Effective Field Theory for Halo Nuclei: Lecture 4

Daniel Phillips
Institute of Nuclear & Particle Physics
Ohio University

OHIO UNIVERSITY

Collaborators: Bijaya Acharya
Matthias Göbel
Hans-Werner Hammer
Chen Ji

INSTITUTE OF NUCLEAR & PARTICLE PHYSICS INPP @OHIO UNIVERSITY

Research supported by the US Department of Energy

The three-body scattering problem

- Consider the Schrödinger equation for the quantum-mechanical three-body scattering problem. Let's start with just pairwise potentials:

$$\left[-\frac{\nabla_1^2}{2m_1}-\frac{\nabla_2^2}{2m_2}-\frac{\nabla_3^2}{2m_3}+V_{12}(\mathbf{r}_{12})+V_{23}(\mathbf{r}_{23})+V_{31}(\mathbf{r}_{31})\right]\Psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)=E\Psi$$

- Two (related) problems:
  - Disconnected diagrams are present in solution for total wave function Ψ
  - What boundary condition should we impose?

Resources:

I.R. Afnan and A. W. Thomas , Fundamentals of Three-body Scattering Theory, in "Modern Three-Hadron Physics" (Springer, 1977)

W. Glöckle, "The Quantum-Mechanical Few-body Problem" (Springer, 1983)

Solving problem I

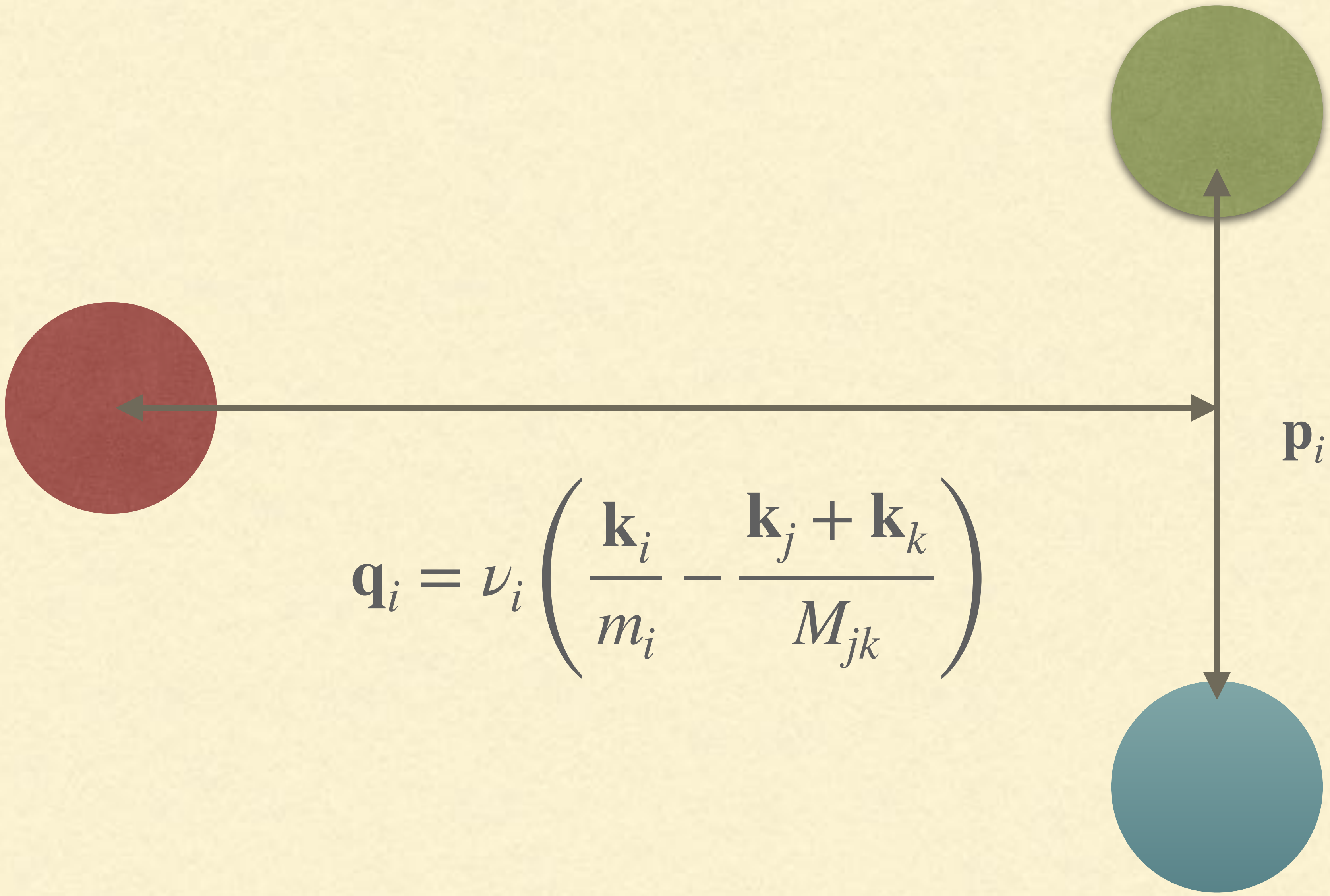
- Let's define 2B t-matrices that are embedded in the 3B Hilbert space
- Or, equivalently, think about states $|\psi_3^{(+)}\rangle \otimes |\mathbf{q}_3\rangle$, where $|\psi_3^{(+)}\rangle$ is the solution of the Schrödinger equation in which particle 3 is a "spectator",while the (12) wave function has spherical outgoing wave boundary conditions and solves

$$\left[-\frac{\nabla_{12}^2}{2\mu_{12}} + V_{12}(\mathbf{r}_{12})\right]\psi_3(\mathbf{r}_{12}) = \left(E - \frac{\mathbf{q}_3^2}{2\nu_3}\right)\psi_3(\mathbf{r}_{12})$$

where $\mu_{12} = \frac{m_1 m_2}{m_1 + m_2}\nu_3 = \frac{(m_1 + m_2)m_3}{m_1 + m_2 + m_3};.$

- $V_3|\psi_3^{(+)}\rangle = t_3\left(E^+ - \frac{q_3^2}{2\nu_3}\right)|\mathbf{p}_{12}\rangle$; 3B operator $= t_3\left(E^+ - \frac{q_3^2}{2\nu_3}\right)_{12} \otimes \mathbf{1}_3$

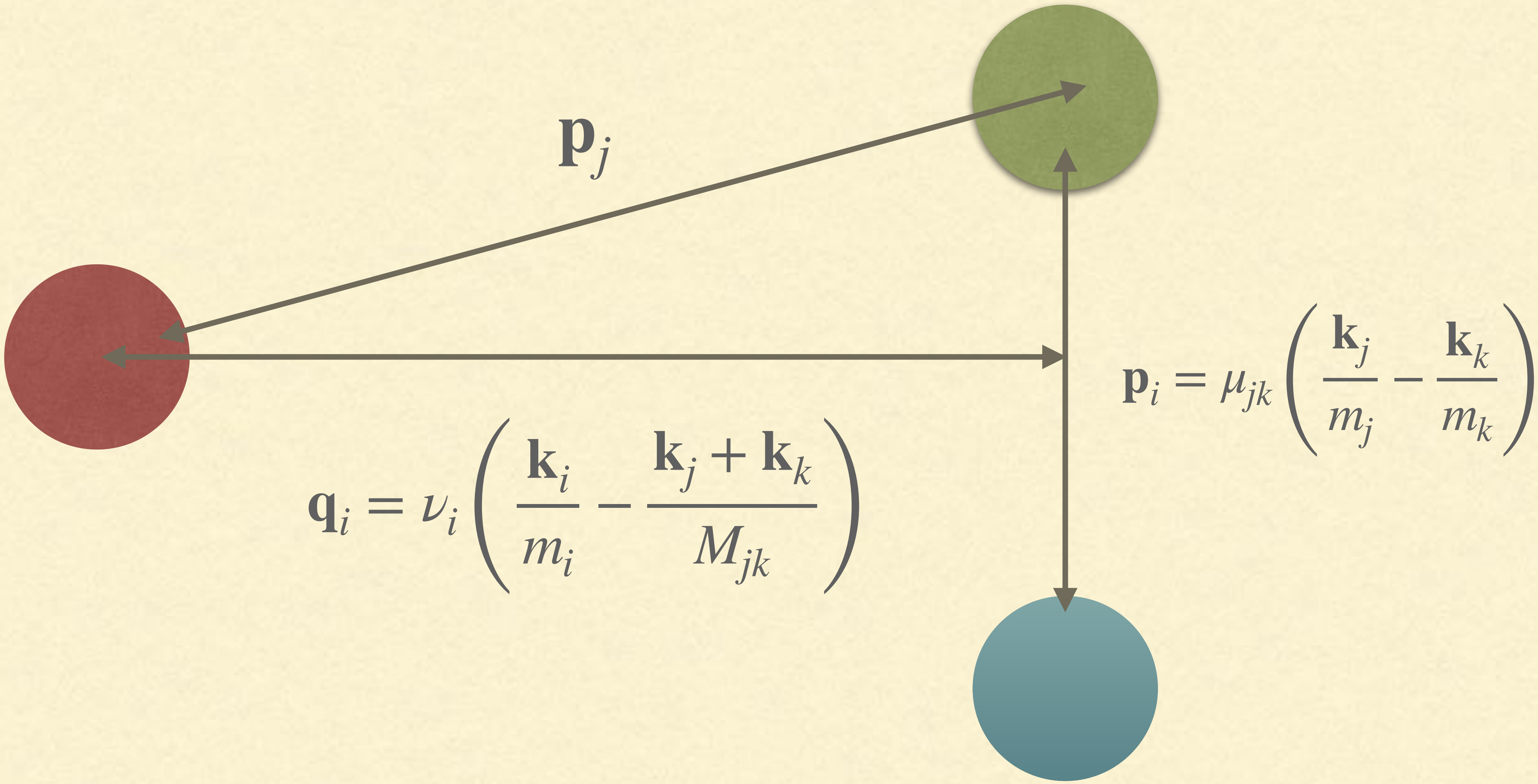
Reminder: Jacobi momenta



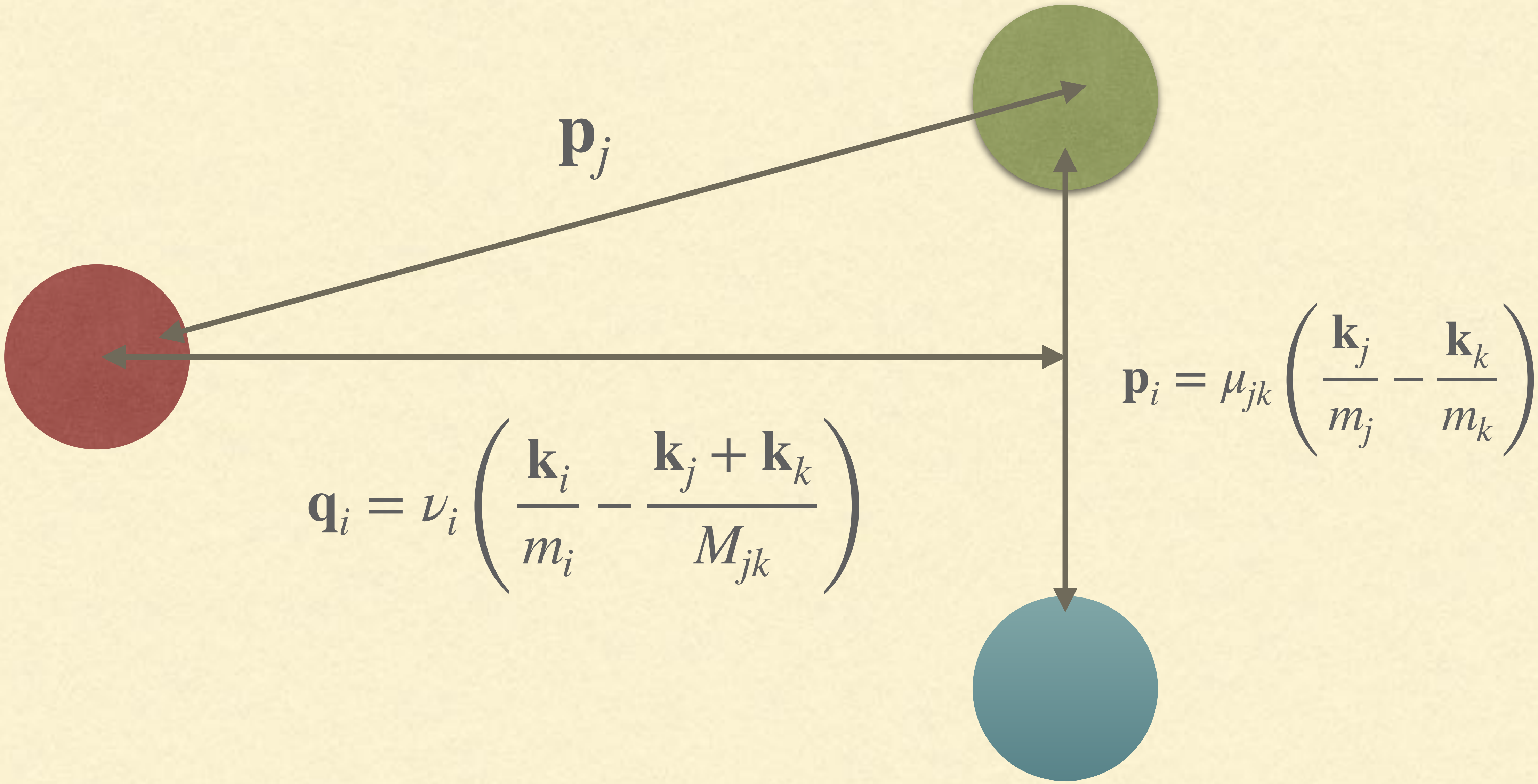
$$\mathbf{p}_i = \mu_{jk}\left(\frac{\mathbf{k}_j}{m_j} - \frac{\mathbf{k}_k}{m_k}\right)$$

$$\mathbf{q}_i = \nu_i\left(\frac{\mathbf{k}_i}{m_i} - \frac{\mathbf{k}_j + \mathbf{k}_k}{M_{jk}}\right)$$

Reminder: Jacobi momenta



Reminder: Jacobi momenta



"Recoupling": achieved by inserting complete sets of eigenstates of two Jacobi momenta, but which two?

