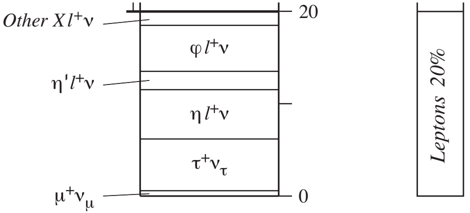


Also: Burch [1502.00675].

D_s semi-leptonic decays

20% of decays involving leptons.

- Leptonic decays, $D_s \to \ell^+\nu$, $\langle 0|A_\mu|D_s\rangle = p_\mu f_{D_s}$. Well measured in expt. and on the lattice. FLAG report [1310.8555] $f_{D_s} = 248.6 \pm 2.7$ MeV for $N_f = 2+1$, used to determine V_{cs}. Expt: $f_{D_s} = 257.5(4.6)$ MeV PDG (2013) (using V_{cs}).
- Semi-leptonic decay $D_s \to \phi\,\ell^+\nu$. Helicity functions measured in expt. On the lattice only HPQCD [1311.6669].
- Semi-leptonic decay $D_s \to \eta^{(\prime)}\,\ell^+\nu$. Only branching fractions measured by CLEO [0903.0601].



Rosner and Wohl (2010) PDG review.

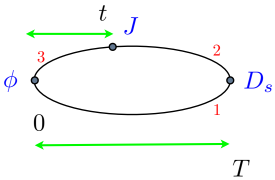
$D_s \to \phi \ell \nu$

HPQCD: [1311.6669], MILC $N_f = 2+1$, $a = 0.09$, 0.12 fm. HISQ charm+strange.

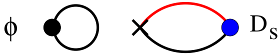
$\langle \phi(p', \varepsilon) | V^\mu - A^\mu | D_s(p) \rangle$, five form factors V, A_0, A_1, A_2, A_3.

Differential decay rate: $m_\ell \to 0$, V, A_1 and A_2 contribute (A_3 is not independent).
CKM: V_{cs}

Computed



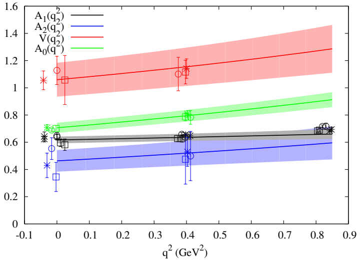
Ignored



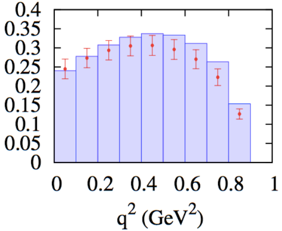
ϕ treated as stable.

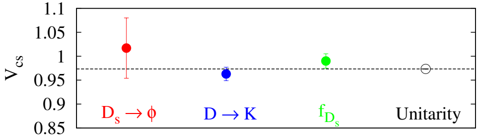
$D_s \to \phi \ell \nu$

Formfactors:

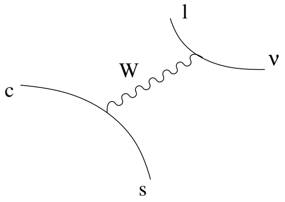


Compare to differential decay rate from BaBar [0807.1599] using V_{cs} from unitarity or use to determine V_{cs}.





$D_s \to \eta\ell^+\nu$, $D_s \to \eta'\ell^+\nu$



$$A = \frac{G_F}{\sqrt{2}} V_{cs} \bar{\ell}\gamma_\mu(1-\gamma_5)\nu\langle\eta^{(\prime)}|\bar{c}\gamma_\mu(1-\gamma_5)s|D_s\rangle$$

$\bar{c}\gamma_\mu\gamma_5 s$ doesn't contribute due to parity.

$$\langle\eta^{(\prime)}|V_\mu|D_s\rangle = f_+(q^2)\left(p_{D_s\,\mu} + p_{\eta^{(\prime)}\,\mu} - \frac{m_{D_s}^2 - m_{\eta^{(\prime)}}^2}{q^2}\, q_\mu\right) + f_0(q^2)\frac{m_{D_s}^2 - m_{\eta^{(\prime)}}^2}{q^2}\, q_\mu$$

