CP-violation in the kaon sector on the lattice

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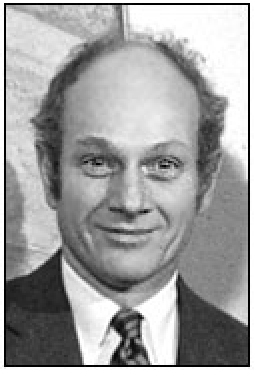
Introduction and Motivation

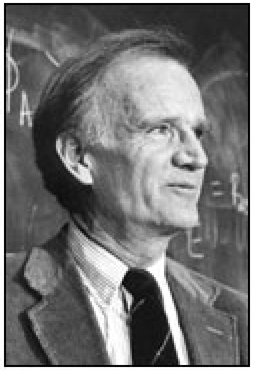
CP-violation in the kaon sector

- CPV is present in the Standard Model in the form of a complex phase in the CKM matrix, δ_{13}=1.2 rad.
- In the kaon sector this manifests as:

Indirect CPV in $K^0 \leftrightarrow \bar{K}^0$ mixing, parametrized by ε=2.233(15)x10^{-3}.

First observed by Cronin and Fitch at BNL, for which they received the Nobel prize in 1980.


James Cronin


Val Fitch

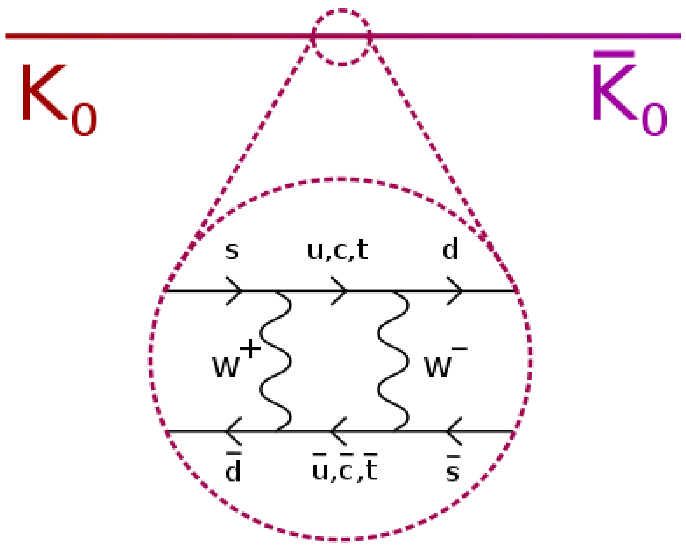
Direct CPV in $K^0 \rightarrow \pi\pi$ decays, parametrized by Re(ε'/ε)=1.67(23)x10^{-3}.

First observed in 1993 at NA31 expt (CERN), later confirmed by NA48 and KTeV (FNAL) in 1999.

- Many BSM models predict new sources of CPV and the small SM value makes it an ideal probe to search for new physics.

The role of the lattice

- Underlying dynamics governed by higher-order Weak interactions, e.g.



- However hadronic-scale QCD interactions typically play important role.
- Example: 450x enhancement of isospin I=0 channel $K \to \pi\pi$ decay over the I=2 channel ($\Delta I=1/2$ rule).
- Therefore vital to accurately determine hadronic contributions.
- SU(3) ChPT provides a useful tool, but difficult to assess model errors.
- On the other hand, lattice QCD provides a **systematically improvable** technique that has been wildly successful.

A sketch of a lattice calculation

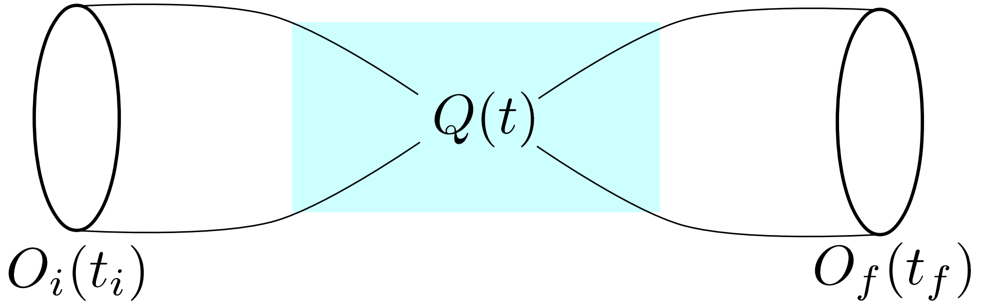
- At energy scales μ«M_W weak interactions accurately described by weak effective theory:

$$\mathcal{A} \propto (G_F)^n \sum_i S_i(\mu) \langle f | Q_i(\mu) | i \rangle$$

Perturbative Wilson coefficients describing high-energy behavior

Matrix elements of Weak effective 4-quark operators (Measure on lattice)

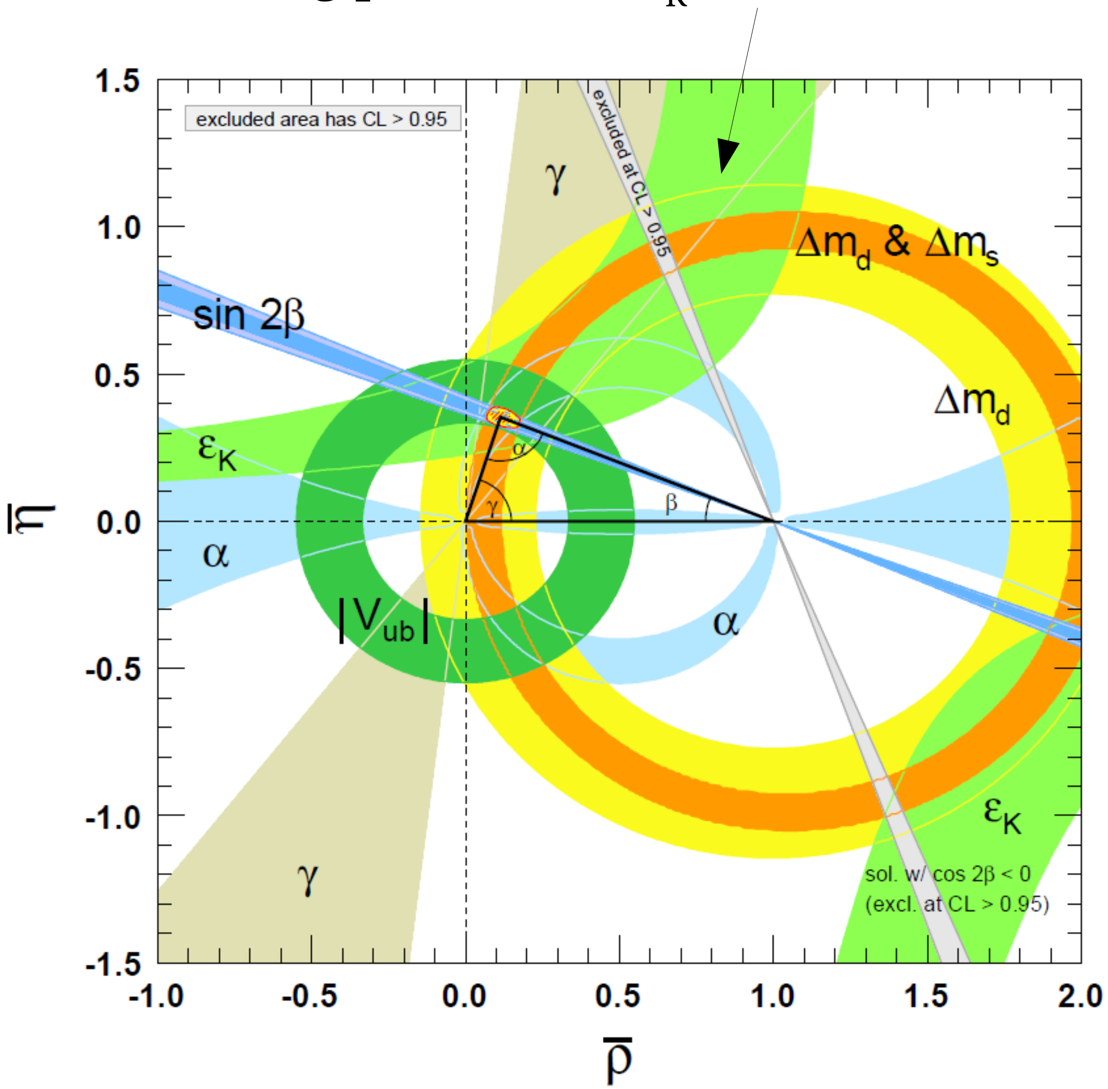
- Both operators and Wilson coeffs are renormalization scheme dependent: must renormalize in a consistent scheme!
- In Euclidean space states evolve as exp(-Ht), so in the large-time limit only the lightest state persists.
- Place operator within temporal region far away from source/sink to obtain ground-state matrix elements.