The Faddeev equations

The Faddeev equations

- Decompose $|\Psi\rangle = |\Phi\rangle + |\Psi_1\rangle + |\Psi_2\rangle + |\Psi_3\rangle$
- Where $|\Psi_i\rangle = G_0 V_i |\Psi\rangle$ and $|\Phi\rangle$ is a three-body plane wave; separate wave function according to the *last* interaction before particles go to the detector: that defines three separate outgoing boundary conditions

The Faddeev equations

- Decompose $|\Psi\rangle = |\Phi\rangle + |\Psi_1\rangle + |\Psi_2\rangle + |\Psi_3\rangle$
- Where $|\Psi_i\rangle = G_0 V_i |\Psi\rangle$ and $|\Phi\rangle$ is a three-body plane wave; separate wave function according to the *last* interaction before particles go to the detector: that defines three separate outgoing boundary conditions
- Show that the resulting $|\Psi\rangle$ solves the 3B Schrödinger equation $(E - H_0)|\Psi\rangle = (V_1 + V_2 + V_3)|\Psi\rangle$

The Faddeev equations

- Decompose $|\Psi\rangle = |\Phi\rangle + |\Psi_1\rangle + |\Psi_2\rangle + |\Psi_3\rangle$
- Where $|\Psi_i\rangle = G_0 V_i |\Psi\rangle$ and $|\Phi\rangle$ is a three-body plane wave; separate wave function according to the *last* interaction before particles go to the detector: that defines three separate outgoing boundary conditions
- Show that the resulting $|\Psi\rangle$ solves the 3B Schrödinger equation $(E - H_0)|\Psi\rangle = (V_1 + V_2 + V_3)|\Psi\rangle$
- Manipulate $|\Psi_1\rangle = G_0 V_1(|\Phi\rangle + |\Psi_1\rangle + |\Psi_2\rangle + |\Psi_3\rangle)$ into a formal equation for $|\Psi_1\rangle$

The Faddeev equations

- Decompose $|\Psi\rangle = |\Phi\rangle + |\Psi_1\rangle + |\Psi_2\rangle + |\Psi_3\rangle$
- Where $|\Psi_i\rangle = G_0 V_i |\Psi\rangle$ and $|\Phi\rangle$ is a three-body plane wave; separate wave function according to the *last* interaction before particles go to the detector: that defines three separate outgoing boundary conditions
- Show that the resulting $|\Psi\rangle$ solves the 3B Schrödinger equation $(E - H_0)|\Psi\rangle = (V_1 + V_2 + V_3)|\Psi\rangle$
- Manipulate $|\Psi_1\rangle = G_0 V_1 (|\Phi\rangle + |\Psi_1\rangle + |\Psi_2\rangle + |\Psi_3\rangle)$ into a formal equation for $|\Psi_1\rangle$

$$|\Psi_i\rangle = G_0 t_i |\Phi > + G_0 t_i \sum_{j \neq i} |\Psi_j\rangle$$

The Faddeev equations

- Decompose $|\Psi\rangle = |\Phi\rangle + |\Psi_1\rangle + |\Psi_2\rangle + |\Psi_3\rangle$

- Where $|\Psi_i\rangle = G_0 V_i |\Psi\rangle$ and $|\Phi\rangle$ is a three-body plane wave; separate wave function according to the *last* interaction before particles go to the detector: that defines three separate outgoing boundary conditions

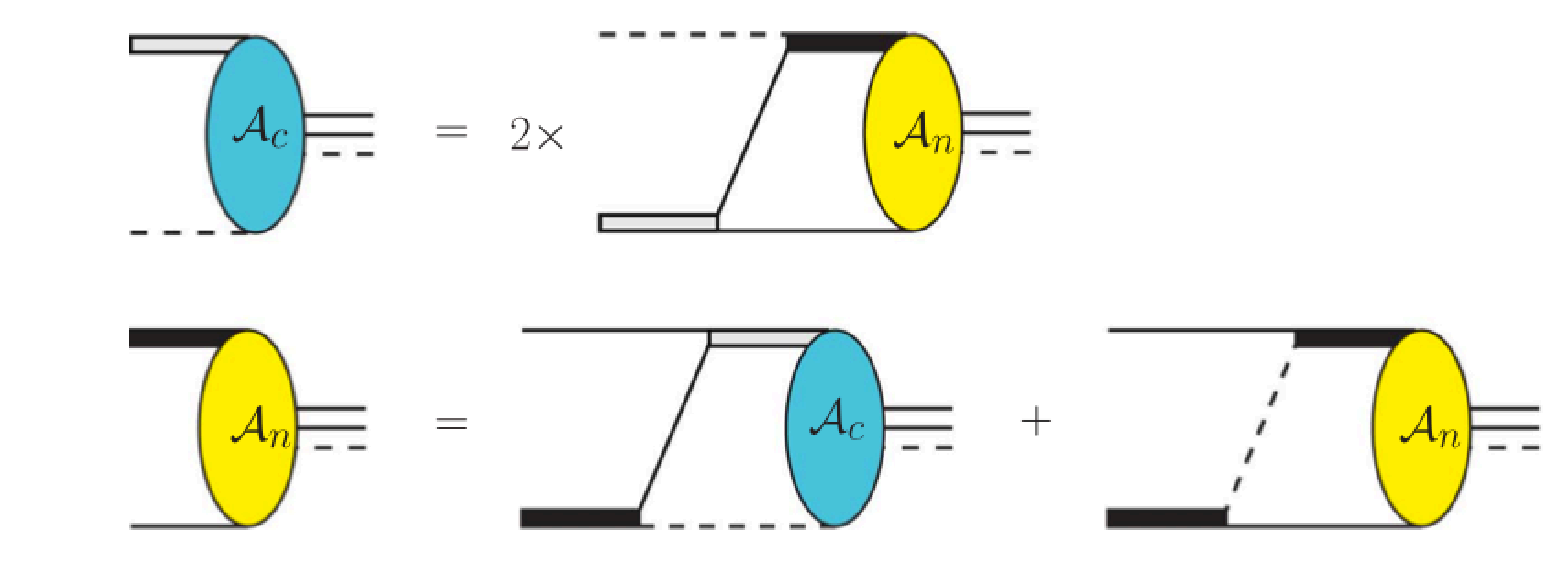
- Show that the resulting $|\Psi\rangle$ solves the 3B Schrödinger equation $(E - H_0)|\Psi\rangle = (V_1 + V_2 + V_3)|\Psi\rangle$

- Manipulate $|\Psi_1\rangle = G_0 V_1 (|\Phi\rangle + |\Psi_1\rangle + |\Psi_2\rangle + |\Psi_3\rangle)$ into a formal equation for $|\Psi_1\rangle$

$$|\Psi_i\rangle = G_0 t_i |\Phi > + G_0 t_i \sum_{j\neq i} |\Psi_j\rangle$$

$$|\Psi_i\rangle = G_0 V_i |\Psi\rangle \equiv G_0 T_i |\Phi\rangle \Rightarrow T_i = t_i + t_i \sum_{j\neq i} G_0 T_j$$

For separable (EFT) interactions: bound-state equations



$$\tilde{A}_c(\boldsymbol{q}) = 2\int \frac{d^3q}{4\pi^2} G_0^c(\pi_2(\boldsymbol{q}', \boldsymbol{q}), q; B_3)\, \tau_\sigma(q'; B_3)\, \tilde{A}_n(\boldsymbol{q}')$$

$$\tilde{A}_n(\boldsymbol{q}) = \int \frac{d^3q'}{4\pi^2} G_0^c(\pi_2(\boldsymbol{q}, \boldsymbol{q}'), q'; B_3)\, \tau_d(q'; B_3)\, \tilde{A}_c(\boldsymbol{q}') + \int \frac{d^3q'}{4\pi^2}\left[G_0^n(\pi_3(\boldsymbol{q}', \boldsymbol{q}), q; B_3) + \frac{H(\Lambda)}{\Lambda^2}\right]\tau_\sigma(q'; B_3)\, \tilde{A}_n(\boldsymbol{q}'),$$

For E=-B_3
=-S_{2n}

$$G_0^n(p, q; B_3) = \left(m_n B_3 + \frac{A+1}{2A}p^2 + \frac{A+2}{2(A+1)}q^2\right)^{-1},$$

$$G_0^c(p, q; B_3) = \left(m_n B_3 + p^2 + \frac{A+2}{4A}q^2\right)^{-1}.$$

$$\tau_d(q; B_3) = \frac{2}{-\gamma_{0,d} + \sqrt{m_n B_3 + \frac{A+2}{4A}q^2}},$$

$$\tau_\sigma(q; B_3) = \frac{(A+1)/A}{-\gamma_{0,\sigma} + \sqrt{\frac{A}{A+1}\left(2m_n B_3 + \frac{A+2}{A+1}q^2\right)}}.$$

Bound-state equations for s-wave 2n halo

Canham, Hammer (2008)

Bound-state equations for s-wave 2n halo

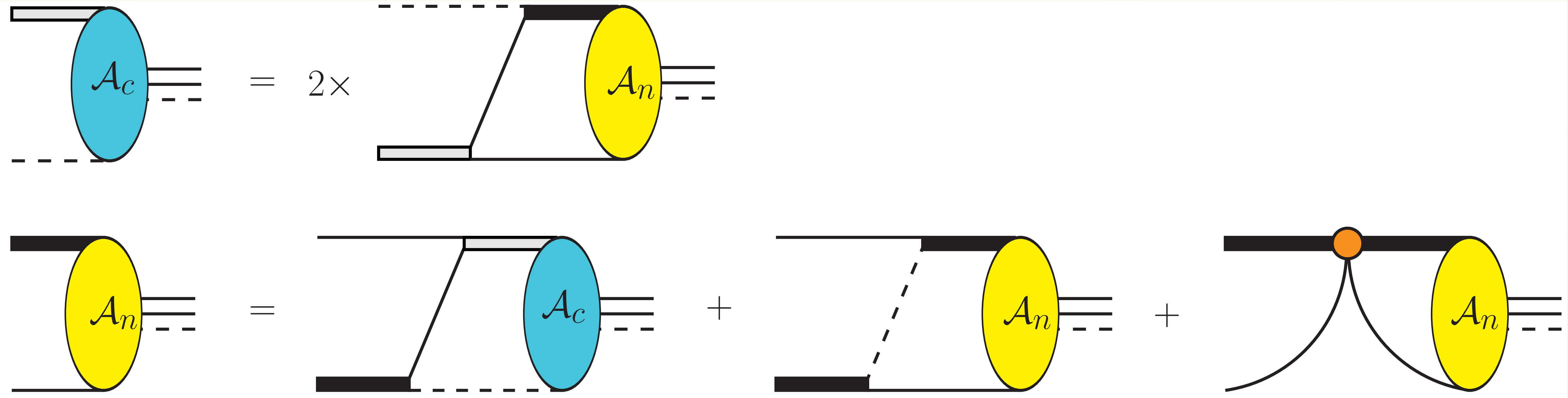
Canham, Hammer (2008)

- Core-n and n-n contact interactions at leading order: solve 3B problem

Bound-state equations for s-wave 2n halo

Canham, Hammer (2008)

