HQET on the lattice. The b quark mass

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Fundamental Parameters from Lattice QCD Workshop

Outline

- Higgs physics
- b quark mass: lattice QCD input to Higgs physics
- (Non-perturbative) HQET on the lattice: a ALPHA Collaboration retrospective
  - Signal to noise problem
  - Non-perturbative matching between QCD and HQET, why and how
- The b-quark mass, results and review
- Extra: reweighting fermionic boundary conditions

S. Dawson (BNL)
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SM Very Predictive

- Very precise predictions
  - Couplings to fermions proportional to mass $\frac{m_f}{v} H \overline{f} f$
  - Couplings to massive gauge bosons proportional to (mass)^2

$$2 m_W^2 \frac{H}{v} W_\mu^+ W^{-\mu} + m_Z^2 \frac{H}{v} Z_\mu Z^\mu$$

  - Couplings to massless gauge bosons at 1-loop

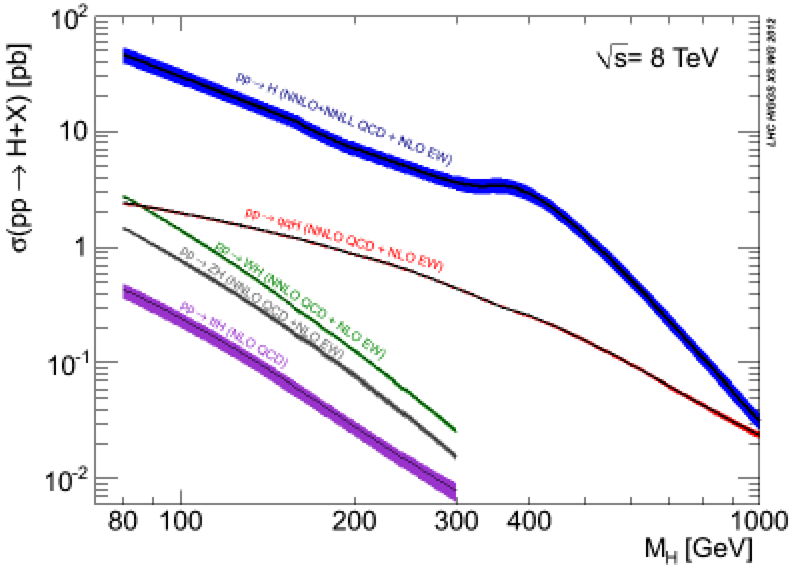
$$\kappa_g \frac{\alpha_s}{12\pi} \frac{H}{v} G_{\mu\nu}^A G^{A,\mu\nu} + \kappa_\gamma \frac{\alpha}{8\pi} \frac{H}{v} F_{\mu\nu} F^{\mu\nu} + \kappa_{Z\gamma} \frac{\alpha}{8\pi s_W} \frac{H}{v} F_{\mu\nu} Z^{\mu\nu}$$

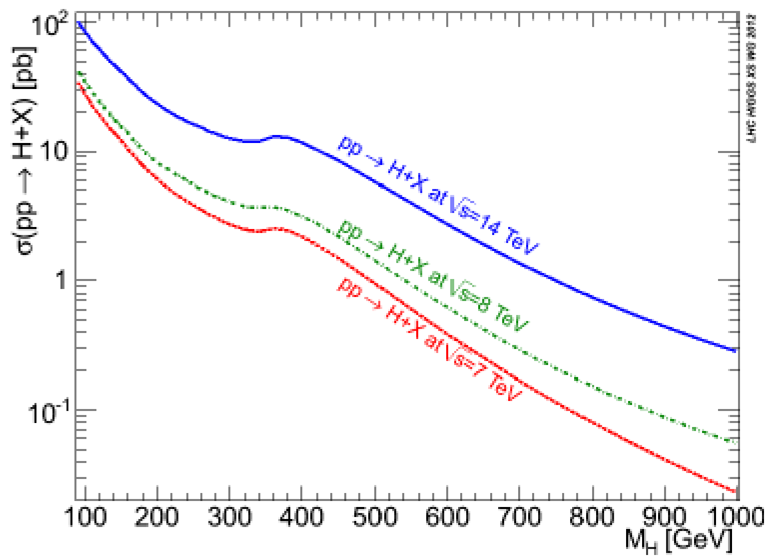
  - Higgs self-couplings proportional to m_H^2

$$\frac{m_H^2}{2} H^2 + \frac{m_H^2}{2v} H^3 + \frac{m_H^2}{8v^2} H^4$$

S. Dawson (BNL)
CP3, May, 2015

Precise Calculations





Reduction of scale uncertainty at N^3LO:

13 TeV $\sigma(m_H=125\ \text{GeV})=43.14\ \text{pb}\ ^{+2.71\%}_{-4.45\%}$ **3 Loops!**

Increase of +2.2% from NNLO rate [Anastasiou, Duhr, Dulat, Herzog, Mistlberger, arXiv:1503.06056]

At 13 TeV: factors 2-4 increases in rates

Largest Higgs BR is to b's

S. Dawson (BNL)
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- Sensitive to m_b: $\Gamma(H \rightarrow b\bar{b}) = \frac{G_F N_c}{4\sqrt{2}\pi} m_H \beta^3 M_b^2$

- QCD included to N^3LO for H→bb predictions

Input values for Higgs BR fits

Parameter	Central Value	Uncertainty
$\alpha_s(M_Z)$	0.119	±0.002 (90%CL)
m_b	4.49 GeV	±0.06 GeV
M_t	172.5 GeV	±2.5 GeV

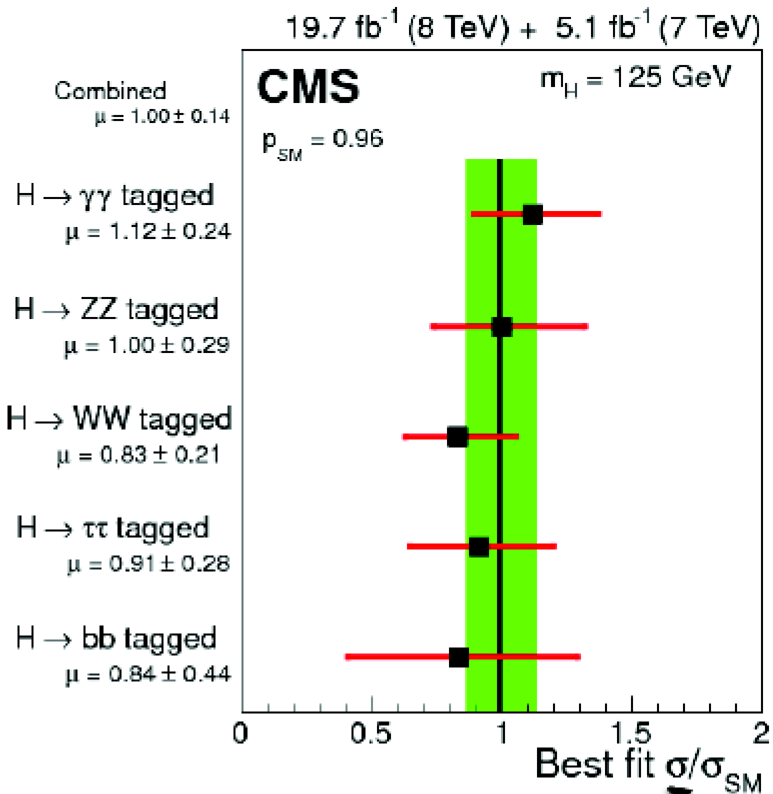
M_b is pole mass calculated with 1 loop running of $m_b(m_b)$=4.16 GeV

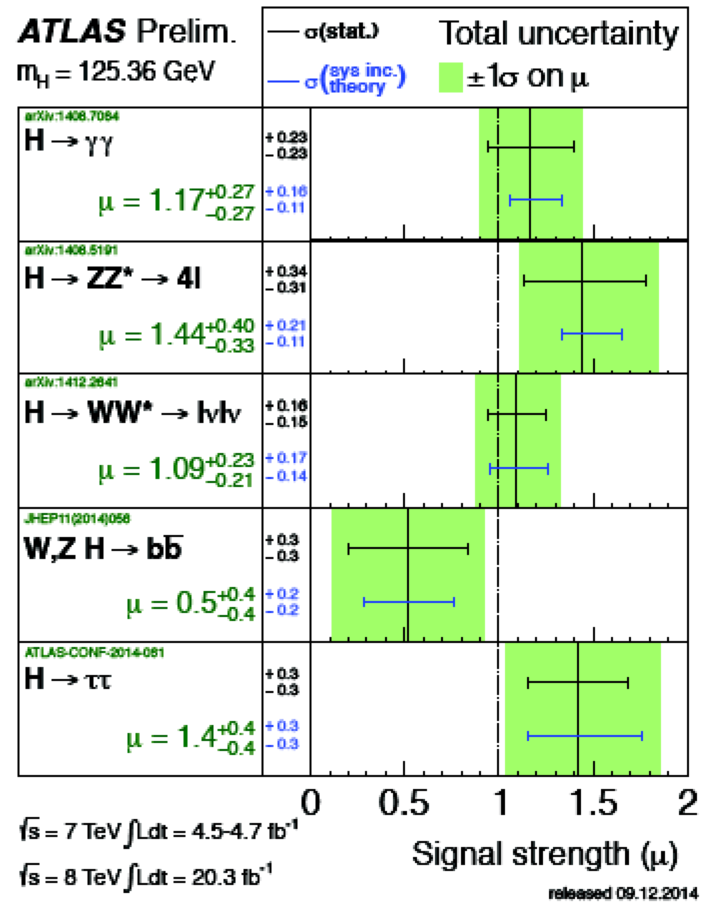
$$\frac{\delta\Gamma_b}{\Gamma_b} \sim \pm 3\%$$

Also, dominating uncertainty in:

$$\Gamma_H(m_H = 125\ GeV) = 4\ MeV \pm 4\%$$

Consistent with SM Hypothesis





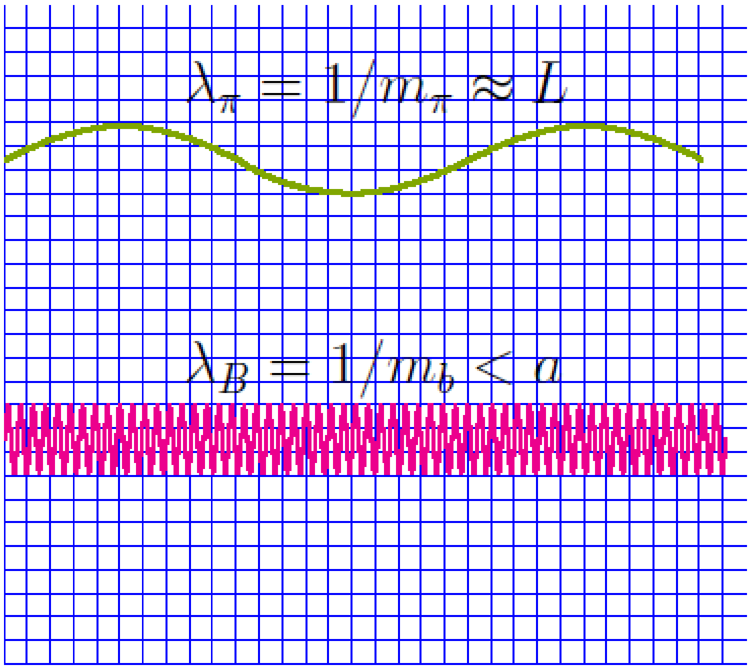
Requires theory input!

