Quark masses, strong CP, and all that

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Three controversial points

1. $m_u = 0$ unrelated to the strong CP problem

2. Topological susceptibility not a physical observable

3. Staggered fermions do not root properly

How did I reach these conclusions?

- Concentrate on m_u
- Susceptibility is a corollary
- Rooting: practitioners don't care and won't listen.

Motivation

- Standard model requires chiral gauge coupling
- lattice formulation unknown
- Lattice is the fully non-perturbative regulator

Is the standard model well defined?

Study chiral symmetry from all angles

MC (1995):

- naive variable change $\psi \longrightarrow e^{i\gamma_5\theta}\psi$
- $\overline{\psi}\psi \longrightarrow \cos(\theta)\overline{\psi}\psi + i\sin(\theta)\overline{\psi}\gamma_5\psi$

Study QCD dependence on m_1 and m_5

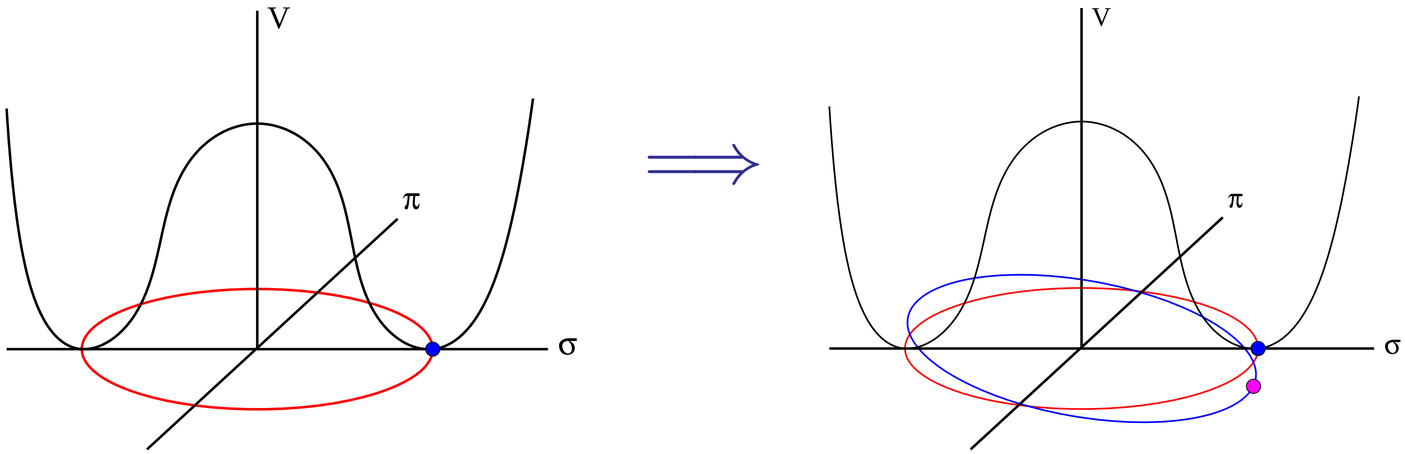
- $m\overline{\psi}\psi \to m_1\overline{\psi}\psi + im_5\overline{\psi}\gamma_5\psi$

Does physics depend only on $\sqrt{m_1^2 + m_5^2}$? X

Tool: effective chiral Lagrangean

Consider 2 flavors with effective potential

- $V = (\sigma^2 + \vec{\pi}^2 - v^2)^2$
- mass term $m_1 \overline{\psi}\psi \longrightarrow m_1 \sigma$ tilts the sombrero



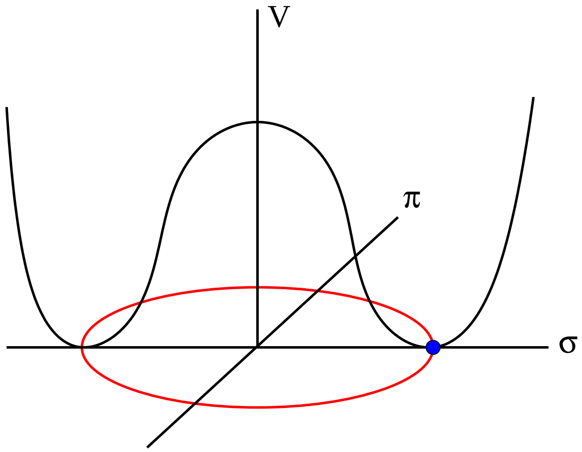
- pion becomes massive $M_\pi^2 \propto m_1$

What does m_5 do?