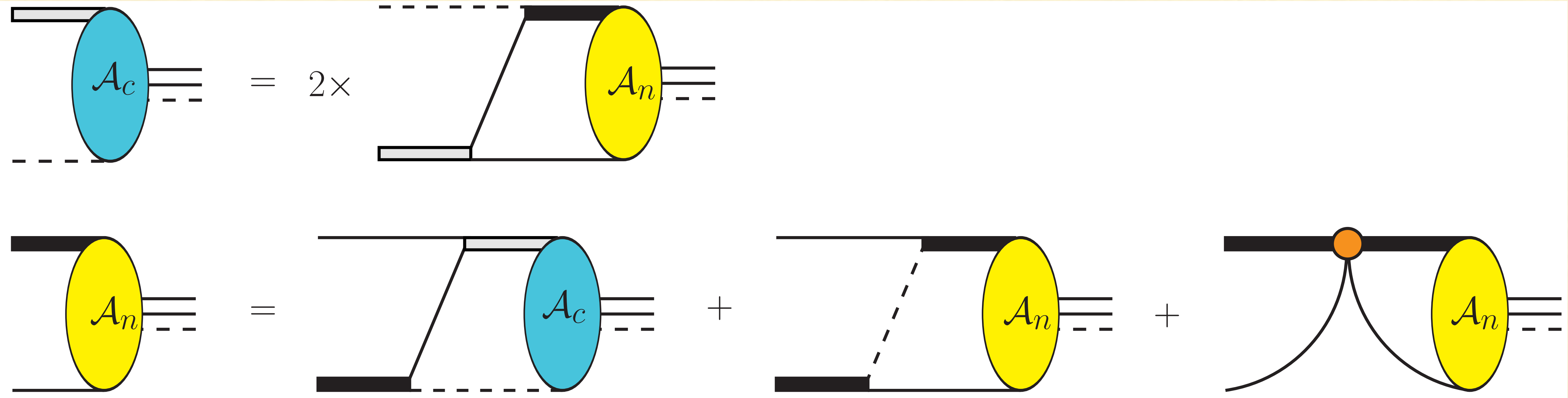
- Core-n and n-n contact interactions at leading order: solve 3B problem



Bound-state equations for s-wave 2n halo

Canham, Hammer (2008)

- Core-n and n-n contact interactions at leading order: solve 3B problem



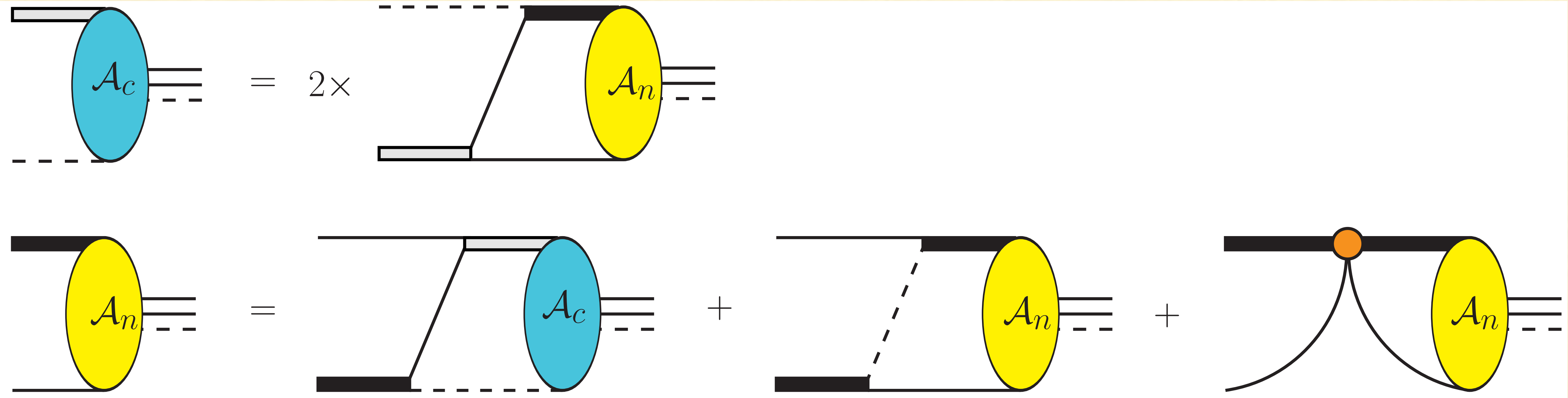
- Efimov-Thomas effects: $\mathscr{A} \sim p^{is_0 - 1}$ for p→∞

Danilov

Bound-state equations for s-wave 2n halo

Canham, Hammer (2008)

- Core-n and n-n contact interactions at leading order: solve 3B problem



- Efimov-Thomas effects: $\mathscr{A} \sim p^{is_0-1}$ for p→∞

Danilov

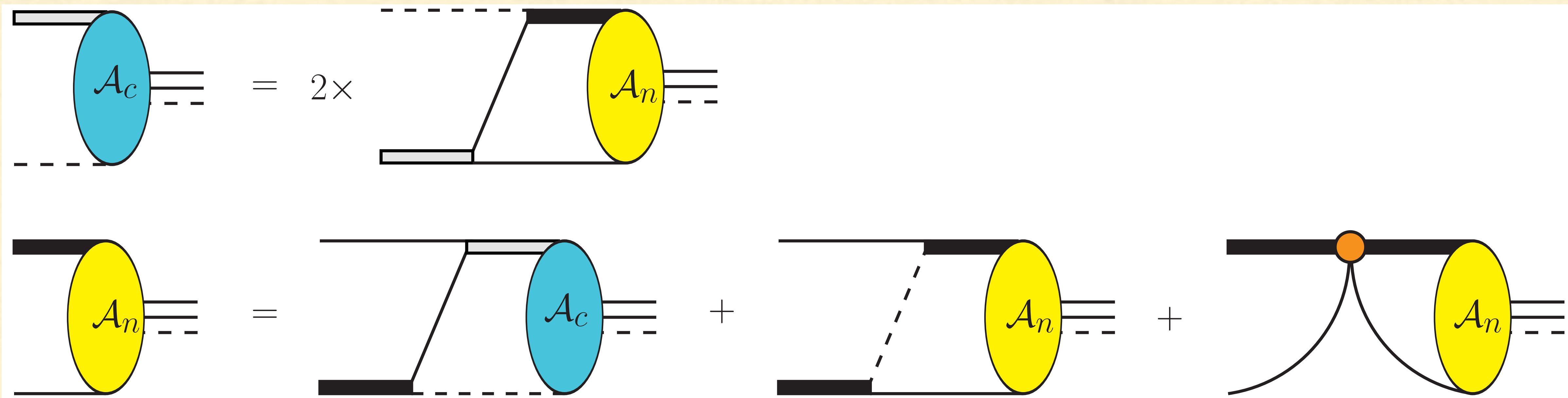
- (cn)-n contact interaction to stabilize three-body system

Bedaque, Hammer, van Kolck (1999)

Bound-state equations for s-wave 2n halo

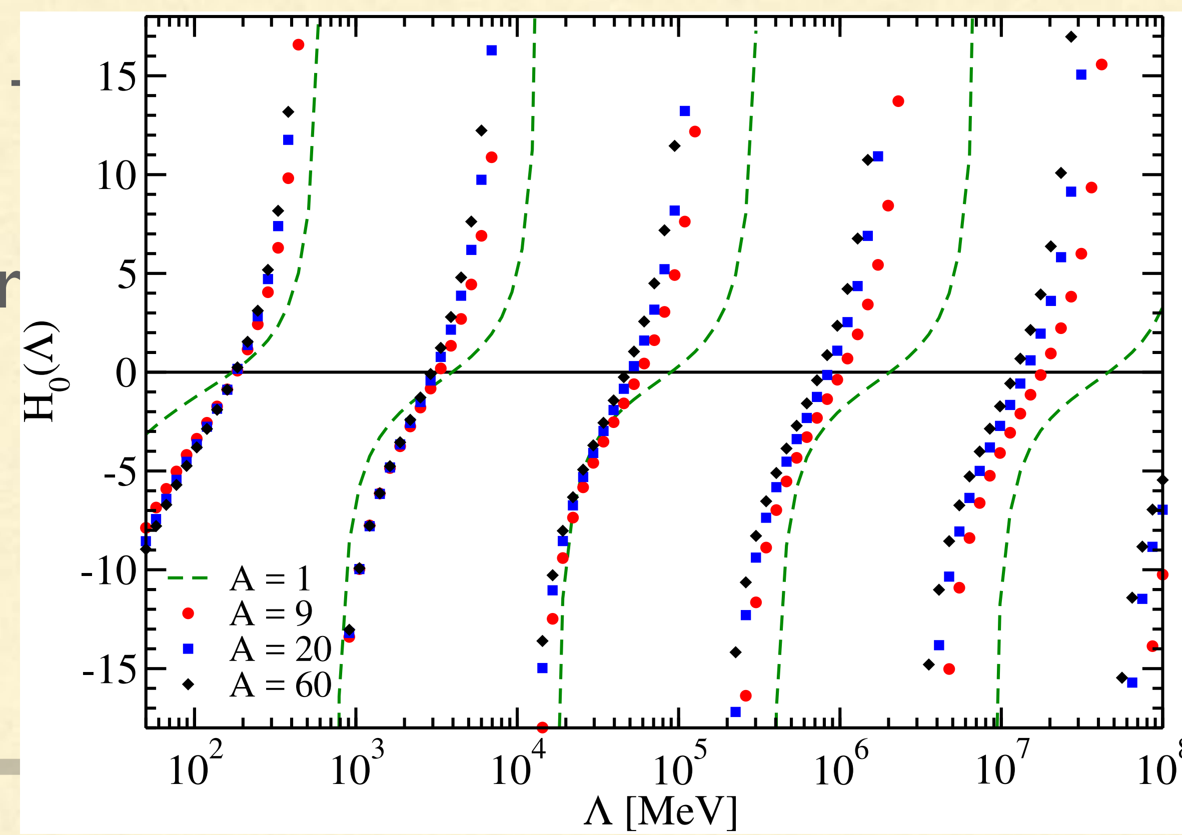
Canham, Hammer (2008)

- Core-n and n-n contact interactions at leading order: solve 3B problem



- Efimov-Thomas effects: $\mathscr{A} \sim p^{is_0-1}$ for p

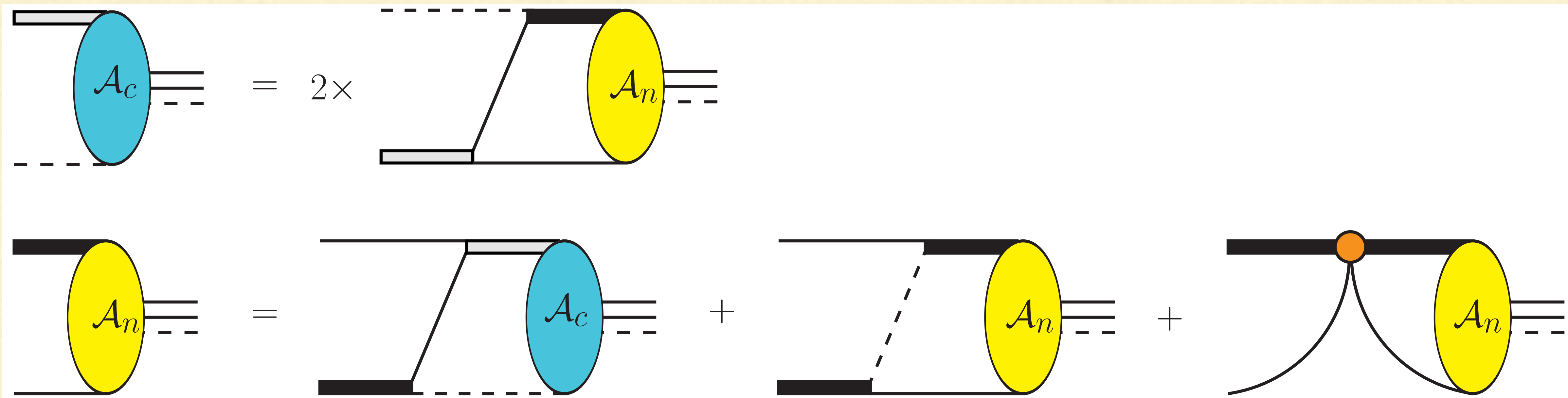
- (cn)-n contact interaction to stabilize thr



Bound-state equations for s-wave 2n halo

Canham, Hammer (2008)

