Lecture 2

Scaling of C_0 with R for a Gaussian

$$V(\vec{r}) = \frac{C_0}{(2\pi R^2)^{3/2}} \exp\left(-r^2/2R^2\right)$$

$$= C_0\, \delta_R^{(3)}(r)$$

$$\longrightarrow \langle \vec{p}\,' | V | \vec{p} \rangle = C_0$$

$$C_0 = \frac{g^2}{\Delta}$$

$$\frac{\Delta}{g^2} = \frac{m_R}{2\pi}\left(\frac{1}{a_0} - \frac{2\Lambda}{\pi}\right)$$

$$\xrightarrow{\Lambda \to \infty} -\frac{2\Lambda m_R}{2\pi^2} = -\frac{\Lambda m_R}{\pi^2}$$

$$\Rightarrow \quad C_0 \sim \frac{1}{\Lambda m_R} \sim \frac{R}{m_R}$$

$$|V(0)| \sim \frac{C_0}{R^3} \sim \frac{1}{m_R R^2}$$

$$\langle V + T \rangle = -B \quad \rightarrow \text{independent of } R.$$

$$\langle T \rangle_R \sim \frac{1}{m_R R^2}$$

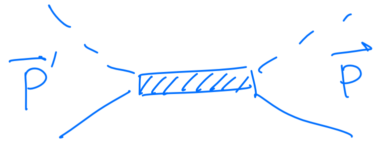
Questions

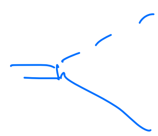
1. $$t(E) = \frac{2\pi}{m_R} \frac{1}{\frac{1}{a_0} - \frac{1}{2} r_0 k^2 + ik}$$

"On-shell" t-matrix

$$\langle \vec{p}\,' | t(E) | \vec{p} \rangle = t(E)$$

$$\frac{\vec{p}\,'^2}{2m_R} = \frac{\vec{p}^{\,2}}{2m_R} = E$$





$-ig$

$k = i\gamma$ in denominator $\Rightarrow$ pole iff

$$\frac{1}{a_0} + \frac{1}{2} r_0 \gamma^2 - \gamma = 0$$

$$\gamma = \frac{1 \pm \sqrt{1 - 2r_0/a_0}}{r_0}$$

One pole at $\gamma_+ = 2/r_0$

$$\gamma_- \simeq 1 - \left(1 - r_0/a_0 + \cdots\right)$$

$$\frac{\quad}{r_0}$$

$$= \frac{1}{a_0} + O\left(\frac{r_0^2}{a_0}\right)$$

$$T(E) = V + V\,G_0(E)\,T(E)$$

$$G_0(E) = \frac{1}{E - H_0}$$

$$G(E) = \frac{1}{E - H_0 - V} = \frac{1}{E - H}$$

$$G = G_0 + G_0 T G_0$$

$$G(E^+) = \sum_n \frac{|\chi_n\rangle\langle\chi_n|}{E + B_n} + \int \frac{d^3p}{(2\pi)^3} \frac{|\chi_{\vec p}^{(+)}\rangle\langle\chi_{\vec p}^{(+)}|}{E^{+} - p^2/2m_R}$$

$E \simeq -B_n$ then $G(E) \simeq \dfrac{|\chi_n\rangle\langle\chi_n|}{E + B_n}$

$$\Rightarrow \quad \langle \vec p\,'|\,G(E)\,|\vec p\rangle = \frac{\langle \vec p\,'|\chi_n\rangle\langle\chi_n|\vec p\rangle}{E + B_n}$$

Poles of t $\equiv$ Poles of G — Bound for $E<0$ for $E<0$ $=$ states

Residue of $\langle \vec{p}' | G | \vec{p} \rangle = \langle \vec{p}' | \psi_n \rangle$
at $E = -B_n$ $\langle \psi_n | \vec{p} \rangle$

Push through calculation to find

$$t(E) \longrightarrow G(E) \longrightarrow \psi(\vec{p}')$$

$$\Rightarrow \psi(\vec{r}) = \frac{A_0}{\sqrt{4\pi}} \frac{e^{-\gamma r}}{r}$$

$$A_0 = \frac{\sqrt{2\gamma}}{\sqrt{1-\gamma r}} \qquad \gamma = \sqrt{2 m_R B}$$

$\xrightarrow{r_0 \to 0}$ $A_0 = \sqrt{2\gamma}$

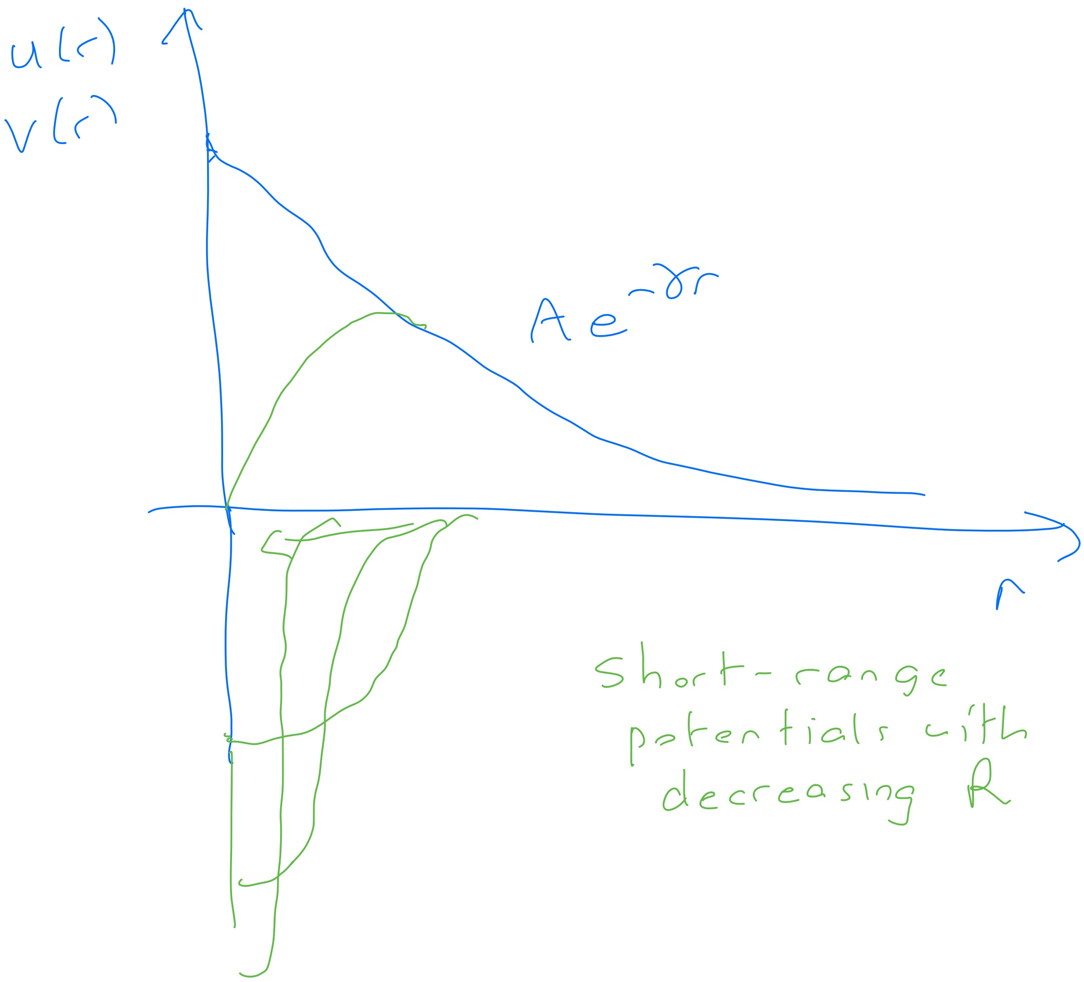
$$\frac{1}{a_0} + \frac{1}{2} r_0 \gamma^2 - \gamma = 0$$

$\rightarrow$ low-momentum root of this

$$B = \frac{1}{2 m_R a_0^2} \left(1 + O\left(\frac{r_0}{a_0}\right) \right)$$

$$\psi(\vec{r}) = Y_{00}(\hat{r}) \frac{u(r)}{r}$$

$$u(r) = A e^{-\gamma r}$$



$$u(r) = A e^{-\gamma r} \qquad 0 < r < \infty$$

$$\int_0^\infty dr\, u^2(r) = 1$$

$$\Rightarrow \int_0^\infty dr\, A_0^2 e^{-2\gamma r} = 1$$

$$\Rightarrow \frac{A_0^2}{2\gamma} = 1$$

$$\Rightarrow \quad A_0 = \sqrt{2\gamma}$$

At leading order this works fine, but at NLO this component of the dimer has norm > 1.