hadronic corrections to electroweak observables

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Fundamental Parameters from Lattice QCD, MITP, Mainz, Sept. 8, 2015

hadronic corrections to electroweak observables

- observables that enter in the consistency tests of the Standard Model in the electroweak sector
  - QED coupling : $\alpha(M_Z)$
- observables that can be probed in low-energy experiments
  - $\sin^2\theta_W(Q^2)$
  - $(g-2)_\mu$ [talk by Taku Izubuchi]

running of QED coupling

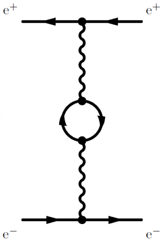
$$\Delta\alpha_{\mathrm{QED}}^{\mathrm{had}}$$

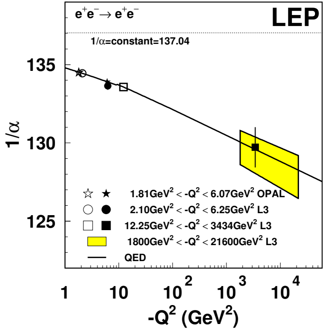
$\Delta\alpha_{\mathrm{QED}}$

$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha_{\mathrm{QED}}(Q^2)}$$

- vacuum polarisation: charge screening $\rightsquigarrow$ running of QED coupling
- Standard Model (SM) precision tests and sensitivity to new physics requires precise knowledge of $\Delta\alpha_{\mathrm{QED}}(Q^2)$: input parameter of SM

experimental evidence of the running of $\alpha(Q^2)$: LEP differential cross-section of Bhabha scattering





[S. Mele, hep-ex/0610037]

$\Delta\alpha_{\rm QED}$

$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha_{\rm QED}(Q^2)}$$

- $\Delta\alpha_{\rm QED}(Q^2)$ receives contributions from leptons and quarks :

$$\Delta\alpha_{\rm QED} = \Delta\alpha_{\rm lep} + \Delta\alpha^{(5)}_{\rm had} + \Delta\alpha_{\rm top}$$

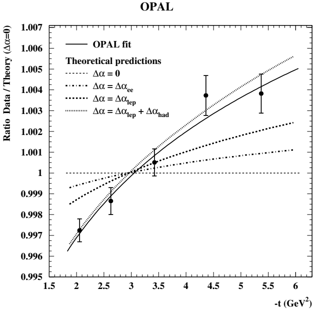
$$\Delta\alpha_{\rm lep}(M_Z) = 0.03150$$

$$\Delta\alpha^{(5)}_{\rm had}(M_Z) = 0.02771(11)$$

$$\Delta\alpha_{\rm top}(M_Z) = -0.00007(1)$$

[PDG, 2014]

OPAL : Bhabha scattering



[OPAL, Eur. Phys. J. C 45 (2006) 1.]

$\Delta\alpha^{\mathrm{had}}_{\mathrm{QED}}$

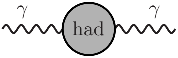
$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha_{\mathrm{QED}}(Q^2)}$$

- $\alpha =$ 1/137.035999074(44) [0.3 ppb] [PDG, 2013]
- $\alpha(M_Z^2) = 1/128.952(14)$ [10^{-4}] $\rightsquigarrow$ 10^5 less accurate . . .

  [M. Davier et al., 1010.4180]

- hadronic effects : $\alpha(Q^2)$ depends strongly on Q^2 at low energies

  hadronic uncertainties propagate . . .

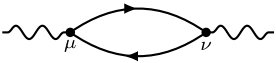


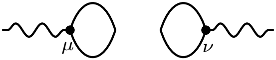
- uncertainty in $\Delta\alpha^{\mathrm{had}}_{\mathrm{QED}}(M_Z^2)$ is comparable to that of $\sin^2\theta_W(M_Z^2)$

$\Delta\alpha_{\rm QED}^{\rm had}$

$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha_{\rm QED}(Q^2)}$$

- leading order (LO) contribution





$$\int \mathrm{d}^4x\, e^{iQx} \langle J_\mu(x)\, J_\nu(0)\rangle = (Q_\mu Q_\nu - Q^2\, \delta_{\mu\nu})\, \Pi(Q^2)$$

$$J_\mu(x) = \sum_{f=1}^{N_{\rm f}} Q_f\, \overline{\psi}_f(x)\gamma_\mu\psi_f(x)$$

$$Q_f \in \{-1/3, 2/3\}$$

- $\Pi(Q^2)$: photon vacuum polarisation function (VPF)

$\Delta\alpha^{\mathrm{had}}_{\mathrm{QED}}$

$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha_{\mathrm{QED}}(Q^2)}$$

- leading order (LO) contribution

$$\int \mathrm{d}^4x \, e^{iQx} \, \langle J_\mu(x) \, J_\nu(0) \rangle \; = \; (Q_\mu Q_\nu - Q^2 \, \delta_{\mu\nu}) \, \Pi(Q^2)$$

$$J_\mu(x) \; = \; \sum_{f=1}^{N_{\mathrm{f}}} Q_f \, \overline{\psi}_f(x) \gamma_\mu \psi_f(x)$$

$$Q_f \in \{-1/3, 2/3\}$$

- $\Pi(Q^2)$: photon vacuum polarisation function (VPF)

$$\Delta\alpha_{\mathrm{QED}}(Q^2) \; = \; 4\pi\alpha \, \Big(\Pi(Q^2) - \Pi(0) \Big) \; = \; 4\pi\alpha \, \Pi_{\mathrm{R}}(Q^2)$$

- Adler function $D(Q^2)$:

$$\frac{D(Q^2)}{Q^2} = 12\pi^2 \, \frac{d\,\Pi\,(q^2)}{dq^2}$$

$$= -\frac{3\pi}{\alpha} \, \frac{d}{dq^2} \Delta\alpha^{\mathrm{had}}_{\mathrm{QED}}(q^2) \qquad Q^2 = -q^2$$

$\Delta\alpha_{\rm QED}^{\rm had}$

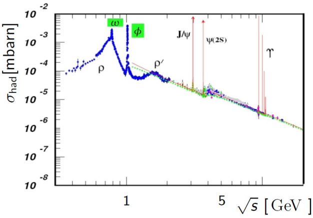
$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha_{\rm QED}(Q^2)}$$

- the VPF $\Pi(Q^2)$ and the Adler function $D(Q^2)$ $\rightsquigarrow$ $\Delta\alpha_{\rm QED}^{\rm had}(Q^2)$ and $a_\mu^{\rm HLO}$