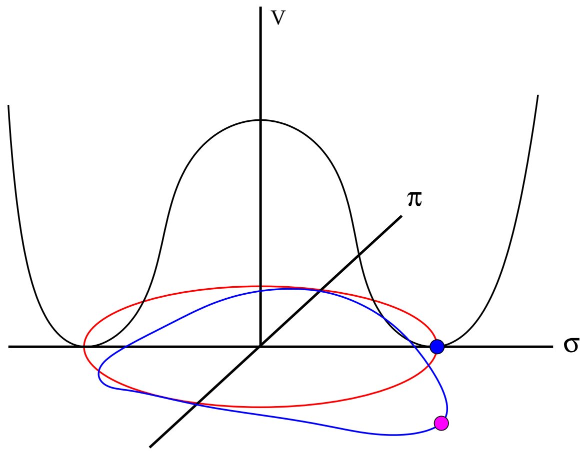
- $i m_5 \overline{\psi} \gamma_5 \psi \longrightarrow m_5 \eta$
  - not in above effective potential
- m_5 will give η an expectation value
  - $\langle \eta \rangle \propto m_5 / M_\eta^2$

Add an effective coupling $(\vec{\pi} \cdot \vec{a}_0 + \sigma\eta)^2 \to \langle \eta \rangle^2 \sigma^2$

$V \to V - \alpha m_5^2 \sigma^2$ (sign related to pi eta mixing)

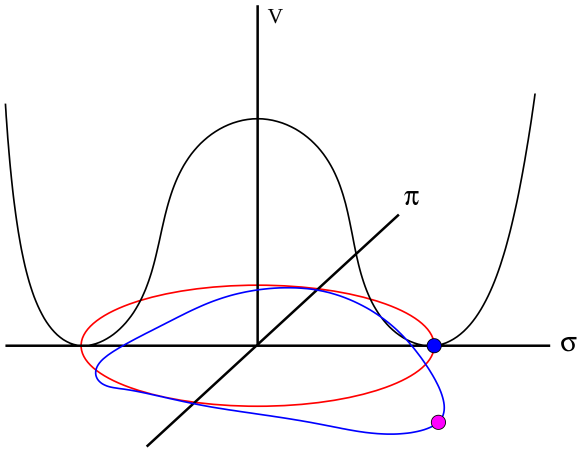


$\Longrightarrow$

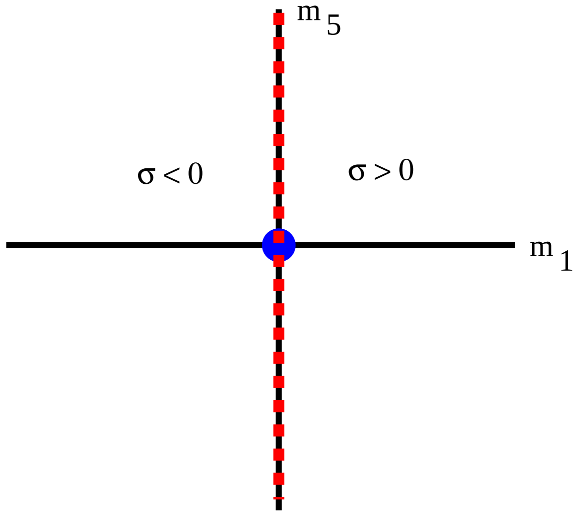


m_5 also gives pions a mass

- $M_\pi^2 \propto m_5^2$ not linear in m_5



Transition at $m_1 = 0$ becomes first order



Transition occurs at conventional $\Theta = \pi$

- $\frac{m_5}{m_1} = \tan(\Theta/2)$
- physics not only dependent on $\sqrt{m_1^2 + m_5^2}$
- variable change is anomalous