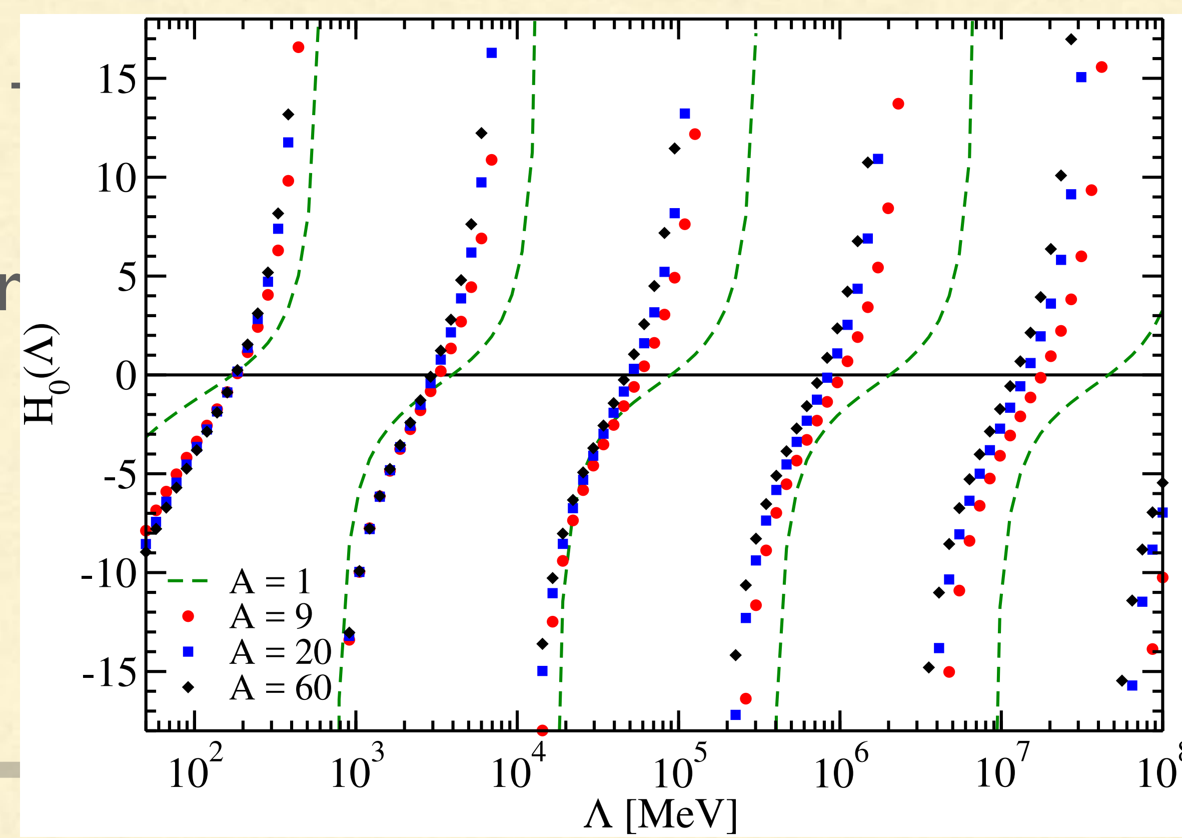
- Core-n and n-n contact interactions at leading order: solve 3B problem



- Efimov-Thomas effects: $\mathscr{A} \sim p^{is_0 - 1}$ for p

- (cn)-n contact interaction to stabilize thr

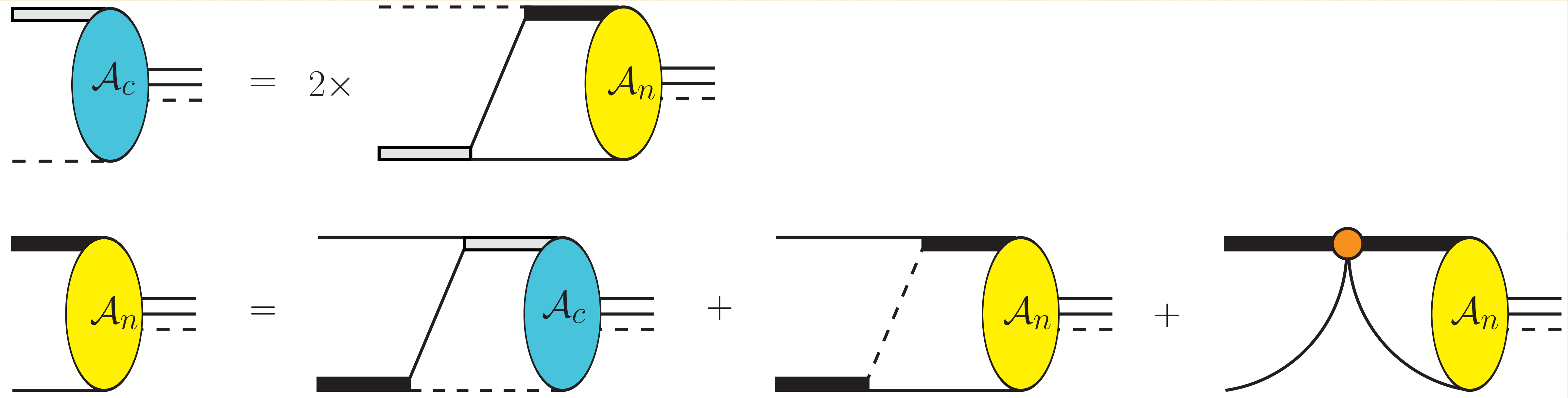
- Inputs: $E_{nn}=1/(m\, a_{nn}^2)$, E_{nc}, S_{2n} (=B)



Bound-state equations for s-wave 2n halo

Canham, Hammer (2008)

- Core-n and n-n contact interactions at leading order: solve 3B problem



- Efimov-Thomas effects: $\mathscr{A} \sim p^{is_0-1}$ for p→∞

Danilov

- (cn)-n contact interaction to stabilize three-body system

Bedaque, Hammer, van Kolck (1999)

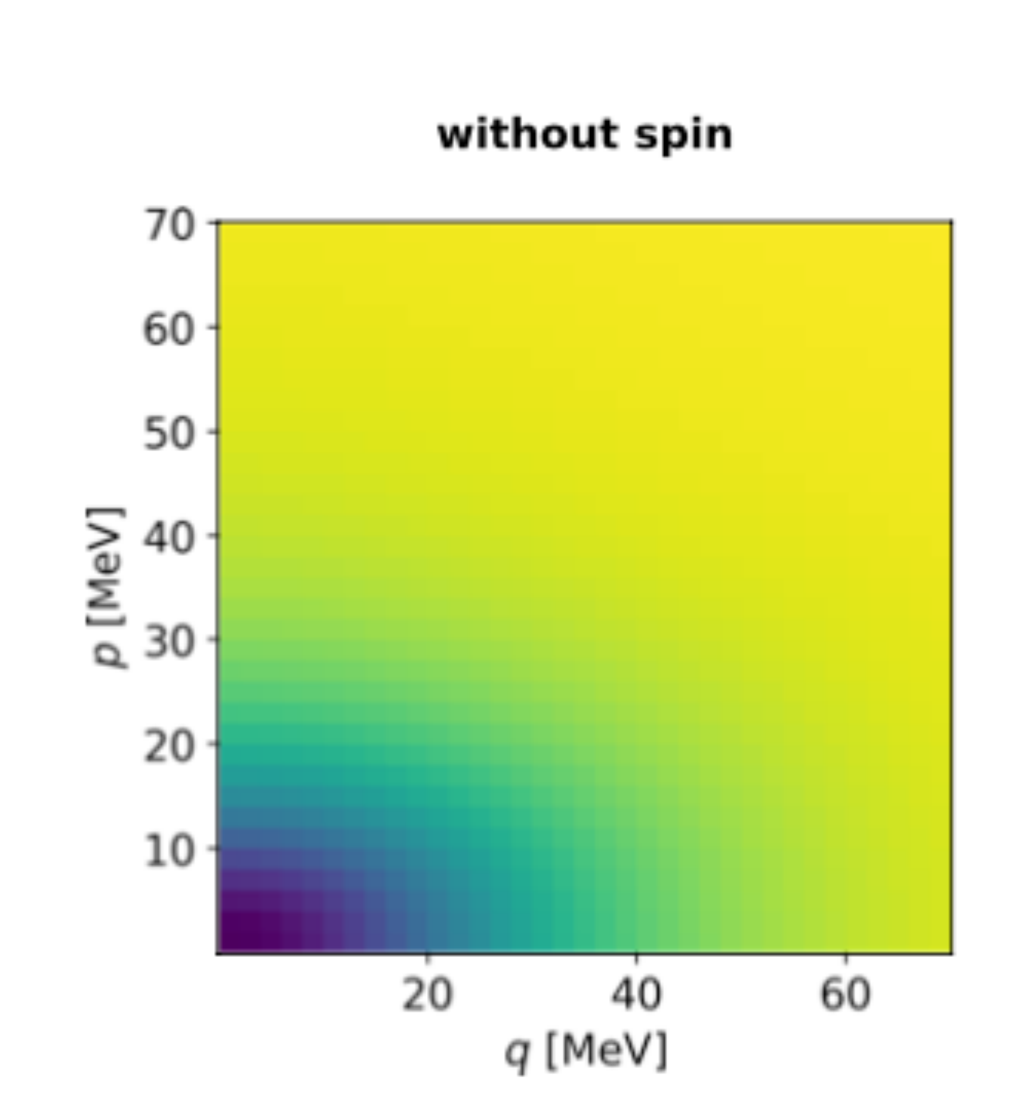
- Inputs: $E_{nn}=1/(m\,a_{nn}^2)$, E_{nc}, S_{2n} (=B)

- Output: everything. Up to R_{core}/R_{halo} corrections.

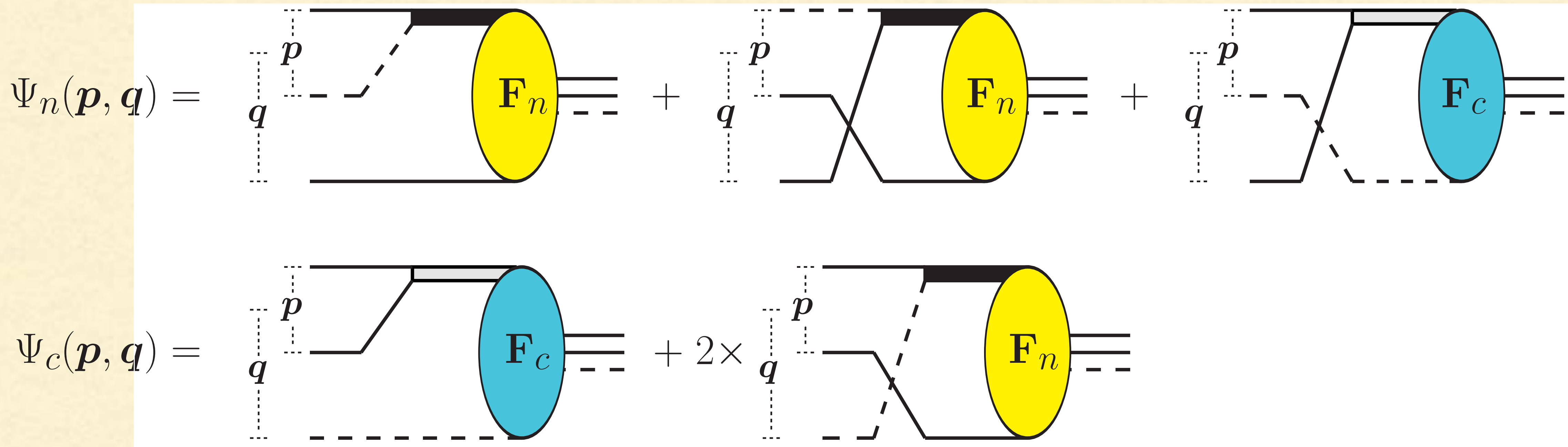
^{11}Li as a 2n halo

- a_{nn}=-18.7 fm, E_{nc}=0.026 MeV
- S_{2n}=369 keV
- Calculations done with a cutoff of 470 MeV, but results checked for a cutoff of 700 MeV
- Here results with a spin-0 core, but we also examined case of spin-3/2 core
- Results identical if spin-1 and spin-2 nc interactions have equal strength

^{11}Li momentum distribution



Matter radii of 2n s-wave halos



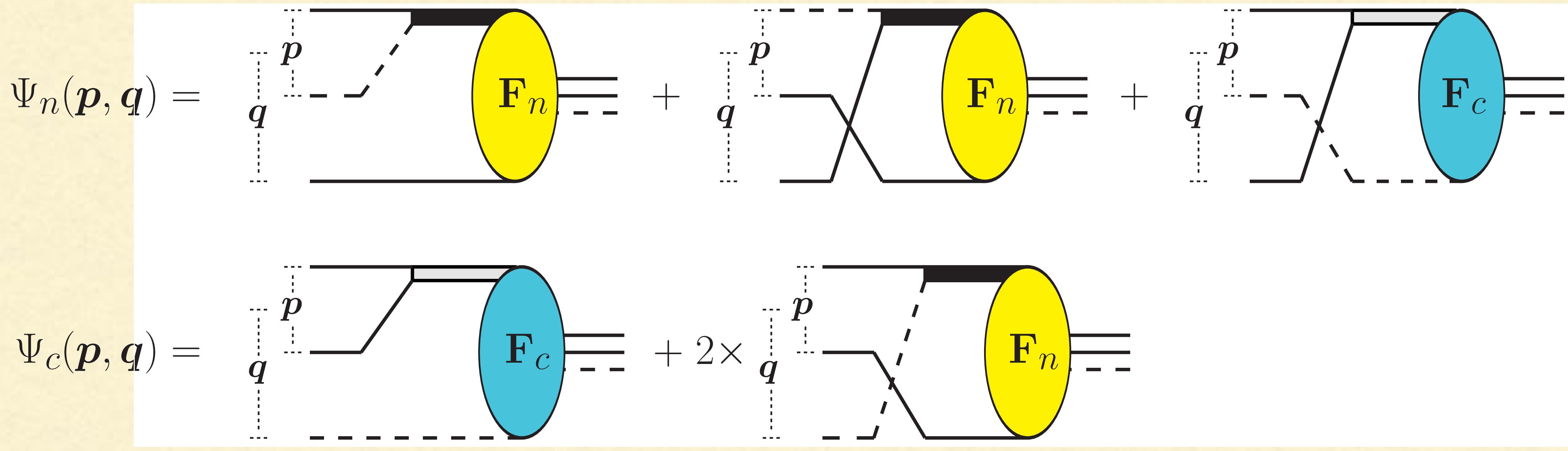
Canham, Hammer (2008)