Indirect CP violation

Why is this interesting?

- Kaon indirect CP-violating parameter ε_K enters in CKM unitarity tests



PDG2014

The bag parameter

- Neutral kaon mixing occurs via the ΔS=2 Weak Hamiltonian:

~1, long-distance effects and effect on |ε| of $\phi_\varepsilon \neq 45$ deg. (est. using ChPT and lattice)

Wilson coeffs and CKM matrix elems

$$\varepsilon = e^{i\phi_\varepsilon} \underbrace{\frac{G_F^2 m_W^2 f_K^2 m_K}{12\sqrt{2}\pi^2 \Delta m_K^{\rm exp}}} B_K \kappa_\varepsilon \, {\rm Im}\Big[\eta_1 S_0(x_c) \left(V_{cs}V_{cd}^*\right)^2 + \eta_2 S_0(x_t) \left(V_{ts}V_{td}^*\right)^2 + 2\eta_3 S_0(x_c, x_t) V_{cs}V_{cd}^* V_{ts} V_{td}^* \Big] ,$$

experimental input

Bag parameter (lattice)

$\phi_\epsilon = 43.52(5)^\circ$

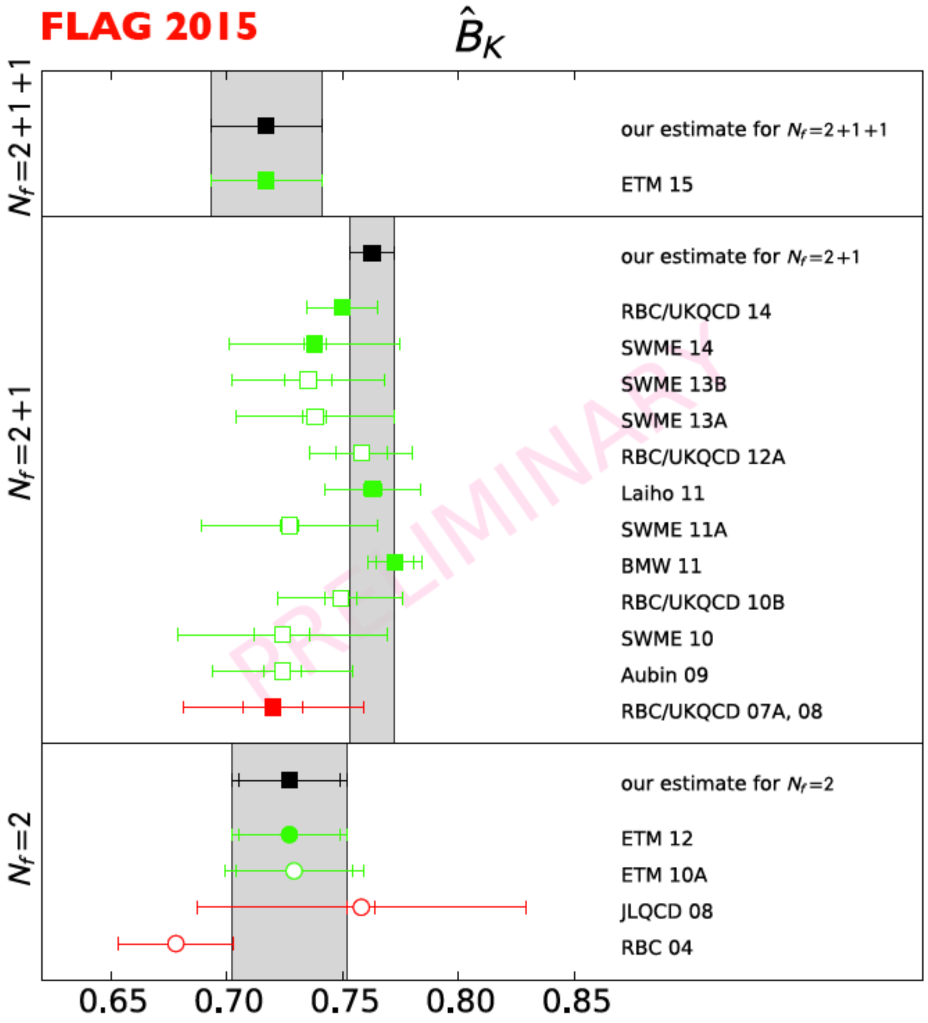
$$B_K^{\rm latt} = \frac{\langle \bar{K}^0 | \mathcal{O}_{VV+AA} | K^0 \rangle}{\frac{8}{3} \langle K^0 | A_0 | 0 \rangle \langle 0 | A_0 | \bar{K}^0 \rangle}$$

$$\mathcal{O}_{VV+AA} = \bar{s}\gamma_\mu(1-\gamma^5)d \; \bar{s}\gamma^\mu(1-\gamma^5)d$$

- Match to scheme used for Wilson coeffs, typically MSbar, (mostly) non-perturbatively, e.g. using RI/SMOM scheme to run to high energy at which perturbative matching reliable.

B_K, a "golden" quantity
(or "gold-plated" if you prefer)

- Lattice measurement is comparatively simple and can be made very precise using modern techniques (RI/SMOM, AMA, physical-point ensembles, etc)



(Courtesy of A.Vladikas)

$$N_f = 2+1+1 \quad \hat{B}_{\rm K} = 0.717(24)$$

$$N_f = 2+1 \quad \hat{B}_{\rm K} = 0.7627(97)$$

$$N_f = 2 \quad \hat{B}_{\rm K} = 0.727(25)$$

[Bailey et al arXiv:1503.06613]

TABLE IX. Fractional error budget for $\varepsilon_K^{\rm SM}$ obtained using the AOF method, the exclusive V_{cb}, and the FLAG $\hat{B}_K$.

source	error (%)	memo
V_{cb}	40.7	FNAL/MILC
$\bar{\eta}$	21.0	AOF
η_{ct}	17.2	$c-t$ Box
η_{cc}	7.3	$c-c$ Box
$\bar{\rho}$	4.7	AOF
m_t	2.5	
ξ_0	2.2	RBC/UKQCD
$\hat{B}_K$	1.6	FLAG
m_c	1.0	
$\vdots$	$\vdots$	

LD correction

angle-only fit

- Dominant source of error on ε_K is V_{cb} determination.
- Errors on B_K completely subdominant!
- It appears that we should now devote our efforts to computing V_{cb}, e.g. via $\bar{B} \to D^* l \bar{\nu}$ form factor (FNAL/MILC method)
- Return later to κ_ε ...