Errors soon to be dominated by theory

Heavy quarks on the lattice

- finite volume effects are mainly triggered by the light degrees of freedom. The usual requirement is $m_{PS}L > 4$ and m_{PS} is now getting to the physical point $\Rightarrow L \simeq 7$ fm.
- cutoff effects are related to the heavy quark mass. $a << 1/m_b \simeq 0.03$ fm .

$\Rightarrow L/a \simeq 200$ is needed to have those systematics under control !!
Integrating out the heavy quark mass in this case is useful !!



In addition the autocorrelation of observables grows as $1/a^n$ with $n \geq 2$ [Schäfer, Sommer and Virotta '10, Lüscher and Schäfer, '11]

Different methods have different systematics. It is crucial to compare results from a variety of them. Briefly:

Most approaches directly apply some EFT (typically valid in a particular kinematic regime)

- NRQCD [Thacker, Lepage 1991]: Expansion in v_h and in $1/m_h$.
  Dim. 5 ops at leading order $\Rightarrow$ non-renormalizable.
  One has to look for a *window* where cutoff effects ($O(a^n)$)and power divergences ($O(1/a^m)$) are both small. Typically $am_h \geq 1$.
- Combinations of HQET and Symanzik effective theory:
  - O(a) improved HQET [ALPHA ...]
  - First Symanzik EFT, then HQET (expand in $1/m_h$ the improvement coefficients) $\rightarrow$ Fermilab action [El-Khadra, Kronfeld, Mackenzie 1996], RHQ [Christ, Li, Lin 2007] and Tsukuba action [Aoki, Kuramashi, Tominaga 2003].

Having introduced operators of higher dimensions all these theories produce power divergences (also in the Fermilab approach when $m_h \rightarrow \infty$ at fixed a).

The continuum limit exists only if these divergences are subtracted non-perturbatively. At any order in g_0^2:

$$\frac{g_0^{2n}}{a} \approx \frac{1}{\ln(a)^n a} \to \infty \quad \text{as } a \to 0$$

We devised a completely non-perturbative setup for lattice HQET.

Non (directly) EFT based approaches

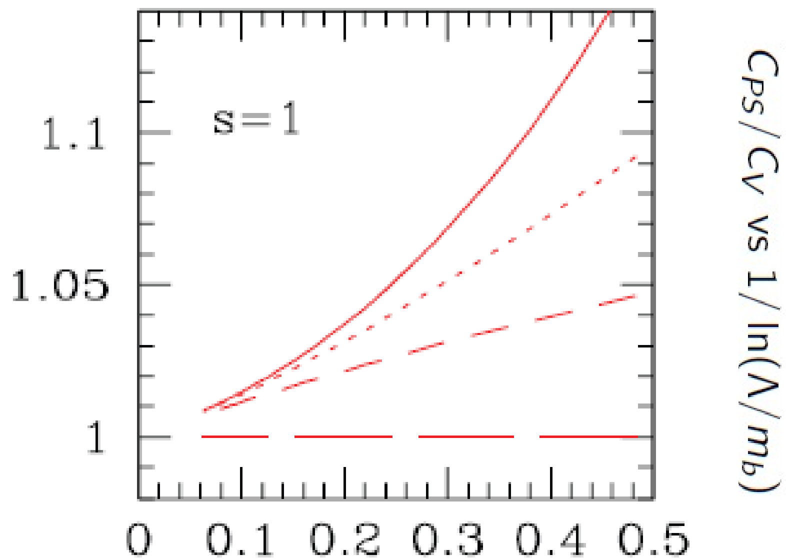
- HISQ [HPQCD 2011]: at lattice spacings of $a \approx 0.05$ fm and $L/a \approx 100$ as currently produced by MILC, $am_b \approx 1$ so one can simulate directly at $m_b/2$ and then extrapolate to the b (using HQET).
  Getting there but autocorrelations seem a severe problem ...
- Interpolation method [Guazzini, Sommer, Tantalo 2006, ETM 2010 ...]: using data around the charm and results in the static limit + fits in powers of $1/m_h$. Ratio Method:

$$P(m_b) = P(m_c)\,\frac{P(\lambda m_c)}{P(m_c)}\,\frac{P(\lambda^2 m_c)}{P(\lambda m_c)}\ldots$$

  The ratios have static limit=1.

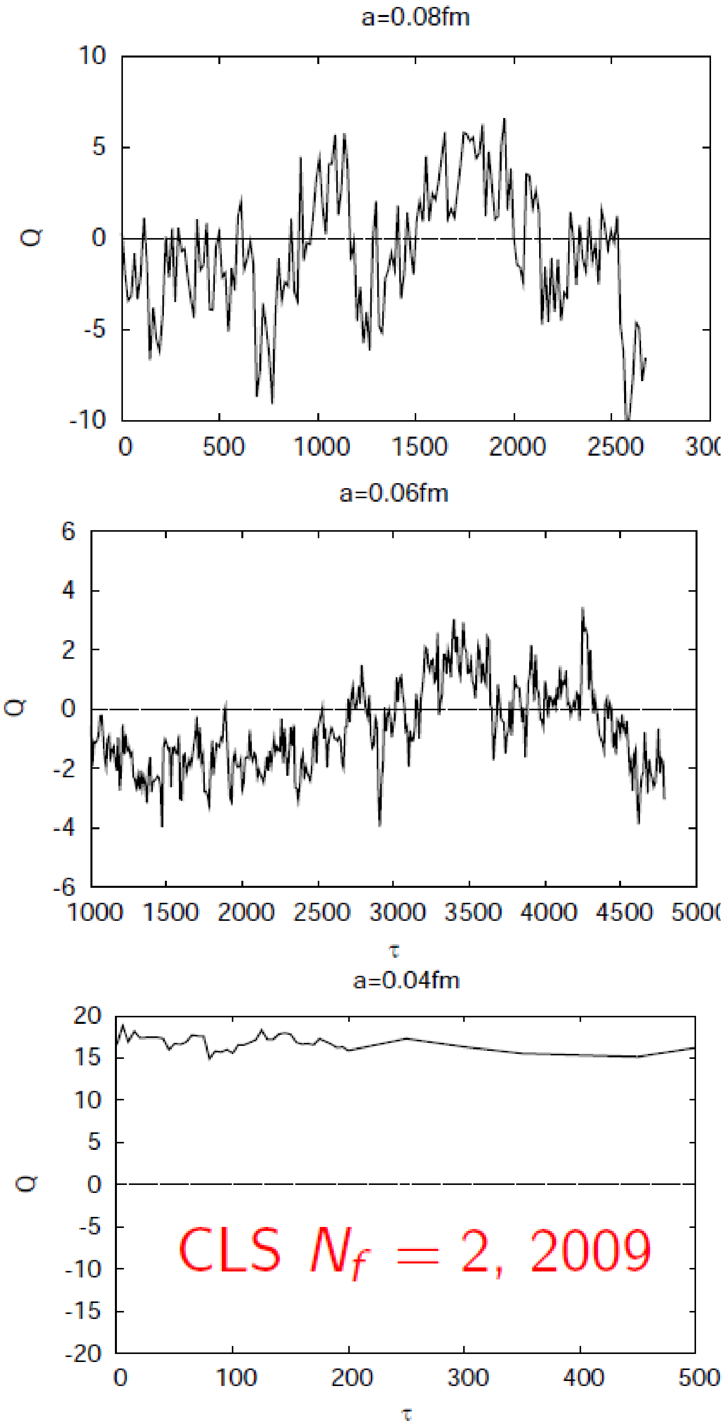
  In both methods matching factors from PT are used to define obervables with the proper scaling (removing $\ln(m_h)$).

[R. Sommer, 2010]

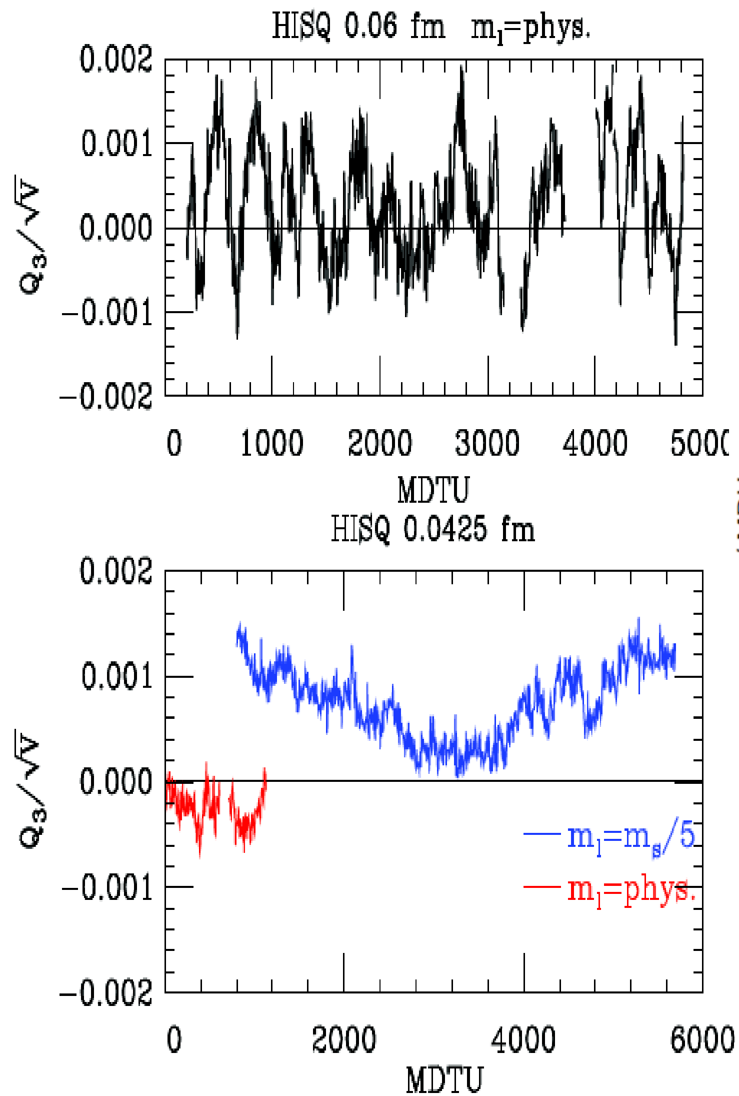


[Chetyrkin and Grozin 2003, Broadhurst and Grozin '91, '95, Bekavac et al. 2010]

