TALENT school on "Effective Field Theories in Light Nuclei: from Structure to Reactions", Week 3, Exercise Sheet 4

Date: Thursday, August 11

Deriving the STM equation

Consider the one-nucleon-exchange amplitude derived in lecture this morning:

$$G_0(\mathbf{p}, \mathbf{q}; E) = \frac{1}{E - \frac{\mathbf{p}^2}{2M} - \frac{\mathbf{q}^2}{2M} - \frac{(\mathbf{p}+\mathbf{q})^2}{2M}}. \tag{1}$$

Note that If we define energy E with respect to the three-body threshold then the three-body singularity does not occur for $E < 0$. Note also that the neutron-halo-nucleus (neutron-deuteron) scattering threshold is then at negative energies with respect to this three-body threshold. It occurs at $E = -B \equiv -S_{1n}$.

1. Explain why Eq. (1) is the amplitude for exchange of a nucleon between two deuterons at an energy E.

2. Evaluate:
$$\frac{1}{2}\int_{-1}^{1} dx \frac{1}{p^2 + q^2 + \mathbf{p}\cdot\mathbf{q} - ME}. \tag{2}$$
Hence justify the form of the driving term used in the code.

3. Explain why we have $E = -B + \frac{3\mathbf{k}^2}{4M}$ where $\mathbf{k}$ and $-\mathbf{k}$ are the momentum of the nucleon and the deuteron respectively in the on-shell external state. What is the minimum value of $|\mathbf{k}|$ at which the propagator (1) has a singularity, i.e, the three-body channel is open?

4. Consider the "dimer propagator" that appears in the three-body equations;
$$\tau(q; E) = \frac{1}{-\gamma + \sqrt{\frac{3q^2}{4} - ME}}. \tag{3}$$
By multiplying top and bottom by $\gamma + \sqrt{\frac{3q^2}{4} - ME}$ and setting $\gamma^2 = MB$ show that this propagator has a pole for any $E > -B$. What intermediate state does this pole represent?

Without a three-body force

1. Pick $\Lambda = 500$ MeV. Evaluate the phase shift δ at a range of energies from 0 to 2 MeV. Plot $k \cot \delta$. Extract the scattering length a.

2. Repeat this exercise for $\Lambda = 1000$ MeV, $\Lambda = 2000$ MeV, and $\Lambda = 3000$ MeV. Do you get a stable value for a?

3. Plot $p * T(p, 0)$ versus $\log(p)$ for each of these different cutoffs. What features are cutoff-independent? Which are not?

With a three-body force

Now add an atom-dimer interaction—a "three-body force"—to the driving term. I.e., replace $Z(p, q; E)$ by $Z(p, q; E) + H$.

1. Pick $\Lambda = 500$ MeV again and a value of H. Extract the scattering length a as before.
2. Then adjust H so that $a = 0.1/\gamma$, where γ is the two-body binding energy.
3. Repeat this adjustment of H to get $a = 0.1/\gamma$ for $\Lambda = 1000$ MeV, $\Lambda = 2000$ MeV, and $\Lambda = 3000$ MeV. Note that you will need to choose a different H at each value of Λ.
4. Is $k \cot \delta$ stable with Λ now?
5. Plot the $p * T(p, 0)$ versus $\log(p)$ for each of these different cutoffs. Now which features are cutoff independent?