Direct CP violation

Direct CPV in K→ππ Decays

$$\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)}, \qquad \eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}.$$

$$\mathrm{Re}(\epsilon'/\epsilon) \approx \frac{1}{6}\left(1 - \left|\frac{\eta_{00}}{\eta_{\pm}}\right|^2\right) = 16.6(2.3) \times 10^{-4} \quad \text{(experiment)}$$

- In terms of isospin states: $\Delta I=3/2$ decay to I=2 final state, amplitude A_2

  $\Delta I=1/2$ decay to I=0 final state, amplitude A_0

$$\omega = \mathrm{Re}A_2/\mathrm{Re}A_0$$

$$A(K^0 \to \pi^+\pi^-) = \sqrt{\frac{2}{3}}A_0 e^{i\delta_0} + \sqrt{\frac{1}{3}}A_2 e^{i\delta_2}\,,$$
$$A(K^0 \to \pi^0\pi^0) = \sqrt{\frac{2}{3}}A_0 e^{i\delta_0} - 2\sqrt{\frac{1}{3}}A_2 e^{i\delta_2}\,.$$

➞

$$\epsilon' = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}}\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$

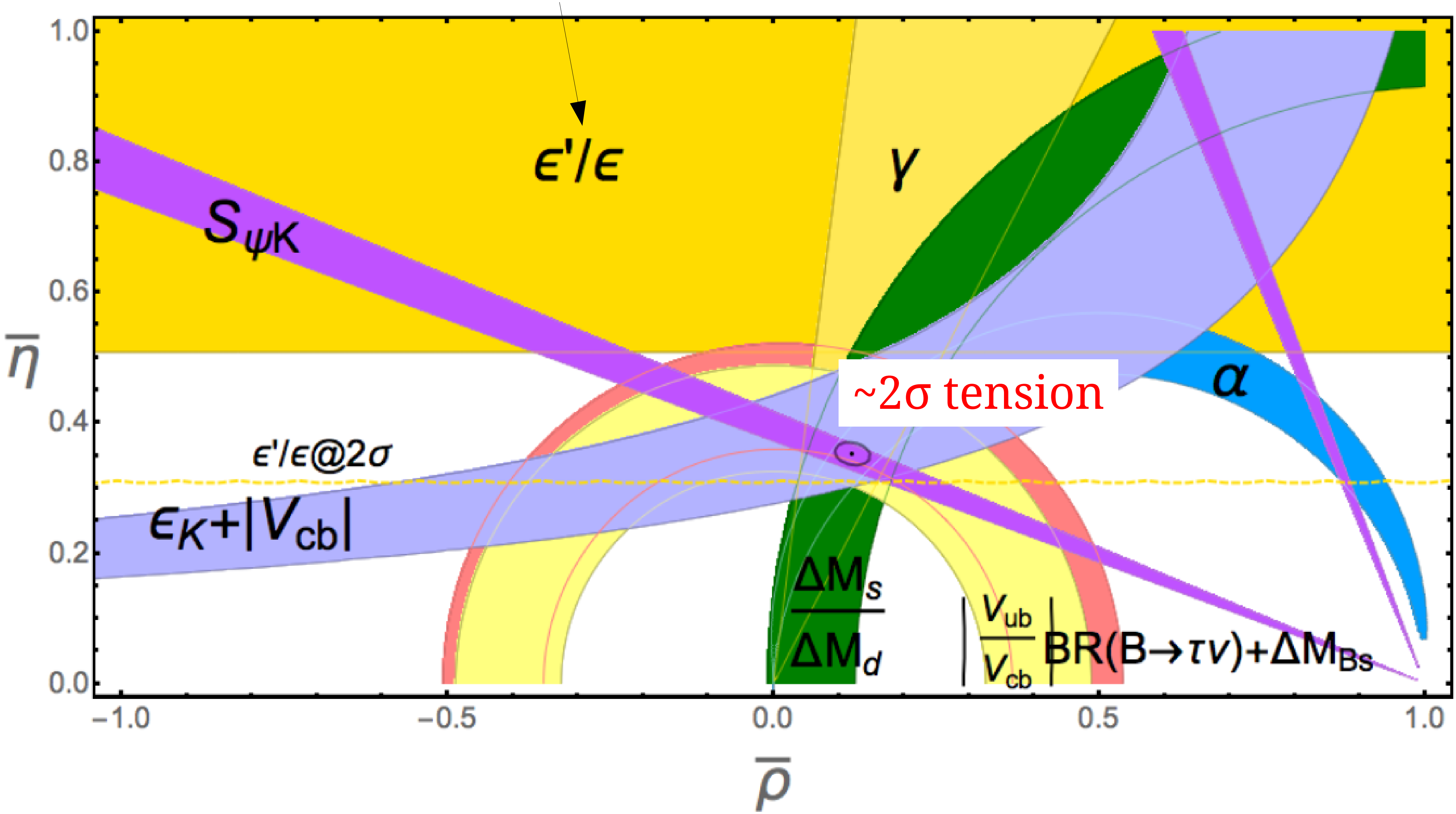
(δ_I are strong scattering phase shifts.)

Why is this interesting?

- ε' is highly sensitive to BSM sources of CPV.
- New horizontal band constraint on CKM matrix:

[Lehner et al arXiv:1508.01801]

new constraint from lattice!



ΔS=1 Weak Effective Hamiltonian

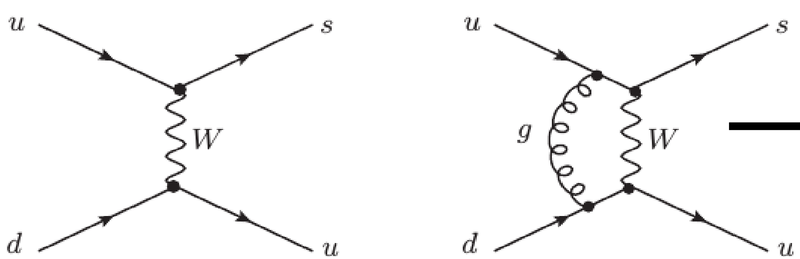
$$H_W^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \sum_{j=1}^{10} [z_j(\mu) + \tau y_j(\mu)] Q_j$$

Wilson coeffs.

$$\tau = -\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} = 0.0014606 + 0.00060408i$$

Imaginary part solely responsible for CPV
(everything else is pure-real)

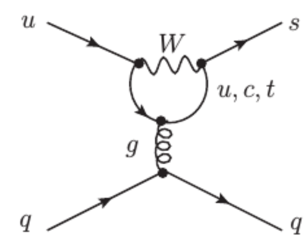
- Q_j are 10 effective four-quark operators:



(a) current-current

$\longrightarrow Q_1, Q_2$

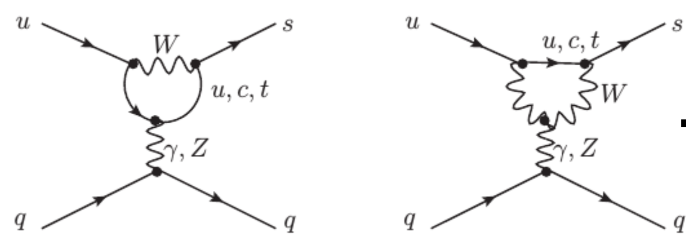
dominate
$Re(A_0)$, $Re(A_2)$



(b) QCD penguin

$\longrightarrow Q_3 - Q_6$

Q_4, Q_6 dominate
$Im(A_0)$



(c) Electro-Weak penguin

$\longrightarrow Q_7 - Q_{10}$

Q_7, Q_8 dominate
$Im(A_2)$

Lattice Determination of K→ππ

- On the lattice compute $M_j = \langle (\pi\pi)_I | Q_j | K \rangle$
- Mixing under renormalization, hence Z is a matrix.

$$A_{2/0} = F \frac{G_F}{\sqrt{2}} V_{ud} V_{us} \sum_{i=1}^{10} \sum_{j=1}^{7} \left[\left(z_i(\mu) + \tau y_i(\mu) \right) Z_{ij}^{\mathrm{lat} \to \overline{\mathrm{MS}}} M_j^{\frac{3}{2} / \frac{1}{2}, \mathrm{lat}} \right] ,$$

- F is finite-volume correction calculated using Lellouch-Luscher method.
- Important to calculate with physical (energy-conserving) kinematics. With physical masses:

$$2 \times m_\pi \sim 270 \text{ MeV} \qquad\qquad m_K \sim 500 \text{ MeV}$$

we require non-zero relative momentum for the pions.