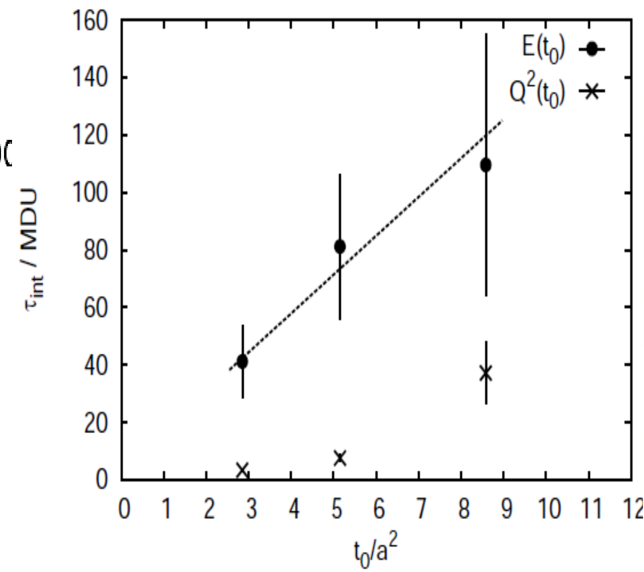
Autocorrelations



CLS $N_f = 2$, 2009



De Tar, Lattice2015



CLS $N_f = 2 + 1$, 2015

open boundary conditions

Noise to signal ratio in HQET

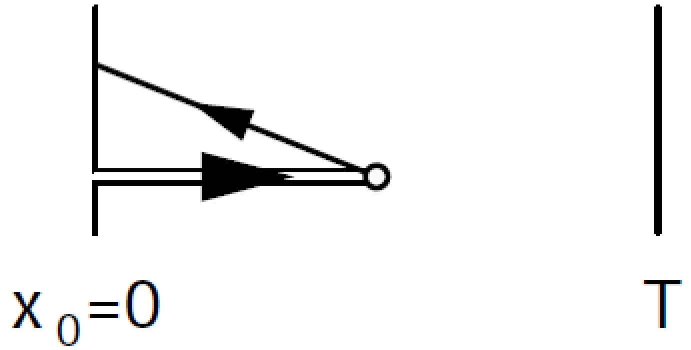
Field content [Eichten and Hill 1990]: $P_+\psi_h = \psi_h\,, \quad \bar\psi_h P_+ = \bar\psi_h\,, \quad P_+ = \frac{1+\gamma_0}{2}$

Action: $S_h^W = a^4 \sum_x \bar\psi_h(x)(D_0^W + \delta m_W)\psi_h(x)$ [MDM, Shindler, Sommer 2005]

with the covariant derivative

$$D_0^W \psi_h(x) = \frac{1}{a}\left[\psi_h(x) - W^\dagger(x - a\hat 0, 0)\psi_h(x - a\hat 0)\right]$$

δm_W cancels the divergence of the self energy. Let us consider a (SF) two-point function with A^{stat} in the bulk, and set $\delta m_W = 0$



$$f_{\rm A}^{\rm stat}(x_0) \propto e^{-E_{\rm stat}x_0}$$

with $E_{\rm stat} \sim E_{self} + \mathrm{O}(a^0) \sim \frac{1}{a}e^{(1)}\, g_0^2 + \ldots$

$e^{(1)}$ depends on the regularization and $\delta m_W = -E_{self} + O(a^0)$.

For the noise one has to "square" the quantity in the gauge average (and subtract the square of the average)

$$\sigma_\mathrm{A}(x_0) = \frac{a^{18}}{4L^6} \sum_{\mathbf{x},\mathbf{y},\mathbf{z},\mathbf{x}',\mathbf{y}',\mathbf{z}'} \langle\, [A_0^\mathrm{stat}]_\mathrm{hl}(x_0,\mathbf{x})\,\overline{\zeta}_\mathrm{h}(\mathbf{y})\gamma_5\zeta_\mathrm{l}(\mathbf{z})\times \{[A_0^\mathrm{stat}]_\mathrm{h'l'}(x_0,\mathbf{x}')\,\overline{\zeta}_\mathrm{h'}(\mathbf{y}')\gamma_5\zeta_\mathrm{l'}(\mathbf{z}')\}^\dagger \,\rangle - [f_\mathrm{A}^\mathrm{stat}(x_0)]^2$$

asymptotically

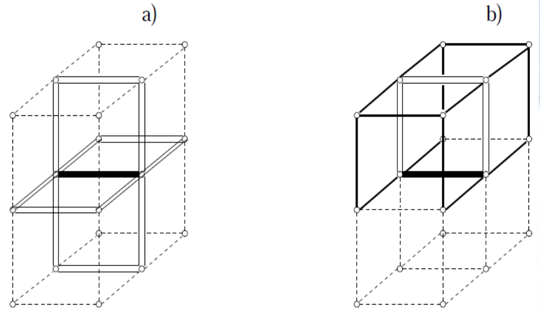
$$\sigma_\mathrm{A}(x_0) \propto \sum_{\mathbf{x},\mathbf{x}'} C(\mathbf{x},\mathbf{x}')\,\mathrm{e}^{-x_0 E_{\mathrm{ll'}}(\mathbf{x},\mathbf{x}')}$$

with $E_{\mathrm{ll'}}(\vec{x},\vec{x}')$ the energy of a state with a static quark-antiquark pair at $\vec{x},\vec{x}'$ and a light quark-antiquark pair with flavors l,l'

$$\mathrm{Min}_{\mathbf{x},\mathbf{x}'}\, E_{\mathrm{ll'}}(\mathbf{x},\mathbf{x}') = V(0) + m_\pi \qquad \Rightarrow \qquad R_\mathrm{NS} \propto e^{[E_\mathrm{stat} - (m_\pi + V(0))/2]\, x_0}$$

Conclusion: The non-universal E_stat controls the noise to signal ratio.

We just changed the gauge parallel transporter (HYP smearing), following [Hasenfratz, Knechtli 2001]



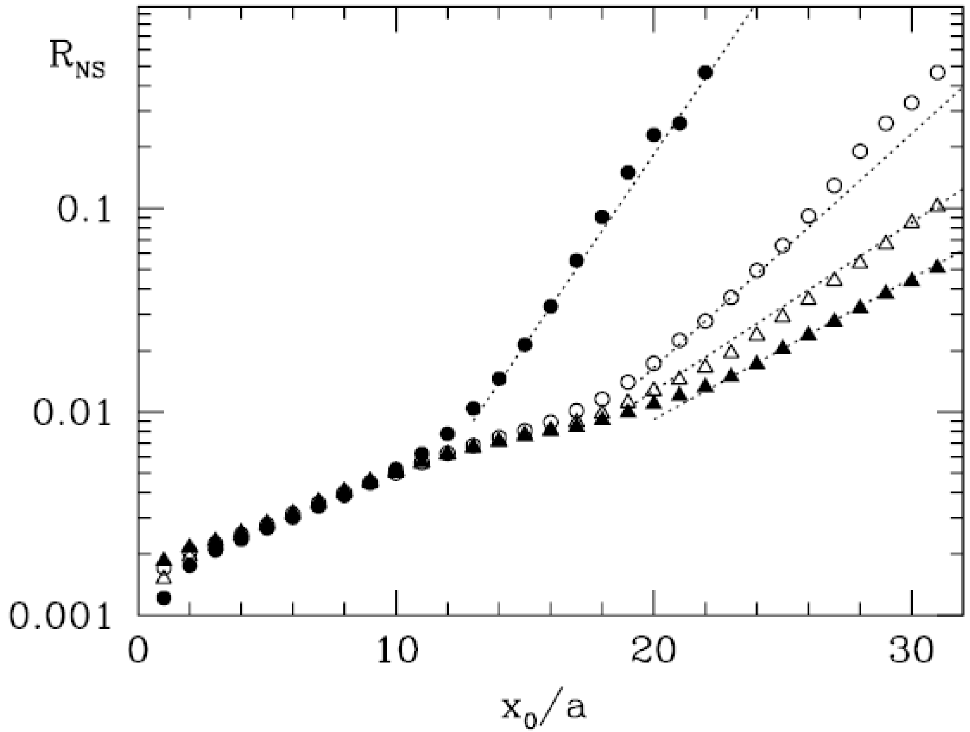


Figure 2: *The ratio $R_{\rm NS}$ for the correlation function $f_{\rm A}^{\rm stat}(x_0)$ for a statistics of 5000 measurements on a $16^3 \times 32$ lattice at $\beta = 6$. Filled circles refer to $S_{\rm h}^{\rm EH}$, empty circles to $S_{\rm h}^{\rm A}$ ($S_{\rm h}^{\rm s}$ gives similar results), empty (filled) triangles to $S_{\rm h}^{\rm HYP1}$ ($S_{\rm h}^{\rm HYP2}$).*

This smearing is now used in all applications of HQET, and also in dynamical simulations [BMW].