Kinematical constraint at $q^2 = 0$: $f_+(0) = f_0(0)$

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2|V_{cs}|^2}{24\pi^3}\frac{(q^2 - m_\ell^2)^2\sqrt{E_{\eta^{(\prime)}}^2 - m_{\eta^{(\prime)}}^2}}{q^4 m_{D_s}^2}$$
$$\left[\left(1 + \frac{m_\ell^2}{2q^2}\right) m_{D_s}^2(E_{\eta^{(\prime)}}^2 - m_{\eta^{(\prime)}}^2)|f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2}(m_{D_s}^2 - m_{\eta^{(\prime)}}^2)^2|f_0(q^2)|^2\right]$$

If set $m_l = 0$, only $f_+(q^2)$ contributes.

Only branching fraction $Br = \Gamma(D_s \to \eta^{(\prime)}))/\Gamma$ is measured so far in expt.

CLEO collaboration [0903.0601]
$Br(D_s^+ \to \eta e^+ \nu_e) = (2.48 \pm 0.29 \pm 0.13)\%$
$Br(D_s^+ \to \eta' e^+ \nu_e) = (0.91 \pm 0.33 \pm 0.05)\%$

To compare, lattice results for $f_+(q^2)$ are needed.

However, we make use of PCVC relation to avoid renormalisation of V_μ operator (HPQCD [1008.4562])

$$q^\mu \langle V_\mu \rangle = (m_c - m_s)\langle S \rangle + O(a^2)$$

which leads to

$$f_0(q^2) = \frac{m_c - m_s}{M_{D_s}^2 - M_{\eta^{(\prime)}}^2} \langle \eta^{(\prime)} | S | D_s \rangle + O(a^2)$$

Only predict $f_0(0) = f_+(0)$.

Will make use of a parameterisation for $f_+(q^2)$ to predict Br.

Flavour singlets

SU(3) flavour symmetry (u,d,s): for mesons, $\bar{q}q$, we have $3 \otimes \bar{3} = 8 \oplus 1$

$$\text{octet}: \pi^0, \pi^\pm, K^\pm, K^0, \overline{K}^0, \eta, \qquad \text{singlet}: \eta'$$

$$\eta = \eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}), \qquad \eta' = \eta_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

Chiral symmetry $(m_q = 0)$: $\mathrm{SU}_A(3)$ symmetry spontaneously broken
$\pi^0, \pi^\pm, K^\pm, K^0, \overline{K}^0, \eta$ Goldstone bosons.

$\mathrm{U}_A(1)$ symmetry anomalously broken

$$\partial_\mu J_{\mu 5} = 2N_f\,\rho(x), \qquad \rho(x) = \tfrac{1}{32\pi^2}\epsilon^{\alpha\beta\mu\nu}\,\mathrm{Tr}(F_{\alpha\beta}F_{\mu\nu})$$

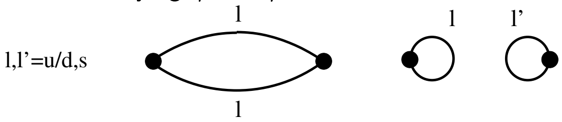
$Q = \sum_x \rho(x) \in \mathbb{Z}$

η' heavier than octet mesons.

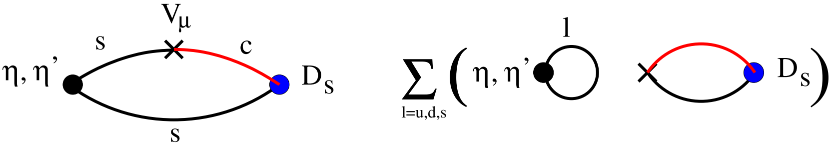
Physical η and η' mixtures of η_8 and η_1.

Technically challenging

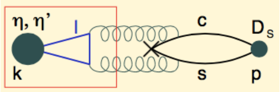
Quark line diagrams for studying η and η'.



For $D_s \to \eta(\eta')$



- Disconnected diagrams which may give a large contribution due to the anomaly also due to sum over $l = u, d, s$.