- This is excited state of the ππ-system. Possibilities:
  - try to perform multi-state fits to very noisy data (esp. A_0 where there are disconn. diagrams) or
  - modify boundary conditions to remove the ground-state

ΔI=3/2 Calculation

[Phys.Rev. D91 (2015) 7, 074502]

(RBC & UKQCD)

The RBC & UKQCD collaborations

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Calculation Strategy

- A_2 can be computed directly from charged kaon decay:

$$\langle(\pi\pi)^{I=2}_{I_3=1}|H_W|K^+\rangle = \sqrt{2}A_2 e^{i\delta_2}$$

- Remove stationary (charged) pion state using antiperiodic BCs on d-quark propagator: $d(x+L) = -d(x)$ → $|p| \in (\pi/L,\, 3\pi/L,\, 5\pi/L \ldots)$

$$\pi^+(x+L) = [\bar{u}d](x+L) = -\pi^+(x)$$ Moving ground state!

$$\pi^0(x+L) = [\bar{u}u - \bar{d}d](x+L) = +\pi^0(x)$$ Stationary ground state....

- Use Wigner-Eckart theorem to remove neutral pion from problem

$$\langle(\pi^+\pi^0)_{I=2}|Q^{\Delta I_z=1/2}|K^+\rangle = \frac{\sqrt{3}}{2}\langle(\pi^+\pi^+)_{I=2}|Q^{\Delta I_z=3/2}|K^+\rangle$$

- APBCs on d-quark break isospin symmetry allowing mixing between isospin states: however $\pi^+\pi^+$ is the only charge-2 state with these Q-numbers hence it cannot mix.

Results

- Calculation performed on RBC & UKQCD 48^3x96 and 64^3x128 Mobius DWF ensembles with $(5\text{ fm})^3$ volumes and a=0.114 fm, a=0.084 fm. Continuum limit computed.
- Make full use of eigCG and AMA to translate over all timeslices. Obtain 0.7-0.9% stat errors on all bare matrix elements!
- Results:

$$\mathrm{Re}(A_2) = 1.50(4)_{\mathrm{stat}}(14)_{\mathrm{sys}} \times 10^{-8}\ \mathrm{GeV}$$
$$\mathrm{Im}(A_2) = -6.99(20)_{\mathrm{stat}}(84)_{\mathrm{sys}} \times 10^{-13}\ \mathrm{GeV}$$

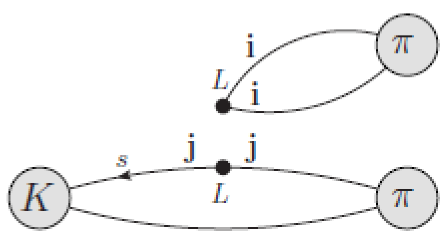
10%, 12% total errors on Re, Im!

- Systematic error completely dominated by perturbative error on NPR and Wilson coefficients.

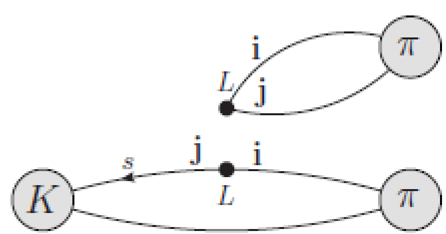
ΔI=1/2 rule

- In experiment kaons approx 450x (!) more likely to decay into I=0 pi-pi states than I=2.
  $$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} \simeq 22.5 \quad \text{(the } \Delta I=1/2 \text{ rule)}$$
- Perturbative running to charm scale accounts for about a factor of 2. Is the remaining 10x non-perturbative or New Physics?
- The answer is **low-energy *QCD!*** RBC/UKQCD [arXiv:1212.1474, arXiv:1502.00263]

  Strong cancellation between the two dominant contractions



Contraction ①.

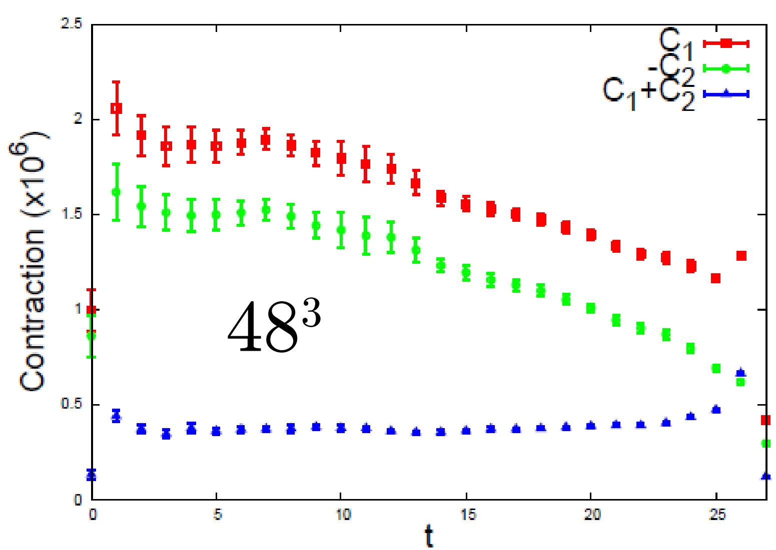


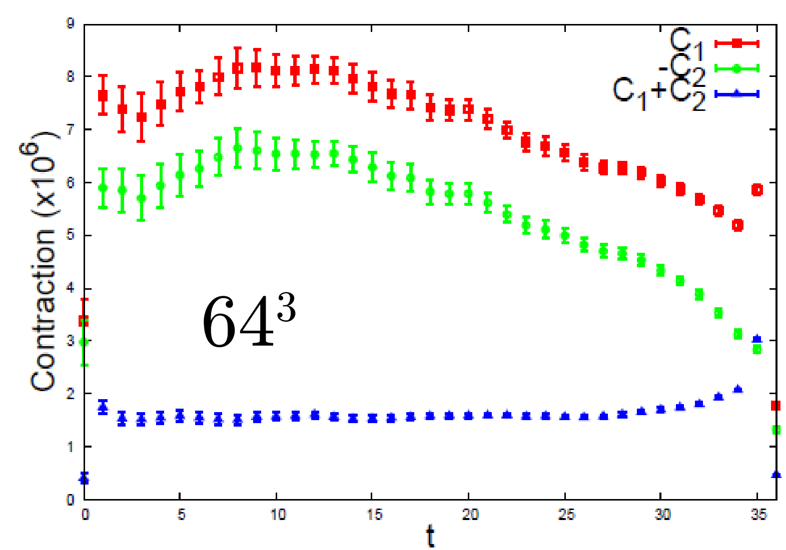
Contraction ②.

$$\mathrm{Re}(A_2) \sim ① + ②$$

$$② \approx -0.7①$$

heavily suppressing $\mathrm{Re}(A_2)$.