NLO in $1/m_h$

Symmetries of the static action

- In the static action no γ-matrices appear, the interactions do not change the spin of the heavy quark ($SU(2)$ spin symmetry).
- also, $\psi_h(x) \to e^{i\eta(\vec{x})}\psi_h(x)$ is a symmetry (local flavor number conservation).

both broken at $O(1/m_h)$:

$$S_{HQET} = a^4 \sum_x \left\{ \bar{\psi}_h (D_0 + m_{\rm bare})\psi_h + \omega_{spin}\bar{\psi}_h(-\sigma\mathbf{B})\psi_h + \omega_{kin}\bar{\psi}_h\left(-\frac{1}{2}\mathbf{D}^2\right)\psi_h + \ldots \right\}$$

Analogously for operators

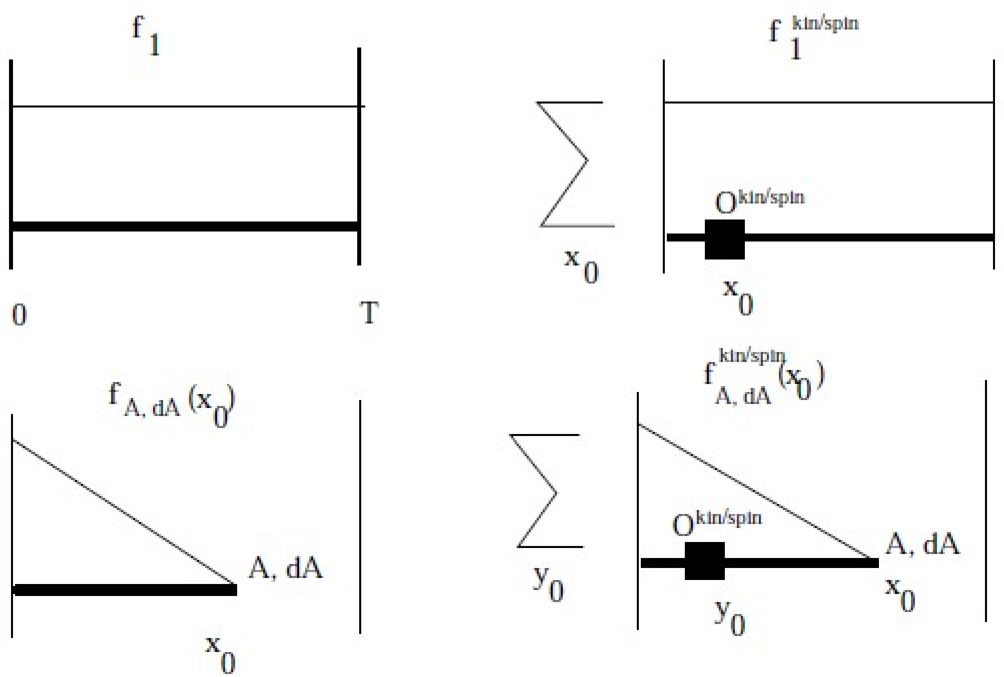
$$A_0^{\rm HQET}(x) = Z_{\rm A}^{\rm HQET}[A_0^{\rm stat}(x) + \sum_{i=1}^{2} c_{\rm A}^{(i)} A_0^{(i)}(x)] \,,$$

$$A_0^{(1)}(x) = \overline{\psi}_{\rm l}\tfrac{1}{2}\gamma_5\gamma_i(\nabla_i^{\rm S} - \overleftarrow{\nabla}_i^{\rm S})\psi_{\rm h}(x) \,,$$

$$A_0^{(2)}(x) = -\tilde{\partial}_i A_i^{\rm stat}(x)/2 \,, \quad A_i^{\rm stat}(x) = \overline{\psi}_{\rm l}(x)\gamma_i\gamma_5\psi_{\rm h}(x)$$

- NB. The static theory is not renormalizable by power-counting (static prop. $\propto \delta(\vec{x} - \vec{y})$). Still, only dim. 4 ops. in the action. In [Grinstein, '90] it has been shown that QCD correlators are reproduced to all orders in α_s at LO in $1/m_h$.

- Next to leading order terms in the $1/m_h$ expansion are not included in the action, that would produce couplings of negative dimension. They are treated as insertions into correlation functions evaluated in the static theory and renormlaized order by order.



Are there enough coeffs to make correlators finite ? YES [MDM, Garron, Papinutto, Sommer 2006], take the expansion of

$$C_{\mathrm{AA}}(x_0) = Z_{\mathrm{A}}^2 a^3 \sum_{\mathbf{x}} \left\langle A_0(x)(A_0)^\dagger(0) \right\rangle$$

- m_{bare} has to absorb a $1/a^2$ divergence from the mixing between the kinetic operator and $\bar{\psi}_h \psi_h$
- Z_A^{HQET} absorbs a $1/a$ divergence from the contact term between the kinetic operator and A_0^{stat}.

Why matching should be performed non-perturbatively ?

Let us consider the example

$$m_{B^*}^2 - m_B^2 = C_{mag}(m_b/\Lambda_{\mathrm{QCD}}) \, \langle B|\bar{\psi}_h \sigma \mathbf{B} \psi_h|B^*\rangle_{RGI} \times (1 + \mathrm{O}(1/m_b))$$

$C_{mag}(m_b/\Lambda_{\mathrm{QCD}})$ has a perturbative expansion. The truncation at O(n-1)

$$\simeq \alpha(m_b)^n \simeq \left\{ \frac{1}{2b_0 \ln(m_b/\Lambda_{\mathrm{QCD}})} \right\}^n >> \frac{\Lambda_{\mathrm{QCD}}}{m_b} \quad \text{as } m_b \to \infty$$

The PT corrections to the leading term are larger than the $1/m_b$ ones !

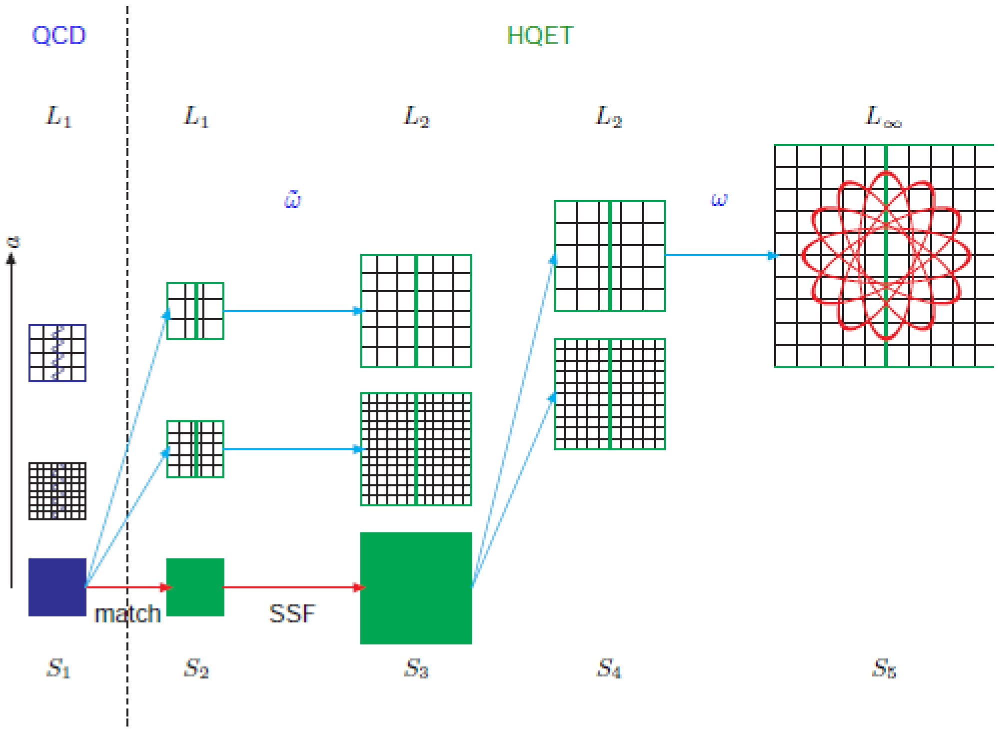
More on the actual matching procedure

[ALPHA, arXiv:1001.4783, 1004.2661, 1006.5816 and 1203.6516, 1311.5498]

- $a = f(\beta)$, $\beta = 6/g_0^2$. large β = small a.
- The parameters are renormalization factors. They depend on a but not on L.
- L/a can't be arbitrarily large.
- Eventually we want them for $a \simeq 0.1 - 0.05$ fm (large volumes for phenomenology).
- Idea: at small L and very fine a we simulate HQET and QCD with a relativistic b-quark. We get the parameters by matching 19 suitable quantities [MDM et al., arXiv:1312.1566]

$$\Phi_i^{\mathrm{QCD}}(m_{\mathrm{b}}, 0) = \Phi_i^{\mathrm{HQET}}(\omega_{\ldots}, c^{(j)}, Z_{\ldots}^{\mathrm{HQET}}, a)$$

- By a sequence of evolution (in L, fixed a) and matching (continuum vs finite a, fixed L) steps in HQET, one can obtain the parameters at larger a.



Let's take the easy one, ω_{spin} as an example. In QCD in a finte volume (Schrödinger Functional) we define VV and AA boundary to boundary correlators and their corresponding HQET expansion.

$$f_1 = -\frac{a^{12}}{2L^6} \sum_{\vec{u},\vec{v},\vec{y},\vec{z}} \left\langle \overline{\zeta}'_\ell(\vec{u})\gamma_5\zeta'_{\rm b}(\vec{v})\,\overline{\zeta}_{\rm b}(\vec{y})\gamma_5\zeta_\ell(\vec{z}) \right\rangle ,$$

$$k_1 = -\frac{a^{12}}{6L^6} \sum_{k} \left\langle \overline{\zeta}'_\ell(\vec{u})\gamma_k\zeta'_{\rm b}(\vec{v})\,\overline{\zeta}_{\rm b}(\vec{y})\gamma_k\zeta_\ell(\vec{z}) \right\rangle ,$$

The expansions read

$$[f_1]_R^{HQET} = Z_{\zeta_h}^2 Z_\zeta^2 e^{-m_{\rm bare}T} \left\{ f_1^{stat} + \omega_{kin} f_1^{kin} + \omega_{spin} f_1^{spin} \right\} ,$$

$$[k_1]_R^{HQET} = Z_{\zeta_h}^2 Z_\zeta^2 e^{-m_{\rm bare}T} \left\{ f_1^{stat} + \omega_{kin} f_1^{kin} - \tfrac{1}{3}\omega_{spin} f_1^{spin} \right\} ,$$

- The matching equation (in a size L_1, usually around 0.5 fm)

$$\Phi_{spin}(L_1, m_b, a) = \frac{3}{4}\ln\left(\frac{f_1}{k_1}\right)(L_1, m_b, a) = \omega_{spin}(m_b, a)\frac{f_1^{spin}}{f_1^{stat}}(L_1, a) + \ldots$$

can be solved for ω_{spin} at a and m_b where the matching is performed. This a is very fine, not suitable for computing the spectrum or decay constants or ...