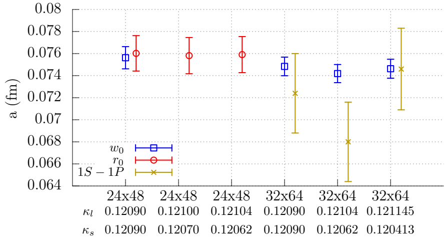


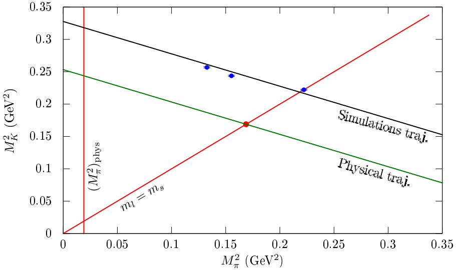
- Sensitivity to the topology of the gauge field configurations.

First step, determine the physical basis for η/η'.

Ensembles, lattice details

- $N_f = 2 + 1$, QCDSF configurations, Stout Link Non-perturbatively improved Clover (SLiNC) fermions, $O(a^2)$ discretisation errors in $f_0(q^2)$.
- Simulate along $\bar{m} = \frac{1}{3}(m_s + 2m_{u/d}) = \text{const.} \propto (X_\pi^{phys})^2 = \frac{1}{3}(2M_K^2 + M_\pi^2)$
- In practice, X_π is 60 MeV heavier due to change in $a \sim 0.083$ fm from average octet baryon mass QCDSF [1003.1114] to $a \sim 0.075$ fm.
- Two ensembles with $V = 24^3 \times 48$
  - Symmetric ($m_s = m_l$): $M_\pi = M_K = 471$ MeV, 939 configs., $LM_\pi = 4.3$
  - Asymmetric ($m_s > m_l$): $M_\pi = 370$ MeV, $M_K = 509$ MeV, 239 configs., $LM_\pi = 3.3$





Extracting η and η' physical states

Start from the SU(3) basis

$$\eta_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}), \qquad \eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$

[flavour basis commonly used: $\eta_l = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$ and $\eta_s = \bar{s}s$]

Construct correlation matrix

$$\langle C_{2\mathrm{pt}}(t,\boldsymbol{p})\rangle = \begin{pmatrix} \langle \eta_8(t;\boldsymbol{p}) \leftarrow \eta_8(0)\rangle & \langle \eta_8(t;\boldsymbol{p}) \leftarrow \eta_1(0)\rangle \\ \langle \eta_1(t;\boldsymbol{p}) \leftarrow \eta_8(0)\rangle & \langle \eta_1(t;\boldsymbol{p}) \leftarrow \eta_1(0)\rangle \end{pmatrix},$$

Optimised smeared operators. Solve the generalised eigenvalue problem

$$\langle C_{2\mathrm{pt}}(t_0,\boldsymbol{p})\rangle^{-\frac{1}{2}} \langle C_{2\mathrm{pt}}(t,\boldsymbol{p})\rangle v_\alpha(t,\boldsymbol{p}) = \lambda_\alpha(t,\boldsymbol{p}) \langle C_{2\mathrm{pt}}(t_0,\boldsymbol{p})\rangle^{\frac{1}{2}} v_\alpha(t,\boldsymbol{p}),$$

Obtain

$$\Longrightarrow \begin{pmatrix} \langle \eta'(t;\boldsymbol{p}) \leftarrow \eta_{\prime}(0)\rangle & 0 \\ 0 & \langle \eta(t;\boldsymbol{p}) \leftarrow \eta(0)\rangle \end{pmatrix}.$$

Parameterise the eigenvectors:

$$v_\eta(t,\boldsymbol{p}) = \left(\cos\theta(t,\boldsymbol{p}), -\sin\theta(t,\boldsymbol{p})\right)^T, v_{\eta'}(t,\boldsymbol{p}) = \left(\sin\theta'(t,\boldsymbol{p}), \cos\theta'(t,\boldsymbol{p})\right)^T.$$