- One-body form factors:

$$\mathcal{F}_x(k^2) = \int_0^\infty \mathrm{d}p\ p^2 \int_0^\infty \mathrm{d}q\ q^2 \int_{-1}^{1} \mathrm{d}\left(\hat{q}\cdot\hat{k}\right)\ \Psi_x(p,q)\ \Psi_x(p,|\vec{q}-\vec{k}|).$$

- Radii: $\mathcal{F}_x(k^2) = 1 - \frac{1}{6}\langle r_x^2\rangle k^2 + O(k^4)$
- Matter radius: $\langle r_m^2\rangle = \frac{2(A+1)^2}{(A+2)^3}\langle r_n^2\rangle + \frac{4A}{(A+2)^3}\langle r_c^2\rangle$

Matter radii of 2n s-wave halos



Canham, Hammer (2008)

- One-body form factors:

$$\mathcal{F}_x(k^2) = \int_0^\infty \mathrm{d}p\; p^2 \int_0^\infty \mathrm{d}q\; q^2 \int_{-1}^1 \mathrm{d}\left(\hat{q}\cdot\hat{k}\right) \Psi_x(p,q)\;\Psi_x(p,|\vec{q}-\vec{k}|).$$

- Radii: $\mathcal{F}_x(k^2) = 1 - \frac{1}{6}\langle r_x^2\rangle k^2 + O(k^4)$

- Matter radius: $\langle r_m^2\rangle = \frac{2(A+1)^2}{(A+2)^3}\langle r_n^2\rangle + \frac{4A}{(A+2)^3}\langle r_c^2\rangle$

Input: E_{cn} and S_{2n}

Output: all radii

Matter radii of 2n halos

Achrya, Ji, Phillips (2013)

- Define: $f\left(\frac{E_{nn}}{S_{2n}}, \frac{E_{cn}}{S_{2n}}; A\right) \equiv 2m_R S_{2n}\langle r_m^2\rangle$
- "Unitary limit", E_{nn}=E_{nc}=0: f becomes a number depending solely on A

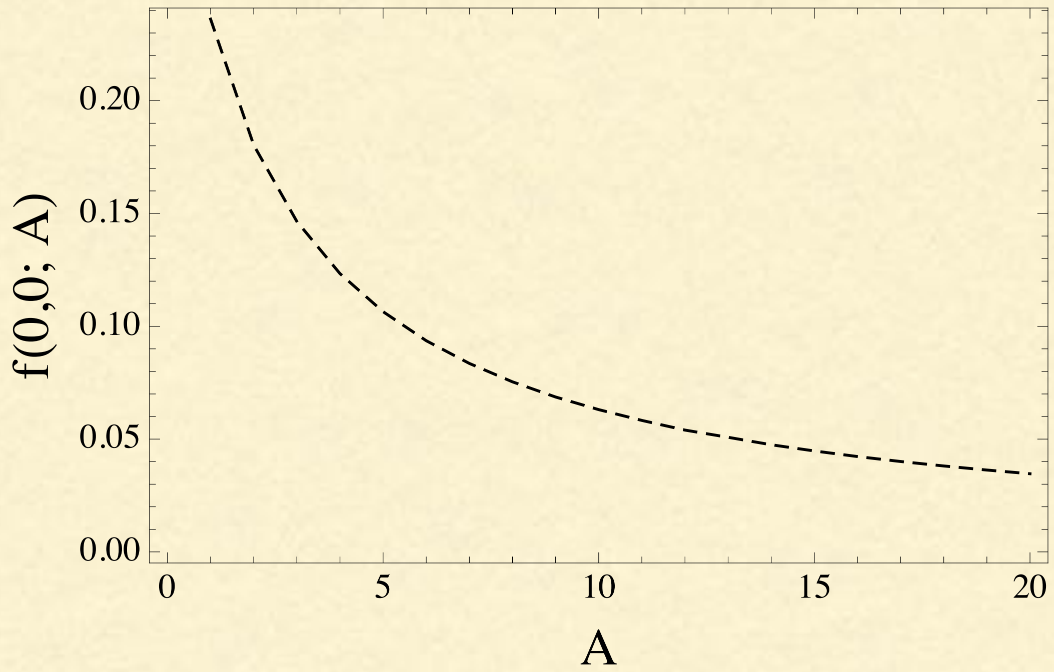
Unification via universality

Matter radii of 2n halos

Achrya, Ji, Phillips (2013)

- Define: $f\left(\frac{E_{nn}}{S_{2n}}, \frac{E_{cn}}{S_{2n}}; A\right) \equiv 2m_R S_{2n}\langle r_m^2\rangle$

- "Unitary limit", $E_{nn}=E_{nc}=0$: f becomes a number depending solely on A



c. f. Yamashita et al. (2004): 15% lower at A=20

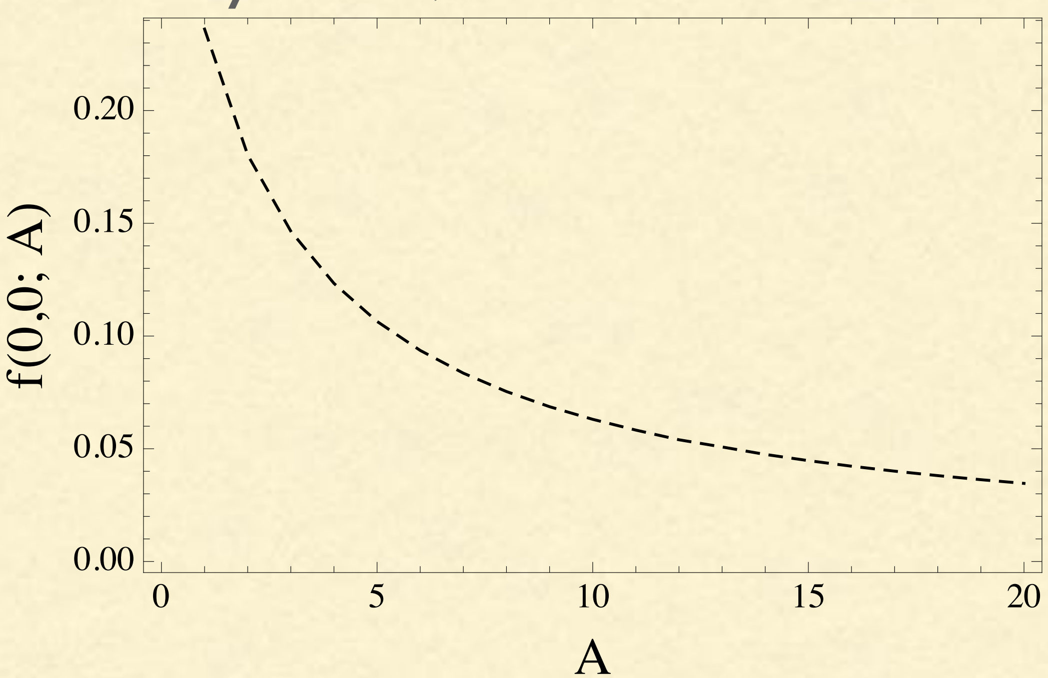
Unification via universality

Matter radii of 2n halos

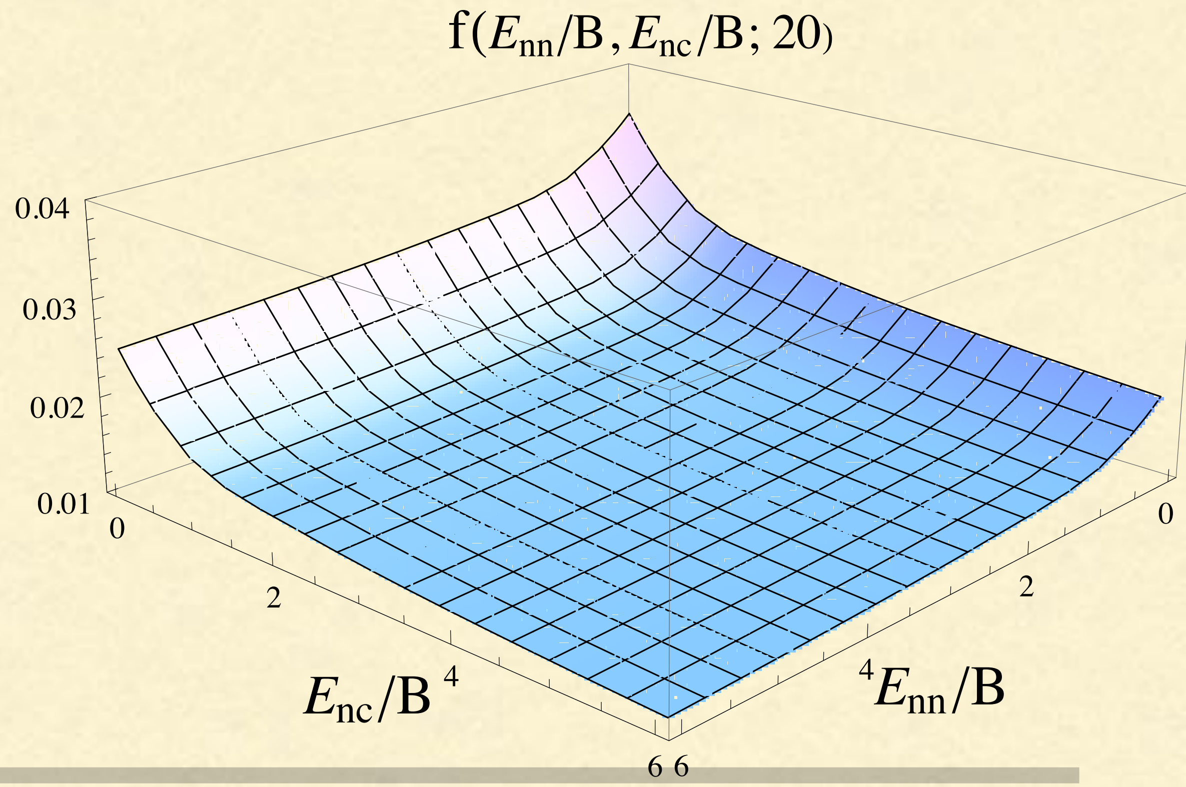
Achrya, Ji, Phillips (2013)

- Define: $f\left(\frac{E_{nn}}{S_{2n}}, \frac{E_{cn}}{S_{2n}}; A\right) \equiv 2m_R S_{2n}\langle r_m^2 \rangle$

- "Unitary limit", E_{nn}=E_{nc}=0: f becomes a number depending solely on A



c. f. Yamashita et al. (2004): 15% lower at A=20



- Fix A=20, plot f as a function of E_{nn} and E_{nc}

Unification via universality

Specific Leading-order results

Canham, Hammer (2011); Hagen, Platter, Hammer (2014); Acharya, Ji, Phillips (2013)

Vanasse (2016)