- Evolution (SSF) an re-matching (from now on in HQET only).

At the same a (and m_b), we consider $L_2 = 2L_1$, simply by doubling the number of points. Using the same ω_{spin} we compute $\Phi_{spin}(L_2, m_b, a)$, with RHS above.

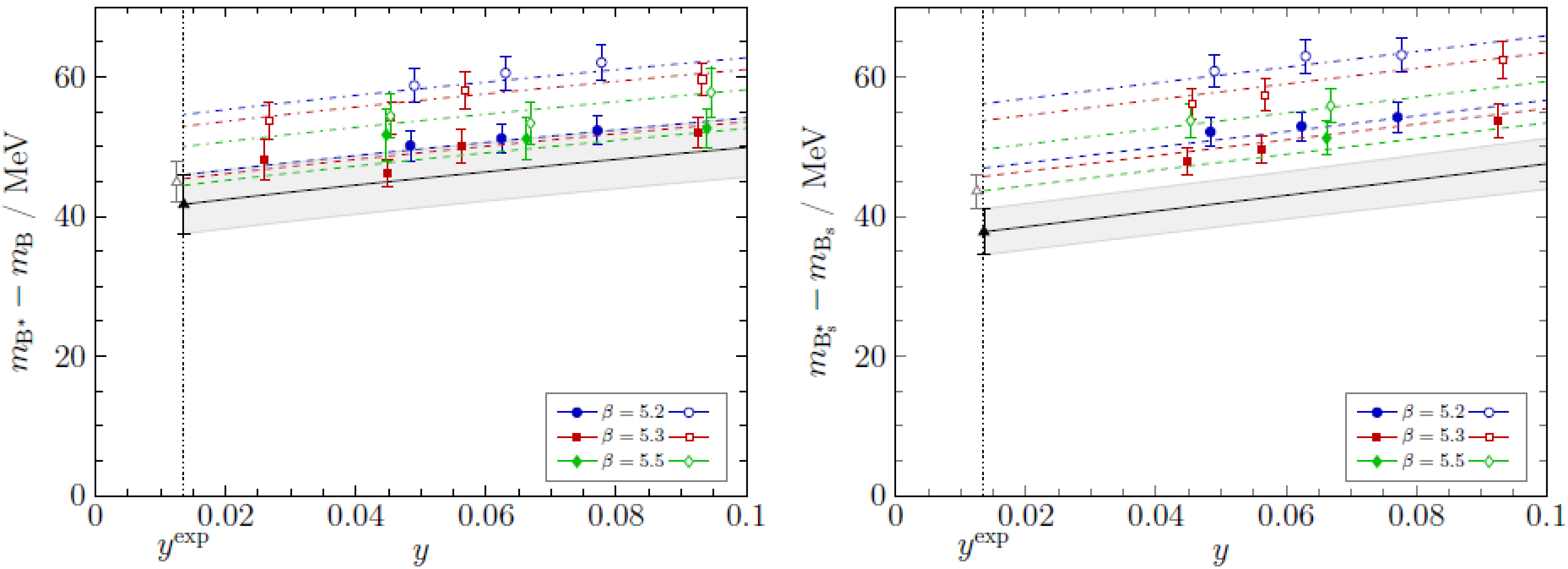
Then we change $a \to 2a$ and solve for $\omega_{spin}(m_b, 2a)$ the equation

$$\Phi_{spin}(L_2, m_b, a) = \Phi_{spin}(L_2, m_b, 2a)$$

so, we set to 0 cutoff effects on Φ_{spin}. One or two steps are usually enough.

Remark. In the LHS of matching equations the $\lim a \to 0$ is usually taken.

Finally, in large volume $\frac{4}{3}\omega_{spin}\langle B|O_{spin}|B\rangle$ give the V-PS splitting



[MDM, ALPHA '15]

Similarly, the parameters entering the b-quark mass and the B-meson decay constant have all been determined non-perturbatively.

Matching-quantites have been defined and studied in perturbation theory far all the 19 parameters in the action and vector and axial currents at $O(1/m_h)$ [MDM, Dooling, Hesse, Heitger and Simma, '13].

The b quark mass in HQET at $O(1/m_h)$

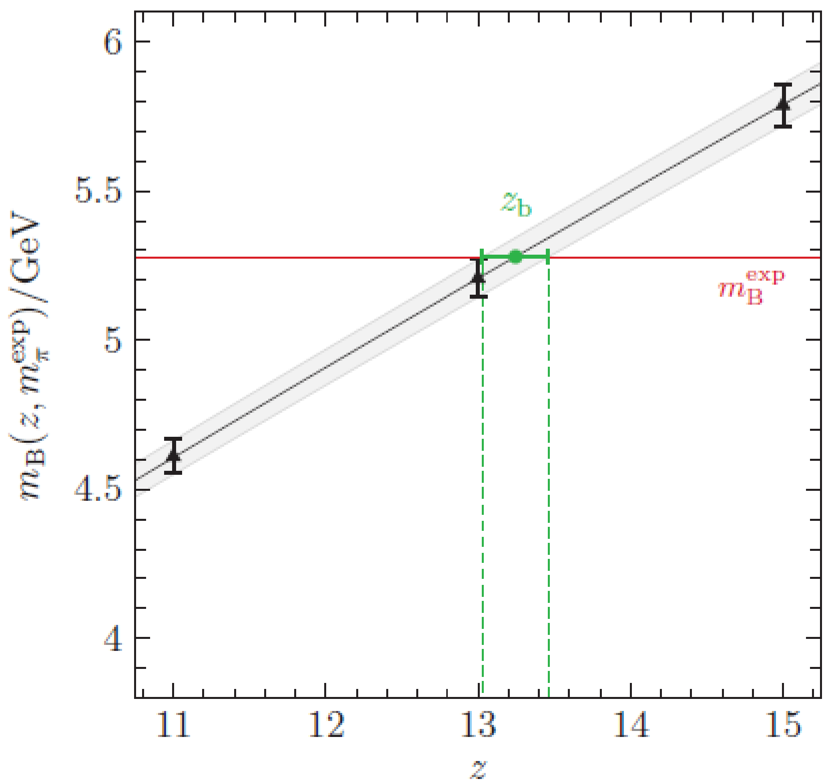
We generate $N_f = 2$ dynamical configurations, with NP $O(a)$ improved Wilson fermions and plaquette gauge action.

β	a[fm]	L/a	m_π[MeV]	$m_\pi L$	#cfgs	$\frac{\#\text{cfgs}}{\tau_{\text{exp}}}$	id
5.2	0.075	32	380	4.7	1012	122	A4
		32	330	4.0	1001	164	A5
		48	280	5.2	636	52	B6
5.3	0.065	32	440	4.7	1000	120	E5
		48	310	5.0	500	30	F6
		48	270	4.3	602	36	F7
		64	190	4.1	410	17	G8
5.5	0.048	48	440	5.2	477	4.2	N5
		48	340	4.0	950	38	N6
		64	270	4.2	980	20	O7

The b-quark mass is determined by computing, as a function of the heavy quark mass m_h used in the matching, the large-volume quantity

$$m_B(m_h) = m_{bare}(m_h) + E^{stat} + \omega_{spin}(m_h)E^{spin} + \omega_{kin}(m_h)E^{kin}$$

and then solving $m_B(m_h) = m_B^{exp}$, with m_h as unknown.



$z = L_1 m_h$, [MDM and ALPHA, arXiv:1311.5498]

N_f	Ref.	M	$\overline{m}_{\overline{MS}}(\overline{m}_{\overline{MS}})$	$\overline{m}_{\overline{MS}}(4\,GeV)$	$\overline{m}_{\overline{MS}}(2\,GeV)$
0	[36]	6.76(9)	4.35(5)	4.39(6)	4.87(8)
2	this work	6.58(17)	4.21(11)	4.25(12)	4.88(15)
5	PDG13 [1]	7.50(8)	4.18(3)	4.22(4)	4.91(5)

Convergence at lower scales may be due to the common low-energy input (m_B).

Other recent computations [see Sanfilippo, LAT14 for less recent ones]

- ETMC [1411.0484], $N_f = 2+1+1$, ratio (or Tor Vergata) method with $P = \frac{m_B(h\ell)}{m_h^{pole}}$. $\mathrm{N^3LO}$ PT is used to convert pole mass to $\overline{\mathrm{MS}}$.

- HPQCD [1408.4169], $N_f = 2+1+1$, HISQ MILC confs. Based on fitting moments of heavy-heavy PS two-point functions to obtain m_c and α_s
  - Low moments (n=4,... 10) should be short distance dominated.
  - (Gluon) condensates enter the OPE.
  - Lattice data for moments up to $n = 10$ are simultaneously fitted including $O(\alpha_s^{15})$ [$\mathrm{N^3LO}$ known] and $O(am_h)^{42}$.
  - Over 40 params, with priors, and 92 data points.
  - Stability of the results including about half the $O(\alpha_s)$ and $O(a)$ terms.
  - The approach gives m_{η_h} as a function of m_h. By extrapolating to m_{η_b} from $\approx 3m_c$ HPQCD gets m_b/m_c and eventually m_b with about 1% error.