Arrive at the physical basis

$$\mathcal{O}_\eta = \cos\theta(\boldsymbol{p})\,\mathcal{O}_8 - \sin\theta(\boldsymbol{p})\,\mathcal{O}_1\,, \qquad \mathcal{O}_{\eta'} = \sin\theta'(\boldsymbol{p})\,\mathcal{O}_8 + \cos\theta'(\boldsymbol{p})\,\mathcal{O}_1\,.$$

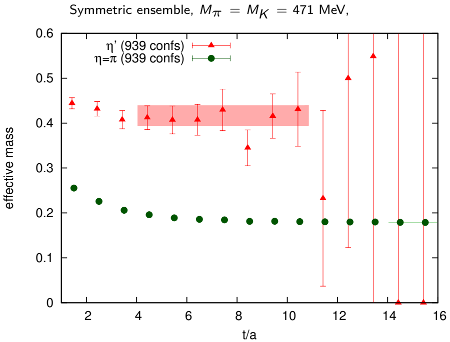
Correlation functions of the physical states.

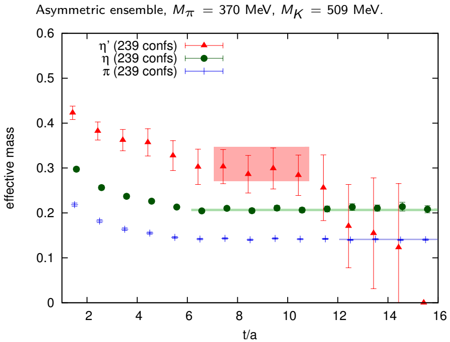
$$\langle C_{2\mathrm{pt}}^{\eta^{(\prime)}}(t,\boldsymbol{p})\rangle = \langle \mathcal{O}_{\eta^{(\prime)}}(t;\boldsymbol{p})\mathcal{O}^\dagger_{\eta^{(\prime)}}(0)\rangle = A_{\eta^{(\prime)}}(\boldsymbol{p})\left(e^{-tE_{\eta^{(\prime)}}(\boldsymbol{p})} + e^{-(T-t)E_{\eta^{(\prime)}}(\boldsymbol{p})}\right)$$

For masses, $\boldsymbol{p} = \mathbf{0}$ sufficient. $\boldsymbol{p} \neq \mathbf{0}$ needed for $D_s \to \eta^{(\prime)}$.

Use: low mode averaging for the connected 2pt function, low modes, stochastic estimation with the hopping parameter expansion, spin and time dilution for the disconnected loops.

Effective masses of η, η' and π, I.Kanamori





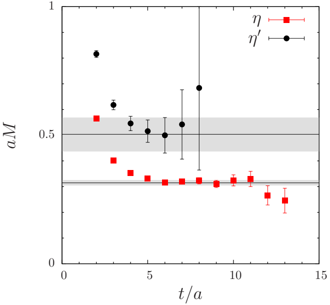
Final masses:

	M_η [MeV]	$M_{\eta'}$ [MeV]	N_{conf}	N_{bin}
Sym.	470.5 (1.8)	1032 (27)	939	5 (25 traj.)
Asym	542.8 (6.2)	946 (65)	239	2 (20 traj.)

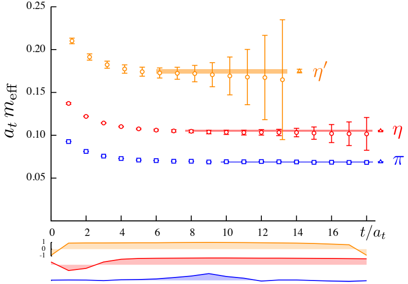
Expt. $M_\eta = 547.8$ MeV $M_{\eta'} = 957.8$ MeV. $t/a = 10$ corresponds to $t = 0.75$ fm.