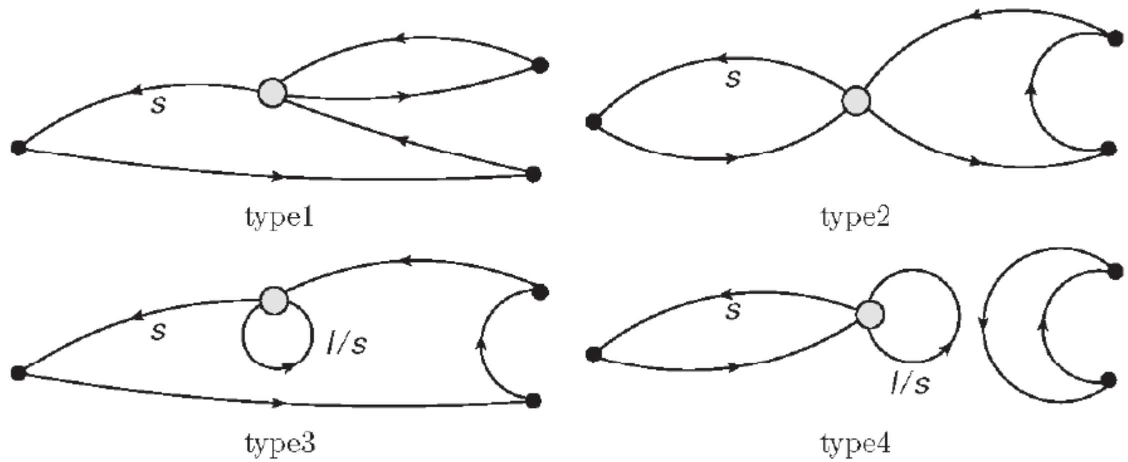
$\Delta I=1/2$ Calculation

arXiv:1505.07863 [hep-lat]

(RBC & UKQCD)

A much more difficult calculation

- A_0 obtained via neutral kaon decays $K^0 \to \pi^+\pi^-$ and $K^0 \to \pi^0\pi^0$
- ~50 distinct contraction topologies → 4 classes:



- Type 4 disconn. diagrams dominate noise. Use Trinity-style all-to-all (A2A) propagators:
    - O(1000) exact low-eigenmodes computed using Lanczos algorithm
    - Stochastic high-modes with full dilution
- Allows all translations of source/sink and operator location to be computed.

Physical Kinematics

- A_2 calculation used APBC on d-quarks, removes stationary **charged pion** state BUT breaks isospin and doesn't work for π^0.
- Solution: Use G-parity BCs:

$$\hat{G} = \hat{C}e^{i\pi \hat{I}_y} \quad : \quad \hat{G}|\pi^{\pm}\rangle = -|\pi^{\pm}\rangle \qquad \hat{G}|\pi^0\rangle = -|\pi^0\rangle$$

- As a boundary condition: (i=+, -, 0)

$$\pi^i(x+L) = \hat{G}\pi^i(x) = -\pi^i(x) \longrightarrow |p| \in (\pi/L,\, 3\pi/L,\, 5\pi/L\ldots)$$

(moving ground state)

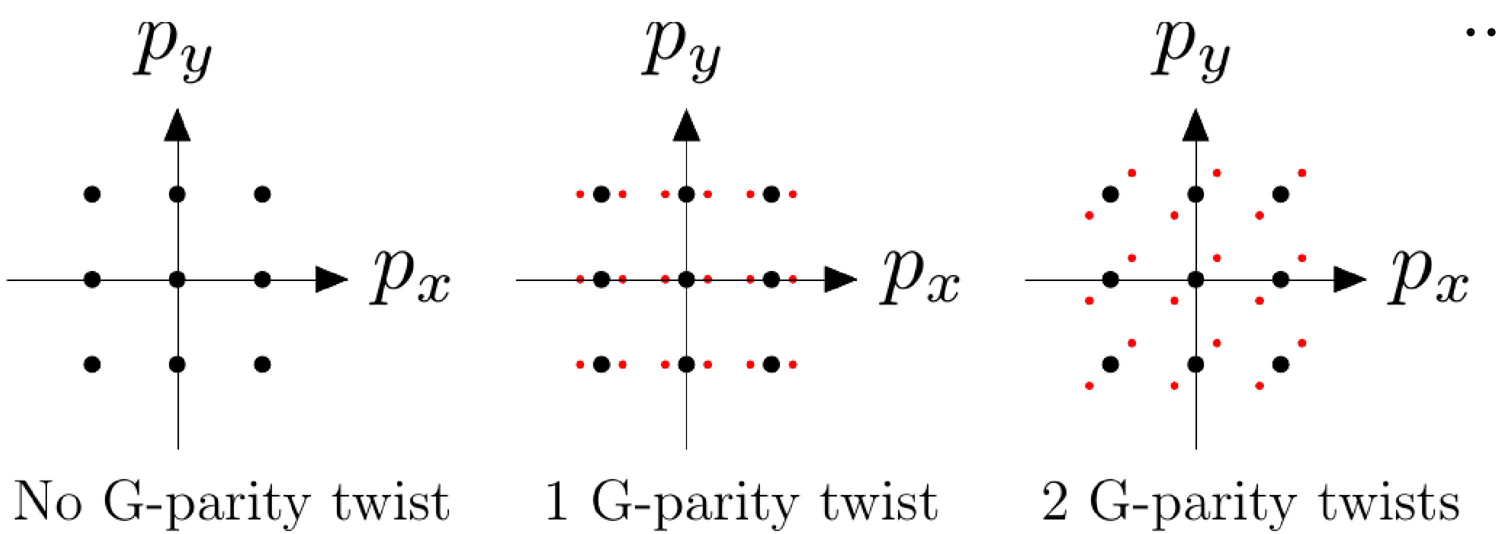
- At quark level: $\hat{G}\begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} -C\bar{d}^T \\ C\bar{u}^T \end{pmatrix}$ where C = $\gamma^2\gamma^4$ in our conventions
- Gauge invariance → gauge field must obey charge conjugation BCs; new ensembles needed.
- For stationary kaon we must introduce fictional degenerate partner to the strange quark: s'

$$|\tilde{K}^0\rangle = (|\bar{s}d\rangle + |\bar{u}s'\rangle)/\sqrt{2}$$ is G-parity even (p=0)

- Coupling of unphysical kaon partner to physical operators exponentially suppressed and can be neglected.

Difficulties of simulating with G-parity BCs

- Additional strange flavor must be rooted out. As Dirac operator intrinsically 2-flavor this is not quite kosher.
- Situation much more benign than staggered quarks - only boundary effect and quarks are degenerate. Using 'replica trick' and EFT can argue effects are exponentially suppressed.
- Intrinsically 2-flavor Dirac operator requires RHMC (kosher) rooting of light-quark $\mathcal{M}^\dagger\mathcal{M}$ (and fourth-rooting of strange $\mathcal{M}^\dagger\mathcal{M}$). Increased computational cost due to floating point overhead of multishift CG.
- GPBC also breaks the cubic rotational symmetry of allowed quark momenta to subgroup of rotations around the 'G-parity vector' (1,1,1) [GPBC in 3 dirs]



- Makes it difficult to form a rotationally symmetric (A1 rep) pipi state needed for Luscher condition.
- Pions just obey regular APBC; even on small volumes have not seen any breaking of degeneracy, but **have** observed different correlator amplitudes for different directions at ~20% level on our 16^3 test ensembles.
- Quark-level symmetry breaking can be suppressed by averaging over combinations of allowed quark momenta.

	p=(2,2,2)	p=(-2,2,2)	p=(2,-2,2)	p=(2,2,-2)
E_π	0.19852(85)	0.19823(82)	0.19839(72)	0.19866(88)
Z_π	6.167(69)e+06	6.081(63)e+06	6.183(50)e+06	6.170(61)e+06

(32^3 ensemble used for K->pipi)

- Aside from norms, can check that there is no mixing between H4 reps in the pipi state

$$\pi\pi_{A1} = \frac{1}{2}\{(+++) \oplus (-++) \oplus (+-+) \oplus (++-)\}$$
$$\pi\pi_{T2_0} = \frac{1}{\sqrt{12}}\{3(+++) \ominus (-++) \ominus (+-+) \ominus (++-)\},$$

- Observed orthogonality to sub-percent scale.
- No evidence for remaining cubic symmetry violation.