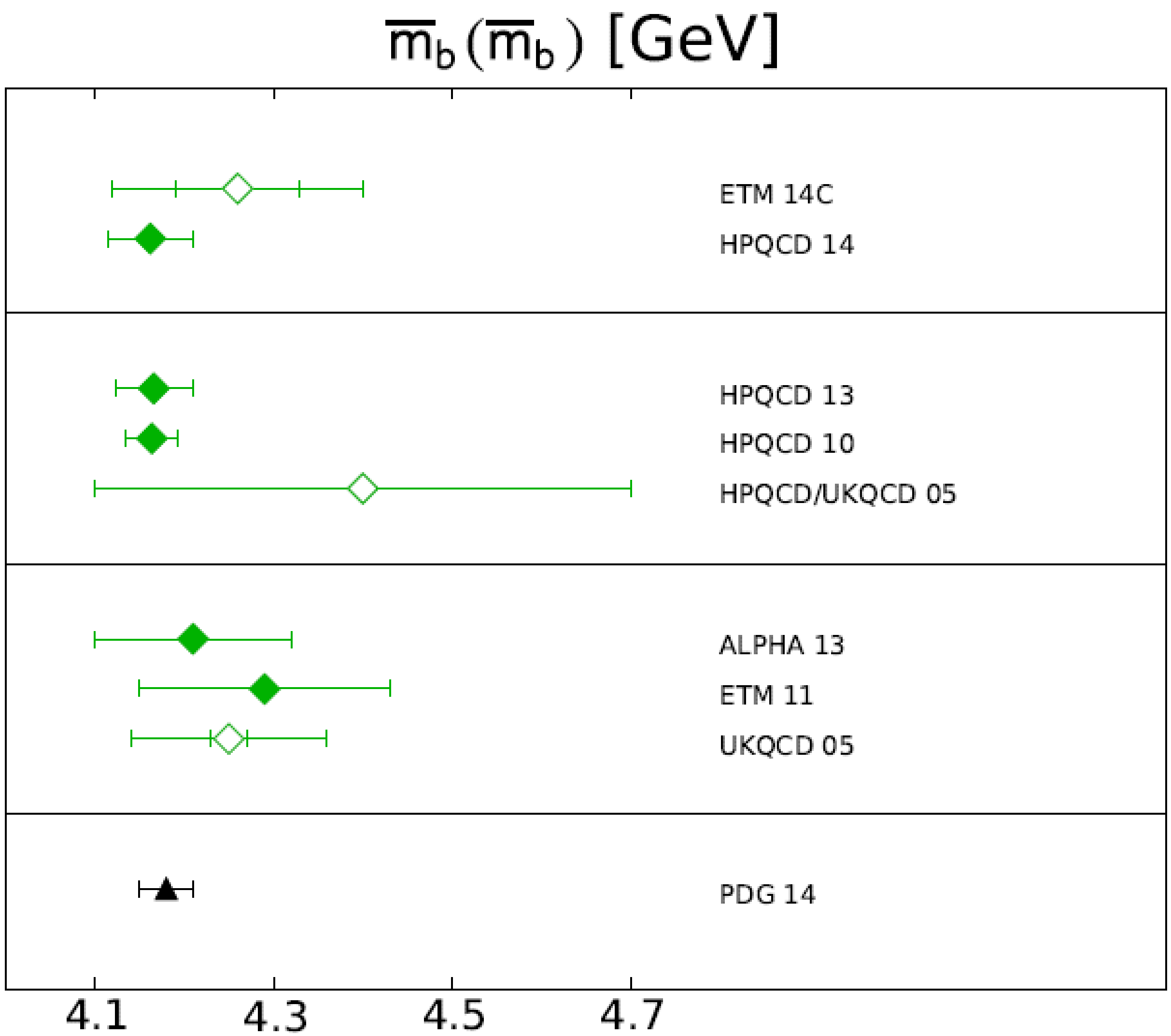
FLAG m_b

Collaboration	Ref.	N_f	publication status	chiral extrapolation	continuum extrapolation	finite volume	renormalization	running	$\overline{m}_b(\overline{m}_b)$ [GeV]
ETM 14C	[1]	2+1+1	C						4.26(7)(14)
HPQCD 14	[2]	2+1+1	A						4.162(48)
HPQCD 13	[3]	2+1	A						4.166(43)
HPQCD 10	[4]	2+1	A						4.164(29)†
HPQCD/UKQCD 05	[5]	2+1	A						4.4(3)
ALPHA 13	[6]	2	A						4.21(11)
ETM 11	[7]	2	A						4.29(14)
UKQCD 05	[8]	2	A						4.25(2)(11)
ALPHA 07	[9]	0	A						4.42(6)
ALPHA 06	[10]	0	A						4.347(48)
Rome 2	[11]	0	A						4.33(10)

† The number that is given is $m_b(10\,\mathrm{GeV}, N_f = 5) = 3.617(25)\,\mathrm{GeV}$.

Table 1: Lattice results for the mass $\overline{m}_b(\overline{m}_b)$ in the $\overline{\mathrm{MS}}$ scheme, together with the colour coding of the calculation used to obtain these. If information about non-perturbative running is available, this is indicated in the column "running", with details given at the bottom of the table.

FLAG m_b


$\overline{m}_b(\overline{m}_b)$ [GeV]
ETM 14C
HPQCD 14
HPQCD 13
HPQCD 10
HPQCD/UKQCD 05
ALPHA 13
ETM 11
UKQCD 05
PDG 14
4.1
4.3
4.5
4.7

Back to matching ...

$$S_{HQET} = a^4 \sum_x \left\{ \bar\psi_h (D_0 + \delta m)\psi_h + \omega_{spin} \bar\psi_h(-\sigma \mathbf{B})\psi_h + \omega_{kin}\bar\psi_h \left(-\frac{1}{2}\mathbf{D}^2\right)\psi_h \right.$$

We also consider the currents

$$\begin{aligned}
A_0^{\mathrm{HQET}}(x) &= Z_{\mathrm{A}}^{\mathrm{HQET}}[A_0^{\mathrm{stat}}(x) + \sum_{i=1}^{2} c_{\mathrm{A}}^{(i)} A_0^{(i)}(x)] \,, \\
A_0^{(1)}(x) &= \overline{\psi}_1 \tfrac{1}{2}\gamma_5\gamma_i(\nabla_i^{\mathrm{S}} - \overleftarrow{\nabla}_i^{\mathrm{S}})\psi_{\mathrm{h}}(x) \,, \\
A_0^{(2)}(x) &= -\tilde\partial_i A_i^{\mathrm{stat}}(x)/2 \,, \quad A_i^{\mathrm{stat}}(x) = \overline{\psi}_1(x)\gamma_i\gamma_5\psi_{\mathrm{h}}(x) \,,
\end{aligned}$$

$$\begin{aligned}
A_k^{\mathrm{HQET}}(x) &= Z_{\mathrm{A}_k}^{\mathrm{HQET}}\,[A_k^{\mathrm{stat}}(x) + \sum_{i=3}^{6} c_{\mathrm{A}}^{(i)} A_k^{(i)}(x)] \,, \\
A_k^{(3)}(x) &= \overline{\psi}_1(x)\frac{1}{2}(\nabla_i^{\mathrm{S}} - \overleftarrow{\nabla}_i^{\mathrm{S}})\gamma_i\gamma_5\gamma_k\psi_{\mathrm{h}}(x) \,, \quad A_k^{(4)}(x) = \overline{\psi}_1(x)\frac{1}{2}(\nabla_k^{\mathrm{S}} - \overleftarrow{\nabla}_k^{\mathrm{S}})\gamma_5\psi_{\mathrm{h}}(x) \\
A_k^{(5)}(x) &= \tilde\partial_i\,\left(\overline{\psi}_1(x)\gamma_i\gamma_5\gamma_k\psi_{\mathrm{h}}(x)\right)/2 \,, \quad A_k^{(6)}(x) = \widetilde{\partial_k}\, A_0^{\mathrm{stat}}/2
\end{aligned}$$

and analogous expressions for the vector current, 19 coeffs in total.

In defining the quantities Φ_i we exploit the possibility of changing the spatial periodicity of the fermions [ALPHA, '96], which can be interpreted as injecting momentum (flavour twisted boundary conditions, [Sachrajda and Villadoro, 2005])

$$\psi(\vec{x} + \hat{k}L) = e^{i\theta_k}\psi(\vec{x})$$

on the torus then

$$S(x) = \langle\psi(x)\bar{\psi}(0)\rangle = \int \frac{dk_4}{2\pi}\frac{1}{L^3}\sum_{\vec{k}} \frac{e^{i(\vec{k}+\vec{\theta}/L)\cdot\vec{x}}}{\not{k}+\frac{\not{\theta}}{L}+m} \ ,$$

We will

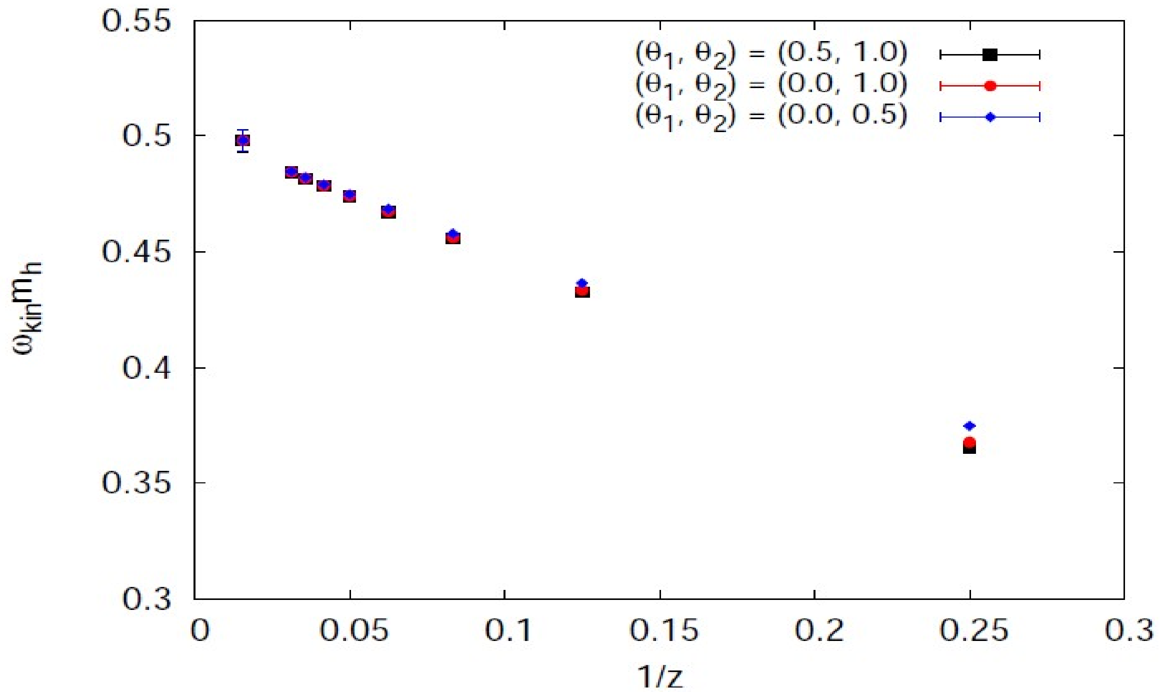
- build ratio of two-point correlators for different $\vec{\theta}_h = \vec{\theta}_l = \theta \cdot \vec{1}$
- consider the kinematics $\vec{\theta}_h \neq \vec{\theta}_l$ (e.g. for total derivative operators)
- use also anisotropic $\vec{\theta}$ angles