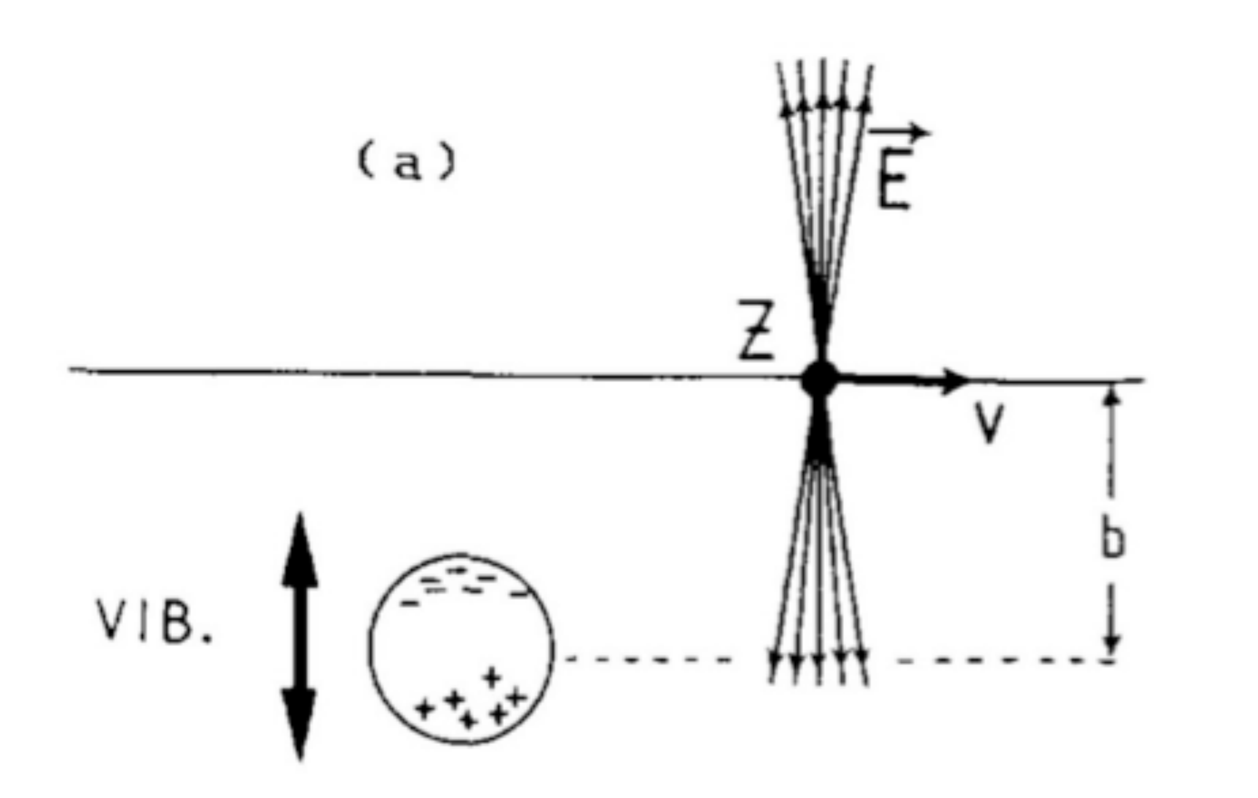
Diagnosing using universality

	E_{nc} (MeV)	S_{2n} (MeV)	R_{core}/R_{halo}	$\langle r_m^2 \rangle$ (fm^2) LO	$\langle r_m^2 \rangle$ (fm^2) Expt
^{11}Li	-0.026(13)	0.3693(6)	0.37	5.76 ± 2.13	5.34 ± 0.15
^{14}Be	-0.510	1.27(13)	0.78	1.23 ± 0.96	4.24 ± 2.42 2.90 ± 2.25
^{22}C	-0.01(47)	0.11(6)	0.26	3.99-∞	21.1 ± 9.7 3.77 ± 0.61

- Input in some two-body subsystems needed
- Errors tend to be dominated by EFT uncertainty though

Coulomb dissociation

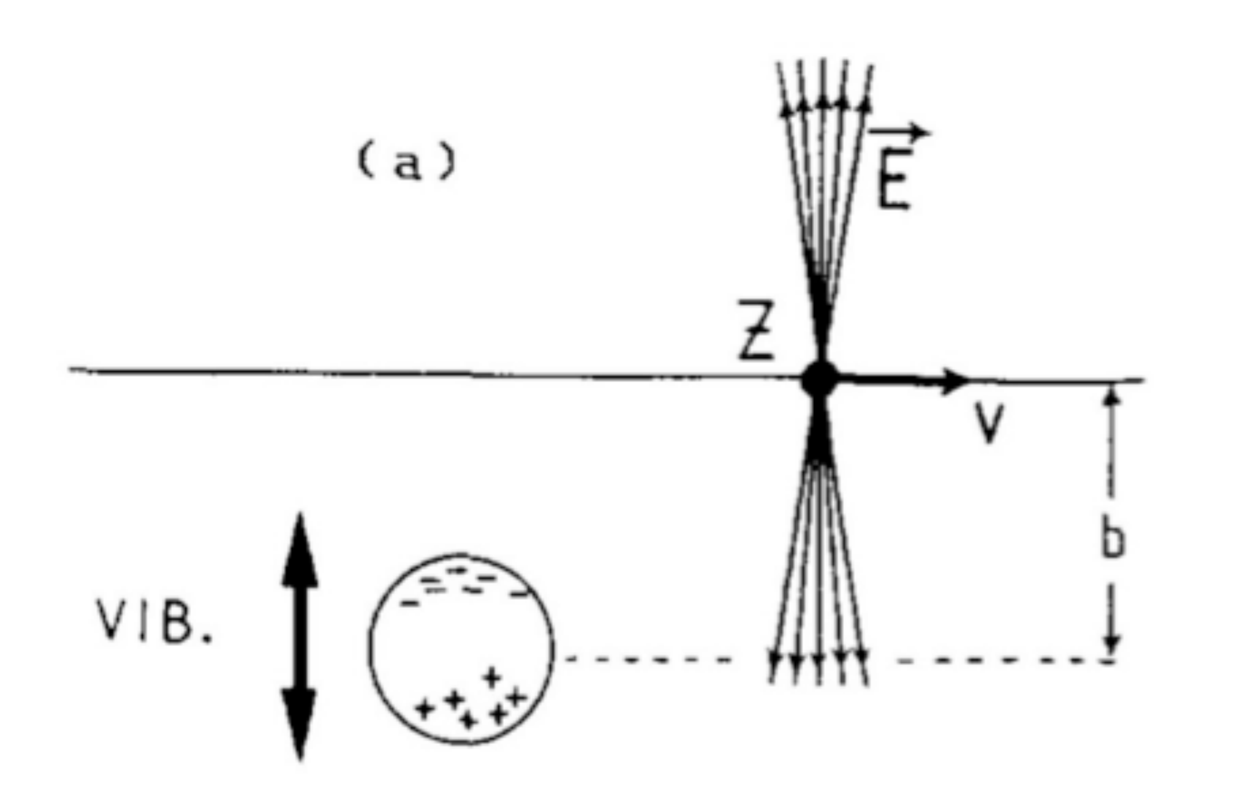
Bertulani, arXiv:0908.4307



- Coulomb dissociation: collide halo (we hope peripherally) with a high-Z nucleus
- Do for different Z, different nuclear sizes, different energies to test systematics

Coulomb dissociation

Bertulani, arXiv:0908.4307



- Coulomb dissociation: collide halo (we hope peripherally) with a high-Z nucleus
- Do for different Z, different nuclear sizes, different energies to test systematics
- Coulomb excitation dissociation cross section (p.v. b≫R_{target})

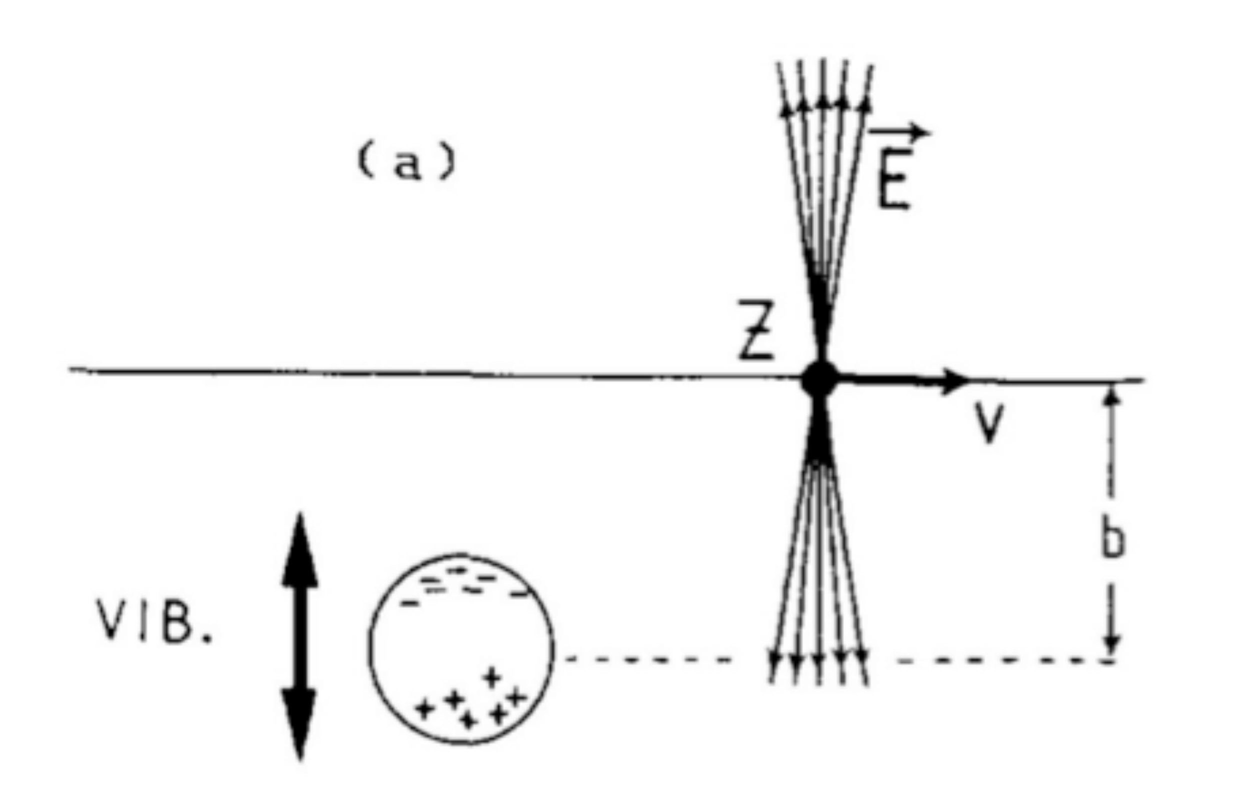
$$\frac{d\sigma_C}{2\pi b db} = \sum_{\pi L} \int \frac{dE_\gamma}{E_\gamma} n_{\pi L}(E_\gamma, b) \sigma_\gamma^{\pi L}(E_\gamma)$$

- $n_{\pi L}(E_\gamma, b)$ virtual photon numbers, dependent only on kinematic factors. Number of equivalent (virtual) photons that strike the halo nucleus

Coulomb dissociation

Bertulani, arXiv:0908.4307

- Coulomb dissociation: collide halo (we hope peripherally) with a high-Z nucleus
- Do for different Z, different nuclear sizes, different energies to test systematics



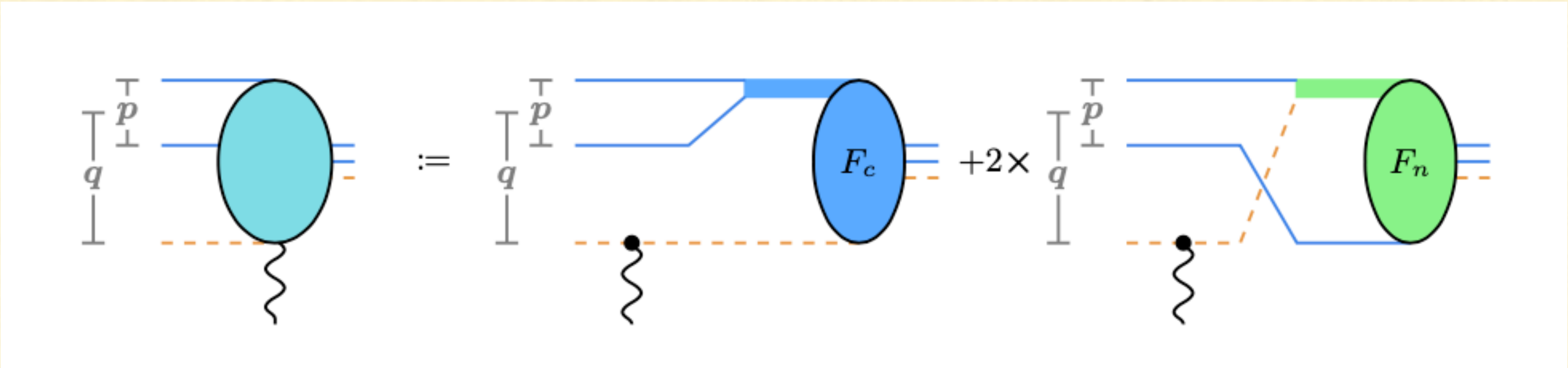
- Coulomb excitation dissociation cross section (p.v. b≫R_{target})

$$\frac{d\sigma_C}{2\pi b db} = \sum_{\pi L} \int \frac{dE_\gamma}{E_\gamma} n_{\pi L}(E_\gamma, b) \sigma_\gamma^{\pi L}(E_\gamma)$$

- $n_{\pi L}(E_\gamma, b)$ virtual photon numbers, dependent only on kinematic factors. Number of equivalent (virtual) photons that strike the halo nucleus
- $\sigma_\gamma^{\pi L}(E_\gamma)$ can then be extracted: it's the (total) cross section for dissociation of the nucleus due to the impact of photons of multipolarity πL

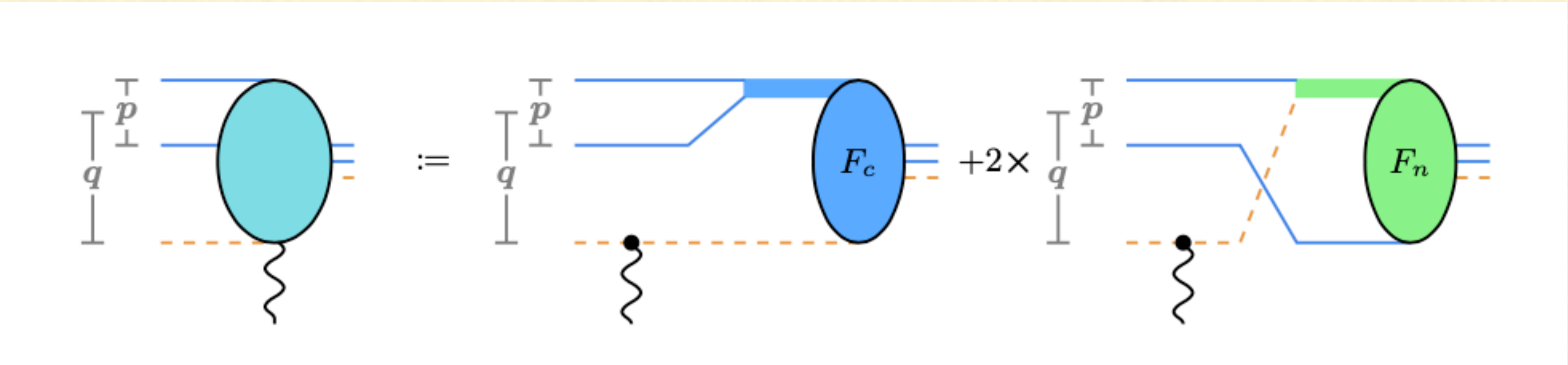
E1 photodissociation of a 2n halo

PWIA

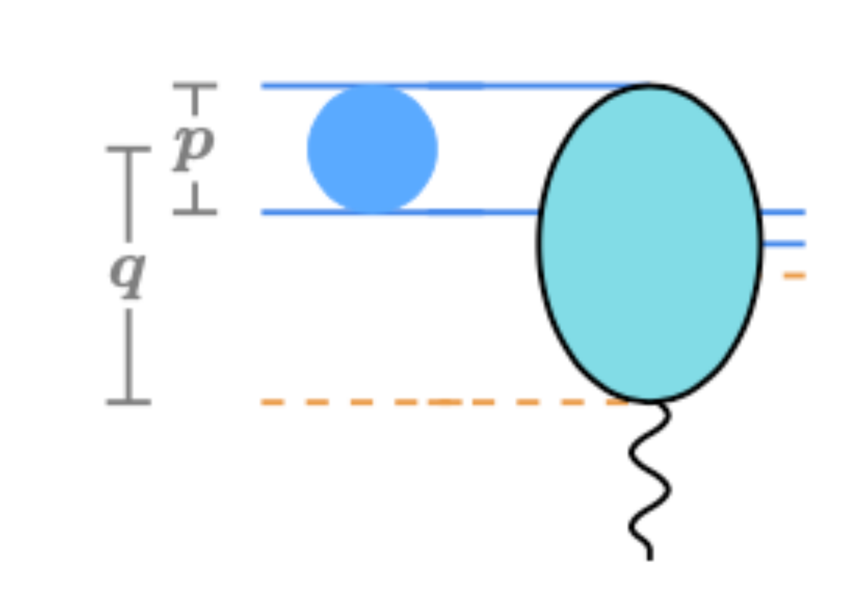


E1 photodissociation of a 2n halo

PWIA

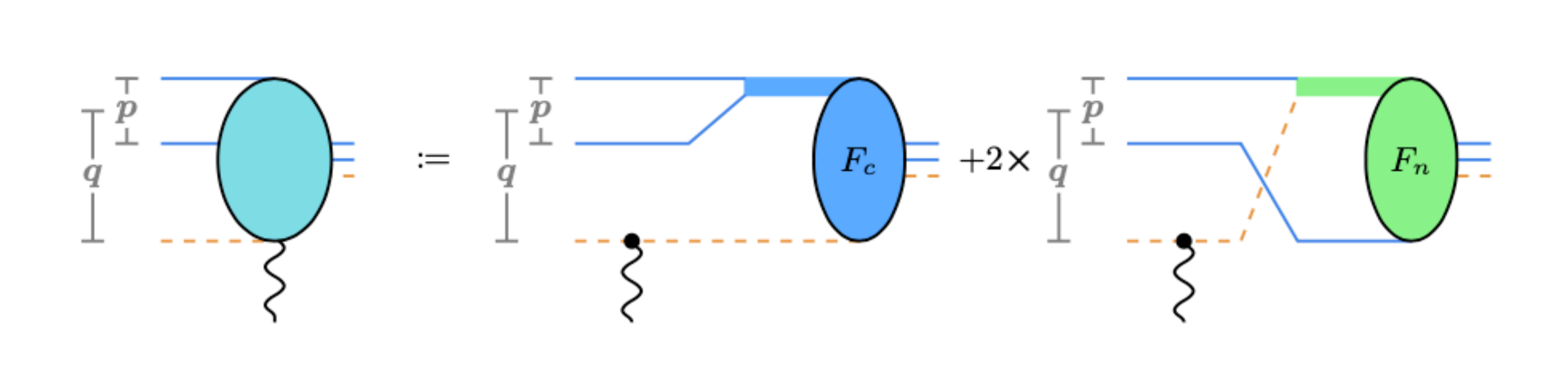


t_{nn} FSI

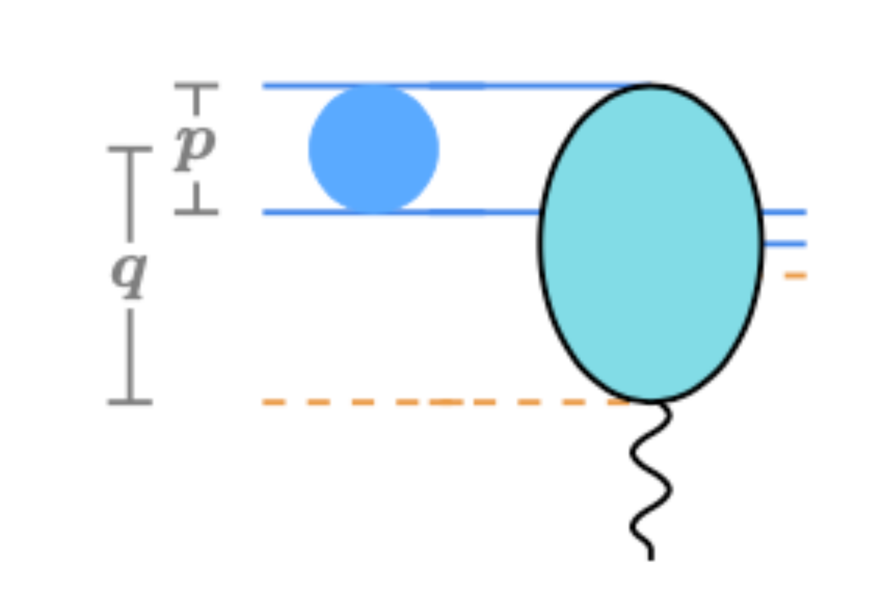


E1 photodissociation of a 2n halo

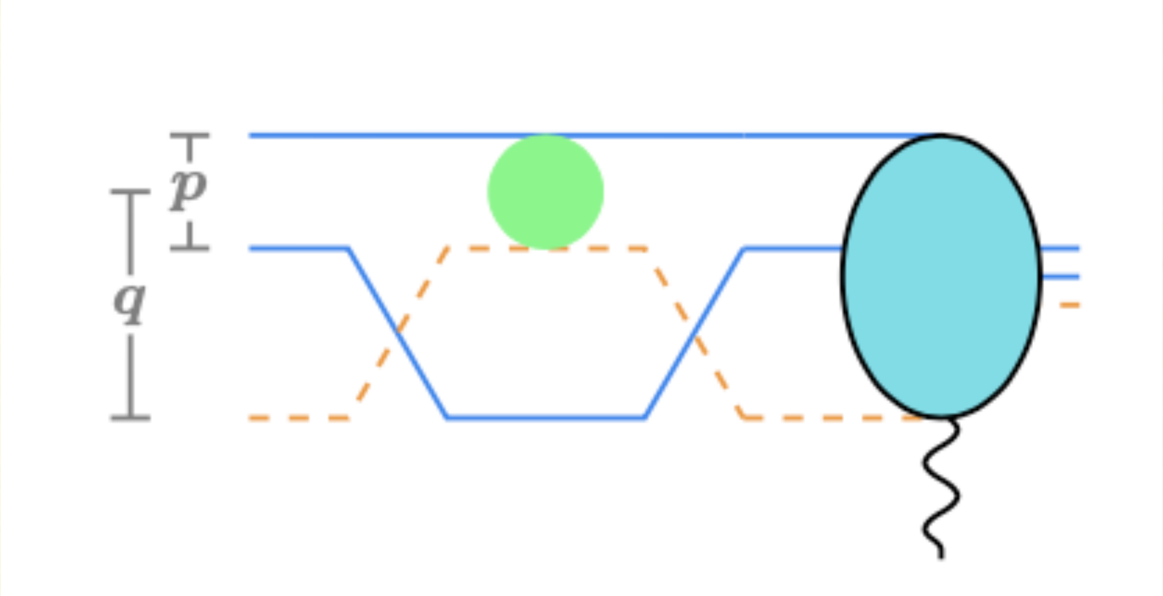
PWIA



t_{nn} FSI



t_{nc} FSI

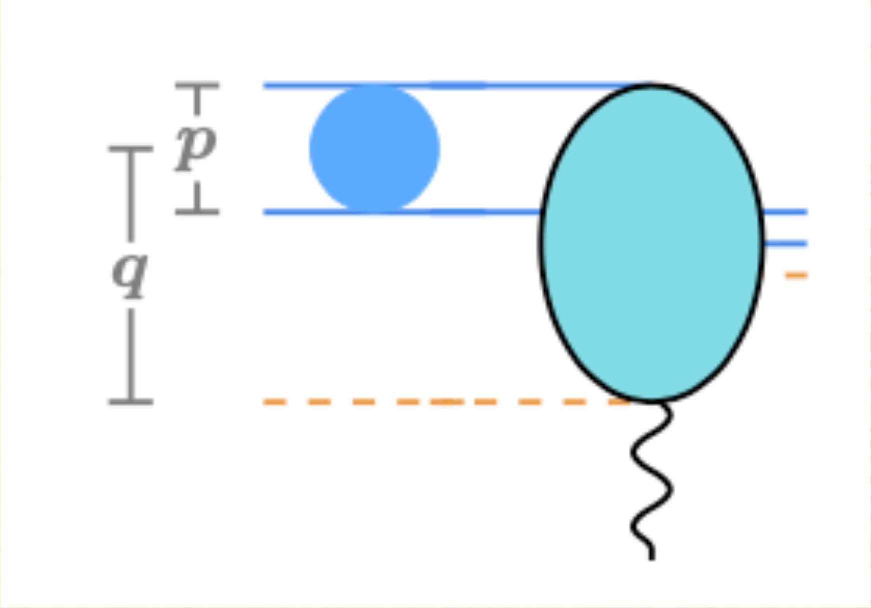


E1 photodissociation of a 2n halo

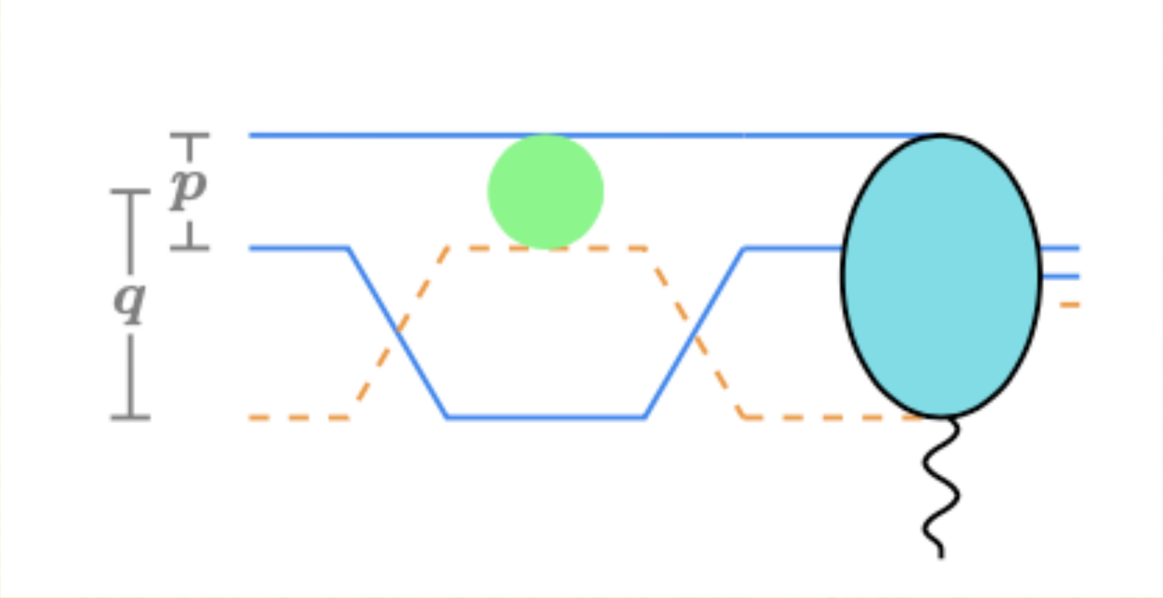
PWIA

$$\frac{dB(E1)}{dE} = \sum_{\mu} \int dp\, p^2 \int dq\, q^2 \,|_c\langle p, q, \Omega_c^{(1,\mu)} \,|\, \mathscr{M}(E1,\mu) \,|\, \Psi \rangle|^2 \, \delta(E_f - E)$$

t_{nn} FSI



t_{nc} FSI



E1 photodissociation of a 2n halo

PWIA

$$\frac{dB(E1)}{dE} = \sum_{\mu} \int dp\, p^2 \int dq\, q^2 \,|\, {}_c\langle p, q, \Omega_c^{(1,\mu)} \,|\, \mathscr{M}(E1,\mu) \,|\, \Psi \rangle \,|^2\, \delta(E_f - E)$$

t_{nn} FSI **t_{nc} FSI**

Modifications to matrix element due to final-state interaction/ wave-function distortion encoded in Møller operators

$${}_c\langle p, q, \Omega_c^{(1,\mu)} \,|\, (1 + t_{nn}(E_p) G_0^{(nn)}(E_p)) \mathscr{M}(E1,\mu) \,|\, \Psi \rangle$$

$${}_n\langle p, q, \Omega_n^{(0,\xi)} \,|\, (1 + t_{nc}(E_p) G_0^{(nc)}(E_p)) \mathscr{M}(E1,\mu) \,|\, \Psi \rangle$$

E1 photodissociation of a 2n halo

PWIA

$$\frac{dB(E1)}{dE} = \sum_{\mu} \int dp\, p^2 \int dq\, q^2 \left| {}_c\langle p, q, \Omega_c^{(1,\mu)} | \mathcal{M}(E1,\mu) | \Psi \rangle \right|^2 \delta(E_f - E)$$

t_{nn} FSI **t_{nc} FSI**

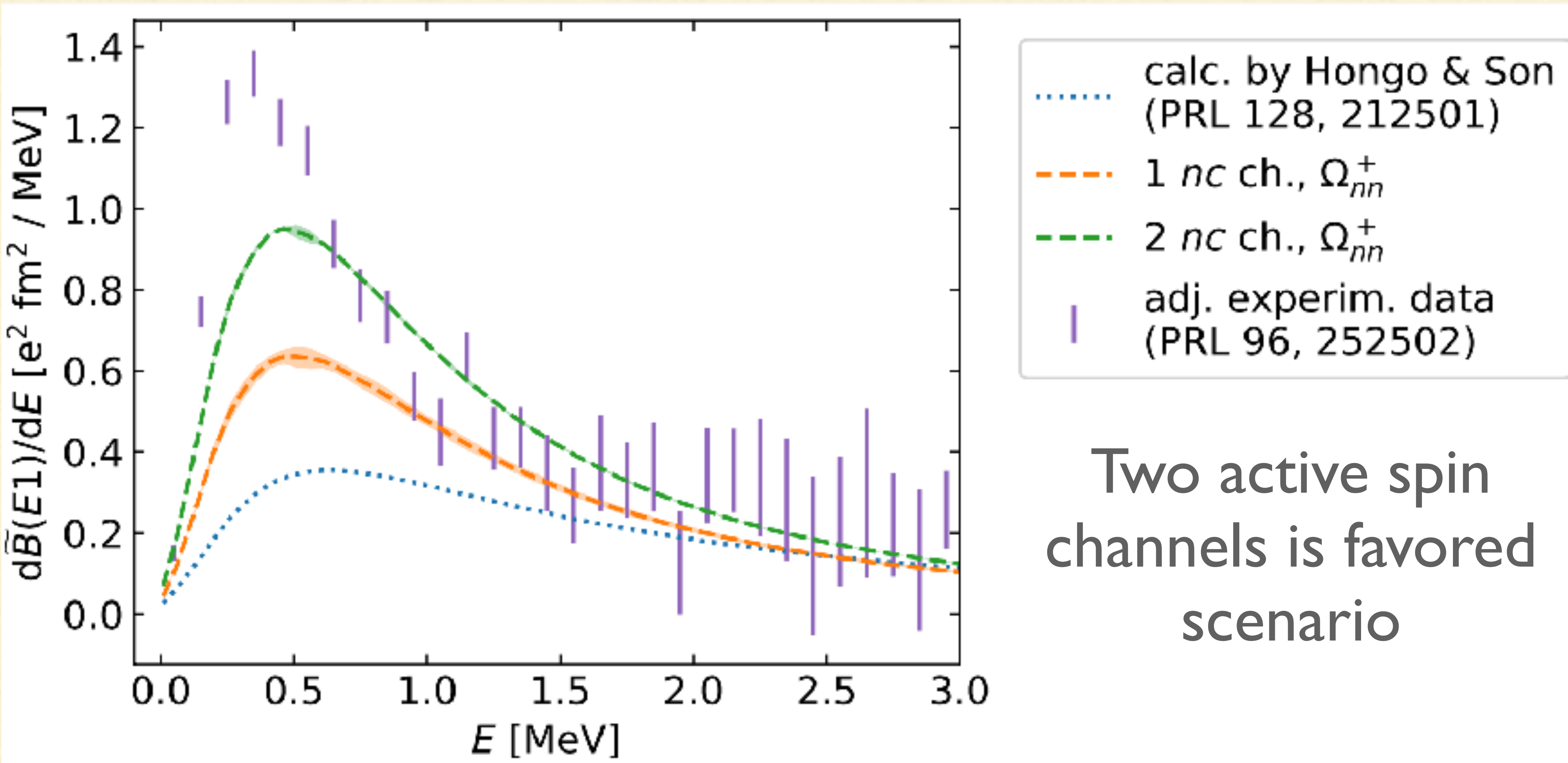
Modifications to matrix element due to final-state interaction/ wave-function distortion encoded in Møller operators

$${}_c\langle p, q, \Omega_c^{(1,\mu)} | (1 + t_{nn}(E_p) G_0^{(nn)}(E_p)) \mathcal{M}(E1,\mu) | \Psi \rangle$$

$${}_n\langle p, q, \Omega_n^{(0,\xi)} | (1 + t_{nc}(E_p) G_0^{(nc)}(E_p)) \mathcal{M}(E1,\mu) | \Psi \rangle$$

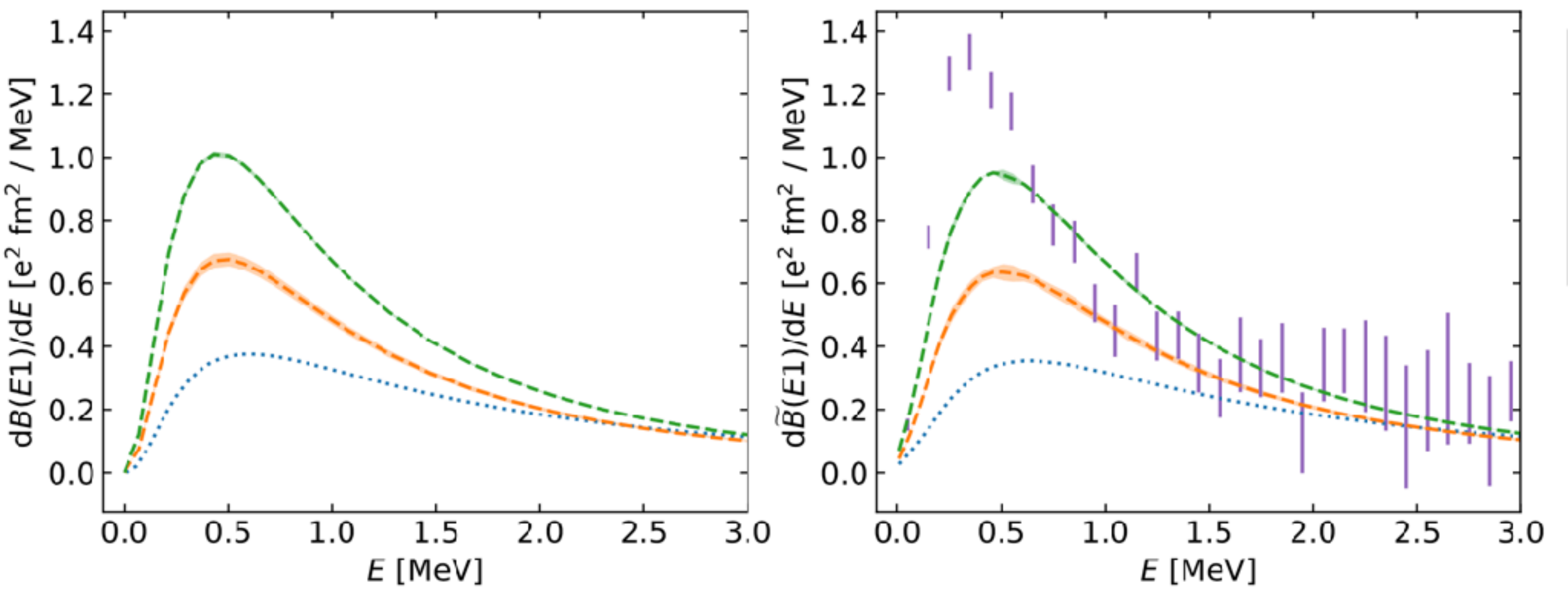
then two t’s in final-state, three t’s in final-state, etc.

Comparison with data



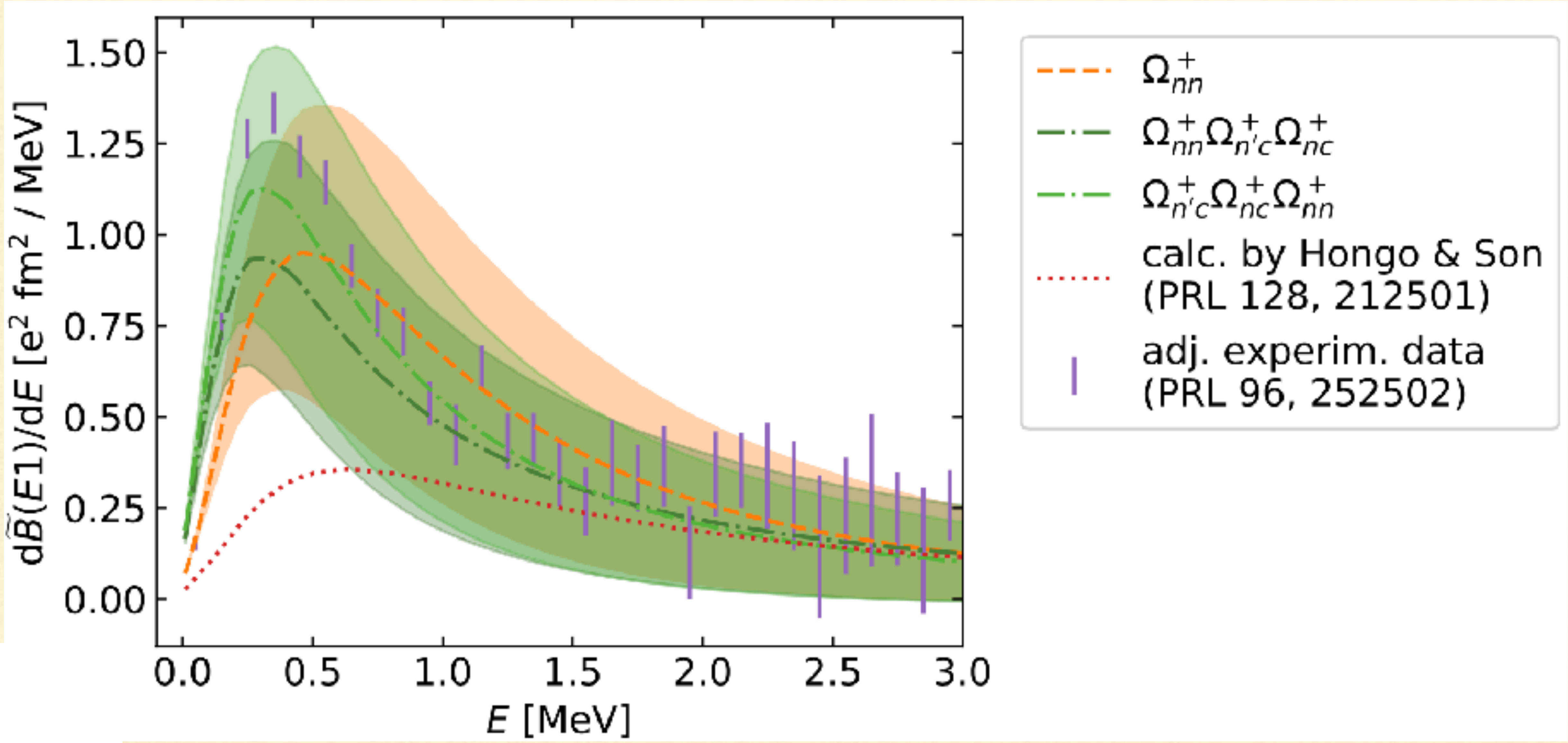
Two active spin channels is favored scenario

Comparison with data



Folding with experimental resolution reduces peak height

Comparison with data



Folding with experimental resolution reduces peak height

Agreement with data is good, given that this is only a leading-order calculation