Comparison with other determinations

ETMC [1310.1207]



HSC [1309.2608]

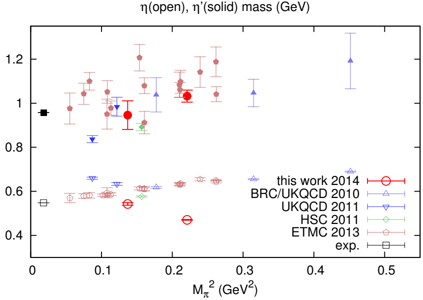


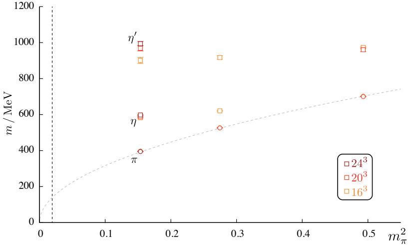
ETMC: $LM_\pi = 5.2$, $24^3 \times 48$, $a \sim 0.09 - 0.10$ fm, $M_\pi = 475 - 427$ MeV.
$N_{conf} \approx 2500$ with $N_{bin} = 10$. $t/a = 7 \to t = 0.67$ fm.
Final results obtained by subtracting off excited states using g.s. determined from connected twopt fn.

HSC: $LM_\pi = 5.7$, $24^3 \times 128$, $a_s \sim 0.12$ fm, $a_t^{-1} \sim 5.6$GeV, $M_\pi = 391$ MeV.
$N_{conf} = 553$ with $N_{bin} = 10$. Distillation method. $t/a = 10 \to t = 0.42$ fm.

Comparison with other mass determinations

HSC [1309.2608]





Our work: flavour average quark mass fixed. Approach physical point $m_\pi \searrow$, $m_K \nearrow$, $\eta \nearrow$.

Consistency with other lattice determinations.

Chiral extrapolations not shown, e.g. ETMC: $M_\eta = 551(8)(6)$ MeV and $M_{\eta'} = 1006(54)(31)(61)$ MeV.

Mixing angle(s)

η and η' are mixtures of the SU(3) basis.

Use pseudoscalar matrix elements (leading order distribution amplitudes)

$$\begin{pmatrix} A_{8\eta} & A_{1\eta} \\ A_{8\eta'} & A_{1\eta'} \end{pmatrix} = \begin{pmatrix} \langle 0|O_8|\eta\rangle & \langle 0|O_1|\eta\rangle \\ \langle 0|O_8|\eta'\rangle & \langle 0|O_1|\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_8 & -\sin\theta_1 \\ \sin\theta_8 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} Z_8 & 0 \\ 0 & Z_1 \end{pmatrix}$$

with local (unsmeared) operators [Alternatively use decay constants - A_8/A_1, diff. angles]

$$\mathcal{O}_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}), \qquad \mathcal{O}_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$

If two states are enough $\theta_8 = \theta_1$. Renormalisation cancels in the ratios:

$$\frac{A_{8\eta'}}{A_{8\eta}} = \tan\theta_8 \,, \qquad \frac{A_{1\eta}}{A_{1\eta'}} = -\tan\theta_1 \,, \qquad \tan^2\bar{\theta} = \tan\theta_8 \tan\theta_1 \,.$$

Extract from fits to twopt correlators

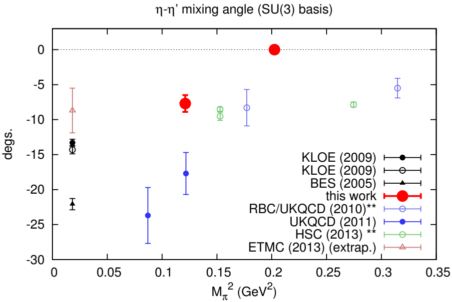
$$\langle 0|\mathcal{O}_j^{\mathrm{local}}(t)\mathcal{O}^{\dagger}_{\eta^{(\prime)}}(0)|0\rangle \to \frac{A_{j\eta^{(\prime)}} Z^S_{\eta^{(\prime)}}}{2M_{\eta^{(\prime)}}} \left(\exp[-M_{\eta^{(\prime)}} t] + \exp[-M_{\eta^{(\prime)}}(T-t)]\right) + \text{add. terms}$$

Smeared amplitude: $Z^S_{\eta^{(\prime)}} = \langle \eta^{(\prime)}|O^{\dagger}_{\eta^{(\prime)}}|0\rangle$

Sym. ensemble: $\theta_8 = \theta_1 = 0$
Asym. ensemble: $\theta_8 = -10.9(1.5)(0.5)$, $\theta_1 = -5.5(1.5)(1.2)$, $\bar{\theta} = -7.7(0.9)(0.8)$

Two angles are needed to describe η and η'. η mostly octet, η' mostly singlet.