Ensemble

- 32^3x64 Mobius DWF ensemble with IDSDR gauge action at β=1.75. Coarse lattice spacing (a^{-1}=1.378(7) GeV) but large, $(4.6\text{ fm})^3$ box.
- Using Mobius params (b+c)=32/12 and L_s=12 obtain same explicit χSB as the L_s=32 Shamir DWF + IDSDR ens. used for ΔI=3/2 but at reduced cost.
- Utilized USQCD 512-node BG/Q machine at BNL, the DOE "Mira" BG/Q machines at ANL and the STFC BG/Q "DiRAC" machines at Edinburgh, UK.
- Performed 216 independent measurements (4 MDTU sep.).
- Cost is ~1 BG/Q rack-day per complete measurement (4 configs generated + 1 set of contractions).
- G-parity BCs in 3 spatial directions results in close matching of kaon and ππ energies:

m_K=490.6(2.4) MeV

$E_{\pi\pi}$(I=0) = 498(11) MeV

$E_{\pi\pi}$(I=2) = 573.0(2.9) MeV

E_π=274.6(1.4) MeV   (m_π = 143.1(2.0) MeV)

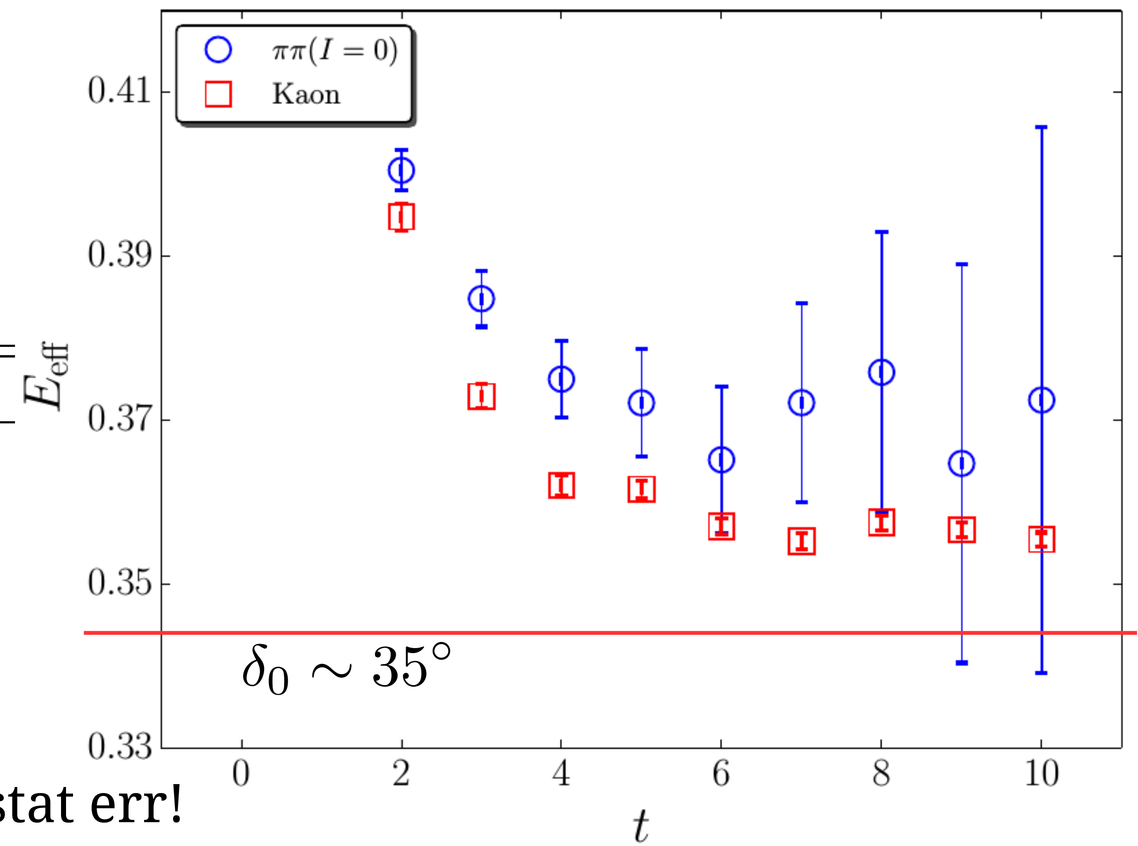
I=0 ππ energy

- Signal/noise deteriorates quickly due to vacuum contrib.
- Difficult to determine plateau start. Performed both 1- and 2-state fits.

$t_{\rm min}$	$E_{\pi\pi}$	$E_{\rm exc}$	$\chi^2/{\rm dof}$
2	0.363(9)	1.04(17)	1.7(7)
3	0.367(11)	1.27(73)	1.8(8)
4	0.364(12)	0.86(39)	1.9(8)

$t_{\rm min}$	$E_{\pi\pi}$	$\chi^2/{\rm dof}$
5	0.375(6)	2.2(9)
6	0.361(7)	1.6(7)
7	0.380(11)	0.9(7)

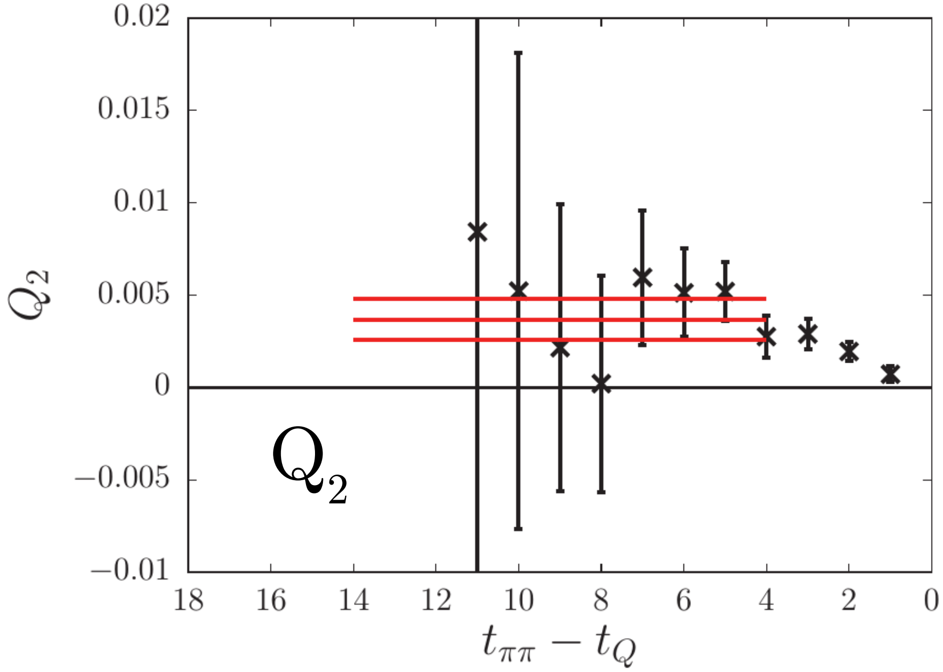
← 2% stat err!