- phenomenological approach :

  dispersion relation + optical theorem + $(e^+e^- \to \text{hadrons})$ cross section

$$\Delta\alpha_{\rm QED}^{\rm had}(Q^2) = -\frac{\alpha\, Q^2}{3\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{R_{\rm had}(s')}{s'(s' - Q^2)}$$



  compared to $a_\mu^{\rm HLO}$, low-energy regions contribute less

  theoretical prediction that relies on experimental data

- lattice QCD

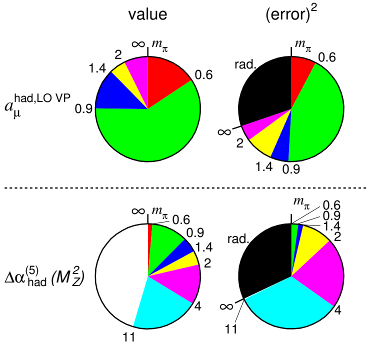
$\Delta\alpha^{\rm had}_{\rm QED}$ and $a_\mu^{\rm HLO}$

- phenomenological approach :

dispersion relation + optical theorem + $(e^+e^- \to \text{hadrons})$ cross section

$$a_\mu^{\rm HLO} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds' \, \frac{R_{\rm had}(s')\, K(s)}{s^2}$$

$$\Delta\alpha^{(5)}_{\rm had}(Q^2) = -\frac{\alpha\, Q^2}{3\pi} \int_{4m_\pi^2}^{\infty} ds' \, \frac{R_{\rm had}(s')}{s'(s'-Q^2)}$$



$$a_\mu^{\rm HLO} = (694.91 \pm 3.72_{\rm exp} \pm 2.10_{\rm rad}) \cdot 10^{-10} \quad [0.6\%]$$

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = (276.26 \pm 1.38) \cdot 10^{-4} \quad [0.5\%]$$

[Hagiwara et al., 1105.3149]

- low energy:

$$\Delta\alpha^{(3)}_{\rm had}(Q^2 = 3.2\,{\rm GeV}^2) = (55.50 \pm 0.78) \cdot 10^{-4} \quad [1.4\%]$$

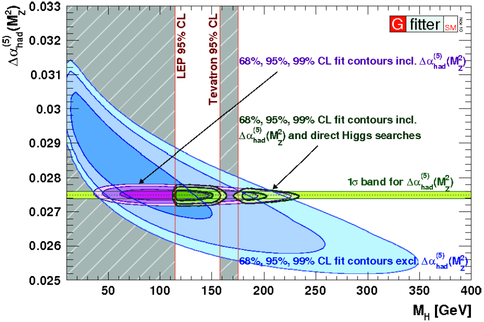
[PDG, 2014]

- use of PT : $Q^2 \sim (2.6\,{\rm GeV})^2$ [Hagiwara et al., 1105.3149]

  $Q^2 \sim (1.8\,{\rm GeV})^2$ [M. Davier et al., 1010.4180]

Global electroweak fit of the SM

- combine measurements of electroweak precision observables to overconstrain the electroweak sector $\rightsquigarrow$ test of SM
- back in 2011 :



[Gfitter, M. Baak et al., 2011]

$$\Delta\alpha^{(5)}_{\rm had}(M_Z) = (276.8 \pm 2.2)\cdot 10^{-4} \quad \rightsquigarrow \quad (274.9 \pm 1.0)\cdot 10^{-4}$$

[Hagiwara et al., 2007; Davier et al., 2010]

$$M_H = 84^{+30}_{-23}\,\text{GeV} \quad \rightsquigarrow \quad 96^{+31}_{-24}\,\text{GeV}$$

Global electroweak fit of the SM

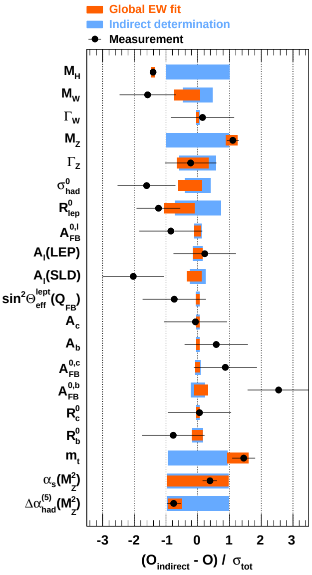
Parameter	Input value	Free in fit	Fit Result	w/o exp. input in line
M_H [GeV]	125.14 ± 0.24	yes	125.14 ± 0.24	93^{+25}_{-21}
M_W [GeV]	80.385 ± 0.015	–	80.364 ± 0.007	80.358 ± 0.008
M_Z [GeV]	91.1875 ± 0.0021	yes	91.1880 ± 0.0021	91.200 ± 0.011
$\sin^2\theta^{\ell}_{\mathrm{eff}}$	0.2324 ± 0.0012	–	0.23150 ± 0.00006	0.23149 ± 0.00007
$A^{0,b}_{\mathrm{FB}}$	0.0992 ± 0.0016	–	0.1032 ± 0.0004	0.1034 ± 0.0004
m_c [GeV]	$1.27\,^{+0.07}_{-0.11}$	yes	$1.27\,^{+0.07}_{-0.11}$	–
m_b [GeV]	$4.20\,^{+0.17}_{-0.07}$	yes	$4.20\,^{+0.17}_{-0.07}$	–
m_t [GeV]	173.34 ± 0.76	yes	173.81 ± 0.85	$177.0\,^{+2.3}_{-2.4}$
$\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$ $[10^{-5}]$	2757 ± 10 [0.4%]	yes	2756 ± 10	2723 ± 44 [1.6%]
$\alpha_s(M_Z)$	–	yes	0.1196 ± 0.0030 [2.5%]	0.1196 ± 0.0030

[Gfitter, 1407.3792]

[FLAG, 1310.8555]: $\alpha_s(M_Z) = 0.1184(12)$ [1%]

$\sin^2\theta^{\ell}_{\mathrm{eff}}$: related to the effective vector and axial couplings of the Z boson to leptons at the Z pole

Global electroweak fit of the SM



- agreement between direct & indirect determinations : p-value = 21%

- no individual value > 3 σ

  $A_{FB}^{0,b}$ with 2.5 σ
  unpolarized Z-pole forward-backward asymmetry : $\bar{g}_V^b$ and $\bar{g}_A^b$

- indirect determination :

$$\sin^2\theta^\ell_{\rm eff} = 0.231488(70) \;\; [0.03\%]$$

  where $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ contributes to 50% of the uncertainty

[Gfitter, 1407.3792]