Tree level study [MDM et al. 1312.1566]

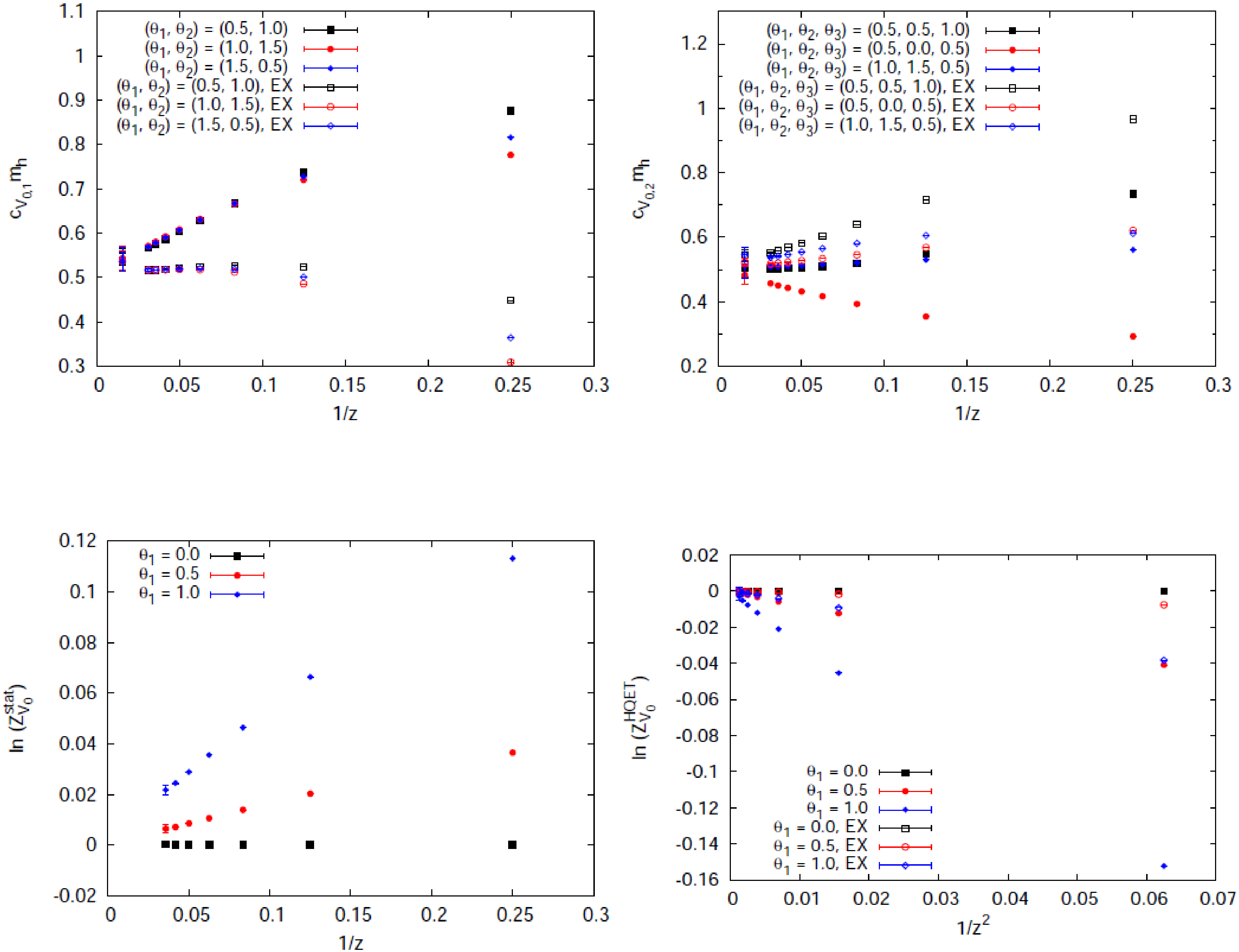
The classical values of the coefficients are known:
$c_A^{(1)} = c_A^{(2)} = -c_A^{(3)} = -c_A^{(5)} = -\frac{1}{2m_b}$ and $c_A^{(4)} = -c_A^{(6)} = \frac{1}{m_b}$.

The single terms in the HQET expansion of a correlator are finite and have a continuum limit. Example ω_{kin}

$$\frac{1}{4}(R_1^P + 3R_1^V) - R_1^{\rm stat} = \omega_{kin} R_1^{kin}\,, \quad (T = L/2)$$



in the non-perturbative study we will have $z_b \simeq 13$.

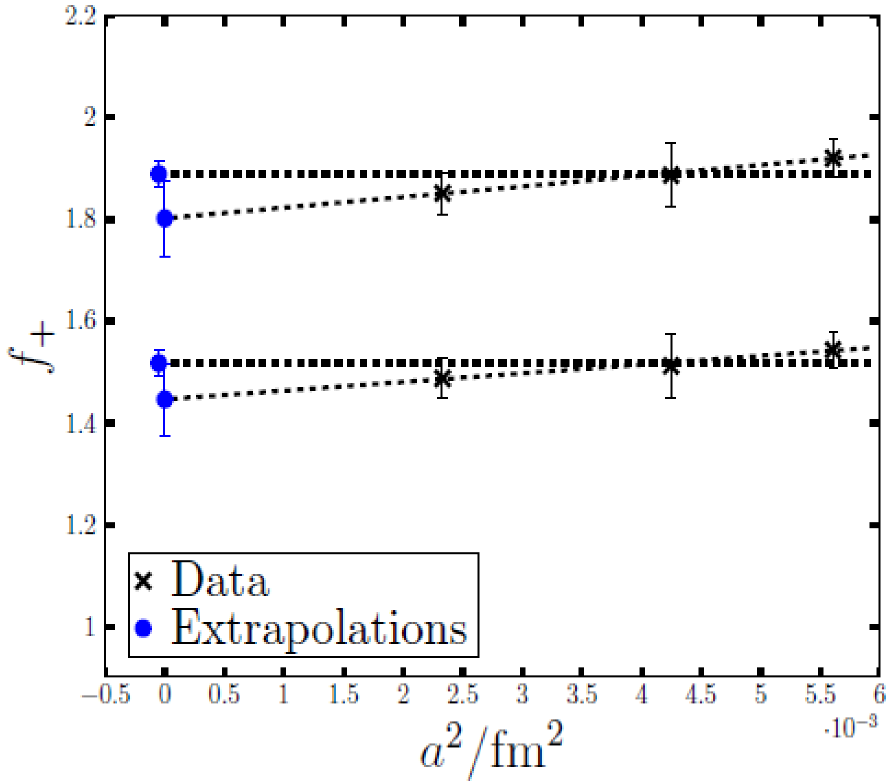


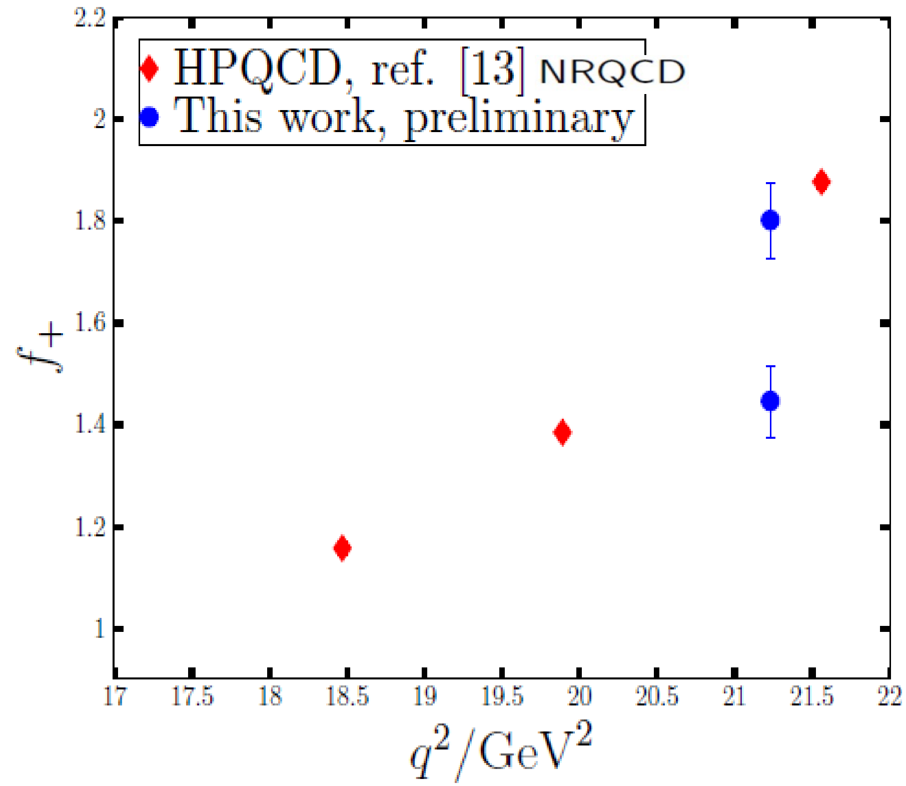
29

Non-perturbative matching ongoing [C. Wittemeier, LAT15]

Goal: form factors

$B_s \to K\ell\nu$ in the static approximation at the moment [F. Bahr et al, 1411.3916]

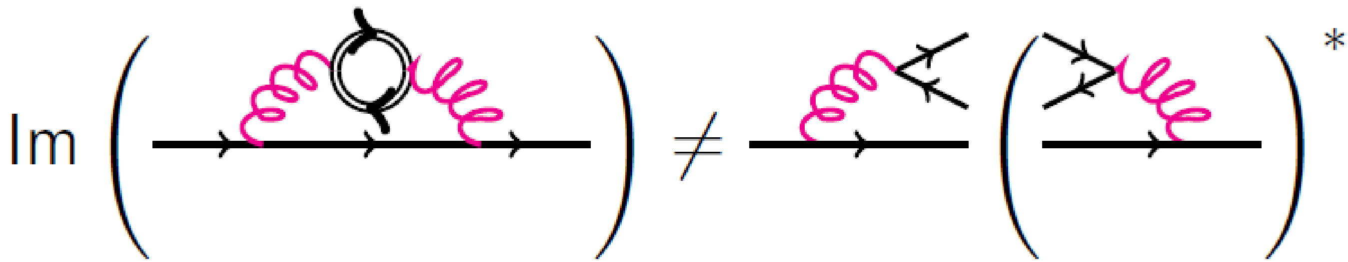




Reweighting twisted boundary conditions [A. Bussone LAT15]