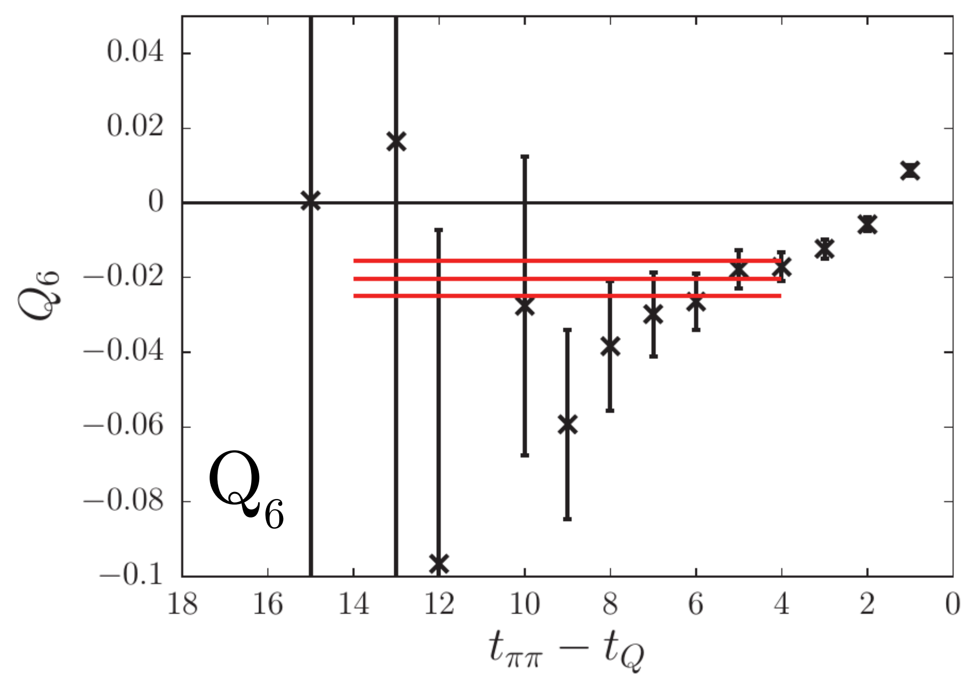
- Our phase shift $\delta_0 = 23.8(4.9)(1.2)^\circ$ lower than most pheno estimates, which prefer $\delta_0 \sim 35^\circ$. More statistics needed to resolve.
- Using 35° → ~3% change in A_0; much smaller than other errs. For consistency we choose to use our lattice value.

Matrix element fits

[Dominant contribution to Re(A_0)]



[Dominant contribution to Im(A_0)]



- Use $t_{min}(\pi \to Q) = 4$ here rather than 6 as signal quickly decays into noise (40% increase in stat. error with t_{min}=5!).
- However comparison to t_{min}=3 shows no statistically resolvable difference, suggesting excited state contamination small.
- Estimate 5% excited state systematic by comparing single-exp fit result for $\pi\pi$(I=0) amplitude with t_{min}=4 to double-exp fit with t_{min}=3.

Systematic errors

- Errors for each separate operator matrix element:

Description	Error	Description	Error
Finite lattice spacing	8%	Finite volume	7%
Wilson coefficients	12%	Excited states	$\leq 5\%$
Parametric errors	5%	Operator renormalization	15%
Unphysical kinematics	$\leq 3\%$	Lellouch-Lüscher factor	11%
Total (added in quadrature)			26%

- Treat as uncorrelated when combining to form $A_{0.}$

- 15% renormalization error dominant due to low, 1.53 GeV renormalization scale. Estimate by comparing two different RI/SMOM intermediate schemes and use the largest observed differences.

- 12% Wilson coefficient error large for same reason. Conservatively estimate as largest observed fractional change between using LO and NLO.

Results for A_0

$$\mathrm{Re}(A_0) = 4.66(1.00)_{\mathrm{stat}}(1.21)_{\mathrm{sys}} \times 10^{-7}\ \mathrm{GeV} \qquad \text{(This work)}$$

$$\mathrm{Re}(A_0) = 3.3201(18) \times 10^{-7}\ \mathrm{GeV} \qquad \text{(Experiment)}$$

- Good agreement between lattice and experiment for $\mathrm{Re}(A_0)$ serves as test for method.
- $\mathrm{Re}(A_0)$ from expt far more precise, and is dominated by tree-level Q_1 and Q_2 hence unlikely to receive large BSM contributions. Use for computing ε'.

$$\mathrm{Im}(A_0) = -1.90(1.23)_{\mathrm{stat}}(1.04)_{\mathrm{sys}} \times 10^{-11}\mathrm{GeV} \qquad \text{(This work)}$$

- ~85% total error on the predicted $\mathrm{Im}(A_0)$ due to strong cancellation between dominant Q_4 and Q_6 contributions:

$$\begin{aligned} \Delta[\mathrm{Im}(A_0), Q_4] &= \quad 1.82(0.62)(0.32) \times 10^{-11} \\ \Delta[\mathrm{Im}(A_0), Q_6] &= -3.57(0.91)(0.24) \times 10^{-11} \end{aligned}$$

despite only 40% and 25% respective errors for the matrix elements.

Lattice results for ε'

Results for ε'

- Using Re(A_0) and Re(A_2) from experiment and our lattice values for Im(A_0) and Im(A_2) and the phase shifts,

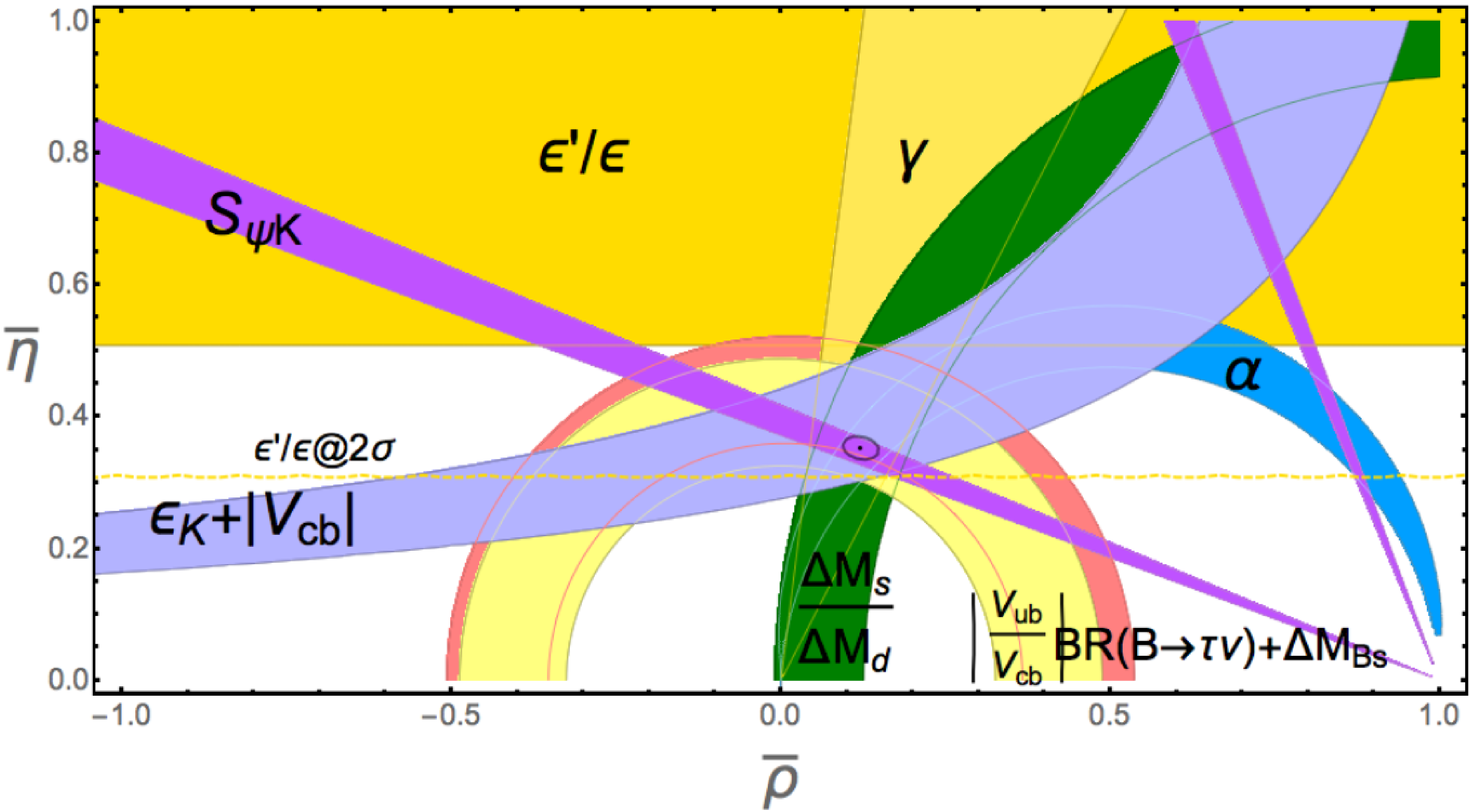
$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$

$$= \quad 1.38(5.15)(4.43)\times 10^{-4}, \qquad \text{(this work)}$$

$$16.6(2.3)\times 10^{-4} \qquad \text{(experiment)}$$

- Find discrepancy between lattice and experiment at the 2.1σ level.
- Total error on Re(ε'/ε) is ~3x the experimental error, and we observe a 2.1σ discrepancy. Strong motivation for continued study!
- Hope to achieve O(10%) errors on Re(ε'/ε) on a timescale of ~5 years.
- We hope these results with spur new efforts in the experimental community to reduce the current 15% error on the experimental number.