Global electroweak fit of the SM : prospects

Parameter	Exp. input [$\pm 1\sigma_{\rm exp}$] Present	ILC/GigaZ	Indirect [$\pm 1\sigma_{\rm exp}, \pm 1\sigma_{\rm theo}$] Present	ILC/GigaZ
M_H [GeV]	0.4	< 0.1	${}^{+31}_{-26}, {}^{+10}_{-8}$	${}^{+6.9}_{-6.6}, {}^{+2.5}_{-2.3}$
M_W [MeV]	15	5	6.0, 5.0	1.9, 1.3
M_Z [MeV]	2.1	2.1	11, 4	2.6, 1.0
m_t [GeV]	0.8	0.1	2.4, 0.6	0.7, 0.2
$\sin^2\theta^{\ell}_{\rm eff}$ [10^{-5}]	16	1.3	4.5, 4.9	2.0, 1.0
$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ [10^{-5}]	10	4.7	42, 13	5.6, 3.0
$\alpha_S(M_Z)$ [10^{-4}]	–	–	40, 10	6.4, 6.9

prospects for ILC/GigaZ : uncertainty on $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ and M_Z

$\rightsquigarrow$ largest contribution to uncertainty in M_H

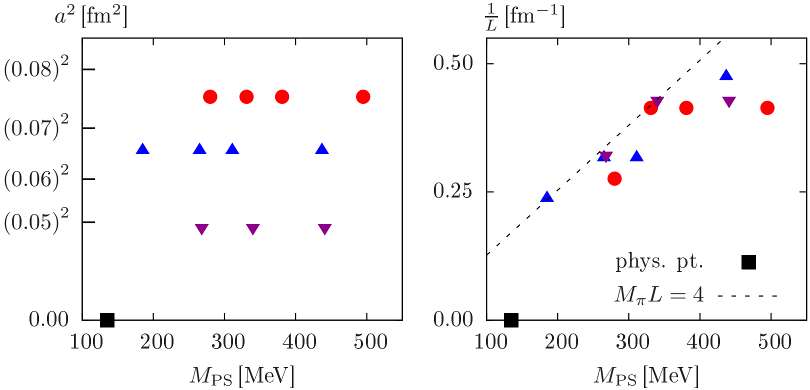
[Gfitter, 1407.3792]

improved precision on the theoretical determination of $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ will be needed

Mainz : electroweak couplings

A. Francis, V. Gülpers, G. H., G. von Hippel, H. Horch, B. Jäger, H. Meyer, H. Wittig

- [Mainz, 1112.2894]



$N_f = 2$ $\mathcal{O}(a)$ improved Wilson fermions [CLS]

strange and charm are quenched : s_Q , c_Q

quark connected + disc. contributions

$1000 \div 4000$ meas. per ensemble

scale from f_K [ALPHA, 1205.5380]

lattice VPF

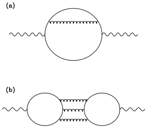
Local current

$$J_\mu^{(1,\,f)}(x) = Z_V\,\overline{\psi}_f(x)\,\gamma_\mu\,\psi_f(x)$$

conserved-local correlator

$$a^6 \left\langle \sum_{f=1}^{N_f} \left(Q_f\, J_\mu^{(\mathrm{ps},\,f)}(x) \right) \sum_{f'=1}^{N_f} \left(Q_{f'}\, J_\nu^{(1,\,f')}(0) \right) \right\rangle$$

$$\Pi_{\mu\nu}(\hat{Q}) = a^4 \sum_x e^{iQ(x+a\hat{\mu}/2)} \langle\, J_\mu^{(\mathrm{ps})}(x)\, J_\nu^{(1)}(0)\,\rangle \quad \rightsquigarrow \quad \Pi(\hat{Q}^2)$$



$$\hat{Q}_\mu = \frac{2}{a}\sin\left(\frac{aQ_\mu}{2}\right)$$

Adler function

the Adler function $D(Q^2)$ is related to the vacuum polarization by

$$D(Q^2) = 12\,\pi^2\, Q^2\, \frac{d\Pi(Q^2)}{dQ^2}$$

compute the Adler function :

- analytic derivative:

  fit a function to $\Pi(Q^2)$ and compute its derivative

  Padé ansatz :

$$\Pi_{fit}(Q^2) = \Pi(0) + Q^2 \left(\frac{p_1}{p_2 + Q^2} + \frac{p_3}{p_4 + Q^2} \right)$$

$$Q^2 \frac{d}{dQ^2} \Pi_{fit}(Q^2) = Q^2 \left(\frac{p_1 p_2}{(p_2 + Q^2)^2} + \frac{p_3 p_4}{(p_4 + Q^2)^2} \right)$$

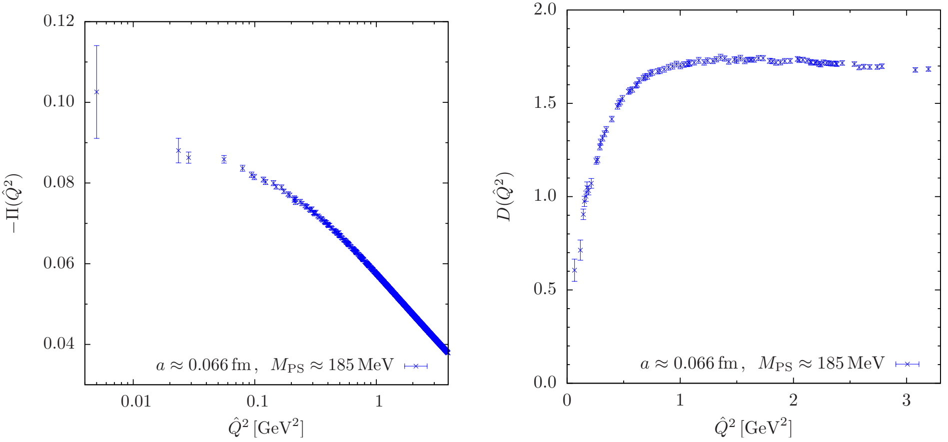
- numerical derivative:

  apply linear or quadratic fits of varying ranges to determine the derivative of $\Pi(Q^2)$

