TALENT school on Few-body Methods, Week 3, Exercise Sheet 3

Date: Wednesday, August 10

Mean-squared radii: LO, NLO, and beyond

1. Pick a couple of different values of R in the LO bound-state wave functions you derived yesterday and use them to compute the mean-square radius of the bound state $\langle r^2 \rangle$. Compare the result to the LO and NLO prediction of Halo EFT. These are:

$$\langle r^2 \rangle^{LO} = \frac{1}{2\gamma^2}; \tag{1}$$

$$\langle r^2 \rangle^{NLO} = \frac{1}{1 - r_0\gamma}\frac{1}{2\gamma^2} = \frac{A_0^2}{4\gamma^3}. \tag{2}$$

2. If you have NLO wave functions, do the same thing at NLO. I.e., pick a couple of different values of R and use the wave functions obtained with the NLO potential at those R's to compute the mean-square radius of the bound state $\langle r^2 \rangle$.

3. Look at your results from Q.1. Also examine the place where your LO wave functions deviate from their asymptotic form. Use those two source of information to answer the following question: if the range of the neutron-core potential is $R_{\rm core}$ then what is the parametric size (i.e. naive-dimensional analysis estimate, don't worry about pre-factors) of the first correction to the NLO result written above?

E1 strength

1. [**Optional**] The Feynman-diagram calculation we did this morning computes the matrix element of the operator $e|Z_c|e^{i\mathbf{q}\cdot\mathbf{r}}$ (in co-ordinate space representation). In contrast, the dipole operator is defined to be (if $\hat{q} = \hat{z}$): $Y_{10}(\hat{r})r\sqrt{\alpha_{em}}$. Explain why you can get the dipole operator matrix element from the calculation with the full A_0-photon-core Feynman rule by:

   (a) Picking out the term linear in $\mathbf{q}$;

   (b) Dividing by $iq\sqrt{\frac{4\pi}{3}}$;

   (c) Replacing e by $\sqrt{\alpha_{em}}$.

2. Starting from:

$$\mathcal{M}_{\rm E1} = 2\sqrt{6\gamma}f\sqrt{\alpha_{em}}Z_c\frac{k}{(\gamma^2 + k^2)^2}\hat{k}\cdot\hat{z}, \tag{3}$$

(the leading-order result) and

$$d\mathrm{B(E1)} = \sum_{m_l} |\mathcal{M}_{\rm E1}^{(l=1,m_l)}|^2\frac{d^3k}{(2\pi)^3}, \tag{4}$$

show that

$$\frac{d\mathrm{B(E1)}}{dE} = \frac{6}{\pi^2}m_R f^2 Z_c^2 \alpha_{em} 2\gamma\frac{k^3}{(\gamma^2 + k^2)^4}. \tag{5}$$

3. Express Eq. (5) in dimensionless units, i.e., convert it to the form:

$$m_R S_{1n}^2\frac{d\mathrm{B(E1)}}{dE} = \frac{3\alpha_{em}Q_{eff}^2}{\pi^2}\frac{x^{3/2}}{(1+x)^4}; \quad Q_{eff} = fZ_c = \frac{Z_c}{A_c+1}; \quad x = E/S_{1n}. \tag{6}$$

Potentially useful information: $2m_R S_{1n} = \gamma^2$ and $2m_R E = k^2$.

4. Pick a couple of different values of R and use your LO potentials to compute the (shape of the) dipole response $\frac{dB(E1)}{dE}$ as a function of the outgoing ^{18}C-neutron relative energy. This should be done in two stages:

   (a) Compute the dipole matrix element in co-ordinate space
   $$\mathcal{M} \propto \int_0^\infty dr r^2 u(r) j_1(kr), \tag{7}$$
   where $u(r)$ is the s-wave reduced radial wave function (properly normalized) obtained from your code, and $j_1(x)$ is the spherical Bessel function for $l = 1$. (The relevant piece of the final-state plane wave.)

   (b) Compute
   $$\frac{dB(E1)}{dE} \propto k|\mathcal{M}|^2. \tag{8}$$

5. Is the position of the peak in the E1 responses you calculated independent (or indepenentish) of R? What about the peak height?

6. [**If time permits**] Compute the ratio of the integrated E1 strength, $\int_0^\infty \frac{dB(E1)}{dE} dE$, to the mean-square radius, $\langle r^2 \rangle$, for each of the LO potentials you constructed. What do you notice? Why do you think that happens?

7. [**If time permits**] Use the NLO potentials to compute $\frac{dB(E1)}{dE}$ at the extreme values of R at which you obtained them.