$|\theta_j|$ likely to become larger for physical m_s/m_l.

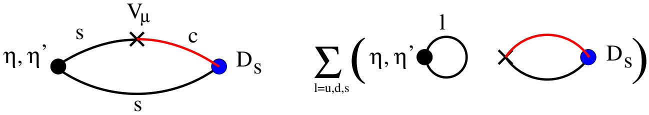


$**$ with non-local operators.

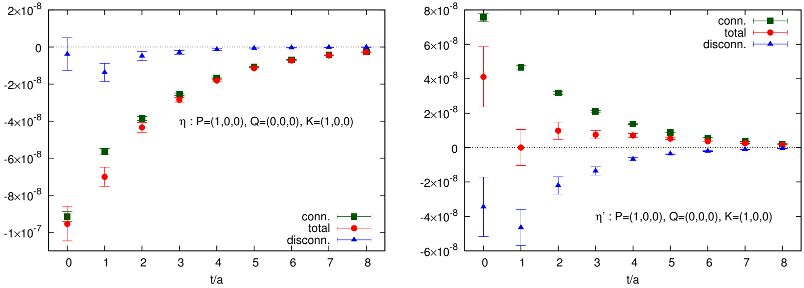
ETMC: results consistent with a single angle.
Most other lattice studies use flavour basis: $\bar{\theta} = \alpha - 54.7^\circ$.

$D_s \to \eta(\eta')\ell\nu$ threept function

$$\left\langle C_{3\text{pt}}^{D_s\to\eta^{(\prime)}}(t,\boldsymbol{p},\boldsymbol{k};t_{\text{sep}})\right\rangle = \langle \mathcal{O}_{\eta(\prime)}(\boldsymbol{k},t_{\text{sep}})S(\boldsymbol{0},t)\mathcal{O}^\dagger_{D_s}(\boldsymbol{p},0)\rangle$$



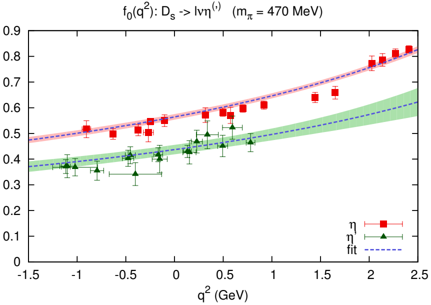
Asymmetric ensemble, $M_\pi = 370$ MeV, $M_K = 509$ MeV (left) η, (right) η'.



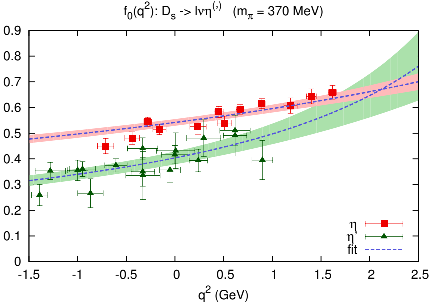
Disconnected contribution is significant, in particular for η'. Dominates error on total threept fn.

$D_s \to \eta(\eta')\ell\nu$ scalar form factor, I. Kanamori

Symmetric ensemble, $M_\pi = M_K = 471$ MeV,



Asymmetric ensemble, $M_\pi = 370$ MeV, $M_K = 509$ MeV.



Interpolate to $q^2 = 0$ using a one-pole ansatz: $f_0(q^2) = f_0(0)/(1 - bq^2)$.

	$f_0^\eta(0)$	$f_0^{\eta'}(0)$
Symm.	0.564(11)	0.437(11)
Asymm.	0.542(13)	0.404(25)
LCSR	0.432(33)	0.520(80)

Comparison with light cone sum rules (LCSR) Offen et al. [1307.2797]:

Phenomenological relevance

Consider comparison to experiment for the ratio

$$\frac{\Gamma(D_s^- \to \eta' e^- \bar{\nu}_e)}{\Gamma(D_s^- \to \eta e^- \bar{\nu}_e)} = 0.36(14), \ \ \mathrm{CLEO}\,[0903.0601].$$

Calculate

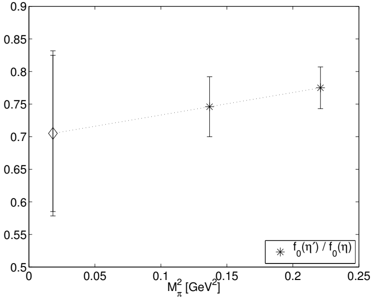
$$\frac{\Gamma(D_s^+ \to \eta' e^+ \nu_e)}{\Gamma(D_s^+ \to \eta e^+ \nu_e)} = \frac{\int_0^{(M_{D_s} - M_{\eta'})^2} \lambda_{D_s,\eta'}^{3/2}(q^2) |f_+^{D_s \to \eta'}(q^2)|^2 dq^2}{\int_0^{(M_{D_s} - M_{\eta})^2} \lambda_{D_s,\eta}^{3/2}(q^2) |f_+^{D_s \to \eta}(q^2)|^2 dq^2},$$

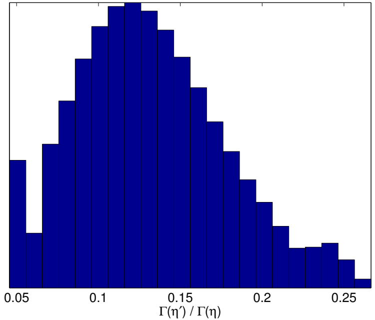
where $\lambda_{D_s,\eta^{(\prime)}}(q^2) = \frac{1}{4M_{D_s}^2}\left((M_{D_s}^2 + M_{\eta^{(\prime)}}^2 - q^2)^2 - 4M_{D_s}^2 M_{\eta^{(\prime)}}^2\right)$.
Use Ball-Zwicky parameterisation [hep-ph/0406232] for $f_+(q^2)$,

$$f_+^{\mathrm{BZ}}(q^2) = f_+(0)\left(\frac{1}{1 - q^2/M_{D_s^*}^2} + \frac{r q^2/M_{D_s^*}^2}{(1 - q^2/M_{D_s^*}^2)(1 - \alpha q^2/M_{D_s^*}^2)}\right)$$

Use LCSR calculation, Offen et al. [1307.2797], for $r = 0.284(142)$ and $\alpha = 0.252(126)$ (with 50% errors).

- Some systematics cancel in the ratio.
- Extrapolate ratio $f_0^{\eta'}(0)/f_0^{\eta}(0) = 0.705(120)(041)$, mild dependence on M_π.
- Vary r_η, α_η, $r_{\eta'}$, $\alpha_{\eta'}$ and $f_0^{\eta'}(0)/f_0^{\eta}(0)$ independently within errors.





Final result 1.6σ below CLEO measurement:

$$\frac{\Gamma(D_s^- \to \eta' e^- \bar{\nu}_e)}{\Gamma(D_s^- \to \eta e^- \bar{\nu}_e)} = 0.128^{+51}_{-42}$$

c.f LCSR Offen et al. [1307.2797] $0.37 \pm 0.09 \pm 0.04$.

Summary

- Heavy-light spectra, heavy(-light)-light, heavy(-heavy)-light spectra is an active area of research within the lattice community.
- Test of systematics: D_s, B, B_s hyperfine splittings, from variety of heavy quark approaches.
- Move to calculate wider spectrum, near threshold states and resonances. $D^*_{s0}(2317)$ consistent with weakly bound state.
- Agreement of lattice results of charmed and bottomed baryons with experimental results, confirming spin/parity assignments.
- Experimental prospects are very good for discovery of further states, LHC, Belle II, BES III.
- Lattice simulations using $\bar{m} = \frac{1}{3}(m_s + 2m_{u/d})$: explore $SU(3)_F$ breaking using Gell-Mann Okubo relations, treating charm as a spectator.
- First lattice calculations of $D_s \to \phi$ and $D_s \to \eta^{(\prime)}$.