Adler function : numerical derivative

$$D(Q^2) = 12\,\pi^2\ \frac{d\Pi(Q^2)}{d\log Q^2}$$

u, *d*



Adler function : combined fit

Adler function:

$$D(Q^2) = 12\,\pi^2\, Q^2\, \frac{d\Pi(Q^2)}{dQ^2}$$

- fit form :

$$D(Q^2) = \text{Padé}(Q^2)\ [\,1 \ + \ \textbf{discr.} \ + \ \textbf{mass}\,]$$

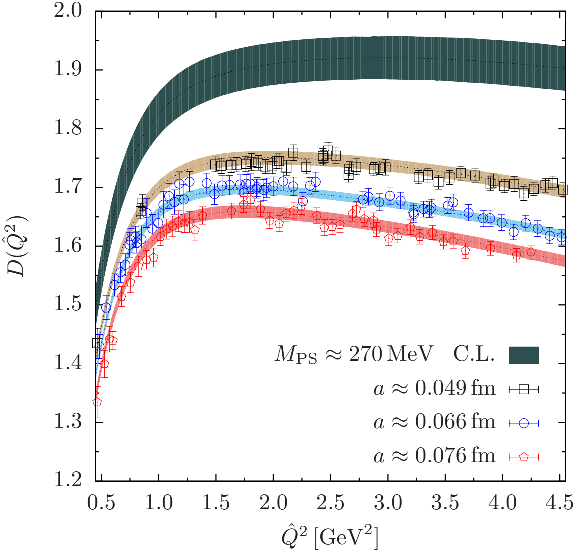
$$D(Q^2) = Q^2 \left(\frac{p_1}{(p_2+Q^2)^2} + \frac{p_3}{(p_4+Q^2)^2} \right) \times$$
$$\left[1 + (d_1\, a + d_2\, |aQ|) + \left(\frac{c_1}{Q^2}\right)\left(M_{\mathrm{PS}}^2 - M_\pi^2\right) + \left(\frac{c_2}{Q^4}\right)\left(M_{\mathrm{PS}}^2 - M_\pi^2\right)^2 \right]$$

- consider 11 ensembles with different a, M_{PS}
- consider also variations over these fit forms
- (u,d) , s_Q and c_Q

Adler function : combined fit $Q^2 \in [0.5, 4.5]\,\mathrm{GeV}^2$

$$D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(Q^2)} \Delta\alpha_{\mathrm{QED}}^{\mathrm{had}}(Q^2)$$

u, d

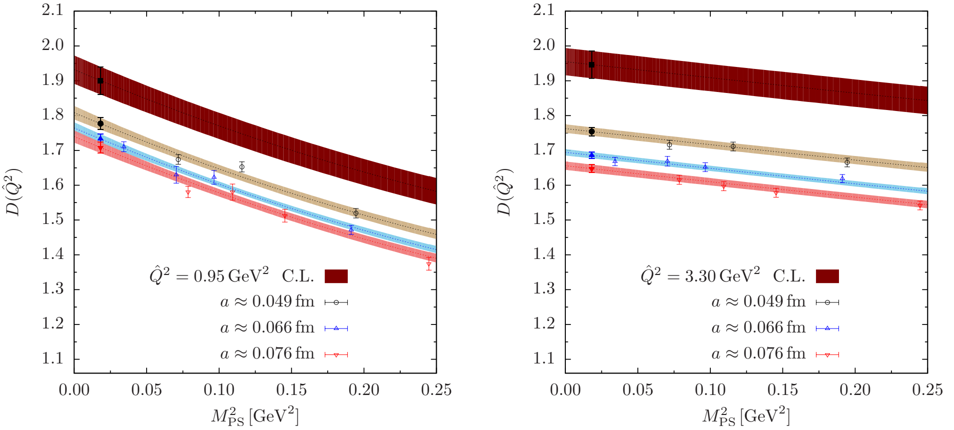


Padé [1, 2] with O(a) lattice artefacts and quadratic form in M_{PS}^2 $\chi^2/\mathrm{d.o.f} = 0.93$

Adler function : $M_{\rm PS}$ dependence

$$D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(Q^2)} \Delta\alpha_{\rm QED}^{\rm had}(Q^2)$$

u, d

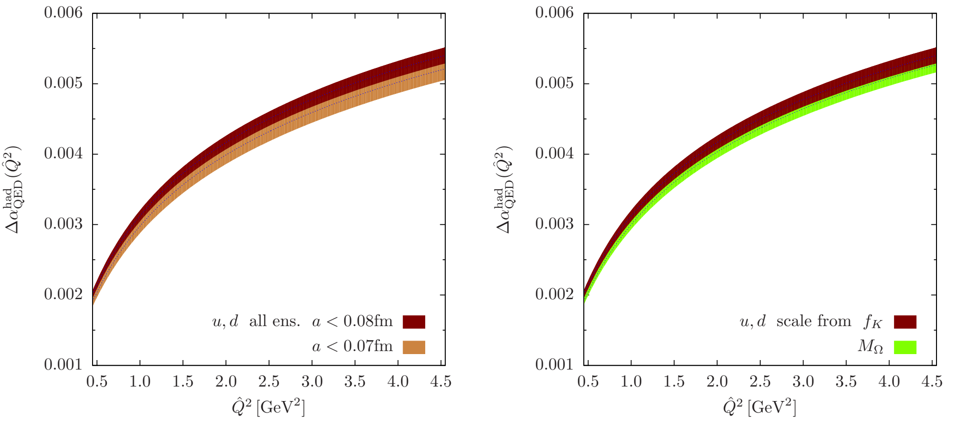


Padé [1, 2] with $O(a)$ lattice artefacts and quadratic form in $M^2_{\rm PS}$ $\quad \chi^2/\mathrm{d.o.f} = 0.93$

$\Delta\alpha_{\rm QED}^{\rm had}(Q^2)$: systematic effects

$$D(Q^2) = \frac{3\pi}{\alpha}\,\frac{d}{d\log(Q^2)}\,\Delta\alpha_{\rm QED}^{\rm had}(Q^2)$$

u, d ; $a \to 0$; $M_\pi^{\rm phys}$

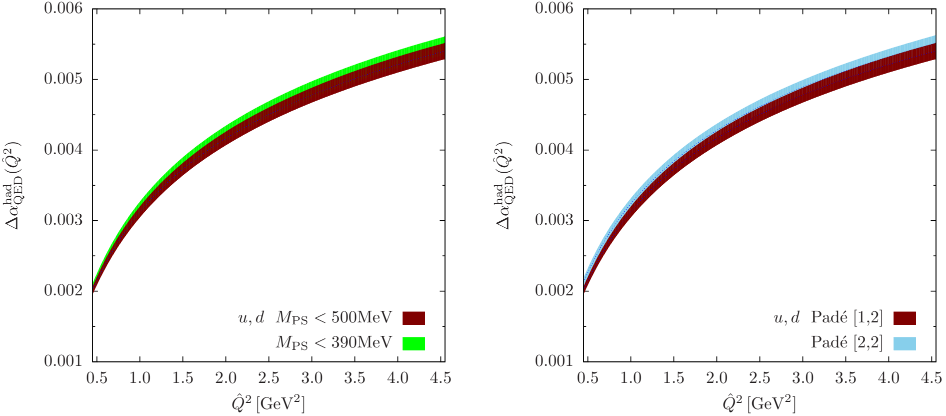


Padé [1, 2] with O(a) lattice artefacts and quadratic form in $M_{\rm PS}^2$

$\Delta\alpha_{\rm QED}^{\rm had}(Q^2)$: systematic effects

$$D(Q^2) = \frac{3\pi}{\alpha}\frac{d}{d\log(Q^2)}\Delta\alpha_{\rm QED}^{\rm had}(Q^2)$$