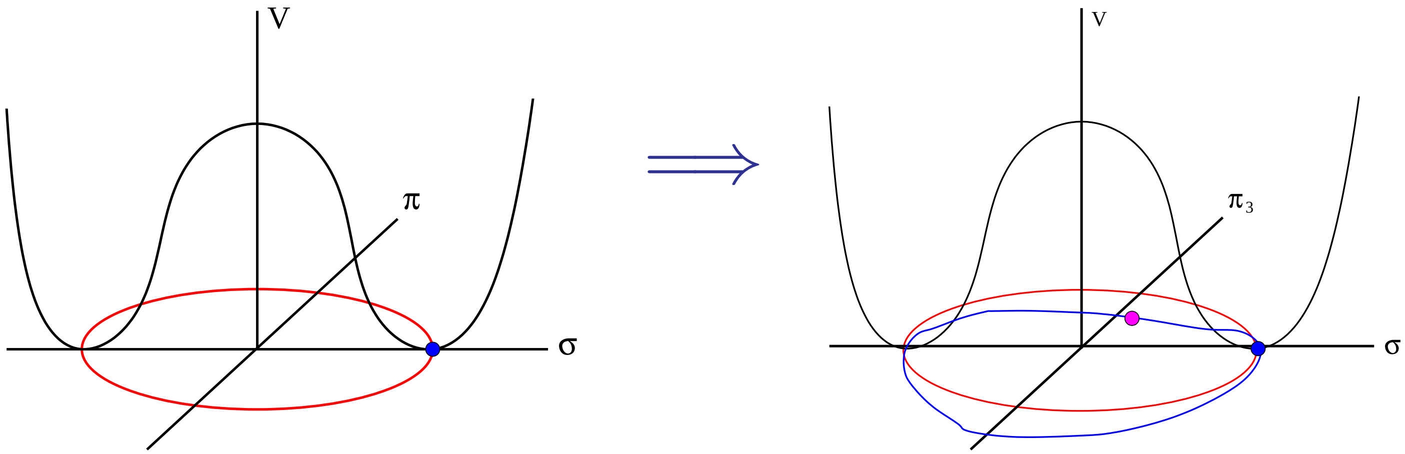
Physics depends non-trivially on Θ

Introduce an up-down mass difference

- $m\overline{\psi}\psi \to m_1\overline{\psi}\psi + m_2\overline{\psi}\tau_3\psi$
- $\overline{\psi}\vec{\tau}\psi \sim \vec{a_0}$ isovector scalar
- also not in starting effective potential
- m_2 will give a_{03} an expectation value
  - $\langle a_{03} \rangle \propto m_2/M_{a_0}^2$

Effective coupling $(\vec{\pi} \cdot \vec{a}_0 + \sigma\eta)^2$ (form dictated by chiral symmetry)

- $V \to V - \beta m_2^2 \pi_3^2$



Without tilt from m_1,

- π_3 gains an expectation value!
- the "Dashen phase"

Conventional QCD parameters α_s m_u m_d Θ

- α_s tied to overall scale: m_p

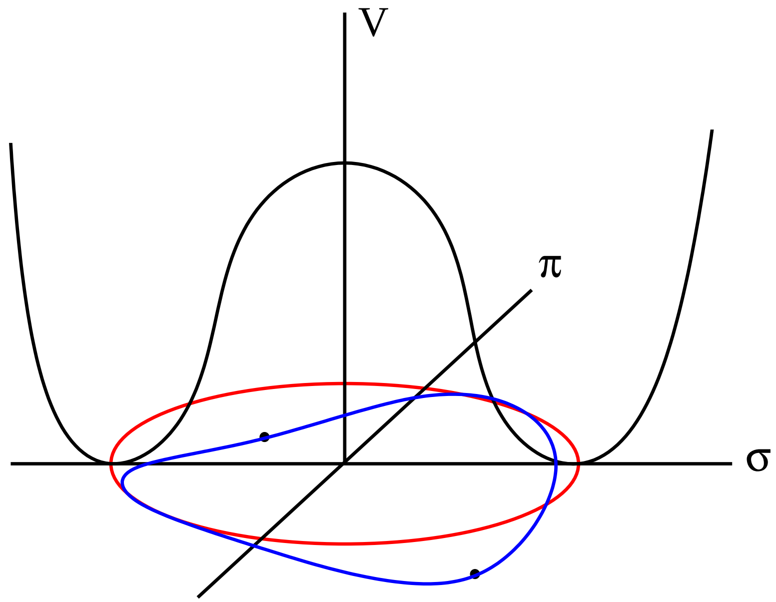
- m_q determines meson spectrum: m_π m_k $\ldots$

- Θ determines neutron electric dipole moment

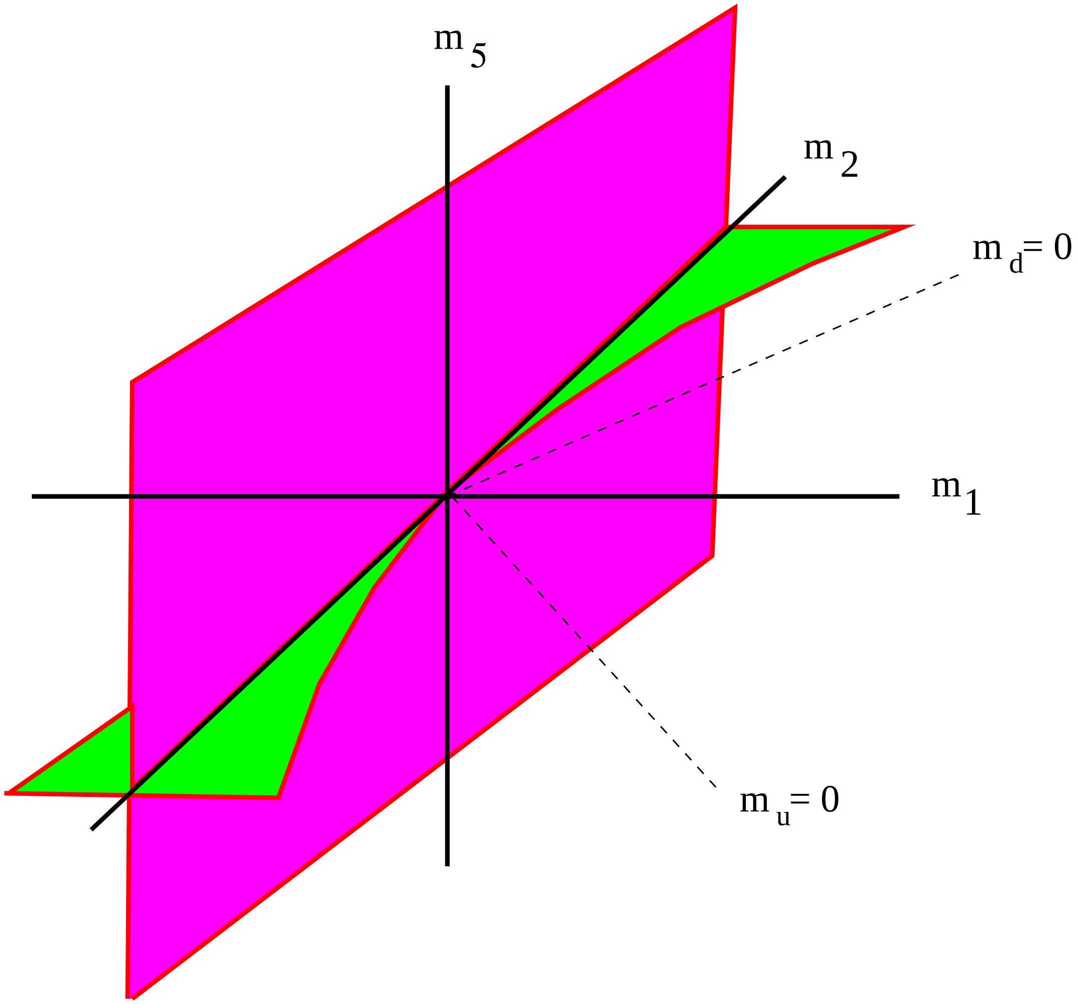
In chiral Lagrangean language:

$\alpha_s \ m_u \ m_d \quad \Theta$ map onto effective potential

- overall scale
- tilt
- warp
- angle between tilt and warp

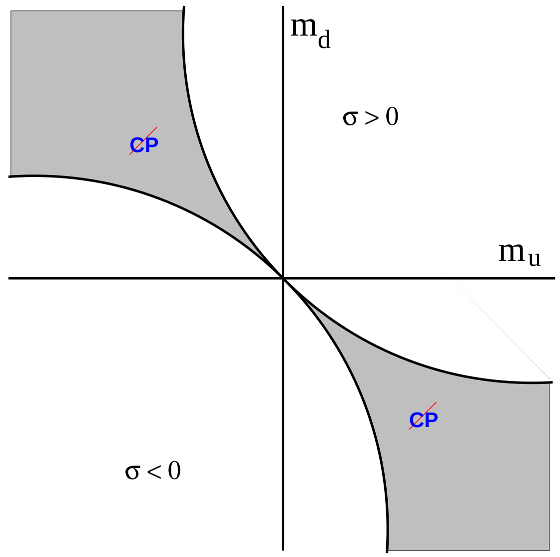


Full 2 flavor phase diagram



M.C (2011)

Concentrate on $m_5 = 0$ plane

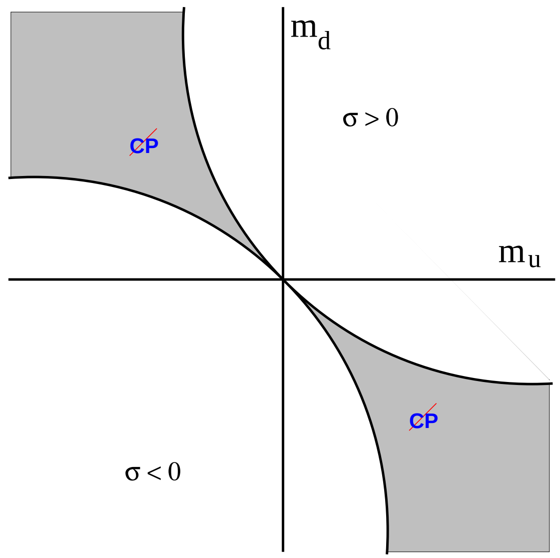


Second order transition at $m_u m_d < 0$; i.e. $\Theta = \pi$

- order parameter $\langle \pi_0 \rangle$
- massless neutral pion along transition line

M.C. (2004), S. Aoki, Sharpe, Horkel

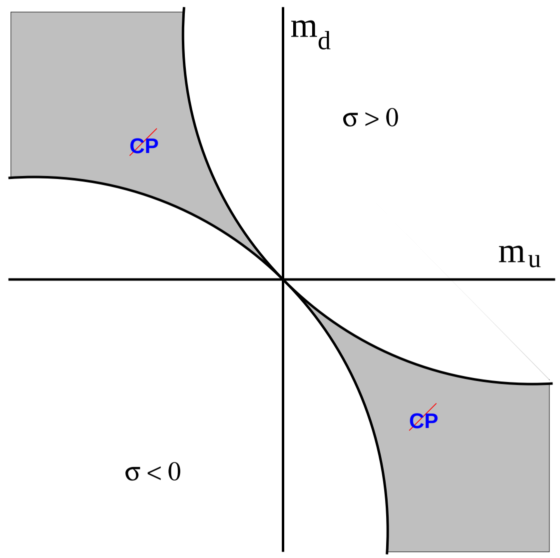
Symmetries



$m_u \leftrightarrow m_d$

- if $m_u = m_d$ isospin is exact
- $m_2 = 0$ protected from additive renormalization

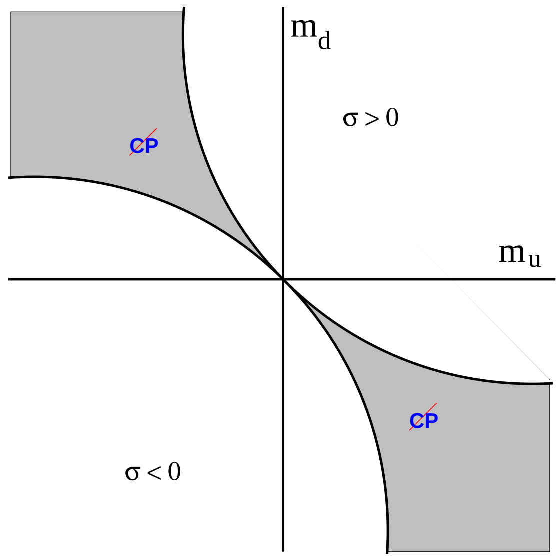
Symmetries



$m_u \leftrightarrow -m_d$

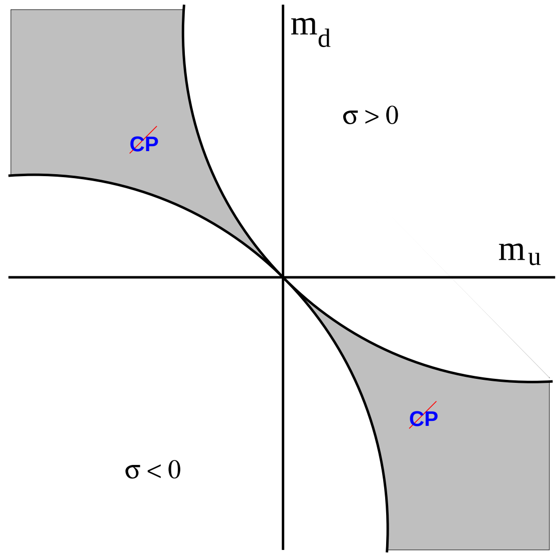
- $m_u = -m_d$ isospin symmetry at $\Theta = \pi$
- $m_1 = 0$ also protected: $m_u + m_d$

Symmetries



$m_u, m_d \leftrightarrow -m_u, -m_d$

- $\psi \rightarrow e^{i\pi\tau_3\gamma_5}\psi$
- flavored chiral symmetry not anomalous
  - $\mathrm{Tr}\tau_3 = 0 \;\Rightarrow\; \left|e^{i\pi\tau_3\gamma_5}\right| = 1$



NO symmetry under $m_u \leftrightarrow -m_u$

- $m_u = 0$ not protected by any symmetry
- Ward identities mix in $F\tilde{F}$

Perturbatively $\overline{\psi}\psi \to \overline{\psi}\ e^{i\pi\gamma_5}\ \psi$ flips sign of the mass

- this rotation is anomalous
- topology gives chiral zero modes
  - $\mathrm{Tr}\gamma_5 = \nu$
- -1^ν measure factor at negative m

The strong CP question

- the m_5 term violates CP: $\langle \eta \rangle \propto m_5$
- no strong CP violation observed
  - $m_5 << \Lambda_{qcd}$

Claim: $m_u = 0$ eliminates the problem

- $m_u \equiv \frac{m_1+m_2}{2} + i m_5$

Symmetries protect m_1, m_2, m_3 separately

- renormalizations not in general equal
- no symmetry to protect $m_1 + m_2$

m_1, m_2, m_3 physically distinct parameters

- independent renormalizations
- $\frac{m_1+m_2}{2} + im_5$ is an artificial construct

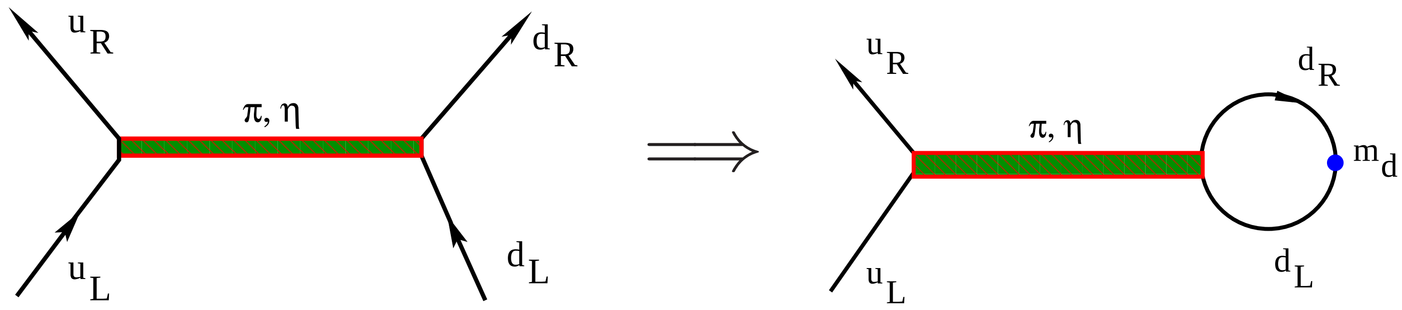
Question: Can any experiment tell if $m_u = 0$?

- is $m_u = 0$ well-defined?
- $\overline{MS}$ is perturbative, cannot answer this

Non-perturbative issues require the lattice

- adjust lattice parameters for hadron spectrum
- read off quark masses and see if $m_u = 0$

Complication: m_d can induce an effective m_u



Mass ratios not renormalization group invariant

$$\frac{m_u}{m_d} \rightarrow \frac{m_u + \epsilon m_d}{m_d + \epsilon m_u}$$

Effect is non-perturbative, from topology

Mass renormalization is not flavor blind !

- Mass independent renormalization?
  - allowed but not natural
  - hides non-perturbative mixing from topology
  - physical particle mass ratios not constant

But there are many lattice formulations

- how do we define the quark mass

Can we use the operator product expansion

- no symmetry to rule out terms
- lattice currents involve neighboring sites
  - details depend on specific formulation

Can we use topology?

- $m_u = 0$ equivalent to vanishing susceptibility

How to define lattice topology?