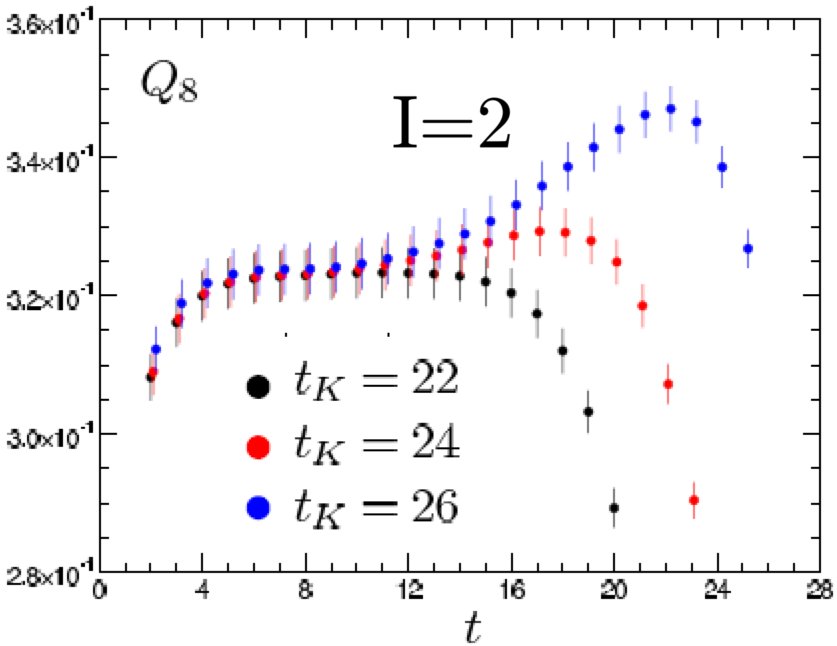
[Lehner et al
arXiv:1508.01801]



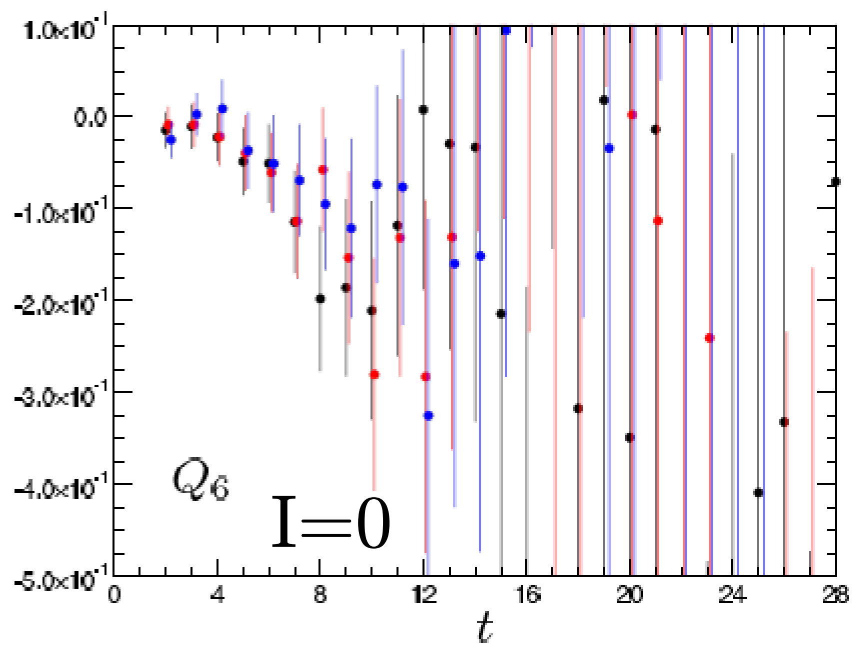
K->ππ with Wilson fermions

Ishizuka et al [arXiv:1505.05289]

- 2+1f O(a) improved Wilson fermions. Calculation performed at threshold.
- 32^3x64 volume (2.9 fm)^3 with a^{-1} = 0.091fm, m_π=280 MeV, m_K=580 MeV.
- Disconnected diagrams using stochastic sources with combine hopping parameter expansion and truncated solver method variance reduction.
- χSB enhances operator mixing, making renormalization difficult.
- Using remaining symmetries C, P, CPS and $SU(3)_V$, authors argue that p-odd components transform as in continuum: use these for NPR.



$\mathrm{Re}A_0 = 60(36) \times 10^{-8}\,\mathrm{GeV}$



$\mathrm{Im}A_0 = -67(56) \times 10^{-12}\,\mathrm{GeV}$

Long-distance contributions to ε_K

Long distance effects

- The factor κ_ε enters the determination of ε_K.
- Originates from absorbtive part of Wigner-Weisskopf formula

$$\Gamma_{21} = \Gamma_{12}^* = \sum_f \mathcal{A}(K^0 \to f)\mathcal{A}(\bar{K}^0 \to f)^*$$

- Roughly a 4% correction to ε_K
- As dominated by 2π intermediate state, we have

$$\frac{\mathrm{Im}\Gamma_{12}}{\mathrm{Re}\Gamma_{12}} \approx -2\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} = -2\xi \longrightarrow \epsilon_K = e^{i\phi_\epsilon}\sin\phi_\epsilon\left[\frac{\mathrm{Im}M_{12}}{\Delta m_K} + \xi\right]$$

- But this neglects LD contribs to $\mathrm{Im}(M_{12})$. Using SU(3) ChPT we can estimate

[Buras et al arXiv:1002.3612]

$$\epsilon_K = \kappa_\epsilon \frac{e^{i\phi_\epsilon}}{\sqrt{2}}\left[\frac{\mathrm{Im}M_{12}^{(6)}}{\Delta m_K}\right] \qquad \kappa_\epsilon = \sqrt{(2)}\sin(\phi_\epsilon)\left(1 + \rho\frac{\xi_0}{\sqrt{2}|\epsilon|}\right)$$

$$\rho = 0.6 \pm 0.3$$

- Can use our lattice calculation of $\mathrm{Im}(A_0)$ to obtain precise correction:

$$\kappa_\varepsilon = 0.963 \pm 0.014$$

[Lehner et al arXiv:1508.01801]

- Can in principle obtain LD corrections to ε_K **directly** from the lattice.
[Christ Lat'2011 arXiv:1201.2065]

Conclusions

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- We are now able to study both indirect and direct CP-violation on the lattice.
- B_K essentially a solved problem; more important to beat down errors on V_{cb}.
- LD contributions can be precisely determined using ChPT with lattice input, and perhaps even directly on the lattice in the future.
- $\Delta I=3/2$ K->pipi amplitude precisely measured, with errors dominated by perturbative systematics in Wilson coeffs and NPR. Step-scaling and higher-order PT necessary.
- First calculation $\Delta I=1/2$ amplitude performed. Both theoretically and computationally difficult calculation due to desire for physical kinematics and presence of disconnected diagrams. Strong need for more statistics.
- $\mathrm{Re}(\varepsilon'/\varepsilon)$ from lattice has 2.1σ tension with expt.
- Work demonstrating viability of calculating ε' using Wilson fermions, may allow for more precise determination.

[Christ arXiv:1201.2065]

- Calculation very similar to RBC/UKQCD calculation of Δm_K: