TALENT school on Few-body Methods, Week 3, Exercise Sheet 2

Date: Tuesday, August 9

Bound-state properties

In this set of exercises you will compute the two-body bound states produced by the potentials you constructed yesterday. You will then check whether the properties of those bound states are well described by the universality formulae derived in lectures and in previous exercises.

Binding energy

1. Compute the binding energy of the two-body bound state for your LO Gaussian potential for the different radii $R = 0.5$ 1, 1.5, 2, 2.5, and 3 fm.

2. How does the deviation of the binding energy from that LO Halo EFT prediction, $B = \frac{1}{2m_R a^2}$, scale with the effective range?

ANC

1. (a) If the code you are using provides the ANC for each of the bound states then, for at least one of the values of R, check that this number is correct by generating a plot where you overlay the asymptotic form of the wave function we derived this morninng against the wave function your code obtains for the bound state of this potential.

   (b) If you are using your own Numerov code then extract the ANC from your solution. (Make sure to check that your answer is stable with respect to the radius at which you extract it.) For at least one of the values of R, check that this number is correct by generating a plot where you overlay the asymptotic form of the wave function we derived this morning against the wave function your code obtains for the bound state of this potential.

2. Identify the radius at which there is significant deviation between the asymptotic form and the numerical solution to the Gaussian potential. How do you think that radius will change with R? Check your prediction!

3. Now extract all the ANCs for all the different R's. Plot the ANC^2 against $\gamma = \sqrt{2m_R B}$. What do you find? Can you explain this result from the (NLO) Halo EFT prediction for the ANC^2?

Neutron-core radii

Consider the mean-square radius between the core and the halo neutron:

$$\langle r_{nc}^2 \rangle = \int d^3r r^2 |\psi(\mathbf{r})|^2. \qquad (1)$$

In this problem we will examine this quantity both in the EFT calculation we looked at this morning, and as obtained numerically using your bound-state wave functions.

1. Insert the $\psi(\mathbf{r})$ derived in class this morning from Halo EFT into Eq. (1) and use it compute $\langle r_{nc}^2 \rangle$.

2. What is the LO piece of this result?

3. Pick a couple of different values of R in the LO wave functions you derived yesterday and use them to compute the mean-square radius of the bound state $\langle r^2 \rangle$. Compare the result to the LO and NLO prediction of Halo EFT, i.e., to the analytic results you obtained in Questions 1 and 2 of this section.

NLO potentials: the struggle and the payoff

Now we add a second-order (in the EFT sense) term to the potential. The NLO form of the "Halo EFT potential" is (note that $\tilde{C}_0 \neq C_0$ in general):

$$V_{NLO}(r;R) = \frac{1}{(2\pi R^2)^{3/2}} \left[\tilde{C}_0(R) \exp\left(-\frac{r^2}{2R^2}\right) + C_2(R) r^2 \exp\left(-\frac{r^2}{2R^2}\right) \right]. \tag{2}$$

1. Show that $\nabla^2 \exp\left(-\frac{r^2}{2R^2}\right) = \frac{r^2}{R^4} \exp\left(-\frac{r^2}{2R^2}\right) - \frac{1}{R^2} \exp\left(-\frac{r^2}{2R^2}\right)$. This justifies Eq. (2).

2. For the different radii $R = 2$ fm, $R = 2.5$ fm and $R = 3$ fm adjust the parameters $\tilde{C}_0(R)$ and $C_2(R)$ so that, no matter which of those radii you pick, you get both the a_0 and r_0 for ^{18}C-n scattering: $a_0 = 7.75$ fm; $r_0 = 2.6$ fm.

3. What happens if you try to do this for $R = 1.5$ fm?

4. Plot the phase shift $\delta(E)$ for s-wave ^{18}C-neutron scattering for all of the NLO potentials you've constructed. Generate a plot of the $k \cot \delta$'s for the different R's using the NLO potentials. Now compare this to the same plot for the LO potentials (the one you generated at the end of Exercise 1). What do you notice?

5. Obtain the bound-state energies for the NLO potentials you've constructed. What do you notice compared to the analogous LO potentials?

6. [**If time permits**] Pick the extreme R values at which you used the NLO potentials to obtain a bound-state wave function. Make sure each wave function is normalized (!) and then use them to compute $\langle r^2_{nc} \rangle$. How stable is your answer with R? How does that compare to the variation with R of the answers you found in Q.3 of the previous subsection? What do you think is causing this change? (**Hint:** you should think about the universal formula you derived for $\langle r^2_{nc} \rangle$ in the previous subsection.)