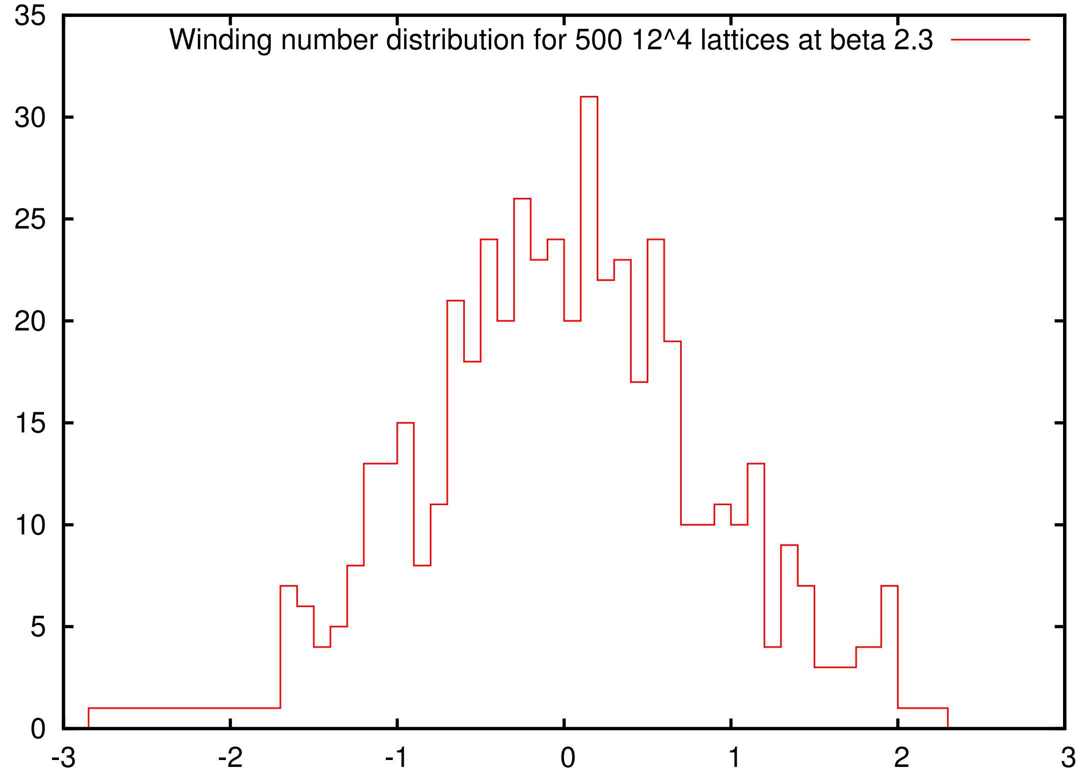
Space of lattice fields simply connected

Topology lost at the outset

- small instantons can fall through the lattice

Combine loops around a hypercube

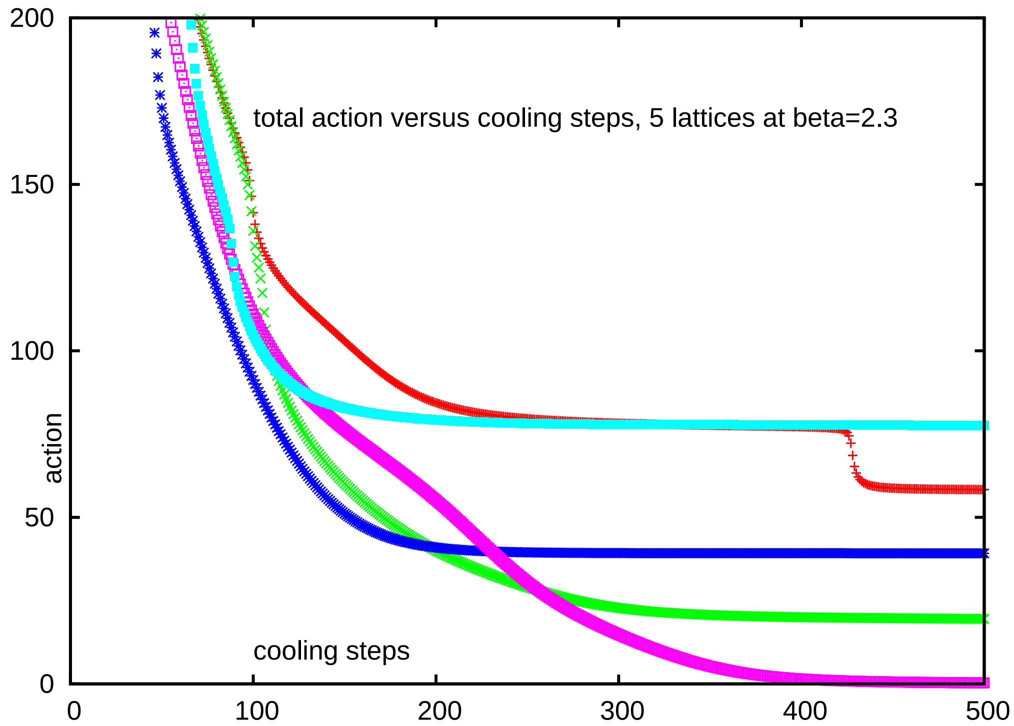
- to give $F\tilde{F}$ in continuum limit
- resulting charge not generally an integer