Twisting only in the valence $\longrightarrow$ Breaking of unitarity

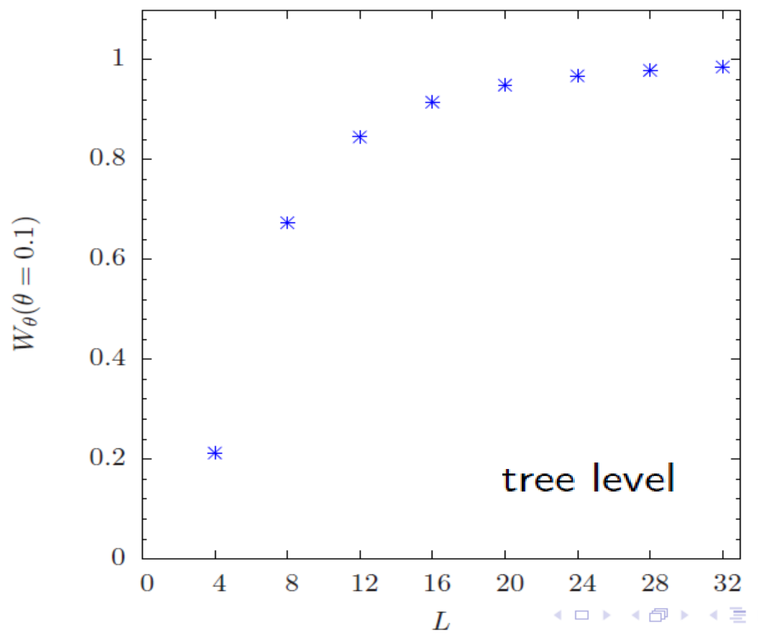
- Sea quark propagator:
- Valence quark propagator:

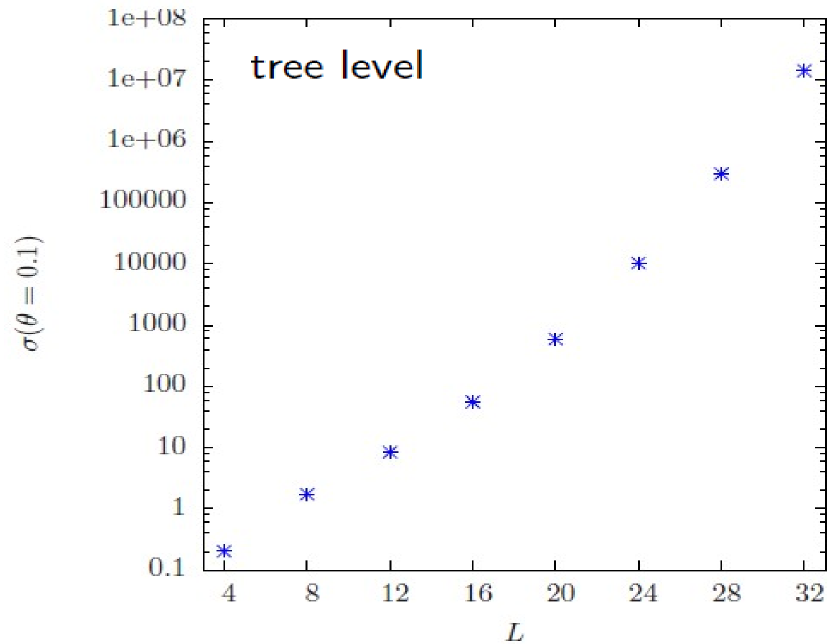


We expect that it is a finite volume effect: in χ-PT this is the case

[Sachrajda, Villadoro Phys. Lett. B 609 (2005) 73]

Reweighting can be used in small volume to compensate breaking of unitarity

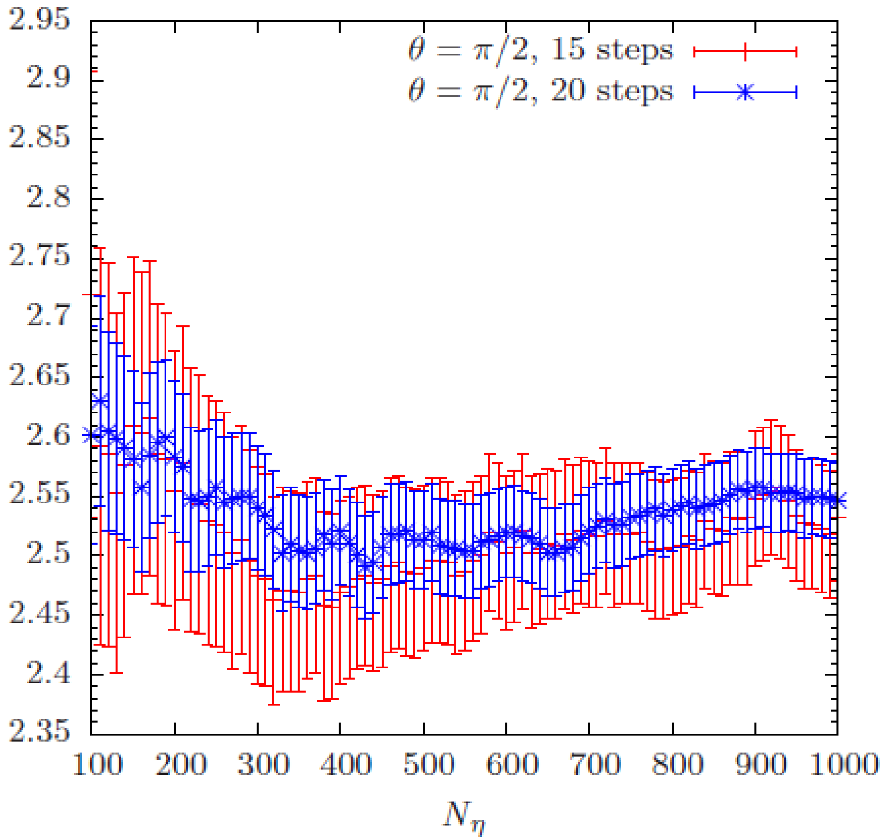


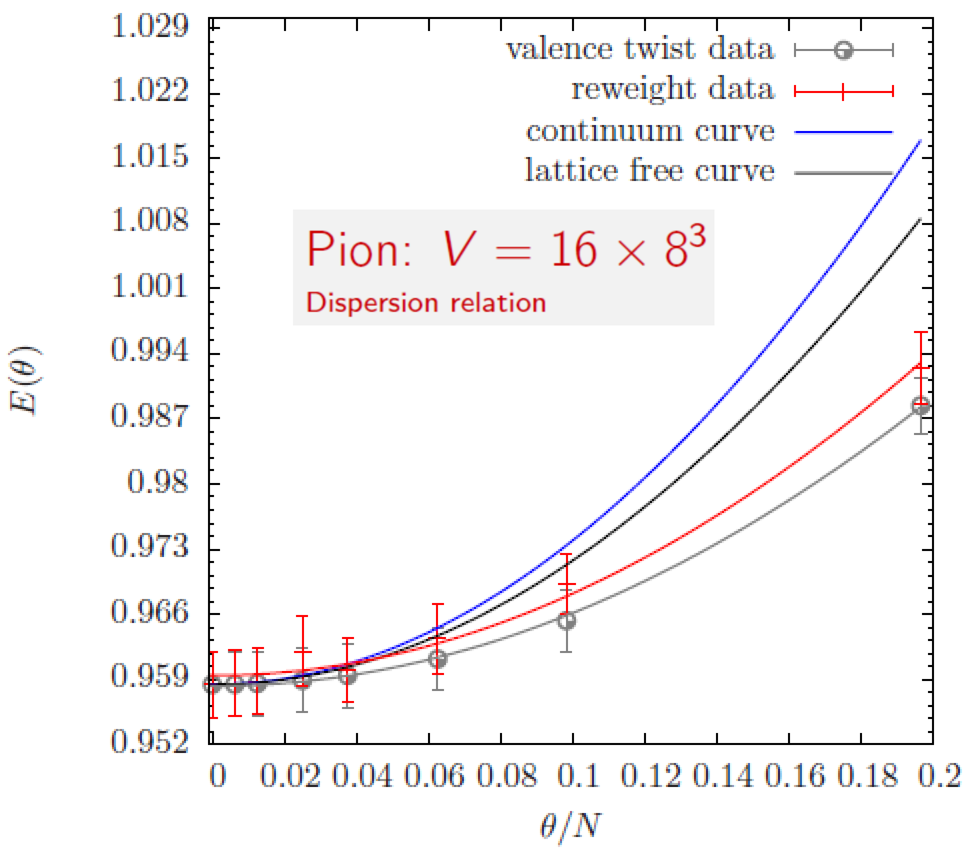


We factorize our matrix in the following way [Finkenrath et al. Nucl. Phys. B 877 (2013) 441]

$$D_W(\theta) D_W^{-1}(0) = A = \prod_{l=0}^{N-1} A_\ell, \quad \text{with } A_\ell \simeq \mathbf{1} + \mathrm{O}\left(\delta\theta_\ell\right)$$

$$\frac{1}{\det A} = \prod_{\ell=0}^{N-1} \left\langle \frac{\exp\left(-\eta^{(\ell),\dagger} A_l \eta^{(\ell)}\right)}{p\left(\eta^{(\ell)}\right)} \right\rangle_{p\left(\eta^{(\ell)}\right)}$$





- **SU**(2), fermions in the fundamental (confinement, χSB)

Conclusions

- Higgs less of a portal to New Physics than we hoped.

- Even to establish that precise lattice results in the b-sector are needed.

- Tensions in B-physics (R(D*), $B^0 \rightarrow K^0 \mu^+\mu^-$) need lattice inputs to assess significances.

- HQET on the lattice:
  NP subtraction of power divergences
  (conceptually a 'must' as $a \rightarrow 0$).

  NP matching at $O(1/m_h)$
  (conceptually a 'must' as $m_h \rightarrow \infty$)

- b-quark mass

- Now tackling form factors