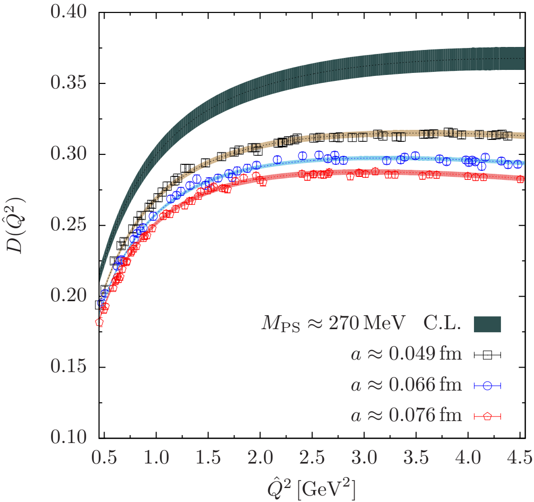
u, d; $a \to 0$; $M_\pi^{\rm phys}$



$O(a)$ lattice artefacts with quadratic form in $M_{\rm PS}^2$

Adler function : strange quark $Q^2 \in [0.5,\ 4.5]\,\mathrm{GeV}^2$

$$D(Q^2) = \frac{3\pi}{\alpha}\,\frac{d}{d\log(Q^2)}\,\Delta\alpha^{\mathrm{had}}_{\mathrm{QED}}(Q^2)$$



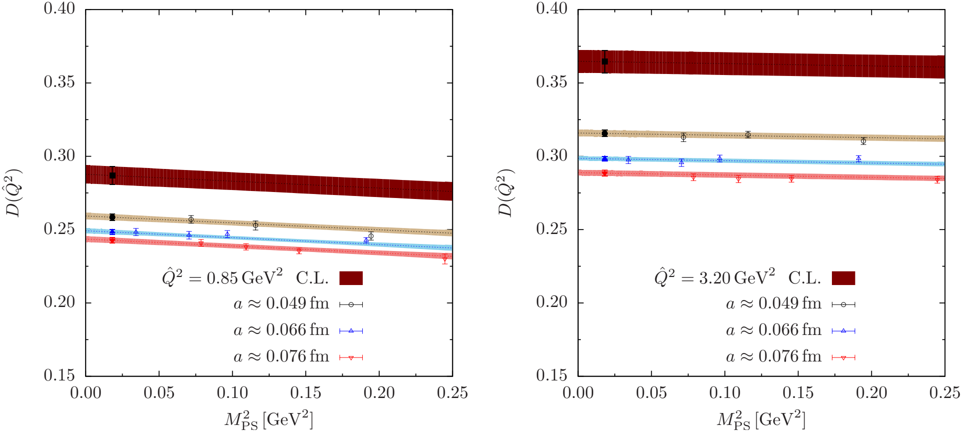
s_Q

Padé [1, 2] with $O(a)$ lattice artefacts and linear form in M^2_{PS} $\chi^2/\mathrm{d.o.f} = 0.87$

Adler function : strange quark

$$D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(Q^2)} \Delta\alpha_{\rm QED}^{\rm had}(Q^2)$$

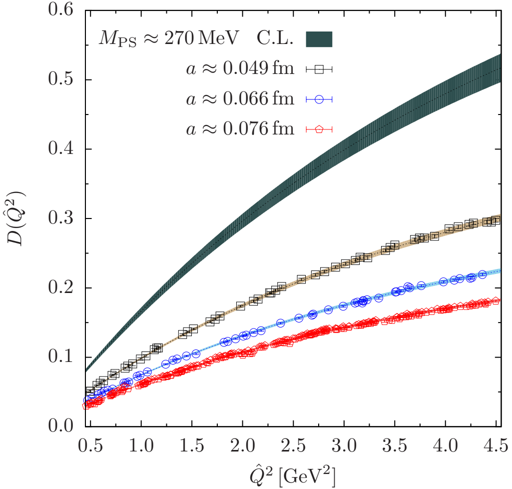
$s_{\rm Q}$



Padé [1, 2] with O(a) lattice artefacts and linear form in $M_{\rm PS}^2$ $\chi^2/\mathbf{d.o.f} = 0.87$

Adler function : charm quark $Q^2 \in [0.5, 4.5]\,\mathrm{GeV}^2$

$$D(Q^2) = \frac{3\pi}{\alpha}\,\frac{d}{d\log(Q^2)}\,\Delta\alpha^{\rm had}_{\rm QED}(Q^2)$$



c_Q

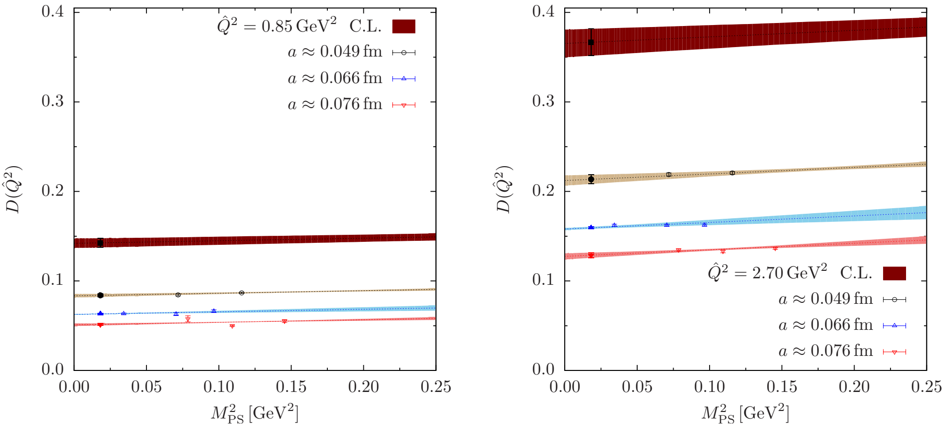
Padé [1, 1] with $O(a)$ lattice artefacts and linear form in $M^2_{\rm PS}$ $\chi^2/\mathrm{d.o.f} = 1.42$

$M_{\rm PS} < 390\,\mathrm{MeV}$

Adler function : charm quark

$$D(Q^2) = \frac{3\pi}{\alpha}\frac{d}{d\log(Q^2)}\Delta\alpha_{\rm QED}^{\rm had}(Q^2)$$

c_Q



Padé [1, 1] with $O(a)$ lattice artefacts and linear form in $M_{\rm PS}^2$ $\chi^2/\mathrm{d.o.f} = 1.42$

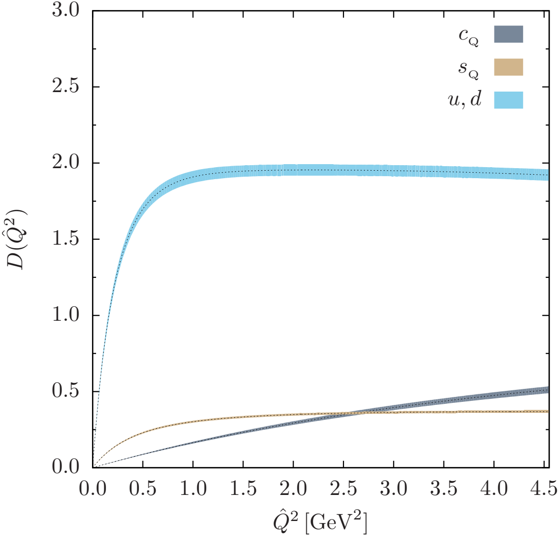
$M_{\rm PS} < 390\,\mathrm{MeV}$

Adler function : flavour contributions

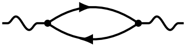
$$D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(Q^2)} \Delta\alpha^{\text{had}}_{\text{QED}}(Q^2)$$

$a \to 0$

M_π^{phys}



u, d, s_Q, c_Q



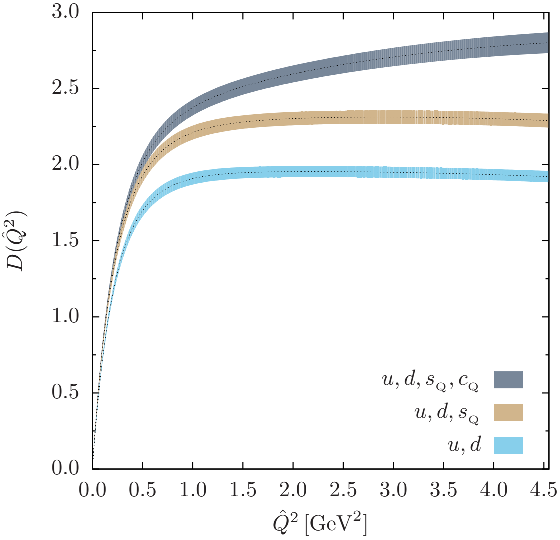
[PRELIMINARY]

Adler function : flavour contributions

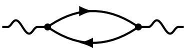
$$D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(Q^2)} \Delta\alpha_{\text{QED}}^{\text{had}}(Q^2)$$

$a \to 0$

M_π^{phys}



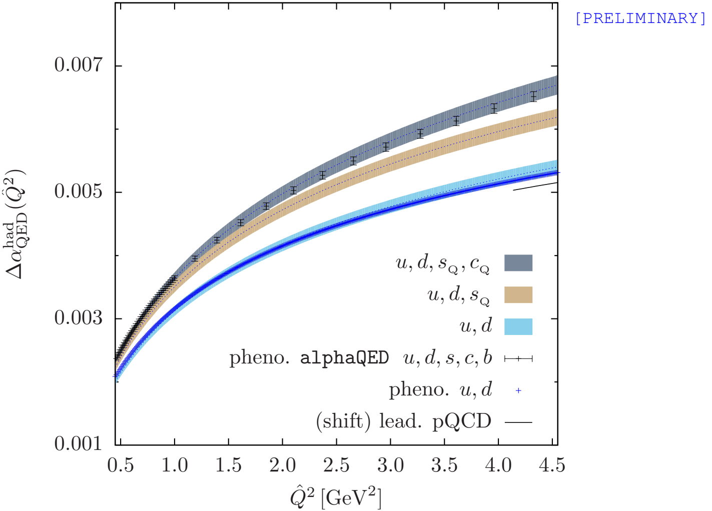
u, d, s_Q, c_Q



[PRELIMINARY]

running QED coupling: $\Delta\alpha_{\mathrm{QED}}^{\mathrm{had}}(Q^2)$

$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha_{\mathrm{QED}}(Q^2)}$$



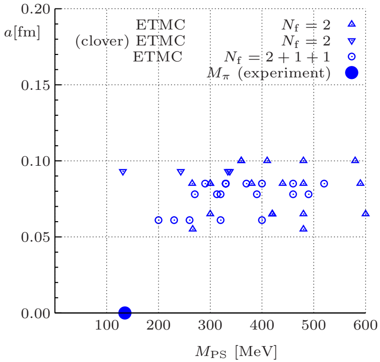
pheno. u, d : [Bernecker & Meyer, 1107.4388]

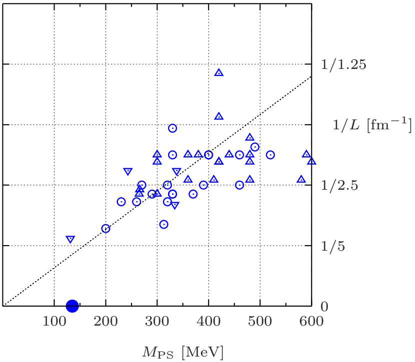
pheno. u, d, s, c, b : [alphaQED package, F. Jegerlehner]

ETMC ensembles

- fermionic lattice action : Wilson twisted-mass
- $N_f = 2$: u, d
- $N_f = 2+1+1$: u, d, s, c

- physical input : M_π , M_K , f_π





- quark connected contribution to $\Delta\alpha_{QED}^{had}$
- conserved current at source and sink

[ETMC, 1505.03283]

ETMC analysis

$$\Pi^{\rm tot}(Q^2) = \frac{5}{9}\Pi^{\rm ud}(Q^2) + \frac{1}{9}\Pi^{\rm s}(Q^2) + \frac{4}{9}\Pi^{\rm c}(Q^2)$$

with

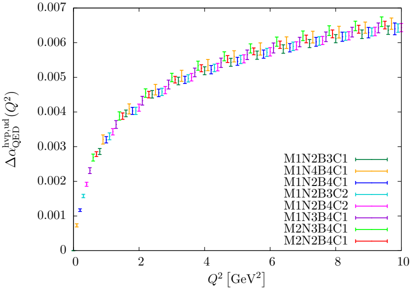
$$\Pi^{\rm f}(Q^2) \;=\; (1-\Theta(Q^2-Q^2_{\rm match}))\,\Pi^{\rm f}_{\rm low}(Q^2) \;+\; \Theta(Q^2-Q^2_{\rm match})\,\Pi^{\rm f}_{\rm high}(Q^2)$$

where

$$\Pi^{\rm f}_{\rm low}(Q^2) \;=\; \sum_{i=1}^{M}\frac{g_i^2 m_i^2}{m_i^2+Q^2} + \sum_{j=0}^{N-1} a_j (Q^2)^j$$

$$\Pi^{\rm f}_{\rm high}(Q^2) \;=\; \log(Q^2)\sum_{k=0}^{B-1} b_k (Q^2)^k + \sum_{l=0}^{C-1} c_l (Q^2)^l\,.$$

with $Q^2_{\rm match} = 2\,{\rm GeV}^2$ & $Q^2_{\rm max} = 100\,{\rm GeV}^2$



- continuum limit and chiral extr. :

$$\Delta\alpha^{\rm had}_{\rm QED}(Q^2)[M_{\rm PS},a] \;=\; A + B\,M^2_{\rm PS} + C\,a^2$$

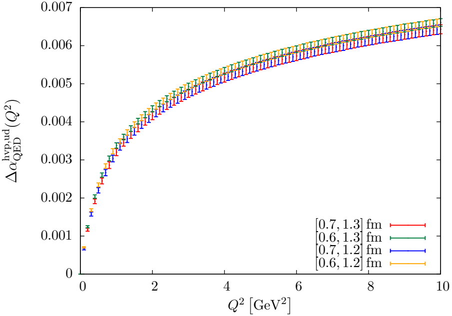
- standard fit : M1N2B4C1

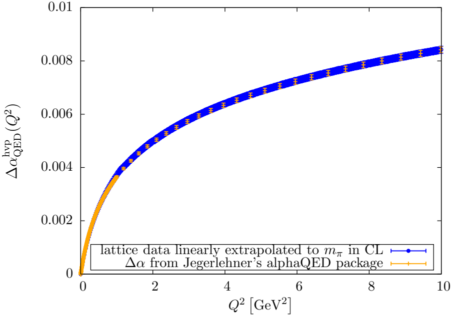
[ETMC, 1505.03283]

ETMC : $\Delta\alpha_{\rm QED}^{\rm had}(Q^2)$

rescaling in the light sector:

$$\Delta\alpha_{\rm QED}^{\rm had}(Q^2) = 4\pi\alpha\, \Pi_{\rm R}\left(Q^2 \frac{M_V^2}{M_{V_{\rm phys}}^2}\right)$$





$\Delta\alpha_{\rm had}^{(5)}(2\,{\rm GeV}^2) =$ $4.916(61)\cdot 10^{-3}$ [1.2%] [alphaQED package]

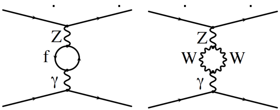
$\Delta\alpha_{\rm had}^{(4)}(2\,{\rm GeV}^2) =$ $4.993(102)(144)\cdot 10^{-3}$ [3.5%] [ETMC]

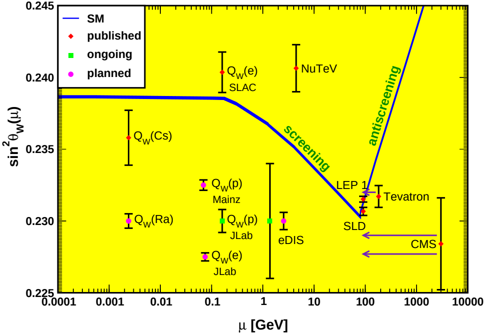
[ETMC, 1505.03283]

see also : HVP from magnetic susceptibilities [G. Bali & G. Endrodi, 1506.08638]

$$\Delta^{\mathrm{had}} \sin^2 \theta_W(Q^2)$$

$\Delta \sin^2 \theta_W(Q^2)$





non-perturbative effects in the SM curve : [Erler & Su, 1303.5522]

- dispersive approach would require separation of up and down type quarks . . .
- $\overline{\text{MS}}$: use threshold quark masses by imposing $\alpha_i^+(\bar{m}_{q_i}) = \alpha_i^+(\bar{m}_{q_i})$

  $\rightsquigarrow$ pheno. estimates : $\bar{m}_u = \bar{m}_d \sim 180\,\text{MeV}$ and $\bar{m}_s \sim 305\,\text{MeV}$

  no connection to other schemes in PT
- assume isospin and absence of singlet contributions
- . . . lattice QCD

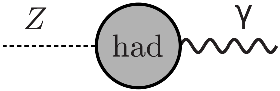
$\Delta^{\rm had} \sin^2\theta_W(Q^2)$

$$\sin^2\theta_W(Q^2) \;=\; \sin^2\theta_W\left(Q^2=0\right)\;\left(1-\Delta\sin^2\theta_W(Q^2)\right)$$

$$\text{with}\quad \sin^2\theta_W\left(Q^2=0\right) \;=\; \alpha/\alpha_2 \;=\; 0.23871(9)$$

[Kumar et al., 1302.6293]

- LO hadronic contribution to the $SU(2)_L$ coupling α_2



$$J^Z_\mu = J^3_\mu - \sin^2(\theta_W)\, J^\gamma_\mu$$

$$J^3_\mu = \frac{1}{4}\sum_f \left(\bar{u}_f\gamma_\mu(1-\gamma_5)u_f - \bar{d}_f\gamma_\mu(1-\gamma_5)d_f\right)$$

- $\Delta^{\rm had}\sin^2\theta_W(Q^2) \;=\; \Delta\alpha^{\rm had}_{\rm QED}(Q^2) \;-\; \Delta\alpha_2^{\rm had}(Q^2)$

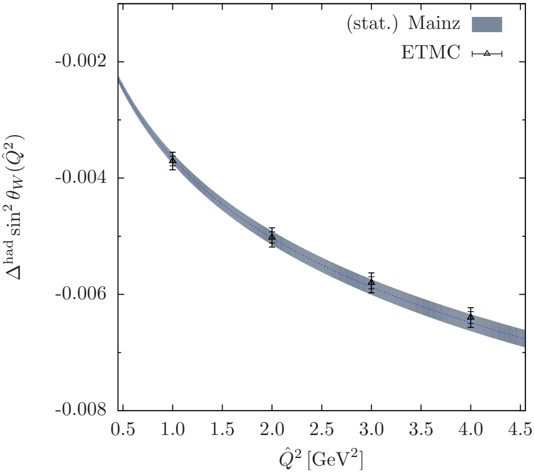
- for instance, (u,d) connected contribution at LO :

$$\Delta^{\rm had}_{ud}\sin^2\theta_W(Q^2) \;=\; \Delta^{ud}\alpha^{\rm had}_{\rm QED}(Q^2)\;\left(1-\frac{9}{20}\frac{\alpha_2}{\alpha}\right)$$

$\Delta^{\mathrm{had}} \sin^2 \theta_W(Q^2)$

u, d, s_Q, c_Q

[PRELIMINARY]



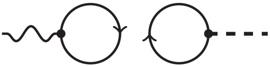
u, d, s, c : [ETMC, 1505.03283]

mixed (time-momentum) representation

[D. Bernecker & H. Meyer, 1107.4388]

$$\Pi_{\mathrm{R}}^{\gamma Z}(Q^2) = = \int_0^\infty dx_0\, G^{\gamma Z}(x_0) \left[x_0^2 - \frac{4}{Q^2} \sin^2\left(\frac{1}{2} Q\, x_0\right)\right]$$

$$G^{\gamma Z}(x_0) = -\int d^3\vec{x}\, \langle\, J_k^Z(x)\, J_k^\gamma(0)\, \rangle$$



- $\ell = (u, d)$ and s disconnected contributions :

$$G_{\mathrm{disc}}^{\gamma Z}(x_0) = \frac{\alpha}{\alpha_2}\, \frac{1}{9}\, G_{\mathrm{disc}}^{(\ell+As),(\ell-s)}(x_0)$$

where

$$G_{\mathrm{disc}}^{(\ell+As),(\ell-s)}(x_0 - y_0) = \frac{Z_V^2}{L^3} \left\langle \left(\sum_{\vec{x}} \mathrm{Tr}\left[\gamma_k D_\ell^{-1}(x,x) + A\, \gamma_k D_s^{-1}(x,x)\right]\right) \times \left(\sum_{\vec{y}} \mathrm{Tr}\left[\gamma_k D_\ell^{-1}(y,y) - \gamma_k D_s^{-1}(y,y)\right]\right) \right\rangle$$

with

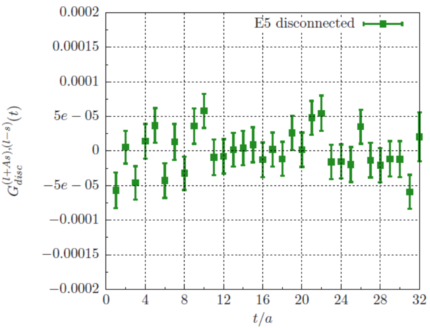
$$A = \frac{3}{4}\, \frac{\alpha_2}{\alpha} - 1$$

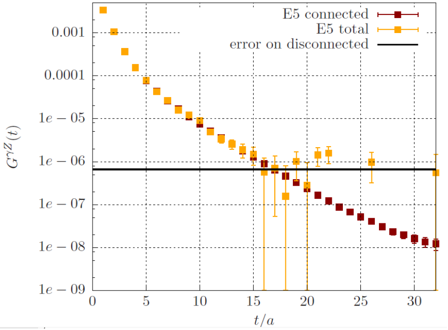
[V. Gülpers et al., lattice 2015]

mixed representation: disconnected contribution

- $a = 0.063\,\text{fm}\,;\quad M_\pi = 455\,\text{MeV}\,;\quad L/a = 32\,;\quad T = 2L$
- 3 stochastic sources and generalized hopping parameter expansion

[G. Bali et al., 0910.3970; V. Gülpers et al., 1309.2104]





[V. Gülpers et al., lattice 2015]

see also recent studies :

[G. Bali & G. Endrodi, 1506.08638]

[lattice 2015 : BMW, HPQCD]

mixed representation: disconnected contribution

- split $J_\mu^Z(x)$ and J_μ^γ into isoscalar and isovector pieces :

$$\rightsquigarrow \quad G^{\gamma Z}(x_0) = G^{\mathrm{I}=0}(x_0) + G^{\mathrm{I}=1}(x_0)$$

where

$$G^{\mathrm{I}=0} = -\frac{\alpha}{\alpha_2}\frac{1}{18}G^{\ell} + \left(\frac{1}{12} - \frac{\alpha}{\alpha_2}\frac{1}{9}\right)G^{s} + \left(\frac{1}{6} - \frac{\alpha}{\alpha_2}\frac{4}{9}\right)G^{c} + \frac{\alpha}{\alpha_2}\frac{1}{9}G_{\mathrm{disc}}^{(\ell+As),(\ell-s)}$$

- spectral representation

$$G^{\gamma Z}(x_0) = \int_0^\infty d\omega\, \omega^2\, \rho^{\gamma Z}(\omega)\, e^{-\omega|x_0|}$$

- $\omega < 3M_\pi$: $\quad \rho^{\mathrm{I}=0}(\omega) = 0$

- asymptotic behaviour : $x_0 \to \infty$

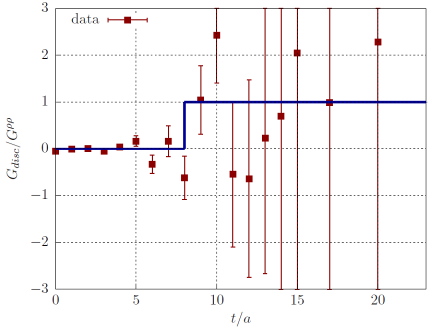
$$\frac{G_{\mathrm{disc}}^{(\ell+As),(\ell-s)}(x_0)}{G^{\rho\rho}(x_0)} \longrightarrow 1$$

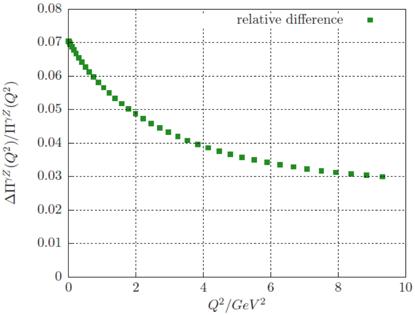
[V. Gülpers et al., lattice 2015]

mixed representation: disconnected contribution

- $x_0 \to \infty$

$$\frac{G_{\rm disc}^{(\ell+As),(\ell-s)}(x_0)}{G^{\rho\rho}(x_0)} \longrightarrow 1$$





4% : conservative estimate for systematic error from neglecting disconnected contribution at $Q^2 \sim 4\,\mathrm{GeV}^2$

[V. Gülpers et al., lattice 2015]

conclusions

- lattice determination of the LO hadronic contribution to the running of the QED coupling and of $\sin^2\theta_W$
- Adler function $\rightsquigarrow$ $\Delta\alpha_{\rm QED}^{\rm had}(Q^2)$, $\Delta^{\rm had}\sin^2\theta_W(Q^2)$, α_s, $a_\mu^{\rm HLO}$
- $\Delta^{\rm had}\sin^2\theta_W(Q^2)$: quark-disconnected diagrams
- $\alpha_{\rm QED}^{\rm had}(Q^2)$: further improvements are needed to reach the accuracy of pheno. results
- $\Delta^{\rm had}\sin^2\theta_W(Q^2)$: needed to confront SM with ongoing experiments
- oblique parameter : S