MC (2011)

Cooling (Wilson flow, ...) to remove UV fluctuations

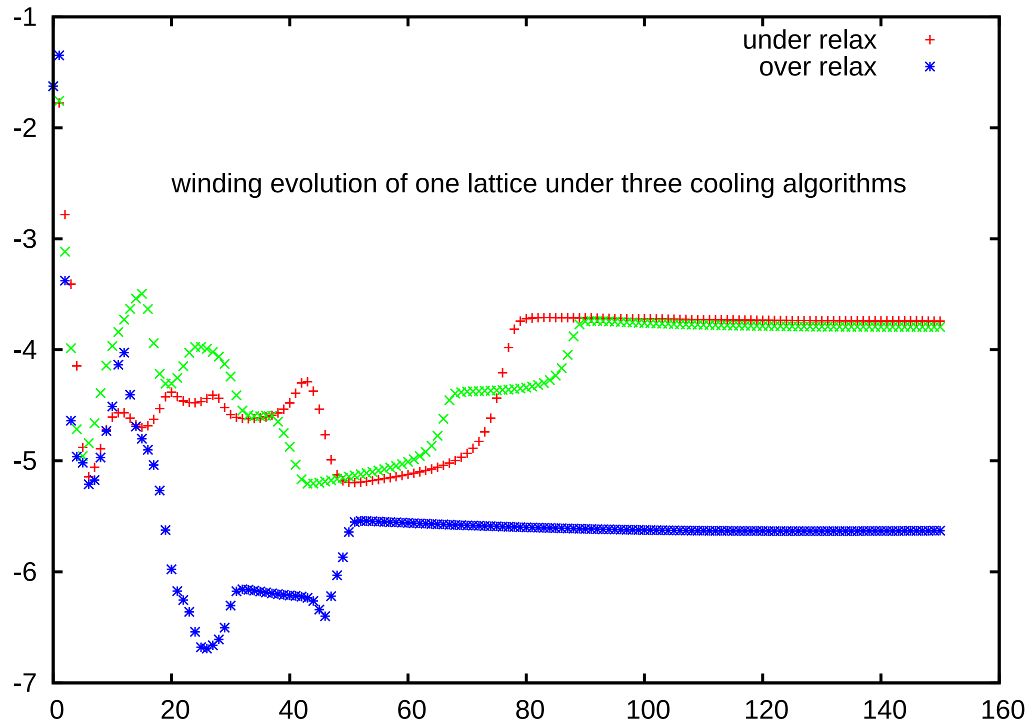
- Action settles to multiple instantons



Many studies over the years: M. Teper (1985); de Forcrand, Garcia-Perez, Stamatescu; Del Debbio, Giusti, Pica; Bruckmann, Gruber, Jansen, Marinkovic, Urbach, Wagner; Ilgenfritz, Martemyanov, Muller-Preussker, Veselov, ...

Often stable but ambiguities appear

- Winding can depend on cooling algorithm
- with which action should we cool? How long?



Can we use the index theorem?

- count small eigenvalues of the Wilson operator
  - At finite cutoff not exact zeros
  - How to define “small”?
  - depends on eigenvalues in first Wilson “circle”

Count zero modes of the overlap operator

- operator not unique: “domain wall height”
- reverts to Wilson eigenvalue distribution

Should we care if topology is ambiguous?

- not measured in laboratory experiments
- concentrate on $M_{\eta'}$, which is physical
  - Witten-Veneziano formula a large N_c result
- tied to the $m_u = 0$ issue

Summary

$m_u = 0$ ambiguous between lattice schemes

- not an appropriate solution to strong CP
- Entangled with ambiguities in defining topology

Intuition from perturbation theory fails

- mass renormalization is not flavor blind
- matching lattice and perturbative quantities tricky

Review: Acta Physica Slovaca 61, 1 (2011), arXiv:1103.3304;